UC Irvine

UC Irvine Previously Published Works

Title

Impulsive pyramid-vertex and double-wedge diffraction coefficients

Permalink

<https://escholarship.org/uc/item/3xn01785>

Journal

IEEE Antennas and Propagation Society, AP-S International Symposium (Digest), 2

ISSN

1522-3965

Authors

Capolino, F
Albani, M

Publication Date

2001-11-26

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

Impulsive Pyramid-Vertex and Double-Wedge Diffraction Coefficients

Filippo Capolino and Matteo Albani

Dip. Ingegneria dell'Informazione, Università di Siena, Via Roma 56, 53100 Siena, Italy.
e-mail: capolino@ing.unisi.it, albanim@ing.unisi.it

Abstract

A new time domain version of the uniform description of double diffraction at a pair of coplanar skew wedges and vertex diffraction at the tip of a pyramid is here presented. The diffracted fields are uniformly evaluated in closed form via wavefront approximations.

INTRODUCTION

Doubly diffracted (DD) fields at a pair of skew wedges illuminated by an impulsive point source are obtained directly in the TD using a TD spectral synthesis, while the vertex diffracted (VD) field at the tip of a pyramid is constructed via a TD version of the Incremental Theory of Diffraction (ITD) [1] representation of the field scattered at a wedge [2]. The two diffraction mechanisms here considered complement, in a TD ray field framework, the field predicted by the TD-Uniform Theory of Diffraction (UTD) [3], leading to *new* uniform field expressions for complex scattering problems involving vertexes and double edges. In both cases, closed form solutions are determined using wavefront approximation for the evaluation of the TD radiation integrals. TD-DD and TD-VD field responses to an impulsive spherical delta excitation $\psi^{inc} = \delta[t - r'/c]/(4\pi r')$ (r' being the distance from the source and c the ambient wavespeed), are valid only for *early times*, on and close to (behind) the wavefronts. The field response $\hat{\psi}^{BL}(t)$ to a more general Band Limited (BL) pulsed excitation with large-but-finite bandwidth are found via convolution between the diffracted field due to an impulsive excitation $\hat{\psi}(t)$ and the illuminating waveform $\hat{G}(t)$. If the exciting signal $\hat{G}(t)$ has no low-frequency components and is thus dominated by high frequencies, the range of validity of the resulting pulsed response is enlarged to later observation times behind the wavefronts. The present TD-DD and TD-VD fields are limited to real time, and matches and compensates the spatial discontinuity of the TD singly diffracted field developed in [3]. Analytic extension of the DD mechanisms to complex time, as in [3],[4] for singly diffracted field, is currently under investigation.

TIME DOMAIN DOUBLY DIFFRACTED FIELDS

Let us consider a pair of wedges with soft/hard boundary conditions (BCs) and coplanar edges, illuminated by a spherical source. It is useful to define a spherical (r_i, β_i, ϕ_i) ray fixed coordinate system at each edge with origin at the diffraction point Q_i ($i = 1, 2$) (see Fig.1a). Our description of the DD mechanism is here constructed entirely in TD, following the procedure in [5] for the FD. In the following, only the TD field diffracted at edge 2 when it is illuminated by the field diffracted at edge 1 (12) will be considered. The DD mechanism (21) can be analogously obtained. The ray geometry for the DD field is depicted in Fig.1 with ℓ the distance between the two diffraction points Q_1 and Q_2 , and ϕ_{12} (ϕ'_{12}) the azimuthal coordinate of Q_2 (Q_1) measured in the system at edge 1 (2). The singly diffracted field at the first wedge illuminated by an impulsive spherical source at $P'(\phi'_1) \equiv (r'_1, \pi - \beta'_1, \phi'_1)$, is expressed as superposition of impulsive α_1 -spectral spherical sources at $P'(\alpha_1 + \phi_{12} + \pi)$ weighted by $G(\phi_1, \phi'_1; \alpha_1, n_1)$ [5], where $n_1\pi$ is the exterior wedge angle, and

$$G(\phi, \phi'; \alpha, n) = \sum_{i,j=1}^2 (\mp 1)^j B[(-1)^i (\phi - (-1)^j \phi'); \alpha, n], \quad B[\phi; \alpha, n] = \frac{-1}{2n} \frac{\sin[(\pi - \phi)/n]}{\cos[\frac{\alpha}{n}] - \cos[\frac{\pi - \phi}{n}]} \quad (1)$$

with the $-/+$ sign referring to soft/hard BC. Each impulsive α_1 -spherical source provides a diffracted field contribution from edge 2 at the observation point $P(\phi_2) \equiv (r_2, \beta_2, \phi_2)$, that is conveniently calculated using reciprocity, i.e., the diffracted field from edge 2 at $P'(\alpha_1 + \phi_{12} + \pi)$ due to a point source at $P(\phi_2)$ is represented as a summation of α_2 -spectral spherical sources at $P(\alpha_2 + \phi'_{12} + \pi)$ weighted by the spectral $G^{s,h}(\phi_2, \phi'_{12}; \alpha_2, n_2)$. Thus, using the α_2 -spectral superposition for the field radiated by wedge 2, for all the α_1 -spectral spherical sources representing the radiation by wedge 1, leads to the double integral representation

$$\hat{\psi}_{12}^{dd} = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\delta[t - R(\alpha_1, \alpha_2)/c]}{4\pi R(\alpha_1, \alpha_2)} G(\phi'_1, \phi_{12}; \alpha_1, n_1) G(\phi_2, \phi'_{12}; \alpha_2, n_2) d\alpha_1 d\alpha_2, \quad (2)$$

Fig. 1. Geometry and reference systems for the two canonical problems: two skew wedges (a); pyramid (b).

in which $R(\alpha_1, \alpha_2) = |P'(\alpha_1 + \phi_{12} + \pi) - P(\alpha_2 + \phi'_{12} + \pi)|$ is the distance between the spectral source and observation whose explicit expression is given in [5], and c is the ambient wavespeed.

The TD-DD field $\hat{\psi}_{12}^{dd}$ is obtained through a *uniform wavefront approximation* of (2) that is dominated by the spectrum around the saddle point $(\alpha_1, \alpha_2) = (0, 0)$ that satisfy $(\partial/\partial\alpha_i)R(\alpha_1, \alpha_2)|_{(0,0)} = 0$, for $i = 1, 2$. Furthermore, it exhibits poles in each variable given by the G functions at $\alpha_1 = \Phi_1^{pq} = (-1)^p\phi'_1 + (-1)^q\phi_{12} + \pi$ ($p, q = 1, 2$), and $\alpha_2 = \Phi_2^{rs} = (-1)^r\phi'_2 + (-1)^s\phi'_{12} + \pi$ ($r, s = 1, 2$). As shown in [5] for the FD case, the integrand is here decomposed in its even and odd part with respect to $(\alpha_1, \alpha_2) = (0, 0)$. Then, $R(\alpha_1, \alpha_2)/c$ is approximated with its Taylor expansion up to the second order in a neighborhood of its saddle point as $R(\alpha_1, \alpha_2)/c \approx t^{dd} + (Au^2 + Huv + Bv^2) + \dots$, with $t^{dd} = R(0, 0)/c = (r'_1 + \ell + r_2)/c$ (see Fig.1), $u = \sin(\alpha_1/2)$, and $v = \sin(\alpha_2/2)$. Furthermore, in each variable, a canonical quadratic polynomial with the same α_i -pole of G is used to regularize each integrand in (2), so that it is regular at the wavefront and can thus directly evaluated at $\alpha_1 = \alpha_2 = 0$. This canonical mapping for the wavefront approximation, permits to reduce (2) to a sum of integrals representing our TD-DD transition functions that can be evaluated in closed form. Summarizing, the TD-DD field is given by $\hat{\psi}_{12}^{dd} \sim A^{inc} A^{dd} \hat{D}_{12}^{dd, s, h}(t - t^{dd})$, where $A^{inc} = 1/(4\pi r'_1)$ is the incident spreading factor at Q_1 , $A^{dd} = \sqrt{r'_1}/\sqrt{\ell r_2(r'_1 + \ell + r_2)}$ is the DD spreading factor, and $\hat{D}_{12}^{dd, s, h}(t) = \hat{D}_{12}^{I, s, h}(t) + \hat{D}_{12}^{II, s, h}(t)$ is the TD double diffraction coefficient evaluated at the retarded time $t - t^{dd}$. Only the first order

$$\hat{D}_{12}^{I, s, h}(t) = \frac{cU(t)}{8\pi n_1 n_2 \sin \beta'_1 \sin \beta_2} \sum_{p, q, r, s=1}^2 (\mp 1)^{p+q+r+s} \cot\left(\frac{\phi_1^{pq}}{2n_1}\right) \cot\left(\frac{\phi_2^{rs}}{2n_2}\right) \hat{T}^I(\hat{a}_{pq}(t), \hat{b}_{rs}(t), w) \quad (3)$$

is here discussed because of space limitation. The TD transition function $\hat{T}^I(\hat{a}, \hat{b}, w)$ is evaluated in closed form closing one integral with the delta Dirac and the other via the Cauchy residue theorem, leading to

$$\hat{T}^I(\hat{a}, \hat{b}, w) = \frac{1}{\hat{c}^2 - 4w^2\hat{a}\hat{b}} \left[\frac{\hat{b}(\hat{c} - 2w^2\hat{a})}{\sqrt{1 + 1/\hat{a}}} + \frac{\hat{a}(\hat{c} - 2w^2\hat{b})}{\sqrt{1 + 1/\hat{b}}} \right] \quad (4)$$

with $\hat{c} = 1 - w^2 + \hat{a} + \hat{b}$, and $U(t) = 0$ or 1 , when $t > 0$ or < 0 , respectively. The parameters of $\hat{T}^I$ in (3) are $\hat{a}_{pq} = 2/(ct) \sin^2 \beta'_1 r'_1 \ell / (r'_1 + \ell) \sin^2 [(\Phi_1^{pq} - 2\pi n_1 N_1^{pq})/2]$, $\hat{b}_{rs} = 2/(ct) \sin^2 \beta_2 r_2 \ell / (r_2 + \ell) \sin^2 [(\Phi_2^{rs} - 2\pi n_2 N_2^{rs})/2]$, in which N_1^{pq} and N_2^{rs} are the integers that most nearly satisfy $\Phi_1^{pq} = 2\pi n_1 N_1^{pq}$ and $\Phi_2^{rs} = 2\pi n_2 N_2^{rs}$, respectively, and $w = (r'_1 r_2)^{1/2} [(r'_1 + \ell)(r_2 + \ell)]^{-1/2}$. Similar expressions apply for the higher order contribution $\hat{D}_{12}^{II, s, h}$. Far from the shadow boundaries (SBs), where $|\hat{a}|$ and $|\hat{b}| \rightarrow \infty$, it can be easily verified that $\hat{T}^I(\hat{a}, \hat{b}, w) \rightarrow 1$, whence the DD wave phenomenology acts like a time integrator. The wavefront approximation here obtained via a direct TD method, exactly coincides with the direct Fourier inversion applied to the high-frequency solutions in [5].

TIME DOMAIN VERTEX DIFFRACTED FIELDS

Let us consider a pyramid (see Fig.1b) with M edges and M faces on which either hard or soft boundary conditions (b.c.) are imposed on each face. The edges are counterclockwise counted when observing the pyramid from the vertex, and the m -th face is that delimited by m -th and $(m+1)$ -th edges. In our formulation the pyramid is thought as the superposition

of m wedges sharing common faces and all intersecting at the pyramid vertex. On each m -th edge both a spherical and a cylindrical reference systems are defined with the origin at the vertex, the x_m axis along the edge and the x_m axis lying on the plane containing the m -th face. The exterior angle of the m -th wedge is denoted by $n_m\pi$. The pyramid is illuminated by a spherical impulsive source $\psi^{inc} = \delta(t - r'/c)/(4\pi r')$. In [2] a useful representation for the field diffracted by a wedge is given. Furthermore, as shown in [1], the field diffracted by a curved or a truncated wedge can be expressed as a superposition of these *incremental field contributions* arising from the edge itself. Thus, the field diffracted by each truncated wedge of the pyramid can be estimated integrating the incremental field contributions along each m -th semi-infinite straight edge as

$$\hat{\psi} = \frac{j}{8\pi^2} \sum_{m=1}^M \int_0^\infty G(\phi_m, \phi'_m; ju_m(\zeta), n_m) \frac{\delta[t - (r'_m(\zeta) + r_m(\zeta))/c]}{r'_m(\zeta)r_m(\zeta)} d\zeta \quad (5)$$

where $G(\phi_m, \phi'_m, ju_m(\zeta), n_m)$ is defined in (1), $r'_m(\zeta)$ [$r_m(\zeta)$] denotes the distance between the source at $P' \equiv (r', \pi - \beta'_m, \phi'_m)$ [observer $P \equiv (r, \beta_m, \phi_m)$] and the incremental source at $z_m = \zeta$ on the m -th edge, $u_m(\zeta) = \ln[\tan \frac{\beta_m(\zeta)}{2}] - \ln[\tan \frac{\beta'_m(\zeta)}{2}]$, in which $\beta_m(\zeta)$ [$\beta'_m(\zeta)$] is the elevation spherical coordinate of P [P'] in a ray fixed system centered at the incremental source at ζ . The TD-VD field ψ^v is obtained through a *uniform wavefront approximation* of (5) that is dominated by the ζ -values of each integrand around the truncation end points $\zeta = 0$. Furthermore, the integrand exhibits poles in each variable given by the G in (1). Then, $(r'_m(\zeta) + r_m(\zeta))/c$ is approximated with its Taylor expansion up to the second order in a neighborhood of its end point as $(r'_m(\zeta) + r_m(\zeta))/c \approx t^v + A\zeta + B\zeta^2 \dots$, with $t^v = (r'(0) + r(0))/c = (r' + r)/c$ (see Fig.1b). Furthermore, a canonical quadratic polynomial with the same ζ -pole of $G(\phi_m, \phi'_m; ju_m(\zeta), n_m)$ is used to regularize each integrand in (5), so that it is regular at the wavefront and can thus directly evaluated at $\zeta = 0$. This canonical mapping for the wavefront approximation, permits to reduce (5) to a sum of integrals with simple poles and delta Dirac with argument a quadratic polynomial, which can be evaluated in closed form. The integral in (5) also describes the TD singly diffracted field at each edge, whose wavefront approximations $\hat{\psi}_m^d(t)$ for each m -edge are obtained by expanding $(r'_m(\zeta) + r_m(\zeta))/c$ around its saddle point ζ_s , which satisfies $(\partial/\partial\zeta)(r'_m(\zeta) + r_m(\zeta))/c|_{\zeta_s} = 0$, up to the second order. This leads to the results already obtained in [3]. The only difference is that here the saddle point ζ_s is subject to the existence condition $\zeta_s > 0$, namely when the wavefront of the m -th singly diffracted field is originated at ζ_s on the actual m -th *truncated* edge. Summarizing, the *total diffracted* field at the pyramid is approximated as

$$\hat{\psi}(t) = \sum_m \hat{\psi}_m^d(t) U(\beta'_m - \beta_m) + \hat{\psi}^v(t), \quad \hat{\psi}^v(t) = A^{inc} \frac{1}{r} \left(\sum_{m=1}^M \hat{D}_m^{v,s,h}(t - t^v) \right), \quad (6)$$

where $U(\beta'_m - \beta_m)$ is the Heaviside step function (see text after (4)) which delimits the region of existence of the m -th edge diffracted field in the region $\beta_m < \beta'_m$. The m th coefficient $\hat{D}_m^{v,s,h}(t - t^v)$, evaluated at the retarded time $t - t^v$, is given by

$$\hat{D}_m^{v,s,h}(t) = \frac{cU(t)}{2\pi(\cos\beta'_m - \cos\beta_m)} \sum_{i,j=1}^2 (\mp 1)^j B[(-1)^i(\phi_m - (-1)^j\phi'_m); ju_m, n_m] \hat{T}_G[\hat{b}_m, \hat{a}^i(\phi_m - (-1)^j\phi'_m)] \quad (7)$$

in which the upper/lower sign applies for soft/hard case, $u_m = u_m(0) = \ln[\tan \frac{\beta_m}{2}] - \ln[\tan \frac{\beta'_m}{2}]$, and $\hat{T}_G(\hat{b}, \hat{a}) = [1 + 1/(\hat{a} + \hat{b})\sqrt{1 + 1/\hat{b}}]^{-1}$ is the vertex TD transition function with arguments $b_m = (ct)^{-1} \frac{rr'}{r+r'} [1 - \cos(\beta'_m - \beta_m)]$, and $a_m^i(\Phi_m) = (ct)^{-1} \frac{rr'}{r+r'} \sin\beta'_m \sin\beta_m [1 + \cos(\Phi_m - 2N_m^i n_m \pi)]$ in which N_m^i , $i = 1, 2$ is the integer that most nearly satisfies the relation $\Phi_m - 2N_m^i n_m \pi = (-1)^i \pi$, $i = 1, 2$. Far from the shadow boundaries (SBs) ($\beta_m = \beta'_m$), where $\hat{b}_m$ and $\hat{a}_m^i(\Phi_m) \rightarrow \infty$, it can be easily verified that $\hat{T}_G(\hat{b}_m, \hat{a}^i) \rightarrow 1$, whence the VD wave phenomenology acts like a time integrator.

VECTORIAL CASE AND ILLUSTRATIVE EXAMPLES

The solution for the problem of an Electromagnetic spherical wave, arbitrarily polarized, illuminating either a couple of wedges or a pyramid can be solved in a ray framework,

Fig. 2. TD-DD field in three distinct wave regimes. Geometry: $r'_1 = 42\text{cm}$; $\ell = 45\text{cm}$; $r_2 = 33\text{cm}$; $\beta'_1 = 100^\circ$; $\beta_2 = 50^\circ$; $\phi'_{12} = 100^\circ$; $\phi_{12} = 100^\circ$, vertical dipole. a) Far from transition regions ($\phi'_1 = 310^\circ$, $\phi_2 = 310^\circ$). b) Observer in transition region ($\phi'_1 = 310^\circ$, $\phi_2 = 281^\circ \approx \phi'_{12} + 180^\circ$). c) Source and observer both in transition region ($\phi'_1 = 281^\circ$, $\phi_2 = 281^\circ$).

using the hard and soft respective diffraction coefficients to build dyadic diffraction coefficients in the ray fixed reference systems [5], [3], $\mathbf{D}^{dd}_m(t) = \hat{\beta}'_1 \hat{\beta}_2 \hat{D}^{dd,s}_m(t) + \hat{\phi}'_1 \hat{\phi}_2 \hat{D}^{dd,s}_m(t)$, $\mathbf{D}^{dv}_m(t) = \hat{\beta}'_m \hat{\beta}_m \hat{D}^{dv,s}_m(t) + \hat{\phi}'_m \hat{\phi}_m \hat{D}^{dv,h}_m(t)$. At those aspects in which a singly wedge diffracted ray abruptly disappears, either because shadowed by another wedge or because of the truncation of the relevant edge at a vertex, the total field preserves its continuity thanks to the uniform description of these transitions provided by the transition functions $\hat{T}$ and $\hat{T}_G$, introduced for DD and VD fields, respectively. The smooth compensation still holds when the abruptly vanishing singly diffracted field is itself in its transition with a direct or reflected field. This emphasizes as both the DD and VD fields can exhibit a two level (simple or double) transitional behavior. To better explain this phenomenon we present a sample-numerical calculation (Fig.2) for the DD field due to an electric dipole illuminating a couple of wedges. The excitation field waveform is a normalized Rayleigh pulse $\hat{G}(t) = \text{Re}[j/(j + 2\pi f_M t/4)^5]$, with spectrum with central frequency $f_M = 3\text{GHz}$ ($\lambda_M = c/f_M = 10\text{cm}$). In the first (from the left) figure both source and observation points lie far from their transition, i.e. they lie sufficiently below the edges plane. In these deep shadow region (Fig.2a), the only contribution arriving at the observer is the DD ray that is shaped as the primitive of the Rayleigh pulse. This is consistent with the $1/\omega$ -frequency dependence of the FD-DD field.

Shadow Boundary Limits. When the observation point crosses the plane containing edges 1 and 2 (Fig.1b), the singly diffracted field from edge 1 is spatially discontinuous at any time, due to the shadowing by edge 2. At this aspect, $\phi_2 \rightarrow \phi'_{12} + \pi$ (see [5]), it can be shown that $\hat{\psi}^{dd}_{12} = -\frac{1}{2}\text{sgn}(\hat{b}_{11})\hat{\psi}^d_1$, with $\hat{\psi}^d_1$ the TD-UTD singly diffracted field [3], so that compensating for the discontinuity of $\hat{\psi}^d_1$ at the SB $\phi_2 = \phi'_{12} + \pi$, at and after the wavefront. At this SB limit, the $\hat{\psi}^{dd}_{12}$ time dependence recovers the well known $1/\sqrt{t}$ behaviour of the singly diffracted field, as shown in Fig.2b. When the source and the observer are both close to the plane containing the edges, both $\hat{a}_{11}, \hat{b}_{11} \rightarrow 0$ and $\hat{\psi}^{dd}_{12} \sim 1/4\text{sgn}(\hat{a}_{11}\hat{b}_{11}) \hat{\psi}^{inc}(P)$, i.e. the DD field reduces to one fourth of the free-space direct contribution of the source, allowing the simultaneous compensation for the appearing/disappearing of the Geometrical Optics and of the two close-to-transition singly diffracted fields, restoring the continuity of the total field at and after the wavefront. At this double transition regime, the DD field time dependence is the same of the exciting pulse (Fig.2c). The VD field has an analogous behavior transitional behavior, not shown here for space limitation.

REFERENCES

- [1] R. Tiberio and S. Maci, "An incremental theory of diffraction: Scalar formulation," *IEEE Trans. Ant. Propagat.*, vol. 42, pp. 600-612, May 1994.
- [2] A. Rubinowicz, "ber Miyamoto-Wolfsche Vektorpotentiale, die mit der Lösung Eines Randwertproblems im Gebiete der Schwingungsgleichung Verknüpft Sind" *Acta Physica Polonica*, vol.28, pp.361-387, 1965.
- [3] P. R. Rousseau and P. H. Pathak, "Time-domain uniform geometrical theory of diffraction for a curved wedge," *IEEE Trans. Ant. Propagat.*, vol. 43, pp. 1375-1382, December 1995.
- [4] R. Ianculescu and E. Heyman, "Pulsed field diffraction by a perfectly conducting wedge: A spectral theory of transients analysis," *IEEE Trans. Ant. Propagat.*, vol.42, pp.781-789, June 1994.
- [5] F. Capolino, M. Albani, S. Maci, and R. Tiberio, "Double diffraction at a pair of coplanar skew edges," *IEEE Trans. Ant. Propagat.*, vol. 45, no. 8, pp. 1219-1226, August 1997.