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A MODEL FOR THE AXIAL CURRENT-THREE PION AMPLITUDE CONSISTENT WITH CURRENT ALGEBRA, THE ADLER CONSISTENCY CONDITION AND THE GENERALIZED VENEZIANO MODEL

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ABSTRACT

Using spurion techniques on the five-point function for $\pi\sigma \to \pi\pi\pi$, we obtain an expression for $A^\mu\pi \to \pi\pi$ for a conserved Axial Current which has poles along the π-A_1 trajectory in the current momentum squared whose residues are Veneziano type strong interaction amplitudes satisfying the Adler consistency condition. Using the soft pion theorem we obtain a simple expression for the pion form factor,

$$F(q^2) = \frac{1}{\pi} \frac{\Gamma(\frac{1}{2})\,\Gamma(-q^2+\frac{1}{2})\,\Gamma(-\gamma_2+\frac{1}{2})}{\Gamma(-q^2-\gamma_2+\frac{1}{2})}$$

where $-\gamma_2$ is a constant, and is related to the s-wave coupling constant in the $A_1\rho\pi$ vertex.

I. INTRODUCTION

Recently there have been several papers on the $A_1\rho\pi$ system in the Veneziano model,[1,2,3] and a few papers on using spurion techniques to obtain off-mass shell amplitudes from scalar n-point functions.[4,5,6] The main defect in the former papers was the arbitrariness in deciding what a consistent minimal number of Beta functions really meant--without such definition any ratio of $A_1\rho\pi$ coupling constants would be obtained. The main problem with the latter works were that the generalized N-point functions in the Veneziano model deal with scalar mesons and have little connection with pions, and the concurrent problems of satisfying the Adler consistency conditions. In this paper we use an invariant amplitude decomposition for $A^\mu\pi \rightarrow \pi\pi$ where $\partial_\mu A^\mu = 0$ (zero mass pions) and where the Invariant Amplitudes are mutilated five-point functions for the reaction $\pi\sigma \rightarrow \pi\pi\pi$. Since the Axial Vector is conserved here, we are automatically guaranteed a conserved isovector pion form factor when we use the soft pion limit to generate our pion form factor. Thus if our mutilated five-point fuction satisfies the Adler condition on the mass shell of the A_1 and the A_1 daughters, we have a consistent theory. We shall show presently how this comes about.

The most general form for the amplitude for a conserved Axial Current and three masssless pion containing no exotic resonances, and having crossing symmetry is

$$A^\mu_{1234} = \mathrm{Tr}\ \tau_1\ \tau_2\ \tau_4\ \tau_3[(P_1\cdot P_3)P_{1\mu} - P_1^2 P_{3\mu}]A(s,\ t,\ P_1^2)$$

$$+ [(P_1\cdot P_3)P_{2\mu} - (P_1\cdot P_2)P_{3\mu}]B(s,\ t,\ P_1^2)$$

$$+ 2 \leftrightarrow 4,\ 2 \leftrightarrow 3,\ 3 \leftrightarrow 4\ . \tag{1}$$

We define all momentum as incoming,

$$(P_1 + P_3)^2 = t = (P_2 + P_4)^2,$$

$$(P_1 + P_2)^2 = s = (P_3 + P_4)^2,$$

$$(P_2 + P_3)^2 = u = (P_1 + P_4)^2.$$

The Axial Vector Current has momentum P_1, isospin index 1 and the pions have momenta P_2, P_3, P_4 and isospin indices 2, 3, 4. The isospin factor

$$\mathrm{Tr}\ \tau_1\ \tau_2\ \tau_4\ \tau_3 = \delta_{12}\delta_{43} + \delta_{13}\delta_{24} - \delta_{14}\delta_{23}$$

guarantees no exotic resonances. The $A(s,t,P_1^2)$ and $B(s,t,P_1^2)$ are to be multilated five-point functions for the process $\pi\sigma \to \pi\pi\pi$ and will contain poles in s,t along the ρf_0 trajectory and poles in P_1^2 along the π-A_1 trajectory. The terms which have poles in the s-t channel are

$$\begin{aligned}\mathrm{Tr}\ \tau_1\ \tau_2\ \tau_4\ \tau_3\ & A(s,t,P_1^2)[P_1\cdot P_3\ P_{1\mu} - P_1^2\ P_{3\mu}] \\ & + A(t,s,P_1^2)[P_1\cdot P_2\ P_{1\mu} - P_1^2\ P_{2\mu}] \\ & + B^{(-)}(s,t,P_1^2)[P_1\cdot P_3\ P_{2\mu} - P_1\cdot P_2\ P_{3\mu}] \qquad (2)\end{aligned}$$

$$B^{(-)}(s,t,P_1^2) = B(s,t,P_1^2) - B(t,s,P_1^2).$$

II. MODEL FOR THE INVARIANT AMPLITUDES

The five-point function for the process $\pi_0\sigma_5 \to \pi_2\pi_3\pi_4$ is of the form[7]

$$\int_0^1 \int_0^1 d\mu_1\, d\mu_4\, \mu_1^{-\alpha(s)} \left(\frac{1-\mu_1}{1-\mu_1\mu_4}\right)^{-\alpha(t)} \mu_4^{-\alpha_{05}-1} \left(\frac{1-\mu_4}{1-\mu_1\mu_4}\right)^{-\alpha_{02}} (1-\mu_1\mu_4)^{-\alpha_{54}-2} \tag{3}$$

where

$\alpha_{05} = (P_0 + P_5)^2 \equiv P_1^2$ is the πA_1 trajectory (see Fig. 1),

$\alpha_{45} = (P_4 + P_5)^2$,

and

$\alpha_{02} = (P_0 + P_2)^2 + \frac{1}{2}$ is the ρ trajectory.

This function has poles in s,t,P_1^2, but also has poles in the variables dual to α_{05}, i.e. α_{45} and α_{02}. To obtain a function which just has poles in s,t,P_1^2, we set $\alpha_{02} = \gamma_2$ and $\alpha_{54} = \gamma_1$. We thus make the ansatz that:

$$A(t,s,P_1^2) = a\int_0^1 \int_0^1 d\mu_1\, d\mu_4\, \mu_1^{-\alpha(s)} (1-\mu_1)^{-\alpha(t)}\, \mu_4^{-P_1^2-1} (1-\mu_4)^{-\gamma_2} \times (1-\mu_1\mu_4)^{\alpha(t)+\gamma_2-\gamma_1-2} , \tag{4}$$

and

$$B^{(-)}(t,s,P_1^2) = \frac{b}{a} [A(t,s,P_1^2) - A(s,t,P_1^2)] . \tag{4a}$$

So we have a theory with four parameters, a, b, γ_1, γ_2. The q^2 pole structure $(q^2 \equiv P_1^2)$ of $A(t,s,q^2)$ is the following:

$A(t,s,q^2)$ =

$$= a \sum_{mn} \frac{B\left(1 + m - \alpha(s),\ 1-\alpha(t)\right)\left(\alpha(t) + \gamma_2 - \gamma_1 - 2\right)!(-\gamma_2)!(-1)^{m+n}}{(-q^2 + m + n)\quad m!\left(\alpha(t) + \gamma_2 - \gamma_1 - 2 - m\right)!(-\gamma_2 - n)!\ n!} . \tag{5}$$

At $q^2 = j$ there are $j + 1$ terms and the residue of each term has the following asymptotic behavior:

$$\begin{aligned} \text{as} \quad & s \to \infty \qquad \text{Residue } A(t,s,m^2=j) \Longrightarrow s^{\alpha(t)-1} \\ & t \to \infty \qquad \text{Residue } A(t,s,m^2=j) \Longrightarrow t^{\alpha(s)-1} . \end{aligned} \tag{6}$$

We also notice that only A has the pion pole since the residue at $q^2 = 0$ is symmetric in s and t. This is in agreement with our invariant amplitude decomposition.

III. ADLER CONSISTENCY CONDITION

The Adler consistency conditions states that for the strong interaction processes $\pi\pi \to \pi\pi$, $\pi A_1 \to \pi\pi$, $\pi A_1^{(n)} \to \pi\pi$ (where $A_1^{(n)}$ is the nth daughter on the π-A_1 trajectory with spin-parity 1^+) if we set any of the pion four-momentum to zero, the amplitude must vanish.

The pion scattering amplitude is the coefficient of P_1^{μ} when we take the residue at $P_1^2 = 0$. We notice that the st pole terms are proportional to $P_1 \cdot P_3$ and $P_1 \cdot P_2$. Thus for zero-mass pions we automatically satisfy the Adler condition for the π-π scattering amplitude.

The scattering amplitude for $\pi A_1^{(n)} \to \pi\pi$ is

$$\epsilon^\mu(P_1)\; T_{(n)}{}^\mu(s,t,u)$$

where

$$\lim_{P_1^2 \Rightarrow m_n^2 = n} (-P_1^2 + m_n^2)\langle \pi_3(-P_3)\pi_4(-P_4)|A_{\mu_1}{}^{(0)}|\pi_2(P_2)\rangle$$

$$= g_A^{(n)} \left(g_{\mu\nu} - \frac{P_{1\mu}P_{1\nu}}{m_n^2} \right) T_{(n)}{}^{\nu}{}_{1234}(s,t,u) \quad . \tag{7}$$

The relation $m_n^2 = n$ is due to the fact the πA_1 trajectory has slope 1 and passes through the origin.

We thus find that the scattering amplitude for $A_1^{(n)}\pi \to \pi\pi$ is

$$(\delta_{12}\delta_{34} + \delta_{13}\delta_{24} - \delta_{14}\delta_{23})$$

$$\frac{\epsilon^\mu(P_1)}{g_A^{(n)}} \Big\{ P_{2\mu}\left(-m_n^2 \text{ Res } A(t,s,m_n^2) + (P_1 \cdot P_3) \text{ Res } B^{(-)}(s,t,m_n^2)\right)$$

$$+ P_{3\mu}\left(-m_n^2 \text{ Res } A(s,t,m_n^2) - (P_1 \cdot P_2) \text{ Res } B^{(-)}(s,t,m_n^2)\right)\Big\}$$

$$+ \text{ permutations} \; . \tag{8}$$

When $P_4 \to 0$ $s=t=0$ and the above expression vanishes automatically.

When $P_3 \to 0$ $t = m_n^2$, $s=0$, and we get the requirement $\mathrm{Res}\, A(t = m_n^2,\ s=0,\ m_n^2) = 0$. This requirement guarantees the Adler condition when $P_2 \to 0$ also.

Now $\mathrm{Res}\, A(t,s,m_j^2)$

$$= a \sum_{m=0}^{j} \frac{B\big(1 + m-\alpha(s),\ 1 - \alpha(t)\big)\ (\alpha(t)+ \gamma_2-\gamma_1-2)!\ (-\gamma_2)!\ (-1)^j}{m!\ (j-m)!\ (-\gamma_2-j+m)!\ (\alpha(t)+ \gamma_2-\gamma_1-2-m)!} \ . \tag{10}$$

Since α is the ρ trajectory,

$$\alpha(s) = s + \frac{1}{2} \ \text{ and } \ \alpha(t) = t + \frac{1}{2} \ . \tag{11}$$

The Adler point is $t = m_j^2, s=0$. Since $m_j^2 = j$, $\alpha(m_j^2) = j + \frac{1}{2}$ and $B\big(1+ m - \alpha(s),\ 1 - \alpha(t)\big)$ vanishes at $\alpha(t) = j + \frac{1}{2}$ $\alpha(s) = \frac{1}{2}$ if $m \leq j - 1$, the only term that does not vanish automatically at the Adler point is the term

$$a\, B\big(1+ j - \alpha(s),\ 1-\alpha(t)\big)\ \frac{(\alpha(t) + \gamma_2 - \gamma_1 - 2)!}{j!\ (\alpha(t)+ \gamma_2-\gamma_1-2-j)!} \tag{12}$$

So by choosing $\gamma_2 - \gamma_1 = \frac{1}{2}$, this term also vanishes at the Adler point. The result $\gamma_2 - \gamma_1 = \frac{1}{2}$ is just the difference in the intercepts of the two trajectories with which γ_1 and γ_2 were associated in the five-point function.

IV. STRONG INTERACTION AMPLITUDES FOR $\pi\pi \to \pi\pi$, $\pi A_1 \to \pi\pi$.

In a massless pion theory we have that the pion scattering amplitude is defined as follows

$$\lim_{-P_1^2 \to 0} -P_1^2 \langle \pi_3(-P_3)\pi_4(-P_4)|A_1^{\mu}(0)|\pi_2(P_2)\rangle$$
$$= P_1^{\mu} f_\pi \langle \pi_3(-P_3)\pi_4(-P_4)|T|\pi_1(P_1)\pi_2(P_2)\rangle \quad . \tag{13}$$

Therefore we have

$$\langle \pi_3(-P_3)\pi_4(-P_4)|T|\pi_1(P_1)\pi_2(P_2)\rangle \tag{14}$$
$$= \frac{(\delta_{12}\delta_{43} + \delta_{13}\delta_{24} - \delta_{14}\delta_{23})}{f_\pi} \frac{(s+t)}{2} \, a \, B\big(1-\alpha(s),\, 1-\alpha(t)\big)$$
$$+ \text{ permutations } .$$

This is the same result as J. Shapiro[8] in the zero pion mass limit.

By looking at the ρ pole in the $I_s = 1$ amplitude one can get a in terms of $g^2_{\rho\pi\pi}$ and f_π. The $I_s = 1$ part of the Axial Current matrix element is:

$$A^{\mu I_s=1}_{1234} = (\delta_{13}\delta_{24} - \delta_{23}\delta_{14})\Big\{2\,A(s,t,P_1^{\,2})(P_{1\mu}\,P_2\cdot P_3 - P_{3\mu}\,P_1^{\,2})$$

$$+ 2\,A(s,u,P_1^{\,2})\quad(P_{4\mu}\,P_1^{\,2} - P_{1\mu}\,P_1\cdot P_4)$$

$$+ 2\,(A(t,s,P_1^{\,2}) - A(u,s,P_1^{\,2}))\,(P_{1\mu}\,P_1\cdot P_2 - P_{2\mu}\,P_1^{\,2})$$

$$+ 2\,B^{(-)}(s,t,P_1^{\,2})(P_{2\mu}\,P_1\cdot P_3 - P_1\cdot P_2\,P_{3\mu})$$

$$+ 2\,B^{(-)}(s,u,P_1^{\,2})(P_{4\mu}\,P_1\cdot P_2 - P_{2\mu}\,P_1\cdot P_4)\Big\}\,. \tag{15}$$

Therefore the $I_s = 1$ π-π scattering amplitude is

$$T^{I_s=1} = \frac{a}{f_\pi}\,(\delta_{13}\delta_{24} - \delta_{23}\delta_{14})[(s+t)\,B\big(1-\alpha(s),\,1-\alpha(t)\big) - (s+u)\,B\big(1-\alpha(s),1-\alpha(u)\big)]. \tag{16}$$

Near the s channel ρ pole $s \approx m_\rho^{\,2}$ and we obtain

$$T^{I_s=1} \approx \frac{a}{f_\pi}\;\frac{(\delta_{13}\delta_{24} - \delta_{23}\delta_{14})(t - u)}{-\,s + m_\rho^{\,2}}\quad. \tag{16a}$$

If we write the vertex for ρ (isospin index m) $\pi_1\pi_2$ as $g_{\rho\pi\pi}\;\epsilon^\mu(q)(P_1-P_2)_\mu\;\epsilon_{12m}$ we obtain

$$T_{\text{pole}} = \frac{(\delta_{13}\delta_{24} - \delta_{23}\delta_{14})(t-u)\, g^2_{\rho\pi\pi}}{-s + m_\rho^2} \,. \tag{17}$$

Therefore we obtain

$$a = g^2_{\rho\pi\pi} \;\; f_\pi \,. \tag{18}$$

From Eqs. (5),(15) and (7) we obtain that the $I_s=1$ scattering amplitude for $A_1\pi \to \pi\pi$ is

$$\begin{aligned}
\langle \pi(-P_3)\pi(-P_4)|T|A_1(P_1)\pi(P_2)\rangle &= \frac{\epsilon^\mu(P_1)}{g_A}\,(\delta_{13}\delta_{24} - \delta_{23}\delta_{14}) \\
\times P_{2\mu}\Big\{ & b(t-m_A{}^2)[(\tfrac{3}{2}-\alpha(s))B(2-\alpha(t),\,1-\alpha(s)) - (\tfrac{3}{2}-\alpha(t))\,B(1-\alpha(t),\,2-\alpha(s))] \\
& + [-2m_A{}^2a + b(t+m_A{}^2)][(\tfrac{3}{2}-\alpha(s))\,B(1-\alpha(s),\,2-\alpha(u)) - (\tfrac{3}{2}-\alpha(u))\,B(1-\alpha(u),2-\alpha(s))] \\
& -2m_A{}^2a\,(\gamma_2\,B(1-\alpha(s),\,1-\alpha(t)) + (\tfrac{3}{2}-\alpha(t))\,B(1-\alpha(t),\,2-\alpha(s))\Big\} \\
+ P_{3\mu}\Big\{ & b(m_A{}^2-s)[(\tfrac{3}{2}-\alpha(s))\,B(2-\alpha(t),\,1-\alpha(s)) - (\tfrac{3}{2}-\alpha(t))\,B(1-\alpha(t),2-\alpha(s)) \\
& + (\tfrac{3}{2}-\alpha(s))\,B(2-\alpha(u),\,1-\alpha(s)) - (\tfrac{3}{2}-\alpha(u))\,B(1-\alpha(u),\,2-\alpha(s))] \\
& - 2m_A{}^2\gamma_2 a\,(B(1-\alpha(s),\,1-\alpha(t)) + B(1-\alpha(s),\,1-\alpha(u))) \\
& - 2m_A{}^2a\,(\tfrac{3}{2}-\alpha(s))\,(B(1-\alpha(s),\,2-\alpha(t)) + B(1-\alpha(s),\,2-\alpha(u)))\Big\} \,.
\end{aligned} \tag{19}$$

We notice the above expression is different from those expressions found in the literature[1,2,3] in that we have terms like $B\left(1-\alpha(s),\ 2-\alpha(t)\right)$. However "nonleading" terms of this form were absolutely necessary for parameterizing $A_1\pi \to A_1\pi$ which also has the ρ pole.[9] So for a consistent answer to the question of the $A_1\rho\pi$ system these terms should be present.

Near the ρ pole in the s channel, $s \approx m_\rho^2$ $\alpha(s) \approx 1$

$$T(\pi A_1 \to \pi\pi) \approx \frac{(\delta_{13}\delta_{24} - \delta_{23}\delta_{14})\ \epsilon^\mu(P_1)}{(-s + m_\rho^2)\ \ g_A} \tag{20}$$

$$\times \left[P_{2\mu}\left\{bt - m_A^2 a - 2m_A^2 a\gamma_2\right\} + P_{3\mu}\left\{b(m_A^2 - m_\rho^2) - 4m_A^2\gamma_2 a - 2m_A^2 a\right\}\right].$$

Defining the $A_1(P_1)\rho_m(q)\,\pi(P_2)$ vertex as

$$\epsilon_{A_1}^{\ \mu}(P_1)\epsilon_\rho^{*\nu}(q)[G_s\ g_{\mu\nu} + G_D P_{1\nu}q_\mu]\ \epsilon_{12m}\ , \tag{21}$$

we obtain for the ρ pole contribution

$$T_{pole} = \frac{\delta_{13}\delta_{24} - \delta_{23}\delta_{14}}{-s + m_\rho^2}\ \epsilon^\mu(P_1)\ g_{\rho\pi\pi}$$

$$\times \left\{P_{2\mu}[(u-t)G_D - G_s] - 2P_{3\mu}G_s\right\}. \tag{22}$$

Comparing we find

$$- 2G_D g_{\rho\pi\pi} = \frac{b}{g_A} \tag{23}$$

$$\frac{- m_A^{\;2}(1 + 2\gamma_2)}{g_A} \quad a = [(m_A^{\;2} - m_\rho^{\;2})G_D - G_s]g_{\rho\pi\pi}$$

$$\frac{b(m_A^{\;2} - m_\rho^{\;2}) - 2m_A^{\;2}a(1 + 2\gamma_2)}{g_A} = -2G_s\, g_{\rho\pi\pi} \quad .$$

We notice the first two equations yield the third. This is different from what happened in Ref. 1,2,3 where making the three conditions compatible determined the ratio $\frac{G_s}{G_D}$. Substituting $a = g^2_{\rho\pi\pi}\, f_\pi$ we have

$$-2G_s\, g_{\rho\pi\pi} = \frac{b(m_A^{\;2} - m_\rho^{\;2}) - 2m_A^{\;2} g^2_{\rho\pi\pi}\, f_\pi(1 + 2\gamma_2)}{g_A} \quad . \tag{24}$$

We see if we knew G_s and G_D we could then determine b and γ_2.

V. CURRENT ALGEBRA LOW ENERGY THEOREM AND THE PION ISOVECTOR FORM FACTOR

The soft pion theorem tells us

$$\lim_{P_3 \to 0} \langle \pi_3(-P_3)\pi_4(-P_4)|A_1^{\;\mu}(0)|\pi_2(P_2)\rangle \Longrightarrow \frac{(P_2 - P_4)_\mu}{f_\pi}$$

$$F(P_1^{\;2})\;(\delta_{12}\delta_{34} - \delta_{23}\delta_{41}) \quad , \tag{25}$$

Where $F(P_1^{\;2})$ is the pion form factor.

Letting $P_3 \to 0$ we obtain

$$2(\delta_{12}\delta_{34} - \delta_{14}\delta_{23})(P_{4\mu} - P_{2\mu})P_1{}^2 A(P_1{}^2, 0, P_1{}^2) \quad . \tag{26}$$

Thus we have

$$F(P_1{}^2) = -2f_\pi P_1{}^2 \, A(P_1{}^2, 0, P_1{}^2). \tag{27}$$

Using the normalization condition $F(0) = 1$,

We obtain

$$1 = 2af_\pi \, B(\tfrac{1}{2}, \tfrac{1}{2}) \tag{28}$$

$$= 2g^2_{\rho\pi\pi} \, f_\pi{}^2 \, \pi$$

Which gives us

$$g^2_{\rho\pi\pi} = \frac{1}{2\pi \, f_\pi{}^2} \quad . \tag{28a}$$

A result obtained in a different manner by other authors.[3]

Using the integral representation for $A(t,s,q^2)$ we have

$$F(q^2) = -\frac{q^2}{\pi}\int_0^1\int_0^1 d\mu_1 d\mu_4 \mu_1^{-\frac{1}{2}}(1-\mu_1)^{-q^2-\frac{1}{2}}(1-\mu_1\mu_4)^{q^2-1}$$

$$\times\ \mu_4^{-q^2-1}(1-\mu_4)^{-\gamma_2}\ . \qquad (29)$$

From the Bateman manuscript 1.51 (12) we have

$$\int_0^1 t^{x-1}(1-t)^{y-1}(1+bt)^{-x-y}\,dt = (1+b)^{-x}\,B(x,y).$$

$$(30)$$

Therefore

$$F(q^2) = -\frac{q^2}{\pi}B(\tfrac{1}{2}, -q^2+\tfrac{1}{2})\,B(-q^2, -\gamma_2+\tfrac{1}{2}) \qquad (31)$$

$$= \frac{1}{\pi}\ \frac{\Gamma(\frac{1}{2})\,\Gamma(-q^2+\frac{1}{2})\,\Gamma(-\gamma_2+\frac{1}{2})}{\Gamma(-q^2-\gamma_2+\frac{1}{2})}\ .$$

$$\text{As } q^2\to-\infty \quad F(q^2)\sim(q^2)^{\gamma_2}\ . \qquad (32)$$

Choosing $-\gamma_2$ half integer gives us a form factor with poles along the ρ trajectory. Choosing $-\gamma_2$ a integer = n gives us an n pole form factor.

The fit of Jengo and Remiddi,[10] to the proton form factor suggests that $-\gamma_2 = \frac{5}{2}$. It is interesting to note that for $-\gamma_2$ = half integer $F(q^2)$ has zeroes at $q^2 = -\gamma_2 + \frac{1}{2}; -\gamma_2 + \frac{1}{2} + n$. If we set $-\gamma_2 = \frac{3}{2}$, the form factor obtained would be similar to that obtained if we used a field current identity for the Axial Current, and related $A^{\mu}\pi \to \pi\pi$ to the Veneziano amplitudes for $\pi\pi \to \pi\pi$, and $\pi A_1 \to \pi\pi$ by vector dominance. This is amusing in light of the fact that the latter calculation completely ignores all the poles in the Axial Current due to the A_1 daughters and therefore is internally inconsistent with the philosophy of the Veneziano model.

VI. SUMMARY

We have shown that using a mutilated five-point function for the invariant amplitudes of the matrix element of the Axial Current provides a description of that process consistent with Current algebra, Adler consistency condition, and the Veneziano model for the four-point function. It leads to a possibly more consistent framework for discussing the $A_1\rho\pi$ system and to a simple expression for the form factor.

What remains to be shown is that these mutilated five-point functions have the same spectrum and same factorization properties as the original five-point functions. If that is true, we can then extract the coupling of the vector current to the ρ daughters, and the coupling of the Axial Current to the A_1 daughters by calculating $\rho_{\text{daughter}} \to \pi\pi$ and $A_{1\ \text{daughter}} \to \pi\pi\pi$ from the five-point function.

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REFERENCES AND FOOTNOTES

* Research supported in part by Office of Naval Research

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1. F. Cooper, P. Carruthers, Phys. Rev. (to be published).

2. C. Goebel, M. Blackman, K. Wali, Phys. Rev. (to be published).

3. S. P. De Alwis, D. Nutbrown, P. Brooker, J. Kosterlitz, Phys. Letters 29B, 362 (1969).

4. H. Sugawara, Tokyo University of Education preprint, TUEP-69-4 (1969).

5. I. Ohba, Waseda University Preprint, WUP-8 (1969).

6. M. Ademollo, E. Del Giudice, Nuovo Cimento (to be published).

7. K. Bardakci and H. Ruegg, Phys. Letters 28B, 342 (1968).

8. J. Shapiro, Phys. Rev. (to be published).

9. F. Cooper, P. Carruthers, (submitted to Phys. Rev.).

10. R. Jengo and E. Remiddi, CERN Preprint TH. 1023 (1969).

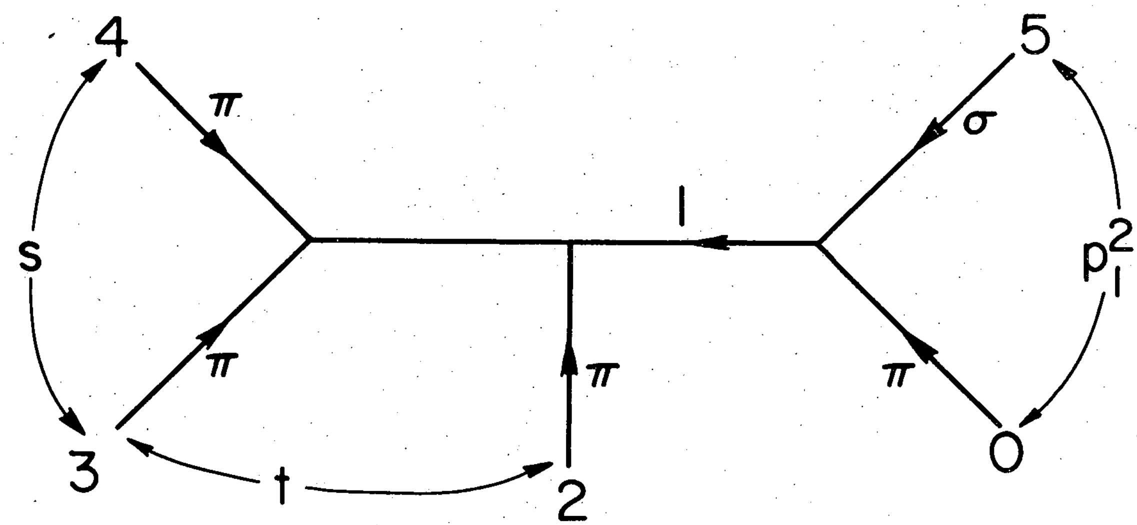

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Fig. 1.

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