

UCLA

UCLA Previously Published Works

Title

Competitive Maps: The Structure Underlying Asymmetric Cross Elasticities

Permalink

<https://escholarship.org/uc/item/3w22j3zd>

Journal

Management Science, 34(6)

ISSN

0025-1909

Author

Cooper, Lee G

Publication Date

1988-06-01

DOI

10.1287/mnsc.34.6.707

Peer reviewed

COMPETITIVE MAPS: THE STRUCTURE UNDERLYING ASYMMETRIC CROSS ELASTICITIES*

LEE G. COOPER

*Anderson Graduate School of Management, University of California,
Los Angeles, California 90024*

A special case of three-mode factor analysis is used to portray the systematic structure underlying asymmetric cross elasticities for a broad class of market-share attraction models. Analysis of the variation over retail outlets and weeks reveals competitive patterns corresponding to sales for the major brands in the market as well as patterns reflecting shelf-price competition. Analysis of the brand domain results in a joint space. One set of brand positions portrays how brands exert influence over the competition. The other set of points portrays how brands are influenced by others. The interset distances (angles) provide direct measures of competitive pressures. Maps are formed as spatial representations of each of the competitive patterns discovered.

(MARKETING; PRICING; COMPETITIVE)

1. Introduction

Elasticities summarize how conditions of the marketplace (e.g. patterns of prices and promotions) translate into changes in market shares or sales. In at least two major ways elasticities provide important linkages for marketing scholars and practitioners. First, they provide the language which links much of marketing theory to marketing models and practice. Optimal advertising expenditures (e.g. Dorfman and Steiner 1954; Clarke 1973; Bultez and Naert 1979; Magat, McCann and Morey 1986), optimal price (e.g. Bass and Bultez 1982; Kalish 1983; Rao and Bass 1985), the relation between price and product life cycle (e.g. Parsons 1975; Simon 1979; Shoemaker 1986), and general issues of optimal marketing mix and marketing effectiveness (e.g. Lambin, Naert and Bultez 1975; Karnani 1983; Morey and McCann 1983), all use the language of elasticities to convey the implications of marketing theory.

Second, elasticities serve as measures of competition—indicators of market structure. But the psychometric approach to market-structure analysis developed mainly without reference to or use of elasticities. This approach uses multidimensional scaling (MDS) to map perceived similarities or preferences among the brands, or to model consumer choice as some function of distance from an ideal point in the brand map (cf. Cooper 1983). MDS's direct-similarities judgment task has one of the least-biasing instructions in all of social science (e.g. "How similar are the brands in each pair?"), thus providing powerful methods for discovering the dimensions underlying consumer perceptions. In new-product research, MDS provides a basis for understanding how consumers might react to new offerings. But, for the management of mature brands, particularly frequently purchased branded goods (FPBGs), this approach's *power of discovery* contributes to an important representational problem. Depending on the context, substitutable and complementary products could both appear close together in the perceptual space. For example, hot dogs and Coke (complements) could be near each other in one perceptual space, while Coke and Pepsi (substitutes) could be close

* Accepted by John R. Hauser; received September 19, 1985. This paper has been with the author 10 months for 3 revisions.

together in another perceptual space.¹ While careful structuring of object sets helps, the essential ambiguity between the treatment of substitutes versus complements can diminish the utility of traditional MDS for brand management.

Panel data may overcome this ambiguity more easily than aggregate market-share data. The record of interpurchase intervals available in panel data can reveal information about substitutes versus complements. In the extreme case, co-purchase of two brands on a single buying occasion indicates complementarity, while switching between brands with equal interpurchase times indicates substitutability.² Fraser and Bradford (1983) used this kind of information in a panel-based index of revealed substitutability which they decomposed using principal components. A stream of research based on panel or discrete-choice data begins with Lehmann's (1972) use of brand-switching data as similarity measures in MDS. The Hendry Model (Kalwani and Morrison 1977), the wandering vector model (Carroll 1980; DeSoete and Carroll 1983), Genfold2 (DeSarbo and Rao 1984), Moore and Winer's (1987) use of panel data in Levine's (1979) pick-any analysis, and the powerful maximum-likelihood procedures in Elrod's (1987) Choice Map can be thought of as part of the general effort to develop market-structure maps from disaggregate choice data. Moore and Winer (1987) distinguish their effort by using a multiple-equation system to integrate panel data with market-level data, but only Fraser and Bradford (1983) specifically address the potential of panel data to resolve the representational ambiguity involving substitutes and complements.

The modeling efforts at the aggregate level have been much more limited. Vanhoneracker (1984) has worked on methods which result in two separate structural maps—one for negative cross elasticities and one for positive cross elasticities. But how we integrate information across these two maps is, as yet, unresolved. Shugan (1986, 1987) has developed methods to represent the market structure specifically implied by the demand function in the Defender model (Hauser and Shugan 1983). This market-structure map contains price-scaled dimensions. The elasticities implied by the Defender model can be computed as simple relations among the angles brands make with these per-dollar dimensions. While brand positions have the advantage of relating directly to the rich strategic implications of the Defender model, choice sets have to be very carefully defined to screen out complements. Otherwise brands may be forced to have negative coordinates on some dimension. There is still uncertainty about the meaning of a negative coordinate on a per-dollar dimension. But even if explanations are forthcoming, developments are needed which map the structural implications of other demand functions, as well as convey, in a single map, the implications of both positive and negative cross elasticities.

The goals of this article are (1) to develop a structural representation of the elasticities from a broad class of market-share attraction models (multiplicative competitive-interaction models and aggregate multinomial-logit models) and (2) to demonstrate the utility of the resulting competitive maps in the management of frequently purchased branded goods.

We proceed by presenting a general matrix representation for the self and cross elasticities from market-share attraction models and discussing the differences in elasticities from progressively more general market-share models. A special case of three-mode factor analysis (Tucker 1969) is then presented which is uniquely suited for revealing the structure implied by these asymmetric (cross) elasticities. The method is

¹ Factor analysis of consumer rating scales has less difficulty here than does MDS, since the attributes of substitutes should be much more highly correlated than the attributes of complements—leading to similar locations for substitutes, but quite dissimilar positions for complements.

² This can be muddled however since panel records are mostly of household purchases which may reflect the preferences of different final consumers. So-called *super-position processes* are discussed by Kahn, Morrison and Wright (1986).

illustrated using price elasticities derived from IRI BehaviorScan records of aggregate weekly sales for grocery chains in the ground, caffeinated coffee market.

2. Elasticities from Market-Share Attraction Models

The family of market-share attraction models includes simple-effect Multiplicative, Competitive-Interaction (MCI) models and the corresponding simple-effect Multinomial Logit (MNL) models (cf. McFadden 1974; Nakanishi and Cooper 1974, 1982; Bultez and Naert 1975), in which the effectiveness³ of a single marketing instrument, e.g. price, is considered the same for all brands in the market. The family also includes differential-effects MCI and MNL models which allow the effectiveness of marketing instruments to vary by brand (cf. Bultez and Naert 1975), asymmetric market-share models (cf. Carpenter, Cooper, Hanssens and Midgley 1987) which allow for estimation of a specific set of cross-competitive effects for each brand, and fully-extended MCI and MNL models (cf. Bultez and Naert 1975; McGuire, Weiss and Houston 1977), which specify the full set of all possible cross-competitive effects for each brand.

A general representation of the demand function for the family of market-share attraction models is:

$$A_{it} = \exp(\alpha_i) \prod_{k=1}^K [f_k(X_{kit})]^{\beta_{ki}} \prod_{(k^*j^*) \in C_i} [f_{k^*}(X_{k^*j^*t})]^{\beta_{k^*ij^*}} \exp(\epsilon_{it}), \tag{1}$$

$$M_{it} = \frac{A_{it}}{\sum_{j=1}^N A_{jt}} \tag{2}$$

where A_{it} is the attraction of brand i ($i, j = 1, 2, \dots, N$) in period t ($t = 1, 2, \dots, T$), α_i is a brand-specific intercept, X_{kit} is the value of marketing instrument k ($k = 1, 2, \dots, K$), $f_k(\cdot)$ represents the positive, ratio-scale transformations of X_{kit} which specify members of this family of models (cf. Cooper and Nakanishi 1983a), β_{ki} is a parameter reflecting the sensitivity of market share, M_{it} , to changes in $f_k(X_{kit})$, $\beta_{k^*ij^*}$ is the parameter for the cross-competitive effect on brand i of marketing instrument k^* for a particular brand j^* , and ϵ_{it} is a random disturbance. Each brand has a set (C_i) of cross-competitive effects, with marketing instrument k^* for brand j^* as a member. For the simple- and differential-effects models, this is the null set. For asymmetric market-share models, this is a specifically estimated set of cross-competitive effects and, for the fully-extended models, this set includes all possible cross-competitive effects for each marketing instrument.

There is a simple, general expression for the self and cross elasticities from this family of market-share attraction models. It is illustrated here using price elasticities, but applies to elasticities for any marketing instrument. Consider a matrix E_t with entries e_{ijt} indicating the influence of brand j 's price on brand i 's market share in time period t ,

$$E_t = (I - JM_t)BS_t, \tag{3}$$

where I is the $N \times N$ identity matrix, J is the $N \times 1$ unit vector, M_t is the $N \times 1$ market-share vector for period t , B is the $N \times N$ price-coefficients matrix (i.e. entry β_{kij} is the influence of brand j 's price on brand i 's market share, and S_t is an $N \times N$ matrix which relates the raw prices to the various functional forms in the family of market-share attraction models. While this illustration deals with price, in general, for marketing instrument k the (ij) elements of S_t are the percent changes in $f_k(X_{kit})$ which go with percent changes in X_{kit} :

³ Effectiveness is reflected in the parameters of these models which indicate the sensitivity of market share for a brand to changes in the value of a marketing instrument.

$$S_{ijt}^{(k)} = \frac{\partial f_t(X_{kit})}{f_t(X_{kit})} \times \frac{X_{kjt}}{\partial X_{kjt}} \tag{4}$$

This expression for the elasticities becomes more intuitive by starting with the simplest models. For a simple-effect, raw-price MCI model, S_t becomes the identity matrix (since $f(X) = X$), B becomes the scalar matrix $\beta \times I$ leading to the familiar expression:

$$E_t = (I - JM'_t)\beta \tag{5}$$

For the simple, raw-price MNL model S_t is D_{X_t} , a diagonal matrix of prices for period t (since $f(X) = \exp(X)$), resulting in:

$$E_t = (I - JM'_t)\beta D_{X_t} \tag{6}$$

For the differential-effects models B is a diagonal matrix. In asymmetric market-share models and the fully-extended models B is a square matrix with cross-effect parameters off the diagonal. For the asymmetric market-share models there would be a non-zero entry wherever a specific cross effect was estimated. For the fully-extended model all off-diagonal terms would be nonzero; although not all would necessarily be statistically significant.⁴

The algebraic expression for the elasticities, from which all the matrix expressions can be derived is given in general for marketing instrument k , the influence of brand j on brand k during period t by:

$$e_{ijt}^{(k)} = \underbrace{\beta_{ki} S_{ijt}^{(k)}}_{\text{Direct Effect}} + \underbrace{\sum_{(kj^*) \in C_t} \beta_{kij^*} S_{j^*jt}^{(k)}}_{\text{Specific Cross Effects}} - \underbrace{\sum_{i'=1}^N M_{i't} (\beta_{ki'} S_{i'jt}^{(k)} + \sum_{(kj^*) \in C_t} \beta_{ki'j^*} S_{j^*jt}^{(k)})}_{\text{General Competitive Effect}} \tag{7}$$

where all terms are as previously defined.

Different functional forms for representing marketing efforts have been used in this family of models. Using relative price or share of advertising expenditures leads to more complex versions of S . This article uses a standard-score representation for prices and other marketing variables in an asymmetric MNL model. This functional form provides contextual information about the distinctiveness of marketing efforts (cf. Carpenter, Cooper, Hanssens and Midgley 1987) and helps ameliorate the collinearity problems which have been associated with extended-forms of market-share models.⁵ But, regardless of the functional form, we are normally interested in the raw-price elasticities. In the case used here these raw-price elasticities come from using the following S_t matrix:

$$S_t = \frac{1}{\sigma_{X_t}} \left[\left(I - \frac{1}{N} JJ' \right) - \frac{1}{N} Z_t Z_t' \right] D_{X_t} \tag{8}$$

where D_{X_t} is an $N \times N$ diagonal matrix of prices, σ_{X_t} is the standard deviation of the prices taken over brands within a time period, and Z_t is an $N \times 1$ vector of standard scores for prices:

⁴ McGuire *et al.* (1977) showed that in the fully-extended model we can only estimate $(\beta_{kij} - \bar{\beta}_{kj})$ where $\bar{\beta}_{kj}$ is the arithmetic mean of β_{kij} over i . But Nakanishi and Cooper (1980) showed that this was all one needed to compute the elasticities.

⁵ See Bultez and Naert (1975) and Cooper (1985) for discussions of the collinearity in these models, and Cooper and Nakanishi (1983a) for the development of this particular functional form as a representation of the competitive context.

$$z_{it} = (X_{it} - \bar{X}_t) / \sigma_{X_t}. \quad (9)$$

This matrix S_t contains scaling factors which relate a percent change in standardized price of row brand i (i.e. $f_i(X_{it}) = \exp(z_{it})$) to a percent change in the raw price of column brand j .

Since prices or promotional patterns change over time, the matrix E_t will be a layer in a three-way matrix describing the market over the period of study. Representing the market structure underlying this three-way array is the topic of the next section.

3. Asymmetric Three-Mode Factor Analysis

Consider a three-way array of the cross-elasticities. The layers summarize the differences over time periods. The entries in each row of a layer of this cube summarize how all the brands' prices (promotions) affect that row brand's sales. The entries in a column from any layer summarize how one brand's prices (promotions) affect all brands' sales.

We desire certain properties in the mapping of cross elasticities. The brands will be represented as vectors from the origin. The stronger the cross elasticity between two brands the more correlated those brand vectors should be. The more complementary two brands are in the market, the more opposite they should be in the map. If two brands do not compete at all (zero cross elasticity), their vectors should be at right angles (orthogonal). Thus the patterns of substitutability, complementarity and independence could be represented in a single map. These properties are achievable when cosines (or scalar products) between brand positions in a multidimensional space represent cross elasticities. Then the elements in an elasticities matrix are represented by the inner (scalar) product of (1) a row from a matrix R reflecting how receptive each brand is to being influenced by some small number of underlying market-place forces, and (2) the elements in a row from a matrix C reflecting how much clout each brand has in the market. In sum, a cross elasticity is the inner (scalar) product of the clout of one brand times the receptivity of the other brand. After developing this representation for a single time period, it is generalized by assuming that the competitive patterns underlying time periods are related by simple nonsingular transformations (i.e. that the dimensions of a common space can be differentially reweighted and differentially correlated to approximate the pattern of influences in any single time period). Establishing a common origin and units of measure for the R and C matrices allow plotting in a joint space.

Equation (10) represents the cross elasticities E in a particular time period t as the scalar product of a row space R for time period t and a column space C for time period t plus a matrix Δ of discrepancies (lack of fit).

$${}_i E_j^{(t)} = {}_i R_q^{(t)} C_j^{(t)} + {}_i \Delta_j^{(t)}. \quad (10)$$

Similar entries in the row space for t indicate similarities between brands in the way they are influenced by competitive pressures—receptivity. Similar entries in the column space for t indicate similarities between brands in how they exert influence on others—clout. So while the inner (scalar) product across R and C reflects cross elasticities, the inner product within R reflects similarity in the pattern of how brands are influenced (i.e. receptivity or vulnerability), and the inner product within C indicates similarity in the pattern of how brands exert influence on other brands (i.e. clout). Think of the row space for t as a nonsingular transformation Q of a common row space \mathbf{R} over time:

$${}_i R_q^{(t)} = {}_i \mathbf{R}_q Q_q^{(t)}. \quad (11)$$

Think of the column space for t as a nonsingular transformation U of common column space \mathbf{C} :

$${}_q C_j^{(t)} = {}_q U_q^{(t)} \mathbf{C}_j. \quad (12)$$

Each time period can differ from the common pattern not only by a differential shrinking and stretching of the underlying dimensions, but also the dimensions of the common space can be differentially correlated over time, so that a wide variety of competitive conditions can be represented. Tucker (1969) developed this basic formulation as an extension of his pioneering work on three-mode factor analysis (Tucker 1963, 1966).⁶ Elasticities in each period are represented, in terms of the common row and column spaces, as the sum (over time-factors $l = 1, \dots, L$) of the triple matrix-product—the row space times the appropriate layer of the core matrix $G^{(l)}$ times the common column space. Each term in the triple product is weighted by a coefficient w_{il} showing the association of each time period with each time-factor (like factor scores for time periods),

$${}_i E_j^{(l)} = \sum_{l=1}^L {}_i R_q G_q^{(l)} C_j w_{il} + {}_i \Delta_j^{(l)}. \quad (13)$$

A joint space represents brand competition on each time-factor (Kroonenberg 1983, pp. 164–167). Each layer of the core matrix is diagonalized using singular value decomposition:

$${}_q G_q^{(l)} = {}_q V_q^{(l)} \Gamma_q^{(l)2} Y_q^{(l)} \quad (14)$$

where $V^{(l)}$ contains the left principal vectors of a particular layer of the core matrix, $Y^{(l)}$ contains the right principal vectors, and $\Gamma^{(l)2}$ is a diagonal matrix of singular values:

$${}_i R_q^{(l)} = {}_i R_q (V_q^{(l)} \Gamma_q^{(l)}), \quad (15)$$

$${}_q C_j^{(l)} = ({}_q \Gamma_q^{(l)} Y_q^{(l)}) C_j. \quad (16)$$

In this joint space, $R^{(l)}$ reflects the similarities in how brands are influenced, $C^{(l)}$ reflects the similarities in how brands exert influence, and the proximity (cosine between the vectors) of row and column points reflects how much the brands compete. From the joint-space coordinates, we approximate the elasticities corresponding to any particular week or any simulated pattern of marketing activity, using:

$${}_i \tilde{E}_j^{(l)} = \sum_{l=1}^L {}_i R_q^{(l)} C_j^{(l)} w_{il}. \quad (17)$$

Thus if the analysis of differences in competitive contexts reveals particularly interesting patterns, we could approximate the elasticities which reflect those competitive conditions. Repeating the analysis on just the approximated elasticities for some special condition, we create a competitive map specific to this context. With only one layer this is a two-mode analysis which amounts to a singular value decomposition of the E matrix, in which the variance is split between the left principal vectors and the right principal vectors. The result is the asymmetric three-mode equivalent of idealized-individual analysis developed by Tucker and Messick (1963) for the individual-differences model for multidimensional scaling. For any idealized competitive pattern t^* in which the elasticities have been approximated by equation (17), we get the simplest representation:

$${}_i \tilde{E}_j^{(t^*)} = {}_i R_q^{(t^*)} C_j^{(t^*)}. \quad (18)$$

This provides a very direct visualization of the idealized elasticities because the inner product of the coordinates in $C^{(t^*)}$ for brand j and the coordinates in $R^{(t^*)}$ for brand i will

⁶ Closely related developments are reported in Tucker (1972) and illustrated in Cooper (1973). In these articles symmetric, brand by brand, arrays make up each layer of the three-mode matrix, making the analysis a very general model for individual differences in multidimensional scaling. In the current context, individual differences are replaced by differences in the competitive mix from one time to another, and the symmetric measures of brand similarity are replaced by asymmetric measures of brand competition.

reproduce the (ij) entry in $\tilde{E}^{(*)}$. These idealized competitive patterns are isolated and interpreted in the illustration which follows.

4. Illustration

For illustration, consider the ground-caffeinated coffee market. Data, provided by Information Resources, Inc. from BehaviorScan stores in two cities, report price, newspaper feature, in-store display and coupon activity for all brands. The small-volume, premium brands were aggregated into an "All Other Branded" (AOB) category, and the small "Private Label" (PL) brands were aggregated into an "All Other Private Label" (AOPL) category. Consequently, 12 brands of coffee were analyzed: Folgers, Regular Maxwell House, Maxwell House Master Blend, Hills Brs., Chock Full O Nuts, Yuban, Chase & Sanborne, AOB, PL 1, PL 2, PL 3, and AOPL. For 18 months, each week's data for a brand were aggregated over package weights, and over stores-within-grocery chains in the two cities. These are aggregate data from stores, not discrete-choice data from BehaviorScan consumer panels. Price for each brand was aggregated into average price per pound, net of coupons redeemed. Feature, display and coupon were represented as percent of volume sold on promotions of each type to allow for aggregation over stores with differing promotional environments. The data were divided into a year for calibration of the market-share model, and six months for cross-validation. The average price and market share of each brand appear in Table 1.

4.1. *The Market-Share Model*

The asymmetric market-share model was estimated by the procedures discussed in Carpenter, Cooper, Hanssens and Midgley (1987). A differential-effects model was estimated with a unique parameter for each brand's price, feature, display and store-coupons, and a brand-specific intercept for the qualitative features of each brand. The brand-specific intercept which was closest to zero was set to zero to avoid singularity. The residuals from this differential-effects model were cross-correlated brand by brand with the contemporaneous explanatory variables for all other brands. The cross-competitive effects which were significant in the residual analysis were entered into the model. The final model was estimated for the development period of 52 weeks and then cross-validated on the remaining 26 weeks.

The resulting model had an R^2 of 0.92 with 133 parameters estimated and 2006 residual degrees of freedom ($F_{2006}^{133} = 168$). Twenty-eight of the 59 differential effects

TABLE 1
Average Prices and Market Shares

Brand	Average Price/lb.	Average Share
Folgers	\$2.33	28.5
Maxwell House	\$2.22	24.2
Master Blend	\$2.72	7.8
Hills Brs.	\$2.13	4.3
Chock Full O Nuts	\$2.02	15.3
Yuban	\$3.11	0.2
Chase & Sanborne	\$2.34	0.3
All Other Branded	\$2.64	2.4
Private Label 1	\$1.99	3.9
Private Label 2	\$1.95	3.6
Private Label 3	\$1.93	3.7
AOPL	\$1.95	5.7

were significant (six price, five feature, five display, and four coupon effects). Thirty-eight of the 74 cross-competitive effects were significant (20 price, five feature, eight display, and five coupon effects). The 74 cross-competitive effects provided a significant increment to the explanation provided by the 59 differential effects.⁷ The cross-validation correlation is 0.86. Without using cross-competitive effects the cross-validation correlation is 0.82.

The price parameters from the market-share models were used to generate market-share-price cross elasticities for each grocery chain in each week. The average elasticities are shown in Table 2. The greatest price elasticity is for Chock Full O Nuts (-4.71). The clear policy of this brand is to maintain a high shelf price and generate sales through frequent promotions. Over 80% of Chock Full O Nuts sales in these two cities are on price promotions. Chock Full O Nuts maintains the third largest market share with this policy. The substantial price elasticities for Folgers, Maxwell House and All Other Branded result from a similar policy, but with less frequent and less predictable price-promotions. Master Blend, Hills Brs. and Yuban have elasticities more like the private label brands (i.e. PL 1, PL 2, PL 3 and AOPL). Since the private-label brands have so little to offer other than price, we might expect them to have greater price elasticities. But with an everyday-low-price strategy these brands do not generate enough variation in price to achieve the elasticities of the more frequently promoted brands. As we see in the subsequent analyses, the average elasticities in Table 2 reflect an aggregation of widely differing competitive conditions. There are shelf-price elasticities which are quite different from the promotion-price elasticities one obtains during sales for the three major brands in these markets.

4.2. Three-Mode Analysis

Three-mode factor analysis is a technique for structured exploration. Although elasticities help researchers understand the raw data, the average elasticities are too aggregate to reflect the diversity of the competitive environment. The first task of the analysis is to signal when particular competitive events are part of a systematic pattern. Knowing that say five particular weeks in particular grocery chains constitute a pattern, we can go back to the original data to seek the meaning of that pattern in the antecedent conditions (e.g. these are weeks of deep price cuts for Folgers). This reduces the noise, so that signals are more easily detected. Next, the three-mode analysis determines the

TABLE 2
Average Market-Share Elasticity of Price

	Folg	MaxHse	MstrBln	Hills	CFON	Yuban	C & S	AOB	PL 1	PL 2	PL 3	AOPL
Folgers	-4.37	0.59	2.53	0.32	1.41	-0.34	-0.44	0.35	-0.06	-0.12	0.06	0.06
Maxwell House	1.95	-3.89	0.56	0.01	0.71	0.19	0.26	-0.05	0.15	-0.06	0.10	0.06
Master Blend	1.48	-0.05	-0.88	0.02	0.68	0.10	-0.73	0.48	-0.07	-0.09	-0.11	0.15
Hills Brs.	-0.53	-0.45	-0.17	-0.54	0.30	0.11	0.09	1.23	0.03	-0.08	0.00	0.00
Chock Full O Nuts	1.50	1.55	-1.90	-0.32	-4.71	0.35	0.78	1.55	0.86	0.35	0.00	0.00
Yuban	0.73	-0.16	-0.25	-0.19	-0.03	0.12	0.07	-0.16	0.00	0.00	-0.02	-0.10
Chase & Sanborne	0.19	0.24	0.09	0.20	0.47	0.05	-1.47	0.19	0.00	0.04	0.00	0.00
All Other Branded	1.43	3.47	-0.15	-0.87	-0.28	0.59	-0.07	-3.46	-0.25	-0.35	-0.47	0.40
Private Label 1	-0.06	0.03	0.02	0.08	0.42	0.00	0.00	-0.01	-0.47	0.00	0.00	0.00
Private Label 2	0.07	0.08	0.22	0.12	0.42	0.00	0.06	-0.09	0.00	-0.89	0.00	0.00
Private Label 3	0.16	0.07	-0.14	0.00	0.00	0.05	0.00	-0.03	0.00	0.00	-0.14	0.03
AOPL	1.29	0.15	-1.00	0.00	0.00	0.18	0.00	0.04	0.00	0.00	0.03	-0.69

⁷ The model has an R^2 of 0.82 before the introduction of cross-competitive effects. The reduced-model test was significant $F_{2006}^{59} = 13.8$, indicating a significant increment in R^2 is associated with the inclusion of these cross-competitive effects.

competitive building blocks for the elasticities. There is a pair of matrices R and C for each time factor ($l = 1, 2, \dots, L$) in the matrix $W = \{w_{il}\}_{T \times L}$. Equation (17) approximates the elasticities for any particular competitive pattern as a linear combination of the building blocks, where the entries in a row of W serve as the linear combining weights. While each pair of matrices R and C can be interpreted, we form the linear combinations implied by particularly interesting competitive patterns and interpret the competitive maps resulting from the approximated elasticities because the building blocks may include more dimensions than are operative in any particular competitive pattern.

The three-mode factor analysis in this illustration was implemented using the matrix-algebra routines in SAS. The limit of 32,767 elements in the maximum array size in SAS's PROC MATRIX meant that only three of the seven grocery chains reporting each week could be analyzed.⁸ This limitation resulted in exclusion of Private Label 3 and All Other Private Labels from the subsequent analyses, because these brands were not distributed in Chains 1–3.

The factor structure over the 52 weeks for the three chains was four-dimensional, accounting for almost 93% of the variation over chains and weeks. The dominant first factor accounted for over 74% of the variance, and the next three factors accounting for 9%, 6.5% and 3% respectively.

Figures 1 and 2 plot the weights w_{il} showing the influence of the time-factors ($l = 1-4$) on the weeks t —with Figure 1 depicting the first two dimensions over time, and Figure 2 portraying dimensions 3 and 4 over time. The symbols \circ (circles) and \diamond (diamonds) correspond to grocery chains 2 and 3 respectively. The \square (boxes), some of which have letters inside, represent Chain 1. To help reflect the dimensionality in these figures the size of the symbol decreases the farther away the observation is from the “Week” axis. The coefficients for the grocery chains are indicators of systematic structure of events in these weeks. Each of these four factors corresponds to a fundamental building block which collectively can represent any pattern of competition in the data. For each factor (column) in W there is a brand map which provides a spatial representation of the competitive pattern implied by the building block. The dimensionality of these brand maps is determined independently from the dimensionality of the structure over time, but in this case was also four dimensional.

The weeks marked by blank boxes (\square) indicate *Shelf Prices*—weeks in which there are no price promotions in Chain 1. “F,” “M” and “C,” indicate big price cuts for Folgers, Maxwell House and Chock Full O Nuts, respectively. Note that the weeks in Chain 1 which have high weights on the first factor reflect shelf-price competition—weeks in which no major brand is being promoted. These weeks had an average loading of about 0.11 on Factor 1 and -0.08 on Factor 2, but very little weight on Factor 3 (-0.02) or Factor 4 (-0.04). These linear combining weights were used to develop approximate or idealized shelf-price elasticities from the basic building blocks. These elasticities were mapped and are discussed below.

The weeks with Folgers on sale have somewhat less weight on Factor 1 than the Shelf-Price weeks, tend to have the most negative weights on Factor 2, have the most positive weights on Factor 3, and have the most negative weights on Factor 4. The positive weights on dimension 3 for Folgers are neutralized when Folgers and Maxwell House promote simultaneously. These weeks, marked “FM” in Figure 3, look much more like shelf-price weeks, indicating head-to-head promotions partially cancel each other.

⁸ The PROC MATRIX routine is available from the author. Real applications of much larger size are currently feasible. First, SAS IML apparently removes the size restriction. Second, a general three-mode program, developed by Pieter Kroonenberg, is available for a small fee from the Department of Data Theory, University of Leiden, P.O. Box 9507, 2300 RA Leiden, The Netherlands.

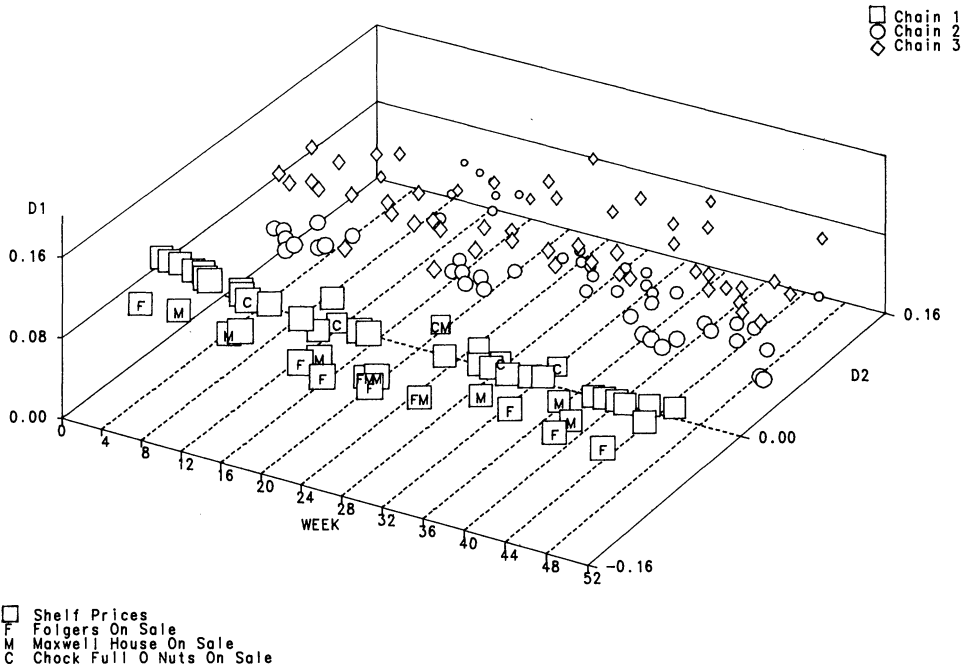


FIGURE 1. Competitive Structure of Stores—Dimensions 1 & 2 over Weeks.

Next, we create elasticities corresponding to the idealized competitive patterns discovered in the structure underlying grocery chains and weeks. Ross (1966) advises that idealized individuals be placed very near the positions of real individuals to minimize the possibility that averaging several locations could create unreal dimensional struc-

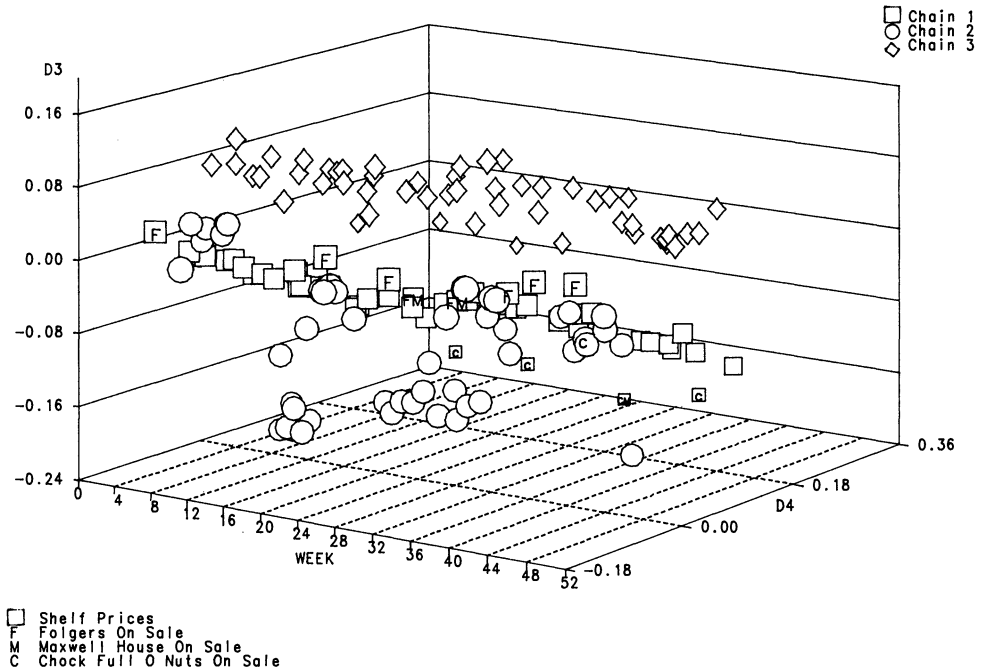


FIGURE 2. Competitive Structure of Stores—Dimensions 3 & 4 over Weeks.

tures. Following this advice, the idealized shelf-price elasticities correspond to the coordinates in the W matrix for Chain 1 Week 2, the idealized Folgers sale elasticities come from the coordinates for Chain 1 Week 1.⁹ Although other interesting patterns can be discovered, the maps for Shelf-Price competition and when Folgers goes on sale provide sufficient illustration. The idealized elasticities for the two competitive patterns appear in Table 3. The competitive maps are developed and interpreted in the next section.

4.3. *Competitive Maps: The Structure over Brands*

The common scaling space developed by a three-mode factor analysis of asymmetric cross elasticities as well as the two-mode representations of idealized competitive patterns, both provide maps of competitive interactions, *rather than necessarily portraying attribute relations among the brands*. These are competitive maps, rather than product perceptual spaces. In this illustration all the maps relate to price as an attribute, since price elasticities are used to develop the maps. In applied contexts the maps derived from all other promotional instruments would be investigated.

A competitive map involves two sets of points plotted in the same space. The first set of points—symbolized by circles \circ —represents the way brands exert influence on one another. Similar positions for two brands indicate they exert a similar pattern of influence on the market place. The second set of points—symbolized by squares \square —represents the way brands are influenced by competitive pressures. Similar positions for two brands indicate they are influenced similarly by pressures from other brands.

TABLE 3
Idealized Shelf-Price Elasticities

	Folg	MaxHse	MstrBln	Hills	CFON	Yuban	C & S	AOB	PL 1	PL 2
Folgers	-3.92	-1.32	2.90	0.98	4.39	-0.64	-2.81	1.14	-0.25	-0.48
Maxwell House	-2.43	-2.78	1.29	0.14	0.80	0.01	2.56	0.98	-0.33	-0.23
Master Blend	0.42	2.56	0.18	0.30	1.29	-0.34	-3.96	-0.61	0.18	-0.04
Hills Brs.	-1.41	-0.93	0.00	0.18	-1.65	0.02	0.80	2.43	0.51	0.05
Chock Full O Nuts	4.63	4.17	-6.82	-1.60	-15.91	1.27	5.68	4.77	2.59	1.24
Yuban	0.12	0.46	-0.25	-0.14	-0.79	0.07	0.51	-0.05	0.05	0.02
Chase & Sanborne	0.45	2.43	0.16	1.16	2.35	-0.74	-8.69	1.75	1.00	0.14
All Other Branded	3.84	8.78	-1.50	-1.58	-1.78	0.32	0.28	-7.31	-0.94	-0.11
Private Label 1	-0.25	-0.14	0.46	0.03	1.05	-0.06	-0.11	-0.63	-0.26	-0.10
Private Label 2	-0.70	0.85	0.39	-0.23	-0.18	0.04	1.06	-0.81	-0.28	-0.15

Idealized Folgers Sale-Price Elasticities

	Folg	MaxHse	MstrBln	Hills	CFON	Yuban	C & S	AOB	PL 1	PL 2
Folgers	-0.90	-0.06	0.76	0.41	1.50	-0.26	-1.91	0.51	0.06	-0.10
Maxwell House	-1.03	-1.46	0.47	-0.06	0.01	0.09	1.99	0.31	-0.22	-0.10
Master Blend	0.70	1.75	-0.21	0.21	0.47	-0.20	-2.89	-0.16	0.27	0.06
Hills Brs.	-0.13	-0.16	-0.56	-0.05	-1.89	0.11	0.81	1.35	0.40	0.13
Chock Full O Nuts	2.91	2.66	-4.61	-1.21	-11.20	0.93	4.83	3.19	1.68	0.82
Yuban	0.19	0.33	-0.27	-0.12	-0.72	0.07	0.44	-0.01	0.05	0.04
Chase & Sanborne	0.78	1.82	-0.18	0.83	1.45	-0.52	-6.50	1.34	0.82	0.17
All Other Branded	2.55	5.17	-1.32	-0.99	-2.01	0.27	0.47	-3.81	-0.36	0.03
Private Label 1	-0.17	-0.11	0.31	0.03	0.72	-0.04	-0.12	-0.40	-0.16	-0.07
Private Label 2	0.15	0.74	-0.24	-0.27	-0.89	0.11	0.97	-0.49	-0.07	-0.01

⁹ Maps were created for other chain-weeks representing major promotions for Folgers. Very similar maps were obtained which should increase our confidence in the reliability of the mapping procedures.

For the \circ brands, the distance of a brand from the origin of the space is a measure of how much *clout* the brand has.¹⁰ For the \square brands, the distance from the origin is a measure of how *vulnerable* or *receptive* a brand's sales are to price competition.¹¹ Hence, two \circ brands on the same vector from the origin exert the same pattern of pressure on the other brands, but differ in the amount of clout each possesses. Two \square brands on the same vector from the origin are pressured by the same competitors, but could be differentially vulnerable or receptive. A \circ brand on the same vector as a \square brand would exert its most direct influence on that \square brand. The influence falls off as the cosine between the angles of the brands drops toward zero (brands at right angles). Brands on opposite sides of the origin (angles greater than 90°) exert complementary influences, rather than competitive pressures.

In the ground-caffeinated coffee market under study there are four major dimensions describing the relations among brands on each time-factor. Eventhough these building blocks are four-dimensional, the linear combinations representing each of these two special cases are three-dimensional. Any particular competitive pattern need not involve all the basic factors.

Figure 3 portrays the competitive map accounting for 98.7% of the idealized shelf-price elasticities in Table 3. The size of the symbol for each brand represents the distance from the origin. This reflects the clout or receptivity/vulnerability of the brand. We see that Folgers and Maxwell House exert a similar pattern of pressure, with Maxwell House having more clout at shelf prices. They are both aligned to exert the greatest pressure on the premium brands in the All-Other-Branded category, which are quite vulnerable to their attack. Even though \square Chock Full O Nuts is separated from \circ Folgers and Maxwell House by a sizeable angle, its extreme receptivity translates into its being strongly pressured by both Folgers and Maxwell House. The almost 180° angle between Folgers \circ and Folgers \square indicates that Folgers helps itself quite directly with price cuts. The most extreme example of this involves Chock Full O Nuts which has a great deal of clout and is very vulnerable. The pattern in Figure 3 shows Chock Full O Nuts competing much more with Folgers than with Maxwell House, while being very receptive to its own price moves. \circ Master Blend is positioned to exert its greatest pressure on Folgers at shelf prices. But Folgers is not in the best position to return the pressure on either Regular Maxwell House or Master Blend. This is a pattern with substantial asymmetries which would not have been revealed by any other market-structure map.

When Folgers goes on sale the pattern in Figure 4 is operative (accounting for 99.8% of the corresponding idealized elasticities in Table 3). First, note that Folgers has less clout on sale. Shelf-price elasticities are like potential energy. When the brand actually goes on sale some of this energy is dissipated, in this case by being translated into sales. The reduction in the angle between \circ and \square for Folgers on sale indicates a dissipation of Folgers' influence on its own market share. An approximately 90° between \circ and \square for Folgers would indicate that Folgers cannot help itself by further price reductions. On sale, Folgers is substantially less vulnerable to both Chock Full O Nuts and AOB. Folgers can still attack the All-Other-Branded category and Chock Full O Nuts, but the reduced shares for these brands during a Folgers promotion provide less incentive to Folgers.

¹⁰ Formally it is a function of the sum of squares of the cross elasticities of other brands' shares with respect to this brand's price.

¹¹ Formally it is a function of the sum of squares of the cross elasticities of this brand's share with respect to the other brands' prices. Elasticities reflect percentage changes. So if a brand with a small share can lose a large percent of its share to other brands, it can appear far from the origin and be very vulnerable in percentage terms.

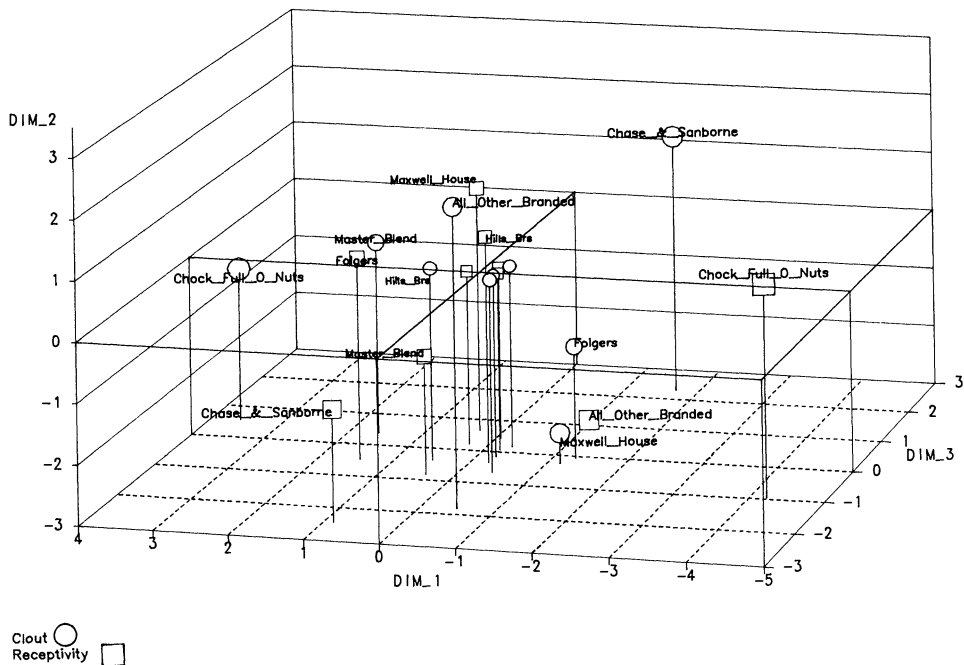


FIGURE 3. The Competitive Map—Shelf Prices.

There are very simple relations between the maps and the elasticities in Table 3. As implied by equation (18), one need only multiply the clout coordinate of brand j times the receptivity coordinate of brand i , and sum over dimensions to produce the elasticity of brand i 's market share with respect to brand j 's price. For those who are more

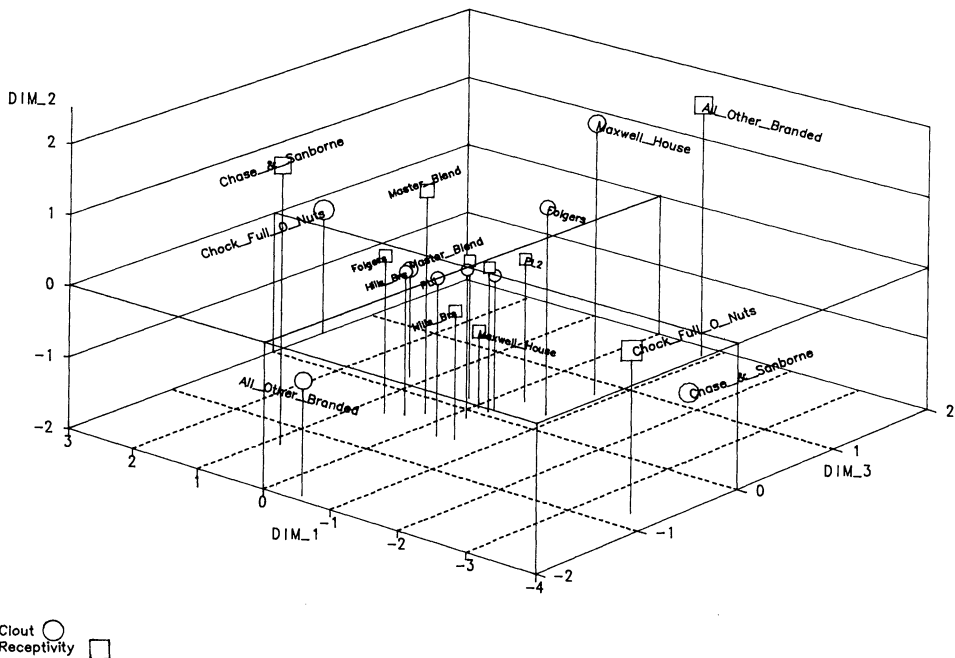


FIGURE 4. The Competitive Map—Folgers on Sale.

comfortable with maps than with matrices, these maps provide a visual representation of the richly asymmetric competitive patterns resulting from price changes in the coffee market. As in other categories of frequently purchase branded goods, price is used as a major weapon of promotional strategy. What one can read from these maps is what brands constitute the major threats to others with their price policy and where the major opportunities for competitive advantage may reside. These maps offer many signals which are new and very different from the market-structure maps of the past. Only a very few of these signals have been mentioned in this illustration. The full meaning of these signals is better interpreted by managers and management scientists involved in these markets, than by academic researchers involved in methods development.

5. Discussion

The value of much of the developments so far rests on the propriety of using cross elasticities to reflect market structure. There are at least three deficiencies to elasticities as measures of competition:¹² (i) they are static measures as they assume no competitive reaction to change in a marketing-mix variable, (ii) because they are static measures, they do not account well for structural change in markets, and (iii) they can be difficult to measure when price changes are infrequent or are of low magnitude.

First, both historic lags and competitive reactions can be included in elasticity calculations. This was indicated in Hanssens (1980), although misspecification of his equation (2) precluded computation. Lagged influences on brand i 's market share can be represented as $e_{ijt}^{(k)}$. This is the influence that brand j 's price (k th marketing instrument) in historic time-period t^* has on brand i 's market share in period t . But for a competitive reaction to influence a current elasticity a combination of events must occur. There must be an action involving marketing instrument k' by some brand i' in historic period t' which produces a significant price reaction by brand j in some historic period t^* , and there must be a nonzero elasticity for the effect of brand j 's price in period t^* on brand i 's market share in period t . We can represent the reaction elasticity as $e_{k' i' t' \bar{R} k j t^*}$, where the subscripts before \bar{R} indicate the antecedents producing the reaction, while the subscripts after \bar{R} indicate where the reaction occurs. Then the market-share cross elasticity is represented as:

$$e_{ijt}^{(k)} = \sum_{t^*=(t-h)}^t e_{ijt^*}^{(k)} + \sum_{t^*=(t-h)}^t \sum_{t'=(t^*-h)}^{t^*} \sum_{k'=1}^K \sum_{i'=1}^N e_{k' i' t' \bar{R} k j t^*} \cdot e_{ijt^*}^{(k)}, \quad (19)$$

where h is the maximum relevant historic lag. Note that if either $e_{k' i' t' \bar{R} k j t^*}$ or $e_{ijt^*}^{(k)}$ is zero, the entire term makes no contribution to $e_{ijt}^{(k)}$. In the current illustration there were no significant lagged effects (or cross effects) on market share, nor were there any significant competitive reactions. Given the highly disaggregate nature of the modeling effort (i.e. modeling marketing shares for brands in each grocery chain each week) and the irregular timing of major promotions, the absence of such effects is not surprising.

Second, exploration of the time mode (chain-weeks in the current illustration) can help minimize the limitations of elasticities in reflecting structural changes. While a regular lag structure may not be evident, one of the attractive features of the three-mode factor analysis is in its ability to highlight structural events which occur at irregular intervals over the study period. In the coffee market, promotions for major brands signalled the big structural changes. These will most likely occur at irregular intervals to minimize competitive reaction as well as consumers delaying purchase in the certain anticipation of a sale for their favorite brand.

¹² Thanks are due to an anonymous reviewer for pointing out these potential problems.

Third, if there is too little variation in a marketing instrument, elasticities can be hard to estimate accurately. In the retail coffee market there are very frequent price promotions, features, displays and considerable couponing activity. About 50% of all sales are made on a promotion of some kind. But in warehouse-withdrawal data, more temporally or regionally aggregated data, or in categories with less frequent retail promotions (e.g. bar soap), lack of variation would be more of a concern.

The limitations of this approach stem mainly from it being descriptive, rather than prescriptive. The dimensions of a competitive map describe the terrain. There is no guarantee that the map can easily be labelled in terms of brand attributes. Although we may always attempt to imbed property vectors in the space to help interpret the dimensions, the fit may not be good even in a good map. A map does not tell us *how* to reposition a brand to avoid competitive pressures. It merely reflects *what* those pressures are.

The benefits of this style analysis become clearer when we consider the task of intelligently using scanner data for brand planning. We could plot sales, prices, features, displays and coupons for each brand, each chain and each week. But the points of information become so numerous that the ability to assimilate soon suffers. In the current illustration this would entail 60 plots with over 21,000 data points. Market-response models provide an enormous concentration of information. But how do we assimilate the implications of a market-response model. Simulations and forecasts are very valuable, but they reverse the concentration of information achieved by the market-response model.

The parameters of the market-response model can be a source of insight. We could even factor the matrix B using the model in equation (18). While we might obtain some sense of the structure of competitive forces, we would have no idea of how that structure changes with changes in competitive patterns. The notion of reflecting changes is one of the most basic and appealing features of elasticities.

Elasticities can provide quantitative understanding of a market. Like simulations, however, elasticities reverse the information concentration achieved by market-response models. Using asymmetric three-mode factor analysis summarizes the 22,464 elasticities (3 Chains \times 52 Weeks \times [12 \times 12] Brands) into only two plots for the factors differentiating grocery chains over weeks, and a plot for each of the idealized competitive situations. The plots representing the structure over time can be helpful in planning the timing of promotions, tracking promotional effectiveness and detecting promotional wear-out. Looking at over-time patterns helps counteract some of the limitations imposed by the static nature of each week's elasticity estimates. In the analysis of these data aggregated to the weekly level over grocery chains, these plots have revealed the expansion of elasticities in key pay-weeks. It is during the key-pay week that all Federal checks (Social Security, Aid to Families with Dependent Children, welfare, government pensions, etc.) arrive. These Federal checks are often cashed at supermarkets and then banked in the form of food purchases for the month. These first-week customers, typically being of limited means, are some of the most price-sensitive shoppers and are purchasing a disproportionate share of their monthly needs in this week.¹³ Other applications could reveal temporal or seasonal patterns of interest. This format signals what are the systematic structural events in a dataset which otherwise might be too large to explore. The structure over brands is contained in figures describing the idealized competitive contexts which characterize this market. Shelf-price competition, and the structure of competition during sales for each of the three largest-selling brands, are differentiated in a manner which could never be detected from the average elasticities.

¹³ Special thanks are due to J. Dennis Bender and John Trotten for helping me understand the meaning and significance of this finding.

The structure over brands could perhaps be made even more useful if ideal points were located in the space. The two logit models in Cooper and Nakanishi (1983b) were designed to do just this. They provide an external analysis of preference. The competitive maps supply the needed pre-existing configuration, while sales data provide the relative choice frequencies required for locating the ideal points for each week. If we imbed ideal points using the positions of brands in the column space (how brands exert influence), we could see not only the opportunities for each brand, but also how those opportunities could be translated into sales.

Asymmetric market-share models, the procedures for estimating elasticities from those models, three-mode factor analysis for representing the structure underlying those elasticities both over time and over brands, and the ideal-point model are a major part of a system of models for competitive analysis. Rather than proposing specific behavioral models of consumer response, these models provide the structural relations between the entities in a market information system. Although this system of models can be used for many purposes; the scientific goal is to provide a more systematic basis for using market information, while the practical goal is to provide a graphic understanding of the competitive structure and dynamics of a marketplace.¹⁴

¹⁴ I gratefully acknowledge the assistance of Penny Baron, Gerald Eskin and John Totten. I wish to thank Masao Nakanishi, Dominique Hanssens, Gregory Carpenter and J. Dennis Bender for their helpful comments and discussions, and thank Wilfried Vanhonacker for discussions during his visit to UCLA in 1984, which prompted some of my initial thinking in this area. I also gratefully acknowledge the careful reading and suggestions of the editors and reviewers, as well as Information Resources, Inc. for providing the data used in this study. This research was supported in part by the MoIS Grant from IBM Corporation to the Anderson Graduate School of Management. Computing was supported by the UCLA Office of Academic Computing through an intramural grant.

References

- BASS, FRANK M. AND ALAIN V. BULTEZ, "A Note on Optimal Strategic Pricing of Technological Innovations," *Marketing Sci.*, 1, 4 (Fall 1982), 371-378.
- BULTEZ, ALAIN V. AND PHILIPPE A. NAERT, "Consistent Sum-Constrained Models," *J. Amer. Statist. Assoc.*, 70, 351 (September 1975), 529-535.
- and —, "Does Lag Structure Really Matter in Optimal Advertising Expenditures?" *Management Sci.*, 25, 5 (May 1979), 454-465.
- CARPENTER, GREGORY S., LEE G. COOPER, DOMINIQUE M. HANSENS AND DAVID F. MIDGLEY, "Asymmetric Market-Share Models," Working Paper No. 137 Revised, Center for Marketing Studies, Graduate School of Management, University of California, Los Angeles, June 1987.
- CARROLL, J. DOUGLAS, "Models and Methods for Multidimensional Analysis of Preferential Choice (or Other Dominance) Data," in *Similarity and Choice*, Ernst D. Lantermann and Hubert Feger (Eds.), Hans Huber Publishers, Bern, 1980, 234-289.
- CLARKE, DARRAL G., "Sales-Advertising Cross-Elasticities and Advertising Competition," *J. Marketing Res.*, 10 (August 1973), 250-261.
- COOPER, LEE G., "A Multivariate Investigation of Preferences," *Multivariate Behavioral Res.*, 8 (April 1973), 253-272.
- , "A Review of Multidimensional Scaling in Marketing Research," *Appl. Psychological Measurement*, 7 (Fall 1983), 427-450.
- , "Sources of and Remedies for Collinearity in Differential-Effects Market-Share Models," Working Paper No. 136 Revised, Center for Marketing Studies, Graduate School of Management, University of California, Los Angeles, July 1985.
- AND MASAO NAKANISHI, "Standardizing Variables in Multiplicative Choice Models," *J. Consumer Res.*, 10 (June 1983a), 96-108.
- AND —, "Two Logit Models for External Analysis of Preferences," *Psychometrika*, 48, 4 (December 1983b), 607-620.
- DESARBO, WAYNE S. AND VITALLA R. RAO, "GENFOLD2: A Set of Models and Algorithms for GENERAL UNFOLDING Analysis of Preference/Dominance Data," *J. Classification*, 1, 2 (1984), 147-186.
- DESOETE, GEERT AND J. DOUGLAS CARROLL, "A Maximum Likelihood Method of Fitting the Wandering Vector Model," *Psychometrika*, 48, 4 (December 1983), 553-566.
- DORFMAN, ROBERT A. AND PETER D. STEINER, "Optimal Advertising and Optimal Quality," *Amer. Economic Rev.*, 44 (December 1954), 826-836.

- ELROD, TERRY, "Choice Map: Inferring Brand Attributes and Heterogeneous Consumer Preferences From Panel Data," *Marketing Sci.*, (1987), forthcoming.
- HANSENS, DOMINIQUE M., "Market Response, Competitive Behavior, and Time Series Analysis," *J. Marketing Res.*, 27 (November 1980), 470-485.
- HAUSER, JOHN R. AND STEVEN M. SHUGAN, "Defensive Marketing Strategies," *Marketing Sci.*, 2, 4 (Fall 1983), 319-360.
- KAHN, BARBARA E., DONALD G. MORRISON AND GORDON P. WRIGHT, "Aggregating Individual Purchases to the Household Level," *Marketing Sci.*, 5, 3 (Summer 1986), 260-268.
- KALISH, SHLOMO, "Monopolistic Pricing with Dynamic Demand and Production Cost," *Marketing Sci.*, 2, 2 (Spring 1983), 135-159.
- KARNANI, ANEEL, "Minimum Market Share," *Marketing Sci.*, 2, 1 (Winter 1983), 75-93.
- KROONENBERG, PIETER M., *Three-Mode Principal Components Analysis: Theory and Applications*, DSWO Press, Leiden, The Netherlands, 1983.
- LAMBIN, JEAN-JACQUES, PHILIPPE A. NAERT AND ALAIN V. BULTEZ, "Optimal Market Behavior in Oligopoly," *European Economic Rev.*, 6 (1975), 105-128.
- MAGAT, WESLEY A., JOHN M. McCANN AND RICHARD C. MOREY, "When Does Lag Structure Really Matter in Optimal Advertising Expenditures?" *Management Sci.*, 32, 2 (February 1986), 182-193.
- McFADDEN, DANIEL, "Conditional Logit Analysis of Qualitative Choice Behavior," in Paul Zarembka (Ed.), *Frontiers in Econometrics*, Academic Press, New York, 1974, 105-142.
- McGUIRE, TIMOTHY W., DOYLE L. WEISS AND FRANK S. HOUSTON, "Consistent Multiplicative Market Share Models," in *Contemporary Marketing Thought*, Barnett A. Greenberg and Danny N. Bellinger (Eds.), American Marketing Association, Chicago, 1977, 129-134.
- MOORE, WILLIAM L. AND RUSSELL S. WINER, "A Panel-Data Based Method for Merging Joint Space and Market Response Function Estimation," *Marketing Sci.*, 6, 1 (Winter 1987), 25-42.
- MOREY, RICHARD C. AND JOHN M. McCANN, "Estimating the Confidence Interval for the Optimal Marketing Mix: An Application to Lead Generation," *Marketing Sci.*, 2, 2 (Spring 1983), 193-202.
- NAKANISHI, MASAO AND LEE G. COOPER, "Parameter Estimation for a Multiplicative Competitive Interaction Model—Least Squares Approach," *J. Marketing Res.*, 11 (August 1974), 303-311.
- AND —, "Parameter Estimation of the MCI and Related Models: Revisited," University of California, Graduate School of Management, Center for Marketing Studies Paper No. 97, October 1980.
- AND —, "Simplified Estimation Procedures for MCI Models," *Marketing Sci.*, 1, 3 (Summer 1982), 314-322.
- PARSONS, LEONARD J., "The Product Life Cycle and Time-Varying Advertising Elasticities," *J. Marketing Res.*, 16 (November 1975), 439-452.
- RAO, RAM C. AND FRANK M. BASS, "Competition, Strategy, and Price Dynamics: A Theoretical and Empirical Investigation," *J. Marketing Res.*, 22 (August 1985), 283-296.
- ROSS, JOHN, "A Remark on Tucker and Messick's *Points of View* Analysis," *Psychometrika*, 31, 1 (March 1966), 27-31.
- SHOEMAKER, ROBERT W., "Comments on *Dynamics of Price Elasticities and Brand Life Cycles: An Empirical Study*," *J. Marketing Res.*, 23 (February 1986), 78-82.
- SHUGAN, STEVEN M., "Brand Positioning Maps from Price/Share Data," University of Chicago, Graduate School of Business, Revised July 1986.
- , "Estimating Brand Positioning Maps From Supermarket Scanning Data," *J. Marketing Res.*, 24 (February 1987), 1-18.
- SIMON, HERMANN, "Dynamics of Price Elasticities and Brand Life Cycles: An Empirical Study," *J. Marketing Res.*, 16 (November 1979), 439-452.
- TUCKER, LEDYARD R., "Implications of Factor Analysis of Three-Way Matrices for Measurement of Change," in Chester W. Harris (Ed.), *Problems in Measuring Change*, University of Wisconsin Press, Madison, 1963, 122-137.
- , "Some Mathematical Notes On Three-Mode Factor Analysis," *Psychometrika*, 31, 4 (December 1966), 279-311.
- , "Some Relations Between Multi-Mode Factor Analysis and Linear Models for Individual Differences in Choice, Judgmental and Performance Data," Paper presented to the Psychometric Society Symposium: Multi-Mode Factor Analysis and Models for Individual Differences in Psychological Phenomena, April 18, 1969.
- , "Relations Between Multidimensional Scaling and Three-Mode Factor Analysis," *Psychometrika*, 37, 1 (March 1972), 3-27.
- AND SAMUEL MESSICK, "An Individual Differences Model for Multidimensional Scaling," *Psychometrika*, 28, 4 (December 1963), 333-367.
- VANHONACKER, WILFRIED, "Structuring and Analyzing Brand Competition Using Scanner Data," Columbia University, Graduate School of Business, April 1984.