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Critical Thermal Conductivity of $^4$He for $T \geq T_\lambda$

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Nonlinear renormalization-group recursion relations are used to show that the thermal conductivity of $^4$He has a crossover between a weak-coupling regime for $t_c \ll t < 1$ and a scaling regime for $t \ll t_c$, where $t = (T - T_\lambda)/T_\lambda$ and $t_c \approx 10^{-3}$. The smallness of $t_c$—an essential feature—is explained by a small bare dynamic coupling constant. The asymmetric spin model (E) provides a quantitative fit to experiment, whereas the symmetric model (F) is only semiquantitative. The borderline dimension $d^*$, below which dynamic scaling breaks down, is shown to satisfy $d^* \approx 2.54$.

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Apart from confirming the exponent predictions of dynamic scaling, the renormalization-group approach has had difficulty in quantitatively explaining the thermal conductivity of $^4$He for $T \geq T_\lambda$. Since it was recognized that the data were not in the asymptotic critical region, a number of attempts were made to account for the effect of transients. Recently Hohenberg et al., and independently Dohm and Folk, suggested that an analysis based on nonlinear recursion relations would provide a simple test of the renormalization-group approach over a wide temperature range. Dohm and Folk carried out such an analysis using the symmetric spin model (E) of Halperin et al., and found agreement with the vapor-pressure data of Ahlers for $10^{-6} < t < 10^{-3}$, claiming to determine the borderline dimension for the breakdown of dynamic scaling from the data as $d^* = 3$. We have performed a similar analysis fitting data which extend over a larger temperature range and at various pressures, and find the following results: (i) The bare dynamic coupling constant in $^4$He is a small parameter, leading to a weak-coupling regime at high temperatures ($t_c \ll t < 1$), and a crossover to the scaling regime near $T_\lambda$ ($t < t_c$) at a small reduced temperature $t_c \approx 10^{-3}$. (ii) The asymmetric model (F), which takes into account the intrinsic effect of the singular specific heat on the dynamics, provides a quantitative fit over more than four decades in reduced temperature. The symmetric model (E), on the other hand, is only semiquantitative. (iii) The effect of the weak-scaling critical fixed point, which has been invoked to explain the data, is shown to have little importance in the experimental range (in fact, we obtain excellent fits with $d^* < 2.54$).

Model (F) (Refs. 3 and 4) has a nonconserved order-parameter mode $\omega_\epsilon(k)$, and a conserved entropy mode $\omega_\eta(k)$, which couple together below $T_\lambda$ to form second sound $\omega_\epsilon(k) = (g_0^2 \rho_s k^2/C_p)^{1/2}$, where $g_0$ is the coupling constant (proportional to the entropy), $\rho_s \propto \xi^{d-4}$ is the superfluid density, and $C_p$ is the constant-pressure specific heat. Using these frequencies and setting $k = \xi^{1/3} = \xi_0^{-1/3}$, and

$$ t = t_0 e^{-1/\nu}, \tag{1} $$

we may define a dynamic coupling constant $f(l) = K_d \omega_\epsilon^2(l)/\omega_\epsilon(l) \omega_\eta(l)$ and a frequency ratio $w(l) = \omega_\epsilon(l)/\omega_\eta(l) \{t_0 \}$ is an arbitrary scale and $K_d = 2^{d-1}\pi^d/2$ $T(d/2)$. The quantities $f(l)$ and $w(l)$ satisfy recursion relations

$$ df/dl = - \beta_f(w, f), \quad dw/dl = - \beta_w(w, f), \tag{2} $$

where the functions $\beta_f$ and $\beta_w$ can be expanded in powers of $f$ with coefficients which depend on $w$ [see Eqs. (3.9) and (3.10) and (3.26)-(3.30) of Ref. 5, or Eqs. (3.1)-(3.5) of Ref. 7]. The more realistic asymmetric model (F) has a singular specific heat $C_p$ coupled dynamically to $f$ and $w$ via its effective exponent

$$ 4w = - d \ln C_p/df, \tag{3} $$

as a result of which $w(l)$ becomes complex, i.e., its imaginary part $w^*(l)$ is an additional dynamical variable. If the real part of the ratio $w(l)$ obtained by solving (2) goes to a positive constant $w^*$ as $l \rightarrow \infty (t = 0)$, then the system obeys dynamic scaling with the thermal conductivity $\lambda$ diverging as $(\xi C_p)^{1/2}$ when $T-T_\lambda$. More generally, we
define\textsuperscript{5,10} \[
\hat{R}_A(t) = \frac{\Gamma}{\rho_A (\xi C_p)}^{1/2},
\]
and find\textsuperscript{3,5,8} \[
\hat{R}_A = K_A^{1/2} \left[ w(l)/l \right]^{1/2} \left[ \frac{1 - f(l)}{4} + O(l^2) \right].
\]
If \(w^* = 0\), then the system is said to obey weak scaling\textsuperscript{4} and \(\lambda\) diverges faster than \((\xi C_p)^{1/2}\). With use of the expansion of (2) to two-loop order, it has been estimated\textsuperscript{3} that \(w^* = 0\) for \(\epsilon = 4 - d\) close to 1, i.e., that \(0 < w^* \approx 0.1\) for \(d = 3\).

We have examined\textsuperscript{11} Model \(E\) theoretically in the high-temperature limit \((t \gg t_0, \eta = \infty)\) and find a weak-coupling regime where \(f \propto e^{-d} \to 0, w \to w_0,\) and \(\lambda = \lambda_\infty > 0\). This behavior is consistent with the fits obtained by Dohm and Folk\textsuperscript{6} and with the observation of Ferrell and Bhattacharjee\textsuperscript{12} that the high-temperature form of the thermal conductivity is \(\lambda = \lambda_\infty (1 + \lambda_\infty t^{-\upsilon} + \lambda_\infty t^{-2\upsilon} + \cdots )\). This limit is physically relevant since we can make a rough estimate of the starting value of \(f\) by using data at a given "high" temperature \((t = 0.1,\) say) and find \(f(t = 0.1) = 0.16 \ll 1\). Thus the "high-temperature" region remains within the domain of validity of Model \(E\) \((t < 1)\). Furthermore, inserting the small \(f(t = 0.1)\) as an initial value in Eq. (2), we obtain a crossover from weak-coupling \((f < 1)\) to scaling \((f = 1)\) behavior at the small reduced temperature \(t_0 = 10^{-2}\), independent of the starting \(w\). Similar, but somewhat more complicated behavior is found in Model \(F\).

Detailed measurements of the singularity in the thermal conductivity were first made by Archibald, Mochel, and Weaver\textsuperscript{14} and by Ahlers.\textsuperscript{15} The cells used by Archibald, Mochel, and Weaver had very small spacings \(h\) and gave rise to an appreciable size effect in the range \(5 \times 10^{-8} \text{ cm} < h < 5 \times 10^{-7} \text{ cm}\). They are therefore not suitable for the present analysis. The measurements of Ahlers\textsuperscript{15} were made in a cell with \(h = 1 \text{ cm}\) and had a precision which was limited by large thermal relaxation times. Subsequent measurements by Ahlers\textsuperscript{8} in a cell with \(h = 0.088 \text{ cm}\) (to be referred to as cell \(D\)) had a higher precision and covered a wide pressure range in the temperature region \(3 \times 10^{-6} < t < 10^{-3}\). Extrapolating from the data of Archibald, Mochel, and Weaver,\textsuperscript{14} however, one might still expect the data in cell \(D\) to be influenced by size effects. Very recently, measurements were performed by Ahlers and Behringer\textsuperscript{16} at vapor pressure in a cell with \(h = 0.265 \text{ cm}\) (cell \(A\)) and over a wide temperature range. These new results differ somewhat from the data in cell \(D\) with the difference increasing at small \(t\). For our purposes the data for cell \(D\) do not extend to sufficiently large \(t\), and so we have generated values in the range \(10^{-3} < t < 10^{-2}\), using the high-\(t\) extrapolation\textsuperscript{12} of \(\lambda\) noted above. As described in detail elsewhere,\textsuperscript{11} we believe that our procedure yields a good representation of the thermal conductivity in cell \(D\), but in any case we have carried out all our analyses on both sets of data (cells \(A\) and \(D\)) and our qualitative conclusions do not depend on which set we use. Moreover, further experimental work is clearly necessary in order to elucidate the nature of the size dependence.\textsuperscript{9,14,16}

Model \(E\) was tested by integrating the nonlinear recursion relations (2) numerically, adjusting the initial values \(f_0\) and \(w_0\) at \(t = t_0\) to obtain the best fit of \(\hat{R}_A(t)\) to experimental values obtained from Eq. (4). The corresponding fit for Model \(F\) [Eqs. (B.11)–(B.15) of Ref. 7] involves the additional parameter \(w_0''\) and values of \(\nu\) [Eq. (3)] extracted from specific-heat measurements. Figure 1

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{The amplitude \(\hat{R}_A\) of the singular thermal conductivity as a function of reduced temperature obtained from fits in the range \(0.003 < t < 0.01\) (between the vertical arrows), with use of (a) Model \(E\) and (b) Model \(F\). The points are experimental values in cell \(A\). Also shown are the functions \(f, w = w',\) and \(w''\) obtained from the fits.}
\end{figure}
shows fits obtained using data in the restricted range $0.003 < t < 0.01$. Also shown are the functions $f(t)$ and $w(t) = w' + iw''$ obtained from these fits, as well as $\tilde{R}_R(t)$ over a wider range. From Fig. 1(a), it is seen that Model E shows perceptible deviations from the data in cell A for both large and small $t$. This is particularly significant since the deviations appear in a region where $f$ is small and the loop expansion is a controlled approximation. Model F, on the other hand (Fig. 1(b)), only deviates for $t < 3 \times 10^{-4}$, where $f \approx 1$ and corrections to the loop expansion cannot be neglected. Model-E fits of $\tilde{R}_R(t)$ to data in the broader ranges $10^{-6} < t < 10^{-2}$ and $10^{-6} < t < 10^{-3}$ are shown in Fig. 2(a), and again the deviations are apparent. These discrepancies lead us to consider as unreliable the conclusion of Dohm and Folk (based on a Model-E fit in the range $10^{-6} < t < 10^{-3}$), that dynamic scaling breaks down at $d^* = 3$.

As a means of assessing the influence of the truncation of the loop expansion on the physical predictions of the theory, we have added a phenomenological three-loop term $w B_3 f^3$ to the function $\beta_w$ in (2) and have treated $B_3$ as an additional adjustable parameter in the fit. The results of a four-parameter least-squares fit to Model F are shown in Fig. 2(b) for the data of both cell A and cell D. Note that a small value of $B_3$ yields an excellent fit in each case ($B_3 = 0.192$ for cell A and 0.162 for cell D). In contrast, the addition of $B_3$ to Model E does not significantly improve the fit in Fig. 2(a). The equations with parameters fitted in the range $t < t_{\text{max}} = 0.01$ may be used to calculate $\tilde{R}_R(t)$ for $t > t_{\text{max}}$. The results for Model F agree with experiment out to $t = 0.04$, and only show appreciable deviations for $t \approx 0.1$, where the model is no longer expected to be quantitative (in particular the squared vertex $v$ extracted from experiment via Eq. (3) becomes negative at $t = 0.2$). Model E, on the other hand, has deviations immediately above $t = 0.01$. We have also used Model F to analyze the finite-pressure data in cell D, fixing $B_3$ at its vapor-pressure value, and have found fits of comparable quality to those in Fig. 2(b). Having used the data to estimate the contribution of higher-loop terms to Eq. (2) in Model F, we have determined the fixed-point values $w' = 0.249$ (cell A) and $w' = 0.189$ (cell D) for $d = 3$. For $B_3 = 0.162$ (cell D) weak scaling ($w* = 0$) sets in at $d^* = 2.54$, whereas for $B_3 = 0.192$ (cell A) $w*$ remains positive for all $d$. In view of the uncertainties in the data for $t < 10^{-3}$ and of the ad hoc nature of the three-loop term, these estimates are rather uncertain; in any case, for $d = 3$, $w'$ remains larger than 0.3 throughout the experimental range for both sets of data, indicating further that the weak-scaling fixed point (even if it is the stable one in the exact theory) has little effect on the data. A similar conclusion holds for the Model-E fits.

The present analysis thus shows that Model F provides a fully quantitative description of the thermal conductivity over a range of $t$ where the theory makes no uncontrolled approximations. This range can be extended phenomenologically to cover more than four decades in reduced temperature. The ensuing fits provide one of the best demonstrations to date of the quantitative power of renormalization-group theory.

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Propagation of Sound in a Spin-Glass

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Sound propagation and damping are studied near the spin-glass transition for a time-dependent Landau-Ginzburg model coupled to phonons. The sound speed decreases linearly with $T-T_{c}$, and the sound damping diverges as $(T-T_{c})^{-1}$, when the spin-glass transition, $T_{c}$, is approached from high temperatures. At the spin-glass transition, the sound speed has a term proportional to $\omega^{1/2}$, and the sound damping is proportional to $\omega^{-1/2}$.

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The spin-glass (SG) phase was proposed by Edwards and Anderson (E-A)\textsuperscript{1} to explain the low-temperature properties of some amorphous magnetic alloys. According to E-A, the phase is characterized by a nonvanishing value of the different time correlation of a spin at a given site so that the spin may be visualized as being frozen in time. In view of this picture, the dynamics of spins are of considerable interest in understanding the phase transition. Several authors\textsuperscript{2–8} have studied spin relaxation and damping near the spin-glass freezing temperature, $T_{c}$. In this Letter, we study propagation of sound with a view to investigating spin dynamics in and just above the spin-glass phase, and we report results on the propagation velocity and damping of sound for a simple model. Our result for the temperature dependence when the SG transition is approached from above $T_{c}$ agrees with the observed decrease in the sound speed.\textsuperscript{6,7} There are no systematic data for the frequency dependence of the speed, nor on the sound damping; we encourage work to test our prediction.

We study a time-dependent Ginzburg–Landau (TDGL)\textsuperscript{9,10} model. If $\sigma(x)$ is the local spin density and $\varphi(x)$ a longitudinal phonon field, then the...