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The Use of Markets to Control Pollution

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The primary purpose of this paper is to study the extent to which prices and markets can be employed as means towards efficient control of pollution. The model which is examined is, however, sufficiently general to apply to any sort of externalities among producers and consumers, including classical public goods. In the first section we consider the problem of efficient means of achieving a specified standard of environmental quality. We then briefly discuss the more difficult problem of choosing the environmental quality standards themselves.

SECTION I. EFFICIENT MEANS OF ACHIEVING SPECIFIED STANDARDS

A. A Simple Model

We first study a rather simple special problem which turns out to be sufficient to illustrate the relevant issues for a very general class of economies. Consider an economy with n consumers, one private good and one pollutant. Let x_i be the quantity of the private good consumed by consumer i and y_i be the amount of pollutant released by i. Let $z = \sum_{i=1}^{n} y_i$ be the total amount of pollutant. Suppose that preferences of each consumer i are represented by the utility function U_i (x_i,

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y_i,z) where U_i is an increasing function of x_i and y_i, and a decreasing function of z. Thus each consumer takes pleasure in the consumption of the private good and in releasing pollution, while he finds increases in the total amount of pollution unpleasant. Suppose that there is a fixed total amount $\mathring{x}$ of the private good which may be distributed in non-negative amounts to each consumer in any way such that $\sum_{i=1}^{n} x_i = \mathring{x}$. We observe that the quantities x_i and y_i are private in the sense that when z is fixed, no consumer other than i cares about these quantities. On the other hand, the quantity z plays a formal role identical to that of a pure public commodity, albeit undesirable, as formulated by Lindahl (1964), Samuelson (1954) and others.

Let us suppose that z is somehow fixed at an arbitrary level $\mathring{z}$. There remains the allocation problem of choosing particular values of $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ in such a way that $\Sigma\, x_i = \mathring{x}$ and $\Sigma\, y_i = \mathring{z}$. A non-negative vector $(x_1, \ldots, x_n, y_1 \ldots, y_n)$ wille be called a *feasible allocation* relative to $(\mathring{x}, \mathring{z})$ of these two equations are satisfied. A feasible allocation will be called *conditionally efficient* relative to $(\mathring{x}, \mathring{z})$ if there is no alternative feasible allocation relative to $(\mathring{x}, \mathring{z})$ which all consumers like as well and some consumer likes better.

The formal set up of the problem of conditional efficiency is easily seen to be the same as that of the problem of finding Pareto efficient allocations for a pure exchange economy with two desirable private goods. Since z is fixed at $\mathring{z}$ we can write $U_i^* (x_i, y_i) = U_i (x_i, y_i, \mathring{z})$ and note that the feasibility constraints are that $\Sigma\, x_i = \mathring{x}$ and $\Sigma\, y_i = \mathring{z}$. This is precisely the set up of the efficiency problem in an exchange economy with purely private goods. We may press the analogy to the exchange model further by supposing that for each i there is an "initial endowment" vector $(\mathring{x}_i, \mathring{y}_i) \geq 0$ such that $\Sigma\, \mathring{x}_i = \mathring{x}$ and $\Sigma\, \mathring{y}_i = \mathring{z}$.

We must, however, establish an economic interpretation for the "initial endowments", $\mathring{y}_i$. This is quite easily performed. Suppose there is an institutional arrangement such that if one is to release y_i units of pollution he must hold a total of y_i "pollution tickets" each of wich allows him to release one unit. Each consumer i originally holds $\mathring{y}_i$ tickets ands $\Sigma\, \mathring{y}_i = \mathring{z}$. No new tickets may be printed but consumers are allowed to trade these tickets (and to divide them into non-integral quantities) just as they are allowed to trade their initial holdings of the private good. What is done is essentially to establish a system of property rights for polluting activities and to enforce these rights in much the same way that property rights in private goods are enforced.

When this is done, there is a complete isomorphism between the problem of allocating a specified total amount of pollution rights and the exchange model for purely private goods. We can therefore apply the results of exchange theory to the problem. An allocation $(x_1, \ldots x_n, y_1, \ldots y_n)$ will be called a *conditional market equilibrium* relative to the endowments $(\mathring{x}_1, \ldots, \mathring{x}_n, \mathring{y}_1, \ldots, \mathring{y}_n)$ if there are prices p and q such that :

(1) For all i : (x_i, y_i) maximizes $U_i (x_i, y_i, \mathring{z})$ subject to $px_i + qy_i \leq p\mathring{x}_i + q\mathring{y}_i$.

(2) $\Sigma\, x_i = \Sigma\, \mathring{x}_i$ and $\Sigma\, y_i = \Sigma\, \mathring{y}_i = \mathring{z}$.

An allocation will be said to have the *core property* relative to $(\mathring{x}, \mathring{y})$ if no coalition consisting of a subset of the set of all consumers can redistribute the initial holdings of its own members in such a way as to harm none of its members and to benefit at least one. More formally, the allocation $(x,y) = (x_1, \ldots, x_n, y_1 \ldots, y_n)$ has the core property relative to $(\mathring{x},\mathring{y})$ if there exists no subset K of the set of all consumers such that for some allocation $(x_1', \ldots, x_n', y_1', \ldots, y_n')$, $\sum_K x_i' = \sum_K \mathring{x}_i$, $\sum_K y_i' = \sum_K \mathring{y}_i$, and $U_i(x_i', y_i', \mathring{z}) \geqq U_i(x_i, y_i, \mathring{z})$ for all $i \epsilon K$ with strict inequality for some $i \epsilon K$.

Due to the isomorphism between this model and the usual theory of exchange for private goods, the following results emerge as immediate consequences of familiar results in the theory of exchange.

Where preferences are convex, locally non-satiated, and represented by continuous utility, then subject to a few well-known technical assumptions :

1) There exists a conditional market equilibrium relative $(\mathring{x},\mathring{z})$.
2) Conditional market equilibria are conditionally efficient and have the core property relative to $(\mathring{x},\mathring{y})$.
3) Any conditionally efficient allocation is a conditional market equilibrium relative to some initial endowment of private goods and pollution tickets.
4) If the economy has "many similar participants" in a certain welll-defined sense, then any allocation which has the core property relative to $(\mathring{x},\mathring{y})$ is "nearly" a conditional market equilibrium relative to (x,z) (1).

Even when preferences are not assumed to be convex, results 1) - 4) will be "essentially" true for economies with "sufficiently many" participants so long as individual non-convexities are not "too large" (2).

Thus we are assured that there exists a conditional market equilibrium in which private commodities and pollution tickets are exchanged at competitive prices. Such an equilibrium allocation is efficient in the sense that there is no feasible way of achieving a universally preferred allocation given the specified level of total pollution.

The interpretation of the core property is slightly unconventional in that strategies which are available to a coalition are restricted to those in which total pollution released by its members does not exceed the total amount of pollution allowed by tickets which its members hold. Yet this seems to be a very reasonable restriction to make, quite analogous to the protection of property rights for private goods in the usual model.

Result 4 is of interest if we wish to predict the outcome of simply making an assignment of property rights to pollution and enforcing them. In particular, if

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(1) Expositions of the theory of exchange can be found in Newman (1965), Quirk and Saposnik (1968) or Arrow and Hahn (1971). Result 4 can be found in Arrow and Hahn, Chapter 8.

(2) This result is explicitly shown for a finite economy in Bergstrom (1973a) ; Similar results are demonstrated in Arrow and Hahn.

we have "many" consumers and we suspect that unhindered bargaining will lead to an allocation which has or nearly has the core property, then we would except the ultimate allocation to be similar to a conditional market equilibrium.

In many of practical situations which we observe, it appears that property rights to pollution emission are not well defined. If one wishes to establish such rights explicitly it is not at all obvious what is the "just" way to do so. One must decide whether historically observed polluting activities should be explicitly recognized as endowing the polluter with "rights" or whether the polluters should be treated as usurpers and be forced to purchase tickets from others if they wish to continue their activities. The model formulated here helps to distinguish between pollution control as a means toward economic efficiency and as a means of redistribution of wealth. At any rate, result 3 assures us that the use of a market system is distributionally neutral in the sense that for any standard, $\mathring{z}$, of environmental quality, any possible distribution of utility can be achieved from a competitive allocation if initial holdings are appropriately chosen.

B. Many Commodities

Here we generalize the model to an economy with many private commodities and many activities which generate externalities. It is assumed that there are n consumers, k private commodities and m non-private activities. Quantities of private commodities consumed by i are represented by vectors $x_i \in E^k$ and levels of non-private activities undertaken by i are represented by vectors $y_i \in E^m$. Aggregate levels of non-private activities are represented by vectors $z \in E^m$ where z is the vector sum of non-private activities y_i over the set of consumers. We will suppose that an individual's level of preference depends on his consumption of private goods, on the vector of non-private activities which he pursues, and on the sum of the vectors of non-private pursued by all members of the economy. Preferences of each consumer i will thus be represented by a utility function $U_i(x_i, y_i, z)$ with domain $C_i C E^k \times E^m \times E^m$.

No prior assumptions are made about the desirability of any of the commodities. Certain non-private activities may be unpleasant for individuals to perform but socially desirable in their effect on the aggregate level of non-private activities. Some components of the vector of aggregate non-private activities may be such that increases are desirable to some consumers and undesirable to others, and so on.

We again suppose that there is no production (or destruction) of private goods. Feasible states of the economy are described by vectors $(x_1, \ldots, x_n, y_1 \ldots, y_n, z)$ such that for each i, $(x_i, y_i, z) \in C_i$, $\Sigma x_i = \mathring{x}$ where $\mathring{x}$ is the vector of initial holdings of private goods and $\Sigma y_i = z$. As before if z is fixed at some level $\mathring{z}$ and if there are vectors $(\mathring{x}_i, \mathring{y}_i)$ of individual initial holdings such that $\Sigma \mathring{x}_i = \mathring{x}$ and $\Sigma \mathring{y}_i = \mathring{z}$ then there is an isomorphism to an exchange economy with k + m private commodities. As previously we can apply the standard results of exchange theory to the resulting allocational problem.

Some of the commodities in the vector (x_i, y_i) may be undesirable to consumer i. This raises no particular difficulties for extending our previous remarks. There are perfectly adequate counterparts to each of the traditional results for economies in which some goods may be undesirable to some or all consumers. [See for example, Rader (1964), Debreu (1956) and (1962), or Bergstrom (1973a)]. The only novel feature is that some prices may be negative.

One interesting case in which negative equilibrium prices occur is the following. Suppose that activity h is socially useful but its performance is privately odious to each consumer. Then we must interpret initial holdings $\mathring{y}_i^h$ of activity h by consumer i to represent an obligation to perform activity h at the level of $\mathring{y}_i^h$. But in a market equilibrium he is able to "sell this obligation at a negative price" which of course amounts to paying someone else to perform it. This interpretation may appear a bit less forced if it is observed that in market equilibrium such holdings amount to an abligation to pay a certain share of the cost of inducing the performance of activity h at level $\mathring{z}^h$.

We have assumed that the social effect of individual non-private activities y_i are represented by $z = \Sigma y_i$. One might question whether this assumption is justified. In particular, two alternative allocations could be imagined in which we have alternative allocations of non-private activities $y_1, \ldots, y_n$, and $y'_1, \ldots, y'_n$, such that $\Sigma y_i = \Sigma y'_i$ but where the environmental effects are markedly different. A facile reply to this query is to argue that all that is needed is further disaggregation. If certain components of the y_i vectors do not interact additively we can salvage the situation by allowing there to be more commodities. In the extreme case we would have to treat emissions of a given pollutant as a different commodity for each consumer. Thus if there are k pollutants we might write y_i as kxn dimensional vector such that y_i contains 0's in all positions except in positions $k(i-1)+1$ through ki and where the number in position $k(i-1)+j$ represents the amount of emission j by agent i. Then, of course, the vector Σy_i contains a full description of the activities of each agent. If we do this, however, we lose both baby and bathwater, since now a specification of the level of z also specifies the level of y_i for each i. There is no advantage in using decentralized pricing for the conditional efficiency problem since there is no conditional efficiency problem. In effect, the entire burden of the problem is placed on the selection of overall standards which becomes an enormously difficult and detailed problem.

It is clearly an empirical question to determine the extent to which it is reasonable to aggregate. There usually will be some costs in realism but the advantage is a considerable reduction of that part of the problem which must be dealt with by centralized means.

C. Production

The previous results can be extended to productive economies at surprisingly low cost. What is required is a slight generalization of a result of Rader (1964) which demonstrates that a straightforward equivalence can be constructed between a productive economy and an exchange economy. We suppose that each agent is able to act both as a producer and as a consumer.

Agents may exhaust supplies of private commodities either by consuming them or by using admissable production processes to transform them into other private commodities which they in turn may either consume or transfer to others. For any agent the possible production transformations may depend on the vector z of aggregate non-private activities. A non-private activity may take the form either of the production of a physical good or of an action.

The formal situation is as follows. Let $\tilde{x}_i$ represent net depletions of existing stocks by consumer i. If a consumer has initial holdings $\mathring{x}_i$ and he exhausts the vector $\tilde{x}_i$ then on net he makes available the vector $\mathring{x}_i - \tilde{x}_i$ to others. For each i, define a correspondence $T_i (z)$ from E^m to the set of subsets of $E^k \times E^m$ which is employed as follows. If consumer i exhausts the vector $\tilde{x}_i$ he is able to consume any vector x_i of private goods and to pursue any vector y_i of non-private activities such that $x_i = \tilde{x}_i + x_i'$ where $(x_i', y_i) \epsilon T_i (z)$.

This indicates the following interpretation of $T_i (z)$. A vector (x_i', y_i) belongs to $T_i (z)$ if where the aggregate of non-private activities is z, agent i is able to pursue non-private activities y_i and to produce on net the vector x_i' of private commodities. Negative components of x_i' denote commodities exhausted in production and positive components denote commodities produced. Writing $\tilde{x}_i = x_i - x_i'$ we see that the vector of commodities exhausted by i consists of his consumption minus his net production.

Suppose that preferences on x_i, y_i and z are represented by the utility function $U_i (x_i, y_i, z)$. Define the derived utility function

$$\tilde{U}_i (\tilde{x}_i, y_i, z) \equiv \underset{\substack{(x_i', y_i) \epsilon T_i (z) \\ (\tilde{x}_i + x_i', y_i, z) \epsilon C_i}}{\text{maximum}} \quad U_i (\tilde{x}_i + x_i', y_i, z) .$$

This function $\tilde{U}_i$ represents the highest utility that i can achieve by means of technical possibilities available to him when the aggregate vector of non-private activities is z and when he pursues non-private activities y_i and exhausts the vector $\tilde{x}_i$. Consider an economy with the following features.

Type I Economy

Preferences are represented by utility functions
$U_i (x_i, y_i, z)$. Feasible allocations are described by
vectors $x_i, ..., x_n, x_i', ..., x_n', y_1, ..., y_n$ and z such that
$\Sigma y_i = z$ and such that for all i :
$(x_i', y_i) \epsilon T_i (z)$ and $\Sigma x_i - \Sigma x_i' = \Sigma \mathring{x}_i$.

Any type I economy can be equally well described as a type II economy as below.

Type II Economy

Preferences are represented by the utility functions. $\tilde{U}_i$ $(\tilde{x}_i, y_i, z)$ where $\tilde{U}_i$ is defined as above. Feasible allocations are described by vectors $\tilde{x}_1, ..., \tilde{x}_n, y_i, ..., y_n$ and z such that $\Sigma \tilde{x}_i = \Sigma \mathring{x}_i$ and $\Sigma y_i = z$.

Observe that a type II economy is isomorphic to an exchange economy just as is the model with no production.(3) Of course, before the results of exchange theory can be applied to this economy, it must be demonstrated that properties which are conventionally assumed for feasible production activities and for consumer preference are sufficient to endow the functions $\tilde{U}_i$ with the properties usually assumed in exchange theory. This task is performed in the Appendix to this paper.

This equivalence result enables us to apply all of the results of exchange theory discussed above to an economy with production. We are thus afforded a considerable economy of notational effort. Of course, all of the usual conditions for efficient production can be derived when one extends his consideration to the detailed specification of the production processes implicitly underlying the derived utility functions. Often in discussions of social policy it is not particularly important to work out these details since they amount to private maximization problems which one can reasonably assume are left to the individual agents. All information relevant to social interaction is contained in the variables $\tilde{x}_i$, y_i and z. There may, however, be reasons for explicitly considering the nature of production processes of it is thought that certain plausible assumptions on the nature of interaction in production may lead to useful simplifications in the structure of derived utility.

D. Applications and Interpretation

One can apply the results so far obtained to a variety of situations where the determination of overall environmental standards is not at stake. For example, suppose that we are confronted with an observed state of the economy in which there is some regulation of pollution but where this regulation is of a piecemeal *ad hoc* nature. There may, for instance, be special restrictions on the activities of certain firms, imposed either by explicit laws or by threats of legal activity. There

(3) There is one unfamiliar feature of isomorphic economy. Some of the $\bar{x}_i$'s may have negative components. So long as these vectors are constrained to lie in a compact set, however, there are no extra difficulties. Very mild assumptions on the limitations of productivity are sufficient to ensure that this is the case. See Rader (1964).

may also be numerous special exemptions and "loopholes". Some inefficiency would be expected on *a priori* grounds simply because of the randomness, from an economic view, of the restrictions. Our results suggest an alternative method of regulation which, without changing the total amount of pollution, would benefit some individuals and harm no one. Suppose that present pollution emissions by each individual or firm are measured and that the central authority issues "pollution tickets" to each economic agent entitling him to emit pollution at his current level. Agents are required to produce no more pollution than is allowed by their ticket holdings. All other restrictions on pollution are eliminated. Agents are allowed to trade among themselves from the existing stock of tickets, but no further tickets are issued.

In a stationary economy, all agents would be at least as well off after the reform as before, since they are given tickets which allow them the option of pursuing exactly the same activities as before and since total pollution is unchanged. But the elimination of the *ad hoc* restrictions may be expected to allow some agents to make mutually beneficial exchanges, again without altering total pollution. In fact unless the initial situation is conditionally efficient, the situation after the reform will be Pareto superior.

The assumption of a stationary economy is, of course, unrealistic. In a growing and changing economy, individuals have plans for future polluting activities which cannot be ascertained by direct observation. If these plans were known, tickets could be issued for future pollution in such a way that after the reform the individual is still allowed the option of pursuing his pre-reform plans. After the reform he may then choose to make trades in future tickets so as to alter his plans. It is, however, difficult to see how accurate information about such plans could be obtained.

In practice, crude rules of thumb are often applied for setting standards. It may be decided to maintain present environmental standards in the future or to allow a fixed rate of increase or decrease in the amounts of certain pollutants over time. See, for example, the discussion of air quality standards in the St. Louis Metropolitan area by Kohn (1971). Conditional efficiency could be attained, subject to these rule of thumb standards, by the market methods discussed above. Extreme losses to individuals could be avoided by issuing to agents current tickets approximately equal in quantity to their current polluting activities and altering their allowance of future tickets in proportion to changes in aggregate standards(4).

It may seem odd to "reward" individuals who currently produce much pollution by offering them many tickets for future pollution. It might be argued that the polluters had no right to pollute in the first place and that they are no more entitled to the revenue generated by use or sale of such tickets than anyone

(4) It is possible that some individuals might be affected adversely by changes in the prices of private goods after the reform. This could be avoided if individuals acquire property rights to private goods for future periods which approximate planned consumptions.

else. The suggested market scheme could be employed equally well if ownership of future tickets were distributed according to a view of "environmental rights" differing from that embodied in current practice. As with all redistributional schemes, enforcement of an alternative distribution of ownership of rights to pollute would encounter strenuous objections from the losers. It could be argued that in the existing economy, traditional patterns of rights to pollute have been capitalized into the market value of existing firms and that the owners of polluting firms may not be the original "usurpers", but relatively innocent investors who have purchased shares whose prices were based on the expectation of no radical changes in the regulation of pollution. These considerations and the requirements for broad consensus in a democracy suggest the desirability of a policy which imposes at least some regularity on the pattern of losses and gains resulting from a new view of the structure of property rights respecting pollution([5]).

There is an extensive litterature on the merits of tax-subsidy schemes as compared to direct regulation of pollution. See, for example, Coase (1960), Ruff (1970) or Freeman, Haveman and Kneese (1973). The procedure which is usually envisioned is one in which a uniform fee is charged for the emission of pollutants. Individuals and firms, when confronted with these fees, can then be expected to adjust their decisions about production methods, emissions, and emission control according to their own knowledge of technology and markets. The level of fees can be adjusted in such a way as to induce alternative levels of pollution. The clearest formal model of this situation is presented by Ruff (1972). In Ruff's model, uniform fees lead to conditional efficiency, much as does the ticket scheme of this paper.

Instead of setting fees at a fixed level and allowing the market to determine total quantities of emissions, the method of this study is to set the amount of emissions to be allowed by fixing the supply of pollution tickets and then to allow market forces to determine the price and the ultimate allocation of these tickets. Though the two methods are similar in their reliance on decentralized decisions and impersonal markets, the ticket proposal is sufficiently different to merit separate study, both for the elegance of its formal strucutre and for its potential usefulness in practical policy ([6]).

In the applications dicussed, the method of issuing a fixed number of tickets has an appealing directness not possessed by the method of fees. No experimentation with setting alternate levels of fees is required to achieve the desired

(5) It has been pointed out to me by Harold Barnett that a firm which obtains a marketable "property right" to perform an activity which it has previously performed without a well-defined right to do so may be a substantial gainer both because its situation has been legitimated and thereby made secure from further restriction and because he may in the future be able to sell such rights at positive prices. For the reasons, some firms may not be harmed even if the pollution tickets originally alloted to them allow considerably less pollution than they currently emit.

(6) Though the idea of marketable tickets is certainly not new in economics, I have not been able to find any substantial discussion of this notion in the litterature.

emission standards. This advantage, should not, however, be overstressed. The dice were loaded in favor of "tickets" when it was supposed that there is a certain standard of environmental quality to be met regardless of the costs of meeting it. Of course, standards can be intelligently proposed only when something is known about the cost of meeting the standards. But accurate knowledge of this cost is learned only after the market price for a given supply of tickets is found. Thus for either method it would be useful to have some prior knowledge of the mutual responses of price and quantity. With either method, one would also expect to make adjustments of standards in response to experience with the actual working of the market.

SECTION II. MARKET MECHANISMS FOR DETERMINING STANDARDS

One might think that since analogy to market solutions for private goods proves so fruitful in dealing with the problem of conditional efficiency, a way could be found to choose the goals themselves by means of a market-like mechanism. In particular, could we not arrange that the supply of "pollution tickets" itself be determined by market forces.

There are two important difficulties with this approach. The first difficulty is the traditional public goods problem that the level of the environmental quality vector z affects the utilities of many consumers simultaneously. Means must be found to reconcile divergent interests of different consumers who must each consume the same amount of certain commodities. A second difficulty is that it does not seem reasonable to assume convexity of preferences with regard to the quantities of externalities. Starret (1972) has convincingly argued the implausibility of this assumption where there are unpleasant externalities. Even in a world with only desirable private commodities no convincing *a priori* or empirical case has been made for the convexity assumption. In the case of private commodities where there are many consumers, the traditional results of equilibrium emerge almost intact if the convexity assumption is substantially weakened. So long as the aggregate level of externalities is held fixed, Starrett's non-convexities are not crucial to the analysis. The isomorphism to an exchange economy allows us to handle non-convexities which are not "too large" When, however, we study the determination of efficient quantities of non-private commodities for which there may be substantial non-convexities of preference, matters become much more difficult.

We approach these difficulties one at a time by first supposing preferences of all consumers to be convex in x_i, y_i and z. We examine possibilities and difficulties for market solutions to the full allocation problem of determining the x_i's the y_i's and z. For simplicity of language we will confine our verbal discussion to the case where there is one pollutant, one private good and where there is no production of private goods. The extension of this model to the case of many commodities, both private and public, and where there is production is entirely straightforward along the lines indicated in the previous section.

Consider an economy in which there is a "central authority" which may either issue or purchase pollution tickets. There is a specified initial holding ($\mathring{x}_i$, $\mathring{y}_i$) for each consumer. Suppose that consumer i receives (pays) some fixed fraction s_i of the revenues (costs) of the net sales (purchases) of pollution tickets by the central authority. If pollution tickets are marketed at a price q while the price of the private good is p, then each consumer i will be faced by a budget constraint of the form :

$$px_i + qy_i \leq p\mathring{x}_i + q\mathring{y}_i + s_i q\,(z - \Sigma\, \mathring{y}_i)$$

where $z - \Sigma\, \mathring{y}_i$ is the net sales (purchases if negative) of tickets by the central authority and z the resulting supply after the central authority has made its sales or purchases.

How would we identify equilibrium prices and quantities ? For any given p, q and s_i each individual could in principle state the value of the vectors (x_i, y_i, z) which maximize his utility relative to the corresponding constraint. But even if we find values of p and q for which $\Sigma\, x_i = \Sigma\, \mathring{x}_i$, it will in general be the case that different individuals will desire different values of z. Any actual social outcome must settle on just one value of z and on values of $y_i, \ldots, y_n$ such that $\Sigma\, y_i = z$. These considerations motivate the following discussion.

To simplify the presentation let us suppose that preferences are *strictly* convex, thus giving us single valued demand functions. (There are extensiohs of the subsequent discussion to the case of demand correspondences, from which we refrain on grounds of expository simplicity). For each i, let $\hat{z}_i\,(p,q,s_i)$ denote the quantity $\hat{z}_i$ such that ($\hat{x}_i, \hat{y}_i, \hat{z}$) maximizes $U_i\,(x_i, y_i, z)$ subject to :

$$px_i + qy_i \leq p\mathring{x}_i + q\mathring{y}_i + s_i\, q(z - \Sigma\, \mathring{y}_i).$$

Thus $\hat{z}_i\,(p, q, s_i)$ is the amount of pollution which i would choose if he were allowed free choice of x_i, y_i and z from his budget set. Define a social decision function $\hat{z}\,(p, q, s)$ which determines the quantity $\hat{z}$ to be selected by the central authority in response to the values $\hat{z}_1, \ldots, \hat{z}_n$ desired by individuals at prices p and q. Thus,

$$\hat{z}\,(p, q, s) \equiv f\,(\hat{z}_1\,(p, q, s), \ldots, \hat{z}_n\,(p, q, s))$$

where $\hat{z}\,(p, q, s)$ is the number of pollution tickets which the central authority would allow in circulation at prices p and q and shares $s_1 \ldots, s_n$. For example $\hat{z}\,(p, q, s)$ might be the median of the $\hat{z}_i$'s or some average of the $\hat{z}_i$'s weighted by the s_i's, as would be the case in various forms of majority voting.

We then define a social equilibrium as a pair of prices $\bar{p}$ and $\bar{q}$ and a vector,

$$(\bar{x}_1, \ldots, \bar{x}_n, \bar{y}_1, \ldots, \bar{y}_n, \bar{z}) \text{ such that :}$$

1) $\bar{z} = \hat{z}\,(\bar{p}, \bar{q}, s) = f\,(\hat{z}_1\,(\bar{p}, \bar{q}, s_1), \ldots, \hat{z}_n\,(\bar{p}, \bar{q}, s_n))$

2) For each i, $(\bar{x}_i, \bar{y}_i)$ maximizes R_i subject to

$$\bar{p}\, x_i + \bar{q}\, y_i \leq \bar{p}\, \mathring{x}_i + \bar{q}\, \mathring{y}_i + s_i\, \bar{q}\, (\bar{z} - \Sigma\, \mathring{y}_i)$$

3) $\Sigma\, \bar{x}_i = \mathring{x}$ and $\Sigma\, y_i = \bar{z}$.

The idea of this definition is simply the following : The values s_i are predetermined. For any price vector, (p, q) each consumer i finds the quantity $\hat{z}_i$ of pollution that he would like best given the budget constraint determined by p, q and s. These quantities are reported and a social decision is made leading to the quantity z (p, q, s). Consumers now must make their choices of the quantities x_i and y_i which they like best given the realized budget constraint

$$px_i + qy_i \leq p\mathring{x}_i + q\mathring{y}_i + q\, (\hat{z}\, (p, q, s) - \Sigma\, \mathring{y}_i).$$

Equilibrium occurs at a price vector $\bar{p}, \bar{q}$ where these decisions are mutually consistent as expressed in condition (3) of the definition.

It is a straightforward matter to show that such an equilibrium exists if

$$\hat{z}\, (p, q, s) \equiv f\, (\hat{z}_1 (p, q, s_1), \ldots, \hat{z}_n\, (p, q, s_n))$$

where f is a continuous function of $\hat{z}_1, \ldots, \hat{z}_n$ and where the usual assumptions on individual preferences are made which guarantee the existence of ordinary competitive equilibrium.(7) In general, there is no reason to suppose that a social equilibrium is Pareto optimal. Since a social equilibrium satisfies the requirements of the previous section for a market equilibrium (with the value z fixed at $\bar{z}$) it does follow that a social equilibrium is conditionally efficient. Properties of such an equilibrium will of course depend on the nature of the function f and on the choice of the values of s_i. Suitable choices in these matters could result in a model of considerable descriptive value. Some might argue that in view of the difficulties in finding Pareto efficient choices of z, refinements of such a model would constitute the best that can be done with the problem.

We continue our search for optimality. Suppose that it would be possible to find certain values of the s_i's, (denote these by $\bar{s}_i$ where $\Sigma\, \bar{s}_i = 1$) such that for some prices $\bar{p}$ and $\bar{q}$ and some allocation $(\bar{x}_1, \ldots, \bar{x}_n, \bar{y}_1, \ldots, \bar{y}_n, \bar{z})$ we have :

1) For all i, $(\bar{x}_i, \bar{y}_i, \bar{z})$ maximizes $U_i\, (x_i, y_i, z)$ subject to

$$\bar{p}x_i + \bar{q}y_i < \bar{p}\mathring{x}_i + \bar{q}\mathring{y}_i + \bar{s}_i\bar{q}\, (z - \Sigma\, \mathring{y}_i).$$

2) $\Sigma\, \bar{x}_i = \mathring{x}$ and $\Sigma\, \bar{y}_i = \bar{z}$.

In this case, unanimous agreement is achieved on the quantity $\bar{z}$, by means of appropriate adjustment in the way in which the receipts or costs of the central

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(7) This problem is studied in greater generality by Denzau (1974).

authorities sales or purchases of tickets are divided. Such an allocation is called a *Lindhal equilibrium*. In a previous paper (1970) I have proved that Lindhal equilibrium exists under assumptions which, except for the assumption of convexity, are very weak and quite plausible. (The definition of Lindhal equilibrium stated in the paper is different in form but includes the above definition as a special case). The Lindhal equilibrium as defined here is in fact Pareto optimal (where preferences are locally nonsatiated). To see this, observe that if an allocation $(x_1, \ldots, x_n, y_1, \ldots, y_n, z)$ is Pareto superior to the equilibrium, then for each consumer it must be that

$$\bar{p}x_i + \bar{q}y_i \geqslant \bar{p}\mathring{x}_i + \bar{q}\mathring{y}_i + \bar{s}_i\bar{q}\,(z - \Sigma\,\mathring{y}_i)$$

with strict inequality in the case of at least one consumer.
Adding these inequalities we obtain

$$p\,(\Sigma\, x_i - \Sigma\, \mathring{x}_i) + \bar{q}\,(\Sigma\, y_i - z) > 0 \text{ which is impossible}$$

$$\text{if } \Sigma\, x_i = \mathring{x} = \Sigma\, \mathring{x}_i \text{ and } \Sigma\, y_i = z.$$

Thus any allocation which is Pareto superior to the Lindhal equilibrium is not feasible which establishes the Pareto optimality of the Lindhal equilibrium.

Following a suggestion of Arrow (1970) which was further developed by Starrett (1972), one can treat a Lindhal equilibrium as a market equilibrium for an economy in which pollution inflicted on different consumers are treated as different commodities. As in our earlier analysis, results on the existence and optimality of equilibrium could be applied.

It can again be shown that such an economy is isomorphic to an exchange economy. As before the results on the existence and optimality of equilibrium can be applied. However, the result that in large economies the only allocations with the core property are nearly market equilibria does not extend naturally to this case. The formal structure requires that the number of commodities be approximately proportional to the number of consumers. Also, unless initial holdings of pollution tickets entitling consumers to pollute any other consumer are widely dispersed, some consumers may have monopoly power in the sale of tickets allowing pollution of themselves. For these reasons it seems too much to hope that a plausible argument can be made for all elements of the core to be nearly competitive equilibria. Any efforts to argue that a *laissez faire* policy with an appropriate system of property rights would lead to a Lindhal equilibrium would have to be based on some bargaining concept other than the core.

Thus, even in the case where preferences are convex, it is difficult to see how decisions about the levels of environmental standards to be enforced can be fully decentralized. More problems for decentralized market solutions appear when there are important non-convexities. Possibilities for central planning in the determination of environmental standards will be examined in a separate paper ([8]).

(8) "Regulation of Externalities," (1973b).

APPENDIX

Properties of Derived Preferences

First, we consider properties of $\widetilde{U}_i\,(\widetilde{x}_i, y_i, \mathring{z})$ where $\mathring{z}$ is fixed. Assume that there is not infinite free production. That is : any subset of $T_i\,(\mathring{z})$ which is bounded from below is also bounded from above. If $T_i\,(\mathring{z})$ is closed and C_i is closed and bounded from below, then

$T_i\,(\mathring{z})$ $\quad (x_i', y_i) \mid (x_i' + \widetilde{x}_i, y_i, \mathring{z}) \in C_i$ is a compact set. Since U_i is continuous,

max. $\quad U_i\,(x_i' + \widetilde{x}_i, y_i, z)$ exists and hence, $\widetilde{U}_i\,(\widetilde{x}_i, y_i, z)$ is well-defined.

$(x_i', y_i) \in T_i\,(\mathring{z})$

$(x_i' + \widetilde{x}_i, y_i, \mathring{z}) \in C_i$

These assumptions also guarantee that for any $(\widetilde{x}_i, y_i)$ the set $\{\, (\widetilde{x}_i', y_i') \mid \widetilde{U}_i\,(\widetilde{x}_i', y_i', \mathring{z}) \geqq U_i\,(\widetilde{x}_i, y_i, \mathring{z}) \,\}$ is closed and that for any $(\widetilde{x}_i, y_i)$ and $(\widetilde{x}_i', y_i')$ the function

$$f\,(\lambda) = \widetilde{U}_i\,(\lambda \widetilde{x}_i + (1 - \lambda)\, \widetilde{x}_i', \lambda y_i + (1 - \lambda)\, y_i', \mathring{z})$$

is a continuous function of λ on the intervals $[0, 1]$.

These continuity properties, while slightly weaker than the assumption that $\widetilde{U}_i$ is a continuous function in x_i and y_i prove to be sufficient for any of the usual applications of equilibrium analysis. Assumptions guaranteeing local non-satiation of $\widetilde{U}_i$ are also easily found, although it is probably just as reasonable to assume this property directly. Convexity of derived preferences on x_i and y_i follows from the convexity of the original preferences on x_i and y_i and the assumption that $T_i\,(\mathring{z})$ is a convex set. All of these results are direct consequences of results proved in detail by Rader (1964). If we add the assumption that T_i (z) is an upper semi-continuous correspondence then it is easy to show that the above continuity properties apply to $\widetilde{U}_i$ as a function of x_i, y_i *and z.* If it is assumed that the original preferences relation is convex in x_i, y_i and z and that the set $(x_i', y_i, z) \mid (x_i', y_i) \in T_i\,(z) \}$ is convex then the drived preferences will be convex in $\widetilde{x}_i$, y_i and z. There appears to be no compelling justification for the realism of the latter assumption but, in fact, neither is there a strong case for the convexity of U_i as a function of x_i, y_i and z.

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