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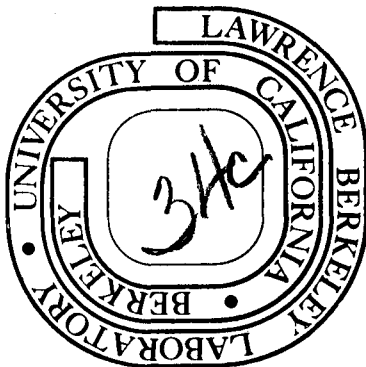
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STATISTICAL DESCRIPTION OF INTERACTING FERMIONS
AND LIMITING ANGULAR MOMENTUM[†]

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ABSTRACT

The statistical properties of interacting fermions have been studied for various angular momentum with the nuclear pairing included. The dependence of the critical temperature on angular momentum for several nuclei, $A=28$ up to $A=200$ have been studied. The yrast energy as a function of angular momentum for ^{28}Si and ^{44}Ti nuclei have been calculated up to 60.0 MeV of excitation energy. The computed limiting angular momentum are compared with the experimental results for ^{26}Al produced by $^{12}\text{C} + ^{14}\text{N}$ reaction.

Key Word:

NUCLEAR STRUCTURE statistical properties of a paired nucleons;
calculated limiting angular momentum for ^{28}Si , ^{44}Ti and ^{26}Al .

I. INTRODUCTION

Results of the statistical treatment of excited nuclei are improved by the use of more realistic sets of single particle levels.^{1,2,3} In this article we shall describe the general methods of treating the statistical nuclear properties which are based on realistic sets of single particle levels with inclusion of pairing effects and angular momentum. A preliminary account of such results has been reported elsewhere.^{4,5} In the first part of the present paper, the mathematical technique consisting of the statistical calculations will be discussed, while in the second part some actual calculations will be presented and at the end calculated limiting angular momentum using a microscopic theory of interacting fermions will be compared with the experimental results.

II. DISCUSSION OF THE MATHEMATICAL TECHNIQUE

For a system of interacting fermions, for example that of neutrons the logarithm of the grand partition-function is given by

$$\Omega = -\beta \sum_k (\epsilon_k - \lambda - E_k) + \sum_k \ln [1 + \exp -\beta(E_k - \gamma m_k)] + \sum_k \ln [1 + \exp -\beta(E_k + \gamma m_k)] - \beta \frac{\Delta^2}{G} \quad (1)$$

where λ, β, γ are the three lagrange multipliers associated with the nucleon number, energy and angular momentum and $E_k = [(\epsilon_k - \lambda)^2 + \Delta^2]^{\frac{1}{2}}$ is the quasi-particle energy while Δ is the gap parameter.

The quantities $\Delta, \lambda, \beta (= \frac{1}{T}, T$ is the nuclear temperature) and γ are connected through the following gap equation,

$$\frac{2}{G} = \sum_k \frac{1}{2E_k} [\tanh \frac{1}{2}\beta(E_k - \gamma m_k) + \tanh \frac{1}{2}\beta(E_k + \gamma m_k)] \quad (2)$$

The saddle point is defined by the following equations,⁶

$$N = \frac{1}{\beta} \frac{\partial \Omega}{\partial \lambda} = \sum_k \left[1 - \frac{\epsilon_k - \lambda}{2E_k} \tanh \frac{1}{2}\beta(E_k - \gamma m_k) + \tanh \frac{1}{2}\beta(E_k + \gamma m_k) \right] \quad (3)$$

$$M = \frac{1}{\beta} \frac{\partial \Omega}{\partial \gamma} = \sum_k m_k \left[\frac{1}{1 + \exp \frac{1}{\beta}(E_k - \gamma m_k)} - \frac{1}{1 + \exp \frac{1}{\beta}(E_k + \gamma m_k)} \right] \quad (4)$$

$$E = \frac{\partial \Omega}{\partial \beta} = \sum_k \epsilon_k \left[1 - \frac{\epsilon_k - \lambda}{2E_k} \left\{ \tanh \frac{1}{2}\beta(E_k - \gamma m_k) + \tanh \frac{1}{2}\beta(E_k + \gamma m_k) \right\} \right] \frac{\Delta^2}{G} \quad (5)$$

N represents the nucleon number, M the projection of the angular momentum on a spaced fixed axis and E the energy of the system. The system of Eqs. (2), (3) and (4) define the values of Δ , λ and γ .

The above theory can be extended to include systems containg both neutrons and protons. For a system of N neutrons and Z protons, let the energy levels be represented by ϵ_k^n and ϵ_k^p ; let also the magnetic quantum numbers be m_k^n and m_k^p . The constants of motions are then the neutron and proton numbers N and Z, the total energy, E and the projection of the total angular momentum on a fixed axis, M.

$$N = \sum_k m_k^n \epsilon_k^n, \quad Z = \sum_k m_k^p \epsilon_k^p \quad (6)$$

$$E = E_n + E_p \quad (7)$$

$$M = M_n + M_p \quad (8)$$

III. DEPENDENCE OF THE GAP PARAMETER UPON THE TEMPERATURE AT ZERO ANGULAR MOMENTUM

At $M = 0$, the gap eq. G then gives the dependence of Δ upon T. It is obtained from Eqs. (2) and (3) by setting $\gamma=0$.

$$N = \sum_k \left(1 - \frac{\epsilon_{k-\lambda}}{E_k} \tanh \frac{1}{2} \beta E_k\right) \quad (6)$$

$$\frac{2}{G} = \sum_k \frac{1}{E_k} \tanh \frac{1}{2} \beta E_k \quad (7)$$

The numerical calculations are done in the following way.

i) For a given nucleon number, N and the gap parameter, Δ_0 (the gap parameter is obtained from the literature⁷), the Eqs. (6) and (7) are solved for λ_0 and the pairing strength, G.

ii) The critical temperature t_c and the corresponding chemical potential λ_c are then evaluated by setting $\Delta_c = 0$, and using the pairing strength G obtained above for that nucleon number N.

iii) The quantities $\Delta(T)$ and $\lambda(T)$ are evaluated for given value of $T < T_c$ by solving again the same Equations with the values of N and G from (i).

iv) The above steps (i)-(iii) are repeated for the second type of particle. A computer program has been developed to do the numerical calculations.⁸ An example of the results is shown in Fig. 1.

The increase in T produces a decrease in Δ up to T_c , Δ becomes zero and then the pairing correlation vanishes. This decrease is because the excitation energy breaks up pairs of particles there by generating quasi-particles which block the occupations of levels close to the Fermi surface.⁶

IV. DEPENDENCE OF THE CRITICAL TEMPERATURE UPON ANGULAR MOMENTUM

The dependence of the critical temperature, T_c upon M , is obtained by setting $\Delta_c = 0$ in Eqs. (2), (3) and (4):

$$\frac{2}{G} = \sum_k \frac{1}{2(\epsilon_k - \lambda_c)} [\tanh \frac{1}{2} \beta_c (\epsilon_k - \lambda_c - \gamma_c m_k) + \tanh \frac{1}{2} \beta_c (\epsilon_k - \lambda_c + \gamma_c m_k)] \quad (8)$$

$$N = \sum_k [1 - \frac{1}{2} \{ \tanh \frac{1}{2} \beta_c (\epsilon_k - \lambda_c - \gamma_c m_k) + \tanh \frac{1}{2} \beta_c (\epsilon_k - \lambda_c + \gamma_c m_k) \}] \quad (9)$$

$$M = \sum_k m_k \left[\frac{1}{1 + \exp \beta_c (\epsilon_k - \lambda_c - \gamma_c m_k)} - \frac{1}{1 + \exp \beta_c (\epsilon_k - \lambda_c + \gamma_c m_k)} \right] \quad (10)$$

Numerical methods are used to solve for the critical temperature, T_c , the corresponding chemical potential λ_c , and γ_c , using the pairing strength G obtained in section 3 for that given neutron number and for each value of M . Calculations are repeated for the protons in the same way. A computer program has been developed for making this calculation. An example of the results is shown in Fig. 2. It is seen that T_c decreases by increasing M and in some cases like this case it becomes double-valued at M close to $M_c (T_c = 0)$. The physical explanation for this is given by Moretto.⁶

For the case of proton system, the calculations are made for the various values of Δ_{op} in steps of 0.5 MeV. The results are shown in Fig. 2b. It is seen that the dependence of T_c upon M stays single-valued up to $\Delta_{op} = 2.19$ MeV, when it becomes double-valued. This

effect of double valuedness has been investigated for several nuclei and the results are shown in Table I. It is seen from this Table that for light nuclei both the neutron and proton systems are double-valued near $M=M_c$ while for heavier ones they are single-valued. In the proton system, it again becomes double-valued for $A = 200$.

V. DEPENDENCE OF Δ UPON ANGULAR MOMENTUM AT $T = 0$

The dependence of Δ upon M for zero temperature is obtained by solving Eqs. (2), (3) and (4) by setting $T = 0$. In this case when the shell model level sequence is used, the calculations become rather complexed because of the discontinuities for β approaching infinity.

The case however can be treated in the following way:

A. $E_k > \gamma m_k$
 In this case, both the expressions $\exp \beta(E_k + \gamma m_k)$ and $\exp \beta(E_k - \gamma m_k)$ approaches to infinity the equations then reduce to:

$$\frac{2}{G} = \sum_k \frac{1}{E_k} \quad (14a)$$

$$N = \sum_k \left(1 - \frac{E_k - \lambda}{E_k}\right) \quad (15a)$$

$$M = \sum_k 0 \quad (16a)$$

The sum k runs over all the levels for which $E_k > \gamma m_k$ is satisfied.

B. $E_k < \gamma m_k$
 In this case the expression $\exp \beta(E_k - \gamma m_k)$ approaches to zero instead. Then the equations become,

$$\frac{2}{G} = \sum_k 0 \quad (14b)$$

$$N = \sum_k 1 \quad (15b)$$

$$M = \sum_k m_k \quad (16b)$$

Here the index, k runs over all the single particle levels for which $E_k < \gamma m_k$ is satisfied.

The numerical calculations are carried out on the computer⁸ solving the three unknowns Δ, λ and γ for values of G_n , N and for each value of M . The computations are repeated for the proton system. A result of such calculations is shown in Fig. 3. It is seen that the gap parameter Δ , decreases for increasing M and is zero for all $M > M_c$. These results were also checked by setting the value of T very small (i.e., not zero) in Eqs. (2), (3) and (4) and by solving them instead, it was found to produce almost the same results.

It should be noted that G_n and G_p depend on the number of single particle levels which are included in the calculations. However, for a given value of Δ_0 the final results are not sensitive to the number of single particle levels used as long as a sufficient number of them considered so that levels of the largest ϵ have very small occupation probabilities.

The complete dependence of Δ upon T and M for neutron and proton system using realistic sets of single particle levels of several nuclei are given in Figs. 4 to 11. It is interesting to see that in some cases like that of ^{196}Pt Δ_p becomes double-valued at angular momentum close to the critical value.

VI. LIMITING ANGULAR MOMENTUM

The yrast line, which gives the energy of the system at $T = 0$, is obtained in the following way.

i) The ground state energy E_n (ground) is calculated by setting $\gamma = 0$ and solving Eqs. (2) and (3) for λ_0 and the pairing strength G for a given number of neutron and gap-parameter Δ_0 . These values are then used in Eq. (5) to compute E_n (ground).

ii) The ground state energy for protons, E_p (ground) is computed in the same way. Thus the ground state energy of the system is defined as $E_{\text{(ground)}} = E_n(\text{ground}) + E_p(\text{ground})$.

iii) The three unknowns $\Delta(M)$, $\lambda(M)$ and $\gamma(M)$ are evaluated for each value of M by solving Eqs. (14), (15) and (16) for the same number of neutrons and the same value of G as in (i). These values of Δ, λ and γ are used to compute E_n using Eq. (5).

iv) The same procedure is used to obtain E_p .

v) From Eqs. (7) and (8) the total excitation energy and the total angular momentum of the combined system is obtained

$$U = (E_n + E_p) - E_{\text{(ground)}} \quad (17)$$

$$M = M_n + M_p$$

Calculations are done on the computer.⁸

VII. RESULTS AND DISCUSSION

The yrast line has been computed for ^{28}Si for M up to 30 units of angular momentum and 60 MeV of excitation energy. The theoretical calculations were performed with the single particle levels of

Seeger et al.⁹, Nilsson et al.¹⁰ The results are almost the same for the two sets of particles.

The yrast line has also been calculated for ^{44}Ti using again Seeger and Nilsson set of single particle levels for spherical deformations. In addition, the yrast line has been computed for the Nilsson potential for prolate and oblate deformations.¹⁰ The ϵ_2 deformation used in obtaining the latter set of levels were ± 0.20 . The results are shown in Fig. 13. The results are somewhat different even for the Seeger and Nilsson spherical deformations specially at high excitation energies.

It is interesting to note that the yrast energy decreases as the nuclear deformation goes from prolate to oblate deformation. When nuclear deformations are introduced the K-projection along the deformation axis replaces the z-projection of the total angular momentum, M. It is found that the energy spacing of rotational members built upon each K-state is much less than that of the energy spacing between different K-states. This ensures the contributions to the yrast line from the single particle levels or the K-states only.

The limiting angular momentum are also calculated for $^{24}_{12}\text{Mg}$ using Nilsson single particle levels for spherical, oblate and prolate shapes and are compared with the experimental values for $^{26}_{13}\text{Al}$ (an odd-odd system) produced by the $^{12}\text{C} + ^{14}\text{N}$ reaction.¹¹ These results are shown in Fig. 14. There is good agreement between experimental and theoretical results if the single particle levels of the prolate deformation is used.

The limiting angular momentum for $^{24}_{12}\text{Mg}$ obtained by using again the Nilsson single particle levels for spherical deformation but with

different values of pairing energy parameter are shown in Fig. 15. The experimental values for the compound nucleus ${}_{13}^{26}\text{Al}$ is also shown. These results are shown for 50% decrease and 50% increase of the original values of the gap parameter obtained from Ref. 4. It is seen that the yrast energy decreases by increasing the gap parameter and the agreement between the theory and experiment has been improved.

It is seen from Fig. 14 that for a fixed excitation energy the oblate deformation holds more angular momentum. This can be explained by the occupational probabilities for the various single particle levels. The neutron single particle levels are excited almost equally. However, there is a large contribution from the $1h_g/2$ proton single particle level. This can be seen from Fig. 16. Where the portion of the level sequences used together with their occupation probabilities are shown. The numbers under the levels represent the occupational probabilities.

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FOOTNOTES AND REFERENCES

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1. Moretto, L. G., R. Stella and V. Caranella Crespi, Nucl. Phys. 17 (1970) 436.
2. Hillman, M., J. R. Grover, Phys. Rev. 185 (1969) 1303.
3. Behkami, A. N., J. R. Huizenga, Nucl. Phys. A217 (1973) 78-92.
4. Behkami, A. N., J. R. Huizenga, Bull. Am. Phys. Soc. 9, 1017 (1974).
5. Behkami, A. N., J. R. Huizenga, Bull. Am. Phys. Soc. 12, 1496 (1975).
6. Moretto, L. G., Nucl. Phys. A185 (1972) 145-165.
7. Nemirowsky, P. E., and Yu. V. Adamchuck, Nucl. Phys. 39 (1962), 551.
8. This code can be obtained from the author upon request.
9. Seeger, P. A., and R. C. Perisho, Los Alamos Scientific Laboratory report LA-3751 (1957).
10. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. L. Lamm, P. Møllor and B. Nilsson, Nucl. Phys. A131 (1969) I.
11. Stokstad, R. G., Proceedings of the International Conference on Reactions between Complex Nuclei, 1, 327 (1974) North-Holland Publishing Co.

Table I. Existence of double-valuedness for the Neutrons and Protons

Nucleus	Δ_{on}^7 (MeV)	Δ_{op}^7 (MeV)	Double-valued	
			Neutron	Proton
^{28}Si	2.93	2.70	No	Yes
^{44}Ti	1.53	2.62	No	No
^{56}Fe	1.33	1.68	Yes	Yes
^{68}Zn	1.88	1.29	Yes	Yes
^{96}Mo	1.18	1.68	Yes	Yes
^{124}Te	1.36	1.34	Yes	Yes
^{136}Ba	1.28	1.19	No	Yes
^{150}Sm	1.18	1.54	No	No
^{152}Sm	1.52	1.80	No	No
^{152}Gd	1.25	1.61	No	No
^{162}Dy	0.96	0.86	No	No
^{166}Er	0.75	0.93	No	No
^{172}Yd	0.78	0.76	No	No
^{178}Hf	0.73	0.96	No	No
^{180}Hf	0.72	0.96	No	No
^{192}Pt	1.02	1.32	No	No
^{200}Hg	0.78	1.02	No	Yes

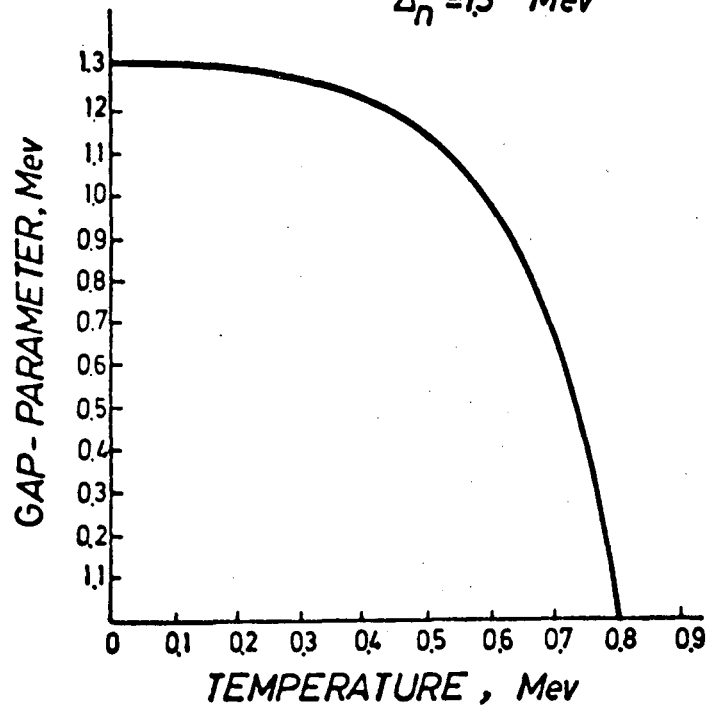
FIGURE CAPTIONS

- Fig. 1 Dependence of the gap parameter upon temperature at zero angular momentum for the ^{136}Ba nucleus. a) For the neutron system.
b) For the proton system.
- Fig. 2 Dependence of the critical temperature upon angular momentum of the ^{136}Ba nucleus. a) For the neutron system. b) For the proton system.
- Fig. 3 Dependence of the gap parameter upon angular momentum at zero temperature for the ^{136}Ba . a) For the neutron system. b) For the proton system.
- Fig. 4 The complete dependence of Δ upon M and T for the ^{28}Si nucleus.
- Fig. 5 The complete dependence of Δ upon M and T for the ^{44}Ti nucleus.
- Fig. 6 The complete dependence of Δ upon M and T for the ^{68}Zn nucleus.
- Fig. 7 The complete dependence of Δ upon M and T for the ^{96}Mo nucleus.
- Fig. 8 The complete dependence of Δ upon M and T for the ^{124}Te nucleus.
- Fig. 9 The complete dependence of Δ upon M and T for the ^{136}Ba nucleus.
- Fig. 10 The complete dependence of Δ upon M and T for the ^{192}Pt nucleus.
- Fig. 11 The complete dependence of Δ upon M and T for the ^{200}Hg nucleus.
- Fig. 12 Yrast line for the ^{28}Si with the single particle levels of Seeger and Nilsson.
- Fig. 13 Yrast line for the ^{44}Ti with the Seeger and Nilsson spherical deformations and Nilsson levels with prolate and oblate deformations.
- Fig. 14 Comparison of the experimental limiting angular momentum for the ^{24}Mg with the theory using Nilsson orbitals obtained for various deformations.

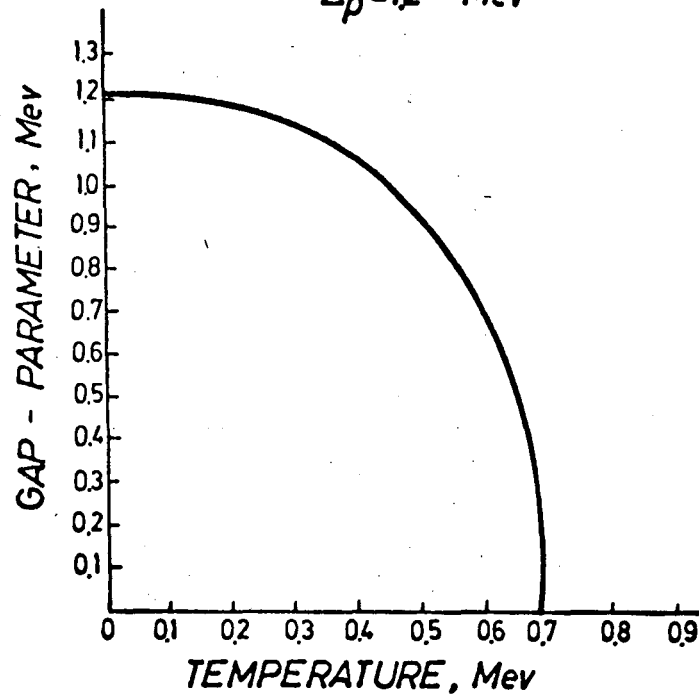
Fig. 15 Comparison of the limiting angular momentum of ^{26}Al with the theory using Nilsson orbitals with the 50% decrease and 50% increase values of the gap parameters.

Fig. 16 Energies of single particle levels of Nilsson with the occupational probabilities of ^{24}Mg for fixed excitation, $E^* + 43.0$ (MeV).

(a) Neutron System
 $\Delta_n = 1.3$ Mev

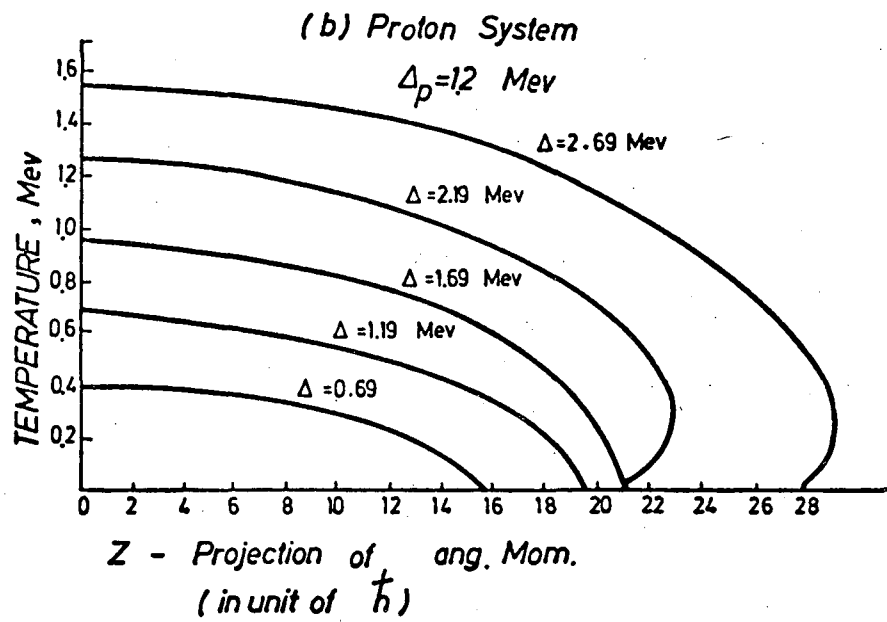
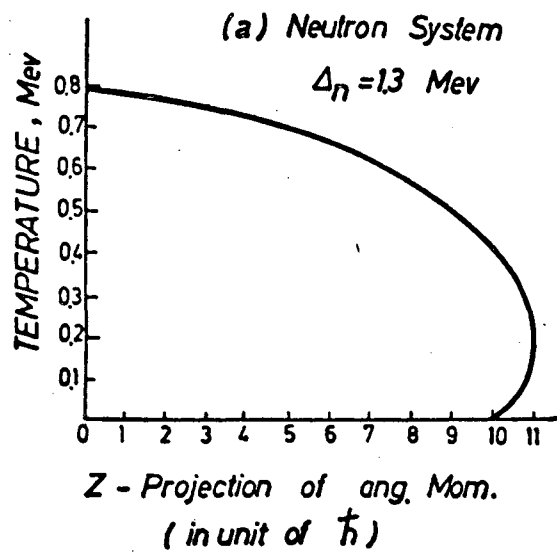


(b) Proton System
 $\Delta_p = 1.2$ Mev



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Fig. 1



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FIG. 2

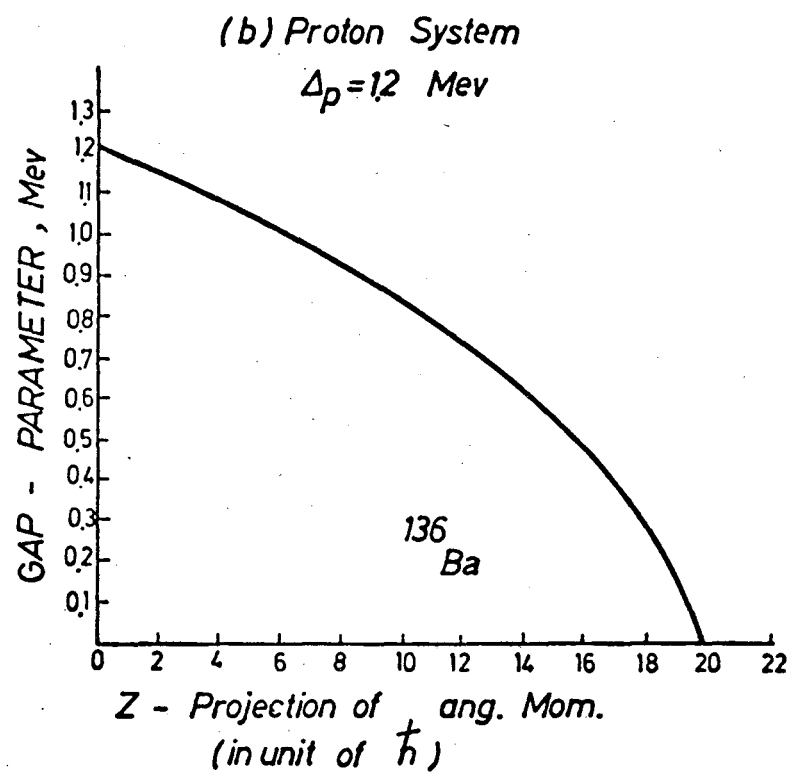
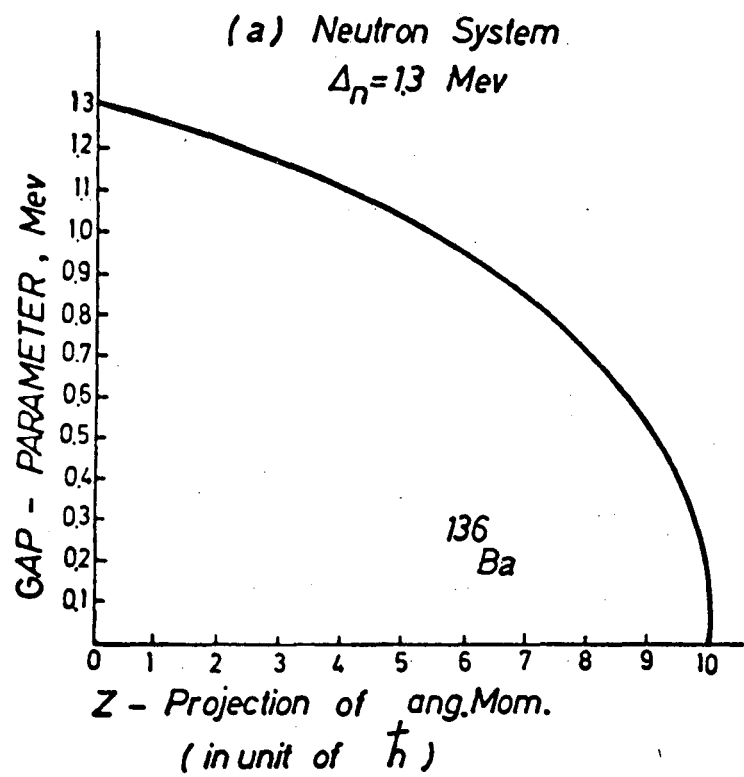
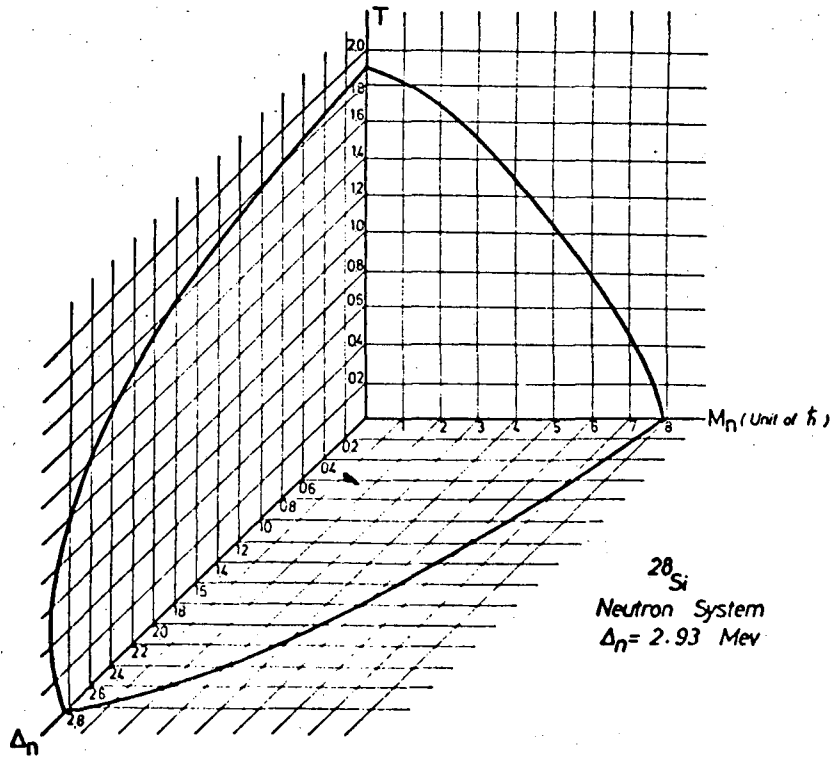
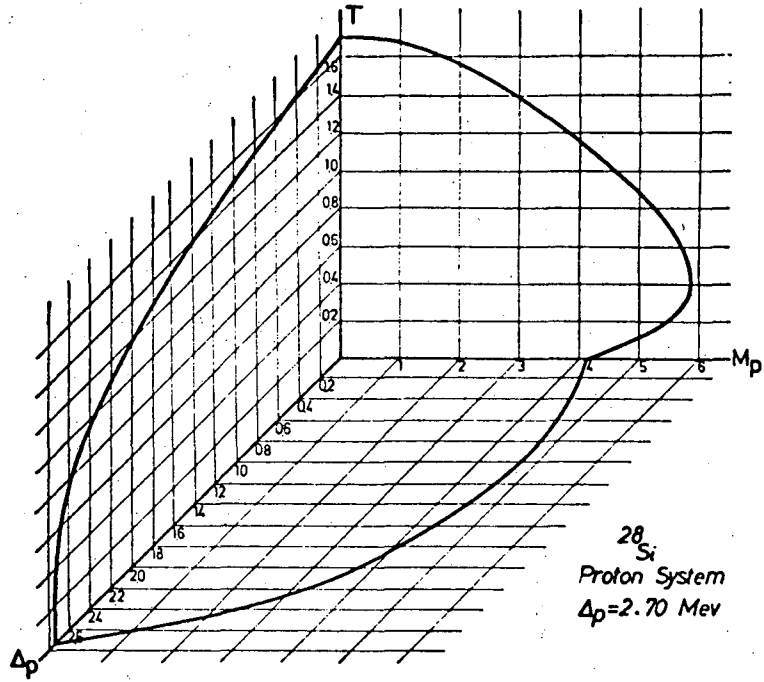


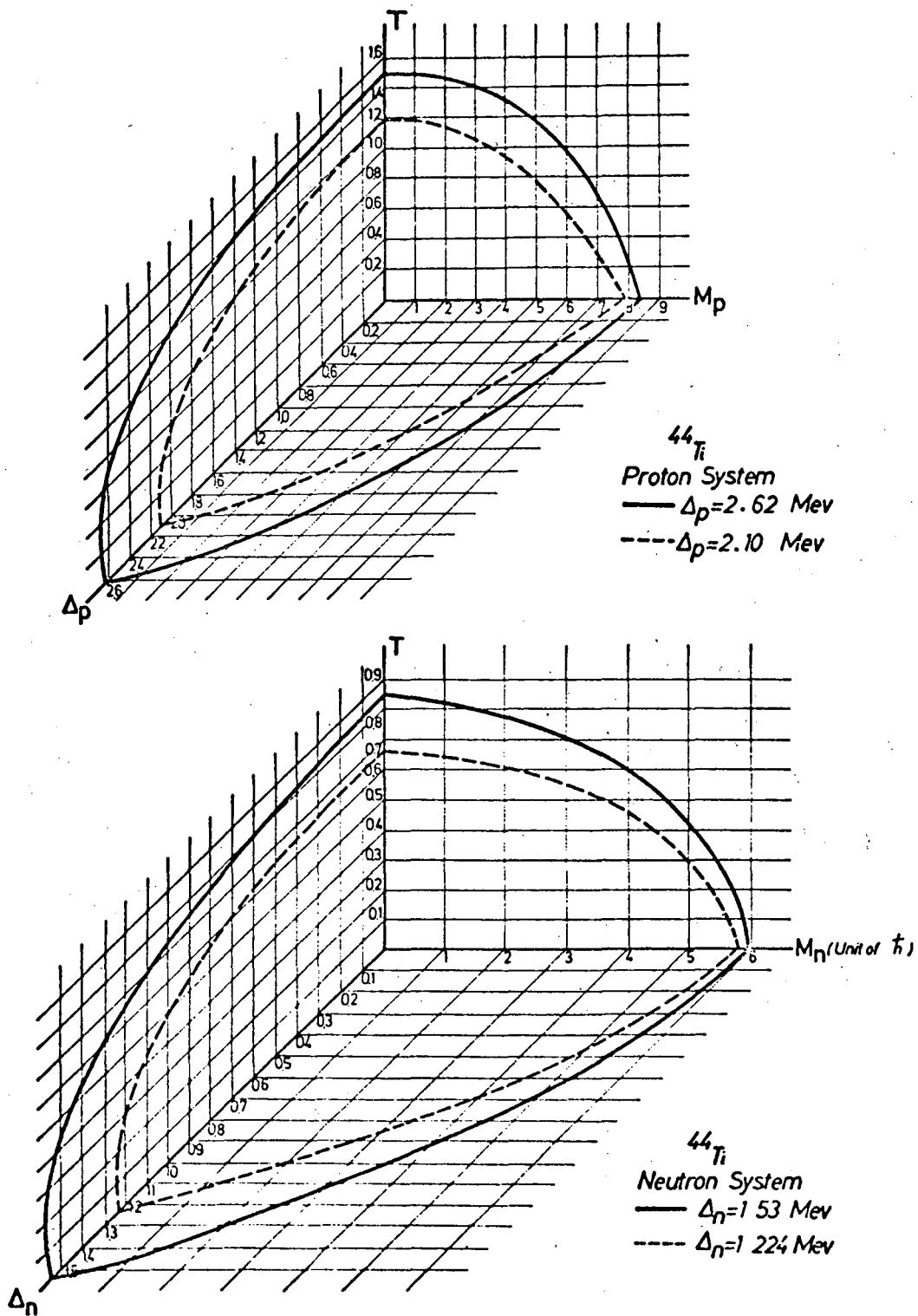
FIG. 3

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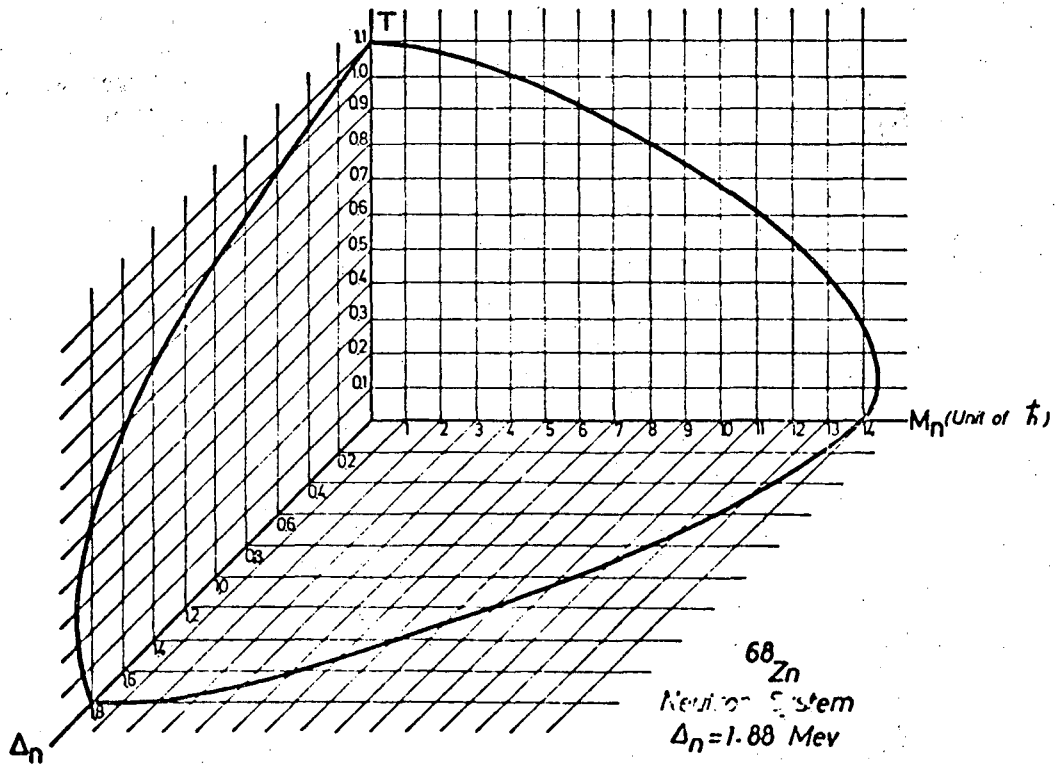
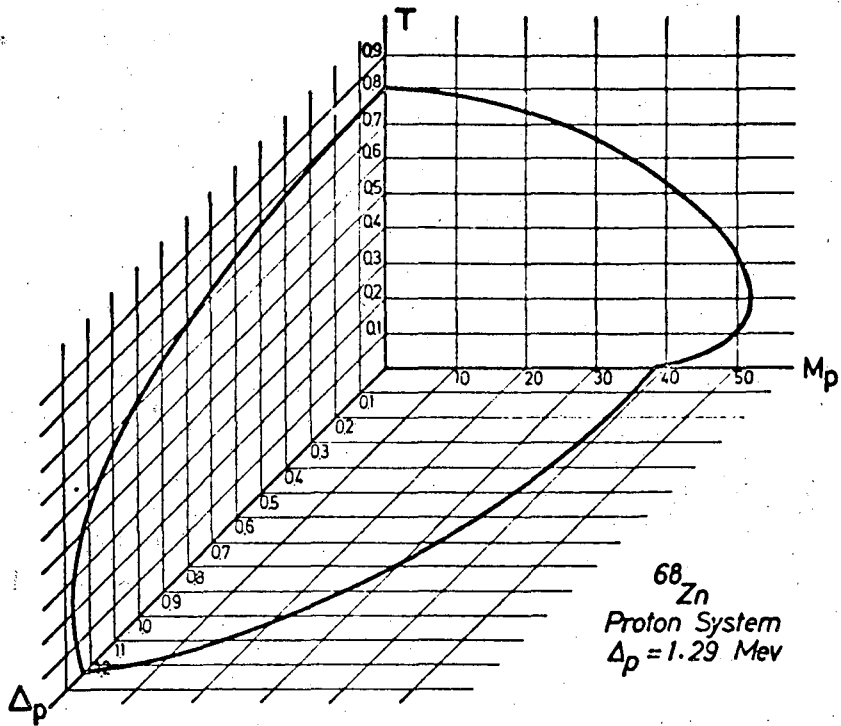
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FIG. 4



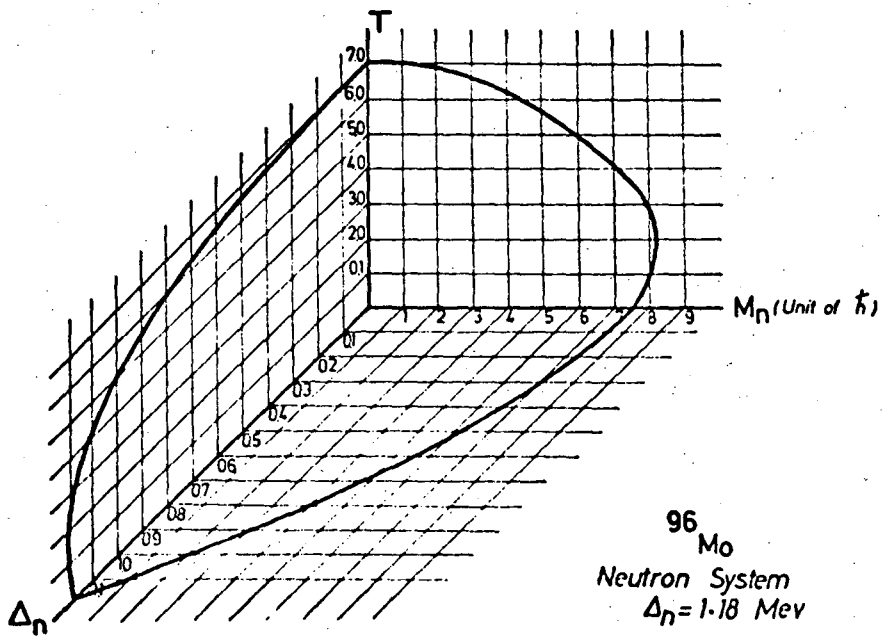
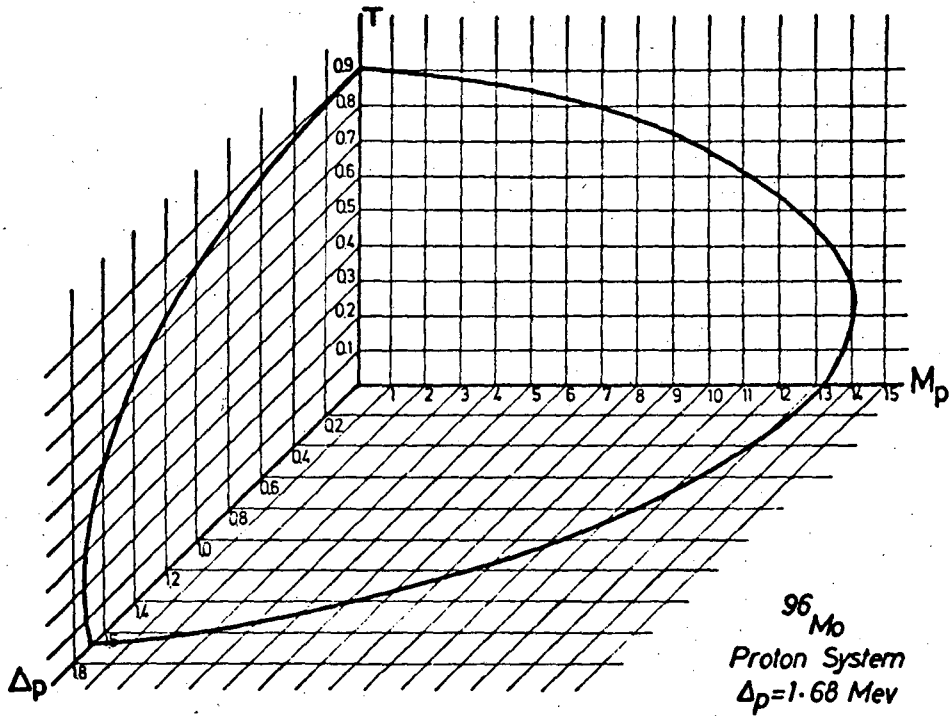
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FIG. 5



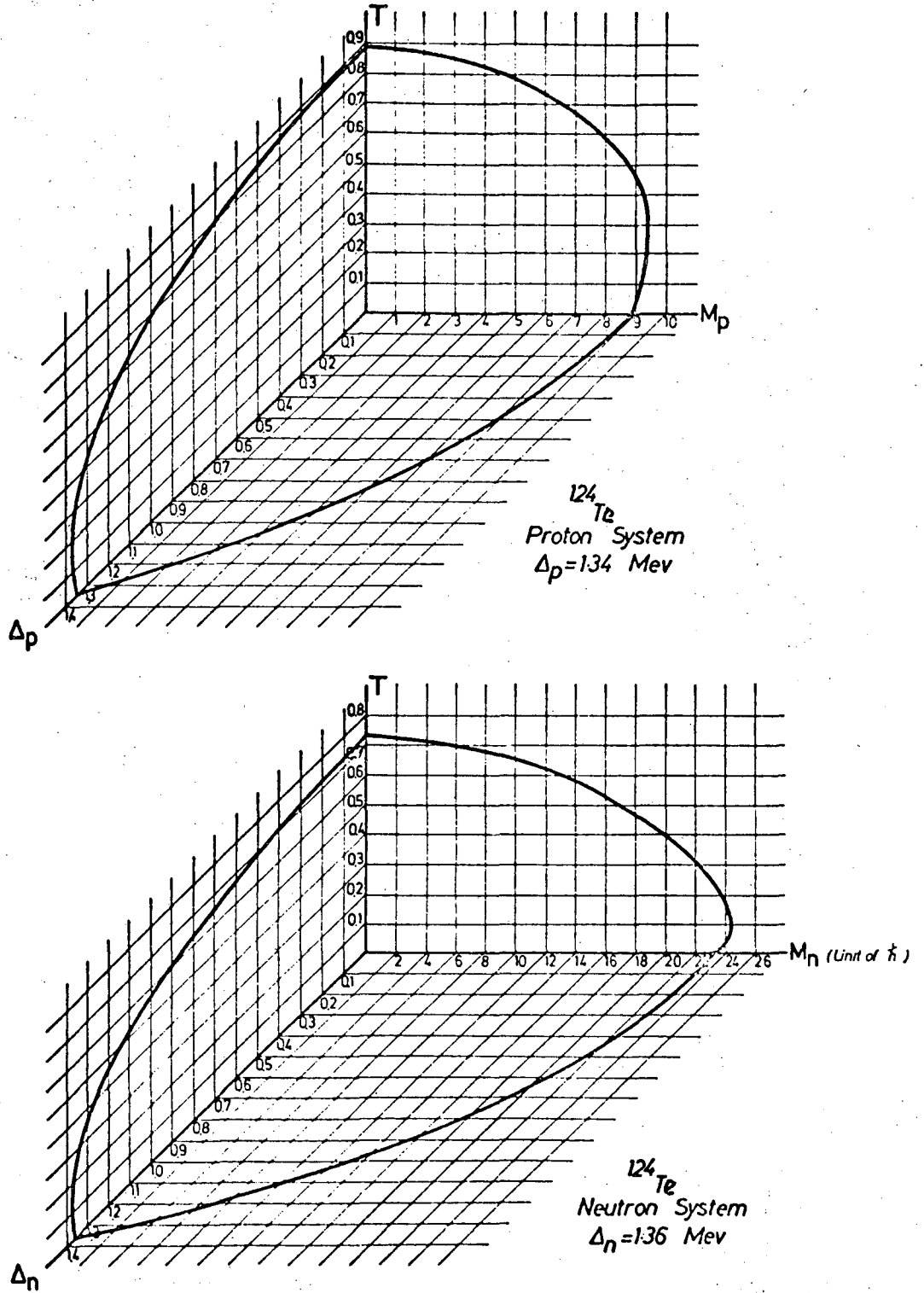
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FIG. 6



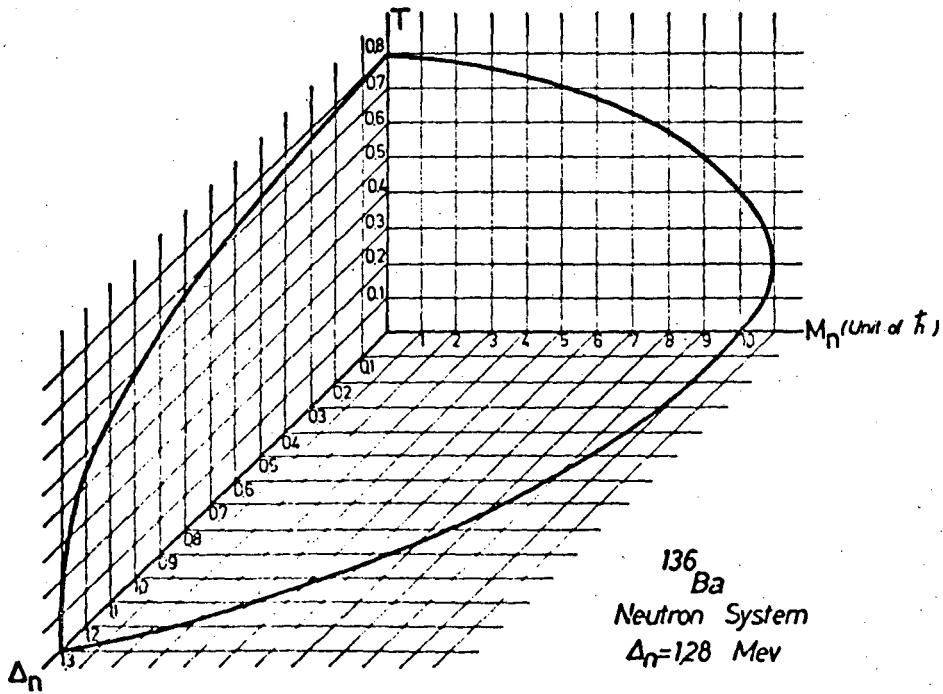
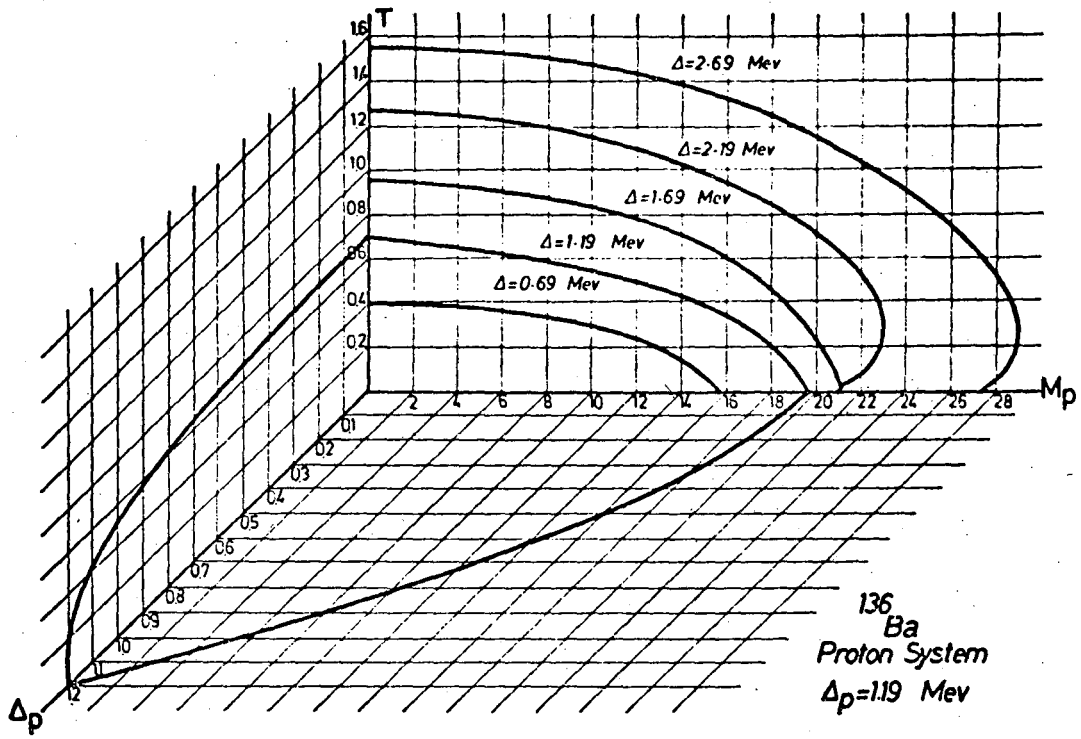
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FIG. 7



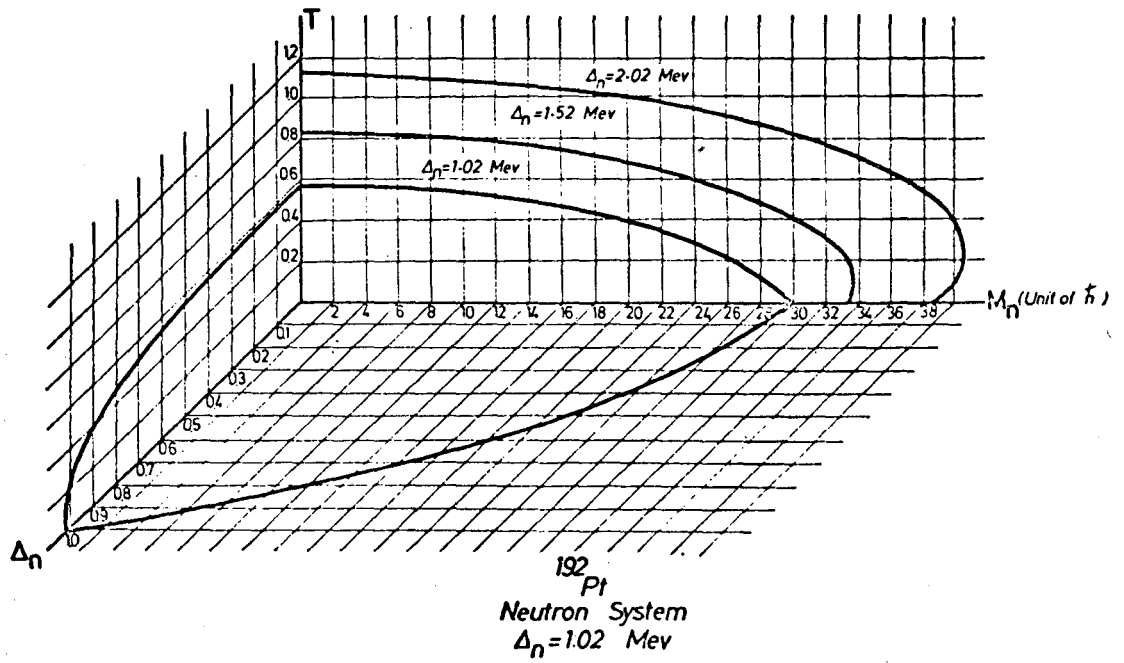
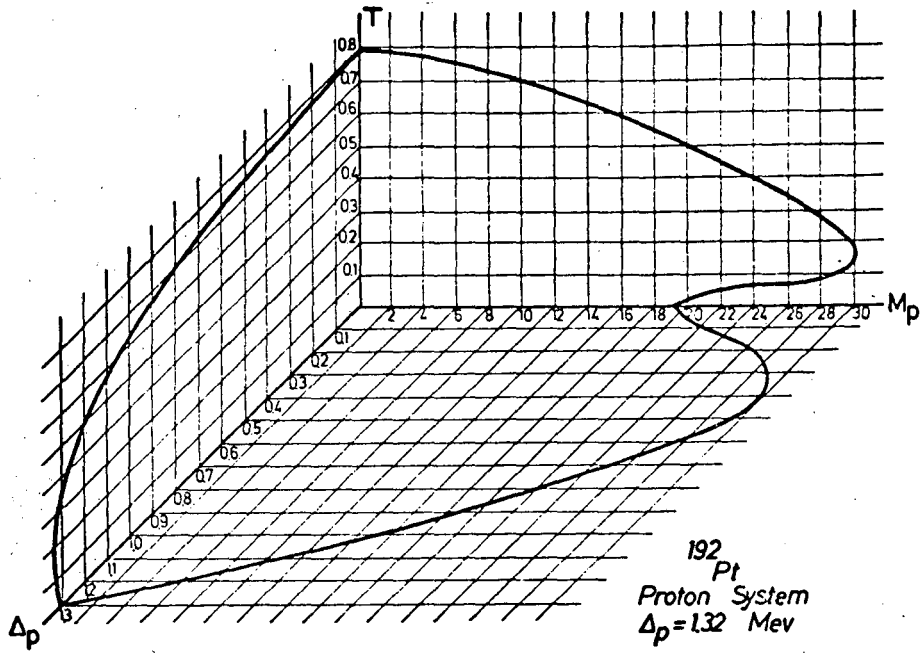
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FIG. 8



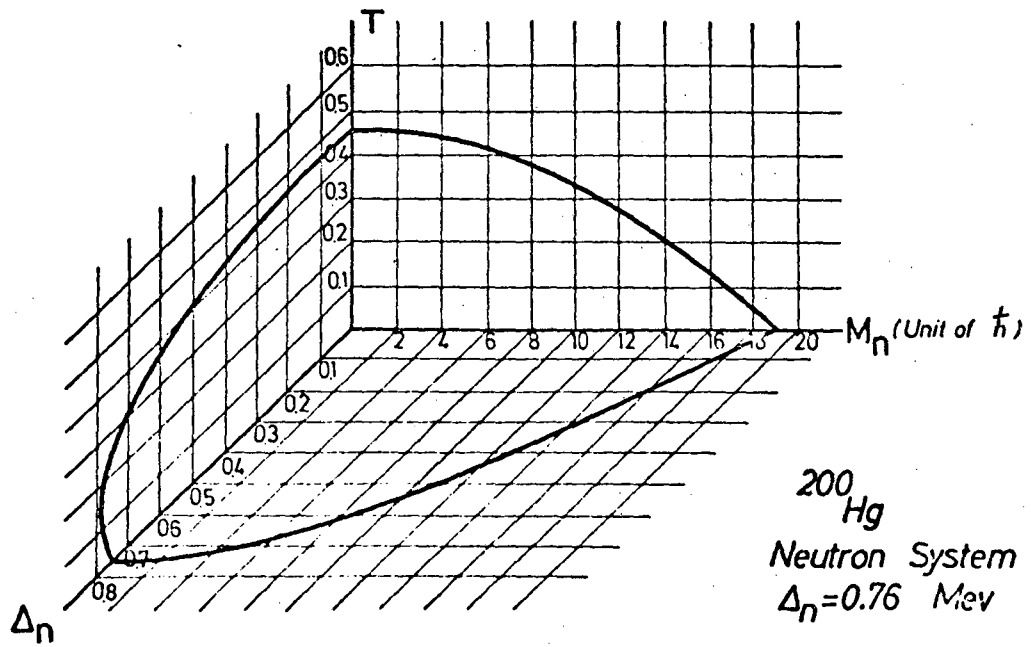
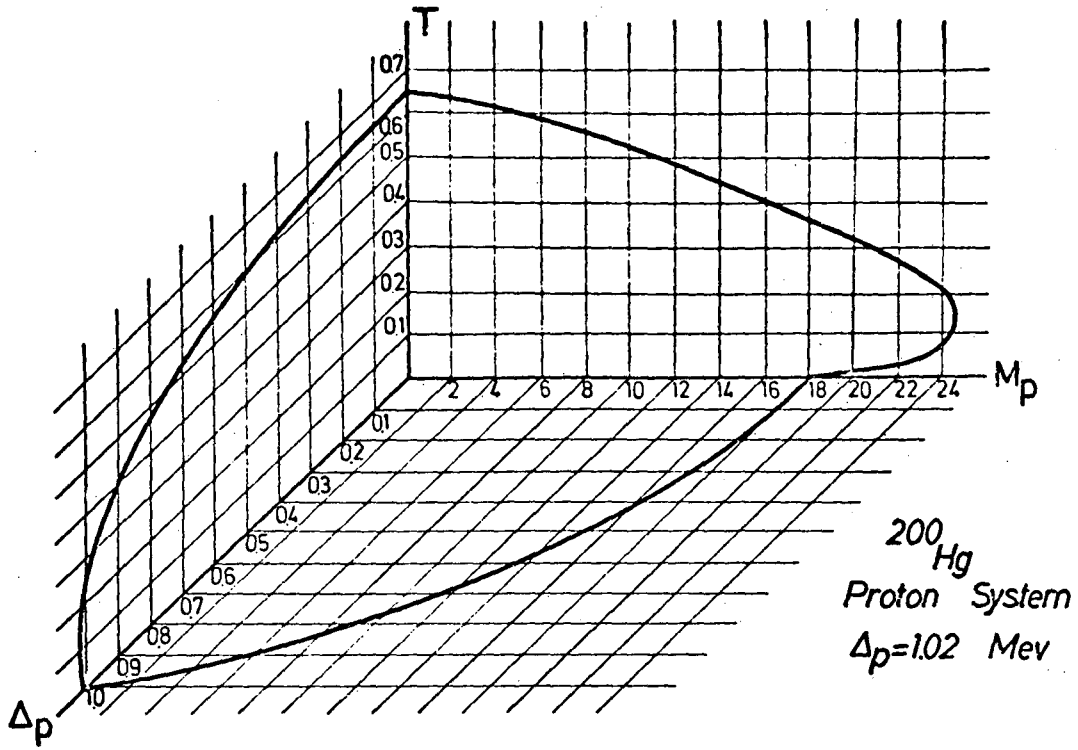
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FIG. 9



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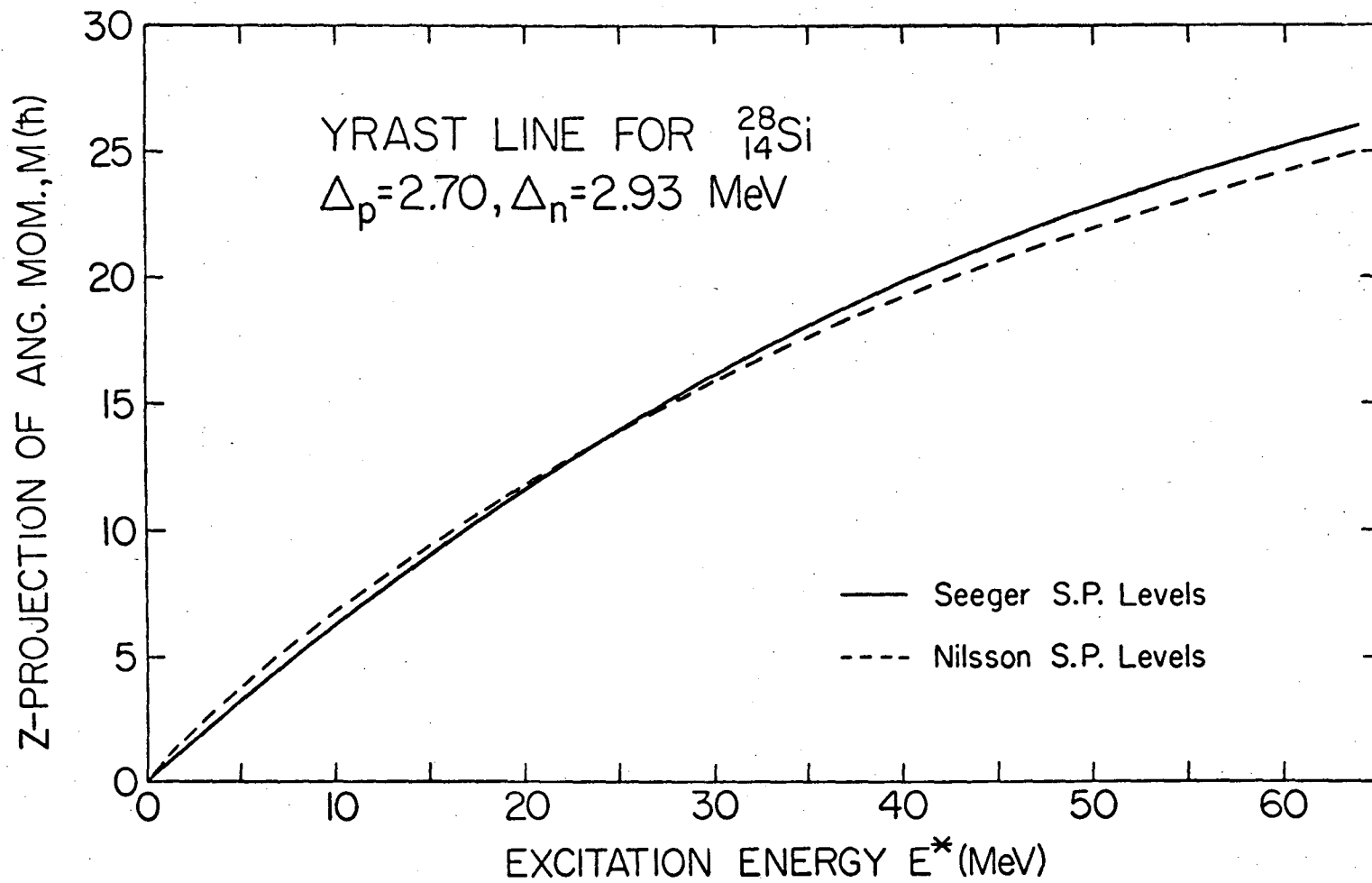
FIG. 10



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FIG. 11

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FIG. 12

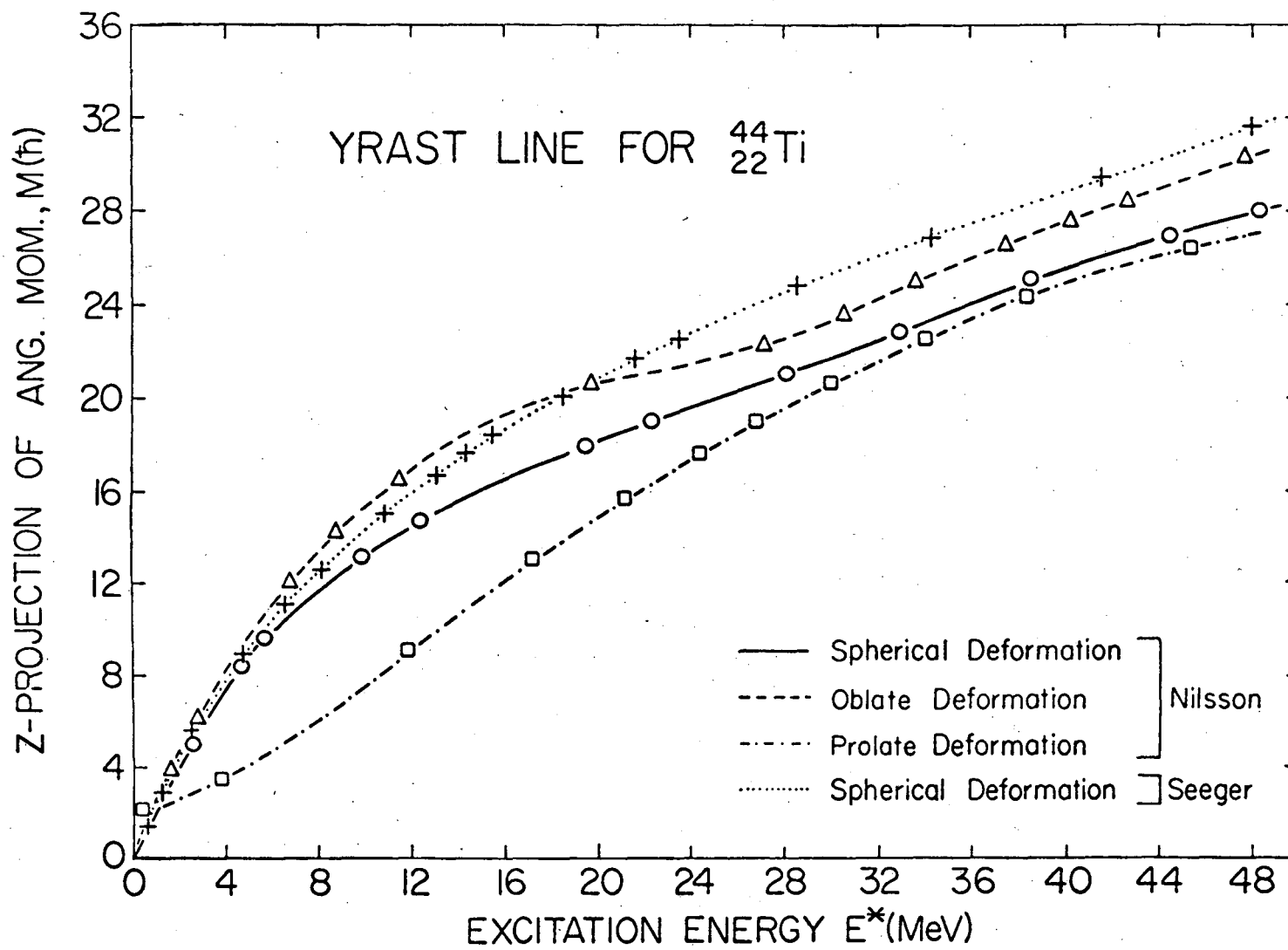
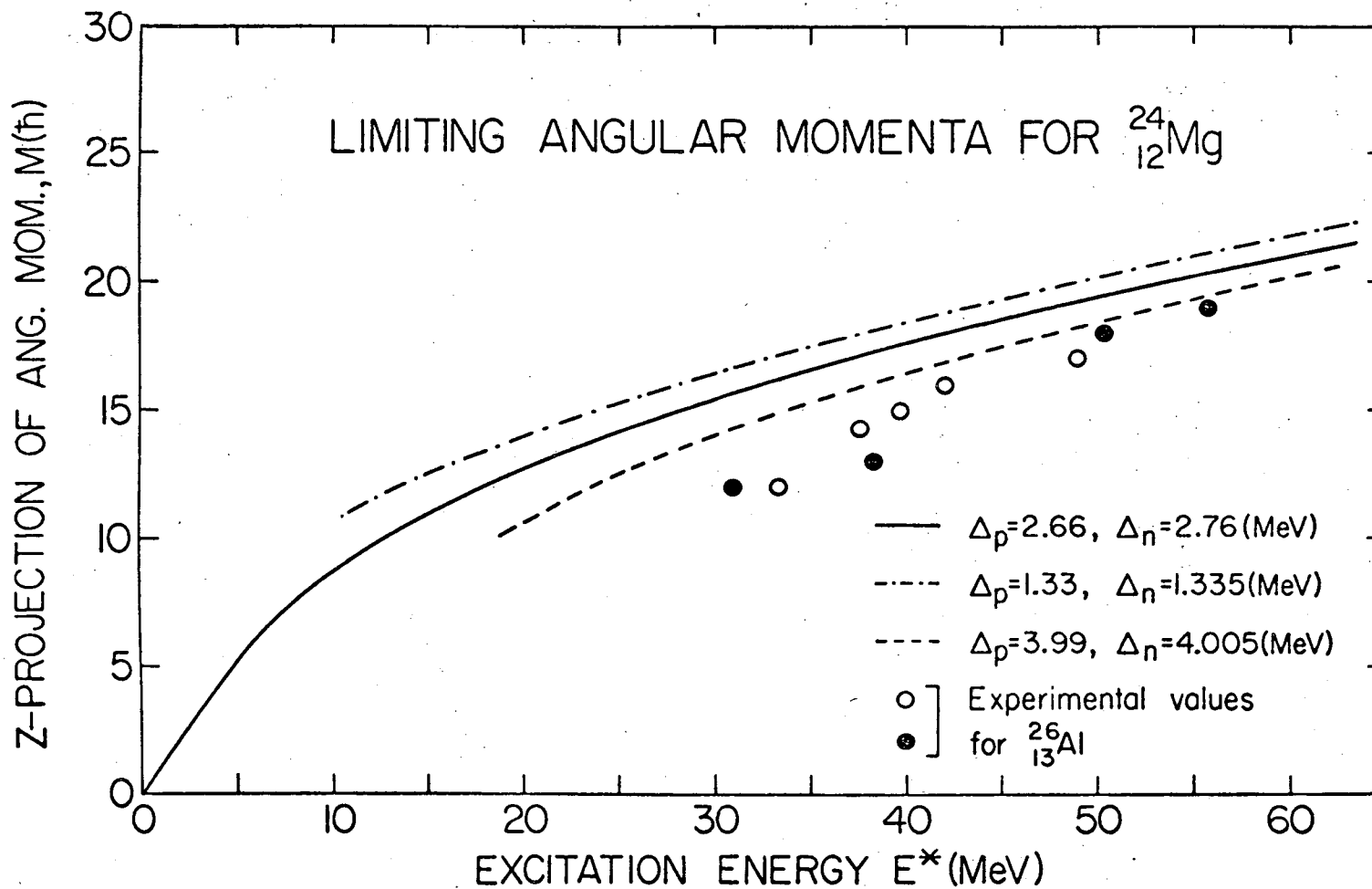


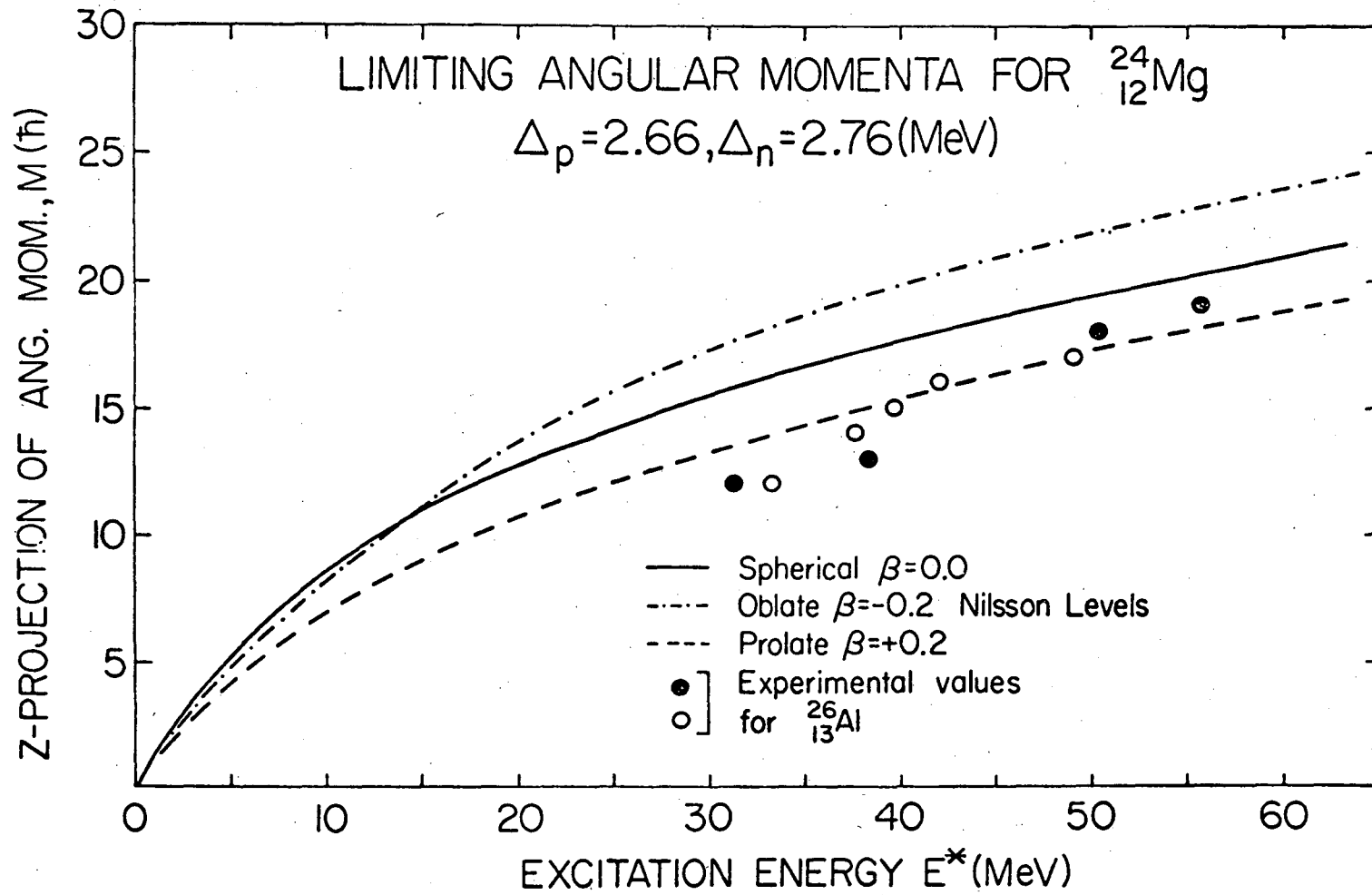
FIG. 13

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XBL 781-7072

FIG. 14

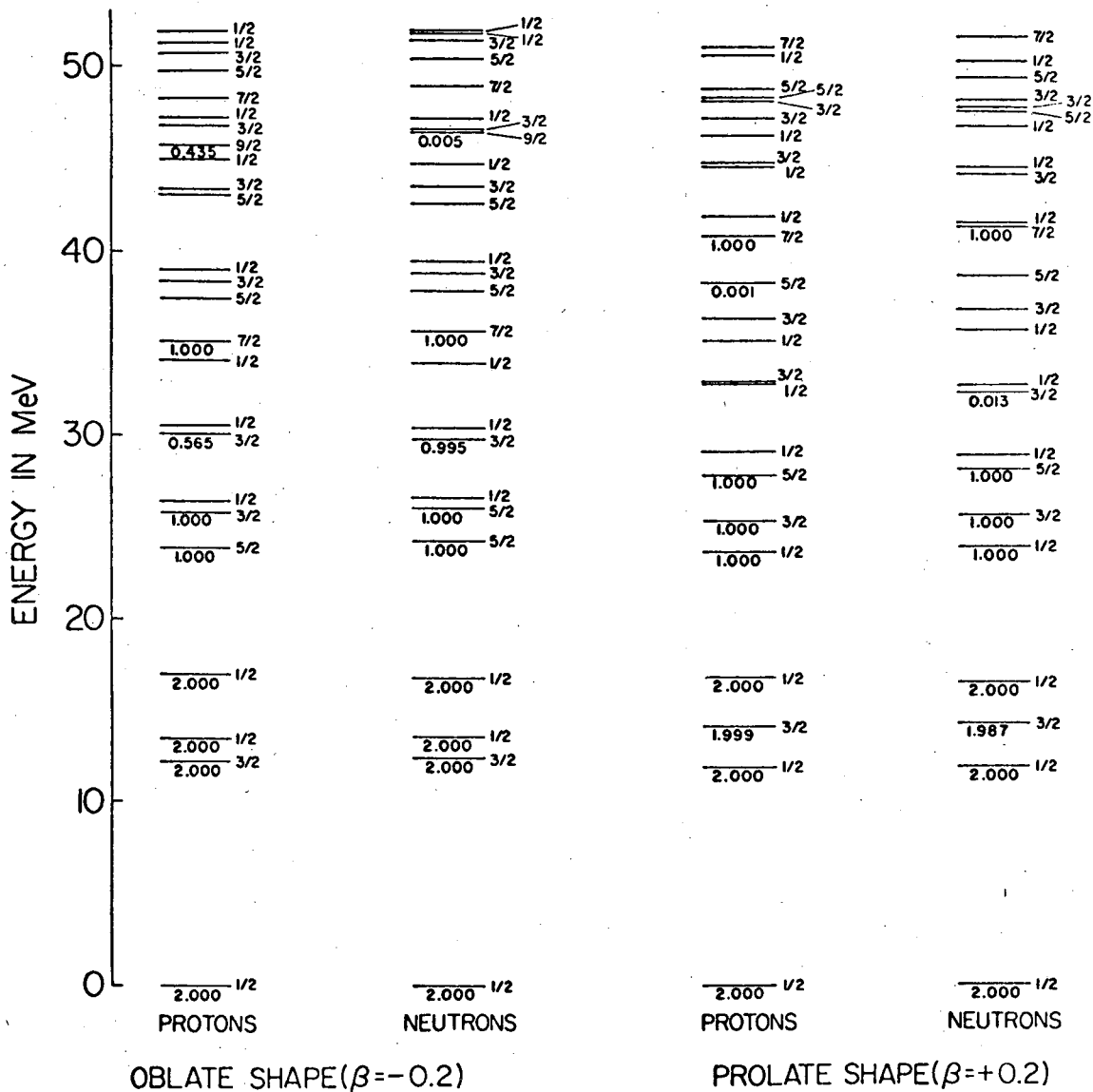


XBL 781-7070

Fig. 15

007-3477

$^{24}_{12}\text{Mg}$ FOR FIXED EXCITATION, $E^* = 43.0$ (MeV)



XBL 782-7085

FIG. 16

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