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In a preceding paper[1] the author has shown that the assumption of SU3 invariance for strong interactions[2,3] allows a general understanding of weak leptonic processes. In this letter we extend to nonleptonic processes the concepts presented in Ref. 1.

An interesting result of this extension is a qualitative understanding of the ratio between the rates of $K^0 \to 2\pi$ and $K^+ \to 2\pi$ without the introduction of $\Delta I = 3/2$ interactions of nonelectromagnetic origin.

The weak current responsible for leptonic processes is assumed[1] to be a member of an octet[4]

$$J_\mu = \cos\theta(j_\mu^{(0)} + g_\mu^{(0)}) + \sin\theta(j_\mu^{(1)} + g_\mu^{(1)}). \tag{1}$$

In addition to the usual frame of reference, F , in the octet space, it is interesting to consider a new frame, F' . This frame is obtained from F through a rotation U , belonging to SU3 , such that it transforms J_μ into the $\Delta S = 0, \Delta Q = 1$ member of the octet

$$U I_\mu U^{-1} = j_\mu^0 + g_\mu^0 , \tag{2}$$

where

$$U = \exp\left[2i\theta F^7\right] . \tag{3}$$

The basic members of the octet in the new frame are linear combinations of the old ones; in the case of pseudoscalar mesons, for example, we have

$$\pi'^{\pm} = \cos\theta\, \pi^{\pm} + \sin\theta\, K^{\pm}$$

$$K'^{\pm} = -\sin\theta\, \pi^{\pm} + \cos\theta\, K^{\pm}$$

$$K'^{0} = 1/2 \left\{ \cos(2\theta)\, (\overline{K}^{0} + K^{0}) - (\overline{K}^{0} - K^{0}) + \sin(2\theta)\, (\pi^{0} - \sqrt{3}\, \eta) \right\} \qquad (4)$$

$$\pi'^{0} = 1/4 \left\{ \pi^{0} \left[3 + \cos(2\theta) \right] + \sqrt{3}\, \eta \left[1 - \cos(2\theta) \right] - \sqrt{2}\, \sin(2\theta)\, (K^{0} + \overline{K}^{0}) \right\}$$

and

$$\eta' = 1/4 \left\{ \eta \left[1 + 3\cos(2\theta) \right] + \sqrt{3}\, \pi^{0} \left[1 - \cos(2\theta) \right] + (6)^{1/2} \sin(2\theta)(K^{0} + \overline{K}^{0}) \right\}$$

In frame F' , we have a strangeness quantum number, S' , which is conserved in leptonic processes. If SU3 were exact (all baryons degenerate, and similarly for the other multiplets), F' would be the natural frame of reference--the one in which leptonic decays conserve strangeness. Apparently strangeness violation is more to be blamed on SU3 breaking than on weak interactions.

It seems desirable to extend this property to nonleptonic processes, and require also that

(a) <u>nonleptonic processes conserve S' , the strangeness in frame F'</u>. We can therefore build the Lagrangian $\mathcal{L}_W$ as strangeness-conserving in frame F' , and obtain the Lagrangian for the physical processes by substitution (4).[5]

How does the Lagrangian transform under SU3? <u>A priori</u>, any representation that appears in the decomposition of $8 \otimes 8 \otimes 8$ will do, i. e., 1, 8, 10, $\overline{10}$, 27, 35, $\overline{35}$, 64.[6] Now the singlet does not accommodate any $\Delta S \neq 0$, while the higher representations contribute $\Delta S \geq 2$ and(or) $\Delta I \geq 3/2$ when we go from frame F' to the physical frame F. Therefore, if we require

(b) <u>no $\Delta S \geq 2$ or $\Delta I \geq 3/2$ in the physical frame F</u>,

we must conclude that

(c) <u>the Lagrangian for nonleptonic processes behaves as a member of an octet.</u>[7]

We have then two possibilities: in frame F' in which, according to (a), $\mathcal{L}_W$ conserves strangeness, it can behave either as the hypercharge Y',[8] or the third component of the isotopic spin, I_3'. For physical processes with $\Delta S = 1$, there is no difference between the two possibilities.

Hypotheses (a) and (c) have a very interesting implication on the $K \to 2\pi$ interaction, if combined with CP invariance.[9] These combined conditions forbid the 2π decay of a neutral K. This could appear shocking, but is in fact extremely gratifying. In fact one of the more serious pieces of evidence[10] against the validity of the $\Delta I = 1/2$ rule is the value of the ratio of the rates for $K_1^o \to 2\pi$ and $K^+ \to \pi^+\pi^o$

$$R = (K_1^o \to 2\pi)\Big/(K^+ \to \pi^+\pi^o) \approx 700 \quad \text{(experimental).} \qquad (5)$$

If $K^+ \to \pi^+\pi^o$ (a $\Delta I = 3/2$ transition) is of electromagnetic origin, we expect it to be depressed by a larger factor, say $(137)^2$. However, if $K^o \to 2\pi$ is forbidden by SU3, we expect it to be supressed too, by some factor of the order of 10 to 20, and the experimental value of R can be qualitatively understood without the introduction of $\Delta I = 3/2$ interactions of nonelectromagnetic origin.

The easiest way to see that $K_1^o \to 2\pi$ is forbidden is to exploit hypothesis (a) and try to build a pseudoscalar, strangeness-conserving interaction among three pseudoscalar mesons in frame F'. We can build the following isoscalars

$$(\bar{K}'K')\,\eta' \ , \ (\bar{K}'\underline{\tau}K')\,\underline{\pi}' \ , \ (\underline{\pi}'\cdot\underline{\pi}')\,\eta' \ , \ \eta'\eta'\eta' \ ; \tag{6}$$

they all have CP = -1. So we cannot build a $K\pi\pi$ interaction that transforms like Y' and does not violate CP. Clearly, we also cannot build an interaction that transforms like I_3' (and therefore belongs to an octet) and conserves CP, because this should have a unitary partner that transforms like Y'.

Since the experimental rate for $K_1^o \to 2\pi$ is quite high, it might seem difficult to conceive of this mode as suppressed. In fact it is very difficult to say what this rate should be. If we introduce a phenomenological coupling

$$M^3\, G\, K_1^o\, (\underline{\pi}\cdot\underline{\pi}), \tag{7}$$

where G is the Fermi coupling constant and M, an adjustable mass parameter. The rate depends on the sixth power of M. This makes any prediction completely unreliable. However, if we try to extract M from the experimental value for the lifetime of K_1, we get $M \sim 1.5\ m_\pi$ which is a rather low value.

As has been discussed by Lee, not much can be said on hyperon decays without resorting to further restrictions, such as R invariance.[8] However, we feel that the success of the theory in explaining, at least qualitatively, the $(K^o \to 2\pi)$-$(K^+ \to 2\pi)$ puzzle is very encouraging.

FOOTNOTES AND REFERENCES

* Work done under the auspices of the U. S. Atomic Energy Commission.

† On leave from Laboratori Nazionali del C. N. E. N., Frascati, Italy.

1. N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
2. Y. Ne'eman, Nucl. Phys. 26, 222 (1961).
3. M. Gell-Mann, California Institute of Technology Report CTSL-20 (1961), unpublished, and Phys. Rev. 125, 1067 (1962).
4. We remember that the superscripts in Eq. (1) refer to the change of strangeness. In the usual notation (that of Ref. 3.) we have

   $$j_\mu^{(0)} = j_\mu^{1} + i j_\mu^{2}, \quad j_\mu^{(1)} = j_\mu^{4} + i j_\mu^{5}.$$

5. We speak of Lagrangians in a phenomenological way, and do not enter the question of possible intermediate bosons.
6. If the Lagrangian is thought as arising from current-current interactions, and the currents belong to the octet representation, then the possible choice of a representation is restricted to 1, 8, or 27.
7. B. W. Lee, to be published, and S. Coleman and S. L. ~~Coleman~~ Glashow, to be published, have also considered this possibility.
8. In this case $\mathcal{L}_W$ is invariant under an SU2 subgroup of SU3 generated by the isotopic-spin operators in frame F', I'. This interesting possibility has been proposed by B. d'Espagnat and J. Prentky, Nuovo Cimento 24, 497 (1962) and by M. Baker and S. L. Glashow, Nuovo Cimento 26, 803 (1962).
9. The CP transformation must be defined in such a way that it leaves unchanged the structure of the Lie-group algebra of SU3. If the commutation relations among the generators, $[F^i, F^l] = i f^{ilm} F^m$, are to be unchanged by CP, assumed to operate on the F as

$CP\, F^{i} (CP)^{-1} = \epsilon^{i} F^{i}$, the coefficients $\epsilon^{i} = \pm 1$ are restricted by the relations $(\epsilon^{i} \epsilon^{\ell} \epsilon^{m} - 1) f^{ilm} = 0$. Among the many solutions we select $\epsilon^{1} = \epsilon^{4} = \epsilon^{7} = +1$, $\epsilon^{2} = \epsilon^{3} = \epsilon^{5} = \epsilon^{6} = \epsilon^{8} = -1$. For the octet of PS mesons we then have $(CP)\, \pi^{i} (CP)^{-1} = \epsilon^{i} \pi^{i}$. Note that we have selected a solution in which F^{7} has $CP = 1$, so that U conserves the CP character of the Lagrangian.

10. Perhaps the only serious evidence; see for example the report by R. H. Dalitz at the Brookhaven Conference on Weak Interactions, September 1963, to be published.