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# Optimal Resource Management under Conditions of 

 Uncertainty: The Case of an Ocean FisheryA dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics
by

Tracy Royal Lewis

Committee in charge:
Professor Richard Schmalensee, Chairman
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1975

The dissertation of Tracy Royal Lewis is approved and it is acceptable in quality and form for publication on microfilm:


University of California, San Diego

1975
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# ABSTRACT OF THE DISSERTATION 

# Optimal Resource Management under Conditions of Uncertainty: The Case of an Ocean Fishery 

## by

## Tracy Royal Lewis

Doctor of Philosophy in Economics

University of California, San Diego, 1975

Professor Richard Schmalensee, Chairman

A Markov Decision Process model is developed for analyzing the socially optimal allocation of a replenishable or non replenishable resource over time. The resource is managed by choosing the rate of extraction in each period to maximize the discounted stream of expected social returns. Elements of uncertainty enter the analysis in three ways: (1) Uncertainties may exist about the current size of the resource, either because of difficulties in observing the stock, as in the case of a fishery, or because of the possibilities of finding new reserves through exploration, as in the case of minerals and oil. (2) The market value of the resource and the cost of extracting it may be random, due to varying
economic conditions. (3) Unpredictable changes in the environment may perturb the natural rate of growth or deterioration of the resource, as well as the effective rate of depletion by man.

The model is used to answer these questions: How do optimal programs for allocating resources in a deterministic environment compare with optimal programs under stochastic conditions? Do different attitudes toward social risk bearing as regards variations in resource rents, have an effect on optimal decision rules? What is the effect of increased uncertainty about resource prices, extraction costs, and resource growth and depletion rates on optimal programs?

The questions posed above are considered in the context of an empirical study. of the Eastern Pacific yellowfin tuna fishery. The main conclusions of the study are that: (l) Optimal programs for resource management are more moderate, with smaller variations in the rate of fishing over time, when society is risk averse rather than risk neutral. This difference betweeen risk neutral and risk averse programs is accentuated with increasing uncertainty about market prices, fishing costs, and the future availability of the resource. (2) "Cyclical" fishing, in which the stock is depleted rapidly over a short time period and then allowed to grow back, is optimal if scale economies exist in the fishing industry. This contrasts with the optimal "steady state" fishing programs that are emphasized in the control theory literature on resource management.

## Chapter I

## INTRODUCTION

## A. INTRODUCTION AND REVIEW OF LITERATURE

The ongoing concern over natural resources has peaked recently with the emergence of possible world wide shortages of energy and food. Extrapolating on current growth trends in population and resource depletion, the now famous Club of Rome studies ${ }^{1}$ predict a sudden and uncontrollable decline in world population and living standards unless these trends are quickly reversed. Of course, this is not the first warning of Mathusian doom, nor are the conclusions from these studies universally accepted. A more optimistic view of the future espoused by Nordhaus (1974) and Smith (1974) is that as some of the valuable nonreplenishable resources become scarce society will find substitutes for them. For example, solar radiation is a substitute source of energy for fossil fuels, and it appears to be virtually inexhaustible.

Probably most economists would agree that the world will survive in spite of the finite supply of natural resources. Yet, there are numerous technical and socio-economic problems still to be solved in the quest for optimal resource utilization. The most basic problem has been to identify a common set of objectives for resource management. This is particularly difficult for marine resources such as fisheries and oil and mineral deposits found in

[^0]the ocean that are owned and used jointly by different countries. ${ }^{2}$ Typically, these countries have disparate social value systems, and different preferences for present versus future consumption of the resource. Common agreement on principles of resource management is not easy to obtain when national economies are organized according to different ideologies as in the case of developed and underdeveloped countries or socialist and capitalist nations.

The common property characteristic of ocean fisheries, various wild animal populations, and certain common underground oil pools presents a special problem. No single user has exclusive property rights to these resource stocks, nor can he prevent others from sharing in its exploitation. Thus, the individual is in competition with all others in an attempt to appropriate a large portion of the stock for himself. Efforts by one user to conserve the resource will be futile, since there is no guarantee that others will do the same. Consequently, the stock is depleted rapidly until further extraction is not economical. To prevent over use of the resource, quotas, extraction taxes, and licensing shcemes are sometimes introduced. Unfortunately, such programs are difficult to institute and enforce since they require the cooperation of all independent resource consumers.

It appears that resources with well defined property rights might offer a welcome relief from the aforementioned common
property situation. This is not true, however, if ownership is concentrated among only a few individuals, thus creating a potential oligopoly market in resources. For example, the majority of current and future world reserves of oil are controlled by a small number of countries in the Middle East. In recognition of their potential bargaining power these countries have formed an oil cartel known as the Organization of Petroleum Exporting Countries. As a result, since 1970 when the Organization began to assert itself in the petroleum market, the price of Middle East light crude has increased by about five-hundred percent. ${ }^{3}$

It would be naive and presumptuous to suggest that these problems are soluble through economic analysis alone. Only an interdisciplinary program enlisting the expertise from various social and physical sciences will contribute to their resolution. Economists can help by describing the resource extraction patterns under varying conditions. For example, it is generally known that resource use differs according to whether markets are competitive or monopolistic and that neither of these market forms will necessarily produce the socially optimal scheme for resource exploitation. ${ }^{4}$ By evaluating resource allocation under different sets of political, social, and institutional constraints, economists can identify those extraction programs most consistent with national goals, as well as suggest the tradeoffs between attaining certain political and economic
objectives. Probably, however, economists have been most productive in specifying optimal models of resource use--optimal in the sense of maximizing certain efficiency criteria. Although efficiency is not the only dimension of resource valuation, ${ }^{5}$ these models are a useful reference for constructing new effective management programs and evaluating those already in existence.

The extensive literature on the economics of optimal resource management is separated naturally into two divisions according to whether the resource is nonrenewable or renewable. The first formal analysis of nonrenewable resources, sometimes referred to as the Theory of the Mine, was presented by Hotelling (1931). Following Hotelling's analysis, Gordon (1967), Herfindahl (1967), Scott (1967), and Cummings (1969) have considered optimal resource extraction from the point of view of an individual firm. Recently, public interest centering on the effects of resource shortages on economic growth has prompted a new body of literature, particularly the works by Anderson (1972), Vousden (1973), Smith (1974), and Schmalensee et al. (1975). These authors are concerned with the problems of resource constrained optimal economic growth. Special attention is paid to analyzing the time horizon over which resource exhaustion occurs and the resulting pattern of extraction.

Unlike the Theory of the Mine, the potential for resource growth is important in the analysis of renewable resources. Primary
emphasis in the literature has been on the optimal use of resources
 by Gordon (1954), Scott (1955), Turvey (1964), Christy and Scott (1965), Crutchfield and Pontecorvo (1969), and Smith (1969) consider static models in which the rate of fish landings is chosen to maximize the net economic yield from the fishery while keeping the stock in biological equilibrium. ${ }^{6}$ Although these studies abstract from the dynamic characteristics of the fishery, they are important in emphasizing and formalizing the idea that competitive harvesting of a common property resource is inefficient unless it is owned or managed by one party.

In the current stage of economic modelling of the fishery Crutchfield and Zellner (1962), Plourde (1970) and (1971), Quirk and Smith (1970), Clark (1973), Brown (1974), Neher (1974), and Spence (1973) extend previous analyses by providing an explicit dynamic treatment of optimal resource use. .These authors employ optimal control theory ${ }^{7}$ in choosing the rate of fish landings in each time period to maximize the sum of discounted rents, subject to changes in the population caused by natural growth and mortality, and predation by man. Some interesting suggestions for coping with the stockflow dynamics of the fishery have evolved from these studies.

## B. PURPOSE OF THE STUDY

The aforementioned analyses assume a deterministic world in which all current and future demands, prices, and costs are known, in which the current reserve of the resource can be observed and measured exactly, in which environmental factors affecting the growth or deterioration of the resource are either unimportant or are perfectly predictable, and in which the entire time path of reserves and extraction rates can be calculated with certainty for a given program of resource management. In reality, of course, there is not only uncertainty regarding current and future resource prices, as well as the effects of environmental changes on resource stocks, but also there is often uncertainty about the existing supply of the resource available for extraction. Deterministic models have dominated the literature thus far not because the elements of uncertainty are unimportant or because they have gone unrecognized, ${ }^{8}$ but because existing stochastic models are either not operable or too difficult to work with. Unfortunately, by neglecting the effects of uncertainty in analyzing and advocating optimal programs for resource management, economists have done little to inspire public confidence in current resource conservation efforts. The purpose of this study is to mitigate some of the deficiencies in the literature by introducing and analyzing a general model of resource management that readily incorporates various aspects of uncertainty.
B. 1. Description of the Model

In our model the resource, whether it be a fishery, a mineral deposit, an oil reserve, etc., is controlled by a hypothetical social manager. It is assumed that the manager chooses the rate of extraction in each period to maximize the expected social utility of the stream of economic rents from the resource. Although we are interested in socially optimal behavior, the model is also appropriate for describing resource use for different market and allocation systems. Elements of uncertainty are accommodated in the analysis in the following forms: (1) Uncertainties may exist about the current size of the resource either because of difficulties in observing and measuring the resource stock, as in the case of fisheries, or because of the possibilities of finding new reserves through exploration, as in the case of minerals and oil; (2) the market price of the resource may vary due to fluctuations in consumer demand and the availability of substitutes. The costs of extracting the resource may also be random; (3) unpredictable changes in th environment may perturb the natural rate of growth or deterioration of the resource as well as the effective rate of depletion by man. For example, variations in the weather and the temperature of the water may have an effect on the natural growth rate of a fish population and the rate at which the fish are caught.

The optimizing technique for this analysis, developed by Howard (1960) is an application of dynamic programming to a discrete

Markov process model. The dynamic structure of the resource extraction program is described in terms of a simple, one period Markov process model with a finite number of states. In its most basic scalor form a state is simply a possible size of the resource stock; in more complicated vector forms a state might contain information on the size of the stock, the season of the year, prevailing economic and political conditions, etc. During a particular time interval, the program is in a certain state if it is described by the value of all the variables that define the state. A state transition occurs when its describing variables change from the values specified for one state to those specified for another. Movements from one state to another, described by the transition probabilities are random as a result of variations in environmental and socioeconomic conditions affecting the natural growth and depletion of the resource. Thus, the transition probabilities in the simplest scalor form depend on the growth of the stock (which is important in the case of renewable resources), and on the rate of extraction. With the states and transition probabilities fully specified, the manager regulates the use of the resource over time to maximize the expected social utility of the stream of future rents from the reserve. It is assumed that there are a finite number of possible extraction rates in each period for the manager to select from. This type of formulation results in a sequential optimization problem that is solved by dynamic programming.

The most important feature of this programming approach is the ease with which elements of uncertainty are incorporated into the model. Stochastic elements are not accommodated at all in the control theory models used in the literature. In addition, there are no restrictions on the form of the criterion function and the equation of motion of the state variables imposed by the Markov model. This enables one to analyze cases where the functions are not concave, a condition usually needed in control theory problems, ${ }^{9}$ and where the criterion function is not quadratic, a necessary condition to invoke "certainty equivalence" theory. ${ }^{10}$ For example, cases of scale economies that are usually ignored because they give rise to nonconcave criterion functions are easily included in the Markov decision model.

Another attractive feature of our model is the relative ease of solution by computer. In contrast to control theory analyses which generally only provide conditions for optimality, the programming method yields complete solutions to the allocation problem. In addition, explicit calculations of the expected present value for different allocations strategies are obtained.

## B.2. Analysis of the Model

Once formulated and tested, the model is used to study the effects of uncertainty on optimal decision rules for the allocation of natural resources over time. In particular, we focus on the following questions: fiow do optimal programs for allocating resources in
a deterministic environment compare with optimal programs derived under stochastic conditions? Do deterministic decision rules serve as a good approximation for optimal stochastic programs? How do different attitudes for risk bearing with regards to variations in resource rents effect optimal decision rules? Is the usual practice of representing the "riskiness" of a project in terms of the social discount rate appropriate for use in stochastic sequential maximization problems such as ours? What is the affect of increased uncertainty about consumer demand, and resource growth and depletion rates on optimal programs?

Following a general specification of the Markov model in Chapter II, an analysis of the questions posed above are applied to a study of a specific renewable resource, the Eastern Pacific yellowfin tuna fishery. Because the resource can replenish itself, models of renewable resources generally are more complex than models of nonrenewable resources. Thus, although the study pertains to fisheries, our model is easily modified for analyzing nonrenewable resource problems.

Apart from demonstrating the use of our model, the purpose of this study is to generate some practical policy reconimendations for the management of the Eastern Pacific yellowfin tuna fishery. The fishery is not only important as a food sorrce, but it also provides incomes for fishermen from the United States, Canada, Japan, and several South and Central American countries. The data on the
fishery needed for implementing the model will be matched against that which is currently available. Where data are lacking, a sensitivity analysis on various parameters is undertaken to identify areas requiring further empirical research.

## C. PLAN OF THE STUDY

In Chapter II the optimal allocation of a natural resource is described in general form in terms of a finite state and action Markovian decision process. First, changes in the resource stock as a function of natural growth (for replenishable resources) and depletion are specified. Various maximization criteria are examined, and the resource allocation problem is formulated as a discrete dynamic programming problem. It is seen that a solution to this problem exists and Howard's algorithm for finding it is presented. Certain limitations of the model are discussed and suggestions for extensions are made.

In Chapter III the economic and biological characteristics of the Eastern Pacific yellowfin tuna fishery are modelled in terms of a discrete Markov system. Assuming deterministic conditions, the allocation problem for this fishery is formulated and solved by the programming procedures introduced in Chapter II. The general form of the optimal strategy is examined and summarized for use later on in comparison with optimal stochastic decision rules. Solutions to this problem are also derived by optimal control methods as an
accuracy check for the Markov decision model.
A probabilistic model of the tuna fishery allowing for variations in consumer demand and uncertainty about population growth and depletion rates is presented and solved in Chapter IV. The resulting optimal stochastic strategies are compared with deterministic decision ruies. We find that in certain cases policies for resource consumption derived for deterministic conditions are also appropriate in a stochastic environment. The effects of increasing uncertainty in consumer demand and population growth rates on optimal allocation programs are assessed. Different attitudes for risk bearing are analyzed for their impact on optimal programs, and the prospects for being able to represent risk via the discount rate are examined. Finally, areas requiring additional empirical research are identified.

## Footnotes

${ }^{1}$ See Meadows et al. (1972) and Forrester (1971).
${ }^{2}$ For a discussion of the problems involved in managing internationally owned resources, see Crutchfield (1972).
${ }^{3}$ For a discussion of the economic implications of the formation of the Organization of Petroleum Exporting Countries, see Schmalensee et al. (1975).
${ }^{4}$ On this point see Burt and Cummings (1969), Hotelling (1931), Lewis (1974), Smith (1968), and Solow (1974).
${ }^{5}$ Other goals that might influence a country's use of the resource would be reducing unemployment and attaining a favorable balance of payments situation.
${ }^{6}$ This is analogous to finding the "Golden Rule" in standard models of capital accumulation.

7 In this context the term "optimal control theory" includes both the Pontryagin Maximum Principle, and the Calculus of Variations optimization methods.
$8_{\text {For example, see the discussion by } \operatorname{Scott}(1967, ~ p .26) . ~}^{\text {. }}$
${ }^{9}$ See Kamien and Schwartz (1971).
10
See Holt et al. (1960).

## Chapter II

## MARKOV DECISION PROCESS MODEL

In this chapter the problem of allocating a renewable or nonrenewable natural resource over time to maximize net economic returns is described in general terms in the form of a finite state and action Markovian decision process. The basic model we introduce allows for the important effects of environmental variation on the natural growth and depletion of the resource and the effect of random changes in socio-economic conditions on market prices and the costs of extraction. This model is subsequently used in Chapters III and IV to analyze the Eastern Pacific yellowfin tuna fishery.

The plan of the chapter is as follows: Changes in the resource stock caused by the natural growth or deterioration of the stock as well as the depletion due to man are specified in section A. Various criteria for allocating the resource over time, that depend on the social attitudes toward risk, are set forth in section $B$. In section $C$ a discrete dynamic programming approach is formulated for analyzing the resource allocation problem. It is seen that a solution exists and Howard's aigorithm ${ }^{1}$ for finding it is presented. In section $D$ the basic model is modified and extended in various ways to allow for (1) seasonal variation in market prices, extraction costs and resource growth (in the case of fisheries), (2) a non stationary stochastic process caused by temporal changes in certain parameter distributions, (3) the possibility of joint resource management, as in the case of two interacting fish populations, or two price or cost

## 1

See footnotes on pages 47-48.
related mineral reserves, and (4) errors in observing and measuring the actual size of the resource pool available for exploitation.

## A. GROW TH CHARACTERISTICS OF THE RESOURCE

In each time interval, the rate of change in the resource reserve depends on the rate of natural growth of the stock and the rate of depletion by man. Thus,

$$
\begin{equation*}
X_{t+1}=X_{t}+f\left(X_{t}\right)-L_{t} \tag{1}
\end{equation*}
$$

where: $\quad X_{t}=$ resource stock at time $t$

$$
L_{t}=\text { rate of extraction at time } t
$$

$f\left(X_{t}\right)=$ a function that measures changes in the stock due to $r$ diural growth or deterioration of the resource.

Equation (1) states that the available stock at time $t+1$ equals the stock at time $t$ enhanced by the rate of growth of the resource during that period minus the rate of extraction. The model is easily modified for the case of petroleum reserves where very large values of $L_{t}$ result in more than proportional drops in recoverable resources due to water seepage effects. ${ }^{2}$

Normally, we assume $f\left(X_{t}\right)=0$ for nonrenewable resources, although nearly all minerals, natural gases and oils, generally conceived of being fixed in supply, are replenishable over a time span of millions of years. On the other hand, some resources such as
uranium, depreciate or deteriorate over time so that $f\left(X_{t}\right)<0$. For renewable resources, $f\left(X_{t}\right)$ is the natural growth function indicating the rate of resource replenishment. In Chapter III the form of $f\left(X_{t}\right)$ is specified in greater detail as it pertains to a fishery population.

Defining $E_{t}$ as a composite input variable representing the capital and labor used in resource extraction at time $t$,

$$
\begin{equation*}
L_{t}=g\left(X_{t}, E_{t}\right) \tag{2}
\end{equation*}
$$

where $g()$ is a production function for extraction. The stock $X_{t}$ enters production essentially as a capital input, which when combined with the variable input $E_{t}$ yields a flow of resource consumption. For most resources we assume

$$
\begin{equation*}
\frac{\partial g}{\partial X} \geq 0 ; \frac{\partial}{\partial X} \frac{\partial g}{\partial E} \geq 0 \tag{3}
\end{equation*}
$$

reflecting the increased difficulty of harvesting or extracting the resource as it becomes more scarce. Obvious examples of this occur in fisheries where catch rates decline (for given $E_{t}$ )with smaller populations since the fish are harder to locate, and in mining where extraction decreases because lower grade ores are encountered as the resource is depleted. Of course, it is possible that the size of the resource reserve has no marginal effect on production in which case $L_{t}$ is a function of $E_{t}$ only, ${ }^{3}$ although we shall retain the more general specification of $L_{t}$ in equation (2).

Allowing for uncertainty in resource growth and depletion rates, and substituting for $L_{t}$ facm equation (2), we obtain

$$
\begin{equation*}
x_{t+1}=x_{t}+\eta_{1 t} f\left(x_{t}\right)-\eta_{2 t} g\left(x_{t}, E_{t}\right) \tag{4}
\end{equation*}
$$

where $\eta_{1 t}$ and $\eta_{2 t}$ are random variables. To illustrate the meaning of equation (4), in the context of ocean fisheries, $\eta_{\text {lt }}$ might represent random changes in the natural rate of growth and $\eta_{2 t}$ might measure variations in the depletion rate of the resource caused by random changes in the water temperature and weather conditions. In terms of mineral and petroleum resources, $\eta_{2 t}$ might describe the effect on extraction rates for varying mining and drilling conditions. For the present, we assume the distributions for $\eta_{\text {It }}$ and $\eta_{2 t}$ are stationary through time with expected values, $\mathcal{C}\left(\eta_{1 t}\right)=$ $c\left(\eta_{2 t}\right)=1$. Thus the expected value of $X_{t+1}$ in equation (4) is equal to its value under deterministic conditions given by equation (1). In section $D$ our model is modified to allow for temporal changes in the distributions for $\eta_{1 t}$ and $\eta_{2 t}$, perhaps as a result of seasonal variation in environmental conditions. Naturally, random disturbances in the resource reserve need not enter multiplicatively as in equation (4); however we retain this form for use later on in analyzing the fishery.

For a given initial stock size, equation (4) specifies which programs of resource extraction are feasible. In turn, the next section examines several criteria for selecting the best of these feasible programs.

## B. CRITERIA FOR OPTIMAL RESOURCE ALLOCATION

In the present literature on resource economics, a certain or riskless world is assumed in which all physical and economic processes are deterministic. According to this analysis, the criteria for optimal resource allocation is to maximize the present value, $V$, of the certain stream of economic rents from the resource over time, represented by

$$
\begin{equation*}
V=\sum_{t=0}^{\infty} B^{t} R_{t} ; \quad B=\frac{1}{1+0} \tag{5}
\end{equation*}
$$

The economic rent, ${ }^{4} R(t)$, is taken as a measure of the net benefit to society from the consumption of the resource. The distribution of the rent is not considered, presumably because a larger total rent can in principle be redistributed so as to make everyone better off than in a situation with a smaller aggregate rent. ${ }^{5}$ The stream of rents is discounted at the rate of $\rho$, which is the social rate of time discount. Assuming efficiency throughout the economy, $\rho$ is the rate of return on private riskless investments.

In reality, however, our physical, political, and economic environment is dominated by risk and uncertainty. There is uncertainty about locating the resource, an important consideration for petroleum reserves and ocean fisheries, and about the costs of extraction. Sudden social and economic changes throughout the world
cause fluctuations in the market price of natural resources as we have witnessed recently with respect to the price of gold and petroleum products. This means that the economic rent from the resource use is a random variable whose distribution is determined by a combination of stochastic social, economic, and physical processes. Under these circumstances, the objective of maximizing present value in equation (5) is not meaningful, and the question arises as to what is an acceptable criterion for the allocation of natural resources over time. This is not a trivial consideration since the welfare of many countries and many individuals depends directly on the receipts from natural resource production. Consideration of this question is best handled by first examining possible attitudes toward risk, and second by representing these attitudes in some analytical or functional form.

## B.1. Attitudes Toward Risk

The net social benefit from the use of a natural resource will depend on society's risk attitude toward variability in economic yields. The approach taken in this paper is that the risk attitude of individuals is important in determining risk preferences of society. It is generally accepted that individuals are not indifferent to risk and that investors must be paid a "risk premium" yield above the expected rate of return as compensation for the costs of risk-bearing. For example in marine fisheries, risk averse behavior serves to explain the prevalence of share contracting ${ }^{6}$ between boat owners and fishermen.

Following the arguments of Cheung (1969), Reid (1974), Stiglitz (1974), and Sutinen (1973) share contracts may be regarded as a device for risk sharing--in this case the variance of the fishery yield is distributed among the contracting parties. Assuming risk aversion, it can be shown that a share contract will be mutually preferred by the boat owner and fishermen. ${ }^{7}$

Accepting the fact that private risk aversion exists, the major issue with respect to evaluating public projects in general and the management of a resource in particular is if the private cost of riskbearing represents a social cost as well. This question has provoked much controversy in the cost-benefit literature on government investment decisions. ${ }^{8}$ The major support for the viewpoint that private risk aversion is socially unimportant stems from the pooling argument due to Samuelson (1964) and Vickrey (1964) and the risk spreading argument due to Arrow and Lind (1970). The pooling argument asserts that the government invests in a large number of diverse projects and is able to pool risks to a much greater extent than the private sector. Although the outcome of any particular investment may be uncertain, the entire investment program taken as a whole is virtually riskless. The risk spreading argument applies to private as well as public investment. Arrow and Lind show that when the risks associated with any project are distributed among a large number of people so that the size of the share born by each individual is a very small component of his income, the total of the costs of risk-bearing
is negligible. The inverse of this result implies that even with public projects, significant social costs of risk-bearing may exist when some benefits and costs of sizeable magnitude accrue to individuals, so that these individuals incur the attendant costs of risk-bearing. With regard to the management of a natural resource the actual distribution of risks and the attendant cost of risk-bearing will depend largely on the structure of the management program. For example, assume that all rents are captured in profits ${ }^{9}$ and suppose that the resource is managed by a government control authority which employs private producers at a fixed wage to extract the resource with the resulting profits distributed equally among the taxpayers. Although total profits may vary considerably, the size of the share going to an individual taxpayer is presumably a negligible component of his income and the total cost of risk-bearing can be ignored. Suppose instead that the rents are redistributed to producers in the form of lump sum transfers. Under these circumstances, resource rents of a large magnitude may accrue to a small number of individuals and the social cost of risk-bearing cannot be ignored.

From this section we see that the social attitude towards uncertainty will depend on the structure of 'he management program and the fashion in which the rents from the resource are distributed.

Economists have proposed numerous systems for resource management, including the imposition of taxes and subsidies on resource use, the sale of extraction licenses, and placing direct quotas and
limitations on resource production. Each of these schemes results in a different distribution of the economic rents. A general theory of natural resource allocation under uncertainty should allow for averse and neutral attitudes towards risk, depending on the institutional structure of the management program. The next section demonstrates how different attitudes towards risk can be represented in an analytical form that serves as a criterion for maximizing the net social benefit from resource exploitation.

## B.2. Maximization Criteria for Different Attitudes Towards Risk

The approach here is to describe the resource allocation problem in terms of the classic Von Neumann-Morgenstern theory of individual decision making under uncertainty. It is assumed that the manager utilizes the resource through time so as to obtain the socially optimal sequence of economic rents $R$.

$$
R=\left(R_{1}, R_{2},,,,,,, R_{t}, \ldots,,,\right)
$$

where $R_{t}$ is the economic rent from the resource at time $t$. Due to various physical and economic uncertainties, the rent sequence is random. Consequently, the manager must select the best of the available probability distributions for $R$ which are called random prospects. If we assume that the manager's behavior in solving this problem conforms to the Von Neumann-Morgenstern axioms, ${ }^{10}$ then it can be inferred that the preference ordering for various random prospects can be represented by a utility function

$$
U(R)=U\left(R_{1}, R_{2},, \ldots,,, R_{t},,,,,,,\right)
$$

and that the best prospect is found by maximizing the expected value of utility.

Assuming that $U(R)$ is an additive separable utility function, and that time preference enters in the form of a constant multiplicative discount factor, the certainty equivalence criterion, C.E., can be written as

$$
C . E .=\sum_{t=0} B^{t} z\left[U\left(R_{t}\right)\right] ; \quad B=\frac{1}{1+0}
$$

where $\rho$ is the riskless interest rate and $\varepsilon\left[U\left(R_{t}\right)\right]$ is the certainty equivalent of the possible economic rents from the resource at time $t$. The social attitude towards risk in resource rents is represented by the form of $U\left(R_{t}\right)$. Strict concavity in the utility function implies risk aversion, while risk neutrality occurs if $U\left(R_{t}\right)$ is a linear function. For purposes of comparison, different forms of the utility function may be used to analyze the optimal resource allocation. The choice of the particular function is based on its risk characteristic in terms of the measures of absolute and relative risk aversion developed by Arrow (1965) and Pratt (1964).

In taking this approach to resource management, we abstract from several issues which should be mentioned. First, the problems associated with "group decision making" are submerged behind our assumption of a single resource manager who makes allocation
decisions for society. Somehow the manager is able to sort out and reconcile different individual objectives and preferences for utilizing the resource in a consistent and equitable manner. Second, in maximizing the expected utility of the stream of rents from the resource we abstract from other possible goals of economic policy such as attaining high employment, acquiring a favorable balance of payments position, etc. While these are important issues pertaining to resource management, a proper treatment of these problems is beyond the scope of this study.

## C. DYNAMIC PROGRAMMING FORMULATION

In section $A$ changes in the resource stock caused by natural growth and depletion have been examined. The appropriate criterion for evaluating resource use has been analyzed in section B. What remains then is to develop a decision process for finding the optimal allocation of resources over time.

Assume the resource is described by a finite number of states, each state corresponding to a certain stock size. In each time period, $t$, the rent from the resource will be a function of the stock at that time, $X_{t}$, and the amount of inputs, $E_{t}$, which we shall call effort, used in the extraction process. The manager seeks to

$$
\begin{equation*}
\underset{E_{t}}{\operatorname{maximize}} \sum_{t=0}^{N} B^{t} \varepsilon\left[U\left(R\left(E_{t}, X_{t}\right)\right)\right] \tag{7}
\end{equation*}
$$

Suppose the resource is in state $i$ with $N$ time periods left in the planning horizon. The manager selects an effort allocation policy, $d$, from a finite number of possibilities. A policy is a rule for choosing the amount of effort used for extraction in each time period. $V_{i}^{d}(N)$ is the expected social value from the resource obtained with policy $d$ given that the resource starts in state i. It is defined by

$$
\begin{equation*}
v_{i}^{d}(N)=\varepsilon\left[\sum_{t=0}^{N} B^{t} U\left(R\left(E_{t}^{d}, X_{t}^{d}\right)\right) / X_{0}=X_{i}\right] \tag{8}
\end{equation*}
$$

It is possible to rewrite equation (8) in recursive form such that

$$
\begin{equation*}
v_{i}^{d}(N)=q_{i}^{d}+B \sum_{j} p_{i, j}^{d} v_{j}^{d}(N-1) \tag{9}
\end{equation*}
$$

where $p_{i, j}^{d}$ is the transition probability that the stock moves from state $i$ to state $j$ for policy $d$. The immediate expected social return from the resource is $q_{i}^{d}=\mathcal{E} U\left(R\left(E^{d}, X_{i}^{d}\right)\right.$ ). Equation (9) means that the expected social value, $\mathrm{V}_{\mathrm{i}}^{\mathrm{d}}(\mathrm{N})$, is equal to the immediate expected retuin $q_{i}^{d}$ plus the sum of the discounted values of being in state j with $\mathrm{N}-1$ periods left, weighted by the probability that the resource will occupy state $j$ in the next time period. If there are $S+1$ possible states for the resource, writing equation (9) in matrix form yields

$$
\begin{equation*}
V^{d}(N)=Q^{d}+B P^{d} \cdot V^{d}(N-1) \tag{10}
\end{equation*}
$$

where:
$V^{d}(N)$ is an $S+1$ dimensional column vector of the $V_{i}^{d}(N)^{\prime} s$ $Q^{d}$ is an $S+1$ dimensional column vector of the $q_{i}^{d / s}$ $P^{d} \quad$ is the Markov transition matrix corresponding to policy $d ; P^{d}=\left[p_{i, j}^{d}\right] ; i, j=0,1,2,3, \ldots, S$.

## C. 1. Steady State Properties, Existence of Solution, and

## Solution Algorithm

We want to consider a planning horizon of indefinite duration for the resource. Consider equation (10) as the number of periods in the planning horizon becomes large. Assume a particular policy, d, has been selected so that a given Markov process is specified.

Dropping the superscript, equation (10) becomes

$$
\begin{equation*}
V(N)=Q+B P V(N-1) \tag{11}
\end{equation*}
$$

Writing $\mathrm{V}(1), \mathrm{V}(2), \mathrm{V}(3) \ldots$ explicitly

$$
\begin{align*}
& V(1)=Q+B P V(0) \\
& V(2)=Q+B P Q+B^{2} P^{2} V(0)  \tag{12}\\
& V(3)=Q+B P Q+B^{2} P^{2} Q+B^{3} P^{3} V(0)
\end{align*}
$$

The general form of these equations is

$$
\begin{equation*}
V(N)=\left[\sum_{j=0}^{N-1}(B P)^{j}\right] Q+B^{\left.N_{P} N_{V(0)}\right)} \tag{13}
\end{equation*}
$$

Since $0 \leq B<1$,

$$
\begin{equation*}
\lim _{N \rightarrow \infty} V(N)=\sum_{j=0}^{\infty}(B P)^{j} Q \tag{14}
\end{equation*}
$$

because $P$ is a stochastic matrix and thus has eigenvalues equal to or less than one. The matrix BP has eigenvalues strictly less than one because $B<1$. Consequently,

$$
\begin{equation*}
\lim _{N \rightarrow \infty} V(N) \equiv V=(I-B P)^{-1} Q \tag{15}
\end{equation*}
$$

V is the vector of expected present values because each of its elements, $V_{i}$, is the present value of an infinite number of future expected social returns discounted by B.

Let $\mathrm{V}^{*}=\max _{\mathrm{d}} \mathrm{V}^{\mathrm{d}}$. A policy, $\mathrm{d}^{*}$, is optimal if $\mathrm{V}^{\mathrm{d}^{*}}=\mathrm{V}^{*}$.
In terms of this formulation, the allocation problem for the resource manager is solved by finding the optimal policy assuming it exists. Ross (1969, pp. 119-24) has shown that in general it is sufficient for an optimal policy to exist if the number of states and policies are finite and $q_{i}^{d}$ is bounded for all $i$ and $d$. For this problem $q_{i}^{d}$ is bounded, as is demonstrated in a later section, and therefore an optimal policy exists. Ross also shows that the optimal policy is stationary, meaning that it is nonrandom and that the action it chooses depends only on the state of the process.

Howard (1960) develops an iterative scheme for finding the optimal policy. ${ }^{11}$ The algorithm consists of calculating the present value vector $V$ in equation (15) for a given policy, $d$, and then
determining a better policy if one exists by using a policy improvement routine. Once the improved policy is determined, its present value vector is calculated and then it too may be improved. Since there are only a finite number of policies, the improvement routine will eventually converge to the optimal policy. A complete description of the algorithm is presented in Appendix II.

Now let us consider the actual specification of the states, policies, social returns, and transition probabilities that comprise this model of the fishery.

## C.2. Specification of States

The size of the stock (measured in some physical units depending on the resource) determines the state that the system occupies. There are $S+1$ possible states, $X_{i}$, for $i=0,1,2, \ldots, S$. $X_{0}$ corresponds to the minimum stock of 0 , and $X_{S}$ represents the largest possible resource size, assumed to be the maximum sustainable population without predation by man in the case of a fishery, or the initial recoverable stock for a nonrenewable resource. The rest of the states occur at regular intervals between $X_{0}$ and $X_{S}$ (see Figure (I-1)


Figure II-1.. Resource States

By increasing $S$ we can represent the stock with greater precision. However, it becomes more expensive to solve the allocation problem as the number of states increase. Consequently, the choice of $S$ involves a tradeoff between accuracy and computational costs.

## C. 3. Specification of Policies and Expected Social Returns

During each time period, the manager selects an effort allocation, $E_{i}^{m}$, from a finite number of possibilities. The " $m$ " refers to the rate of effort and the "i"represents the state occupied by the resource. The possible allocations for each state $i$ are represented by the array $\left(E_{i}^{1}, E_{i}^{2}, \ldots, E_{i}^{m}, \ldots, E_{i}^{M_{i}}\right.$ ) where $E_{i}^{m}<E_{i}^{m+1}$. The maximum rate of effort for state $i$ is $E_{i} M_{i}$. It is equal to or less than the rate which maximizes the expected immediate returns from the resource. ${ }^{12}$

A policy is a rule for selecting an effort allocation in each state. To simplify the notation, assume policy $d$ selects the "m th" effort rate in each state. Thus the allocations corresponding to policy d are represented by the array ( $\mathrm{E}_{0}^{\mathrm{m}}, \mathrm{E}_{1}^{\mathrm{m}}, \mathrm{E}_{2}^{\mathrm{m}}, \ldots, \mathrm{E}_{\mathrm{S}}^{\mathrm{m}}$ ). The expected one period return from the resource in state $i$ using policy $d$ is

$$
\begin{equation*}
q_{i}^{d}=\varepsilon U\left(R\left(E_{i}^{m}, X_{i}\right)\right. \tag{16}
\end{equation*}
$$

where $U$ is the social utility function and $R$ is given by

$$
\begin{equation*}
R\left(E_{i}^{m}, X_{i}\right)=G\left(\eta_{2} g\left(X_{i}, E_{i}^{m}\right), \gamma_{1}\right)-C\left(E_{i}^{m}, \gamma_{2}\right) \tag{17}
\end{equation*}
$$

The function $G()$ is the total revenue plus consumer surplus generated by the resource extracted, $\eta_{2} g\left(X_{i}, E_{i}^{m}\right)$, and $C()$ is the total cost of effort. The random shift parameters, $\gamma_{1}$ and $\gamma_{2}$ appearing in $G()$ and $C()$ capture the effects of random changes in economic and social conditions on revenues and costs. For the case where costs and revenues are nonrandom but there is variation in the rate of extraction

$$
\begin{equation*}
q_{i}^{d}=\int_{z_{1}}^{z_{2}} U\left[G\left(\eta_{2} g\left(X_{i}, E_{i}^{m}\right)-C\left(E_{i}^{m}\right)\right] h_{2}\left(\eta_{2}\right) d \eta_{2}\right. \tag{18}
\end{equation*}
$$

where $\eta_{2}$ varies between $z_{1}$ and $z_{2}$. For situations where $\eta_{2}$ is a constant equal to one, but costs and revenues are random

$$
\begin{equation*}
q_{i}^{d}=\int_{x_{1}}^{x_{2}} \int_{1}^{w} U\left[G\left(g\left(X_{i}, E_{i}^{m}\right), \gamma_{1}\right)-C\left(E_{i}^{m}, \gamma_{2}\right)\right] \ell\left(\gamma_{1}, \gamma_{2}\right) d \gamma_{1} d \gamma_{2} \tag{19}
\end{equation*}
$$

where $\ell\left(\gamma_{1}, \gamma_{2}\right)$ is the joint probability density function for $\gamma_{1}$ and $\gamma_{2}$ and $\gamma_{1}$ varies between $x_{2}$ and $x_{1}$ and $\gamma_{2}$ varies between $w_{2}$ and $w_{1}$. The array of expected social returns corresponding to policy $d$ are represented by $\left(q_{0}^{d}, q_{1}^{d}, q_{2}^{d}, \ldots, q_{S}^{d}\right)$. Recall that $q_{i}^{d}$ must be bounded for the existence of an optimal policy in equation (15). This condition is satisfied by making the reasonable assumptions that $G$ and $C$ are bounded for finite $E_{i}^{m}$, and $U$ is bounded for all finite $R$.

## C.4. Specification of Transition Probabilities

This model is a "simple" or "one period" Markov process. The probability of making a transition to each state of the process depends on the state presently occupied and the policy $d$. The transitions occur at regular discrete time intervals. For convenience, define the time units so that the interval between transitions equals one. Then according to equation (4)

$$
\begin{equation*}
X_{t+1}=X_{t}+\eta_{1} f\left(X_{t}\right)-\eta_{2} g\left(X_{t}, E_{t}\right) \tag{20}
\end{equation*}
$$

For convenience we have dropped the time subscript on $\eta_{1}$ and $\eta_{2}$. Suppose at time $t, X_{t}=X_{i}$, and policy $d$ is being utilized. Then

$$
\begin{equation*}
X_{t+1}=X_{i}+\eta_{1} f\left(X_{i}\right)-\eta_{2} g\left(X_{i}, E_{i}^{m}\right) \tag{21}
\end{equation*}
$$

By knowing the probability density functions for $\eta_{1}$ and $\eta_{2}$ is is possible to calculate the transition probabilities $p_{i, j} d$ for all $i$ and $j$, as we explain below.

## a. Deterministic Case

The simplest case occurs when $\eta_{1}$ and $\eta_{2}$ are degenerate random variables with a mean equal to one. Then

$$
\begin{equation*}
X_{t+1}=X_{i}+f\left(X_{i}\right)-g\left(X_{i}, E_{i}^{m}\right) \tag{22}
\end{equation*}
$$

If $X_{t+1}=X_{j}$ then $p_{i, j}^{d}=1$ and $p_{i, n}^{d}=0$ for all $n \neq j$. Suppose instead that $X_{t+1}$ falls in between two states $X_{j}$ and $X_{j+1}$ such that $\mathrm{X}_{\mathrm{j}}<\mathrm{X}_{\mathrm{t}+1}<\mathrm{X}_{\mathrm{j}+1}$. Under these circumstances it is not clear how to
calculate the transition probabilities. This problem arises because we are trying to represent a continuous variable by specifying only a finite number of values for that variable.

For instance when $\mathrm{X}_{\mathrm{j}}<\mathrm{X}_{\mathrm{t}+1}<\mathrm{X}_{\mathrm{j}+1}$ I have adopted the following convention. The probabilities $p_{i, j}^{d}$ and $p_{i, j+1}^{d}$ are calculated by the relative distance that $\mathrm{X}_{\mathrm{t}+1}$ is from $\mathrm{X}_{\mathrm{j}}$ and $\mathrm{X}_{\mathrm{j}+1}$. In particular

$$
\begin{align*}
& p_{i, j}^{d}=\frac{x_{j+1}-x_{t+1}}{x_{j+1}-X_{j}} \\
& p_{i, j+1}^{d}=\frac{x_{t+1}-x_{j}}{X_{j+1}-X_{j}} ; p_{i, j}^{d}+p_{i, j+1}^{d}=1 \tag{23}
\end{align*}
$$

The reason for choosing this method is as follows: Suppose that for two different effort rates $E_{i}^{m}$ and $E_{i}^{m+1}$, the corresponding stock sizes in the following period shown in Figure II-2 and denoted by $X_{t+1}^{m}$ and $X_{t+1}^{m+1}$ both fall in the same interval between $X_{j+1}$ and $X_{j}$. Since $E_{i}^{m}<E_{i}^{m+1}$ by equation (22) $X_{t+1}^{m}>X_{t+1}^{m+1}$


Figure II-2. Bounding Off Procedure

Using the method I have suggested, the fact that $X_{t+1}^{m}>X_{t+1}^{m+1}$ is indicated by the values of the transition probabilities. The probabilities act as weights to indicate the size of the stock relative to the states $X_{j}$ and $X_{j+1}$. It is easy to verify that

$$
\begin{align*}
& x_{t+1}^{m}=p_{i, j}^{d} x_{j}+p_{i, j+1}^{d} x_{j+1} \\
& x_{t+1}^{m+1}=p_{i, j}^{d+1} x_{j}+p_{i, j+1}^{d+1} x_{j+1} \tag{24}
\end{align*}
$$

where the transition probabilities are calculated according to equation (23). In this way it is easy to accurately monitor changes in the stock size corresponding to different policies.

Now let us consider some more complicated cases.
b. Random Growth Rate Case

Assume that only $\eta_{1}$ is random with probability density function $h_{1}\left(\eta_{1}\right)$ and $\eta_{2}$ is equal to one. From equation (21) the conditional probability density function for $X_{t+1}$ given $t_{2}$ at $X_{t}=X_{i}$ for policy $d$, denoted by $H^{d}\left(X_{t+1} / X_{i}\right)$, can be derived for the following conditions ${ }^{14}$ :
(a) If $f\left(X_{i}\right) \neq 0$ then
$H^{d}\left(X_{t+1} / X_{i}\right)=h_{1}\left(\frac{X_{t+1}+g\left(X_{i}, E_{i}^{m}\right)-X_{i}}{f\left(X_{i}\right)}\right)\left|\frac{1}{f\left(X_{i}\right)}\right|$
(b) If $f\left(X_{i}\right)=0$ then the distribution for $X_{t+1}$ is degenerate and the transition probabilities are calculated according to equation (23).

## c. Random Depletion Rate Case

Assume that only $\eta_{2}$ is random with probability density function, $h_{2}\left(\eta_{2}\right)$ and $\eta_{1}$ is equal to one. By equation (20),
(a) If $g\left(X_{i}, E_{i}^{m}\right) \neq 0$ then
$H^{d}\left(X_{t+1} / X_{i}\right)=h_{2}\left(\frac{f\left(X_{i}\right)+X_{i}-X_{t+1}}{g\left(X_{i}, E_{i}^{m}\right)}\right)\left|\frac{1}{g\left(X_{i}, E_{i}^{m}\right)}\right|$
(b) If $g\left(X_{i}, E_{i}^{m}\right)=0$ then the distribution for $X_{t+1}$ is degenerate and the transition probabilities are calculated according to equation (23).
d. Random Depletion and Growth Rate Case $\left(\eta_{1}=\eta_{2}\right)$

Assume that variations in the environment have the same effect on the growth rate and depletion rate of the stock. For this situation equation (20) becomes

$$
\begin{equation*}
x_{t+1}=x_{t}+\eta_{1}\left[f\left(X_{i}\right)-g\left(x_{i}, E_{i}^{m}\right)\right] \tag{20}
\end{equation*}
$$

and therefore,
(a) If $f\left(X_{i}\right)-g\left(X_{i}, E_{i}^{m}\right) \neq 0$ then,

$$
\begin{equation*}
H^{d}\left(X_{t+1} / X_{i}\right)=h_{1}\left(\frac{X_{t+1}-X_{i}}{f\left(X_{i}\right)-g\left(X_{i}, E_{i}^{m}\right)}\right)\left|\frac{1}{f\left(X_{i}\right)-g\left(X_{i}, E_{i}^{m}\right)}\right| \tag{27}
\end{equation*}
$$

(b) If $f\left(X_{i}\right)-g\left(X_{i}, E_{i}^{m}\right)=0$ then the distribution for $X_{t+1}$ is degenerate and the transition probabilities are calculated according to equation (23).
e. $\frac{\text { Random Depletion and Growth Rate Case }}{\text { are independent })}\left(\eta_{1}\right.$ and $\eta_{2}$

Assume that $\eta_{1}$ and $\eta_{2}$ are completely independent of each other with probability functions $h_{1}\left(\eta_{1}\right)$ and $h_{2}\left(\eta_{2}\right)$.
(a) If $f\left(X_{i}\right) \neq 0$ then

$$
\begin{equation*}
H^{d}\left(X_{t+1} / X_{i}\right)=\int_{-\infty}^{\infty} h_{i}\left(\frac{\eta_{2} g\left(X_{i}, E_{i}^{m}\right)-X_{i}+X_{t+1}}{f\left(X_{i}\right)}\right)\left|\frac{1}{f\left(X_{i}\right)}\right| h_{2}\left(\eta_{2}\right) d \eta_{2} \tag{28}
\end{equation*}
$$

(b) If $f\left(X_{i}\right)=0$ then $H^{d}\left(X_{t+1} / X_{i}\right)$ is calculated according to the variable depletion rate case in section C.4.c.

Suppose the condition probability density function $H^{d}\left(X_{t+1} / X_{i}\right)$ is represented in Figure II-3. In general the distribution of $X_{t+1}$


Figure II-3. Conditional Distribution of $\mathrm{X}_{\mathrm{t}+1}$
will be continuous since $h_{1}\left(\eta_{1}\right)$ and $h_{2}\left(\eta_{2}\right)$ are continuous. To transform $H^{d}\left(X_{t+1} / X_{i}\right)$ into a discrete density function, we define $\Phi_{j}(X)$ to be the function that assigns transition probabilities to state j such that

$$
\begin{array}{ll}
\text { if } X_{j} \leq X \leq X_{j+1} & \text { if } X_{j-1} \leq X \leq X_{j} \\
\text { then } p\left(X_{j}\right)=\frac{X_{j+1}-X}{X_{j+1}-X_{j}} \quad \text { then } p\left(X_{j}\right)=\frac{X-X_{j-1}}{X_{j}-X_{j-1}}
\end{array}
$$

where $p\left(X_{j}\right)$ is the probability that the fishery occupies $X_{j}$. Using $H^{d}\left(X_{t+1} / X_{i}\right)$ and $\Phi_{j}(X)$, the transition probability $p_{i, j}^{d}$ is calculated as

$$
\begin{array}{r}
P_{i, j}^{d}=\int_{X_{j-1}}^{X} H^{d+1}\left(X_{t+1} / X_{i}\right) \Phi_{j}\left(X_{t+1}\right) d X_{t+1}  \tag{30}\\
\text { for } i, j=0,1,2, \ldots, S .
\end{array}
$$

In summary, the procedure for calculating the transition probabilities is:

1. If $\eta_{1}$ and $\eta_{2}$ are both constant, then the transition probabilities are calculated by equation (23).
2. If one or both of the variables, $\eta_{1}$ and $\eta_{2}$ are random then
a. Derive the continuous probability density function given by $H^{d}\left(X_{t+1} / X_{i}\right)$.
b. Use the function $\Phi_{j}(X)$ to transform $H^{d}\left(X_{t+1} / X_{i}\right)$ into a discrete density function with the probabilities given by equation (30).

## D. EXTENSIONS OF THE MARKOV MODEL

The basic model presented in sections A-C will be used to study the Eastern Pacific yellowfin tuna fishery in Chapters III and IV. In specifying the model we have deliberately abstracted from certain complexities in order to isolate the effects of uncertainty about prices, and growth and depletion rates on optimal resource use. Nevertheless, our analysis is easily extended to accommodate other factors that presumably have an impact on resource allocation. Possible modifications in the model to account for the effects of seasonal variation, of systematic parameter changes over time, of interdependence between different resources, and of problems with observing the resource stock, on optimal management programs are presented below.

## D. 1. Seasonal Variation ${ }^{15}$

The rate of growth for some resources, particularly the fishery, varies with the time of the year. Expected cost and revenues from the resource may also change with the season, as in the case of heating fuels. To account for these changes in our model each state could be characterized by the size of the stock as well as the season of the year. Then depending on the state, a particular growth equation for that season could be used to measure stock changes, and season specific cost and revenue functions could be employed to calculate profits.

## D.2. Systematic Changes in Parameters over Time

Two limiting assumptions of our model are that (1) all of the parameter distributions are known with certainty, and (2) these distributions do not change over time. The first assumption is necessary because there is no feedback mechanism in our model for modifying parameter distribution estimates if these estimates are in error. 16 The second assumption, however, is unduly restrictive. Our model is easily modified to accommodate periodic, but predictable, changes in parameter distributions.

To account for these changes, each state could be characterized by the size of the stock as well as a description of the parameter distributions assumed. To illustrate, suppose that price is the only random variable in the model and it is uniformly distributed around a mean $\overline{\mathrm{p}}$. Let us assume, that the mean price itself is variable, taking on values $\bar{p}_{j}$ for $\mathrm{j}=1,2,3, \ldots, \mathrm{~J}$ with $\overline{\mathrm{p}}_{\mathrm{j}}<\overline{\mathrm{p}}_{\mathrm{j}+1}$. Note that the entire distribution of prices shifts for a change in mean price. A typical state $S_{i, j}$ would be designated by a set of ordered pairs $\left(X_{i}, \bar{p}_{j}\right)$ for all $i$ and $j$.

Suppose prices are expected to rise steadily over time due to a growing world demand for fish products. This effect could be incorporated in our analysis by specifying that the probability of moving from an initial state to one with a higher mean price is greater than moving to one with a lower price. The more rapid the expected rise in price is, the greater the probabilities would become in the direction
of states with higher prices. In a similar fashion, by manipulating the transition probabilities, we could also capture the effects of decreasing and cyclical movements in prices. ${ }^{17}$ Of course variations in other parameters besides price could be handled in the same way.

## D.3. Interaction Between Different Resources ${ }^{18}$

In the marine ecosystem two or more populations may interact directly through the familiar predator-prey relationship, or more indirectly through the food chain. Besides this biological dependence, different species of fish may be economically related if they are good substitutes for each other in consumption. In the same fashion, the costs $\mathrm{c}_{\mathfrak{2}}$ extracting different minerals may be related since a variety of ores and metals may come from a single mine.

For simplicity suppose there are just two interrelated resources denoted by $X$ and $Z$. Each stock may assume one of a finite number of sizes $X_{i}$ for $i=0,1,2, \ldots, I$, and $Z_{j}$ for $j=0,1$, 2,..., J. Each state in our model could be described by the current sizes of $X$ and $Z$. Thus a typical state $S_{i, j}$ could be designated by a set of ordered pairs $\left(X_{i}, Z_{j}\right)$ for all $i$ and $j$.

Because of the mutual dependence between resources, a joint management program is advisable to achieve efficient resource use. Thus, in each time period optimal allocations of effort $E^{X}$ and $E^{z}$ for stocks $X$ and $Z$ respectively would be determined by the resource manager. For state $S_{i, j}$, the resulting payoff, $q_{i, j}$ would be

$$
\begin{equation*}
q_{i, j}\left(E^{x}, E^{z}\right)=z U\left(R\left(X_{i}, Z_{j}, E^{X}, E^{z}\right)\right) \tag{31}
\end{equation*}
$$

The right hand side of (31) is the expected utility of the rents from $X$ and $Z$ allowing for variation in revenues, costs, and/or depletion rates. Rents are equal to the joint revenue of the resources extracted from $X$ and $Z$, minus the total cost of effort $E^{X}$ and $E^{Z}$.

In the case of fisheries, if $X$ and $Z$ are biologically interrelated then the one period changes in population size,

$$
\begin{align*}
& X_{t+1}-X_{t}=f\left(X_{t}, Z_{t}, E^{z}\right)  \tag{32}\\
& Z_{t+1}-Z_{t}=g\left(X_{t}, Z_{t}, E^{z}\right) \tag{33}
\end{align*}
$$

would depend on the current populations $X_{t}$ and $Z_{t}$, and the amount of effort allocated for catching either X or Z . If, for example, $Z$ was a predator of $X$ then $\partial f / \partial Z_{t}<0$ and $\partial g / \partial X_{t}>0$. Assuming the functions $f$ and $g$ are known, calculation of the transition probabilities of moving from one state $S_{i, j}$ to another $S_{i, j},{ }^{\prime}$, would be straightforward.

## D.4. Nonobservable Resource Stocks

Situations frequently occur where the size of the recoverable resource can't be observed directly. For example, one cannot directly observe the population of a fishery, although indications of the stock size can be obtained by noting the relative abundance of the population encountered while fishing. We shall consider situations
for which a distribution of possible values for the resource stock can be derived in each period by a process described below, but that the exact size of the resource is not known.

Suppose initially, the distribution for $X_{0}$ is known, perhaps. by some sampling routine. Although, the true stock, $\bar{X}_{0}$, is uncertain, we can calculate $\bar{X}_{0}$ for subsequent use in determining future stock sizes after observing resource production in period 0. Assuming the random variable $\eta_{2}$ is an observable quantity, such as water temperature or weather conditions in the case of fisheries, then for a given $E_{0}, \bar{X}_{0}$ is uniquely determined by observing the extraction rate $\eta_{2} g\left(\bar{X}_{0}, E_{0}\right)$.

The resulting stock in period 1 is

$$
\begin{equation*}
X_{1}=\bar{X}_{0}+\eta_{1} f\left(\bar{X}_{0}\right)-\eta_{2} g\left(\bar{X}_{0}, E_{0}\right) \tag{34}
\end{equation*}
$$

Only if $\eta_{1}$ is observable can $X_{1}$ be calculated exactly. From this it follows that if the size of the resource is known in each period, as assumed in the basic model of sections A-C, we require that $\eta_{1}$ and $\eta_{2}$ be observed directly. If, however, $\eta_{1}$ is not observable, as we assume here, then one can only specify a distribution of possible values for $X_{1}$ from equation (34). In general, at any time $t$, we will only know the distribution of possible values for $X_{t}$ from previous knowledge of $X_{t-1}$. Then, with separate observations on $\eta_{2}$, and $\eta_{2} g\left(\bar{X}_{t}, E_{t}\right)$, (for given $E_{t}$ ) we can determine the true stock $\bar{X}_{t}$. A distribution of values for $X_{t+1}$ is then obtained by equation (35)

$$
\begin{equation*}
x_{t+1}=\bar{x}_{t}+\eta_{1} f\left(\bar{X}_{t}\right)-\eta_{2} g\left(\bar{X}_{t}, E_{t}\right) \tag{35}
\end{equation*}
$$

and the process continues.
To incorporate this process in terms of a Markov model the states of the system would represent the distribution of possible stock sizes. Rewriting equation (35) we obtain

$$
\begin{equation*}
X_{t+1}=\varepsilon\left(X_{t+1}\right)+\left(\eta_{1}-1\right) f\left(\bar{x}_{t}\right) \tag{36}
\end{equation*}
$$

where $\varepsilon\left(X_{t+1}\right)$ is the expected value of the stock at time $t+1$. Thus, a typical state $S_{i, j}$ could be designated by a set of ordered pairs $\left(X_{i}, V_{j}\right)$ where $X_{i}$ is the expected value of the stock, $\varepsilon\left(X_{t+1}\right)$ for $i=0,1,2, \ldots, I$ and $V_{j}$ is the distribution of the stock around its mean given by the second term on the right hand side of equation (36), for $j=0,1,2, \ldots, J$. Since the distribution of $\eta_{1}$ is fixed, the dis tribution $V_{j}$ is determined by $f\left(\bar{X}_{t}\right)$ which can assume one of a finite number of values. Assuming the distributions for $\eta_{1}$, and $\eta_{2}$ are known, calculation of the transition probabilities of moving from one state $S_{i, j}$ to another $S_{i}{ }^{\prime}, j$, would be straightforward.

For a given allocation of effort, E, the resulting payoff in state $S_{i, j}$ would be

$$
\begin{equation*}
q_{i, j}(E)=\int_{z_{1}}^{z_{2}} \varepsilon U\left(R\left(X_{i}+z, E\right) V_{j}(z) d z\right. \tag{37}
\end{equation*}
$$

where $\mathcal{E U}\left(R\left(X_{i}+z, E\right)\right)$ is the expected utility of rents for a given
stock size, $X_{i}+z$, assuming there is variation in revenues, costs, and/or extraction rates, and the distribution of stocks around $X_{i}$ ranges between $z_{1}$ and $z_{2}$.

## APPENDIX II

The Howard iteration scheme consists of calculating the present value vector, $V$, for a given policy; and then determining a better policy, if one exists by using a policy improvement routine. The entire method is shown schematically in the two boxes below.


For each cycle, the policy improvement routine is able to make policy improvements until the policies on two successive iterations are the same. At this point the optimal policy or (policies, there may be more than one) has been determined and the problem is completed. The proof that the iteration cycle converges on the optimal policy is presented in Howard (1960, p. 84).

## Footnotes

${ }^{1}$ See Howard (1960, p. 84).
${ }^{2}$ See Kuller and Cummings (1974).
${ }^{3}$ The assumption that resource depletion has no effect on extraction costs is usually introduced to simplify the analysis, but not because it is realistic. See Scott (1967, p. 27).
${ }^{4}$ The economic rent is defined to be the total revenue from the fish caught plus the producer's and consumer's surplus minus the total cost of fishing. Some care is needed in defining producer's surplus, see Mishan (1968).
${ }^{5}$ This is a partial equilibrium argument that assumes operations in the fishery do not significantly affect relative prices throughout the rest of the economy. Also, in principle it may be costly to redistribute income in which case it is not clear that maximizing the economic rent from the fishery is an optimal policy. For a discussion of this point see Bishop (1970).
${ }^{6}$ We are informed by Zoetewejj (1956) that share contracts between boat owners and fishermen predominate in marine fisheries.
${ }^{7}$ Contrary to this view that fishermen are risk averters, Gordon (1955, p. 132) states, "As those who know fishermen well have often testified, they are gamblers and incurably optimistic. " This contention is hardly convincing however, as it is only supported by Gordon's personal observation.

8
For a discussion of this question see Samuels on (1964) and Vickrey (1964), Hirshleifer (1966), Hirshleifer (1970), Baumol (1970) and Arrow and Lind (1970).
${ }^{9}$ In this case, the demand for the resource and the supply of $E_{t}$ are both perfectly elastic and consumer's and producer's surplus are zero.
${ }^{10}$ See Luce and Raiffa (1957, p. 29).
11
A linear programming approach, first suggested by Manne in "Linear Programming and Sequential Decisions," Management Science 6, No. 3, pp. 259-67 (1960), is also available for solving the Markov decision problem.

12 It will never be optimal to allocate effort in excess of that which maximizes the immediate returns from the resource.
${ }^{13}$ We are assuming that policy $d+1$ selects the $m+1{ }^{\text {th }}$ effort allocation.
${ }^{14}$ To derive equation (25) solve for $\eta_{1}$ as a function $X_{t+1}$ from equation (21) with $\eta_{1}=\psi\left(X_{t+1}\right)$. The density function for $X_{t+1}$ can then be written as $H\left(X_{t+1}\right)=h_{1}\left(\psi\left(X_{t+1}\right)\right)\left|d \eta_{1} / d X_{t+1}\right|$; see Hoel (1962, pp. 381-83). Equations (26), (27), and (28) are derived analogously.
${ }^{15}$ To my knowledge the effects of seasonal variation on the fishery have not been analyzed in the fishery economics literature. Despite its title, the paper by Bradley (1970) entitled 'Some Seasonal Models of the Fishing Industry" does not deal with seasonal variaion either.
${ }^{16}$ For a discussion of "feedback" or "adative expectation" models in economics see Nerlove (1972) and Rothschild (1972). An interesting analysis of adaptive decision making in the context of marine economics appears in Devanney (1971).
${ }^{17}$ By properly specifying the transition probabilities one can simulate a large variety of different parameter changes over time. For a discussion of the use of Markov models to simulate different processes over time see Feller (1950, Chapters XV and XVI) and Breiman (1969, Chapters 6 and 7).
${ }^{18}$ Some interesting examples of multiple species models appear in Quirk and Smith (1969) and Lampe (1967).

## Chapter III

A DETER MINISTIC MODEL OF THE FISHERY

We are now ready to use the basic Markov decision process model introduced in sections A-C of Chapter II to study the Eastern Pacific yellowfin tuna fishery. Although our discussion will pertain to the fishery, the application of our model to the analysis of other resource problems should be apparent.

This chapter provides the framework for evaluating optimal programs of resource allocation in the yellowfin tuna fishery. In section A the economic and biological processes of the fishery are modelled in terms of a finite state and action Markov system. The model is intended to approximate conditions in the real world where, of course, a continuum of states and policies exist. Consequently, an accuracy check of the discrete Markov model is undertaken in sections B and C: First, in section B a control theory model which assumes a continuous "state variable," (population) and a continuous "control variable," (effort allocation) is employed to derive optimal allocation rules for the fishery under deterministic conditions. These rules are then compared with the optimal strategies obtained from the Markov decision model presented in section $C$, to determine the effects of discretizing the state and control variables. The Markov model performs quite well in that the solutions yielded by the programming and control theory methods are nearly identical. Finally, in section $D$ the general form of the optimal deterministic policies are analyzed for their implications on current fishery management
policy, and summarized for subsequent use in comparison with optimal stochastic decision rules appearing in Chapter IV.

## A. A MARKOV MODEL OF THE EASTERN PACIFIC YELLOWFIN TUNA FISHERY

The Eastern Pacific yellowfin tuna provides an important source of income for fishermen from the United States, Canada, and several South and Central American countries. It is one of few international fisheries where the rate of fishing has been effectively c ntrolled by a regulatory body, in this case the Inter-American Tropical Tuna Commission. Each year since 1966, the Commission has established a catch quota on yellowfin tuna to maintain the stock at a level producing the maximum sustainable physical yield. ${ }^{l}$ For management purposes, the Tropical Tuna Commission, which is only one of three fishery commissions with an independent research staff, collects and analyzes data on the fishery so that the yellowfin tuna is one of the most extensively studied populations in the world. Consequently, besides the obvious reason that the tuna is a valuable resource, I decided to study this fishery because of the availability of reliable biological data.

In specifying a Markov model of the tuna fishery we begin by presenting the biological foundations of the fishery. A description

[^1]of the economic conditions of the resource follows, and the model is completed by specifying the Markov states, policies, and transition probabilities.

## A. 1. Biological Characteristics of the Fishery

The population dynamics of the Eastern Pacific yellowfin tuna are described by the Schaefer Stock Production model, ${ }^{2}$ which is a special case of the general model of resource growth introduced in section A of Chapter II. According to the Schaefer model, the population at time $t+1$, denoted by $X_{t+1}$ is ${ }^{3}$

$$
\begin{equation*}
X_{t+1}=X_{t}+\bar{\Delta} f\left(X_{t}\right)-\bar{\Delta} L_{t} \tag{1}
\end{equation*}
$$

where $\bar{\Delta}$ is the length of each time interval. The population is expressed in terms of weight or biomass units. In the absence of predation by man the growth rate of the population is described by $f\left(X_{t}\right)$. Increases in the population include new "recruitment" to the fishable stock ${ }^{4}$ and the "growth" of those fish already in the population. Natural decreases in the stock are caused by disease, predation by other fish, aging, and starvation.

Following Lotka (1956), $f(X)$ has the properties that $f\left(X^{1}\right)=f\left(X^{2}\right)=0, f^{\prime}(\hat{X})=0, f^{\prime \prime}(X)<0$, and $0 \leq X^{1}<\hat{X}<X^{2} .4$ Referring to Figure II-1, $X^{1}$ is the "critical population, "populations below $X^{l}$ not being feasible because of inadequate reproduction and because of vulnerability to disease and predators.


Figure III-1. Population Growth Curve

At $\mathrm{X}^{2}$ the population is in natural equilibrium. The marginal increment to the population from recruitment and growth are exactly offset by the decrease caused by natural mortality. For the special case where $f(X)$ is a quadratic $(f(X)=(a-b X) X)$ the population increases according to the well known logistic law of growth.

When the population is exploited by a fishing industry, the rate of fish landings, $L_{t}$ (measured in biomass units) is given by the function

$$
\begin{equation*}
L_{t}=g\left(X_{t}, E_{t}\right) \tag{2}
\end{equation*}
$$

where fishing effort, $E_{t}$ (measured in efficiency equivalent units) is a composite input variable representing the capital and labor used in fishing.

The Schaefer model assumes a particular functional form for
$f(X)$ and $L$. The complete model is:

$$
\begin{align*}
& f\left(X_{t}\right)=\left(a-b X_{t}\right) X_{t} ; a, b>0  \tag{3}\\
& L_{t}=g\left(X_{t}, E_{t}\right)=k X_{t} E_{t} ; k>0  \tag{4}\\
& X_{t+1}=X_{t}+\bar{\Delta}\left(a-b X_{t}\right) X_{t}-\bar{\Delta}_{t} k X_{t} E_{t} \tag{5}
\end{align*}
$$

Equation (4) describes a "mass contact" fishing technology where the catch rate is proportional to the physical contact between the fish and fishing effort. The constant, $k$, is called the "catchability coefficient" and is the per cent of the total fish population removed by one unit of effort.

The stochastic analogue to equation (5) suggested by Pella and Tomlinson (1969, p. 426) is

$$
\begin{equation*}
x_{t+1}=x_{t}+\bar{\Delta}\left[\eta_{1}\left(a-b X_{t}\right) x_{t}-\eta_{2} k X_{t} E_{t}\right] \tag{6}
\end{equation*}
$$

The variable, $\eta_{1}$, represents random variation in the rate of production from the stock due to changes in recruitment, growth, and natural mortality caused by random changes in the environment. Variation in the landings rate due to random changes in availability and catchability is represented by the variable $\eta_{2}$. It is assumed that $\eta_{1}$ and $\eta_{2}$ are distributed independently over time with expected values $\varepsilon\left(\eta_{1}\right)=\varepsilon\left(\eta_{2}\right)=1$. The distributions for $\eta_{1}$ and $\eta_{2}$ denoted by $h_{1}\left(\eta_{1}\right)$ and $h_{2}\left(\eta_{2}\right)$ are described in greater detail in section $C$.

Estimates for the population parameters, $a, b$, and $k$, presented in Table III-1, are based on historical time series data for catches and effort, and were provided by the Inter-American Tropical Tuna Commission. Detalls of the estimation procedure are described in Pella and Tomlinson (1969). According to Pella and Tomlinson, for these parameter values, a $\bar{\Delta}$ equal to 0.10 is sufficiently small such that the finite difference equation in (5) provides a good approximation for the instantaneous changes in population given by $X=(a-b X) X-k X E$.

Table III-1. Population Parameter Estimates

| Parameter | Estimated Value |
| :---: | :--- |
| a | 3.057 |
| b | $1.035 \times 10^{-8}$ |
| k | $7.85 \times 10^{-5}$ |
| $\bar{\Delta}$ | 0.10 |

## A.2. Economic Characteristics of the Fishery

a. Specification of the Rent Function

The rent from fishing is

$$
\begin{equation*}
R\left(X_{t}, E_{t}\right)=\tilde{G}\left(L\left(X_{t}, E_{t}\right)\right)-C\left(E_{t}\right) \tag{7}
\end{equation*}
$$

where $\widetilde{G}(L)$ is the total revenue and consumer surplus as a function of the catch, $L$, given by $L=k X_{t} E_{t}$, and $C\left(E_{t}\right)$ is the cost of fishing
effort.
The price of tuna is determined on the world market, and the quantity of yellowfin taken from the Eastern Pacific is only a small fraction of the total world supply of tuna. Tuna products are also close substitutes for other fish products. Thus one would expect that the demand curve for yellowfin tuna from the Eastern Pacific is quite elastic. To test this hypothesis we regressed the price received for yellowfin tuna on the quantity purchased from the Eastern Pacific for the years 1958-1972; the data and estimation results appear in Appendix III-A. We found prices to be insensitive to the quantity of tuna purchased and consequently we shall assume

$$
\begin{equation*}
\tilde{G}(L)=p L \tag{8}
\end{equation*}
$$

where $p=\$ 0.15$, is the mean price per pound for unprocessed yellowfin tuna during the 1966-1972 period, expressed in terms of 1956 dollars.

Several remarks need to be made concerning the estimation of the cost of effort function. Statistics on the costs of operation for tuna boats are generally not available. This is guarded information to most boat owners who are reluctant to reveal their financial situations. ${ }^{6}$ Besides a lack of data, other factors complicate the estimation of $C\left(E_{t}\right)$. For example, most boats catch various fish besides yellowfin tuna, making it difficult to calculate the fraction of total cost attributed to fishing for yellowfin tuna. In addition, because of
certain accounting procedures, reported costs often do not reflect the true opportunity costs of effort. ${ }^{7}$ With these problems in mind, I have decided to assume three hypothetical specifications for $C\left(E_{t}\right)$ enabling us to study the effect of different cost conditions on optimal resource use.

The first of these specifications is

$$
\begin{equation*}
C\left(E_{t}\right)=0 \tag{9}
\end{equation*}
$$

The cost of effort is assumed to be zero. Although this is not representative of cost conditions in the yellowfin fishery, it is included here for general interest. If boats fishing for population $A$ are simultaneously able to catch fish from another population $B$ (without reducing the catch of $A$ ), then the opportunity cost of fishing for $B$ is zero. As an example, boats that catch yellowfin spend most of their time trying to locate the yellowfin schools. During this time, other species of tuna such as the skipjack are often sighted and caught. Consequently, the opportunity cost of fishing for skipjack is virtually zero, assuming it does not interfere with the catch of yellowfin. ${ }^{8}$

The second specification is

$$
\begin{equation*}
C\left(E_{t}\right)=C_{1} E_{t}+C_{2} E_{t}^{2} ; C_{1}, C_{2}>0 \tag{10}
\end{equation*}
$$

For this case $C\left(E_{t}\right)$ is convex, with costs increasing more than proportionately with the amount of effort. In the Schaefer model, $E$ is strictly defined by the catch equation, $L=k X E$. It is an aggregate
input with constant "fishing power," such that each unit removes a fraction $k$ of the total population. To calculate $C\left(E_{t}\right)$ we must know the inputs comprising a unit of effort. An aggregate measure of these inputs is the number of days spent at sea, denoted by $E^{s}$, necessary to generate a given amount of total fishing effort, E. Expressing $\mathrm{E}^{\mathbf{s}}$ as a function of $E$ we have

$$
\begin{equation*}
E^{s}=h(E), h^{\prime}>0 \tag{11}
\end{equation*}
$$

The cost of $E$ is defined by

$$
\begin{equation*}
C(E)=C^{s}\left(E^{s}\right)=C^{s}(h(E)), C^{s,}>0 \tag{12}
\end{equation*}
$$

where $C^{s}$ is the cost of $E^{s}$. The rate of increase in marginal cost given by

$$
\begin{equation*}
C^{\prime \prime}(E)=C^{S_{\pi}}(h(E)) h^{\prime}(E)+C^{S^{\prime}}(h(E)) h^{\prime \prime}(E) \tag{13}
\end{equation*}
$$

is positive if the derivatives $h^{\prime \prime}(E)$ and $C^{S^{\prime \prime}}(\mathrm{h}(\mathrm{E}))$ are both non negative with one strictly positive. If the minimum earnings needed to attract labor and capital at the margin rises as more of these inputs are employed in fishing then $C^{s_{n}}>0$. The following arguments suggest we may also assume that $h^{\prime \prime}(E)>0$.

Within given fishing grounds the amount of $E^{s}$ needed to generate $E$ depends on the travel time required to reach the area, prevailing weather conditions, ocean turbulence, the density of the stock in that area, etc. Fishermen naturally prefer areas that are close to
port and that have favorable fishing conditions. As total effort increases these preferred areas become saturated with vessels and additional effort must be allocated in less desirable fishing grounds where more $E^{s}$ is required to generate a certain amount of $E$. Also, the number of vessels operating in a given area may increase, reducing the efficiency of each boat because of crowding and congestion externalities.

The third functional form for $C\left(E_{t}\right)$

$$
\begin{equation*}
C\left(E_{t}\right)=C_{3} E_{t}^{1 / 2} \tag{14}
\end{equation*}
$$

has properties quite different from the previous specification. For this function $C\left(E_{t}\right)$ is concave, as average and marginal costs decrease with greater allocations of effort. From equation (13) $C^{\prime \prime}(E)<0$ if the derivatives $C^{S "}$ and $h^{\prime \prime}$ are non positive with one strictly negative. If there are economies of scale in producing fishing gear or providing labor services then $C^{S_{"}}<0$. Assuming the number of vessels fishing in a given area increases as $E$ increases, if there are gains in efficiency due to sharing information among boats about the location of the fish, then $h^{\prime \prime}(E)<0$. in support of this possibility, Orbach (1975) reports that the practice of different boats sharing information with each other is quite common in the Eastern Pacific fleet.

Naturally, for the reasons we mentioned before, the information necessary to estimate the values of $C_{1}, C_{2}$, and $C_{3}$ is not
available. Therefore, we assume a range of different values for these parameters to determine the optimal response in resource allocation to different levels of cost.

In all of these specifications the fixed costs of effort are zero. The large purse seiners ${ }^{9}$ that dominate fishing in the Eastern Pacific are quite mobile and can operate in numerous fisheries throughout the world. Because of the availability of other species in the same area, such as the skipjack, and the easy access to other fishing grounds, the fixed costs of fishing for yellowfin in the Eastern Pacific are minimal.

## b. Specification of Utility Functions

With the equation for fishery rents in (7) formulated in terms of the revenue and cost functions presented above, we have only to specify some suitable forms for the social welfare function to complete the economic component of our model. To accommodate different social attitudes for risk bearing in our analysis, an important consideration in Chapter IV, two specifications of the welfare function are assumed.

$$
\begin{equation*}
U(R)=R \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{U}(\mathrm{R})=\ln (\mathrm{R}+\mathrm{G}) ; \quad G=4.5 \times 10^{8} \tag{16}
\end{equation*}
$$

Due to the curvature of these functions, (15) reflects a risk neutral
attitude and (16) exhibits a risk averse attitude toward variability in the returns from the fishery. The natural log function in (16) was chosen because it is easy to work with computationally. In stochastic models, where variations in rents occur, it is possible for $R$ to be negative. To insure that $\ln (R+G)$ exists, the constant $G$ is specified to be large enough such that $R+G>0$ for all possible values of R. The procedure for determining $G$ is discussed in Appendix III-B. Of course these welfare functions are only hypothetical. The problem of actually estimating and providing a consistent representation of social risk preferences is a very difficult one, and we shall not attempt to resolve it here. I shall defer a discussion of this problem until the stochastic model is introduced in Chapter IV.

The two utility specifications combined with the various forms of the rent function we have presented yield the six classes of objective functions appearing in Table III-2. The analysis that follows is carried through for each of these six classes. Each function is characterized by cost conditions in the fishery as well as the social attitudes toward risk bearing that exist.

Table III-2. Classes of Objective Functions

| Function | Class |
| :--- | :---: |
| $p k X_{t} E_{t}$ | I |
| $p k X_{t} E_{t}-C_{1} E_{t}-C_{2} E_{t}^{2}$ | II |
| $p k X_{t} E_{t}-C_{3} E_{t}^{1 / 2}$ | III |
| $\ln \left(p k X_{t} E_{t}+G\right)$ | IV |
| $\ln \left(p k X_{t} E_{t}-C_{1} E_{t}-C_{2} E_{t}^{2}+G\right)$ | $V$ |
| $\ln \left(p k X_{t} E_{t}-C_{3} E_{t}^{1 / 2}+G\right)$ | VI |

## A.3. Specification of States, Allocation Policies, and Transition Probabilities

The description of the biological and economic characteristics of the Eastern Pacific yellowfin fishery is complete, and we are ready to form the structure for the discrete Markov model.

## a. Specification of States

There are 31 possible states for the fishery, denoted by $X_{i}$, for $i=0,1,2, \ldots, 30$. Each state corresponds to a certain population size given by $X_{i}=i \times 10^{7}$ pounds. $X_{0}$ represents the minimum stock of 0 pounds, and $X_{30}$ is the largest population. The parameter estimates in. Table III-1 indicate a maximum sustainable stock of $29.4 \times 10^{7}$ pounds so that $X_{30}=30 \times 10^{7}$ pounds is large enough to include all probable values of the population. Since the Tropical Tuna

Commission generally tries ${ }^{10}$ to maintain the stock at a level producing the maximum sustained yield, the current population is probably in the neighborhood of $15 \times 10^{7}$ pounds. 11

## b. Specification of Allocation Policies

There are a finite number of possible effort allocation corresponding to each state of the system. These allotments of effort are measured in terms of boat days at sea, and range in multiples of 250 from a minimum of 0 days to a maximum amount determined by economic conditions. Obviously, an upper bound on effort in each state, $i$, is the level that maximizes $R\left(X_{i}, E\right)$, since it is never optimal to allocate effort beyond that point.

## c. Specification of Transition Probabilities

The probability of the system moving from one state to another is a function of the state currently occupied, and the allocation policy of the resource manager. For deterministic systems the calculation of transition probabilities is according to equation (23) in section C. 4. a of Chapter II. For stochastic models, depending on the type of stochastic variation, the transition probabilities are specified according to equations (25) - (30) in sections C. 4.b. - C. 4.d. of Chapter II.

## B. CONTROL THEORY SOLUTION TO THE <br> ALLOCATION PROBLEM

With a descriptive model of the yellowfin tuna industry we proceed to a consideration of optimal resource use in the fishery, over time, under deterministic conditions. In formal terms, the allocation problem is to

$$
\begin{equation*}
\underset{E_{t}}{\operatorname{maximize}} \sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(X_{t}, E_{t}\right)\right)\right] \bar{\Delta} \tag{17}
\end{equation*}
$$

subject to

$$
\begin{align*}
& X_{t+1}=X_{t}+\left[\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\right] \bar{\Delta}  \tag{18}\\
& X_{t} \geq 0  \tag{19}\\
& X_{0}=\hat{X}  \tag{20}\\
& 0 \leq E_{t} \leq E_{\max } \tag{21}
\end{align*}
$$

where $U\left(R\left(X_{t}, E_{t}\right)\right)$ belongs to one of the six classes of objective functions listed in Table III-2, and $B$ is the social discount factor equal to the reciprocal of one plus the social discount rate. Equations (19) and (20) represent the non-negativity and initial conditions for the stock, and (21) restricts effort allotments to be non-negative and less than or equal to some maximum amount denoted by $E_{\text {max }}$. As me..cioned before, the problem is amenable to solution by
dynamic programming methods or optimal control theory. For each class except III and VI, solutions to the allocation problem are derived using both procedures. Then in section $C$, they are compared to evaluate the accuracy of the finite state and action Markov decision process. Certain concavity conditions on the objective function sufficient for the control theory solution to be optimal are not satis-. fied for Classes III and VI, so that these cases can only be solved with the Markov decision model.

Proceeding with the control theory analysis, we first describe the optimal time paths of effort and catches in general terms for Classes I, II, IV, and V. Then, a method to compute these paths for specific cases is introduced. The results of these computations are presented in section $C$ for comparison with the Markov model solutions.

## B. 1. Control Theory Formulation

The allocation problem posed above is well defined. One seeks to maximize the discounted stream of social returis from the fishery by choosing the correct value for the control, $E_{t}$, subject to adjustments in the population described by equation (18). The convergence of the welfare functional is assured if the initial population size is finite, and $B<1$. If $X_{0}$ is less than or equal to the maximum sustainable population, $a / b$, then

$$
\begin{gather*}
\sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(X_{t}, E_{t}\right)\right] \Delta s \sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(a / b, E_{t}\right)\right)\right] \bar{\Delta}\right. \\
<\frac{U^{*}(R(a / b, E))}{1-B} \tag{22}
\end{gather*}
$$

where $E_{t}$ is contained in the set of admissible controls defined by (21), and $\mathrm{U}^{*}=\max _{\mathrm{t}} \mathrm{U}\left(\mathrm{R}\left(\mathrm{X}_{\mathrm{t}}, \mathrm{E}_{\mathrm{t}}\right)\right.$. If $\mathrm{X}_{0}$ is greater than $a / b$ but finite then,

$$
\begin{gathered}
\sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(X_{t}, E_{t}\right)\right)\right] \bar{\Delta} \leq \sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(X_{0}, E_{t}\right)\right)\right] \bar{\Delta} \\
<\frac{U^{*}\left(R\left(X_{0}, E\right)\right) \Delta}{1-B}
\end{gathered}
$$

Applying Pontryagin's Maximum Principle, the necessary conditions for maximization of (17) subject to (18) - (21) are that there exists a costate variable, $\lambda_{t+1}$, such that the control, $E_{t}$, maximizes, H , the Hamiltonian at each instant in time where, ${ }^{13}$

$$
\begin{equation*}
H=B^{t}\left[U\left(R\left(X_{t}, E_{t}\right)\right)+B \lambda_{t+1}\left(\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\right)\right] \tag{24}
\end{equation*}
$$

with

$$
\frac{\partial H}{\partial E_{t}}=B^{t}\left|U^{\prime} \frac{\partial R}{\partial E_{t}}-B \lambda_{t+1} k X_{t}\right| \bar{\Delta}=0 \text { if }\left\{\begin{array}{l}
E_{t}=0  \tag{25}\\
0<E_{t}<E_{\max } \\
\\
\\
E_{t}=E_{\max }
\end{array}\right.
$$

$$
\begin{align*}
& X_{t+1}-X_{t}=\left[\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\right] \bar{\Delta}  \tag{26}\\
& \lambda_{t+1}-\lambda_{t}=-U^{\prime} \frac{\partial R}{\partial X_{t}}+B \lambda_{t+1}\left(a-2 b X_{t}-k E_{t}-\frac{1-B}{\bar{\Delta} B}\right) \bar{\Delta} \tag{27}
\end{align*}
$$

Sufficient conditions for an optimum are the transversality condition

$$
\begin{equation*}
\lim _{T \rightarrow \infty} B^{T+1} \lambda_{T+1} X_{T}=0 \tag{28}
\end{equation*}
$$

and that the maximized Hamiltonian be concave with respect to the state variable $X$. The concavity requirement is trivially satisfied for Classes I and IV. That the condition also holds for Classes II and $V$ is readily verified once the optimal time paths for effort and catches are computed.

## B.2. Economic Interpretation

If at some time, $t$, we have an interior solution with $\left(X_{t}>0\right.$,
$\lambda_{t+1}>0,0<E_{t}<E_{\max }$ ) then equation (25) implies

$$
\begin{equation*}
U^{\prime} \frac{\partial R}{\partial E_{t}}-B \lambda_{t+1} k X_{t}=0 \tag{25}
\end{equation*}
$$

where $\lambda_{t+1}$ is the marginal value of the fishery stock at time $t+1$. This condition implies that along the optimal time path of the control variable, $E_{t}$, the marginal addition to current utility, $U^{\prime} \frac{\partial R}{\partial E_{t}}$, from increasing the catch is exactly offset by the resulting marginal
decrease in population available next period with the stock valued at its marginal worth discounted for time, $B \lambda_{t+1}$. Equation (26) specifies the net growth in the stock as a function of natural growth and the reduction in the population attributed to fishing. Rewriting equation (27) yields

$$
\begin{equation*}
\frac{-\left(B \lambda_{t+1}-\lambda_{t}\right)}{\bar{\Delta}}=U^{\prime} \frac{\partial R}{\partial X}+B \lambda_{t+1}\left(a-2 b X_{t}-k E_{t}\right) \tag{27}
\end{equation*}
$$

The left hand side of (27)' is the rate at which the value of the stock depreciates in present value terms. It equals the rate an additional unit of the stock adds to current utility, $U^{\prime} \frac{\partial R}{\partial X}$, plus the rate it enhances the value of the existing stock.

## B.3. Description of the Optimal Time Paths

a. Classes II, IV, and V

Assuming an interior solution for (25) it is possible under certain conditions to represent ${ }^{14} \mathrm{E}_{\mathrm{t}}$. as a function of $\mathrm{X}_{\mathrm{t}}$ and $\lambda_{\mathrm{t}+1}$. The Jacobian of (25)'

$$
\begin{equation*}
J=U^{\prime \prime}\left(\frac{\partial R}{\partial E}\right)^{2}+U^{\prime} \frac{\partial^{2} R}{\partial E^{2}}<0 \tag{29}
\end{equation*}
$$

is non vanishing for Classes II, IV, and $V$ since either $U$ or $R$ are strictly concave. Substituting for $E_{t}$ in terms of $X_{t}$ and $\lambda_{t+1}$ from (25)', we generate a system of two autonomous first order difference equations for $\Delta X_{t}$ and $\Delta \lambda_{t} \cdot{ }^{15}$ A stationary equilibrium
$\left(X^{*}, \lambda^{*}\right)$ for this system exists for ${ }^{16}$

$$
\begin{equation*}
\Delta X_{t}=\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\left(X_{t} ; \lambda_{t+1}\right)=0 \tag{30}
\end{equation*}
$$

and

$$
\begin{align*}
\Delta \lambda_{t} & =-\left[U^{\prime} \frac{\partial R}{\partial X_{t}}\left(X_{t}, E_{t}\left(X_{t}, \lambda_{t+1}\right)\right)\right. \\
& \left.+B \lambda_{t+1}\left(a-2 b X_{t}-\frac{1-B}{\bar{\Delta} B}-k E_{t}\left(X_{t}, \lambda_{t+1}\right)\right)\right]=0 \tag{31}
\end{align*}
$$

Equation (30) implies equilibrium between the natural growth rate of the stock and the harvest rate by the fishermen. Rewriting equation (31) yields

$$
\begin{equation*}
U^{\prime} \frac{\partial R}{\partial X_{t}}+B \lambda_{t+1}\left(a-2 b X_{t}-k E_{t}\right)=\frac{(1-B)}{\bar{\Delta}} \lambda_{t+1} \tag{31}
\end{equation*}
$$

The term $\frac{(1-B)}{\bar{\Delta}} \lambda_{t+1}$ is the rate at which the value of the stock is discounted. The total value discounted in one period is $(1-B) \lambda_{t+1}$ and thus $\frac{(1-B)}{\bar{\Delta}} \lambda_{t+1}$ is the rate of discount. Equation (31)' implies that the interest on the stock, $\frac{(1-B)}{\bar{\Delta}} \lambda_{t+1}$, equals the rate which an addition to the population increases current utility, $U^{\prime} \frac{\partial R}{\partial X_{t}}$, plus the rate which it enhances the current discounted value of the stock available next period, $\lambda_{t+1} B\left(a-2 b X_{t}-k E_{t}\right)$. We now prove that if a stationary point exists, then

$$
\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}<X^{*}<\frac{a}{b}
$$

As we shall see $\frac{a-\frac{l-B}{\bar{\Delta} B}}{2 b}$ is the stationary equilibrium for the costless effort case, Class $I$, and $\frac{a}{b}$ is the equilibrium occurring when the fishery is left unexploited. Rewriting (31) we obtain

$$
\begin{equation*}
U^{\prime} \frac{\partial R}{\partial X_{t}}-B \lambda_{t+1} k E_{t}=-B \lambda_{t+1}\left(a-2 b X_{t}-\frac{(1-B)}{\bar{\Delta} B}\right) \tag{32}
\end{equation*}
$$

From (25) ${ }^{\prime}$

$$
B \lambda_{t+1} k=\frac{U^{\prime} \frac{\partial R}{\partial E_{t}}}{X_{t}}
$$

Substituting for $B \lambda_{t+1}$ above yields

$$
\begin{equation*}
U^{\prime}\left[\frac{\partial R}{\partial X_{t}}-\frac{\partial R}{\partial E_{t}} \frac{E_{t}}{X_{t}}\right]=-B \lambda_{t+1}\left(a-2 b X_{t}-\frac{(1-B)}{\overline{\Delta B}}\right) \tag{33}
\end{equation*}
$$

Recalling that $R\left(X_{t}, E_{t}\right)=p k X_{t} E_{t}-C\left(E_{t}\right)$, the left hand side of (33) becomes $U^{\prime} C^{\prime}\left(E_{t}\right) \frac{E_{t}}{X_{t}}$ which is strictly positive for $E_{t}>0$. This implies

$$
a-2 b X_{t}-\frac{(1-B)}{\bar{\Delta} B}<0 \quad \text { or } \quad X_{t}>\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b} \text { since } \lambda_{t+1}>0
$$

If $\left(a-\frac{1-B}{\bar{\Delta} B}\right)<0$, a stationary equilibrium for this case does not
exist; however $\left(a-\frac{1-B}{\bar{\Delta} B}\right)>0$ for the examples in this chapter. From (30) with $E_{t}>0, \Delta X_{t}=0$ only for $X_{t}<\frac{a}{b}$. Therefore we have

$$
\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}<X^{*}<\frac{a}{b}
$$

This result is significant in that it implies that $\mathrm{X}^{*}$, the steady state population, may equal $a / 2 b$, the maximum sustainable yield population. However, as we will see, there is no a priori reason to suspect that the two populations will coincide.

The intersection of the $\Delta X=0$ and the $\Delta \lambda=0$ curves and the resulting stationary point $\left(X^{*}, \lambda^{*}\right)$ are illustrated in Figure III-2. To see that at least one stationary point exists in the interval, $\left(\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}, \frac{a}{b}\right)$, consider the values of $\lambda_{t+1}$ evaluated at
$X_{t}=\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}$ and $X_{t}=\frac{a}{b}$ along the $\Delta X=0$ and $\Delta \lambda=0$ curves.
From equations (30) and (31) it is readily verified that

$$
\begin{align*}
& \left.E_{t}\right|_{\Delta X=0}>\left.E_{t}\right|_{\Delta \lambda=0} \text { for } X_{t}=\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}  \tag{34}\\
& \left.E_{t}\right|_{\Delta X=0}<\left.E_{t}\right|_{\Delta \lambda=0} \text { for } X_{t}=\frac{a}{b}
\end{align*}
$$

By equation (25)

$$
\begin{equation*}
\frac{\partial E_{t}}{\partial \lambda_{t+1}}=\frac{B k X_{t}}{J}<0 \tag{35}
\end{equation*}
$$

implying that

$$
\begin{align*}
& \left.\lambda_{t+1}\right|_{\Delta X=0}<\left.\lambda_{t+1}\right|_{\Delta \lambda=0} \text { for } X_{t}=\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b} \\
& \left.\lambda_{t+1}\right|_{\Delta X=0}>\left.\lambda_{t+1}\right|_{\Delta \lambda=0} \text { for } X_{t}=\frac{a}{b} . \tag{36}
\end{align*}
$$



Figure III-2. Phase Diagram

Since the $\Delta \lambda=0$ and $\Delta X=0$ curves are continuous, they must inter. sect at one or more points in the $\lambda_{t+1}-X_{t}$ plane for $X_{t}$ in the interval $\left(\frac{a-\frac{1-B}{\overline{\Delta B}}}{2 b}, \frac{a}{b}\right)$, thus proving the existence of $a$ stationary equilibrium. Furthermore, the equilibrium is unique, for the examples considered here, as we shall see once the optimal time paths are computed.

Consider the direction of motion of a point ( $X_{t}, \lambda_{t+1}$ ) lying above or below each of the $\Delta \lambda=0$ and the $\Delta X=0$ curves. For points above the $\Delta X=0$ curve, $\Delta X>0$, and for points below the curve $\Delta X<0$ since

$$
\begin{equation*}
\left.\frac{d \Delta X}{d \lambda_{t+1}}\right|_{X_{t}=\text { constant }}=-k X_{t} \frac{\partial E_{t}}{\partial \lambda_{t+1}}>0 \tag{37}
\end{equation*}
$$

If $X_{t}<\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}$ then $\Delta \lambda<0$. Otherwise, for $X_{t} \geq \frac{a-\frac{1-B}{\overline{\Delta B}}}{2 b}$

$$
\left.\frac{d \Delta \lambda}{d \lambda_{t+1}}\right|_{X_{t}=\text { constant }}=-\left[\frac{\partial E_{t}}{\partial \lambda_{t+1}}\left[U^{\prime \prime} \frac{\partial R}{\partial X_{t}} \frac{\partial R}{\partial E_{t}}+U^{\prime} \frac{\partial^{2} R}{\partial E_{t} \partial X_{t}}-B \lambda_{t+1} k\right]\right.
$$

$$
\begin{equation*}
\left.+B\left[a-\frac{1-B}{\bar{\Delta} B}-2 b X_{t}-k E_{t}\right]\right] \tag{38}
\end{equation*}
$$

From (25)' we obtain

$$
\begin{equation*}
\frac{\partial E_{t}}{\partial \lambda_{t+1}}\left[U^{\prime \prime} \frac{\partial R}{\partial E_{t}} \frac{\partial R}{\partial X_{t}}+U^{\prime} \frac{\partial^{2} R}{\partial E_{t} \partial X_{t}}-B \lambda_{t+1} k\right]=-B k X_{t} \frac{\partial E_{t}}{\partial X_{t}} \tag{39}
\end{equation*}
$$

and thus

$$
\begin{equation*}
\left.\frac{d \Delta \lambda}{d \lambda_{t+1}}\right|_{X_{t}=\text { constant }}=B\left(k E_{t}+k X_{t} \frac{\partial E_{t}}{\partial X_{t}}\right)-B\left(a-\frac{1-B}{\overline{\Delta B}}-2 b X_{t}\right) \tag{40}
\end{equation*}
$$

The second term on the right hand side of (40) is non negative since
$X_{t} \geq \frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}$. Consequently $\left.\frac{d \Delta \lambda}{d \lambda_{t+1}}\right|_{X_{t}=\text { constant }}>0$ for $\frac{\partial E_{t}}{\partial X_{t}} \geq 0$

Suppose that $\frac{\partial E_{t}}{\partial X_{t}}<0$, then from (25)'

$$
\begin{align*}
& \left|\frac{\partial E_{t}}{\partial X_{t}}\right|=\left|-\frac{U^{\prime \prime} \frac{\partial R}{\partial E_{t}} \frac{\partial R}{\partial X_{t}}+U^{\prime} \frac{\partial^{2} R}{\partial E_{t} \partial X_{t}}-B \lambda_{t+1} k}{U^{\prime \prime}\left(\frac{\partial R}{\partial E_{t}}\right)^{2}+U^{\prime} \frac{\partial^{2} R}{\partial E_{t}^{2}}}\right|  \tag{41}\\
& <\frac{U^{\prime \prime} \frac{\partial R}{\partial E_{t}} \frac{\partial R}{\partial X_{t}}}{U^{\prime \prime}\left(\frac{\partial R}{\partial E_{t}}\right)^{2}}=\frac{\frac{\partial R}{\partial X_{t}}}{\frac{\partial R}{\partial E_{t}}} \\
& \text { since }\left[U^{\prime \prime} \frac{\partial R}{\partial E_{t}} \frac{\partial R}{\partial X_{t}}+U^{\prime} \frac{\partial^{2} R}{E_{t} \partial X_{t}}-B \lambda_{t+1} k\right]<0 \text { by virtue of assuming } \\
& \frac{\partial E_{t}}{\partial X_{t}}<0 \text {, and }\left[U^{\prime} \frac{\partial^{2} R}{\partial E_{t} \partial X_{t}}-B \lambda_{t+1} k\right]=U^{\prime} \frac{C^{\prime}\left(E_{t}\right)}{X_{t}}>0 \text {. Therefore, }
\end{align*}
$$

for points on the $\Delta \lambda=0$ curve

$$
\left.\begin{array}{rl}
\left.\frac{d \Delta \lambda}{d \lambda_{t+1}}\right|_{X_{t}}=\text { constant }
\end{array} \quad>\mathrm{BkE}_{t}-B k X_{t}\left(\frac{\frac{\partial R}{\partial X_{t}}}{\frac{\partial R}{\partial E_{t}}}\right)-B\left(a-\frac{1-B}{\overline{\Delta B}}-2 b X_{t}\right)\right] \quad \begin{aligned}
& \left.\Delta \lambda E_{t}-U^{\prime} \frac{\frac{\partial R}{\partial X_{t}}}{\lambda_{t+1}}-B\left(a-\frac{1-B}{\overline{\Delta B}}-2 b X_{t}\right)\right]=\frac{\Delta \lambda}{\lambda}=0
\end{aligned}
$$

Consequently for points lying to the left and below the $\Delta \lambda=0$ curve $\Delta \lambda<0$, and for points lying above and to the right of the curve then $\Delta \lambda>0$.

The arrows in Figure III-2 indicate the direction of movements of points in the phase space. If $\mathrm{X}_{0}$ is greater than (less than), $\mathrm{X}^{*}$ there exists a $\lambda_{1}$ in region $I$, (III) such that the path beginning at ( $\mathrm{X}_{0} \lambda_{1}$ ) converges to $\left(\mathrm{X}^{*} \lambda^{*}\right)$. This is the optimal path as it satisfies the necessary conditions (25) - (27), and the transversality condition (28). All other paths either begin in or eventually enter region II or IV from where it is impossible to reach the stationary equilibrium. To show that $\left(\mathrm{X}^{*} \lambda^{*}\right)$ is a saddle point expand, $\Delta \mathrm{X}$ and $\Delta \lambda$ about $\left(X^{*} \lambda^{*}\right)$ as a Taylor series, including only linear terms.

$$
\left[\begin{array}{l}
\Delta X \\
\Delta \lambda
\end{array}\right]=A\left[\begin{array}{l}
X^{*}-X \\
\lambda^{*}-\lambda
\end{array}\right]
$$

where

$$
A=\left[\begin{array}{cc}
\frac{\partial \Delta X}{\partial X} & \frac{\partial \Delta X}{\partial \lambda} \\
& \\
\frac{\partial \Delta \lambda}{\partial X} & \frac{\partial \Delta \lambda}{\partial \lambda}
\end{array}\right]
$$

with all the elements of $A$ evaluated at $\left(X^{*} \lambda^{*}\right)$. Since

$$
\frac{\partial \Delta X}{\partial X}=-\left.\frac{d \lambda}{d X}\right|_{\Delta X=0} \frac{\partial \Delta X}{\partial \lambda}
$$

and

$$
\begin{aligned}
& \frac{\partial \Delta \lambda}{\partial X}=-\left.\frac{d \lambda}{d X}\right|_{\Delta \lambda=0} \frac{\frac{\partial \Delta \lambda}{\partial \lambda}}{A=\left[\begin{array}{lll}
-\left.\frac{d \lambda}{d X}\right|_{\Delta X=0} & \frac{\partial \Delta X}{\partial \lambda} & \frac{\partial \Delta X}{\partial \lambda} \\
-\left.\frac{d \lambda}{d X}\right|_{\Delta \lambda=0} & \frac{\partial \Delta \lambda}{\partial \lambda} & \frac{\partial \Delta \lambda}{\partial \lambda}
\end{array}\right] .} .
\end{aligned}
$$

The eigenvalues for $A, \xi_{1}$ and $\xi_{2}$ are determined by the equation,

$$
\begin{equation*}
\xi^{2}+\xi\left[\left.\frac{\mathrm{d} \lambda}{\mathrm{dX}}\right|_{\Delta \mathrm{X}=0} \frac{\partial \Delta \mathrm{X}}{\partial \lambda}-\frac{\partial \Delta \lambda}{\partial \lambda}\right]+\left[\left.\frac{\mathrm{d} \lambda}{\mathrm{dX}}\right|_{\Delta \lambda=0}-\left.\frac{\mathrm{d} \lambda}{\mathrm{dX}}\right|_{\Delta \mathrm{X}=0}\right] \frac{\partial \Delta \mathrm{X}}{\partial \lambda} \frac{\partial \Delta \lambda}{\partial \lambda}=0 \tag{43}
\end{equation*}
$$

Looking at the constant term, $\left[\left.\frac{d \lambda}{d X}\right|_{\Delta \lambda=0}-\left.\frac{d \lambda}{d X}\right|_{\Delta X=0}\right]<0 \quad$ since.the $\Delta \lambda=0$ curve intersects the $\Delta X=0$ curve from above. The entire term is negative since $\frac{\partial \Delta X}{\partial \lambda}$ and $\frac{\partial \Delta \lambda}{\partial \lambda}$ are positive. This implies that
the eigenvalues are real and of opposite sign so that $\left(X^{*} \lambda^{*}\right)$ is a . saddle point.

## b. Class 1

Spence (1973) has identified certain classes of control problems that have solutions of the following type. First, one determines an optimal stationary level for the state variable (for our problem it is the population). Then, regardless of the initial population, one proceeds as rapidly as possible to the stationary state and remains there forever. Time paths of this sort are referred to as most rapid approach paths. Formally, in terms of our model, a path $\tilde{X}_{t}$ is called a most rapid approach path with respect to $X_{0}$ and $X^{*}$ if $\tilde{X}_{0}=X_{0}$, and the appropriate condition below is satisfied:
(a) for $\tilde{X}_{t}<X^{*}: \tilde{X}_{t+1}=\min \left\{X^{*}, \tilde{X}_{t}+\left(a-b \tilde{X}_{t}\right) \tilde{X}_{t} \bar{\Delta}\right\}$
(b) for $\tilde{X}_{t}>X^{*}: \tilde{X}_{t+1}=\max \left\{X^{*}, \tilde{X}_{t}+\left[\left(a-b \tilde{X}_{t}\right) \widetilde{X}_{t}-k \tilde{X}_{t} E_{\text {max }}\right] \stackrel{\rightharpoonup}{\Delta}\right\}$
(c) for $\tilde{X}_{t}=X^{*}: \tilde{X}_{t+1}=X^{*}$.

There may be an intermediate period in which $E_{t}$ equals neither of its extreme values, nor the level which well sustain the state variable at $X^{*}$. It is the period in which the transition from rapid ascent or descent is made to $\mathrm{X}^{*}$.

Spence demonstrates for discrete control problems that a sufficient condition for most rapid approach paths to be optimal is
that the objective function be impliciciy additively separable in $X_{t}$ and $X_{t+1}$. In terms of our model the appropriate condition is that $R\left(X_{t}, E_{t}\right)$ may be written in terms of equation (45),

$$
\begin{equation*}
R=a\left(X_{t}\right)+b\left(X_{t+1}\right) ; \tag{45}
\end{equation*}
$$

where $a(*)$ and $b(*)$ are functions. This condition is satisfied for control problems having a Class I specification of rents. First, we write $E_{t}$ in terms of $X_{t}$ and $X_{t+1}$ from equation (26)

$$
\begin{equation*}
E_{t}=\frac{X_{t}+\bar{\Delta}\left(a-b X_{t}\right) X_{t}-X_{t+1}}{\bar{\Delta} k X_{t}} \tag{46}
\end{equation*}
$$

Substituting for $E_{t}$ in the rent function we obtain,

$$
\begin{equation*}
R\left(X_{t}, X_{t+1}\right)=\frac{p\left(X_{t}+\bar{\Delta}\left(a-b X_{t}\right) X_{t}\right)}{\bar{\Delta}}-\frac{p X_{t+1}}{\bar{\Delta}} \tag{47}
\end{equation*}
$$

Equation (47) is additively separable in $X_{t}$ and $X_{t+1}$ indicating that most rapid approach paths are optimal for Class I problems. From equations (30) and (31) it is easy to verify that the optimal stationary state occurs for $X^{*}=\frac{a-\frac{1-B}{\bar{\Delta} B}}{2 b}$.

## B.4. Computation of Optimal Paths

To calculate the optimal paths for Class I problems first one determines the value for the optimal stationary state $X^{*}$. Then
depending on the initial population, one selects the level of effort according to the conditions in equation (44) in order to reach $\mathrm{X}^{*}$ as quickly as possible.

For Classes II, IV, and V the calculation of optimal paths begins by determining the values for the steady state population $\mathrm{X}^{*}$, and the costate variable $\lambda^{*}$, from equations (30) and (31). For all the examples considered in this chapter, the stationary point ( $X^{*} \lambda^{*}$ ) is unique. Then for a given initial population $X_{0}$, one chooses an initial value for the costate variabie $\lambda_{1}$, as depicted in the phase diagram of Figure III-3. The resulting path consisting of a series of discrete points is simulated on the computer according to equations (25) - (27). If the path converges to $\left(\mathrm{X}^{*} \lambda^{*}\right)$ it is the optimal one, otherwise if the path enters region IV (II) the simulation is repeated


Figure III-3. Selection of Optimal Paths
with a larger (smaller) value for $\lambda_{1}$-- see Figure III-3. This process is continued until a "resonably convergent" path is located -- one which comes within a very close range of reaching $\left(X^{*} \lambda^{*}\right)$ without leaving regions I or III. ${ }^{18}$

The optimal path corresponding to a given $X_{0}$ consists of discrete jumps in the state and costate variable, and thus contains only a certain number of points in the phase plane. Therefore, optimal paths originating from various initial populations are determined in order to generate a locus of optimal points in the fashion illustrated in Figure III-4.

Clearly, the optimal effort allocation at time $t$ depends only on $X_{t}$. According to equation (25), $E_{t}$ is a function of $X_{t}$ and $\lambda_{t+1}$; however, $\lambda_{t+1}$ is also a function of $X_{t}$. The graph of $\lambda_{t+1}$


Figure III-4. Determinịng Locus of Optimal Points
as a function of $X_{t}$ is the locus of optimal points in Figure III-4. Thus the control problem becomes one of choosing the optimal "policy" for allocating effort depending only on the size of the population.

> C. AN AIVAL YSIS AND COMPARISON OF OPTIMAL POLICIES DERIVED FROM THE DISCRETE MARKOV AND CONTROL THEORY MODELS

The purpose of this discussion is twofold. First, by comparing Markov and control theory policies we can determine the effects on optimal allocation schemes of discretizing the state and control variables. If the accuracy lost by employing a discrete model is small, the Markov model can safely be used to study optimal use of resources under conditions of uncertainty. Second, the general form of optimal deterministic policies are analyzed and summarized for subsequent use in comparison with stochastic decision rules derived in Chapter IV. An interesting result of this analysis is the possibility of optimal "cyclical fishing" for Classes III and VI. To my knowledge, only "steady state" fishing strategies that we found for Classes I, II, IV, and $V$ have been discussed in the fishery economics literature.

## C.1. Comparison of Optimal Policies

Dynamic Programming and control theory solutions to the allocation problem in (17) are compared for Classes I, II,•IV, and V to evaluate the accuracy of the discrete Markov model. The procedure for calculating optimal solutions using the Markov model is
discussed in Appendix III-C. The method of control theory computations is presented in section B. 4. of this chapter.

The Markov model performs quite well, yielding solutions nearly identical to the control theory results. For both models, optimal policy is to allocate effort so that the population tends towards a steady state gradually as in the Class II, IV, and V cases, or at a maximum rate as in the Class I programs.

Optimal steady state populations $\mathrm{X}^{*}$ derived by both models for various Class I, II, IV, and V cases appear in Table III-3. For Classes I and IV, where costs are zero, the results are reported for different values of the discount rate, in order to focus on the effect that varying preferences for consumption over time have on optimal allocation policies. To assess the impact of different cost conditions on optimal programs, Class II and V results are reported for various magnitudes of the cost parameter $C_{2}$.

With both models, Classes (I and IV) and (II and V) have identical steady state populations. This occurs because optimal programs for classes with objective functions differing by a positive concave transformation converge to the same steady state, as we prove in Appendix III-D. The close agreement between the values of $X^{*}$ for the two models indicates the accuracy of the Markov model. For every case reported in Table III-3 the Markov value of $X^{*}$ falls within a range of $\pm 3.3 \%$ of the control theory value.

Table III-3. Comparison of Optimal Steady State Values


As expected for Classes $I$ and IV, $X^{*}$ increases with the dis count factor $B$ (although over a certain range the Markov value of $X^{*}$ is insensitive to changes in $B$ ), since it is desirable to harvest less of the resource the higher society weights the consumption of future generations. With Classes II and $V, X^{*}$ varies directly with $C_{2}$ as it becomes less profitable to fish for the resource as the cost of effort increases.

A comparison between Markov and control theory policies for certain of the Class $I$, II, IV, and V cases is presented in Tables III-4 - III-7. The first column in each table represents the current state of the fishery in terms of total population. For the Markov model, there are 31 states ranging in size from 0 to $30 \times 10^{7}$ pounds in multiples of $10^{7}$ pounds. The optimal Markov and control theory allocations of effort corresponding to each state are listed in columns 2 and 3 respectively. The Markov effort allotments are limited to 250 mul tiples. Given the current state of the fishery and the corresponding effort allocation, column four lists the stock size next period for the Markov model calculated by equation (18). The resulting one period changes in population appear in column five. For a given initial state, the expected present value of the stream of returns yielded by the optimal effort policy for the Markov model is reported in column six. As expected, present value figures vary directly with the initial fishery state.

Looking at columns 2 and 3 of Tables III-4 - III-7, we observe the close similarity between Markov and control theory policies, particularly for Classes II and V. In each case with both models the population tends toward the same steady state as indicated in Table III-3, and at approximately the same rate. Although there is some discrepancy in the Markov and control theory effort allocations for the Class I program, the general form of the optimal program for the two models are alike. A most rapid approach path to equilibrium is indicated by the Markov policy (see column 4 of Table III-4) which we have seen is consistent with the control theory solutions. For this particular Class I case, larger values of effort for the Markov program occur because the population jumps to a smaller steady state with the Markov model than it does for the control theory model.

In conclusion, based on the results summarized in Tables III-3 - III-7, it appears that the optimal Markov and control theory policies are generally the same. Of course greater accuracy in the Markov solution is possible by increasing the number of states and policies in the model for a better approximation to a continuous system. However, the model performs satisfactorily for our purposes in that the Markov solution is not distorted or made artificial by using a discrete state and action system.

Table III-4. Optimal Markov and Control Theory Policies for Class I; $B=0.9906$

| State $10^{7}$ <br> Pounds | $\begin{gathered} \text { Markov } \\ E_{t} \end{gathered}$ | Control <br> Theory $E_{t}$ | $\mathrm{X}_{t+1}$ | $\Delta X_{t}$ | $\mathrm{V}(\mathrm{X})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0.0000 E 00 | 0.0000 E 00 | 0.00000E 00 |
| 1 | 0 | 0 | 1.2954 E 07 | 2.9535 E 06 | 3. 19413 E 08 |
| 2 | 0 | 0 | 2.5700E 07 | 5.7000 E 06 | 3.29675 E 08 |
| 3 | 0 | 0 | 3.8240 E 07 | 8.2395 E 06 | 3. 35163 E 08 |
| 4 | 0 | 0 | 5.0572 E 07 | 1.0572 E 07 | 3. 39023 E 08 |
| 5 | 0 | 0 | 6.2698 E 07 | 1.2698 E 07 | 3.42090E 08 |
| 6 | 0 | 0 | 7.4616E 07 | 1.4616 E 07 | 3.44713E 08 |
| 7 | 0 | 0 | 8.6328E 07 | 1.6328 E 07 | 3.47023E 08 |
| 8 | 0 | 0 | 9.7832 E 07 | 1.7832 E 07 | 3.49106 E 08 |
| 9 | 0 | 0 | 1.0913 E 08 | 1.9130 E 07 | 3.51018E 08 |
| 10 | 0 | 0 | 1.2022 E 08 | 2.0220E 07 | 3.52806E 08 |
| 11 | 0 | 0 | 1.3110E 08 | 2.1104E 07 | 3.54496E 08 |
| 12 | 1750 | 0 | 1.4013 E 08 | 2.0132 E 07 | 3.56120E 08 |
| 13 | 12000 | 8969 | 1,4000E 08 | 1.0004 E 07 | 3.57690E 08 |
| 14 | 20500 | 17666 | 1,3998E 08 | -1.7500E 04 | 3.59229E 08 |
| 15 | 27500 | 25028 | 1,4019E 08 | -9.8138E 06 | 3.60738 E 08 |
| 16 | 33750 | 31304 | 1,4003E 08 | -1.9974E 07 | 3.62215 E 08 |
| 17 | 39000 | 36689 | 1.4001 E 08 | -2.9988E 07 | 3.63661 E 08 |
| 18 | 43500 | 41326 | 1.4003 E 08 | -3.9974E 07 | 3.65076 E 08 |
| 19 | 47250 | 45339 | 1.4025 E 08 | -4.9754E 07 | 3.66460E 08 |
| 20 | 50750 | 48817 | 1.4006 E 08 | -5.9938E 07 | 3.67814E 08 |
| 21 | 53750 | 51839 | 1.3995E 08 | -7.0053E 07 | 3.69136 E 08 |
| 22 | 56250 | 54467 | 1.4002 E 08 | -7.9984E 07 | 3.70427E 08 |
| 23 | 58500 | 56750 | 1.3994E 08 | -9.0062E 07 | 3.71686E 08 |
| 24 | 60250 | 58734 | 1. 4024 E 08 | -9.9759E 07 | 3.72915 E 08 |
| 25 | 62000 | 60453 | 1.4006 E 08 | -1.0994E 08 | 3.74113E 08 |
| 26 | 63500 | 61940 | 1.3991E 08 | -1.2009E 08 | 3.75280E 08 |
| 27 | 64500 | 63218 | 1.4038 E 08 | -1.2962E 08 | 3.76416E 08 |
| 28 | 65750 | 64311 | 1. 3993 E 08 | -1.4007E 08 | 3.77520 E 08 |
| 29 | 66500 | 65237 | 1.4022 E 08 | -1.4978E 08 | 3.78594 E 08 |
| 30 | 67250 | 66014 | 1.4019 E 08 | -1.5981E 08 | 3.79637 E 08 |

Table III-5. Optimal Markov and Control Theory Policies for Class II; $B=0.9906, C_{1}=500, C_{2}=0.06$

| $\begin{gathered} \text { State } \\ 10^{7} \end{gathered}$ <br> Pounds | $\begin{gathered} \text { Markov } \\ E_{t} \end{gathered}$ | $\begin{gathered} \text { Control } \\ \text { Theory } \\ E_{t} \end{gathered}$ | $\mathrm{X}_{t+1}$ | $\Delta X_{t}$ | V (X) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0.0000 E 00 | 0.0000 E 00 | 0.00000 E 00 |
| 1 | 0 | 0 | 1.2954 E 07 | 2.9535 E 06 | 1. 36545 E 08 |
| 2 | 0 | 0 | 2.5700 E 07 | 5.7000 E 06 | 1.40932 E 08 |
| 3 | 0 | 0 | 3. 8240 E 07 | 8.2395 E 06 | 1.43278 E 08 |
| 4 | 0 | 0 | 5.0572 E 07 | 1.0572 E 07 | 1.44928E 08 |
| 5. | 0 | 0 | 6.2698 E 07 | 1.2698 E 07 | 1.46239E 08 |
| 6 | 0 | 0 | 7.4616 E 07 | 1.4616 E 07 | 1.47360E 08 |
| 7 | 0 | 0 | 8.6328 E 07 | 1.6328 E 07 | 1.48348E 08 |
| 8 | 0 | 0 | 9.7832 E 07 | 1.7832 E 07 | 1.49238 E 08 |
| 9 | 500 | 607 | 1.0878 E 08 | 1.8776 E 07 | 1. 50056 E 08 |
| 10 | 1250 | 1305 | 1.1924E 08 | 1. 9239 E 07 | 1.5082018 08 |
| 11 | 2000 | 2005 | 1.2938 E 08 | 1.9377 E 07 | 1.51541E 08 |
| 12 | 2750 | 2684 | 1.3919E 08 | 1.9190E 07 | 1.52227E 08 |
| 13 | 3500 | 3443 | 1.4868 E 08 | 1.8678 E 07 | 1.52884E 08 |
| 14 | 4250 | 4260 | 1.5784E 08 | 1.7841 E 07 | 1.53515E 08 |
| 15 | 5000 | 5100 | 1.6668E 08 | 1.6680 E 07 | 1.54125E 08 |
| 16 | 5750 | 5874 | 1.7519E 08 | 1.5194 E 07 | 1.54716E 08 |
| 17 | 6500 | 6503 | 1.8338 E 08 | 1.3383E 07 | 1.55291 E 08 |
| 18. | 7250 | 7156 | 1.9125E 08 | 1. 1248E 07 | 1.55850E 08 |
| 19 | 8000 | 8004 | 1.9879E 08 | 8.7875 E 06 | 1.56396 E 08 |
| 20 | 8750 | 8640 | 2.0600E 08 | 6.0025 E 06 | 1.56929E 08 |
| 21 | 9500 | 9418 | 2.1289E 08 | 2.8928E 06 | 1.57453E 08 |
| 22 | 10000 | 10184 | 2.1989E 08 | -1.1000E 05 | 1.57967E 08 |
| 23 | 11000 | 10928 | 2.2570 E 08 | -4.3010E 06 | 1.58468 E 08 |
| 24 | 11750 | 11615 | 2.3162E 08 | -8.3850E 06 | 1.58963E 08 |
| 25 | 12250 | 12350 | 2.3770⿺𠃊 08 | $-1.2303 E 07$ | 1.59449E 08 |
| 26 | 13250 | 13123 | 2.4247E 08 | -1.7527E 07 | 1.59928 E 08 |
| 27 | 13750 | 13813 | 2.4794E 08 | -2.2056E 07 | 1.60401E 08 |
| 28 | 14500 | 14670 | 2.5258 E 08 | -2.7419E 07 | 1.60867E 08 |
| 29 | 15250 | 15353 | 2.5689E 08 | -3.3107E 07 | 1.61327 E 08 |
| 30 | 16000 | 16057 | 2.6088 E 08 | -3.9120E 07 | 1.61782 E .08 |

# Table III-6. Optimal Markov and Control Theory Policies for Class IV; $\mathbf{B}=0.9906$ 

| $\begin{gathered} \text { State } \\ 10^{7} \\ \text { Pounds } \end{gathered}$ | $\begin{gathered} \text { Markov } \\ E_{t} \end{gathered}$ | Control <br> Theory $E_{t}$ | $\mathrm{X}_{t+1}$ | $\Delta X_{t}$ | $\mathrm{V}(\mathrm{X})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0.0000E 00 | 0.0000 E 00 | 2.11966E 03 |
| 1 | 0 | 0 | 1.2954 E 07 | 2. 9535 E 06 | 2.12650E 03 |
| 2 | 0 | 0 | 2.5700E 07 | 5.7000 E 06 | 2.12672E 03 |
| 3 | 0 | 0 | 3.8240 E 07 | 8.2395 E 06 | 2.12684E 03 |
| 4 | 0 | 0 | 5.0572 E 07 | 1.0572 E 07 | 2.12692E 03 |
| 5 | 0 | 0 | 6.2698 E 07 | 1.2698 E 07 | 2.12699E 03 |
| 6 | 0 | 0 | 7.4616E 07 | 1.4616 E 07 | 2.12704E 03 |
| 7 | 0 | 0 | 8.6328E 07 | 1.6328 E 07 | 2.12709E 03 |
| 8 | 0 | 0 | 9.7832 E 07 | 1.7832 E 07 | 2.12714E 03 |
| 9 | 0 | 0 | 1.0913 E 08 | 1.9130E. 07 | 2.12718E 03 |
| 10 | 250 | 157 | 1.2002 E 08 | 2.0024E 07 | 2.12722E 03 |
| 11 | 1500 | 4261 | 1.2981E 08 | 1.9808E 07 | 2.12725E 03 |
| 12 | 12500 | 10430 | 1.3001E 08 | 1.0005E 07 | 2.12729E 03 |
| 13 | 12000 | 15177 | 1.4000E 08 | 1.0004E 07 | 2. 12732 E 03 |
| 14 | 20500 | 18982 | 1. 3998 E 08 | -1.7500E 04 | 2.12735E 03 |
| 15 | 23250 | 22404 | 1.4519E 08 | -4.8094E 06 | 2.12738E03 |
| 16 | 25750 | 25540 | 1.5007E 08 | -9.9260E 06 | 2.12741E 03 |
| 17 | 27250 | 27841 | 1.5569E 08 | -1.4308E 07 | 2.12744E 03 |
| 18 | 29500 | 29958 | 1.5981E 08 | -2.0192E 07 | 2.12747E 03 |
| 19 | 34000 | 31998 | 1.6001E 08 | -2.9992F 07 | 2.12750E 03 |
| 20 | 33000 | 33705 | 1.6793E 08 | -3.2070E 07 | 2.12752E 03 |
| 21 | 35500 | 35154 | 1.7003E 08 | -3.9968E 07 | 2.12755E 03 |
| 22 | 35750 | 36469 | 1.7542 E 08 | -4.4580玉 07 | 2.12757E 03 |
| 23 | 36500 | 37534 | 1.7966 E 08 | -5.0341E 07 | 2.12759E 03 |
| 24 | 39000 | 38485 | 1.8028E 08 | -5.9724E 07 | 2.12762E03 |
| 25 | 40000 | 39824 | 1.8324 E 08 | -6.6763E 07 | 2.12764E 03 |
| 26 | 39000 | 39989 | 1.8992 E 08 | -7.0083E 07 | 2.12766 E 03 |
| 27 | 41000 | 40616 | 1.9019 E 08 | -7.9812E 07 | 2.12768E 03 |
| 28 | 42500 | 41222 | 1.9104E 08 | -8.8963E 07 | 2.12770E 03 |
| 29 | 40750 | 41657 | 1.9884E 08 | -9.1158E 07 | 2.12771E03 |
| 30 | 41750 | 42096 | 2.0024E 08 | -9.9761E 07 | 2.12773E 03 |

Table III-7. Optimal Markov and Control Theory Policies
for Class $V$; $B=0.9906, C_{1}=500, C_{2}=0.06$

| $\begin{gathered} \text { State } \\ 10^{7} \\ \text { Pounds } \end{gathered}$ | $\begin{gathered} \text { Markov } \\ E_{t} \end{gathered}$ | Control Theory $E_{t}$ | $\mathrm{X}_{\mathrm{t}+1}$ | $\Delta \mathrm{X}_{\mathrm{t}}$ | $\mathrm{V}(\mathrm{X})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0.0000 E 00 | 0.0000 E 00 | 2.11966E 03 |
| 1 | 0 | 0 | 1.2954 E 07 | 2.9535 E 06 | 2.12264 E 03 |
| 2 | 0 | 0 | 2.5700E 07 | 5.7000 E 06 | 2.12274E 03 |
| 3 | 0 | 0 | 3.8240E 07 | 8.2395 E 06 | 2.12279E 03 |
| 4 | 0 | 0 | 5.0572E 07 | 1.0572 E 07 | 2.12282 E 03 |
| 5 | 0 | 0 | 6.2698E 07 | 1.2698 E 07 | 2.12285E 03 |
| 6 | 0 | 0 | 7.4616E 07 | 1.4616 E 07 | 2.12288E 03 |
| 7 | 0 | 0 | 8.6328 E 07 | 1.6328 E 07 | 2.12290E 03 |
| 8 | 0 | 0 | 9.7832E 07 | 1.7832 E 07 | 2.12292E 03 |
| 9 | 500 | 607 | 1.0878E 08 | 1.8776E 07 | 2.12294E03 |
| 10 | 1250 | 1305 | 1.1924E 08 | 1.9239E 07 | 2.12295 E 03 |
| 11 | 2000 | 2005 | 1.2938E 08 | 1.9377 E 07 | 2.12297E 03 |
| 12 | 2750 | 2684 | 1. 3919 E 08 | 1.9190E 07 | 2.12298E 03 |
| 13 | 3500 | 3443 | 1.4868E 08 | 1.8678 E 07 | 2.12300 E 03 |
| 14 | 4250 | 4260 | 1.5784E 08 | 1.7841 E 07 | 2.12301 E 03 |
| 15 | 5000 | 5100 | 1.6668 E 08 | 1.6680 E 07 | 2.12303 E 03 |
| 16 | 5750 | 5874 | 1.7519E 08 | 1.5194 E 07 | 2. 12304 E 03 |
| 17 | 6500 | 6503 | 1.8338E 08 | 1.3383E 07 | 2.12305 E 03 |
| 18 | 7250 | 7156 | 1.9125E 08 | 1.1248E 07 | 2.12306E 03 |
| 19 | 8000 | 8004 | 1.9879E 08 | 8.7875 E 06 | 2.12307E 03 |
| 20 | 8750 | 8640 | 2.0600 E 08 | 6.0025 E 06 | 2.12309 E 03 |
| 21 | 9500 | 9418 | 2.1289E 08 | 2.8928 E 06 | 2.12310 E 03 |
| 22 | 10000 | 10184 | 2.1989E 08 | -1.1000E 05 | 2.12311 E 03 |
| 23 | 11000 | 10928 | 2.2570 E 08 | -4.3010E 06 | 2.12312 E 03 |
| 24 | 11750 | 11615 | 2.3162E 08 | -8.3850E 06 | 2.12313E 03 |
| 25 | 12250 | 12350 | 2.3770E 08 | $-1.2303 \mathrm{E} 07$ | 2.12314 E 03 |
| 26 | 13000 | 13123 | 2.4298E 08 | -1.7017E 07 | 2.12315 E 03 |
| 27 | 13750 | 13813 | 2.4794 E 08 | -2.2056E 07 | 2.12316 E 03 |
| 28 | 14500 | 14670 | 2.5258 E 08 | -2.7419E 07 | 2.12317E 03 |
| 29 | 15000 | 15353 | 2.5746 E 08 | -3.2538E 07 | 2.12318 E 03 |
| 30 | 16000 | 16057 | 2.6088 E 08 | -3.9120E 07 | 2.12319 E 03 |

## C.2. Optimal Fishing Strategies for Classes III and VI

The Markov model enables one to solve allocation problems of the Class III and VI variety where the objective function is not concave in the control variable. We have determined the optimal policies for certain of the Class III and VI cases, and they are presented in Tables III-8 and III-9.

Looking at columns 2 and 3 in each table, it is apparent that the population does not converge to an equilibrium unlike the previous classes. Instead, for large populations, effort allocation are large causing rapid depletion in the stock. As the population decreases all fishing is stopped and the stock is allowed to grow until it reaches a sufficiently large size to begin the harvest once again. This type of "cyclical" fishing takes advantage of the decreasing average cost of effort peculiar to Classes III and VI by employing large amounts of $E_{t}$ whenever fish are harvested.

Changes in fishing strategy for different magnitudes of the cost parameter, $\mathrm{C}_{3}$, are illustrated in Figures III-5 and III-6. For larger values of $C_{3}$ the fishing cycles are more pronounced. The minimum population where fishing begins is larger along with the size of the harvest and the amount of effort employed. With larger values of $C_{3}$ the average cost of effort at all levels of $E_{t}$ increases. Consequently, to produce at the same average cost it is necessary to employ greater amounts of effort resulting in larger harvests and

Table III-8. Optimal Markov Policy for Class III;

$$
B=0.9906, C_{3}=100,000
$$

| State $10^{7}$ <br> Pounds | $E_{t}$ | $\mathrm{X}_{t+1}$ | $\Delta \mathrm{X}_{\mathrm{t}}$ | V (X) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.0000 E 00 | 0.0000 E 00 | 0.00000 E 00 |
| 1 | 0 | 1.2954 E 07 | 2.9535E 06 | 2.54881E 08 |
| 2 | 0 | 2.5700E 07 | 5.7000E 06 | 2.63070 E 08 |
| 3 | 0 | 3.8240E 07 | 8.2395 E 06 | 2.67449 E 08 |
| 4 | 0 | 5.0572 E 07 | 1.0572 E 07 | 2.70529 E 08 |
| 5 | 0 | 6.2698 E 07 | 1.2698 E 07 | 2.72977 E 08 |
| 6 | 0 | 7.4616 E 07 | 1.4616 E 07 | 2.75070E 08 |
| 7 | 0 | 8.6328 E 07 | 1.6328 E 07 | 2.76913 E 08 |
| 8 | 0 | 9.7832 E 07 | 1.7832 E 07 | 2.78575E 08 |
| 9 | 0 | 1.0913 E 08 | 1.9130 E 07 | 2.80102 E 08 |
| 10 | 0 | 1.2022 E 08 | 2.0220 E 07 | 2.81527 E 08 |
| 11 | 0 | 1. 3110 E 08 | 2.1104 E 07 | 2.82877E 08 |
| 12 | 0 | 1.4178 E 08 | 2.1780 E 07 | 2.8417 lE 08 |
| 13 | 0 | 1.5225 E 08 | 2.2250 E 07 | 2.85426E 08 |
| 14 | 0 | 1.6251 E 08 | 2.2512 E 07 | 2.86652E 08 |
| 15 | 0 | 1.7257 E 08 | 2.2568 E 07 | 2.87864E 08 |
| 16 | 0 | 1.8242 E 08 | 2.2416 E 07 | 2.89067E 08 |
| 17 | 0 | 1.9206E 08 | 2.2058 E 07 | 2.90280E 08 |
| 18 | 0 | 2.0149E 08 | 2.1492E 07 | 2.91507 E 08 |
| 19 | 74250 | 9.9976 E 07 | -9.0024E 07 | 2.92764E 08 |
| 20 | 76250 | 1.0003 E 08 | -9.9973E 07 | 2.94080E 08 |
| 21 | 78000 | 9.9971 E 07 | -1.1003E 08 | 2.95371E 08 |
| 22 | 79500 | 9.9864 E 07 | -1.2014E 08 | 2.96637E 08 |
| 23 | 80750 | 9.9765E 07 | -1.3023E 08 | 2.97875E 08 |
| 24 | 81750 | 9.9735 E 07 | -1.4027E 08 | 2.99087E 08 |
| 25 | 87500 | 9.0019 E 07 | -1.5998E 08 | 3.00271 E 08 |
| 26 | 87750 | 9.0418 E 07 | -1.6958E 08 | 3.01430 E 08 |
| 27 | 88250 | 9.0042E 07 | -1.7996E 08 | 3.02561 E 08 |
| 28 | 88250 | 9.0479 E 07 | -1.8952E 08 | 3.03662E 08 |
| 29 | 88500 | 9.0139 E 07 | -1.9986E 08 | 3.04734E 08 |
| 30 | . 88500 | 9.9143E 07 | -2.0986E 08 | 3.05776E 08 |

Table III-9. Optimal Markov Policy for Class IV;
$B=0.9906, C_{3}=100,000$

| State $10^{7}$ <br> Pounds | $\mathrm{E}_{\mathrm{t}}$ | $\mathrm{X}_{\mathrm{t}+1}$ | $\Delta X_{t}$ | V(X) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0.0000 E 00 | 0.0000 E 08 | 2.11966E 03 |
| 1 | 0 | 1.2954 E 07 | 2.9535 E 06 | 2.12468E 03 |
| 2 | 0 | 2.5700 E 07 | 5.7000E 06 | 2. 12484E 03 |
| 3 | 0 | 3.8240E 07 | 8.2395 E 06 | 2. 12492 E 03 |
| 4 | 0 | 5.0572 E 07 | 1.0572 E 07 | 2.12498E 03 |
| 5 | 0 | 6.2698 E 07 | 1.2698 E 07 | 2.12503E 03 |
| 6 | 0 | 7.4616 E 07 | 1.4616 E 07 | 2.12507E 03 |
| 7 | 0 | 8.6328 E 07 | 1.6328 E 07 | 2.125l1E 03 |
| 8 | 0 | 9.7832 E 07 | 1.7832 E 07. | 2.12514E 03 |
| 9 | 0 | 1.0913 E 08 | 1.9130E 07 | 2.12517E 03 |
| 10 | 0 | 1.2022 E 08 | 2.0220E 07 | 2.12520E 03 |
| 11 | 0 | 1. 3110 E 08 | 2.1104E 07 | 2.12523 E 03 |
| 12 | 0 | 1.4178 E 08 | 2.1780E 07 | 2.12525E 03 |
| 13 | 0 | 1.5225E 08 | 2.2250 E 07 | 2.12528E 03 |
| 14 | 0 | 1.6251 E 08 | 2.2512 E 07 | 2.12530E 03 |
| 15 | 0 | 1.7257E 08 | 2.2568 E 07 | 2.12533E 03 |
| 16 | 0 | 1.8242E 08 | 2.2416 E 07 | 2.12535E 03 |
| 17 | 54000 | 1.1999E 08 | -5.0006E 07 | 2.12537E 03 |
| 18 | 57500 | 1.2024 E 08 | -5.9756E 07 | 2.12540E 03 |
| 19 | 58500 | 1.2347 E 08 | -6.6533E 07 | 2.12542E 03 |
| 20 | 57250 | 1.2986 E 08 | -7.0143E 07 | 2.12544E 03 |
| 21 | 59750 | 1. 3006 E 08 | $-7.9944 \mathrm{E} 07$ | 2.12547E 03 |
| 22 | 57250 | 1. 3829 E 08 | -8.1711E 07 | 2.12549E 03 |
| 23 | 58500 | 1. 3994 E 08 | -9.0062E 07 | 2.12551E03 |
| 24 | 58000 | 1.4448E 08 | -9.5520E 07 | 2.12553E 03 |
| 25 | 55000 | 1.5380 E 08 | -9.6200E 07 | 2.12555E 03 |
| 26 | 53750 | 1.5981E 08 | -1.0019E 08 | 2.12557E 03 |
| 27 | 45750 | 1.8012 E 08 | -8.9880E 07 | 2.12559E 03 |
| 28 | 47500 | 1.8005 E 08 | -9.9953E 07 | 2.12561E 03 |
| 29 | 48500 | 1.8120E 08 | -1.0880E 08 | 2.12562E 03 |
| 30 | 47250 | 1.8729 E 08 | -1.1271E 08 | 2.12564E 03 |



Figure III-5. Class III Fishing Policies

$c_{3} 100,000$

$c_{3} 1,000,000$


Figure II-6. Class VI Fishing Policies
greater fluctuations in the population. With smaller values of $C_{3}$, the opposite occurs and fluctuations in the population are less pronounced. Comparing Figures III-5 and III-6 we notice that the decreasing marginal utility of rents causes Class VI cyclical fishing to be less severe than Class III programs. 22

Most management programs are based on "steady state" fishing. Usually an ideal steady state population is determined from biological and economic considerations. Then with a program of controlled resource use the population is allowed to converge to its optimal value. Although there is not always common agreement among economists and biologists as to the ideal value of the stationary population, the principle of steady state fishing is widely accepted.

The theoretical basis for steady state fishing comes from the control theory analysis of optimal fishing behavior. These models only yield solutions for certain specifications of the rent function which unfortunately preclude the possibility of economies of scale in supplying effort. Our results suggest that cyclical as opposed to steady state fishing is optimal for situations where there are decreasing average costs of supplying effort. Since there has been little estimation of cost functions for various fisheries, the case for steady state fishing may have arisen partially because of the analytical convenience of assuming convex cost functions for use in control theory analysis.

## D. POLICY DISCUSSION

In Chapter IV, the foregoing analysis will be modified to allow for conditions of uncertainty. However, assuming for the present that the economic and biological processes of the fishery are non stochastic, what policy recommendations arise from our deterministic models of optimal resource use? We will focus on the results of Classes II and III. Class I programs are not applicable to the yellowfin tuna fishery since the cost of effort is not zero, and Classes IV - VI are only pertinent to the analysis of resource management under uncertainty. It is apparent that optimal allocation policies are sensitive to the specification of the cost of effort function. However, even without a specific knowledge of the cost structure, certain general conclusions can be inferred from our analysis.

The most important implication of our models is that the tuna Commission's policy of maximizing the sustainable physical yield from the fishery is generally not optimal. The policy is deficient in that it fails to specify the rate at which the fishery should approach its optimal steady state population, if it is initially displaced from it. However, even ignoring this problem, and assuming for the present that steady state as opposed to cyclical fishing is optimal, looking at Table III- 3 we find that for Class II programs the stock tends towards a steady state size that generally differs from the maximum yield population of $14.7 \times 10^{7}$ pounds. Of course there is
no reason to believe the two steady state populations will coincide since one population is based on maximizing the physical yield from the fishery, and the other is derived by maximizing the discounted economic benefit of the resource. ${ }^{23}$

Although the inefficiencies of the maximum yield policy have been discussed repeatedly in the literature, 24 the poitoy sin sinetained by most fishery commissions because it is deemed to be "workable." Proponents of the policy argue that if under the optimal economic program the stock converges to a steady state size close to the maximum sustainable yield population, then the maximum yield policy should be adopted since it is probably easier for managers to follow and to understand. Considered pragmatically, the argument is appealing although it rests primarily on two as sumptions that require further consideration.

First, the discrepeancy between optimal economic steady state populations and the maximum yield population may not be small as we assumed above. Looking at Table III-3, the steady state stocks $\mathrm{X}^{*}$ for Class II may be considerably greater than the maximum sustainable yield population, particularly as costs increase. From the standpoint of economic efficiency, the allocation of effort and the resulting annual catch are excessive under the maximum yield policy and they should be reduced allowing the population to reach its optimal level at $\mathrm{X}^{*}$. As a political matter however, such a program of effort reduction may be very difficult to introduce given the current
regulatory policy in the Eastern Pacific yellowfin tuna fishery. Historically the Commission has adhered to an annual catch quota by restricting the season during which fishing is allowed. This system only regulates the size of the catch, but not the number of vessels oper ating in the fishery. As a result, the fishery has become vastly over capitalized. The current size of the tuna fleet is much greater than the number of ships required to catch the annual quota efficiently. ${ }^{25}$ The Commission also grants special quotas to lesser developed countries with relatively small fishing fleets. Recently, the demand for special quotas has increased to allow the developing fishing countries such as Mexico to build a larger and more modern fleet, and to accommodate new countries wishing to enter the fishery. In light of this increased competition among countries for quota shares, and the fact that there is already excess capacity in the fleet, it seems unlikely that a program designed to reduce the allocation of effort and the resulting total annual catch would be well received.

One alternative to the current system would be to reduce the size of the fleet, the allocation of effort, and the annual catch through the sale of a limited number of licenses. Presumably if the licenses were sold competitively, only the most efficient ships could afford to fish and the proceeds from the licenses could be used to compensate those boats forced out of the fleet. 26

The second major assumption behind the argument for choosing a maximum sustained yield policy is that steady state fishing is
optimal. However, our results for Class III suggest that cyclical as opposed to steady state fishing is optimal for situations where there are decreasing average costs of supplying effort. Presumably under these conditions, the fleet would fish intensively for yellowfin tuna over a short period of time, then move on to other fisheries, returning to the tuna fishery once the stock had replenished itself. In this way, the stock size would fluctuate within certain limits, depending on the cost conditions, in the manner depicted in Figure III-5.

In conclusion, we suggest that the current policy of maximizing the sustainable physical yield from the yellowfin tuna fishery is generally not optimal from the viewpoint of economic efficiency. Arguments for the policy stem from the belief that it is easier to understand and more readily accepted by the decision making hierarchy than alternative programs. If this is true, the administrative advantages of the maximum yield policy need to be compared with the loss in economic rents from the fishery that are incurred by adopting this program. Where there is a wide discrepancy between the optimal economic maximum sustained yield programs, the loss in economic rents may be quite large.

One group of fishery economists maintain that whatever difference there may be between optimal economic and maximum sustained yield policies is likely to be swamped by the uncertainty existing with regard to the population dynamics of the fishery. This contention will be examined in more detail in Chapter IV.

## APPENDIX III-A

Estimation of the Yellowfin Demand Curve

Most of the yellowfin tuna purchased by U.S. canneries comes from the Eastern Pacific fishery. Table III-10 shows the U.S. price of raw yellowfin and the annual catch of yellowfin from the Eastern Pacific for the years of 1958-1972. Prices were relatively stable from 1958 until 1966 when they increased significantly. Since then they have fluctuated betwee 13 and 17 cents per pound.

For the Eastern Pacific yellowfin fishery we hypothesize that the price is not influenced by the size of the total catch. This is because the price of tuna is determined on the world market and the

Table III-10. Catches and Prices of Yellowfin Tuna from 1958-1972

| Year | Yellowfin Catch in the Eastern Pacific (millions of pounds) | $\begin{gathered} \text { Real Price per }{ }^{27} \text { Pound } \\ (1956 \text { dollars }) \end{gathered}$ |
| :---: | :---: | :---: |
| 1958 | 148.4 | 0.130 |
| 1959 | 140.5 | 0.125 |
| 1960 | 244.3 | 0.120 |
| 1961 | 230.9 | 0.123 |
| 1962 | 174.1 | 0.146 |
| 1963 | 145.5 | 0.127 |
| 1964 | 203.9 | 0.124 |
| 1965 | 180.1 | 0.132 |
| 1966 | 182.3 | 0.167 |
| 1967 | 179.3 | 0.128 |
| 1968 | 229.1 | 0.138 |
| 1969 | 252.8 | 0.138 |
| 1970 | 283.5 | 0.148 |
| 1971 | 226.3 | 0.167 |
| 1972 | 304.1 | 0.166 |

quantity of yellowfin taken from the Eastern Pacific is only a small fraction of the total world supply of tuna. Using ordinary least squares and the price and landings data in Table III-10 for 1958-1972 we tested this hypothesis by estimating the following equation expressing price, $p$, as a function of the total landings, $X$, and time, $t$.

$$
\begin{align*}
& \mathrm{p}= 12.6-0.0034 \mathrm{X}+0.29 \mathrm{t}  \tag{48}\\
&(8.00) \\
&(-0.04)(2.71) \\
& F= 5.98 \\
& \text { D. W. }=2.096
\end{align*}
$$

The $t$ statistics appear in the parenthesis directly below each of the estimated coefficients. The coefficient for landings is not significant; this confirms our hypothesis that prices are not affected by landings. The time coefficient, which was significant and positive, presumably measures the increase in price caused by population growth and better living standards. Despite this, the time trend for prices is difficult to predict. The primary increase in price is only between two periods, (1958-1965) and (1966-1972), with no systematic price increases occurring within each period.

Based on these results, price is assumed to be independent of total landings with $p=\$ 0.15$, the mean price during the 1966 1972 period. While our model allows for random variations in price, the mean price level is assumed to be constant. In Chapter II, we suggest modifications in the present model that incorporate systematic increases or decreases in the price level over time.

## APPENDIX III-B

## Determination of G

In Chapter IV, $R$ is a random variable given by

$$
\begin{equation*}
R\left(X_{t}, E_{t}\right)=p\left(\eta_{2} k X_{t} E_{t}\right)-C\left(E_{t}\right) \tag{49}
\end{equation*}
$$

where $\eta_{2}$ is a random variable with $\varepsilon\left(\eta_{2}\right)=1$. A lower bound for $R$ is $-C\left(E_{t}\right)$ which occurs for $\eta_{2}=0$. If $R+G>0$ for all $R$ then $G$ must be greater than the maximum value of $C\left(E_{t}\right)$. An upper bound for $C\left(\mathrm{E}_{\mathrm{t}}\right)$ is determined as follows: A necessary condition for $E_{t}>0$ is that

$$
\begin{equation*}
\varepsilon(U(R+G))-U(G)>0 \tag{50}
\end{equation*}
$$

Now $U(\varepsilon(R)+G)>\varepsilon U(R+G))$ since $U=\ln (R+G)$ is strictly concave. Thus

$$
\begin{equation*}
U(\varepsilon(R)+G)-U(G)>\varepsilon U(R+G)-U(G)>0 \tag{5l}
\end{equation*}
$$

which implies

$$
\begin{equation*}
p k X_{t} E_{t}-C\left(E_{t}\right)>0 \tag{52}
\end{equation*}
$$

since $U^{\prime}>0$. In particular,

$$
\begin{equation*}
\bar{\Delta} p k X_{t} E_{t}-\bar{\Delta} C\left(E_{t}\right)>0 \tag{53}
\end{equation*}
$$

so that in any time period with $E_{t}>0$, the expected revenue from the harvest must exceed the costs of production. In section A. 3. a. we specify that the largest possible population in the fishery is
denoted by $X_{30}$, where $X_{30}=30 \times 10^{7}$ pounds. Clearly the maximum catch cannot exceed the maximum population, $X_{30}$, so that an upper bound for total revenue is

$$
\mathrm{pX}_{30}=(\$ 0.15)\left(30 \times 10^{7}\right)=4.5 \times 10^{7}
$$

From equations (53) and (54) we obtain

$$
\begin{equation*}
4.5 \times 10^{7} \geq \mathrm{p} \bar{\Delta} \mathrm{k} X_{t} \mathrm{E}_{\mathrm{t}}>\bar{\Delta} \mathrm{C}\left(\mathrm{E}_{\mathrm{t}}\right) \tag{55}
\end{equation*}
$$

and that therefore an upper bound (not necessarily least upper bound) for $C\left(E_{t}\right)$ is $4.5 \times 10^{8}$, which determines the value of $G$.

APPENDIX III-C<br>Description of the Markov Decision Program

The computer program used to find the computational solution for Classes I - VI is described in this section. The program implements Howard's Algorithm (see Appendix II) to select the optimal policy for allocating effort in the fishery. The algorithm involves calculating the present value of the stream of economic rents accruing to the fishery for a given policy and then determining a better policy if one exists by using a policy improvement routine. A flow chart for the program is presented in Figure III-7. Each procedure follows in the order indicated by the numbers 1-14, and an explanation of each step is given below. The program was written in ALGOL and is designed to run on the Burroughs 6700 at the University of California, San Diego. A complete listing of the program may be obtained from the author.

## 1. Program begins.

2. The values for the parameters, $a, b, C_{1}, C_{2}, C_{3}, B, k$, $p$, and $G$ are read into the program.
3. There are 31 possible states stored in the one dimensional array denoted by $X[I] ; I=0,1,2, \ldots, 30$. Each state corresponds to a certain population size given by $X[I]=I \times 10^{7}$ pounds. $X[0]$ represents the minimum stock of 0 pounds and $X[30]$ is the largest


Figure III-7. Flow Chart
possible population. $\mathrm{X}[30]$ is greater than the maximum sustainable stock, $29.4 \times 10^{7}$ pounds and thus is large enough to include all probable values that the stock may assume.
4. In each state there are a finite number of possible effort allocations. Each allocation is stored in the two dimensional array, $E[I, L]$ where $I$ represents the state and $L$ is an index of the amount of effort used (measured in terms of boat days at sea). The effort allocations are expressed as multiples of 250 boat days ranging from a minimum of 0 days to a maximum amount which is determined by the economic or technological constraints for each case considered. For example, the maximum effort allocation will obviously never exceed the rate of effort which maximizes the immediate returns from the fishery, i.e.

$$
E_{t} \leq E_{M} \text { where } E_{M} \text { is defined by } \frac{\partial E\left(X_{1} E_{t}\right)}{\partial E_{t}}=0
$$

However, $E_{t}<E_{M}$ if there are technological limits on the maximum amount of effort which can be used during a given time interval.
5. In each state, I, the payoff or utility from the effort allocation $L$, is stored in the 2-dimensional array, $Q[I, L]$ where

$$
Q[I, L]=U(R(X[I], E[I, L])) \stackrel{\Delta}{\Delta}
$$

and $R$ is the rate at which rents accrue to the fishery and $\bar{\Delta}$ is the length of each time interval.
6. The transition probability of moving from state I to state $J$ in one time period if the effort allocation during the interval is $E[I, L]$ is stored in the two dimensional array denoted by $P[I, J]^{\text {L }}$. The method for computing these transition probabilities is discussed in section C. 4. of Chapter II.
7. At this stage the states, effort allocations, payoffs, and transition probabilities have been specified and stored in the computer's memory system, and we are ready to begin the Value Determination Routine (see Appendix II). The initial policy $K$ is read into the program. A policy is a rule for selecting an effort allocation in each state. Thus $K$ is a $1 \times 31$ row vector, $K=\left[k_{0}, k_{1}\right.$, $k_{2}, \ldots, k_{30}$ ] where the $i^{\text {th }}$ entry specified the effort allocation for the $i^{\text {th }}$ state. The entries of $K$ are selecting according to reasonable guesses about the optimal amount of effort to allocate in each state.
8. The column vector $Q^{K}$ and matrix $P^{K}$ to be used in step 9 are formed by selecting elements $Q\left[I, K_{I}\right]$ and $P[I, J]^{K_{I}}$ respectively from the memory storage for all $I, J=0,1,2, \ldots, 30$.
9. A procedure for solving systems of linear equations called FORMOPAK, available from the U.C.S.D. computing center, is used to solve a system of 31 equations in 31 unknowns, $\mathrm{V}[\mathrm{I}]^{\mathrm{K}} ; \mathrm{I}=0,1$, $2, \ldots, 30$, where the system is given by,

$$
\begin{aligned}
& =\left[\begin{array}{c}
Q\left[0, K_{0}\right] \\
Q\left[1, K_{1}\right] \\
\cdot \\
\cdot \\
\vdots\left[30, K_{30}\right]
\end{array}\right]
\end{aligned}
$$

$\mathrm{V}[\mathrm{I}]^{\mathrm{K}}$ is the present value of the economic returns to the fishery for policy $K$ assuming the initial stock equals $\mathrm{X}[\mathrm{I}]$.
10. The values of $\mathrm{V}[\mathrm{I}]{ }^{\mathrm{K}_{\mathrm{I}}}, \mathrm{E}\left[\mathrm{I}, \mathrm{K}_{\mathrm{I}}\right]$ are printed for $I=0,1,2, \ldots, 30$.
11. The Policy Improvement Routine begins by calculating the quantity,

$$
\Phi[I, L]=Q[I, L]+B \sum_{j=0}^{30} P[I, J]^{L} v[I]^{K}
$$

12. A new policy $K^{\prime}$ is formed by selecting the effort allocation in each state, $\underline{I}$, that maximizes $\Phi[I, L]$ for all possible $\underline{L}$.
13. The new policy $K^{\prime}$ is compared to the previous policy $K$. If $K^{\prime}$ and $K$ are identical then by the convergence property of the Howard Algorithm $K$ is the optimal policy and the program ends; otherwise the program returns for another iterative cycle beginning at step 14.
14. $K^{\prime}$ becomes the new policy for the Value Determination Rouṭine and steps ( $8-14$ ) are repeated until the optimal policy is determined.

## APPENDIX III-D

## Steady State Properties

Consider the problem

$$
\begin{equation*}
\max _{E_{t}} \sum_{t=0}^{\infty} B^{t} U\left(R\left(X_{t}, E_{t}\right)\right) \bar{\Delta} \tag{54}
\end{equation*}
$$

subject to

$$
\begin{equation*}
X_{t+1}=X_{t}+\left[\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\right] \bar{\Delta} ; X_{0}=\bar{X} \tag{55}
\end{equation*}
$$

Assume the solution to (54) indicates that the optimal program converges to the same steady state $\left(X^{*}, \lambda^{*}, E^{*}\right)$ for all initial stocks $\mathrm{X}_{0}$ defined by the conditions

$$
\begin{align*}
& U^{\prime} \frac{\partial R}{\partial E}-B \lambda^{*} k X^{*}=0  \tag{56}\\
& \Delta X_{t}=\left(\left(a-b X^{*}\right) X^{*}-k X^{*} E^{*}\right) \bar{\Delta}=0  \tag{57}\\
& \Delta \lambda_{t}=-U^{\prime} \frac{\partial R}{\partial X}-B \lambda^{*}\left(a-2 b X^{*}-\frac{1-B}{\bar{\Delta} B}-k E^{*}\right)=0 \tag{58}
\end{align*}
$$

Clearly the solution to

$$
\begin{equation*}
\underset{E_{t}}{\operatorname{Max}} \sum_{t=0}^{\infty} B^{t}\left[k_{1} U\left(R\left(X_{t}, E_{t}\right)+k_{2}\right] \bar{\Delta} ; k_{1}, k_{2} \text { constants, } k_{1}>0\right. \tag{59}
\end{equation*}
$$

$$
=\max _{E_{t}} k_{1} \sum_{t=0}^{\infty} B^{t} U\left(R\left(X_{t}, E_{t}\right)\right) \bar{\Delta}+\frac{k_{2} \bar{\Delta}}{1-B}
$$

subject to (55) is identical to the solution to (54). Solutions are invariant to positive linear transformations of the objective function. Consider the problem

$$
\begin{equation*}
\max _{E_{t}} \sum_{t=0}^{\infty} B^{t}\left[\phi\left(U\left(R\left(X_{t}, E_{t}\right)+K_{2}\right)\right] \bar{\Delta} ; \phi^{\prime}>0\right. \tag{60}
\end{equation*}
$$

subject to (55), where $\phi$ is a concave function. Generally solutions to (54) and (60) will not coincide; however, we can show that the optimal steady state values of $X^{*}$ are identical. For (60) a stationary equilibrium occurs for values of $X^{*}, \lambda^{*}$, and $E^{*}$ satisfying the three equations:

$$
\begin{gather*}
\phi^{\prime} U^{\prime} \frac{\partial R}{\partial E}-B \lambda^{*} k X^{*}=0  \tag{61}\\
\Delta X_{t}=\left(\left(a-b X^{*}\right) X^{*}-k X^{*} E^{*}\right) \bar{\Delta}=0  \tag{62}\\
\Delta \lambda_{t}=-\phi^{\prime} U^{\prime} \frac{\partial R}{\partial X}-B \lambda^{*}\left(a-2 b X^{*}-\frac{1-B}{\overline{\Delta B}}\right)-k E^{*}=0 \tag{63}
\end{gather*}
$$

Since solutions are insensitive to positive linear transformations of the objective function we may arbitrarily set $\phi^{\prime}\left(\mathrm{U}\left(\mathrm{R}\left(\mathrm{X}^{*}, \mathrm{E}^{*}\right)\right)\right.$ equal to one in the equations above. Now it is obvious that these equations also define a steady state equilibrium for (54) as well. One can also . show that the optimal program for problem (60) converges to the steady state $\left(X^{*}, \lambda^{*}, E^{*}\right)$ for all initial stock $X_{0}$.
${ }^{1}$ For a discussion of the Commission's policy in regard to setting quotas, see the Annual Report of the Inter-American Tropical Tuna Commission, 1966-1974.
${ }^{2}$ The formulation of this model appears in Schaefer (1957).
${ }^{3}$ In order to be consistent with the rest of this chapter, the equations appearing in this section are written in discrete form, although these equations most often appear in continuous form in the references that I have cited.
"The "fishable stock" includes all of those fish that are of sufficient size and age for capture by the fishery.
${ }^{5}$ Without dealing specifically with the determinants of growth, new "recruitment," existing population "growth, " "fishing mortality," and "natural mortality," equation (5) is an aggregate description of a commercial fishery. It is beyond the scope of this discussion to mention conditions under which (5) is a good approximation to the dynamics of an exploited population. However, these conditions are examined thoroughly in Beverton and Holt (1957, pp. 329-30), and Schaefer (1957).
${ }^{6}$ The difficulties of obtaining costs of operation data from coat owners is discussed in Green and Broadhead (1964).
$7^{7}$ For example, a large portion of the costs of operation reported by owners is the labor bill. The crew is paid a fixed percentage of the total value of the catch; see Green and Broadhead (1964). In accounting terms, labor costs vary with the value of the catch, while the opportunity cost of labor is presumably insensitive to this value.
${ }^{8}$ Another example where the cost of effort might be zero is in the case of sport fishing where people fish for the sake of relaxation and enjoyment.
${ }^{9}$ For a description of the purse seine fishing technology, see Green, Perrin, and Petrich (1971).
${ }^{10}$ Recently the Commission has increased catch quotas to allow the population to reach a smaller size. This was done to obtain more information on the population dynamics of the stock.

11
For the Schaefer model, the population producing the maximum sustained yield, is one half the size of the maximum sustainable population.

12 Unless $E_{\text {max }}$ is given by technological constraints, it is logically determined by the following restriction on the total catch during each time period. Clearly, the total catch cannot exceed the size of the stock available at the beginning of the period, i.e.

$$
\bar{\Delta} k X_{t} E_{t} \leq X_{t}
$$

implying that $E_{t} \leq \frac{1}{\bar{\Delta} k}=E_{\max }$.
${ }^{13}$ The necessary conditions set forth by the Maximum Principle of Pontryagin are discussed in Pontryagin et al. (1962) and Lee and Marcus (1967). A discussion of the discrete maximum principle is found in Halkin (1966) and Fan and Wang (1964).

14 See Intrilligator (1971).
${ }^{15} \Delta X_{t}=\frac{X_{t+1}-X_{t}}{\bar{\Delta}}$, and $\Delta \lambda_{t}=\frac{\lambda_{t+1}-\lambda_{t}}{\bar{\Delta}}$,
$16_{\text {In equations ( }}$ (30) and (31), $E_{t}=E_{t}\left(X_{t}, \lambda_{t+1}\right)$ to emphasize
is a function of $X_{t}$ and $\lambda_{t+1}$. that $E_{t}$ is a function of $X_{t}$ and $\lambda_{t+1}$.
${ }^{17}$ If $\frac{l-B}{\overline{\Delta B}}>a$, then it is optimal to deplete the stock to the point where the immediate profit from further harvesting is negative. If an immediate profit can be made from harvesting the last remaining animals, then it is optimal to drive the population to extinction; see Clark (1973).
${ }^{18}$ Because $\left(\mathrm{X}^{*} \lambda^{*}\right)$ is a saddle point only one path converges to the stationary equilibrium. Since the path never reaches ( $X^{*} \lambda^{*}$ ) one can never confirm whether or not he has located the convergent path. However, arbitrarily close approximations to the optimal path can be achieved by determining "reasonably convergent" paths. For our purposes, paths which come within one thousandth of one percent of the value of $X^{*}$ without leaving regions $I$ and III are considered reasonably convergent.
${ }^{19}$ The values used for the biological parameters in each program we analyzed correspond to the figures reported in Table III-1. For the economic parameters, $p=\$ 0.15, C_{1}=\$ 500$, and $G=4.5 \times 10^{8}$. The values assumed for $C_{2}, C_{3}$ and $B$ are specified with the results for each case.
${ }^{20}$ To compute the steady state population for the Markov model we fir st obtain optimal effort levels as a function of the states. Effort allocations for populations falling between two states are calculated by linear interpolation. A steady state is then determined by finding the population for which the optimal allocation of effort and the level of effort necessary to maintain the population at its current size coincide.
${ }^{21}$ Note that the control theory effort allocations repcrted are the same for Classes II and V. Calculating the optimal paths for population and effort for Class $V$ programs is quite difficult requiring the repeated solution to a 7 th order polynomial in $E_{t}$. To simplify the procedure we approximated $E_{t}$ with the optimal effort allocation derived for the Class II case. The approximation was excellent for Class $V$ yielding optimal paths for $X_{t}$ and $\lambda_{t+1}$ that were reasonably convergent.
${ }^{22}$ In fact, for relatively small values of $C_{3}$ the cycles decrease and the optimal policy degenerates into a steady state fishing strategy. For example, see the Class VI program in Figure III-6 for $C_{3}=10,000$.
${ }^{23}$ It is not surprising to find that steady state stocks for Class I are quite close to the maximum sustained physical yield population of $14.7 \times 10^{7}$ pounds. Since the costs of effort are zero, maximizing the physical yield from the fishery is nearly identical to maximizing the discounted stream of economic rents.
${ }^{24}$ In particular, see Christy and Scott (1965).
${ }^{25}$ An indication of the dramatic increase in the fleet and in the competition among fishermen for yellowfin is reflected in the fact that the length of the legal fishing season has decreased from nine months to three months in the last ten years.
${ }^{26}$ A complete discussion of various regulatory programs that might be applied in the Eastern Pacific yellowfin tuna fishery is beyond the scope of this study. However, for a good discussion of fishery regulation in general, see Christy and Scott (1965), Coase (1969), and Scott (1969).
${ }^{27}$ Real prices in terms of 1956 dollars are obtained by dividing the current price of tuna by the wholesale price index for all commodities where $1956=100$.

28
Data on the catch of yellowfin in the Eastern Pacific was obtained from the Annual Report of the Inter-American Tropical Tuna Commission, 1972.

29
Data on current yellowfin prices was obtained from Historical Statistics, U.S.F.W.S., Bureau of Commercial Fisheries and Market News, U.S. Dept. of Commerce.

30
The wholesale price index was obtained from the Economic Report of the President, 1973.

Chapter IV
A STOCHASTIC MODEL OF THE FISHERY

## A. INTRODUCTION

From the literature on optimal resource allocation under stochastic conditions, our model is most closely related to the important contributions of Leland (1974), Brock and Mirman (1972), Levhari and Srinivasan (1966), and Phelps (1962). These analyses deal with the problem of finding optimal consumption or investment policies from the point of view of an individual or a single sector economy (in the case of Brock and Mirman), when the rate of growth of the capital stock is random. For the case of an ocean fishery, we are interested in determining the optimal resource consumption strategies where the capital stock is represented by the size of the fish population.

There are two aspects of our model distinguishing it from previous analysis. First, with a given allocation of fishing effort, the total value of the resource consumed is random, varying either with fluctuations in the catch rate or changes in the market value of output. In the papers mentioned above, consumption is non random and completely determined by the decision maker. Second, the net value of consumption in the fishery is the value of the catch minus the cost of effort. The net value not only varies with the caich but also depends on the stock since the effort required for a given catch decreases with population. In contrast, the net value of consumption is independent of the stock in standard models of capital
accumulation. Because of this second difference, the standard techniques appearing in the literature to solve for optimal consumption policies are not applicable to our analysis. However, with the Markov Decision Process model developed in Chapters II and III, we can characterize optimal allocation programs in the fishery using numerical methods.

## A.1. Questions Concerning the Effect of Uncertainty on Optimal Resource Allocation

The following questions will be of particular interest to us in analyzing these programs:

1. How do different attitudes toward social risk bearing as regards variations in resource' rents, affect optimal decision rules when there is uncertainty about:
a. Consumer demand for the fishery products,
b. The rate of resource depletion (for given effort allocations), and the rate of natural growth of the population?
2. What is the effect of increased uncertainty about consumer demand, and growth and depletion rates on optimal programs?
3. Is it possible to account for the social attitudes towards risk bearing in the social discount rate?
4. What are the practical policy implications for the Eastern Pacific yellowfin tuna fishery obtained from analyzing the effect of uncertainty on optimal resource allocation in the fishery? Are decision rules for resource consumption derived by assuming
deterministic conditions in the fishery appropriate in a stochastic environment? Which economic and biological parameters are crucial in determining optimal programs, and what data are needed to estimate these parameters accurately?

Analyzing these questions will provide us with a convenient means of organizing and discussing the results we have obtained. However, before beginning our discussion it would be helpful to describe and list the programs included in our study.

## A.2. Description of Cases

Each of the cases analyzed simulates a different set of conditions in the fishery. These programs are characterized by the kind of cost conditions for effort, by the types of stochastic variation in the economic and biological parameters, and by the social attitudes toward risk assumed in the model.

The resource allocation problem in each case is to

$$
\underset{E_{t}}{\operatorname{maximize}} \sum_{t=0}^{\infty} B^{t} \varepsilon U\left[R\left(X_{t}, E_{t}\right)\right] \bar{\Delta}
$$

subject to $X_{t+1}=X_{t}+\left[\eta_{1 t}\left(a-b X_{t}\right) X_{t}-\eta_{2 t} k X_{t} E_{t}\right] \Delta$

The notation and designation of variables employed here is consistent with that used in Chapter III. Recall that $E$ represents the expectation operator, and $\eta_{l t}$ and $\eta_{2 t}$ are the growth and depletion rate parameters respectively.

Solutions to the allocation problem are characterized for the cases listed in Table IV-1. For convenience we will refer to a particular program by the label corresponding to it in the table. The Roman numerals in each label refer to the specification of the utility and rent functions, reflecting the cost conditions and attitudes toward risk bearing that prevail in the fishery. The letters indicate the type of stochastic variation in the model, for which we assume there are four possibilities. In the yellowfin tuna and other fisheries ${ }^{1}$ fluctuations in the consumer demand and changes in the availability of substitute food products cause the price of the resource to vary. To assess the impact of these variations on resource allocation we consider cases where price is random, denoted by the letter $P$ in the labels appearing in Table IV-1. Because of changes in environmental conditions, the rate of fish landings (for a given allocation of effort), and the natural rate of growth of the stock may fluctuate. Therefore, we also include situations where the depletion rate is variable, denoted by $\underline{D}$; where depletion and growth rates are random and completely dependent, denoted by $\overline{\mathrm{DG}}$; and where they are independent, denoted by DG. Summarizing, the programs listed in Table IV-1 are derived by permuting each of the six specifications of $U(R)$ with the four kinds of stochastic variation in the model.

[^2]| Table IV-1. List of Cases |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Random Prices $\eta_{1 t}, \eta_{2 t}$ <br> Non Random | Random Depletion Rates $\eta_{1 t}, p_{t}$ <br> Non Random | Random Growth and Depletion Rates (Complete Dependence) $\mathrm{p}_{\mathrm{t}}$ <br> Non Random | Random Growth and Depletion Rates (Complete Independence) $p_{t}$ <br> Non Random |
| $\mathrm{p}_{t} \eta_{1 t} \mathrm{kX}_{t} \mathrm{E}_{\mathrm{t}}$ | I-P | I-D | $\mathrm{I}-\overline{\mathrm{DG}}$ | I-DG |
| $\begin{aligned} & p_{t} \eta_{1 t} k X_{t} E_{t} \\ & -C_{1} E_{t}-C_{2} E_{t}^{2} \end{aligned}$ | II-P | II-D | II- $-\overline{\mathrm{DG}}$ | II-DG |
| $\begin{gathered} P_{t} \eta_{1 t} k X_{t} E_{t} \\ -C_{3} E_{t}^{1 / 2} \\ \hline \end{gathered}$ | III-P | III-D | III- $\overline{\mathrm{D} G}$ | III-DG |
| $\operatorname{Ln}\left(G+p_{t} \eta_{1 t} k X_{t} E_{t}\right)$ | IV-P | IV-D | IV- $\overline{\mathrm{DG}}$ | IV-DG |
| $\begin{gathered} \operatorname{Ln}\left(G+p_{t} \eta_{1 t} k X_{t} E_{t}\right. \\ \left.\quad+C_{1} E_{t}+C_{2} E_{t}^{2}\right) \end{gathered}$ | V-P | V-D | $\mathrm{V}-\overline{\mathrm{DG}}$ | V-DG |
| $\begin{aligned} & \operatorname{Ln}\left(G+p_{t} \eta_{1 t} k X_{t} E_{t}\right. \\ & \left.-C_{3} E^{1 / 2}\right) \end{aligned}$ | VI-P | VI-D | VI- $\overline{\mathrm{DG}}$ | VI-DG |

## A. 3. Frequency Distributions for Random Variables

The frequency distributions for the random variables in our model, price, $p_{t}$, the depletion parameter, $\eta_{2 t}$, and the growth parameter, $\eta_{1 t}$, are not known because of a lack of data. ${ }^{2}$ Therefore, we make the following assumptions: All random variables are distributed independently over time. For computational convenience, in the $\overline{D G}$ cases we assume $\eta_{1}$ and $\eta_{2}$ are identical. The expected value of the variables is equal to the value they assume under deterministic conditions with $\varepsilon\left(p_{t}\right)=\$ 0.15$, and $\varepsilon\left(\eta_{1}\right)=\varepsilon\left(\eta_{2}\right)=1$. We are able to simulate a rich variety of stochastic conditions in the fishery by assuming that the variables have either a truncated triangular or uniform frequency distribution. Like the uniform distribution, the truncated triangular distribution is completely determined by the specification of two range parameters, assuming the distribution mean is fixed. For instance, the distribution, $h\left(\eta_{1 t}\right)$, for $\eta_{1 t}$, an example of which appears in Figure IV-1, is constructed in the following manner:

For all values of $d_{1}$ and $d_{2}$, which specify the range of the distribution, with $0 \leq \mathrm{d}_{1}<1<\mathrm{d}_{2}$, the frequency function for $\eta_{\text {It }}$ is completely determined by the two conditions:

$$
\begin{equation*}
\varepsilon\left(\eta_{1 t}\right)=1 \tag{2a}
\end{equation*}
$$

The distribution is symmetric when $\left(1-d_{1}\right)=\left(d_{2}-1\right)$ or skewed when the equality doesn't hold. Truncated triangular distributions for the other two parameters, $P_{t}$, and $\eta_{2 t}$ are constructed similarly. Hopefully, determining the effect on resource allocation for different distributions of $p_{t}, \eta_{1 t}$, and $\eta_{2 t}$ will help to indicate what empirical information on these random variables is needed for fishery management.


Figure IV-1. Example of a Typical Truncated Triangular Frequency Distribution

To identify a particular program, we will refer to it by the appropriate label in Table IV-1 and include a description of the stochastic conditions that are simulated. Once the type of distribution is specified only the "range parameters" are necessary to completely characterize the frequency function. For example, the label [I-D, Skd. L., (.3, 1.2)] denotes the case I-D where the distribution for the depletion parameter, $\eta_{2 t}$, is truncated triangular and skewed to the left with $\eta_{2 t}$ ranging between 0.3 and 1.2. All distributions are truncated triangular, unless " $U$ " appears in the description indicating a uniform distribution. Thus, [I-D, Sym. U., (.2, 1.8)] refers to the situation where the distribution for $\eta_{2 t}$ is symmetric and uniform with $\eta_{2 t}$ ranging between 0.2 and 1.8 .

A total of 266 programs were considered. For the reader's convenience, a complete list and description of all the programs studied for each of the six specifications of the utility and rent functions appears in Table IV-2. The same set of cases were evaluated for all Classes I-VI of utility and rent functions. Three types of distributions, skewed left, skewed right, and symmetric are included in our analysis. We are interested in determining if the optimal consumption policies are sensitive to the different types of distributions. For each kind of distribution, the programs are arranged in increasing order according to the range of variation for the stochastic parameters. As we will see shortly, this allows us to study the effects of increased uncertainty on resource allocation.
Table IV-2. Program Listings Corresponding to Rent and Utility Specifications I-VI

| Stochastic <br> Variation | Random <br> Prices | Random <br> Depletion <br> Rates | Random Growth <br> and Depletion Rates <br> (Complete Dependence) | Random Growth <br> and Depletion Rates |
| :--- | :---: | :---: | :---: | :---: |
| (Complete Independence) |  |  |  |  |

For $D G$ and $\overline{D G}$ conditions, only one range of variation is indicated since we are assuming that $\eta_{1 t}$ and $\eta_{2 t}$ have the same distribution. Recall however, that $\eta_{1 t}$ and $\eta_{2 t}$ are identical for $D G$, and independently distributed for $\overline{D G}$. For DG programs, only uniform distributions are analyzed. The cost of simulating DG conditions is more than for the other cases because of the computational and memory storage demands on the computer in generating two independently distributed random variables. After reviewing the other cases we did not find that the optimal consumption program for the skewed and symmetric distributions were sufficiently different to warrant including both distributions in the DG programs.

Earlier, we stated that the probability density functions for the stochastic variables in our model, $\eta_{1 t}, \eta_{2 t}$, and $p_{t}$ are not known. Except for observations on $p_{t}$, there is also no information about the range over which these parameters vary. For the purposes of our study, we assume that each of the parameters varies at its widest limits within a range of 0.2 and 1.8 its expected value. Although the price variation for some programs is greater than the fluctuations in price commonly observed for the yellowfin tuna fishery (see Appendix III-A), these cases are included in our analysis for general interest.

The values for the rost of the parameters in the model are the same as the values used for our analysis of optimal consumption
programs for deterministic conditions in the fishery discussed in Chapter III.

## Biological Parameters

$$
\begin{aligned}
& a=3.057 \\
& b=1.035 \times 10^{-8} \\
& k=7.85 \times 10^{-5}
\end{aligned}
$$

Economic Parameters

$$
\begin{array}{rlrl}
\overline{\mathrm{p}} & =\varepsilon(\mathrm{p})=\$ 0.15 & \mathrm{~B}=0.9906 \\
\mathrm{C}_{1} & =5.0 \times 10^{2} & \mathrm{G}=4.5 \times 10^{8} \\
\mathrm{C}_{2} & =6.0 \times 10^{-2} & \bar{\Delta}=0.1 \\
\mathrm{C}_{3} & =1.0 \times 10^{5} &
\end{array}
$$

The optimal effort allocations derived with our Markov Decision Process model for each of the programs listed in Tables IV-1 and IV-2 appears in Appendix IV. The results are organized in accord with the program listings and descriptions of Tables IV-1 and IV-2.

Rather than discuss the computational mechanics of our model in the body of this chapter, the interested reader is referred to Appendix III-C for a complete description of the program used to run the Markov model on the Burroughs 6700 computer.

## A. 4. Introductory Remarks Concerning the Questions

Having described and identified the set of cases to be examined, we are ready to answer the four questions posed earlier concerning the effect of uncertainty on optimal consumption rules in the fishery. Obviously, it is not our purpose to present an exhaustive study of all possible stochastic environments for the fishery. As a first attempt at solving a difficult stochastic optimization problem-one for which analytical techniques are not available--our objectives are (1) to develop a computational method for selecting optimal decision rules, and (2) to use this method to study an interesting subset of the possible conditions that might prevail in the fishery.

The number of cases we consider is limited by the trade-off between obtaining more information and the additional cost of computer time and analysis required to study each program. As with all computational or simulation models, there is some question as to what extent the results are specific to the model or parameter values assumed. Hopefully, though, the conclusions from our study will help us to formulate general propositions about allocation rules for the fishery. A valuable by-product of our results will be information on other cases that need to be considered. Whenever appropriate, I have indicated additional programs and modifications in the model that might help to extend our results.

Other analysis of optimal consumption and production policies under stochastic conditions have been performed by Leland (1974),

Levhari and Srinivasan (1966), Mills (1959), Phelps (1962), and Zabel (1970) and (1971). These studies are not sufficiently similar to our own analysis for a meaningful comparison of results. However, one finding common to all of these studies ${ }^{3}$ and applicable to our own model is that optimal rules for allocating resources may be sensitive to the specification of the welfare function and the distribution of the stochastic parameters in the model. Consequently, wherever possible we have tried to indicate which assumptions are critical for certain results in our model to hold.

Before beginning the discussion of Questions 1-4 we have two general observations to make on the results we obtained after analyzing all the programs in Table IV-2.

Observation 1: The qualitative nature of our results is the same for all different parameter distributions we employ, whether they be symmetric, uniform symmetric, or skewed.

Although it does not appear that the type of distribution has a significant impact on resource allocation, we will see that the amount of variation in the random parameters has a pronounced effect on optimal decision rules. To be consistent throughout the chapter, in discussing each question we will illustrate our results with examples of optimal programs where the random variables have a uniform symmetric distribution. This is done with the understanding that our observations apply to all the different types of frequency distributions.

Observation 2: Optimal effort allocations corresponding to variable depletion rate cases (specification $D$ ) are virtually unaffected by allowing simultaneous variation in the natural growth rate (specification $D G$ and $\overline{D G}$ ).

Conceivably, for the parameter values of $\eta_{1 t}$ and $\eta_{2 t}$ we analyzed, fluctuations in the depletion rate dominate any effects of growth rate variation. Another explanation might be that random changes in the growth rate have no impact on allocation programs. To test these hypotheses we analyzed programs where only the growth rate was random for each of the six classes, assuming a variation for the parameter $\eta_{l t}$ of [Sym. U., (.2, 1. 8)]. The optimal effort allocations for these programs, listed in Appendix IV, differs only slightly from the deterministic program for each class. This tends to confirm our hypothesis that variations in the growth rate have a negligible effect on allocation programs.

Throughout the rest of this chapter, we will illustrate our results about growth and depletion rate cases with examples of optimal depletion rate programs, D. However, one should understand that these results also apply in equal force to $D G$ and $\overline{D G}$ cases.

## B. QUESTION. 1

Question 1: How do different attitudes toward social risk bearing as regards variations in resource rents, affect optimal decision rules
when uncertainty exists regarding the price of fishery products, the rate of depletion (for a given effort allocation), and the rate of population growth?

To answer Question 1, we contrast optimal decision rules for risk neutral (Class I-III) programs with risk averse (Class IV-VI) policies under identical stochastic conditions, for cases $P, D, D G$, and $\overline{\mathrm{DG}}$. Our results are summarized in Observation 3.

Observation 3: For all variations, $P, D, \overline{D G}$, and $D G$ listed in Table IV-2, a comparison of optimal risk averse programs with risk neutral policies indicates that:
a. At small populations the optimal allocation of effort and the resulting catch for risk averse programs are equal to or larger than the corresponding values for risk neutral policies.
b. At large populations the allocation of effort and the resulting catch for risk averse programs are generally less than the corresponding values for the risk neutral policies.
c. Except for Class III and VI policies, risk averse and risk neutral programs converge toward the same steady state population.

Our observations apply to all variations listed in Table IV-2, although results will be illustrated with certain cases. Sample comparisons between risk neutral and risk averse policies for conditions
where price is variable [P, Sym., U., (.6, 1.4)] and where depletion rates are random [D, Sym. U., (.6. 1.4)] appear in Figures IV-2a, b, c and IV-3a, b, c, respectively. Each policy is described by the effort allocation corresponding to population size. In Figures IV-2a, b, c deterministic policies are optimal for the risk neutral social maximizer since price variation doesn't affect resource allocation if the expected price remains unchanged.

## B. 1. Concavity or Moderation Effect

Parts (a) and (b) of Observation 3 are explained by what we call the "concavity effect." The difference in effort allocation for risk averse and risk neutral policies is schematically represented in Figure IV-4. This difference measures the changes in effort caused by transforming utility from a linear into a concave function of $R$, or equivalently by changing Class I, II, and III functions to Class IV, V, and VI functions respectively.

In each period, the decision maker selects an allocation of effort based on the trade off between consuming a larger portion of the current stock, but at the expense of reducing the expected future stream of returns from the fishery. Roughly, the optimal policy is to limit current consumption for small stock sizes where the fishing conditions are poor in return for larger expected future revenues from the fishery once the population has increased. Of course for larger populations current consumption increases to take advantage


Figure IV-2a. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Prices for Classes I and IV


Figure IV-2b. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Prices for Classes II and V


Figure IV-2c. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Prices for Classes III and VI


Figure IV-3a. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes I and IV


Figure IV-3b. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes II and $V$


Figure IV-3c. Comparison of Optimal Risk Averse and Risk Neutral Programs with Variable Depletion Rates for Classes III and VI
of improved fishing conditions.
Because of decreasing marginal utility, the effect of making utility a concave function of $R$ is to place greater weight on current consumption for small stock sizes where $R_{t}$ is typically small, and less weight on consumption for larger populations where $R_{t}$ is greater. This occurs because the addition to utility for a small increase in $R_{t}$ is greater when returns are small, and vice versa. Another description of this effect, which we shall call the "concavity effect, $\because$ is that it tends to moderate or smooth out the consumption policy as a function of stock size. That is, the difference in effort allocations and consumption rates corresponding to large and small population sizes is reduced by transforming the utility function. This effect is quite pronounced for Classes I and III as illustrated in Figures IV-2a, IV-2c, IV-3a, and IV-3c.


Figure IV-4. Schematic Difference between Risk Neutral and Risk Averse Policies

Notice that $R_{t}$ is a strictly concave function of $E_{t}$ for
Class II. By arguments similar to those used above, we expect a more moderate consumption policy for II than for the other Classes, I and III. This is verified by examining Figures IV-2a, b, c and IV $-3 a, b, c$. In addition, the concavity effect will be less pronounced for Class II cases since $R_{t}$ is already strictly concave in $E_{t}$.

## B.2. Dynamic Properties

In each figure, the points on the line labeled "Steady State Effort" indicate the effort required to maintain the stock at a steady state, assuming population growth and depletion are deterministic. ${ }^{5}$ The notion of a steady state becomes obscured under stochastic conditions, when population is perturbed constantly by random variations in growth and depletion rates. However, even in Figures IV-3a, $b, c$, where depletion rates are random the population tends to increase (decrease) for effort levels lying below (above) the steady state effort line. For example, on average, the population will decrease if the effort allocated exceeds the steady state quantity of effort.

According to Figures IV-2a,b and IV-3a,b, Class (I and IV) and (II and V) programs tend to converge to the same steady state population. Because of the concavity effect, risk averse programs generally converge to equilibrium at a slower rate than risk neutral policies. This suggests a generalization of the result derived in

Appendix III-D. Recall that for deterministic conditions, if the optimal program corresponding to a specific objective function converges to a particular steady state, then the optimal program for any positive concave transformation of the original objective function converges to the same steady state. Our observations suggests that the result might also hold under stochastic conditions.

For Class III and VI programs, as depicted in Figures IV-2c and IV-3c, "cyclical" as opposed to "steady state" fishing is optimal. The cycles are less pronounced for risk averse policies than for risk neutral programs due to the concavity effect.

## C. QUESTION 2

Question 2: What is the effect on optimal consumption strategies for increased uncertainty regarding prices, and depletion and growth rates?

To answer this question we analyze changes in consumption policies for different distributions of prices, and growth and depletion rates. All these distributions belong to the class of mean-preserving. spreads (the mean of the random variable is unchanged for all distributions), and are ordered according to how "risky" or "uncertain" they are. Adopting the definition of "increasing uncertainty" from Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970), we say that one distribution, $f$, is more uncertain than another, $g$, if

$$
\begin{equation*}
\int U(x) f(x) d x \leq \int U(x) g(x) d x \tag{3}
\end{equation*}
$$

for all risk averters--those with concave utility functions, U. It can be shown ${ }^{6}$ that (3) is formally equivalent to

$$
\begin{equation*}
T(Y)=\int_{a}^{b}(F(x)-G(x)) d x, T(Y) \geq 0 \text { and } T(b)=0 \tag{4}
\end{equation*}
$$

where $F$ and $G$ are the cumulative density functions corresponding to $f$ and $g$, and it is assumed that the points of increase for $F$ and $G$ are contained in the closed interval [a,b].

Looking at Table IV-2 the distributions, symmetric uniform, symmetric, skewed right, and skewed left for each of the random parameters, $p_{t}, \eta_{1 t}$, and $\eta_{2 t}$ are arranged according to the length of the interval over which the variable is allowed to range. It is easy to verify that according to condition (4) the distributions become more risky or uncertain as the range of variation for each of the parameters increase. For example, the distribution of price is more uncertain for [Sym. U., (.4, 1.6)] than it is for [Sym. U., (.6, 1.4)], and [Skd. L., (.4, 1.2)] is more uncertain than [Skd. L., (.6, 1. i) ]. These distributions and their corresponding cumulative density functions are represented in Figures IV-5a and IV-5b. In each case the distribution with the wider range is more risky by the conditions stated in (4).


Figure IV-5a. Mean Preserving Distributions - Symmetric Uniform


Figure IV-5b. Mean Preserving Distributiọns - Skewed Left

The social attitudes for risk bearing are important in assessing the impact on optimal allocation programs of increasing uncertainty regarding the key parameters in our model. To sharpen our analysis we consider the effect of increasing uncertainty in two sections--the first dealing with risk neutral social planners, and the second dealing with risk averse social maximizers. Within each section effects of increased variations in prices, and increased uncertainty regarding depletion and growth rates are analyzed separately.

From the results obtained here we can also compare resource use under deterministic conditions with resource allocation in a stochastic environment where uncertainty exists about prices, and growth and depletion rates.
C. 1. Increased Variations in Price--Risk Neutral Social Planner

Variations in price have no effect on resource allocation for the risk neutral social maximizer as long as the expected price remains unchanged.
C.2. Increased Variations in Growth and Depletion Rates--Risk Neutral Social Planner

Changes in resource allocation for Classes I-III caused by increased uncertainty in growth and depletion rates are analyzed for all the cases of variations listed in Table IV-2 for $D, D G$, and $\overline{D G}$.

Observation 4: For the risk neutral maximizer the effect of increased variation in growth and depletion rates is characterized by the following comments:
a) Increasing variation in growth and depletion rates tend to decrease the optimal allocation of effort and the resultant catch corresponding to each population size.
b) This dampening effect on effort is greatest for Classes I and III rent functions. Within each class, the effect is more pronounced at the high end of the population scale where catches are typically large.

The effect of increasing uncertainty in the depletion rate on optimal programs for Classes $I$ - III is represented in Figures IV-6a, b, c.

Comparing the small variation cases [Sym. U., (.6, 1.4)] with the large variation situations [Sym. U., (.2, 1.8)] the change in effort increases with greater absolute variation in catch. This variation is proportional to the expected catch $L$, since $L_{t}=\eta_{2 t} k X_{t} E_{t}=\eta_{2 t} \bar{L}$. As noted in Observation 4, and looking at Figures IV-6a, b, c we find that (1) the greatest effort adjustment occurs for Classes I and III programs, and (2) within each class, changes in effort increase with population size. Note that the expected value and variation in catch is large for each of these situations. Generally $L$ increases with population, and compared to Class II programs, the expected catch and the variation in $L_{t}$ are greater for Classes I and III policies particularly at the upper end of the population scale.


Figure IV-6a. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class I


Figure IV-6b. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class II


Figure IV-6c. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class III

## C.3. Increased Variation in Prices, Growth and Depletion Rates-Risk Averse Social Planner

The effect of increasing variation in prices, growth and depletion rates on Class IV - VI programs are analyzed for all the $P$, $D$, $\overline{\mathrm{DG}}$, and DG variations listed in Table IV-2.

## Observation 5:

a. With increasing variation in prices or growth and depletion rates, the allocation of effort and resultant expected catch for small (large) populations are the same or increasing (decreasing) for Classes IV and V.
b. With increasing variation, the allocation of effort is generally decreasing and more evenly distributed over population states for Class VI.
c. The greatest change in effort caused by increasing uncertainty in prices or growth and depletion rates occurs for Classes IV and VI.

Changes in optimal programs for Classes IV - VI caused by increasing variation in prices and depletion rates are presented in Figures IV-7a, b, c and Figures IV-8a,b, c, respectively. A plausible explanation for our findings in Observation 5 is that the dispersion in rents, $R_{t}$, is proportional to the size of the catch when either $p_{t}$ or $\eta_{2 t}$ are random. For all programs the optimal catch generally increases with population. Since the decision maker is averse to variations in $R_{t}$ he tends to increase his catch for small populations, despite poor fishing conditions, since the dispersion in returns is


Figure IV-7a. Comparison of Optimal Stochastic Programs with Increasing Variation in Price for Class IV


Figure IV-7b. Comparison of Optimal Stochastic Programs with Increasing Variation in Price for Class V


Figure IV-7c. Comparison of Optimal Stochastic Programs with Increasing Variation in Price for Class VI


Figure IV-8a. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class IV

Effort


Figure IV-8b. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class V


Figure IV-8c. Comparison of Optimal Stochastic Programs with Increasing Variation in the Depletion Rate for Class VI
smaller, and to decrease his catch at larger populations, because of the greater dispersion in returns. ${ }^{7}$ A simple method to verify this hypothesis, which we save for a later study, would be to analyze the effect on consumption programs of variations in returns that are independent of the size of the catch. Examples of these are fluctuations in repair and maintenance costs for breakdowns in radar and navigational equipment on fishing vessels, which presumably occur at random points in time.

Effort allocations are distributed more evenly over population sizes for Classes II and $V$ than for the other classes, due to the concavity of the rent function. Consequently, variations in the depletion rate, which tend to even out effort allocations, have less of an impact on Class V optimal programs.

## D. QUESTION 3

Question 3: Is it possible to account.for the social attitudes towards risk bearing in the social discount rate?

This question is not to be confused with the issue of whether or not private costs of risk bearing represent social costs as well, and should therefore be taken into account in judging the desirability of public projects. Rather, our concern is with evaluating different analytical methods for representing risk aversion, assuming that the variability in fishery rents is a social cost that effects resource allocation decisions.

An alternative to capturing attitudes for risk bearing in the form of the utility function, is to employ a "risk adjusted"interest rate for discounting future uncertain returns. In this case the optimal consumption strategy is determined by choosing effort in each period to maximize the present value of fishery rents,

$$
\begin{equation*}
\sum_{t=0}^{\infty} B^{\prime} \varepsilon\left[R\left(X_{t}, E_{t}\right) \bar{\Delta}\right] ; B^{\prime}=\frac{1}{1+p^{\prime}} \tag{5}
\end{equation*}
$$

where the rate of discount $\rho^{\prime}$ includes a "risk premium" yield over and above the "riskless" rate of interest $\rho$.

In the literature on cost-benefit analysis, by far the most common method of adjusting for risk is through the discount rate. Proponents of this procedure argue that the alternative of representing risk preferences with different forms of the utility functions requires direct knowledge of consumer's utility functions, and is therefore more difficult to implemeni. ${ }^{8}$ However, the simplicity of the present value criteria in equation (5) is deceptive. In practice it is not easy to determine the correct value of م'. Several methods for calculating the social discount rate have been proposed, but all of them are difficult to implement. ${ }^{9}$ However, a more serious objection to employing risk adjusted discounting is that risk is not a simple compounding function of time. ${ }^{10}$ For example, in our model, variations in rent are independent of time and correspond to the size of the expected catch when there is uncertainty about prices or the rate of
depletion. Our general conclusion, stated formally in Observation 6 is that programs for exploitation of the fishery that are derived from maximizing the risk adjusted present value of returns, appearing in equation (5) are non optimal.

Observation 6: For each type of variation in our model, P, D, $\overrightarrow{\mathrm{DG}}$, and.DG, there does not exist a $B^{\prime}$ such that the solution to the problem

$$
\begin{equation*}
\operatorname{Max}_{E_{t}} \sum_{t=0}^{\infty} B^{\prime t} \varepsilon\left[R\left(X_{t}, E_{t}\right) \bar{\Delta}\right] \tag{6a}
\end{equation*}
$$

$$
\text { subject to } X_{t+1}=X_{t}+\left[\eta_{1 t}\left(a-b X_{t}\right) X_{t}-\eta_{2 t} k X_{t} E_{t}\right] \bar{\Delta}
$$

yields a set of $E_{t}^{\prime} s$ which are optimal for the problem

$$
\operatorname{Max}_{E_{t}} \sum_{t=0}^{\infty} B^{t} \epsilon\left[U\left(R\left(X_{t}, E_{t}\right)\right)\right] \bar{\Delta}
$$

where $U\left(R\left(X_{t}, E_{t}\right)\right)=\ln \left(G+R\left(X_{t}, E_{t}\right)\right)$. It is convenient to organize our discussion of Observation 6 into two sections; one dealing with optimal programs for price uncertainty, the other with optimal strategies for growth and depletion rate variation.

## D. 1. Risk Adjusted Discounting for Variable Price Programs

As a general result, we found that the effect of increasing the social discount rate $\rho$ (equivalent to decreasing the discount factor, B), for all programs, regardless of the type of utility function or
parameter variations involved, was to encourage a higher level of current consumption of the fishery resource. This is apparent for example in a comparison of optimal effort allocations corresponding to different discount rates for Class I- III deterministic programs appearing in Figures IV-9a, b, c. Intuitively, it seems natural for society to consume a larger portion of the resource currently as future felicities become less important.

Turning now to specific cases, what can we infer about the effect of increasing price uncertainty on optimal consumption programs? If the social aversion to variations in returns is represented analytically in higher discount rates, then current consumption at all population levels tends to increase as price becomes more uncertain as we.see from Figures IV-9a,b, c. This is contrasted with the alternative convention of representing risk preferences in the form of the utility function. In this case current consumption increases at small populations and decreases for large populations as price variation increases. This disparity is not surprising since the adjusted discounting procedure treats risk as a simple compounding function of time, even though the variations in rent are proportional to the catch and independent of time.

## D.2. Risk Adjusted Discounting for Variable Depletion and Growth Rate Programs

The same conclusion we reached above, that risk adjustment via the discount rate causes distortions in optimal resource allocation,


Figure IV-9a. Comparison of Optimal Deterministic Programs with Annual Discount Rates of $10 \%$ and $15 \%$ for Class I


Figure IV-9b. Comparison of Optimal Deterministic Programs with Annual Discount Rates of $10 \%$ and $15 \%$ for Class II


Figure IV-9c. Comparison of Optimal Deterministic Programs with Annual Discount Rates of $10 \%$ and $15 \%$ for Class III
applies here as well. This is illustrated by the examples appearing in Figures IV-10a, b, c, where the optimal programs derived by maximizing the two different welfare functionals in (6a) and (6b) are compared for the variable depletion rate case [Sym. U., (.6, 1.4)]. The arrows in each figure indicate that with the risk adjusted discountirig model, the effort allocation increases as the discount rate is adjusted upward to account for risk aversion. As before in the price variation case, there is no single overall adjustment in the interest rate that can serve as a consistent and accurate measure of the riskiness in fishery returns.

## D. 3. Another Approach to Risk Adjustment Through the Discount Rate

Using a different procedure, Leland ${ }^{11}$ (1974) has shown that under special circumstances the solution to a certainty problem, where the discount rate $\rho$ is adjusted to $\rho^{\prime}$ for risk, will coincide to the optimal solution for a corresponding stochastic problem. Translating Leland's analyses in terms of our model, we wish to know if there exists a $B^{\prime}=\frac{1}{1+\rho^{\prime}}$ such that the solution to the certainty problem

$$
\begin{equation*}
\max _{E_{t}} \sum_{t=0}^{\infty} B^{\prime t}\left[U\left(R\left(X_{t}, E_{t}\right)\right)\right] \bar{\Delta} \tag{7a}
\end{equation*}
$$

subject to $X_{t+1}=X_{t}+\left[\left(a-b X_{t}\right) X_{t}-k X_{t} E_{t}\right] \bar{\Delta}$


Figure IV-10a. Alternative Risk Adjusted Programs with Variable Depletion Rates for Classes I and IV


Figure IV-10b. Alternative Risk Adjusted Programs with Variable Depletion Rates for Classes II and V


Figure IV-10c. Alternative Risk Adjusted Programs with Variable Depletion Rates for Classes III and VI
yields a set of $E_{t}^{\prime}$ 's which are optimal for the stochastic problem

$$
\begin{aligned}
& \max _{E_{t}} \sum_{t=0}^{\infty} B^{t} \varepsilon\left[U\left(R\left(X_{t}, E_{t}\right)\right)\right] \bar{\Delta} \\
& \text { subject to } X_{t+1}=X_{t}+\left[\eta_{1 t}\left(a-b X_{t}\right) X_{t}-\eta_{2 t} k X_{t} E_{t}\right] \bar{\Delta}
\end{aligned}
$$

Notice the same utility function is employed in (7a) and (7b) but in (7a) the discount rate is adjusted to account for stochastic variation in the fishery.

Referring to Observation 5, the optimal effort allocation is greater at small populations and less at large populations with stochastic programs (for either price variation or depletion and growth rate variation) than it is for deterministic programs. Note, however, that the optimal effort levels for the deterministic program increase for all population sizes as the discount rate is adjusted upward to account for risk. Consequently, the simple approach of solving the certainty problem with the risk modified discount rate will not yield the optimal consumption strategy for the corresponding stochastic problem.

## D. 4. Concluding Remarks

Unfortunately, in our model, the simple approach of increasing the discount rate to capture risk is not operable. The reason for this is clear. Risk is not a simple compounding function of time and
so no overall adjustment in the interest rate is suitable. Of course, there is nothing to preclude us from using a different rate for discounting returns in each period. The calculation of these rates would require knowledge of the variation in returns which is proportional to the expected catch in each period. However since this information is available, only after the optimal consumption programs have been determined, the use of different discount rates to adjust for risk is not practical.

The difficulties with trying to account for risk through the. discount rate are of course not peculiar to our analysis. They occur whenever the risk associated with a particular project or activity is not a simple compounding function of time. The variation in net social returns for many consumption and production processes depends on the level of the activity and not on the time during which it occurs. For these types of projects, the use of risk adjusted discounting is clearly inappropriate.

## E. QUESTION 4

Question 4: Based on the analysis of Questions I-3 and our results concerning optimal resource allocation for the fishery in a stochastic environment, what practical policy recommendations can be made for the yellowfin tuna fishery? In particular, (1) how does the policy of maximizing the sustained physical yield from the fishery compare with optimal stochastic policies, (2) do the solutions to deterministic
problems yield a sufficiently good approximation to the stochastic solutions to ignore probabilistic modeling all together, and (3) what additional information and data on the biological and economic processes of the fishery would be most useful for resource management?

## E. 1. Evaluation of Maximum Sustained Yield Policy under Stochastic Conditions

In the policy discussion of Chapter III we concluded that maximum physical yield policies are not optimal for fishery management under deterministic conditions. The same conclusion holds with regard to managing the fishery under stochastic conditions. With continual variations in the growth and the depletion rates, the population is rarely, if ever, in a steady state. The maximum sustained yield policy is deficient in that it abstracts from the non steady state or transient behavior of the fishery. However, even if the population tends to fluctuate around a certain stock size, as it does for Class I, II, IV, and V programs, this stock value will generally differ from the maximum sustained yield population. Of course, the concepts of steady state or maximum yield fishing are not applicable to Class III and VI programs since cyclical fishing is optimal.

## E.2. Deterministic Results as an Approximation to Stochastic Solutions

Until only recently, Economist in particular, and Social Scientists in general, have avoided an explicit treatment of probabilistic models. Instead they have relied on deterministic results to
provide an approximation for stochastic solutions. The reasons for this are clear. When uncertainty is introduced into the analysis, the formulation of and solution to most problems becomes more involved and sometimes unmanageable. Finally, once the problem is resolved, the results of the stochastic model are often of a subtle and obscure nature and consequently are difficult to interpret for the policy maker. Besides this, there is the widely held belief that most conclusions of deterministic studies remain basically the same when a stochastic treatment is employed. Indeed, if the probabilistic answer to problems differs only slightly from the deterministic solution the large investment required for analyzing stochastic models may not be warranted.

However, regardless of the extent to which solutions differ, the adoption of stochastic methods is desirable if they effect an increase in the social returns from the resource that exceeds the attendant costs of research. Formalizing this notion we define the present value of the resource,

$$
V^{d}\left(X_{0}\right)=\sum_{t=0} B^{t} \varepsilon\left[U\left(R\left(X_{t}, E^{d}\left(X_{t}\right)\right)\right)\right] \bar{\Delta}
$$

to be the expected value of the sum of discounted utilities attainable from an initial population $X_{0}$ and following a policy denoted by 'd'. A policy is a rule or strategy for selecting an effort allocation depending on the size of population, such that $E_{t}^{d}=E_{t}^{d}\left(X_{t}\right)$. Assume $\quad D^{\prime}$ is the optimal deterministic policy chosen to

$$
\max _{d} \sum_{t=0}^{\infty} B^{t}\left[U\left(R\left(X_{t}, E_{t}^{d}\left(X_{t}\right)\right)\right)\right] \bar{\Delta}
$$

where price is non random, and

$$
x_{t+1}=x_{t}+\left[\left(a-b x_{t}\right) x_{t}-k X_{t} E_{t}\right] \bar{\Delta}
$$

Let 'S' be the optimum stochastic strategy chosen to

$$
\max _{d} \sum_{t=0}^{\infty} B^{t} \varepsilon\left[U\left(R\left(X_{t}, E_{t}^{d}\left(X_{t}\right)\right)\right)\right] \bar{\Delta}
$$

where price may vary, and

$$
X_{t+1}=X_{t}+\left[\eta_{1 t}\left(a-b X_{t}\right) x_{t}-\eta_{2 t} k X_{t} E_{t}\right] \bar{\Delta}
$$

For a probabilistic environment, the increase in present value achieved by employing an optimal stochastic policy, $S$, rather than the deterministic consumption rule, $D$, is

$$
\begin{equation*}
v^{S}\left(x_{0}\right)-v^{D}\left(x_{0}\right) \tag{11}
\end{equation*}
$$

Obviously, we are not prepared to recommend whether or not stochastic modeling of the yellowfin tuna industry is warranted based on the type of cost-benefit criteria suggested above. Information on the empirical structure of the fishery is incomplete, and the programs we are investigating only simulate hypothetical situations in the fishery. Yet the following observations should provide us with a useful starting point for future policy analysis.

Observation 7: For the uncertainty programs $P, D, \overline{D G}$, and $D G$ the increases in the present value of the resource for a given initial population realized by employing the optimal stochastic policy is
a.) greatest for Classes I, III, IV, and VI and almost negligible for Classes II and V,
b) increases with larger initial populations and,
c) increases with greater variations in either price or the growth and depletion rates.

Examples of present value increases, calculated by equation (11) and corresponding to Classes I and III for variable depletion rate programs are plotted in Figures IV-11a,b. The additions to present value for Class II programs are only of the order of magnitude of $10^{3}$ dollars. Because our computer print outs were only designed to report numerical results up to five significant figures, the Class II figures are too small to report accurately. Similar computations for Classes IV - VI are not included here because they are more difficult to interpret, since present value figures are in terms of natural logs.

Looking at Figures IV-1la,b we see that present value increases are substantial for Classes I and III, that they increase according to the initial population size and that they become larger as the rate of depletion is more uncertain. ${ }^{12}$ As a general result, it is not surprising to find that the magnitude of the present value increase depends on the extent to which optimal stochastic and deterministic policies differ. This explains the small increase for Classes II and V,


Figure IV-11a. Present Value Increases for Class I Programs with Variable Depletion Rates

Present Value Increaso
( $10^{6}$ àollars)


Figure IV-1lb. Present Value Increases for Class III Programs with Variable Depletion Rates
and that the increase in present values are larger for programs subject to more uncertainty.

At least for the range of parameter values we have analyzed, solutions to deterministic problems serve as excellent approximations for stochastic solutions in the Class II and V cases. On the other hand, our tentative conclusion for Classes I, III, IV, and VI is that a probabilistic treatment of the resource allocation problem is needed since deterministic consumption rules are poor substitutes for optimal stochastic strategies.

## E.3. Suggestions for Additional Empirical Analysis

With our Markov model we have been able to assess the impact of various resource allocation programs on the fishery for a variety of different environmental and economic conditions. In effect, new policies and decision rules for operating the fishery have been tested under simulated conditions without running the risk of experimenting on the real systems. At the same time, our knowledge of the empirical structure of the fishery is only fragmentary. On the biological side, the nature of the variations in growth and depletion rates are as yet unknown, and on the economic side, information on the cost of fishing effort and the social attitudes toward risk bearing is incomplete.

Hopefully though, this study has yielded some valuable insights into which variables are more important than others in the
analysis of the fishery and which topics deserve the highest priority in future research endeavors. We have observed that optimal decision rules for operating the fishery are more sensitive to changes in certain variables and components of the model than others. This sort of information indicates the types of biological and economic data that will prove to be most useful for managing the fishery.

## a. Biological Data

Without knowing the frequency functions for the rate of growth and depletion parameters, $\eta_{1}$ and $\eta_{2}$ we have assumed that they are distributed according to a uniform or triangular density function. If these serve as good approximations for the real distributions of $\eta_{1}$ and $\eta_{2}$ then our results indicate that we should be most concerned with gathering data to determine the range over which these parameters vary. We have observed that the optimal consumption strategies are sensitive to the amount of variations in these parameters. On the other hand, the type of distribution, whether it be uniform or triangular, symmetric or skewed does not seem to effect the optimal decision rules significantly.
b. Economic Research

Our results indicate that optimal allocation policies differ according to the specification of the cost of effort function. In estimating costs it will be particularly interesting to determine if marginal costs increase with greater allocations of effort, as is the case
for Classes II and V. Under these conditions we observed that deter ministic or certainty equivalent policies provide excellent approximations to optimal stochastic decision rules for the fishery.

One should realize that in gathering cost data, for our purposes effort is defined biologically in terms of efficiency units. It is a type of aggregate input which when applied to the fishery will remove or catch, on average, a certain percentage of the population. The components comprising a unit of effort need to be specified in order to estimate the quantity of capital goods and labor services used in the fishing process. Additionally, as with all cost estimations, some care is needed to insure that one is measuring the true opportunity cost of inputs rather than the accounting cost. This is particularly important in the yellowfin fishery, since the boats operating in this industry have a number of alternative opportunities for employment in other fisheries as well.

Optimal decision rules are also sensitive to society's attitudes toward risk bearing. Our approach has been to represent risk preferences in the form of the social welfare function as opposed to the more common, but as we argued less valid procedure of adjusting for risk through the social discount rate. The problem of actually estimating and providing a consistent representation of social risk preferences is a very difficult one, and we shall not attempt to resolve it here. The object of our study is much more modest: to determine the effect of different attitudes for risk bearing on optimal resource
allocation in the fishery. Beyond this, however, we have a few comments pertaining to the choice of the social welfare function.

Naturally the social attitudes towards variations in fishery rents will depend on how these rents are distributed among individuals in the economy, or for the case of an international fishery, how they are dispersed among the member countries. Consequently, implicit in the choice of a social utility function must be some provision for distribution of the fishery rents. At the same time, a number of regulatory schemes to prevent individuals from over exploiting the fishery have been proposed, including quota and licensing systems, each resulting in a different distribution of rents. This suggests that the two problems of: (1) formulating optimal consumption rules based on maximizing expected social utility and, (2) devising schemes to enforce these rules, must be solved simultaneously, as they are interrelated. The type of regulatory procedure will have an impact on distribution, which in turn will effect the choice of the social welfare function.

Once the means for dividing the returns has been established the problem of choosing a welfare function that reflects social risk preferences still remains. The idea of constructing an aggregated social welfare function from a weighted sum of individual utility functions is perhaps theoretically possible, but impractical. It appears to us that the ohoice might best be made politically. For example,
using our Markov Decision model, one could determine various optimal allocation plans for the fishery derived by maximizing different social weifare criteria. These plans could then be reviewed and voted on by an electorate composed of individuals (countries), who were to receive a share of the rents from the fishery.

## F. SUMMARY STATEMENTS

The effect of uncertainty regarding prices, and growth and depletion rates on optimal allocation strategies have been analyzed for situations where society is averse and indifferent to variations in the returns from the fishery. The impact of uncertainty on consumption programs is directly related to the amount of variation in fishery rents determined by the size of the expected catch and the degree of fluctuation in the price and depletion rates. The variability of rents increases with population since the expected catch is typically greater. Key changes in optimal effort allocations occurring in a stochastic environment for different populations are of the following form:

For the risk neutral social planner, the allocation of effort and the resultant expected catch tend to remain constant or decrease as the price and growth and depletion rates become more uncertain. The largest changes in effort take place at the upper end of the population scale. The risk averse fishery manager increases effort at small populations to take advantage of the small fluctuations in rents, and decreases effort for larger populations to avoid greater risk in
returns. The effect of uncertainty on optimal Class II and V programs is relatively insignificant since the allocations of effort, expected catches, and thus the fluctuations in rents are small compared to the other classes.

The qualitative nature of our results are the same for all different forms of the frequency function analyzed. The most important element of the parameter distributions affecting allocation strategies is the range over which the variables are allowed to fluctuate. Optimal effort allocations corresponding to variable depletion rate cases are affected only slightly by allowing simultaneous variation in the natural growth rate.

The policy implications evolving from this analysis are:
(1) The policy of maximizing the sustainable physical yield from the fishery, currently followed by the Tuna Commission, is not an efficient device for allocating resources under deterministic or stochastic conditions. The policy should be retained only if the political and social costs of switching to a new program are prohibitive. (2) Using the discount rate to capture risk is inappropriate since risk is not a simple compounding function of time and no overall adjustment in the interest rate is suitable. (3) Solutions to deterministic problems serve as excellent approximations for stochastic solutions in Class II and V cases. However, for the other classes a probabilistic treatment of the resource allocation problem is needed since deterministic consumption rules are poor substitutes for optimal stochastic strategies.

## APPENDIX IV

Optimal Effort Allocations

## Contents:

Deterministic Programs

Class I Programs
Class II Programs
Class IV Programs
Class V Programs
Class VI Programs
Growth Variation Programs

## PROGRAI:

|  | I-DET. | ( II-DET) | (III-DET) |
| :---: | :---: | :---: | :---: |
| Ponulation |  | Effort Allocation |  |
| ( $10^{7}$ pounds) |  | ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 500 | 0 |
| 10 | 0 | 1,250 | 0 |
| 11 | 0 | 2,000 | 0 |
| 12 | 1,750 | 2,750 | 0 |
| 13 | 12,000 | 3,500 | 0 |
| 14 | 20,500 | 4,250 | 0 |
| 15 | 27,500 | 5,000 | 0 |
| 16 | 33,750 | 5,750 | 0 |
| 17 | 39,000 | 6,500 | 0 |
| 18 | 43,500 | 7,250 | 0 |
| 19 | 47,250 | 8,000 | 74,250 |
| 20 | 50,750 | 8,750 | 76,250 |
| 21 | 53.750 | 9,500 | 70,000 |
| 22 | 56,250 | 10,000 | 79,500 |
| 23 | 52,500 | 11,000 | 80,750 |
| 24 | 60,250 | 11,750 | 81,750 |
| 25 | 62,000 | 12,250 | 87,500 |
| 26 | 63,500 | 13,250 | 87,750 |
| 27 | 64,500 | 13,750 | 88,250 |
| 28 | 65.750 | 11:, 500 | ¢8,250 |
| 29 | 66,500 | 15,250 | \&., 500 |
| 30 | 67,250 | 16,000 | 88,500 |

PROGRSA:

|  | (iv-ber.) | (V-DET. ) | (VI-DET.) |
| :---: | :---: | :---: | :---: |
| Pooulation |  | Effort Allocetion |  |
| ( $10^{7}$ pounds ) |  | ( days ) |  |
| 0 | 0 |  |  |
| 1 | 0 | $0$ | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | - 0 | 0 | 0 |
| 9 10 | 0 250 | 500 | 0 |
| 10 | $\begin{array}{r}250 \\ \hline, 500\end{array}$ | 1,250 | c |
| 11 | 1,500 12,500 | 2,000 | 0 |
| 12 | 12,500 12,250 | 2,750 3,500 | 0 |
| 14 | 20,500 | 3,500 | 0 0 |
| 15 | 23,500 | 5,0r0 | 0 |
| 26 | 25,750 | 5,750 | 0 |
| 1.7 | 27,750 | 6,500 | 54,000 |
| 18 | 29,500 | 7,250 | 57,500 |
| 19 | 314,000 33,000 | 8,000 | 58,500 |
| 20 | 33,000 35,500 | 8,750 | 57,250 |
| 21 | 35,500 35,750 | 9,500 | 50,750 |
| 23 | 36,500 | 10,000 11,000 | 57,250 |
| 24 | 39,000 | 11,750 | 58,500 58,000 |
| 25 | 40,000 | 12,250 | 55,000 |
| 26 | 39,000 | 13,000 | 53,750 |
| 27 | 47,000 | 13,750 | 45,750 |
| 28 | 42,500 | 14,500 | 47,500 |
| 29 | 41,000 | 15,000 | 48,500 |
| 30 | 41,750 | 16,000 | 47,250 |

## CLASS I PRUGRLL:S

## PROGRAR:

I-D, Sym.U.
$(.8,1.2)$
$(.6,1.4)$
(.4, 1.6)
(.2, 1.8 )

| Ponulation ( $10^{7}$ pounds ) | $\frac{\text { EEfort Allocation }}{(\text { days })}$ |  |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | . 0 | 0 | 0 |
| 12 | 1,750 | 1,500 | 1,750 |
| 13 | 10,750 | 10,000 | .12,250 |
| 14 | 18,250 | 17,500 | 18,000 |
| 15 | 24,500 | 25,000 | 24,000 |
| 16 | 30,750 | 30,750 | 28,750 |
| 17 | 36,250 | 35,750 | 32,750 |
| 18 | 47,250 | 10,000 | 36,500 |
| 19 | 45,250 | 43,750 | 39,250 |
| 20 | 48,500 | 46,750 | 41,750 |
| 27. | 51,250 | 49,500 | 43,750 |
| 22 | 54,000 | 52,000 | 45. -00 |
| 23 | 56,250 | 54,000 | 47, 000 |
| 24 | 58,250 | 55,500 | 48,250 |
| 25 | 60,000 | 57,000 | 49,250 |
| 26 | 6],250 | 58,250 | 50,000 |
| 27 | 62,500 | 59,250 | 50,750 |
| 2.8 | 63,500 | 60,250 | 51,500 |
| 29 | 64,250 | 60,750 | 52,000 |
| 30 | 65,000 | 61,500 | 52,250 |

PROGRAL:

| I-D, Sym. | (.8, 1.2) | (.6, 1.4$)$ | (.4, 1.6$)$ | $(.2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Population }}{\left(10^{7} \text { pounds }\right)}$ | ( days) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 12 | 1,750 | 1,750 | 1,500 | 1,500 |
| 13 | 11,250 | 10,500 | 10,250 | 10,250 |
| 14 | 19,000 | 18,000 | 17,750 | 18,000 |
| 15 | 25,750 | 25,000 | 24,750 | 24,500 |
| 16 | 31,500 | 31,000 | 30,503 | 30,000 |
| 17 | 36,500 | 36,250 | 35,500 | 34,250 |
| 18 | 4, 250 | 40,750 | 39,750 | 38,500 |
| 19 | 45,250 48,750 | 44,500 47,750 | 13,250 16,500 | 41,750 $4!500$ |
| 20 | 48,750 51,750 | 47,750 50,750 | $1,6,500$ 49,000 | 4i, 47000 |
| 22 | 54,250 | 53,250. | 51,250 | 49,000 |
| 23 | 56,500 | 55,250 | 53,250 | 50,750 |
| 24 | 58,500 | 57,000 | 54,750 | 52,000 |
| 25 | 60,250 | 58,750 | 56,250 | 53,250 |
| 2.6 | 61,200 | 60,000 | 57,500 58,500 | 54,250 55,250 |
| 27 28 | 64,000 | 62,000 | 59,250 | 56,000 |
| 29 | 64,750 | 63,000 | 60,000 | 56,500 |
| 30 | 65,500 | 63,500 | 60,500 | 57,000 |

PRCGRAI:

| I-D, Skd. R. | (.8, 1.5) | $(.8,1.7)$ | (.8, 1.9$)$ |
| :---: | :---: | :---: | :---: |
| Pooulation |  | Effort Allocetion |  |
| ( $10^{7}$ pounds ) |  | (days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 1,750 | 1,750 | 1,750 |
| 13 | 10,250 | 10,500 | 10,500 |
| 14 | 17,750 | 18,000 | 17,250 |
| 15 | 24,000 | 27,000 | 23,500 |
| 16 | 30,750 | 30,750 | 30,250 |
| 3.7 | 36,250 | 36,000 | 35,500 |
| 18 | 40,750 | 40,000 | 39,250 |
| 19 | 44, 500 | 43,500 | 42,500 |
| 20 | 47,500 | 46,500 | 45,500 |
| 21 | 50,500 | 49,250 | 48,250 |
| 22 | 53,000 | 51,500 | 50,000 |
| 23 | 55,000 | 53,500 | 51,750 |
| 24 | 56,750 | 55,000 | 53,250 |
| 25 | 58,250 | 56,500 | 54,500 |
| 26 | 59,500 | 57,500 | 55,000 |
| 27 | 60,500 | 58,500 | 50́,250 |
| 28 | 61,500 | 59,250 | 57,000 |
| 29 | 62,250 | 60,000 | 57,500 |
| 30 | 62,750 | 60,500 | 57,750 |

PROGRAR:

| I-D, Skd. L. | (.6, 1.2) | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Population |  | Effort : 27 ll ocation |  |
| ( $10^{7}$ pounds) |  | (days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 1,750 | 1,750 | 1,750 |
| 13 | 10,500 | 10,500 | 10,500 |
| 14 | 18,000 | 18,000 | 18,000 |
| 15 | 24,750 | 24,500 | 24,500 |
| 16 | 30,750 | 30,500 | 30,250 |
| 17 | 36,000 | 35,750 | 35,250 |
| 18 | 40,750 | 40,500 | 40,000 |
| 19 | 45,000 | 44,750 | 44,250 |
| 20 | 48,250 | 48,000 | 47,750 |
| 21 | 51,000 | 50,750 | 50,250 |
| 22 | 53,500 | 53,000 | 52,750 |
| 23 | 55,750 | 55,250 | 54,750 |
| 24 | 57,750 | 57,250 | 56,500 |
| 25 | 59,500 | 59,000 | 58,250 |
| 26 | 61,000 | 60,500 | 50,500 |
| 26 27 | 62,250 | 61,750 | 61,000 |
| 28 | 63,000 | 62,750 | 62,000 |
| 29 | 64,000 64,500 | 63,500 | 63,000 |
| 30 | 64,500 | 64,000 | 63,500 |

## PROGRAS:

| I- $\overline{\mathrm{DG}}, \mathrm{Sym}$. ${ }^{\text {U }}$ | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Pomulation }}{\left(10^{7} \text { pounds }\right)}$ | ( days) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| $\begin{array}{r}9 \\ \hline 10\end{array}$ | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 1,500 | 3,250 |
| 13 | 11,000 | 10,250 | 10,000 | 10,750 |
| 14 | 20,500 26,750 | 20,500 | 20,500 | 20,500 |
| 16 | 26,750 32,000 | 25,750 30,500 | 25,250 29,750 | $2 / 2,500$ 28,750 |
| 17 | -36,500 | 35,500 | 29,750 34,500 | 33,250 |
| 18 | 40,750 | 40,000 | 38,500 | 36,500 |
| 19 | 4,4,750 | 43,750 | 42,000 | 40,000 |
| 20 21 | 48,500 51,500 | 47,000 49,750 | 45,000 | 42,250 44,500 |
| 22 | 54,000 | 49,750 52,000 | 57,250 49,500 | 46, 4250 |
| 23 | 56,000 | 54,000. | 51,000 | 47,500 |
| 24 25 | 58,000 59,750 | 55,750 57,250 | 52,250 53500 | 48,750 49,500 |
| 26 | 61,250 | 57,250 58,250 | 53,500 54,250 | 49,250 |
| 27 | 62,500 | 59,250 | 55,000 | 50,750 |
| 28 | 63,250 | 60,000 | 55,500 | 51,000 |
| 29 | 64,250 | 60,500 | 56,000 | 51, 250 |
| 30 | 64,750 | 61,000 | 56,250 | 51,250 |

## PROGRLL:

| I- $\overline{\mathrm{DG}}$, Sym. | (.8, 1.2) | (.6, 1.4$)$ | (.4, 1.6$)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Pomilation |  | Erfor | ation |  |
| ( $10^{7}$ pouncis) |  |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | G | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | - ${ }^{\circ}$ |
| 3.2 | 500 | 0 | 250 | 1,250 |
| 13 | 11,250 | 10,750 | 10,750 | 10,500 |
| 14 | 20,500 | 20,500 | 20,500 27 | 20,500 25,500 |
| 15 36 | 27,000 | 26,500 | 27,000 30,750 | 25,500 30,250 |
| 16 17 | 32,500 37,250 | 31,500 36,250 | 30,750 35,750 | 34,750 |
| 18 | 47,500 | 40,750 | 40,000 | 38,750 |
| 19 | 45,250 | 44,500 | 43,500 | 42,250 |
| 20 | 48,750 | 48,000 | 46,750 | 4,5,250 |
| 27 | 51,750 | 50,750 | 49,500 | 47,500 |
| 22 | 54,250 | 53,250 | 51,500 | 49,500 |
| 23 | 56,500 58,500 | 55,500 57,250 | 53,500 55,000 | 52,500 |
| 23 25 | 68,500 | 57,250 58,750 | 56,500 | 52,500 |
| 26 | 61,500 | 60,000 | 57,500 | 54,500 |
| 27 | 62,750 | 61,000 | 58,250 | 55,250 |
| 28 | 64,000 64,750 | 62,000 62,750 | 59,000 59,500 | 55,750 56,000 |
| 29 30 | 65,500 | 63,250 | 60,000 | 56,250 |

PRCGRAM:
$I-\overline{D G}$, Skd. R.
$(.8,1.5)$
$(.8,1.7)$
(.8, 1.9)


## PROGRAL:

| I- $\overline{\mathrm{DG}}$, Skd. L. | (.6, 1.2) | (.4, 1.2) | $(.2,1.2)$ |
| :---: | :---: | :---: | :---: |
| Population |  | Effort Allocation |  |
| ( $10^{7}$ pounds) |  | ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0. | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 500 |
| 13 | 11,000 | 11,000 | 11,500 |
| 3.4 | 20,500 | 20,500 | 20,500 |
| 15 | 26,500 | 26,500 | 26,500 |
| 16 | 31,750 | 31,500 | 31,500 |
| 17 | 36,500 | 36,250 | 36,000 |
| 18 | 40,750 | 40,250 | 40,000 |
| 19 | 4,4,500 | 44,000 | 43,500 |
| 20 | 48,000 | 47,250 | 46,750 |
| 21 | 51,250 | 50,500 | 49,750 |
| 22 | 53,750 | 53,250 | 52,500 |
| 23 | 55,750 | 55,250 | 54,750 |
| 24 | 57,500 | 57,000 | 56,500 |
| 25 | 59,250 | 58,500 | 58,000 |
| 26 | 60,750 | 60,000 | 59,250 |
| 27 | 62,000 | 61,250 | -0,250 |
| 28 | 63,000 | 62,250 | 61,500 |
| 29 | 63,750 | 63,250 | $62,250$ |
| 30 | 64,500 | 63,750 | 63,000 |

## PROGRAA:

I-DG, Sym.U. (.8, 1.2) (.6, 1.4) (.4, 1.6) (.2, 1.8)

| Pobulation | Effort Alzocation |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ( $10^{7}$ pounds) | ( days) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | O | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 |  | 0 | 0 |
| 11 | 0 | 0 | ${ }_{0}^{0}$ |  |
| 12 | - $\begin{array}{r}0 \\ 9,250\end{array}$ | 9,500 | 10,250 | 3,250 11,250 |
| 14 | 17,500 | 17,750 | 18,000 | 18,000 |
| 15 | 25,000 | 24,750 | 24,250 | 23,750 |
| 16 | 31,000 | 30,750 | 29,750 | 28,500 |
| 17 | 36,500 | 35,500 | 34,000 | 32,250 |
| 18 | 47, 000 | 39,750 | 38,000 | 35,750 |
| 19 | 45,000 | 43,500 | 41,250 | 38,750 |
| 20 | 48,000 | 46,750 | 4, 250 | 4, 4,250 |
| 21 | 51,250 54,000 | 49,500 51,750 | 40,500 48,500 | 4, 4,250 |
| 23 | 56,250 | 53,750 | 50,250 | 46,750 |
| 24 | 58,000 | 55,500 | 51,750 | 48,000 |
| 25 | 59,750 | 57,000 | 53,000 | 49,000 |
| 26 | 61,250 | 58,250 | 54,250 | 50,000 |
| 27 | 62,250 | 59,250 | 55,000 | 50,750 |
| 28 29 | 63,500 64,250 | 60,250 60,750 | 55,750 56,250 | 51,250 51,750 |
| 29 30 | 65,000 | 61,250 | 56,750 | 52,250 |

PROGRAL:

| II-D, Sym.li. | (.8, 2.2) | $(.6,1.4)$ | (.4, 2.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Pomuletion | Enfart Allocation |  |  |  |
| ( $10^{7}$ pouncs ) | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 | 4,250 | 4,250 | 4, 250 | 4,250 |
| 15 | 5,000 | 5,000 | 5,000 | 5,000 |
| 26 | 5,750 | 5,750 | 5,750 | 5,750 |
| - 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,500 8,000 |
| 19 | 8,000 | 8,000 | 8,000 | 8,000 8,750 |
| 20 | 8,750 9,500 | 8,750 9,500 | 8,750 9,500 | 8,750 9,500 |
| 21 | 9,500 10,250 | 9,500 10,250 | 10,250 | 10,250 |
| 23 | 11,000 | 11,000 | 11,000 | 11,000 |
| 24 | 11,750 | 11,750 | 11,750 | 11,500 |
| 25 | 12,500 | 12,250 | 12,250 | 12,250 |
| 26 | 13,000 | 13,000 | 13,000 13,750 | 13,000 |
| 27 | 13,750 | 13,750 | 13,750 | 14,500 |
| 28 | 14,500 | 14,500 | 15, 550 | 15,250 |
| 29 30 | $15,2,0$ 16,000 | 15,200 | 16,000 | 16,000 |
| 30 |  |  |  |  |

## PROGRith:



PHOGRSLI:

| II-D, Skd.R. | (.8, 1.5$)$ | $(.8,1.7)$ | $(.8,1.9)$ |
| :---: | :---: | :---: | :---: |
| Pooulation |  | Effort AlIocation |  |
| ( $10^{7}$ pounds) | . | ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| . 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 |
| 14 | 4,250 | 4,250 | 4,250 |
| 15 | 5,000 | . 5,000 | 5,000 |
| 16 | 5,750 | - 5,750 | 5,750 |
| 3.7 | 6,500 | 6,500 | -6,500 |
| 18 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 | 9,500 | 9,500 |
| 22 | 10,250 | 10,250 | 10,250 |
| 23 | 17,000 | 11,000 | 11,000 |
| 24 | 11,750 | 11,750 | 11,750 |
| 25 | 12,500 | 12,500 | 12,500 |
| .26 | 13,000 | 13,000 | 13,000 |
| 27 | 13,750 | 13,750 | 13,750 |
| 28 | 14,500 | 14,500 | 14,500 |
| 29 | 15,250 | 15,250 | 15,250 |
| 30 | 16,000 | 16,000 | 16,000 |

PROGRAL:

| II-D, Skd. L. | (.6, 1.2) | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Population |  | Efiont fllocation |  |
| ( $10^{7}$ pounds) |  | ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| $\cdots 4$ | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 |
| 10. | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 |
| 3.4 | 4,250 | 4,250 | 4,250 |
| 15 | 5,000 | 5,000 | 5,000 |
| 16 | 5,750 | 5,750 | 5,750 |
| 17 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 | 9,500 | 9,500 |
| 22 | 10,250 | 10,250 | 10,250 |
| 23 | 11,000 | 11,000 | 11,000 |
| 24 | 11,750 | 11,750 | 11,750 |
| 25 | 12,500 | 12,500 | 12,590 |
| 26 | 13,000 | 13,000 | 13,000 |
| 27 | 13,750 | 13,750 | 13,750 |
| 28 | 14,500 | 14,500 | 14,500 |
| 29 | 15,250 | 15,250 | 15,250 |
| 30 | 16,000 | 16,000 | 16,000 |

PROGRAR:

| II- $\overline{\mathrm{DG}}$, Sym.U. | (.8, 1.2) | $(.6,1.4)$ | (.4, 1.6) | $(.2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| Ponulation ( $10^{7}$ pouncis) |  | Fircon | ntion |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | - 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | . 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 | 4,2.50 | 4,250 | 4,250 5,000 | 4,250 |
| 15 | 5,000 | 5,000 | 5,000 | 5,000 |
| 16 | 5,750 | 5,750 | 5,750 | 5,750 |
| 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 | 8,750 |
| 27 | 9,500 | 9,500 | 9,500 | 9,500 |
| 22 | 10,000 | 10,000 | 10,000 | 10,000 |
| 23 | 11,000 | 11,000 | 111,000 | 11,000 |
| 24 | 11,750 | 11,750. | 11,750 | 11,750 |
| 25 | 12,250 | 12,500 | $12,500$ | 12,250 |
| 26 | 13,250 | 13,000 | 13,000 | 13,000 |
| 27 | 13,750 | 13,750 | 13,750 | 13,750 |
| 28 | 14,500 | 14,500 | $14,500$ | 14,500 |
| 29 | $15,250$ | $15,250$ | 15,250 | $15,250$ |
| 30 | 16,000 | 16,000 | 16,000 | 16,000 |

PROGRAM:

| II- $\overline{D G}$, Sym. | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Ponulation $\text { ( } 10^{7} \text { pounds ) }$ | ( فąys ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 3,500 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 4,250 | 3,500 | 3,500 | 3,500 |
| 14 15 | 4,250 5,000 | 4,250 5,000 | 4,250 | 4,250 5,000 |
| 16 | 5,750 | 5,750 | 5,000 5,750 | 5,1750 |
| 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,250 |
| 19 20 | 8,000 8,750 | 8,000 | 8,000 |  |
| 20 | 8,750 9,500 | 8,750 9,500 | 8,750 9,500 | 8,750 |
| 22 | 10,500 | 9,500 10,000 | 9,500 10,000 | 9,500 10,000 |
| 23 | 11,000 | 11,000 | 11,000 | 11,000 |
| 24 | 11,'750 | 11,750 | 11,750 | 11,750 |
| 25 | 12,250 | 12,250 | 12,500 | 12,500 |
| 26 | 13,250 | 13,000 | 13,000 | 13,000 |
| 27 | 13,750 14,500 | 13,750 14,500 | 13,750 14,500 | 13,750 14,500 |
| 29 | 15,250 | 15,250 | 15,250 | 15,250 |
| 30 | 16,000 | .16,000 | 16,000 | 16,000 |

## PROGR:ST:



PRocRis:


PROGRAM:

| II-DG, Sym.U. | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Populction }}{\left(10^{7} \text { pounds }\right)}$ | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 50 | 500 | 8 | 0 |
| 10 | 1,250 | 500 1,250 | 500 1,250 | 500 1,250 |
| 12 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 | 4,250 | 4,250 | 4,250 | 4,250 |
| 15 | 5,009 | 5,000 | 5,000 | 5,000 |
| 16 | 5,750 6,500 | 5,750 | 5,750 6,500 | 5,750 6,500 |
| 17 | 6,500 7,250 | 6,500 | 6,500 7,250 | 6,500 7,250 |
| 19 | 7,250 8,000 | 7,250 8,000 | 7,250 | 7,250 |
| 20 | 3,750 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 | 9,500 | 9,500 | 9,500 |
| 22 | 10,250 | 10,250. | 10,250 | 10,000 |
| 23 | 11,000 | 11,000 | 11,000 | 10,750 |
| 24 | 11,750 | 11,750 | 11,500 | 11,500 |
| 25 26 | 12,500 13,000 | 12,250 13,000 | 12,250 13,000 | 12,250 13,000 |
| 27 | 13,750 | 13,750 | 13,750 | 13,750 |
| 28 | 14,500 | 14,500 | 3.4,500 | 14,500 |
| 29 | 15,250 | 15,250 | 15,250 | 15,250 |
| 30 | 16,000 | 16,000 | 16,000 | 16,000 |

## CIASS III PRCGRY:S

PROGRAM:

| IIII-D, Sym.U. | (.8, 2.2) | (.6, 1.4) | (.4, 1.6$)$ | (.2, 1.8$)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulntion }}{\left(10^{7} \text { pounds }\right)}$ | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 8 | 0 | 0 | 0 | 0 |
| 8 | - | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | - 0 |
| 12 | 0 | 0 | 0 | 0 |
| 32 |  | 0 | 0 |  |
| 13 | 0 | 0 | 0 | 0 |
| 14. | 0 | 0 | 0 | 0 |
| 15 16 | 0 | 0 | 0 | 0 |
| 16 17 | 0 | 0 | 0 | 0 |
| 18 | 0 | ${ }_{0}^{0} 0$ | 0 | - |
| 19 | 72,750 | 67,750 | 61,750. | 56,250 |
| 20 | 75,000 | 69,500 | 63,750 | 57,500 |
| 21 | 77,000 | 71,250 | 64,750 | 58,500 |
| 22 23 | 78,500 79,750 | 72,500 73,500 | 65,500 66,250 | 58,500 59,250 |
| 24 | 61,000 | 74,000 | 66,750 | 60,000 60,500 |
| 25 | 81,750 | 74,500 | 67,000 | 60,750 |
| 26 | 82,250 | 75,000 | 67,250 | 60,750 |
| 27 | 82,750 83,000 | 75,250 | 67,500 | 61,000 |
| 28 | 83,000 83,250 | 75,250 | 67,500 67,500 | 61,000 |
| 29 30 | 83,250 83,250 | 75,250 75,250 | 67,500 | 61,000 |
| 30 | 83,250 | 75,250 | 67,250 | 61,000 |

PROGRAM:

| III-D, Sym. | (.8, 1.2$)$ | (.6, 1.4 ) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulation }}{\left(10^{7} \text { pounds }\right)}$ |  | Defor |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | - 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 112 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 | 0 |
| 18 | ${ }^{73} 7$ | - ${ }^{0}$ | 0 | 6, 0 |
| 19 | 73,750 76,550 | 70,750 73 | 66,750 | 62,500 |
| 20 | 76,250 78,250 | 73,000 74,750 | 68,750 70,000 | 64,000 65,000 |
| 22 | 80,000 | 76,250 | 71,250 | 65,000 |
| 23 | 81,250 | 77,250 | 72,000 | 66,500 |
| $2{ }_{4}$ | 82,500 | 78,250 | 72,750 | 67,000 |
| 25 26 | 83,250 84,000 | 78,750 79,250 | 73,000 | 67,250 |
| 26 27 | 84,000 84,500 | 79,250 79,500 | 73,250 | 67,250 67,250 |
| 28 | 84,750 | 79,750 | 73,500 | 67,250 |
| 29 | 85,000 | 79,750 | 73,250 | 67,000 |
| 30 | 85,000 | 79,750 | 73,250 | 66,750 |

PRCGRSH:

III-D, Shed.R.
(.8, 1.5)
$(.8,1.7)$
(.8, 2.9)

| $\frac{\text { Pooulation }}{\left(10^{7} \text { pounds }\right)}$ | Effort Allocatica |  |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 |  | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 |
| 19 | 70,250 | 67,500 | 64,750 |
| 20 | 72,250 | 69,250 | 6é, 000 |
| 21 | 74,000 | 70,750 | 67,000 |
| 22 | 75,250 | 71,650 | 67,500 |
| 23 | 76,250 | 72,250 | 67,750 |
| 24 | 77,000 | 72,750 | 67,750 |
| 25 | 77,500 | 73,000 | 67,750 |
| 26 | 78,000 | 73,000 | 67,250 |
| 27 | 78,250 | 73,000 | 66,750 |
| 28 | 73,250 | 72,750 | 66,250 |
| 29 | 73,250 | 72,500 | 65,750 |
| 30 | 78,000 | 72,250 | 65,000 |

## PRCGRAL:

| III-I, Skd. I. | $(.6,1.2)$ | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Ponulation |  | Effort in]ocation |  |
| ( $10^{7}$ pounds) |  | (days) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0. | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 |
| 19 | 72,000 | 70,500 | 69,500 |
| 20 | 74,500 | 73,000 | 71,750 |
| 21 | 76,250 | 75,250 | 73,750 |
| 22 | 77,750 | 76,500 | 75,750 |
| 23 | 79,000 | 77,500 | 75,750 |
| 24 | - 0,000 | 7\%,250 | 77,500 |
| 25 | 80,750 | 79,000 | 72,250 |
| 26 | 81,500 | 79,750 | 7e,750 |
| 27 | 81,750 | 80,000 | 79,000 |
| 28 | 82,000 | 80,500 | 79,250 |
| 29 | 82,250 | 30,750 30,750 | 79,500 |
| 30 | 82,250 | 80,750 | 79,500 |

PROGRALA:

|  | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulation }}{\left(10^{7} \text { pounds }\right)}$ | ( deys ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | $\bigcirc$ |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | . 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | - | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 17 | 0 0 | 0 | 0 | 0 |
| 17 18 | 0 | 0 | 0 | 0 0 |
| 19 | 73,000 | 69,000 | 64,750 | 59,750 |
| 20 | 75,500 | 71,250 | 66,000 | 61,000 |
| 21 | 77,500 | 72,750. | 67,000 | 61,750 |
| 22 | 79,000 | 73,750 | 67,750 | 62,000 |
| 23 | 80, 250 | 74, 500 | 68, 000 | 62,000 |
| 24 24 25 | 81,250 82,000 | 75,250 75,500 | 68,250 68,000 | 62,000 62,000 |
| 2. 26 | 82,500 | 75,500 | 68,000 62,000 | 62,000 61,750 |
| 27 | 82,750 | 75,500 | 67,150 | 61,500 |
| 28 | 83,000 | 75,250 | 67,500 | 61,000 |
| 29 | 83,000 | 75,000 | 67,000 | 60,500 |
| 30 | 83,000 | 74,750 | 66,500 | 59,750 |

PROGRAR:

| $\text { III- } \overline{D G}, \text { syma }_{0}$ | (.8, 1.2) | (.6, 1.4$)$ | (.4, 1.6$)$ | (.2, 1.8$)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Population }}{\left(10^{7} \text { pounds }\right)}$ | Eifort Alocation |  |  | ( days) |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 |
| 17 18 | 0 | O | 0 | 0 |
| 19 | 73,750 | 72,750 | 68,500 | 65,000 |
| 20 | 76,500 | 73,750 | 70,500 | 65,500 |
| 21 | 78,500 | 75,500 | 71,750 | 67,500 |
| 22 | 80, 000 | 77,000 | 72,750 | 60, 250. |
| 23 | 81,500 | 78,000 | 73,500 | 68,500 |
| 24 25 | 82,500 83,500 | 78,750 79,250 | 73,750 | 69,500 |
| 2.6 | 8,4,000 | 79,750 | 74,000 | 68,500 68,250 |
| 27 | 84,500 | 79,750 | 73,750 | 68,000 |
| 28 | 84.750 | 79,750 | 73,500 | 67,500 |
| 29 | 85,000 | 79,750 | 73,250 | 66,750 |
| 30 | 85,000 | 79,500 | 72,750 | 66,250 |

## PRCGR:G1:



## PROGR:M:

| III- $\overline{D G}$, Sicc. I . | (.6, 1.2) | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Population |  | Effort Allocation |  |
| ( $10^{7}$ pounds) |  | (days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 |
| 19 | 72,250 | 71, 250 | 70,750 |
| 20 | 74,750 | 73,500 | 72,750 |
| 21 | 77,000 | 75,500 | 74,500 |
| 22 | 78,250 | 77,250 | 76,000 |
| 23 | 79,500 | 78,250 | 77,250 |
| 24 | 80,250 | 79,000 | 78,250 |
| 25 | E1,000 | 70,500 | 78,750 |
| 26 | 81,750 | 80,000 | 79,000 |
| 27 | 82,000 | 80,250 | 79,250 |
| 28 | \&2,250 | 80,500 | 79,250 |
| 29 | ع2,000 | 80, 500 | 79,250 |
| 30 | 82,000 | 80,500 | 79,250 |

PROGRini:

| III-DG, Symu. | $(.8,1.2)$ | (.6, 1.4$)$ | (.4, 1.6) | (.2, 1.8$)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Pomuliation }}{\left(10^{7} \text { pounds }\right)}$ | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |  |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 17 | 0 | 0 | 0 | 0 |
| 17 18 | 0 | 0 | 0 | 0 |
| 19 | 72,500 | 67,250 | 61,250 | 55,250 |
| 20 | 75,000 | 69,500 | 62,750 | 56,750 |
| 27 | 77;000 | 72,000 | 64,250 | 57,750 |
| 22 | 78,500 | 72,250 | 65,250 | 58,500 |
| 23 | 79,750 | 73,250 | 65,750 | 59,250 |
| 2.4 <br> 25 <br> 25 | 80,750 81.750 | 74,000 74,500 | 66,500 66,750 | 59,750 60,250 |
| 26 | 82,250 | 75,000 | 67,000 | 60,500 |
| 27 | 82, 750 | 75,250 | 67,250 | 60,750 |
| 28. | 83,000 | 75,250 | 67,250 | 60,750 |
| 29 | 83,250 | 75,250 | 67,250 | 60,750 |
| 30 | 83,250 | 75,000 | 67,250 | 60,500 |

## CI,ASS IV PRCGRQ:S

PROGRAM:


PRGGRibi:

| IV-P, S.Jm. | $\begin{array}{r} \$ .15 x \\ (.8,1.2) \end{array}$ | $\begin{gathered} 3.15 x \\ (.6,1.4) \end{gathered}$ | $\begin{gathered} \text { \$.15 x } \\ (.4,1.06) \end{gathered}$ | $\begin{gathered} \$ .15 x \\ (.2,1.8) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Ponulation | Fifort Allocntion |  |  |  |
| ( $10^{7}$ pounds ) | ( Cays ) |  |  |  |
| 0 |  | 0 | 0 | 0 |
| 1 |  | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | - 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | $0$ | 0 | 0 | 0 |
| 9 | $0$ | 0 | 0 | 0 |
| 1.0 | 250 | 250 | 250 | 250 |
| 11 | 1,500 | 1,500 | 1,750 | 2,750 |
| 12 | 12,500 | ].2,500 | 12,500 | 12,500 |
| 13 | 12,000 | 12,000 | 12,000 | 12,250 |
| 14 | 20,500 | 20,500 | 20,500 | 20,500 |
| 15 | 23,000 | 23,000 | 23,000 | 23,000 |
| 16 | 25,750 | 25,750 | 23,750 | 25,750 |
| 17 | 27,500 | 27,250 | 27,250 | 27,000 |
| 18 | 29,500 | 29,500. | 20,500 | 29,500 |
| 19 | 34, 000 | 31.000 | 33,750 | 33,500 |
| 20 | 32,750 | 32,250 | 32,250 | 31,750 |
| 21 | 35,500 | 35,500 | 35,500 | 35,500 |
| 22 | 36,500 | 35,250 | 35,000 | 34,500 |
| 23 | 36,250 | 36,250 | 36,250 | 36,250 |
| 24 | 39,000 | 39,000 | 39,000 | 39,000 |
| 25 | 39,750 | 39,500 | 39,250 | 38,750 |
| 26 | 39,000 | 39,000 | 39,000 | 39,000 |
| 27 | 41,000 | 41,000 | 41,000 | 41,000 |
| 28 | 42,250 | 42,000 | 41,500 | 40,500 |
| 29 | 40,750 | 40,500 | 40,250 | 40,2.50 |
| 30 | 41,750 | 41,750 | 41,750 | 42,750 |

PRCGRATI:


RROGRAL:

| IV-P, Skd.I. | $\begin{gathered} \$ .15 \mathrm{x} \\ (.6,1.2) \end{gathered}$ | $\begin{gathered} 8.15 x \\ (.4,1.2) \end{gathered}$ | $\begin{gathered} 8.15 x \\ (.2,1.2) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Population |  |  |  |
| ( $10^{7}$ pounds) |  | ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 250 | 250 | 250 |
| 11 | 1,500 | 1,500 | 1,500 |
| 12 | 12,500 | 12,500 | 12,500 |
| 13 | 12,000 | 12,000 | 12,000 |
| 14 | 20,500 | 20,500 | 20,500 |
| 15 | 23,000 | 23,000 | 23,000 |
| 16 | 25,750 | 25,750 | 25,750 |
| 17 | 27,250 | 27,250 | 27,250 |
| 18 | 29,000 | 29,500 | 29,500 |
| 19 | 34,000 | 34,000 | 34,000 |
| 20 | 32,500 | 32,500 | 32,250 |
| 21 | 35,500 | 35,500 | 35,500 |
| 22 | 35,500 | 35,250 | 35,000 |
| 23 | 36,250 39,000 | 36,250 | 36,250 |
| 24 | 39,000 39,750 | 39,000 39,500 | 39,000 39,250 |
| 25 | 39,000 | 39,000 | 39,000 |
| 26 27 | 47,000 | 41,000 | 41,000 |
| 27 28 | 42,000 | 41,750 | 41,500 |
| 28 29 | 40,740 | 40,750 | 40,250 |
| 30 | 41,750 | 41,750 | 41,750 |

PROGRiLi:

| IV-D, Sym. ${ }_{\text {c }}$ | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8$)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Pomulition }}{\left(10^{7} \text { pounds }\right)}$ | (days) | Eifort Allocation |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 250 | 250 | 250 | 250 |
| 11 | 1,750 | 3,250 | 5,250 | 7,250 |
| 3.2 | 11,250 | 10,500 | 10,500 | 11,750 |
| 13 $1 / 4$ | 1/4,000 | 15,500 | 15,250 | 15,250 18,750 |
| $1 / 4$ 15 | 19,000 22,250 | 18,750 22,000 | 19,250 22,000 | 18,750 21,503 |
| 16 | 25,000 | 24,750 | 24,250 | 24,000 |
| 17 | 27,750 | 27,250 | 26,750 | 26,250 |
| 18 | 30,000 | 29,250 | 28,500 | 27,750 |
| 19 | 31,500 | 31,000 | 30,750 | 29,750 |
| 20 | 33,500 | 32,750 | 32,000 | 31,000 |
| 27 | 35,000 | 34,250 | 33,250 | 32,000 |
| 22 | 36,000 | 35,500 | 34,250 | 33,250 33,200 |
| 23 24 | 37,000 38,250 | 35,500 37,500 | 35,500 36,250 | 33,500 34,750 |
| 24 25 | 30,250 | 38,250 | 37,000 | 35, $\times 2$ |
| 26 | 30,750 | 39,250 | 37,750 | 36,2.50 |
| 27 | 40,250 | 39,750 | 38,500 | 36,250 |
| 28 | 40,750 | 40,000 | 38,750 | 37,000 |
| 29 | 41,250 | 40,250 | 39,020 | 37,000 |
| 30 | 41,500 | 40,750 | 39,250 | 37,250 |

## PROGRAL:



PRGGRAII:

IV-D, Skd. R.
(.8. 1.5)
$(.8,1.7)$
$(.8,1.9)$


PROGR:A:


PROGRAli:

| IV- $\overline{D G}$, Sym. ${ }_{\text {e }}$ | (.8, 1.2) | (.6, 1.4) | (.14; 1.6$)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Pomulction }}{\left(10^{7} \text { pounds }\right)}$ |  | $\frac{\text { Eifort }}{}$ |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0. |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 500 |
| 11 | 3,750 | 5,2,50 | 5,500 | 6, 250 |
| 12 | 10,750 13,250 | 10,000 | 10,000 | 11, 600 |
| 13 | 13,250 20,500 | $1 / 4,000$ 20,500 | 14,750 20,500 | 15,500 20,500 |
| 15 | 23,000 | 23,000 | 23,000 | 20,500 22,750 |
| 16 | 25,500 | 25,500 | 25,000 | 22, 21,500 |
| 17 | 27,750 | 27,750 | 27,000 | 26,750 |
| 18 | 29,750 | 29,750 | 29,250 | 28,500 |
| 19 | 31,750 | 31,250 | 30,750 | 30,000 |
| 20 | 33,750 35,000 | 32,750 34,250 | 32,250 | 31,500 |
| 21 | 35,000 36,250 | 34,250 35,500 | 33,500 34,500 | 31,500 33,500 |
| 22 | 37,250 | 36,500 | 34,500 | 33,500 34,000 |
| 24 | 32,250 | 37,750 | 36, 250 | 34,750 |
| 25 | 39,000 | 38,250 | 37,250 | 35,500 |
| 26 | 39,750 | 38,750 | 37,750 | 35,500 |
| 27 | 40,250 41,050 | 39,250 40,000 | 38,250 38,250 | 36,000 |
| 28 29 | 4, 41,050 | 40,000 40,000 | 38,250 38,250 | 36,250 37,00 |
| 29 30 | 41,500 | 40,500 | 38,500 | 36,500 |

PROGRAM:

| IV-DG, Syn. | (.8, 1.2) | (.6, 1.4$)$ | $(.4, ~ 1.6)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Pobulation | Effort Allocation |  |  |  |
| ( $10^{7}$ pounḋs) | ( deys ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 |  | 0 | 0 | 0 |
| 12 | 1,750 | 3,500 | 4,500 | 4;750 |
| 12 | 9,000 | 9,250 | 9,500 | 10,250 |
| 13 | 12,750 | 13,250 | 14,000 | 1.4,500 |
| 14 | 20,500 | 20,500 | 20,500 | 20,500 |
| 15 | 24,750 | 24,950 | 24,750 | 24,250 |
| 17 | 27,750 .29 | 26,250 28,500 | 26,000 24,500 | 25,750 27,750 |
| 18 | 29,750 30,250 | 30,250 | 30,000. | 29,750 21,50 |
| 19 | 33,000 | 32,000 | 31,500 | 31., 06 |
| 20 | 34,250 | 33,750 | 33,000 | 32,500 |
| 27 | 35,500 | 35,250 | 312,250 | 33,750 |
| 22 | 36,750 | 36,250 | 35,750 | 35,000 |
| 23 | 37,500 | 37,250 | 36,750 | 35,750 |
| $2{ }^{2}$ | 38,750 39,500 | 38,250 39,000 | 37,500 38,250 | 36,750 37,500 |
| 25 26 | 40,000 | 39,500 | 38,750 | 37,750 |
| 27 | 40,750 | 40,250 | 39,250 | 38,250 |
| 28 | 41,250 | 40,500 | 39,750 | 38,750 |
| 29 | 41,500 | 41,250 | 40,000 | 39,0:0 |
| 30 | 4,2,000 | 41,500 | 40,250 | 39,250 |

## PiRCGRAM:



RRCGRAI:

| IV-DG, Skd. I. | (.6, 1.2) | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Population |  | Effori hilacation |  |
| ( $10^{7}$ pounds) |  | . (days ) |  |
|  |  |  |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 4,000 | 4,250 | 4,000 |
| 12 | 10,750 | 10,750 | 10,750 |
| 13 | 13,500 | 13,500 | 13,500 |
| 14 | 20,500 | 20,500 | 20,500 |
| 15 | 23,000 | 23,000 | 23,000 |
| 16 | 25,250 | 25,000 | 25,000 |
| 1.7 | 28,000 | 28, 250 | 28,250 |
| 18 | 29,750 | 29,250 | 28,750 |
| 19 | 31,750 | 31,500 | 31,250 |
| 20 | 34,000 | 34,000 | 33,750 |
| 21 | 34,500 | 34,500 | 34,500 |
| 22 | 35,750 | 35,750 | 35,500 |
| 23 | 37,250 | 37,000 | 36,750 |
| 24 | 38,000 | 37,750 | 37,250. |
| 25 | 3i, 750 | 38,250 | 39,000 |
| 26 | 39,500 | 39,250 | 39,000 |
| 27 | 40,000 | 39,750 | 39,500 |
| 28 | 40,750 | 40,250 | 40,250 |
| 29 | 41,250 | 40,?50 | 40,500 |
| 30 | 41,750 | 4, 3,000 | 40,750 |

## PROGRAL:

| IV-DG, Sym.U. | (.8, 1.2) | (.6, 1.4$)$ | (.4, 1.6) | ( $2,2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| Pomulation | Effort Allocation |  |  |  |
| ( $10^{7}$ pounds ) | ( days) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 12 | 5,000 | 5,250 | 6,000 | 6,250 |
| 12 | 10,000 | 10,250 | 10,750 | 11,250 |
| 13 | 14,750 | 14,750 | 15,500 | 15,750 |
| 14 | 19,000 | 16,750 | 19,500 | 17,750 |
| 15 | 22,000 | 21,750 | 21 m 750 | 21,500 |
| 16 17 | 24,750 27,750 | 24,500 27,750 | 21,750 27,500 | 21, 2.50 |
| 18 | 29,500 | 29,500 | 29,250. | 27,000 |
| 19 | 32,250 | 30,750 | 30,250 | 29,500 |
| 20 | 32,750 | 32,750 | 32,500 | 30,250 |
| 27 | 35,000 | 34, 350 | 32,500 34,250 | 31,250 32,500 |
| 22 23 | 36,250 37,000 | 35,500 36,770 | 34, 34,50 | 32,500 32,750 |
| 23 24 | 37,000 38,250 | 36,730 36,500 | 34,750 36,250 | 32,750 34,500 |
| 25 | 39,000 | 37,500 | 36,750 | 35,000 |
| 26 | 39,500 | 39,000 | 37,250 | 35,250 |
| 27 | 40,500 | 38,750 | 37,750 | 35,750 |
| 28 | 41,000 | . 40,250 | 37,750 | 36,500 |
| 29 30 | 41,500 | 40,500 40,750 | 38,750 38,750 | 37,000 37,250 |
| 30 | 41,750 | 40,750 | 38,750 | 37,250 |

PROGRAR:

| V-P, Sym.U. | (.8, 1.2) | (.6, 1.4$)$ | (.4, 7.6$)$ | (.2, 1.8$)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulation }}{\left(10^{7} \text { pounds }\right)}$ | . ( deys ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | $\bigcirc$ |
| 5 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 32 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 15 | 4,250 5,000 | 4,250 5,000 | 4,250 5,000 | 4,250 |
| 16 | 5,750 | 5,750 | 5,750 | 5,750 |
| 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 | :5,000 |
| 20 | \&,750 | 2,750 | 8,750 | 8,500 |
| 21 | 9,500 10,000 | 9,500 10,000 | 9,500 10,000 | 9,250 10,000 |
| 22 23 | 11,000 | 10,750 | 10,750 | 10,750 |
| 21 | 11,750 | 11,500 | 11,500 | 11,500 |
| 2.5 | 12,250 | 12,250 | 12,250 | 12,000 12,750 |
| 26 | 13,000 13,750 | 13,000 13,500 | 13,000 | 12,750 13,500 |
| 28 | 14,500 | 14,250 | 1.4,250 | 14,000 |
| 29 | 15,000 | 15,000 | 15,000 | 14,750 |
| 30 | 15,750 | 15,750 | 15,750 | 15,500 |

PROGRAM:

| V-P, Sym. | (.8, 1.2 ) | (.6, 1.4) | (.4, 1.6$)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulition }}{\left(10^{7} \text { pounds }\right)}$ | ( days) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 6 | 0 0 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | O |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 15 | 4,250 5,000 | 4,250 5,000 | 4,250 5,000 | 4,250 5,000 |
| 16 | 5,750 | 5,750 | 5,750 | 5,750 |
| 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 10,000 | 9,500 10,000 | 9,500 10,000 | 9,500 10,000 |
| 23 | 11,000 | 12,000 | 10,750 | 10,750 |
| 24 | 11,750 | 11,750 | 11,500 | 11,500 |
| 25 | 12,250 13,000 | 12,250 13,000 | 12,250 13,000 | 12,250 13,000 |
| 27 | 13,750 | 13,750 | 13,500 | 13,500 |
| 28 | 14,500 | 14,500 | 14,250 | 14,250 |
| 29 | 15,000 | 15,000 | 14,750 | 14,750 |
| 30 | 15,750 | 15,750 | 15,500 | 15,500 |

## PRGGRAH:



EROGROL:


PROGRAL:

| V-D, Sym.U. | (.8, i.2) | (.6, 1.4) | (.4, 1.6) | $(.2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| Ponulation | Effort Alרocation |  |  |  |
| ( $10^{7}$ pounds ) | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 7 | 0 | 0 | 0 0 | 0 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 1.5 | 4,250 5,000 | 4,250 5,000 | 4,250 5,000 | 4,250 5,000 |
| 16 | 5,750 | 5,750 | 5,750 | 5,750 |
| 17 | 6,500 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 | 9,500. | 9,250 | 9,250 |
| 22 23 | 10,250 10,750 | 10,250 10,750 | 10,000 10,750 | 10,000 10,750 |
| 24 | 11,500 | 11,500 | 11,500 | 11,500 |
| 25 | 12,250 | 12,250 | 12,250 | 12,000 |
| 26 | 13,000 | 13,000 | 13,000 | 12,750 |
| 27 | 13,750 | 13,750 | 13,500 | 13,500 |
| 28 | 14,250 | 14,250 | 14,250 | 14,000 |
| 29 | 15,000 15,750 | 15,000 15,750 | 14,750 15,500 | 14,750 15,250 |
| 30 | 15,750 | 15,750 | 15,500 | 15,250 |

## PROGRAY:

| V-D, Sym. | (.8, 2.2 ) | $(.6,1.4)$ | (.4, 1.6) | $(.2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| Ponulation | Eifort Allocation |  |  |  |
| $\text { ( } 10^{7} \text { pounds ) }$ | (days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5. | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 250 | 500 | 0 500 | 0 |
| 9 | 500 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 | 1,250 |
| 11 | 2,000 | 2,000 | 2,000 | 2,000 |
| 3.2 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 | 3,500 |
| 14 | 4,250 | 4,250 | 4,250 | 4,250 |
| 15 | 5,000 | 5,000 | 5,000 | 5,000 |
| 16 | 5,750 | 5,750 | 5,750 6,500 | 5,750 |
| 37 | 6,500 | 6,500 | 6,500 7,250 | 6, 500 |
| 18 | 7,250 | 7,230 | 7,250 8,000 | 7,250 |
| 19 | 8,000 | 8,000 | 8,000 8,750 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 9,500 | 8,750 |
| 21 | 9,500 | 9,500 10,250 | 9,500 10,000 | 9,500 10,000 |
| 22 | 10,250 11,000 | 10,250 10,750 | 10,000 10,750 | 10,000 10,750 |
| 23 24 | 11,000 | 11,500 | 11,500 | 11,500 |
| 24 25 | 12,250 | 12,250 | 12,250 | 12,250 |
| 26 | 13,000 | 13,000 13,750 | 12,750 13,500 | 12,750 13,500 |
| 27 | 13,750 14,500 | 13,1500 | 14,250 | 14,2, 50 |
| 28 29 | 15,250 | 15,000 | 15,000 | 15,000 |
| 30 | 15,750 | 15,750 | 15,750 | 15,500 |

## PRCGRAIT:

V-D, Skd. R.
(.8, 1.5)
$(.8,1.7)$
(.8, 2.9)

| Ponulation <br> ( $10^{7}$ pounds ) |  | 2fitort Allocation (days) |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 |
| 10 | 1,250 | 1,250 | 1,250 |
| 11 | - 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 |
| 13 | 3,500 | 3,500 | 3,500 |
| 14 | 4,250 | 4,250 | 4,250 |
| 15 | 5,000 | 5.000 | 5,000 |
| 1.6 | 5,750 | 5,750 | 5,750 |
| 1.7 | 6,500 | 6,500 | 6,500 |
| 18 | 7,250 | 7,250 | 7,250 |
| 1.9 | 8,000 | 8,000 | 8,000 |
| 20 | 8,750 | 8,750 | 8,750 |
| 21 | 9,500 | 9,500 | 9,500 |
| 22 | 10,250 | 10,000 | 10,250 |
| 23 | 10,750 | 10,750 | 10,750 |
| 24 | 11,500 | 11,500 | 11,500 |
| 25 | 12,250 | 12,250 | 12,250 |
| 26 | 13,000 | 13,000 | 13,000 |
| 27 | 13,750 | 73,750 | 13,750 |
| 28 | 14,250 | 14,250 | 14,250 |
| 29 | 15,000 | 15,000 | 15,000 |
| 30 | 15,750 | 15,750 | 15,500 |

RROGRAM:


PROGRA:


## PROGRAR:



## PRCGRAM:



RROGRII:


PROGRiAS:

| $\underline{V}-\mathrm{DG}, \mathrm{Sym} . \mathrm{U}$. | (.8, 2.2) | (.6; 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Ponulition } \\ \left(10^{7} \text { pounds }\right) \end{gathered}$ |  | ERTOR | ation ) |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | - 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | $0$ | 0 | 0 | 0 |
| 9 | 500 | 500 | 500 | 750 |
| 10 | 1,500 | 1,500 | 1,250 | 1,250 |
| 17 | 2,000 | 2,000 | 2,000 | 2,000 |
| 12 | 2,750 | 2,750 | 2,750 | 2,750 |
| 13 | 3,750 | 3,750 | 3,500 | 3,500 |
| 14 | 4,250 | 4,500 | 4,250 | 4, 250 |
| 15 | 5,000 | 5,250 | 5,000 | 5,000 |
| 16 | 5,750 | 5,750 | 5,750 | 6,000 |
| 17 | 6,500 | 6,750 | 6,500 | 6,750 |
| 18 | 7,500 | 7,250 | 7,250. | 7,500 |
| 19 | 8,000 | 8,000 | 8,000 | 7,500 |
| 20 | 8,750 | 8,750 | 8,500 | 8,500 |
| 21 | 9,500 | 9,250 | 9,250 | 9,500 |
| 2.2 | 10,000 | 10,250 | 10,000 | 10,000 |
| 23 | 11,000 | 10,750 | 11,000 | 10,750 |
| 24 | 11,500 | 11,500 | 11,750 | 11,250 |
| 25 | 12,250 | 12,000 | 12,000 | 12,000 |
| 26 | 13,000 | 13,000 | 12,750 | 12,500 |
| 27 | 13,500 | 13,750 | 13,500 | 13,250 |
| 28 | 14,250 | 14,500 | $14,250$ | $14,250$ |
| 29 | 15,000 | 15,000 | $14,750$ | $14,500$ |
| 30 | 15,750 | 15,500 |  |  |

## PFOGRili:

| VI-P, Sym.U. | (.8, 1.2) | (.6, 1.4) | $(.4,1.6)$ | $(.2,1.8)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Ponulation } \\ \left(10^{7} \text { pounds }\right) \end{gathered}$ | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 |
| 17 | 54,000 | 54,000 | 51,000 | 54,000 |
| 18 | 57,500 | 57,500 | 57,250. | 55,000 |
| 19 | 57,750 | 56,500. | 55,000 | 54,250 |
| 20 | 57,250 | 57,250 | 57,000 | 57,000 |
| 27 | 59,500 | 58,500 | 56,500 | 56,000 |
| 22 | 57,500 | 56,500 | 56,250 | 55,000 |
| 23 | 58,500 | 58,250 | 57,000 | 55,000 |
| 24 | 57,500 | 56,000. | 54,000 | 53,250 |
| 25 | 51,500 | $54,000{ }^{\circ}$ | 52,000 | 51,000 |
| 26 | 53,750 | 53,500 | 52,750 | 51,000 |
| 27 | 45,750 | 45,750 | 45,750 | 45,750 |
| 28 | 47,500 | 47,500 | 47,500 | 45,500 |
| 29 | 48,250 | 47,500 | 46,250 | 45,000 |
| 30 | 46,750 | 46,250 | 46,250 | 46,000 |

PROGRAR:

| VI-P, Sym. | (.8, 1.2) | (.6, 1.4) | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulation }}{\left(100^{7} \text { pouncis }\right)}$ |  | Effor | $\frac{\text { setion }}{}$ | , |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 17 | 0 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 0 | 0 |
| 13 | 0 0 | 0 | 0 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0. |
| 17 | 54,000 | 54,000 | 54,000 | 54,000 |
| 18 | 57,500 | 57,500 | 57,500 | 57,500 |
| 19 | 58,000 | 57,250 | 56,250 | 54,750 |
| 20 | 57,250 | 57,250 | 57,250 | 57,250 |
| 21 | 59,500 57,250 | 59,250 | 58,500 | 57,750 |
| 22 | 57,250 58,500 | 56,750 58,500 | 56,250 58,000 | 56,250 56,250 |
| 23 | 57,750 | 57,000 | 55,750 | 55,000 |
| 25 | 54,500 | 54, 250 | 54,000 | 53,250 |
| 26 | 53,750 | 53,750 | 53,500 | 51,500 |
| 27 | 45,750 47,500 | 45,750 47,500 | 45,750 47,500 | 45,750 |
| 28 29 | 48,250 | 48,000 | 47,250 | 46,250 |
| 30 | 46,750 | 46,500 | 46,250 | 46,250 |

## PRCGRAR:

| VI-P, Skȧ.R. | $(.8,1.5)$ | $(.8,7.7)$ | (.8, 1.9) |
| :---: | :---: | :---: | :---: |
| Poculation |  | Effort Allocation |  |
| ( $10^{7}$ pounds ) |  | $\therefore$ ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| J 6 | 0 | 0 | 0 |
| 17 | 54,000 | 54,000 | 54,000 |
| 18 | 57,500 | 57,500 | 57,500 |
| 19 | 57,750 | 57,000 | 56,750 |
| 20 | 57,250 | 57,250 | 57,250 |
| 21 | 59,250 | 59,000 | 58,750 |
| 22 | 57,250 | 56,750 | 56,500 |
| 23 | 58,500 | 58,250 | 58,250 |
| 24 | 57,000 | 56,750 | 56,250 |
| 25 | 54,250 | 54,250 | 54,000 |
| 26 | 53,750 | 53,500 | 53,500 |
| 27 | 45,750 | 45,750 | 45,750 |
| 28 | 47,500 | 47,500 | 17,500 |
| 29 | 48,000 | 47,7,0 | 47,750 |
| 30 | 46,500 | 46,250 | 45,250 |

PRCGRin:

| VI-P, Skd.L. | (.6, 1.2) | (.4, 1.2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| $\frac{\text { Pcpulation }}{\left(10^{7} \text { pounds }\right)}$ |  | Effort illocation <br> (days) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0. | 0 | 0 |
| 9 | 0 | 0 | 0 |
| 10. | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 3.4 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 54,000 | 54,000 | 54,000 |
| 18 | 57,500 | 57,500 | 57,500 |
| 19 | 57,500 | 57,000 | 56,500 |
| 20 | 57,250 | 57,250 | 57,250 |
| 21 | 59;250 | 59,000 | 53,500 |
| 22 | 57,000 | 56,750 | 56,500 |
| 23 | 58,500 | 58,250 | 58,250 |
| 24 | 57,250 | 56,750 | 56,000 |
| 25 | 54,500 | 54,250 | 54,000 |
| 26 | 53,750 | 53,500 | 53,500 |
| 27 | 45,750 | 45,750 | 45,750 |
| 28 | 47,500 | 47,500 | 47,500 |
| 29 | 43,000 | 47,750 | 47,500 |
| 30 | 46,500 | 46,250 | 40,250 |

Progridi:

| VI-D, Sym.U. | (.8, 1.2) | $(.6,2.4)$ | (.4, 1.6$)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Ponulation } \\ & \left(10^{7} \text { pounds }\right) \end{aligned}$ |  | EROR | ation ) |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 |
| 17 | 54,000 | 51,750 | 48,250 | 14,750 |
| 18 | 55,250 | 53,000 | 4i,000 | 45, 250 |
| 29 | 56,500 | 53,500 | 50,250 | 46,500 |
| 20 | 57,250 | 54,000 | 50,000 | 46,500 |
| 21 | 57,250 | 51,750 | 50,250 | 47,000 |
| 22 | 57,000 | 53,750 | 51,000 | 47,250 |
| 23 | 56,750 | 54,000 | 50,500 | $17,750$ |
| 24 | 56,000 | 53,250 | 51,000 | 47,500 |
| 25 | 54,000 | 52,000 | 50,500 | 47,500 |
| 26 | 52,000 | 52,250 | 50,750 | 47,250 |
| 27 | 50,500 | 51,500 | 50,000 | 47,500 |
| 28 | 49,750 | 5.1,500 | 49,500 | 47,500 |
| 29 | 49,000 | 50,500 | 19,750 | 46,750 |
| 30 | 48,000 | 50,000 | 49,250 | 47,000 |

PROGRaL:

| VI.-D, Sym. | $(.8,1.2)$ | (.6, 1.4). | (.4, 1.6) | ( $.2,1.8$ ) |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Ponulation }}{\left(10^{7}\right. \text { pounds ) }}$ | ( days) | Eefort Allocation |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | $\stackrel{0}{0}$ | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| ¢ | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | - |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 0 | 0 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 17 | 54,750 | 53,250 | 51,500 | 49,000 |
| 18 | 56,500 | 55,000 | 52,500 | 50,250 |
| 19 | 57,000 | 55,500 | 53,250 | 50,750 |
| 20 | 57,500 | -50, 500 | 53,500 | 51,500 |
| ? 21 | 57,750 | 55,750 | 51,000 | 51,000 |
| 23 | 57,250 | 55,750 | 54,000 | 51,000 |
| 24 | 50,750 55,000 | 54,750. | 53,000 | 51,250 |
| 25 26 | 52,750 | 52,750 | 52,000 | 50,000 |
| 27 | 48,750 | 51,250 | 51,000 | 49,750 |
| 28 | 48,000 | 50,000 | 50,500 | 49,250 |
| 29 | 47,500 47,250 | 49,500 48,750 | 49,750 49,500 |  |
| 30 | 47,250 | 48,750 | 49,500 |  |

## PRCGR4:

| VI-D, Skd.R. | (.8, 1.5$)$ | $(.3,1.7)$ | (.8, 1.9) |
| :---: | :---: | :---: | :---: |
| $\frac{\text { Pooulation }}{\left(10^{7} \text { pounds }\right)}$ |  | $\frac{\text { 3ffort inlocation }}{\text { (days) }}$ |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 |
| 9 | . 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0. |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 53,250 | 52,000 | 50,750 |
| 18 | 54,750 | 53,500 | 52,000 |
| 19 | 55,250 | 54,250 | 52,750 |
| 20 | 56, 250 | 54,500 | 53,250 |
| 27 | 56,000 | 54, 250 | 52,750 |
| 22 | 55,750 | 54,000 | 53,000 |
| 23 | 55,000 | 54,250 | 53,000 |
| 24 | 54,750 | 52,500 | 50,250 |
| 25 | 52,250 | 50,500 | 49,500 |
| 26 | 50,500 | 50,000 | 49,000 |
| 27 | 50,500 | 49,750 | 49,250 |
| 28 | 49,750 | 49,750 | 49,250 |
| 29 | 49,750 | 49,500 | 49,250 |
| 30 | 49,250 | 49,750 | 49,250 |

830GRil:


## PROGRRA:

\begin{tabular}{|c|c|c|c|c|}
\hline Vİ- $\overline{\mathrm{DG}}$, Sym.U. \& (.8, 1.2) \& (.6, 2.4 ) \& (.4, 1.6) \& (.2, 1.8) <br>
\hline Population \& \multicolumn{4}{|c|}{Effont Allccation} <br>
\hline ( $10^{7}$ pounds) \& \multicolumn{4}{|c|}{( days)} <br>
\hline 0 \& 0 \& 0 \& 0 \& 0 <br>
\hline 1 \& 0 \& 0 \& 0 \& 0 <br>
\hline 2 \& 0 \& 0 \& 0 \& 0 <br>
\hline 3 \& 0 \& 0 \& 0 \& 0 <br>
\hline 4 \& 0 \& 0 \& 0 \& 0 <br>
\hline 5 \& 0
0 \& 0 \& 0 \& 0 <br>
\hline 7 \& 0 \& 0 \& 0 \& 0 <br>
\hline 8 \& 0 \& 0 \& 0 \& 0 <br>
\hline 9 \& 0 \& 0 \& 0 \& 0 <br>
\hline 10 \& 0 \& 0 \& 0 \& 0 <br>
\hline 11 \& 0 \& 0 \& 0 \& 0 <br>
\hline 12 \& 0 \& 0 \& 0 \& 0 <br>
\hline 13 \& 0
0 \& 0 \& 0 \& 0 <br>
\hline 14 \& 0 \& 0 \& 0 \& 0 <br>
\hline 16 \& 0 \& 0 \& 0 \& 0 <br>
\hline 17 \& 54,250 \& 52,250 \& 50,250 \& 47,0.00 <br>
\hline 18 \& 55,750 \& 54,000 \& 51,250. \& 48,500 <br>
\hline 19 \& 57,000 \& 51,750 \& 51,500
50

5, \& 48,500 <br>
\hline 20 \& 57,250 \& 55,250
55,250 \& 52,250
51,750 \& 48, 4 , 250 <br>
\hline 27 \& 57,750
57,500 \& 55,250
54,500 \& 51,750
51,750 \& 18,500
48,500 <br>
\hline 22
23
23 \& 57,500
57,000 \& 54,500
53,500 \& 51,250 \& 43,250 <br>
\hline 214 \& 56,000 \& 53,000 \& 50,750 \& 47,750 <br>
\hline 25 \& 542,500 \& 53,000 \& 51,000
50,000 \& 47,500 <br>
\hline 26 \& 51,500
50,250 \& 51,500
51,500 \& 50,000
4,750 \& 47,500
47,250 <br>
\hline 27 \& 50,250
49,000 \& 51,500
51,250 \& 49,750 \& 47,250 <br>
\hline 28
29 \& 48,500 \& 50,500 \& 49,000
49,000 \& 46,000 <br>
\hline 30 \& 48,500 \& 50,250 \& 49,000 \& 46,250 <br>
\hline
\end{tabular}

PROGRARI:

| VI- $\overline{D G}$, Sym. | $(.8,1.2)$ | $(.6,1.4)$ | (.4, 1.6) | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Pomulation | Effort A7location |  |  |  |
| ( $10^{7}$ pouncis ) | ( days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | - 0 | 0 |
| 9 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | - 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 | 0 |
| 17 | 54,750 | 53,500 | 52,250 | 50,250 |
| 18 | 56,000 | 55,250 | 53,750. | 52,000 |
| 19 | 57,250 | 56,000 | 54,500 | 52,250 |
| 20 | 57,750 | 56,750 | 54,750 | 52,500 |
| 21 | 58,000 | 56,500 | 54,500 | 52,250 |
| 22 | 56,000 | 56,500 | 51,750 | 52,000 |
| 23 | 57,250 | 56,000 | 53,750 | 52,000 |
| 24 | 56,500 | 55,000 | 53,000 | 51,250 |
| 25 | 55,250 | 54,000 | 52,500 | 50,2.50 |
| 26 | 53,250 | 51,750 | 51,500 | 50,250 |
| 27 | 48,250 | 50,750 | 50,500 | 50,000 |
| 28 | 47,750 | 49,750 | 50,000 | 49,500 |
| 29 | 47,250 | 49,250 | 49,500 | 48,750 |
| 30 | 47,500 | 49,000 | 49,250 | 48,250 |

## PRCGRAH:



PROGRAR:

| VI- $\overline{\mathrm{DG}}$, S Srd.I. | (.6, 1.2) | (.4, 3. .2) | (.2, 1.2) |
| :---: | :---: | :---: | :---: |
| Population |  | Effort Allocation |  |
| ( $10^{7}$ pounds) |  | (deys) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | c | 0 |
| 9 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 |
| 11. | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 |
| 16 | 0 | 0 | 0 |
| 17 | 54,250 | 54,000 | 53,250 |
| 1.8 | 55,000 | 55,000 | 54, 500 |
| 19 | 56,000 | 55,000 | 55,000 |
| 20 | 57,250 | 56,500 | 56,000 |
| 21 | 57,500 | 50,750 | 55,750 |
| 22 | 57,000 | 56,500 | 55,500 |
| 23 | 56,250 | 56,250 | 55,750 |
| 24 | 55,000 | 55,000 | 55,000 |
| 25 | 54,000 | 54,250 | 54,000 |
| 26 | 52,500 | 52,750 52,000 | 53,000 |
| 27 | 51,000 | 52,000 | 52,500 |
| 28 | 50,000 | 51,750 | 51,500 |
| 29 | 48,750 | 49,500 49,000 | 50,500 |
| 30 | 48,500 | 49,000 | 48,250 |


| Vİ-DG, Sym.U. | (.e., 1.2) | (.6, 1.4$)$ | $(.4,1.6)$ | (.2, 1.8) |
| :---: | :---: | :---: | :---: | :---: |
| Pozulation | Efrort Allocation |  |  |  |
| ( $10^{7}$.pounds ) | (days ) |  |  |  |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5. | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 3 | 0 | 0 | 0 | 0 |
| 1.0 | 0 | 0 | 0 | 0 |
| 112 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 |
| 16. | 0 | 0 | 0 | 0. |
| 17 | 54,500 | 52,250 | 47,500 | 45,250 |
| 18 | 52,500 | 53,000 | 49,750. | 46,000 |
| 19 | 56,500 | 54,250 | 49,250 | 4i,250 |
| 20 | 56,750 56,750 | 53,750 | 49,750 | 47,000 48,000 |
| 21 | 57,000 | 55,250 51,500 | 50,000 49,500 | 48,000 |
| . 23 | 57,250 | 51, 2,500 53,500 | 50,750 | 48,250 |
| 24 | 55,500 54,250 | 52,750 | 50,500 | 48,000 |
| 25 | 54,250 52,000 | 52,500 | 49,500 | 47,250 47,500 |
| 27 | -3,250 | 51,250 51,250 | 50,500 50,250 | 47,500 46,250 |
| 28 | 50,250 | 51,250 | 49,500 | 47,500 |
| 29 | 49,250 48,500 | 50,250 | 49,000 | 47,000 |
| 30 | 42,500 | 49,750 | 48,500 | 46,750 |

PROGR:IT:

| . | $\begin{gathered} I \\ (.2,1.8) \end{gathered}$ | $\begin{gathered} \text { II } \\ (.2,1.8) \end{gathered}$ | $\begin{gathered} \text { III } \\ (.2,1.8) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Pooulation <br> ( $10^{7}$ pounds ) |  | $\begin{aligned} & \text { Effort Allocetion } \\ & (\text { days }) \end{aligned}$ |  |
| - |  |  |  |
| - 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | $0$ | 0 |
| 5 | $0$ | 0 | 0 |
| $6$ | $0$ | 0 | 0 |
| 7 | 0 | 0 | 0 |
| 8 | 0 |  | 0 |
| 9 | 0 | - 500 | 0 |
| 10 | 0 | 1,250 | 0 |
| 11 | 0 | 2,000 | 0 |
| 12 | 0 | 2,750 | 0 |
| 13 | 8,750 | 3,500 | 0 |
| 14 | 17,500 | 4,250 | 0 |
| 15 | 25,000 | 5,000 | 0 |
| 16 | 31,250 | 5,750 | 0 |
| 17 | 36,500 | 6,500 | 0 |
| 18 | 41,250 | 7,250 | - 0 |
| 19 | 45,250 | 8,000 | 72,750 |
| 20 | 48,750 | \%,750 | 75,750 |
| 27 | 52,000 | 9,500 | .78,000 |
| 22 | 54,750 | 10,250 | 80, 000 |
| 23 | 57,000 | 11,000 | 81,500 |
| 24 | 58,750 | 11,750 | $83,000$ |
| 25 | 60,500 | $12,250$ | $84,250$ |
| 26 | 61,750 | $13,000$ | 85,000 |
| 27 | 63,000 | 13,750 | 85,750 |
| 28 | 64,750 | 14,500 | 86,750 |
| 29 | 66,250 | 15,250 | 88,000 |
| 30 | 67,250 | 16,000 | 86,500 |

## GROMTH VATEATIOM PRGGRAS

PRCGRALS

|  | $\begin{gathered} \text { IV } \\ (.2,1 \end{gathered}$ | $\begin{aligned} & v \\ & (.2,1.8) \end{aligned}$ | $\begin{aligned} & \text { VI. } \\ & (.2,1.8) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\frac{\text { Podulation }}{\left(10^{7} \text { pounds }\right)}$ |  | Effort Allocetion ( days ) |  |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 |
| 7 | 0 | - | 0 |
| 8 | 0 | 0 | 0 |
| 9 | 0 | 500 | 0 |
| 10 | 0 | 1,250 | 0 |
| 11 | 5,000 | 2,000 | 0 |
| 12 | 2,750 | 2,750 | 0 |
| 13 | 15,000 | 3,500 | 0 |
| 14 | 18,500 | 4,250 | 0 |
| 15 | 22,250 | 5,000 | $\bigcirc$ |
| 16 | 25,250 | 5,750 6,500 | 53, 750 |
| 17 | 27,250 29,750 | 6,500 7,250 | 53,750 55,500 |
| 18 | 29,750 31,750 | 7,250 8,000 | 55,500 57,000 |
| 19 | 33,500 | 8,750 | 57,500 |
| 22 | 34,750 | 9,500 | .57,750 |
| 22 | 36,250 | 10,250 | 58,000 |
| 23 | 37,250 | 10,750 | 58,000 |
| 24 | $38,2.50$ 39,000 | 11,500 12,250 | 57,750 57,250 |
| 25 26 | 39,000 40,000 | 12,250 13,000 | 57,250 57,000 |
| 26 27 | 40,500 | 13,750 | 46,000 |
| 28 | -47, 250 | 14,500 | 46,750 |
| 28 | 40,750 | 15,000 | 47,000 |
| 30 | 41,750 | 15,750 | 46,250 |

${ }^{1}$ An important example where price fluctuations are quite large and are believed to have a significant impact on resource allocation occurs in the Peruvian Anchovy Fishery; see Segura (1972).
${ }^{2}$ Unfortunately, the problem of insufficient data is not peculiar to the yellowfin tuna fishery. To my knowledge, at the present time there is little available information to estimate these frequency distributions for any of the ocean fisheries.
${ }^{3}$ For example, Leland (1974), Levhari and Srinivasan (1966), and Phelps (1962) in their analyses of optimal consumption-saving rules discover that individuals with a constant elasticity utility function given by

$$
U(C)= \begin{cases}\frac{C^{\gamma}}{Y} & \text { for } \gamma<1 ; \gamma \neq 0 \\ \log (C) & \text { for } \gamma=0 .\end{cases}
$$

will increase, decrease or not change their consumption of total wealth for $0<\gamma<1, \gamma<0$ or $\gamma=0$ as the rate of return on investment becomes riskier. In other studies, Mills (1959) and Zabel (1970) have noted the strong dependence of results on stochastic specifications assumed in their models.
${ }^{4}$ Please note that the scale for effort on the vertical axis of all graphs corresponding to Classes.II and $V$ is enlarged. Also, the effort allocations for all programs are identical for those populations where only one line appears on the graph.
${ }^{5}$ In equilibrium $(a-b X) X=k X E$ or $E=\frac{(a-b X)}{K}$. Equilibrium values for $E$ are represented by points on the "Steady State Effort ${ }^{\text {" }}$ line in each figure.
${ }^{6}$ See Rothschild and Stiglitz (1970).
${ }^{7}$ This is partially offset by the fact that the utility function $U(R)=\ln (R+G)$ displays decreasing absolute risk aversion. Thus as $R$ increases the resource manager should become less averse to risk.
${ }^{8}$ For a discussion of this point see Hirshleifer and Shapiro (1970).
${ }^{9}$ On this point see Baumol (1970, and Hirshleifer and Shapiro (1970).
${ }^{10}$ See Prest and Turvey (1965).
${ }^{11}$ Leland's analysis is done in the context of a portfolioconsumption framework.

12 Although the absolute increase in present value is large for Class I programs, the percentage increase is less than $1 \%$. However for the Class III programs the percentage increases range from $1 \%$ to $30 \%$.

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    See footnotes on page 14.

[^1]:    ${ }^{1}$ See footnotes on pages $112-115$.

[^2]:    ${ }^{1}$ See footnotes on pages 252-253.

