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The Effects of Tortuosity on Flow through a Natural Fracture

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1 INTRODUCTION

Fractures can play a dominant role in the movement of fluid through rock media. As a result, much work has been done, both theoretically and experimentally, on laminar fluid flow through open fractures consisting of parallel plates of varying degrees of surface roughness. In many cases the behavior of the fluid can be described using the expression developed by Lomize (1951) and used by others such as Witherspoon et al. (1980);

$$Q = \frac{1}{f} \frac{b^3}{12\mu} \frac{dp}{dx} \tag{1}$$

where Q is the flow rate per unit width of fracture normal to the flow direction, x; b is the aperture between the plates; μ is the viscosity of the fluid; $\frac{dp}{dx}$ is the pressure gradient in the direction of flow; and $\frac{1}{f}$ is an empirical factor to account for the effects of surface roughness.

This equation has become generally known as the "cubic law equation" because flow rate is proportional to the cube of the aperture. Although the relation of fracture aperture to flow is obvious, tortuosity of the flow paths in a fracture may also have a significant effect on flow. In this paper the results of a series of flow tests performed on a naturally fractured siltstone sample taken at depth from an oil reservoir are presented. The results of flow measurements over a wide range of apertures showed that tortuosity was a primary factor in explaining the difference between the measured flow rates and those predicted by the cubic law equation.

2 LABORATORY PROCEDURES

The siltstone sample was a rectangular parallelepiped, the top surface of which measured 25mm in width by 75mm in length. The sample was about 25mm thick. A single natural fracture lay in a plane at mid-height approximately parallel to the top surface. Each half of the sample was embedded into a steel casing using a low melting temperature metal alloy called Cerrosafe®. The casings were machined in such a way that the separation between the fracture surfaces of the sample could be controlled by shims over a range of apertures, from conditions where the two surfaces of the fracture were mechanically held apart by shims up to 0.25mm thick, to conditions where the two surfaces were in contact and under moderate stress up to 11 MPa. Except for the inlet and outlet parts the sides and ends of the fracture were sealed to prevent leakage.

Decane, which did not interact chemically or physically with the siltstone, was used as the fluid in these experiments. For the tests in which the fracture was held open by shims, at least eight different heads, ranging from 0.005m to 1.2m were used. For tests with the fracture under load, nitrogen gas was applied to the fluid providing equivalent heads ranging from 23m to 34m. For each aperture, flow rates were plotted against head and found to vary linearly with head for all apertures, indicating that laminar flow conditions had been maintained.

By aperture is meant fracture displacement relative to a reference state, which was taken as the highest stress level used in the experiments. Linear variable differential transformers (LVDT's) were used to measure fracture displacement between zero load on the fracture and the highest stress level. The LVDT measurements were corrected for effects of sample cycling and intact rock deformation. For the tests in which the fracture was held apart by shims, a micrometer was used to measure the distance between the surfaces of the steel casings. These measurements were used to determine the fracture displacement relative to the zero load condition. It should be noted that, according to this convention, shim thickness and aperture are not synonymous; rather, the aperture used in the "cubic law" calculations was equal to the shim thickness plus the fracture displacement which resulted from loading the fracture up to 11 MPa.

3 EXPERIMENTAL RESULTS

Experimental results are shown in Figure 1, which also shows as a dotted line the flow calculated for each aperture based on the cubic law assuming f in Eq. (1) equals 1. In plotting the experimental data it was assumed that the aperture at the highest stress level was 5 μm. The figure shows that the flow rate was about proportional to the cube of the aperture when the fracture surfaces were not in contact, though the curve for these data is displaced above the cubic law curve by a nearly constant amount. A very good fit was obtained at the large apertures by assuming (unrealistically since flow was non zero) a zero aperture at the highest stress level. Even so the experimental data points were slower by 5% - 20% than those predicted by the cubic law.

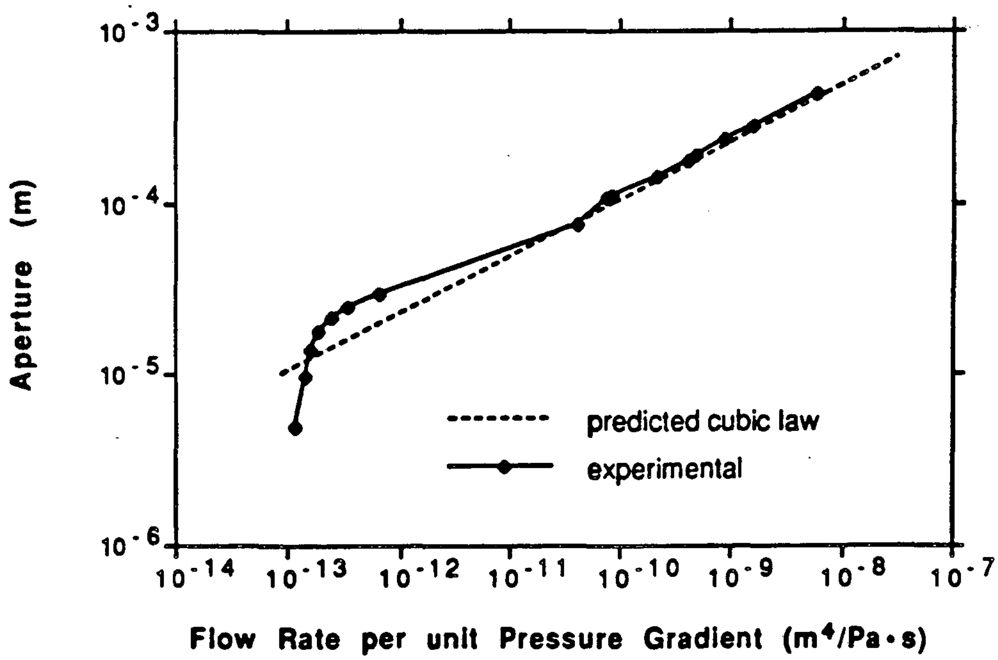

Figure 1. Comparison of experimental data with those predicted assuming smooth parallel plates. Residual aperture is assumed to be 5.0μm.

At the lower apertures, where the fracture surfaces were in contact and under moderate stress, the flow rate was not proportional to the cube of the aperture, but instead to some higher and variable power. In other words, the experimental flow rates decreased at a higher power than those predicted from the cubic law relationship. At higher stresses flow rate asymptoted toward a constant value and this value did not change even with further decreases in aperture. The presence of this irreducible flow (Raven and Gale, 1985 and Pyrak-Nolte et al., 1987) indicates that a residual aperture exists even at very high stresses. Although the values of fracture aperture depend upon the value assumed for the residual aperture, the shape of the curve formed by the experimental data points does not. Thus, the generally observed discrepancies between the experimental data and those predicted from the cubic law relationship will not be affected by the absolute value chosen for the residual aperture.

4 AN IDEALIZED MODEL FOR TORTUOSITY

An explanation for the departure from the cubic equation is motivated by observations that the rough surfaces of many natural fractures have rugged topographies made up of a rich spectrum of wavelengths and amplitudes (e.g., Brown, 1987). At the longer wavelengths the topographies of the two surfaces of a fracture are correlated. However, at the shorter wavelengths they are uncorrelated, and it is these shorter wavelengths and amplitudes that produce asperities of contact between the fracture surfaces. The presence of these long wavelengths introduces out-of-plane tortuosity, which accounts for both the increase in flow path length and the variation in cross sectional area of flow due to surface roughness. In-plane tortuosity arises due to the presence of areas of contact. Fluid can only flow through connecting void spaces around the areas of contact and therefore experiences increased path lengths.

In order to quantitatively evaluate the effects of these factors on flow, an idealized model was created to represent the rough surface topography of a natural fracture. To model out-of-plane tortuosity, a cosine wave of amplitude A, and frequency ω (equal to $2\pi/L$ where L is the wavelength) is used to describe the longer wavelengths at which the two fracture surfaces are correlated. Figure 2 shows two cosine curves, greatly exaggerated with respect to scale for the purpose of clarity, each separated from an imaginary midline curve by a distance equal to a/2. The length of the flowpath is denoted by l. Assuming unit width for the aperture, the total flow rate between the surfaces can be given as

$$Q = \frac{b^3}{12\mu}\frac{dp}{dl} \tag{2}$$

where $\frac{dp}{dl}$ is the pressure gradient along the flowpath. Rearranging and integrating over a linear distance X gives;

$$\frac{P}{Q} = 12\mu \int_0^x \frac{dl}{b^3} \tag{3}$$

Since the flow path is described by a cosine curve, the length of the segment, dl, can be given by,

$$dl = (1 + A^2\,\omega^2\,(\sin\omega x)^2)^{1/2}\,dx \tag{4}$$

The expression for dl is substituted into Eq. (3) giving, for a unit distance,

$$\frac{P}{Q} = 12\mu \int_0^1 \frac{(1 + A^2\omega^2\,(\sin\omega x)^2)^{1/2}\,dx}{b^3} \tag{5}$$

This form of the cubic law equation accounts for increase in flowpath length.

The effect of variations in cross sectional aperture for flow between two cosine surfaces (Fig. 2) is found using the relationship between the vertical separation, a, and the separation, b, locally perpendicular to the direction of flow. Since b = acosθ and θ = arctan (-A ωsinωx), Eq. (5) can now be written as,

$$\frac{P}{Q} = \frac{12\mu}{a^3} \int_0^1 \frac{(1 + A^2\,\omega^2\,(\sin\omega x)^2)^{1/2}\,dx}{(\cos\,(\arctan\,(-A\omega\sin\omega x)))^3} \tag{6}$$

or, by denoting the integral as f,

$$\frac{P}{Q} = \frac{12\mu}{a^3}\, f \tag{7}$$

This expression incorporates both the effects of increased flow path length and the variation in cross sectional area of flow due to surface roughness. Examination of the above equation shows these effects to be independent of aperture. The friction factor used in Eq. (1) by Lomize and Witherspoon et al. corrects for a constant ratio between observed flow rates and those predicted from the cubic law. It appears that this constant, and therefore aperture independent, ratio between the flow rates can be attributed to the effects of out-of-plane tortuosity.

In-plane tortuosity effects arise because the flow path is obstructed by areas of contact between the fracture surfaces. For a given separation distance, h, between the fracture surfaces with no areas of contact, assume that flow can be approximated by equivalent parallel plates so that

$$Q = k_o\,\frac{dp}{dx} \cdot \frac{1}{f} \tag{8}$$

where k_o is equal to $h^3/12\mu$. For the case where areas of contact exist between the fracture surfaces it can be postulated that the flow can be approximated by

$$Q = k_e\,\frac{dp}{dx} \cdot \frac{1}{f} \tag{9}$$

where k_e is the effective flow conductance. Using the analogy of two-dimensional laminar flow of an incompressible fluid, and heat conduction, Walsh (1981) developed an expression for effective flow conductance to account for the effect of areas of contact across the plane of the fracture on flow conductivity. The effective flow conductance, k_e, can be given as

$$k_e = \frac{1-d}{1+d}\, k_o \tag{10}$$

where k_o is the conductance of the fracture with no asperities, and d is the ratio of the contact area of the asperities to the total area of the fracture.

To model in-plane tortuosity when the fracture surfaces are in contact, a "carpet" of very small, equally sized cones, whose bases form a hexagonal close-packed arrangement, is laid on top of the longer wavelength cosine wave forming the lower surface of the fracture in the out-of-plane tortuosity model (see Fig. 3). These cones represent the asperities of contact formed by the shorter uncorrelated wavelengths. Calculations determining the effects of the asperities of contact on tortuosity are based on the assumption that the volume of rock is conserved during interpenetration of the asperities into the fracture surface (Pyrak-Nolte et al., 1988). Figure 4a shows the upper fracture surface in point contact with the cones used to represent asperities. The aperture under this condition is a_o. Now assume a far field stress is applied. Figure 4b shows the change in displacement, D, which would be measured between two reference points external to the fracture and on either side of it, as is typically done in laboratory measurements of fracture deformations. Assuming conservation of

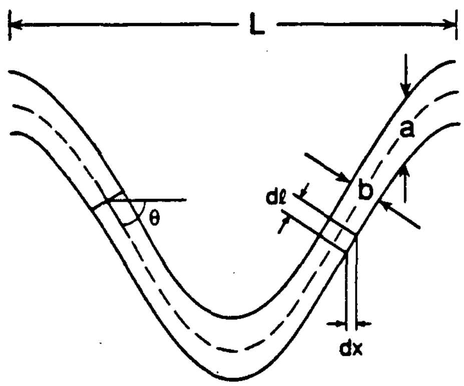

Figure 2. Two cosine waves that represent the surfaces of a fracture.

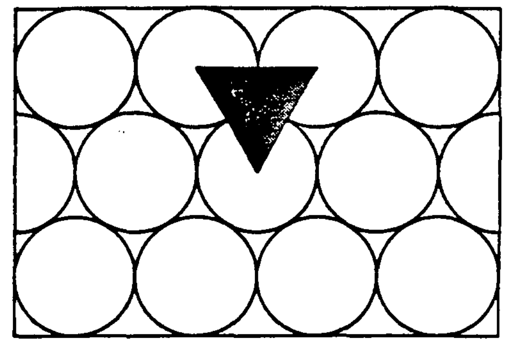

Figure 3. Plan view of the arrangement of the bases of the "carpet" of cones. Triangle denotes a unit area.

volume, the volume displaced by the penetration of the cones is redistributed over the remaining portion of the upper fracture surface. This yields a thickness, z, shown in Figure 4c. As a result, the change in fracture aperture is equal to D plus z; i.e., the change in fracture aperture is greater than the measured fracture displacement. The radius of the cones at any height above the baseline can be given in terms of D, a_o and the base radius R. A unit area such as shown in Figure 3 would be equal to 2 sin60°R^2. The thickness, z, can be given as

$$z = \left[\frac{1}{6}\,\pi\,\frac{D^3}{a_o^2}\right] \Big/ \left[2\sin 60^\circ - \frac{1}{2}\,\pi\,\frac{D^2}{a_o^2}\right] \tag{11}$$

The base radius cancels out in the above equation which means that z is independent of base radius, and therefore independent of the pitch of the cones. The aperture of the fracture, h, can now be given as

$$h = a_o - D - z \tag{12}$$

To account for the effects of areas of contact across the plane of the fracture on flow conductivity, Eq. (10) is substituted into Eq. (9). The factor, d in Eq. (10) is the ratio of the contact area of cylindrical asperities to the fracture surface area. After redistributing the volume of the interpenetrating tips of the cones over the area of the upper fracture surface remaining between the cones, the cones can be considered frustrums with a height equal to the aperture of the fracture, as in Figure 4c. These frustrums can be transformed into equivalent cylindrical asperities whose height is also equal to the aperture by equating the volume of the frustrums per unit area to the volume of the cylinders per unit area. The contact area for the equivalent cylindrical asperities per unit area can then be given as,

$$d = \frac{\frac{1}{6}\pi}{2\sin 60^\circ}\left[\frac{(a_o-h)^2}{a_o^2} + \frac{(a_o-h)}{a_o} + 1\right] \tag{13}$$

where h is given in Eq. (12).

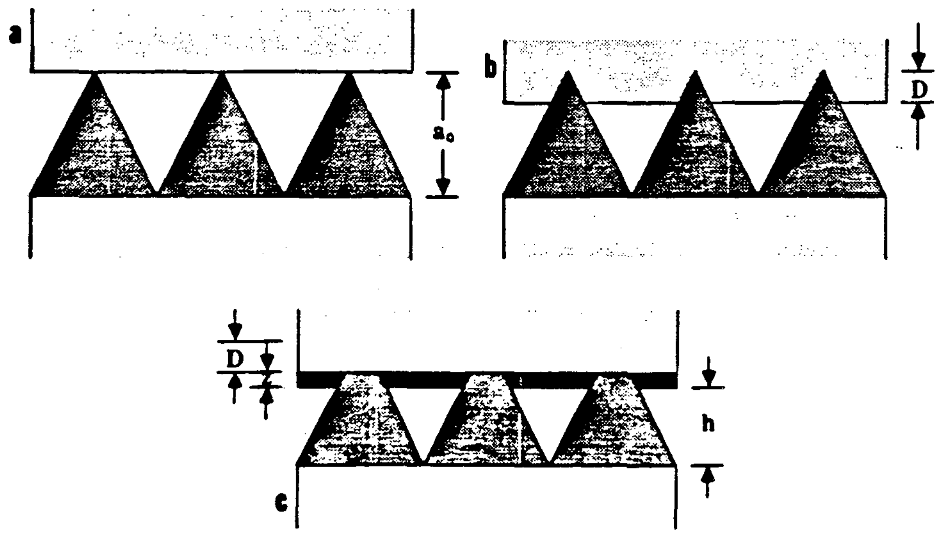

Figure 4. (a) Point contact between fracture surface and cones. (b) measured displacement, D, due to far field stress. Tips of cones have interpenetrated the fracture surface. (c) Volume of the interpenetrating cones is redistributed over the remaining fracture surface area between the cones, resulting in a thickness, z.

The equation for flow through the idealized model can now be given as,

$$\frac{Q}{\frac{dp}{dx}} = \frac{h^3}{12\mu} \cdot \frac{1-d}{1+d} \cdot \frac{1}{f} \tag{14}$$

Finally to account for an irreducible flow at high stress levels Eq. (14) becomes:

$$\frac{Q}{\frac{dp}{dx}} = \frac{h^3}{12\mu} \cdot \frac{1-d}{1+d} \cdot \frac{1}{f} + \frac{Q\infty}{\frac{dp}{dx}} \tag{15}$$

5 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

Using Eqs. (15) and (6) developed for the idealized model, a set of theoretical data points was generated and compared to those obtained experimentally. Figure 5 shows the theoretical and experimental data points as well as the predicted cubic law relationship. To generate the theoretical data points values of aperture were chosen that fell in the same range as those used in the flow experiments, i.e., from 2.5μm to 500μm. The value of a_o was estimated to be 50μm. Values of D were chosen to correspond with the values of displacement measured during the flow experiments and ranged from 15μm to 35μm. The value of $Q\infty/\frac{dp}{dx}$ was taken to be 1.26e-13m^4/Pa·s for a residual aperture of 2.5μm. Calculations to determine f in Eq. (7) were based on an estimate of the geometry of the fracture surface used in the flow experiments and resulted in a factor whose value equaled 1.12. The figure shows very good agreement between the theoretical and experimental data, particularly in the regions of the smaller apertures and irreducible flow. When the two surfaces of the fracture are not in contact the ratio of asperity contact area to fracture surface area is zero and the factor involving the contact area in Eq. (10) is 1. Thus only out-of-plane tortuosity affects flow when the

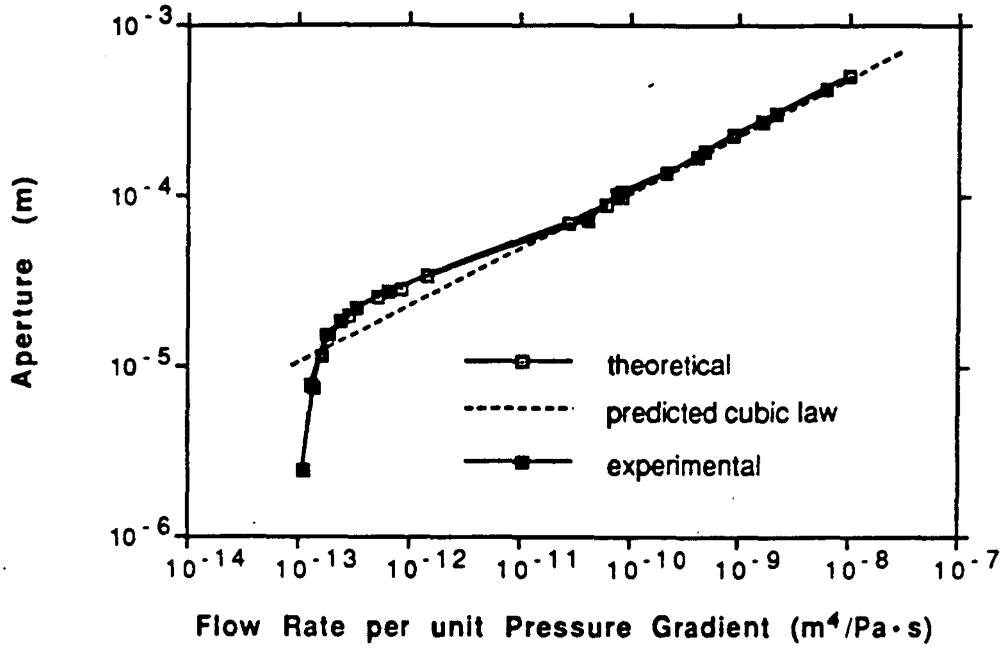

Figure 5. Comparison between data generated using theoretical equations and experimental data, showing the effect of taking the irreducible flow into account.

fracture surfaces are not touching. Evidence of these effects is shown by the constant ratio between both the experimental and theoretical data points and those predicted by the cubic law confirming that the effects of out-of-plane tortuosity are aperture independent.

The theoretical data points generated for flow through the idealized model of a fracture under conditions where the surfaces are in contact, reflect the effects of both in-plane and out-of-plane tortuosity. Conservation of volume during the interpenetration of the asperities into the fracture surface, and the resulting effect on aperture closure, cause the effects of in-plane tortuosity to outweigh those of out-of-plane tortuosity. It is evident that the effect of contact area increases with decreasing aperture.

6 CONCLUSIONS

By generating equations for a simple theoretical model, the effects of tortuosity on fluid flow through a fracture have been evaluated. The significance of these effects is such that they must be included in flow rate calculations. The effects of out-of-plane tortuosity have been found to be independent of aperture and result in a constant ratio between observed flow rates and those predicted from the cubic law relationship. In-plane tortuosity effects, which arise when the fracture surfaces are in contact, outweigh the effects of out-of-plane tortuosity. Because the volume of the asperities is conserved, the area of contact between the asperities and the fracture surface increases very quickly with decreasing aperture. As a result flow rate decreases rapidly with aperture closure to an irreducible, aperture independent value.

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