Submitted to Nuclear Physics

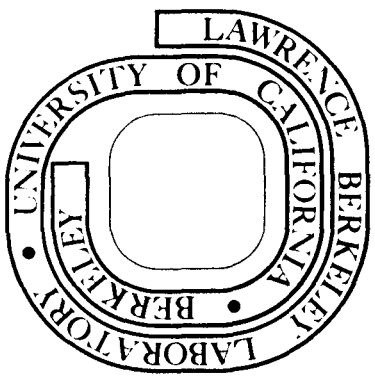
LBL-1693
Preprint c.1

CONFIGURATION MIXING OF TWO-QUASI-PARTICLE STATES IN EVEN-EVEN DEFORMED NUCLEI

H. Massmann, J. O. Rasmussen, T. E. Ward,
P. E. Haustein, and F. M. Bernthal

June 1973

Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48



DISCLAIMER

CONFIGURATION MIXING OF TWO-QUASI-PARTICLE STATES IN EVEN-EVEN DEFORMED NUCLEI*

H. Massmann† and J. O. Rasmussen††
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

and

T. E. Ward

Cyclotron Laboratory
Indiana University
Bloomington, Indiana 47401

and

P. E. Haustein
Department of Chemistry and
Heavy Ion Accelerator Laboratory
Yale University
New Haven, Connecticut 06520

and

F. M. Bernthal
Departments of Chemistry and Physics
and Cyclotron Laboratory
Michigan State University
E. Lansing, Michigan 48873

June 1973

Nuclear Structure: ^{174,176,178,180}Hf, ^{174}Yb; calculated configuration mixing higher K bands.

ABSTRACT

Evidence of configuration mixing of higher K bands in deformed even nuclei is surveyed. A general formulation for configuration mixing due to a two-body neutron-proton force is developed. A fit to the energy-splittings of Gallagher-Moszkowski pairs in odd-odd nuclei is made to obtain an effective Gaussian, central force except for the undetermined Wigner component. With this force, off-diagonal band-mixing matrix elements are calculated for various configurations in ^{176}Hf, ^{178}Hf, and ^{174}Yb. By solving BCS equations, the relevant occupation amplitudes are calculated. The effective n-p Wigner force component is fixed to give best over-all agreement to experimental band-mixing information. The resulting force is compared with the Anantaraman Schiffer force for spherical nuclei.

LBL-1693

I. INTRODUCTION

Among certain classes of two-quasi-particle states of deformed nuclei configuration mixing has been extensively treated, while for other classes there has been almost no attention to this problem.

On the one hand, the excited bands of $K^{\pi} = 0^+, 1^+, 2^+, 0^-, 1^-, 2^-$, and 3^- in even-even nuclei have been extensively treated microscopically. These treatments are usually carried out with some simple separable interactions (quadrupole-quadrupole, octupole-octupole, spin-quadrupole, or surface delta interaction). Some bands in the systems treated may become "collective" and consist of a linear combination of many two-quasi-particle basis states.

On the other hand, there has been little theoretical attention to the question of configuration mixing of higher K-bands than those mentioned above, and the general question of the effective nucleon-nucleon force appropriate in this context is quite open.

II. EXPERIMENTAL EVIDENCE

In the past several years interesting measurements have been made concerning band-mixing of two-quasi particle states in even-even nuclei. The even-even nuclei, in the region around ^{178}Hf, are interesting because of their prolific isomerism, associated with the availability of only large-Ω Nilsson orbitals near the Fermi energy: For protons the orbitals involved are $7/2^+[404]$, $9/2^-[514]$, and $5/2^+[402]$, and for neutrons they are $5/2^-[512]$, $7/2^-[514]$ and $9/2^+[624]$. Thus, relatively low-lying $K^{\pi} = 6^+, 8^-$, and 7^- states can be formed either as two-quasi-proton or two-quasi neutron states.

Khoo et al.[1] have carried out impressive measurements of excited bands in ^{176}Hf. Their analysis shows that the $K^{\pi} = 6^{+}$ bands at 1333.1 and 1761.5 keV are highly mixed between two-quasi-proton and two-quasi-neutron configurations with a 2p-2n mixing ratio of 38:62. Ejiri et al.[2] have independently made similar measurements and come to a similar conclusion. Ejiri et al. analyzed the 1549 keV $K^{\pi} = 6^{+}$ band in ^{174}Hf as being at least 90% two-quasi-proton, whereas we determine, from the comparison of the E2 hindrance factors of ^{174}Yb with those of ^{176}Hf, that the $K^{\pi} = 6^{+}$ isomeric state in ^{174}Yb is nearly pure two-quasi-neutron.

Ejiri et al. and Khoo et al. differ somewhat in their analysis of mixing of the $K^{\pi} = 8^{-}$ bands in ^{176}Hf, due in part to the additional complication of Coriolis mixing. Because of the Coriolis complication, we shall exclude this case from our quantitative analyses to follow. That is, one needs to consider mixing of more than two bands, and we wish here to confine ourselves to cases of two band mixing.

In ^{178}Hf the two $K^{\pi} = 8^{-}$ states have been known from beta decay properties to be highly mixed. Studies by Helmer and Reich[3] and by Ward, Chu, and Cumming[4a] indicate that the 1147-keV state (mainly two-quasi-neutron) and the 1480-keV state (mainly two-quasi-proton) have mixing ratios of 33:67 and 35:65 respectively. The ^{178m}Lu beta decay rates measured by Tamura[4b] give mixing of 36:64 in good agreement.

Körner, Wagner, and Dunlap[5] measured the magnetic moment of the 1142-keV $K^{\pi} = 8^{-}$ state in ^{180}Hf as 8.6 ± 1.0 nuclear magnetons, signifying nearly pure two-quasi-proton configuration.

Another case of such configuration mixing occurs in ^{174}Yb. The ^{174}Tm (5.2 min) ground state has been assigned a Nilsson configuration of $1/2^+[411]_p$, $7/2^-[514]_n$ (which is consistent with general systematics). The ^{174}Tm ground state beta decays to two states of ^{174}Yb at 1886 keV ($\sim$ 80%) and 2383 keV ($\sim$ 20%) with log ft values of 4.90 and 4.65, respectively. The log ft values of less than 5.0 would require that these transitions involve the $[514]\downarrow n$, $[514]\uparrow p$ orbitals. The strong gamma transition between the two states in ^{174}Yb, the observed log ft values, and the general energy systematics of two-quasi-particle states would suggest that the state at 1886 keV (mainly two-neutron) and the state at 2383 keV (mainly two-proton) are highly mixed. These two states can be assigned to the $1/2^-[521]_n$, $9/2^+[624]_n$, and $1/2^+[411]_p$, $9/2^-[514]_p$ orbitals, respectively, with $K^\pi = 5^-$. From the log ft values one would deduce a 35:65 mixing ratio for these bands.

Bernthal *et al.*[6] measured electron capture log ft values to ^{176}Hf of 7.21 and 6.85, to the $K^\pi = 1^+$ states at 1672.3 and 1862.8 keV, respectively. With the reasonable ^{176}Ta ground state assignment by Valentin and Santoni[7] of $K^\pi = 1^-$ ($7/2^+[404]_p$, $5/2^-[512]_n$), it is easy to see that the beta decay can proceed via a first-forbidden unhindered transition to the two-quasi-neutron component of the final $K^\pi = 1^+$ state but not to the two-quasi-proton part. This conclusion is the same even if configuration mixing of ($5/2^+[404]_p$, $7/2^-[514]_n$) is allowed in the initial state. Thus, the ratio of ft values tells us that the lower 1^+ state is 70% two-quasi-proton and 30% two-quasi-neutron, while the upper 1^+ state is the reverse. We note that the mixing of the 1^+ states is also implied by the strong 190.4-keV γ-ray transition between these levels. There might be some reservations about treating these two $K^\pi = 1^+$ states

in isolation from all other $K^{\pi} = 1^{+}$ poles. Gabrakov et al.[8], and Hamamoto et al.[9] have made random phase approximation calculations of 1^{+} states taking into account the spurious state problem associated with the rotational degree of freedom. We feel that our isolated treatment here may be approximately justified, since the j_{+} matrix elements between our 5/2 and 7/2 states are very small (hence, little coupling with the spurion) and since other 1^{+} poles should be considerably higher lying in ^{176}Hf.

III. GENERAL THEORY AND RESULTS

Can we obtain from theory at least a qualitative explanation of the phenomena so far observed? Very generally, when considering two states which interact with one another (but don't interact with any other state) the Schrödinger equation can be written in the form:

$$\begin{pmatrix} H_{11} & m \\ m & H_{22} \end{pmatrix} \psi(\vec{r}) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \lambda \, \psi(\vec{r}) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{1}$$

$$H_{12} = H_{21} \equiv m \quad ; \quad \alpha^2 + \beta^2 = 1$$

We introduce χ as the mixing ratio between the two states, letting $\chi = \beta^2/\alpha^2$. If $m = 0$ (i.e. no mixing) then $\chi = 0$ or ∞. When solving eq. (1), one easily finds:

$$m = \frac{\Delta\lambda\sqrt{\chi}}{1 + \chi} \tag{2}$$

where $\Delta\lambda = \lambda_> - \lambda_<$ is the difference of the eigenvalues of the 2 × 2 matrix, i.e. is the energy difference of like-spin members of the two bands of the configurations considered. In fig. 1 we have graphed the relationship of eq. (2). The different mixing ratios form a family of straight lines relating the proportionality of the energy separation of the bands with the absolute value of the mixing matrix element $|m|$. The lines have been labeled with mixing ratios less than unity, but those ratios χ greater than unity correspond to the line of their reciprocals $1/\chi$.

When the mixing ratio is near unity ($0.5 \lesssim \chi \lesssim 2$), the mixing matrix element $|m|$ is very insensitive to χ and dependent on $\Delta\lambda$.

We now analyze matrix element m in more detail. The quasi-particle operator string for the band-mixing matrix element for even-even states, with parallel angular momentum projection $\Omega_p + \Omega_n$, is:

$$\langle V_{np} \rangle \; \alpha^+_{n_1} \alpha^+_{n_2} \alpha_{p_1} \alpha_{p_2} \tag{3}$$

Substituting the inverse Bogoliubov transformation and retaining only those terms conserving charge and particle number yields:

$$u_{n_1} v_{n_2} v_{p_1} u_{p_2} a^+_{n_1} a^+_{\bar{p}_1} a_{\bar{n}_2} a_{p_2} \langle n_1 \bar{p}_1 | V_{np} | \bar{n}_2 p_2 \rangle + v_{n_1} u_{n_2} u_{p_1} v_{p_2} a^+_{n_2} a^+_{\bar{p}_2} a_{\bar{n}_1} a_{p_1} \langle n_2 \bar{p}_2 | V_{np} | \bar{n}_1 p_1 \rangle + \tag{4}$$

$$- u_{n_1} v_{n_2} u_{p_1} v_{p_2} a^+_{n_1} a^+_{\bar{p}_2} a_{\bar{n}_2} a_{p_1} \langle n_1 \bar{p}_2 | V_{np} | \bar{n}_2 p_1 \rangle - v_{n_1} u_{n_2} v_{p_1} u_{p_2} a^+_{n_2} a^+_{\bar{p}_1} a_{\bar{n}_1} a_{p_2} \langle n_2 \bar{p}_1 | V_{np} | \bar{n}_1 p_2 \rangle \; ,$$

where u and v are the usual BCS amplitudes and a and a^+ are the nucleon annihilation and creation operators (the bar over a subscript denotes the time

reversed orbital). But $\langle n_1\bar{p}_1|V_{np}|\bar{n}_2p_2\rangle = \langle n_2\bar{p}_2|V_{np}|\bar{n}_1p_1\rangle$ and $\langle n_1\bar{p}_2|V_{np}|\bar{n}_2p_1\rangle = \langle n_2\bar{p}_1|V_{np}|\bar{n}_1p_2\rangle$. Therefore we get the following:

$$\langle V_{np}\rangle \alpha^+_{n_1}\alpha^+_{n_2}\alpha_{p_1}\alpha_{p_2} = \left(u_{n_1}v_{n_2}v_{p_1}u_{p_2}a^+_{n_1}a^+_{\bar{p}_1}a_{\bar{n}_2}a_{p_2} + v_{n_1}u_{n_2}u_{p_1}v_{p_2}a^+_{n_2}a^+_{\bar{p}_2}a_{\bar{n}_1}a_{p_1}\right)$$

$$\times \langle n_1\bar{p}_1|V_{np}|\bar{n}_2p_2\rangle - \left(u_{n_1}v_{n_2}u_{p_1}v_{p_2}a^+_{n_1}a^+_{\bar{p}_2}a_{\bar{n}_2}a_{p_1} + v_{n_1}u_{n_2}v_{p_1}u_{p_2}a^+_{n_2}a^+_{\bar{p}_1}a_{\bar{n}_1}a_{p_2}\right)$$

$$\times \langle n_1\bar{p}_2|V_{np}|\bar{n}_2p_1\rangle \qquad (5)$$

In the case of antiparallel angular momentum projections $|\Omega_p - \Omega_n|$ for the pairing factors of even-even excited states we get the following:

$$\langle V_{np}\rangle \alpha^+_{n_1}\alpha^+_{\bar{n}_1}\alpha_{p_1}\alpha_{\bar{p}_2} = -\left(u_{n_1}v_{n_2}u_{p_1}v_{p_2}a^+_{n_1}a^+_{p_2}a_{n_2}a_{p_1} + v_{n_1}u_{n_2}v_{p_1}u_{p_2}a^+_{\bar{n}_2}a^+_{\bar{p}_1}a_{\bar{n}_1}a_{\bar{p}_2}\right)$$

$$\times \langle n_1p_2|V_{np}|n_2p_1\rangle - \left(u_{n_1}v_{n_2}v_{p_1}u_{p_2}a^+_{n_1}a^+_{\bar{p}_1}a_{n_2}a_{\bar{p}_2} + \right. \qquad (6)$$

$$\left. + v_{n_1}u_{n_2}u_{p_1}v_{p_2}a^+_{\bar{n}_2}a^+_{p_2}a_{\bar{n}_1}a_{p_1}\right)\langle \bar{n}_2p_2|V_{np}|\bar{n}_1p_1\rangle$$

In the pairing factors only the combination uv occurs. Thus, the matrix elements can only be large when the two orbitals are on opposite sides of the Fermi energy. This condition is approximately fulfilled for the 72-proton configuration of Hf, since the Fermi energy should be nearly at the degenerate 7/2+ and $9/2^-$ orbitals with the 5/2+ orbital lying slightly higher (cf. diag. 2c and 2d of Nilsson et al.[10]).

A. The Pairing Correction Factors

The uv factors for the cases here considered are shown in Table 1. They have been calculated by solving the BCS equations with the single particle energies obtained with the same program used in ref. 10, taking into account for each nucleus its quadrupole and hexadecapole deformation as given in Fig. 12a in the same reference. However, close to the Fermi energy we used energies obtained by interpolating the empirical single particles energies obtained from experiment by Ogle et al.[11] in order to get the most realistic uv factors. We used the experimental gap parameters Δ_{exp} given in Fig. 4a and 4b of ref. 10 and then adjusted the pairing strength parameter G so that Δ_{exp} was in agreement with the value of Δ obtained by solving the BCS equations.

For the specific case of $K^\pi = 8^-$ states in Hf, with n_1 the $9/2^+$, n_2 the $7/2^-$, p_1 the $9/2^-$ and p_2 the $7/2^+$ orbitals, the first two terms involve $K^\pi = 0^-$ couplings for the n-p matrix elements, and the second two terms involve $K^\pi = 1^+$ couplings.

More specifically, for the $K^\pi = 8^-$ states in ^{178}Hf, one has from eq. 5 and Table 1

$$|m_{8^-}(^{178}\mathrm{Hf})| = \left|0.471\langle 9/2_n\ \bar{9}/2_p|V_{np}|\bar{7}/2_n\ 7/2_p\rangle_{0^-} - 0.501\langle 9/2_n\ \bar{7}/2_p|V_{np}|\bar{7}/2_n\ 9/2_p\rangle_{1^+}\right| \tag{7a}$$

All the contributing matrix elements are of large momentum change (or exchange interaction) type, like those of the odd-even shift terms in odd-odd nuclei. Similarly, one has

$$|m_{6^+}(^{176}\mathrm{Hf})| = \left|0.515\langle 7/2_n\ \bar{7}/2_p|V_{np}|\bar{5}/2_n\ 5/2_p\rangle_{0^-} - 0.395\langle 7/2_n\ \bar{5}/2_p|V_{np}|\bar{5}/2_n\ 7/2_p\rangle_{1^-}\right| \tag{7b}$$

$$|m_{5^-}(^{174}\mathrm{Yb})| = \left|0.254\langle 1/2_n \bar{1}/2_p|V_{np}|\bar{9}/2_n 9/2_p\rangle_{0^-} - 0.725\langle 1/2_n \bar{9}/2_p|V_{np}|\bar{9}/2_n 1/2_p\rangle_{4^+}\right| \quad (7c)$$

For the $K^\pi = 1^+$ state in ^{176}Hf one has to use eq. (6), since in this case we have antiparallel angular momentum projections. We get the following:

$$|m_{1^+}(^{176}\mathrm{Hf})| = \left|0.395\langle 7/2_n 5/2_p|V_{np}|5/2_n 7/2_p\rangle_{6^-} + 0.515\langle \bar{5}/2_n 5/2_p|V_{np}|\bar{7}/2_n 7/2_p\rangle_{0^-}\right| \quad (7d)$$

For those cases where only one of the levels is known (in other words, $\Delta\lambda$ is not known), we have insufficient data to solve for the matrix element m. However, if the mixing is small, then we may make a theoretical estimate for $\Delta\lambda$ and still get a good value for m. This follows because, as pointed out before, m is not very sensitive to $\Delta\lambda$ if mixing is small (see Fig. 1)

B. The n-p Force

We consider now the matrix elements of the effective neutron-proton potential, taking it to be of Gaussian central form

$$V_{np} = \exp(-r^2/r_o^2)\ (V_{TE}P_{TE} + V_{TO}P_{TO} + V_{SE}P_{SE} + V_{SO}P_{SO}) \quad (8)$$

where P_{TE}, P_{TO}, P_{SE}, and P_{SO} are projection operators for the spin triplet (T) or singlet (S) and even (E) or odd (O) relative orbital angular momentum. We will use two different forces in our calculations, force I with range r_o=1.5 fm, force II with range r_o=1.0 fm. Following Jones *et al.*[12] and Ogle[13] we derive the potential strengths (up to a common additive constant) by doing a least square fit of energy splittings of Gallagher-Moszkowsky pairs. The arbitrary additive constant of the potential strength arises because a pure Wigner force does not

contribute to the splittings. Recently Ogle[13] has pointed out that there is still no satisfactory effective force for calculating the energy splitting of Gallagher-Moszkowski pairs. We are reluctant to use the force of Jones et al.[12] as we differ by a factor ~ 2 with their finite range calculations. It is not clear why the first order splittings of the effective n-p force give such poor quantitative fits to experiment, despite the universal applicability of the Gallagher-Moszkowski rule on the sign of the splitting. It may be that the off-diagonal couplings with other bands of the same K (n-p force coupling) or K ± 1 (Coriolis coupling) are responsible for the inability to obtain detailed agreement in first-order splitting calculations.[14] From the general consideration of the greater occurrence of low Ω orbitals in an oscillator shell, it is evident that the level density of two-quasi-particle bands is a rapidly decreasing function of K. Thus we have sought an effective force by excluding from the fit all cases involving K = 0 or 1 bands, which would be most susceptible to the higher order corrections. This exclusion meant we did not attempt to fit the shift terms in K = 0 bands. These shift terms have been shown by Jones et al[12] to be sensitive to tensor components, another reason for not demanding that a central force fit them. Table 2 shows the result of the fitting procedure. The optimum force strengths at the different ranges are shown in Table 5. The mixture of our forces (specially force II) has some features in common with the force obtained by Nunberg and Prosperi.[15] From Table 2 we see a scatter in fit to experiment of about ± 20 keV. The fit depends only weakly on range of force, and as Jones et al.[12] showed, even a delta force is rather satisfactory in fitting pair splittings.

Table 3 gives for the two optimal forces at different range the theoretical values of odd-diagonal, single-particle n-p matrix elements entering into the band-mixing cases for which we reviewed the experimental evidence in the first part of this paper. These are the single-particle matrix elements

entering into the eq. (7) with appropriate weighting factors from the BCS wave functions. The matrix elements in Table 3 were calculated from a sophisticated program due to Dr. Gordon Struble. In this program the Nilsson wave functions are calculated by truncating the set of basis states to a single oscillator shell in the isotropic harmonic oscillator basis. The deformation parameters used were: for Table 2 approximate $\zeta_n = \zeta_p = 3.9$, for Table 3 approximate $\zeta_n = 3.70$, $\zeta_p = 3.75$. The Nilsson potential parameters μ_n, μ_p, κ_n, κ_p, were given values from equations on page 14 of ref. 10. The major shell spacings $\hbar\omega_n$ and $\hbar\omega_p$ were obtained by using equations on page 6 of ref. 10.

With three of the four force strength parameters fixed by the Gallagher-Moszkowski pair fitting of Table 2 we allowed only the freedom to adjust the fourth component (Wigner force) strength to obtain best agreement with experimental band mixing matrix elements from eqs. (7) as shown in Table 4. In the last three cases there are data showing band mixing is small, but with the energy separation of the admixed band unknown only limits can be set on mixing. The agreement is quite satisfactory, though somewhat better for the shorter range force II.

Table 5 summarizes the strengths of our optional Gaussian central forces at the two different, somewhat arbitrary ranges.

We are not able to make a detailed comparison with the remarkably successful spherical shell-model n-p force of Anantaraman and Schiffer[16] (A-S force), since our code will not handle a tensor force component. Furthermore, their force has partially cancelling components of different ranges in triplet-odd and singlet-even parts. We tried fitting the Gallagher-Moszkowski doublet splittings with a force with short- and long-range components, the same ranges as those

of ref. 16, but the fit was not much improved, even with the doubling of free parameters.

It is not clear whether the differences with the A-S force, particularly the different V_{SO}/V_{TE} ratio, arise from our neglect of tensor components or from core polarization effect differences between their spherical nuclei and our spheroidal nuclei.

When an effective force of some reliability is available, one can address the problem of predicting the energies and band-mixing character of the multi-quasiparticle bands expected in the region. Soloviev has summarized[17] many calculations on three- and four-quasiparticle states of deformed nuclei; these calculations are for the most part done in the framework of the independent quasi-particle picture. There is also need for more data on such states. Awaiting discovery are surely other four-quasiparticle isomers like the 16+ isomer[3] of ^{178}Hf and perhaps five- quasiparticle isomers like the 37/2$^-$ state[18] in ^{177}Hf.

IV. CONCLUSIONS

An interesting pattern emerges from analysis of band-mixing of two quasi-neutron and two quasi-proton basis states of deformed even nuclei. The conditions for strong mixing to occur are 1) that the zero-order energies of the 2qp and 2qn basis states be reasonably close and 2) that the two proton (neutron) orbital energies be on opposite sides of the Fermi surface. The first condition follows from the requirement of a small energy denominator in the mixing determinant, and the second from the appearance of BCS weighting factors of form $u_1 v_2$. From analysis of these mixing ratios we can deduce a

value for the Wigner force strength in the effective n-p residual interaction. The Wigner component does not affect the splitting of Gallagher-Moszkowski doublets in odd-odd nuclei. The off-diagonal matrix elements extracted here also may manifest themselves in configuration mixing in odd-odd nuclei. It will be valuable to have more experimental information to further constrain the effective force.

ACKNOWLEDGMENTS

We are especially indebted to Dr. Gordon Struble, who provided the sophisticated computer code for calculating Gaussian force matrix elements among Nilsson states. We acknowledge helpful discussions with Dr. T. L. Khoo.

FOOTNOTES AND REFERENCES

*Work supported by the U. S. Atomic Energy Commission and the National Science Foundation.

†On leave from Facultad de Ciencias, U. de Chile, Santiago Chile, with a fellowship from the Convenio U. de Chile - U. of California.

††J. S. Guggenheim Fellow, 1973.

1. T. L. Khoo, J. C. Waddington, R. A. O'Neil, Z. Preibisz, D. G. Burke, and M. W. Johns, Phys. Rev. Letters 28, 1717 (1972).

2. H. Ejiri, G. B. Hagemann, and I. Hammer, Contributions to the International Conference on Nuclear Moments and Nuclear Structure, Osaka, Japan, Sept. 1972; J. Phys. Soc. Japan 34, Suppl. (1973).

3. R. G. Helmer and C. W. Reich, Nucl. Phys. A114, 649 (1968).

4a. T. E. Ward, Y. Y. Chu, and J. B. Cumming (private communication).

4b. T. Tamura, J. Phys. Soc. (Japan) 23, 691 (1967).

5. H. J. Körner, F. E. Wagner, and B. D. Dunlap, Phys. Rev. Letters 27, 1593 (1971); Argonne National Laboratory Report ANL-7971, 153 (March, 1972).

6. F. M. Bernthal, J. O. Rasmussen, and J. M. Hollander, Phys. Rev. C3, 1294 (1971).

7. J. Valentin and A. Santoni, Nucl. Phys. 47, 303 (1967).

8. S. I. Gabrakov, A. A. Kuliev, N. I. Pyatov, D. I. Salamov, and H. Schultz, Nucl. Phys. A182, 625 (1972).

9. I. Hamamoto, Nucl. Phys. A117, 484 (1971); cf. also B. L. Birbrair, Nucl. Phys. A108, 449 (1968).

10. S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. Lamm, P. Möller, B. Nilsson, Nucl. Phys. A131, 1 (1969).

11. W. Ogle, S. Wahlborn, P. Piepenbring, S. Fredriksson, Rev. Mod. Phys. 43, 424 (1971).

12. H. D. Jones, N. Onishi, T. Hess, and R. K. Sheline, Phys. Rev. C3, 529 (1971).

13. W. Ogle, Research Institute of Physics Annual Report, Stockholm, Sweden p. 154 (1972).

14. cf. G. L. Struble and J. O. Rasmussen, Phys. Letters 17, 283 (1965).

15. P. Nunberg and D. Prosperi, Nuovo Cimento, 40B, 318 (1965).

16. N. Anantaraman and J. P. Schiffer, Phys. Letters 37B, 229 (1971).

17. V. G. Soloviev, Theory of Complex Nuclei, Science Press, USSR (1971) (in Russian), (English translation in progress, Pergamon Press); cf. Tables 4.11, 4.12, and 4.14.

18. T. E. Ward and P. E. Haustein, Phys. Rev. Letters 27, 685 (1971); Y. Y. Chu, P. E. Haustein, and T. E. Ward, Phys. Rev. C6, 2259 (1972).

Notes Added in Proof

1. We have recently received a letter from Dr. N. Onishi to the effect that the program used in Ref. 12 is correct; however, in deducing the force strengths appearing in Table IV (of Ref. 12) a slight error was made. They plan to publish an erratum in the Phys. Rev. listing the correct force strengths for their Table IV.

2. J. I. Zaith and R. K. Sheline (Phys. Rev. C6, 506 (1972)) have studied the levels of ^{176}Hf populated in the (d,t) reaction. In their level scheme (Fig. 2) they do not show the levels of Khoo's upper $K^{\pi} = 6^{+}$ band, which, if pure two-quasi-proton, should not be populated by the (d,t) reaction. (The first two levels of this upper $K^{\pi} = 6^{+}$ band are at $E_{6^+} = 1761.5$ keV and $E_{7^+} = 1926.7$ keV.) However to the extent that this is mixed with the $K^{\pi} = 6^{+}$ band at 1333 keV (which is mainly two-quasi-neutron) then the levels of the upper $K^{\pi} = 6^{+}$ band should be populated relative to the lower $K^{\pi} = 6^{+}$ in proportion to their mixing ratios. In Fig. 1 (of the reference cited above) there are two unassigned peaks (peak number 11 and 16) with the appropriate energies and intensities to be the members of the upper $K^{\pi} = 6^{+}$band, consistent with the mixing ratio of Khoo et al.[1] The $K^{\pi} = 1^{+}$ bands are evidently populated too weakly to be seen in the spectrum of Fig. 1. Thus, we can gain no evidence to supplement the beta decay information on the 1^{+} band mixing.

Table 1. Values of u and v Amplitudes from BCS Wave Function.

Configuration		^{174}Yb	^{174}Hf	^{176}Hf	^{178}Hf	^{180}Hf
$1/2^+ (411)_p$	U	0.490	0.292	0.292	0.292	0.292
	V	0.871	0.956	0.956	0.956	0.956
$7/2^+ (404)_p$	U	0.928	0.747	0.747	0.747	0.747
	V	0.372	0.665	0.665	0.665	0.665
$9/2^- (514)_p$	U	0.938	0.819	0.819	0.819	0.819
	V	0.347	0.574	0.574	0.574	0.574
$5/2^+ (402)_p$	U	0.972	0.911	0.911	0.911	0.911
	V	0.233	0.412	0.412	0.412	0.412
$1/2^- (521)_n$	U	0.308	0.614	0.449	0.346	0.261
	V	0.951	0.789	0.893	0.938	0.965
$5/2^- (512)_n$	U	0.483	0.685	0.507	0.397	0.285
	V	0.876	0.728	0.862	0.918	0.958
$7/2^- (514)_n$	U	0.886	0.904	0.812	0.590	0.466
	V	0.465	0.427	0.584	0.807	0.885
$9/2^+ (624)_n$	U	0.904	0.944	0.898	0.778	0.619
	V	0.427	0.331	0.440	0.628	0.785

Table 2. Fit of Energy Splittings of Gallagher-Moszkowski Pairs.

Nucleus	Configuration Proton	Configuration Neutron	K^{π} $\Sigma_n+\Sigma_p=0$	K^{π} $\Sigma_n+\Sigma_p=1$	ΔE_{th} $r_0=1.5$ fm (keV)	ΔE_{th} $r_0=1.0$ fm (keV)	ΔE_{exp} (keV)	Ref.
HO 164	$7/2^-$ (523)↑	$1/2^+$ (400)↑	3^-	4^-	118	124	102	12
		$3/2^+$ (402)↓	5^-	2^-	-123	-131	- 85	12
		$3/2^-$ (521)↑	2^+	5^+	165	159	171	12
Ho 166	$7/2^-$ (523)↑	$1/2^-$ (521)↓	4^+	3^+	-147	-156	-171	12
Tm 168	$1/2^+$ (411)↓	$5/2^-$ (512)↑	3^-	2^-	-242	-231	-234	12
		$7/2^+$ (633)↑	4^+	3^+	-138	-145	-157	12
Tm 170	$1/2^+$ (411)↓	$5/2^-$ (512)↑	3^-	2^-	-239	-236	-232	12
Lu 174	$7/2^+$ (404)↓	$3/2^-$ (521)↑	5^-	2^-	- 59	- 58	- 90	12
		$1/2^-$ (521)↓	3^-	4^-	93	69	80	12
Lu 176	$7/2^+$ (404)↓	$1/2^-$ (510)↑	4^-	3^-	-130	-138	-118	12
Ta 182	$7/2^+$ (404)↓	$1/2^-$ (510)↑	4^-	3^-	-127	-135	$^+$ -174	15
		$3/2^-$ (512)↓	2^-	5^-	166	168	$^\#$ 154	15

More recent experimental values (R. G. Helmer, R. C. Greenwood and C. W. Reich, Nucl. Phys. A168, 449 (1971)) are: $^+$ -100; $^\#$ 140.

Table 3. V_{np} Matrix-Elements

Matrix-Element	Force I (keV)	Force II (keV)
$\langle 1/2_n\ \bar{1}/2_p \vert V_{np} \vert \bar{9}/2_n\ 9/2_p \rangle$	-152	-148
$\langle 1/2_n\ \bar{9}/2_p \vert V_{np} \vert \bar{9}/2_n\ 1/2_p \rangle$	319	292
$\langle 7/2_n\ \bar{7}/2_p \vert V_{np} \vert \bar{5}/2_n\ 5/2_p \rangle$	-151	-165
$\langle 7/2_n\ \bar{5}/2_p \vert V_{np} \vert \bar{5}/2_n\ 7/2_p \rangle$	318	288
$\langle 7/2_n\ 5/2_p \vert V_{np} \vert 5/2_n\ 7/2_p \rangle$	191	140
$\langle \bar{5}/2_n\ 5/2_p \vert V_{np} \vert \bar{7}/2_n\ 7/2_p \rangle$	-123	-155
$\langle 9/2_n\ \bar{9}/2_p \vert V_{np} \vert \bar{7}/2_n\ 7/2_p \rangle$	- 61	-108
$\langle 9/2_n\ \bar{7}/2_p \vert V_{np} \vert \bar{7}/2_n\ 9/2_p \rangle$	209	213

Table 4. Comparison of Theoretical and Experimental Mixing Matrix Elements (keV).

Case	exp. ref.	$\lvert m_{th} \rvert$ (Force I)	$\lvert m_{th} \rvert$ (Force II)	$\lvert m_{exp} \rvert$
m_{8-} (^{178}Hf)	a	133	158	158
m_{6+} (^{176}Hf)	b	203	199	208
m_{5-} (^{174}Yb)	c	270	249	235
m_{1+} (^{176}Hf)	d	12	25	88
m_{8-} (^{180}Hf)		122	144	
m_{6+} (^{174}Hf)		194	190	$< 0.3\ \Delta\lambda$
m_{6+} (^{174}Yb)		129	126	

Table 5. Potential Strength of Forces Used in this Work and Comparison with the One Used in ref. [16].

Force Component	Force I (r_0 = 1.5 fm) (MeV)	Force II (r_0 = 1.0 fm) (MeV)	Anantaram-Schiffer Force Short-range (r_0 = 1.0 fm)	Anantaram-Schiffer Force Long-range (r_0 = 3.2 fm)
V_{TE}	-117	-262	-195±30	
V_{SO}	- 72	+ 35	-195±30	
V_{TO}	- 62	-169	-230±20	6.5±10
V_{SE}	- 25	- 51	-165±20	6.5±10

FIGURE CAPTIONS

Fig. 1 Relation between $\Delta\lambda$, m, and χ. See text for definitions. The experimental points correspond to: a) $K^{\pi} = 8^{-}$ states in ^{178}Hf; b) $K^{\pi} = 6^{+}$ states in ^{176}Hf; c) $K^{\pi} = 5^{-}$ states in ^{174}Yb; and d) $K^{\pi} = 1^{+}$ states in ^{176}Hf.

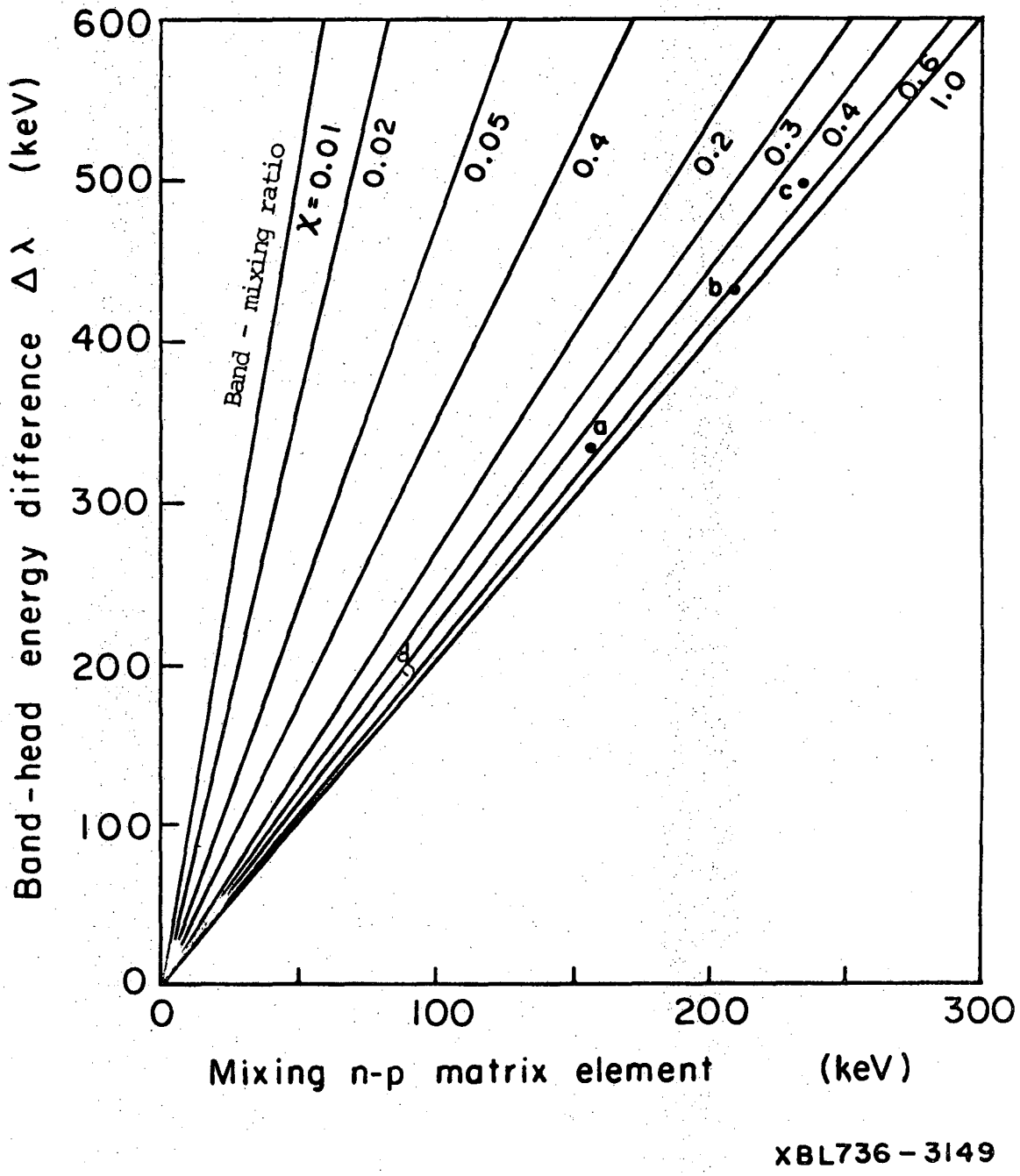

Band-head energy difference Δλ (keV)
600
500
400
300
200
100
0
0
100
200
300
Mixing n-p matrix element (keV)
Band - mixing ratio
χ=0.01
0.02
0.05
0.4
0.2
0.3
0.4
0.6
1.0
a
b
c
d
XBL736-3149

Fig. 1

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720