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Author
Goh, Hock-Seng

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A Little Twin Higgs Model*

Hock-Seng Goh[1,2] and Christopher A. Krenke[1]

[1]Department of Physics, University of Arizona, Tucson, AZ 85721
[2]Lawrence Berkeley National Laboratory, Berkeley, CA 94720

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A Little Twin Higgs Model

Hock-Seng Goh and Christopher A. Krenke

Department of Physics, University of Arizona, Tucson, AZ 85721

We present a twin Higgs model based on left-right symmetry with a tree level quartic. This is made possible by extending the symmetry of the model to include two Z_2 parities, each of which is sufficient to protect the Higgs from getting a quadratically divergent mass squared. Although both parities are broken explicitly, the symmetries that protect the Higgs from getting a quadratically divergent mass are broken only collectively. The quadratic divergences of the Higgs mass are thus still protected at one loop. We find that the fine tuning in this model is reduced substantially compared to the original left-right twin Higgs model. This mechanism can also be applied to the mirror twin Higgs model to get a significant reduction of the fine tuning, while keeping the mirror photon massless.

I. INTRODUCTION

The standard model (SM) is so far the most successful theory that describes physics at energies below the TeV scale. Its predictions are consistent with all precision electroweak measurements. However, the model is unsatisfactory since the Higgs field, which plays a crucial role in electroweak symmetry breaking, receives quadratically divergent radiative corrections to its mass and thus destabilizes the electroweak scale. Hence, it is unnatural to treat the SM as an effective theory with a cutoff scale much higher than a TeV. On the other hand, the cutoffs of nonrenormalizable operators that contribute to precision electroweak measurements are required by experiment to be greater than 5-10 TeV. Such a high cutoff tends to destabilize the electroweak scale and leads to a fine tuning of a few %. This problem is known as the little hierarchy problem or the LEP paradox [1].

The idea that the Higgs is a pseudo-Nambu-Goldstone boson (PNGB) corresponding to a spontaneously broken global symmetry was proposed in refs. [2, 3]. Since the mass of a PNGB tends to be lighter than the UV scale, this idea explains why the Higgs is light. However, using this idea to solve the little hierarchy problem is not quite straightforward. A PNGB Higgs by itself is not sufficient since the global symmetry is, by definition, not exact and the couplings that break the global symmetry will still generate a quadratically divergent mass to the Higgs. Thus, the situation is no better than that in the standard model and more structure is needed. The extra structure required to achieve naturalness is the main challenge for model building. One successful mechanism along this line is known as the little Higgs [4, 5]. In this class of models, the Higgs mass is protected by two separate global symmetries and every term in the Lagrangian breaks at most one of them. In order to break both global symmetries, radiative corrections to the mass have to involve at least two such terms and thus, quadratic divergences are postponed to two loops. This little Higgs mechanism is also known as collective symmetry breaking. To achieve a certain level of naturalness, a special operator is also introduced to provide a tree level quartic without generating a tree level mass to the Higgs.

Another mechanism that has been shown to solve the little hierarchy problem is the twin Higgs [6, 7, 8, 9] (see also [10, 11, 12]). The twin Higgs mechanism is quite different from that of the little Higgs. In twin Higgs models, the Higgs mass is protected by a discrete Z_2, or twin, symmetry instead of multiple global symmetries. The exact twin symmetry guarantees that all gauge invariant dimensionful terms have, up to all orders in perturbation theory, a form which is invariant under a global SU(4) symmetry. The mass of the PNGB Higgs is then protected from receiving quadratically divergent contributions. It was shown that this mechanism alleviates the little hierarchy problem to about the 10% level for the cut off scale $\Lambda = 10$ TeV without introducing a tree level quartic.

In this class of models where the quadratic divergences are naturally suppressed, one would expect less fine-tuning if the quartic coupling of Higgs λ is large. In the original twin Higgs models, both the squared mass and the quartic for the SM Higgs come from the one-loop Colemann-Weinberg (CW) potential [13]. The quartic coupling is thus not a free parameter and loop suppressed. In order to improve the naturalness, one should try to find a tree level operator that will give the PNGB a quartic coupling without giving it a tree level mass term. In order to not upset the cancellation of radiative corrections, the tree level operator one introduces must preserve the twin parity. To summarize, in order to improve the fine tuning, the following criteria must be satisfied.

- A tree level operator that generates a quartic for the SM Higgs, but not a mass.

- This operator must preserve the discrete symmetry that protects the Higgs mass.

- Reduce as much as possible the mass squared that arise at loop level. For example, reduce top contribution by making the top Yukawa interaction SU(4) invariant.

One very simple operator which satisfies the first criterion has been constructed and is used in the twin Higgs model [8]. The basic idea is a mismatched alignment

of two vevs. It was shown in ref. [8] that the mirror twin Higgs model [6] improves when this type of tree level quartic is added. The mismatched alignment of the vevs necessarily breaks the mirror $SU(2) \times U(1)$ gauge symmetry to nothing and so the mirror photon becomes massive. Because of this feature, the mechanism seems difficult to implement in the left-right twin Higgs model [9] since the mismatched vev alignment would break $U(1)_{EM}$ and the SM photon would become massive.

However, there is actually more than one type of parity which can be identified as a twin parity, i.e. the original twin parity, known also as **P**, and charge conjugation, **C**[14]. Under these parities, scalar and Dirac fermion in the left-right model transform as

$$\mathbf{P} : \left\{ \begin{array}{ccc} H_L & \to & H_R \\ Q_L & \to & Q_R \end{array} \right\} \tag{1}$$

$$\mathbf{C} : \left\{ \begin{array}{ccc} H_L & \to & H_R^* \\ Q_L & \to & C\bar{Q}_R^T \end{array} \right\} \tag{2}$$

In this paper, we show that by using this fact and the idea of collective symmetry breaking, a new type of quartic operator can be constructed. This new quartic has all the properties we mentioned above, but does not break $U(1)_{EM}$. Most importantly, it preserves one of the parities that will maintain the cancellation of quadratic divergences to one loop. The quadratic divergences are no longer protected to all orders in perturbation theory as in the original twin Higgs model. However, cancellation to one loop is sufficient to address the little hierarchy problem.

The paper is organized as follows: In section II, we review the twin Higgs mechanism and the left-right twin Higgs model. We then explore the possibilities in introducing a tree level quartic and extend the top sector, making it $SU(4)$ invariant. In section III, we analyze the radiative corrections and electroweak symmetry breaking. We then apply the same mechanism to the mirror model and reanalyze its naturalness in section IV. In section V, some phenomenology is discussed and our results summarized.

II. CONSTRUCTION OF THE MODEL

The scalar field H in twin Higgs models is in the fundamental representation of a $U(4)$ global symmetry. After acquiring a vev, $\langle H \rangle = (0, 0, 0, f)$, $U(4)$ is broken to $U(3)$, which yields 7 Goldstone bosons including the standard model (SM) Higgs doublet $h = (h_1, h_2)$. The global symmetry is explicitly broken by gauging only a subgroup $SU(2)_A \times SU(2)_B$ (we ignore $U(1)$ factors here since they are not relevant to present discussion). Under this gauge symmetry, H can be represented by $H = (H_A, H_B)$ where $H_{A,B}$ transform as doublets of $SU(2)_{A,B}$. Since the global symmetry is broken explicitly by the gauge couplings and the breaking is 'hard', masses

of the Goldstone bosons will be radiatively generated and be quadratically divergent. However, by imposing the discrete symmetry (twin parity) that interchanges the two gauged $SU(2)$ symmetries, the quadratic divergences cancel. The simplest way to understand this is the following. First write down the most general gauge invariant mass terms for the linear fields H_A and H_B

$$\alpha_A H_A^\dagger H_A + \alpha_B H_B^\dagger H_B, \tag{3}$$

where $\alpha_{A,B}$ are not required to be related by the gauge symmetry. After imposing the twin symmetry on all the interactions, however, α_A is forced to be equal to α_B and so the form given above is invariant under the global $U(4)$ transformation. Therefore, this term, which is quadratically divergent, does not contribute to potential of the Goldstone bosons. Higher order terms, the quartic term $(|H_A|^4 + |H_B|^4)$ for example, can contribute even though they preserve the twin symmetry since twin symmetry does not require these terms to have a $U(4)$ invariant form. These contributions can have at most logarithmic divergences and so are under theoretical control. Additional interactions such as Yukawa couplings can be added to the theory consistent with the discrete twin symmetry, and the argument above shows that they do not lead to quadratic divergences.

The fine-tuning in twin Higgs theories can be further reduced if there are terms in the Lagrangian which respect the twin symmetry and contribute to the quartic self-coupling of the light pseudo-Goldstone Higgs but not to its mass. In the case of the model discussed above, with a single Higgs field H, there are no such operators consistent with the symmetries of the theory. However, such terms can be written down in theories with more than one set of Higgs fields. We consider the theory with an extra scalar field $\hat{H}$, which has its vev residing in a different direction, $\langle \hat{H} \rangle = (0, 0, \hat{f}, 0)$[8]. After the global $U(4)$ symmetry is spontaneously broken by f and $\hat{f}$, and the massive radial modes are integrated out, we can write down a non-linear sigma model which contains the interactions of the light degrees of freedom. The light fields of the non-linear sigma model can be parametrized as

$$H = \begin{pmatrix} h_1 \\ h_2 \\ C \\ f + i\phi - \frac{h^\dagger h}{2f} \end{pmatrix} + \cdots$$

$$\hat{H} = \begin{pmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{f} + i\hat{\phi} - \frac{\hat{h}^\dagger \hat{h}}{2\hat{f}} \\ \hat{C} \end{pmatrix} + \cdots \tag{4}$$

Notice that a quartic term like $|H|^4$ would give a mass term to the Goldstone boson h because H contains a component $\sim (f - h^2/2f + ...)$. The quartic operator $|H|^4$ therefore contains a term like $(f - h^2/2f + ...)^4$, which

gives a mass term for h. This is why the second Higgs field $\hat{H}$ is required. With the mismatched alignment of vevs as in eq. (4), the operator $|H^\dagger \hat{H}|^2$ gives mass only to C and $\hat{C}$, and gives rise to a quartic term for h and $\hat{h}$ without a corresponding mass term.

The above discussion is general for twin Higgs models. The phenomenological consequences of the additional vev $\hat{f}$, however, depend on the model's U(1) structure. In the mirror twin Higgs model, the gauged subgroup of global U(4) is $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$. Two identical electroweak gauge symmetries are introduced to two sectors of the model. Sector A is identified with the standard model and sector B is a mirror world of the standard model. An extra scalar multiplet $\hat{H} = (\hat{H}_A, \hat{H}_B)$ is added to the model in order to implement the above mechanism [8]. H_B and $\hat{H}_B$ are both singlets under $SU(2)_A \times U(1)_A$ and have the same nontrivial charge under $SU(2)_B \times U(1)_B$. The mismatched vevs thus break the mirror $SU(2)_B \times U(1)_B$ gauge symmetry to nothing but preserve the entire standard model $SU(2)_A \times U(1)_A$. The mirror photon therefore becomes massive, in contrast to the case in the original mirror twin Higgs model where the mirror photon remains massless after U(4) symmetry is broken.

In the left-right twin Higgs (LRTH) model, the gauged subgroup is that of the left-right model: $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \mathbf{P}$[15]. There are two Higgs fields, $H = (H_L, H_R)$ and $\hat{H} = (\hat{H}_L, \hat{H}_R)$, both of which transform as a fundamental representation under the SU(4) global symmetry. Under the gauge symmetry, these scalars transform as

$$H_L \text{ and } \hat{H}_L : (\mathbf{2}, \mathbf{1}, \mathbf{1}), \quad H_R \text{ and } \hat{H}_R : (\mathbf{1}, \mathbf{2}, \mathbf{1}) \quad (5)$$

In this model, the scalar fields acquire the vevs, $\langle H_R \rangle = (0, f)$ and $\langle \hat{H}_R \rangle = (0, \hat{f})$, which break the SU(4) global symmetry as well as the gauge symmetry $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$ hypercharge. Without introducing any extra scalar fields, can we apply the mismatched mechanism to this model to obtain a tree-level quartic coupling to the pseudo Goldstone Higgs? The previous discussion seems to suggest that we need to change the vev of $\hat{H}_R$ to $\langle \hat{H}_R \rangle = (\hat{f}, 0)$. These new vevs would break $U(1)_Y$ and hence, $U(1)_{EM}$. Therefore, this mechanism can not be applied to the left-right twin Higgs model in its simplest form. The question we would like to answer is whether there exists a different operator or a certain assignment of charges that achieves the same goal, while leaving $U(1)_{EM}$ unbroken.

A. Quartic for the left-right model

The charge assignment for H and $\hat{H}$ given in eq. (5) is unique. All other charge assignments which are consistent with the symmetry breaking $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to U(1)_{EM}$ and preserve the left-right symmetry are, up to a set of field redefinitions, equivalent

to this assignment. With this charge assignment, the vevs that preserve the hypercharge $U(1)_Y$ is the one given in the original LRTH model: $\langle H \rangle = (0, 0, 0, f)$ and $\langle \hat{H} \rangle = (0, 0, 0, \hat{f})$. In order to have a tree level quartic, we add to the LRTH model the following terms that break the global SU(4) symmetry

$$\Delta V = \lambda(|(H_R^T \tau_2 \hat{H}_R)|^2 + |(H_L^\dagger \hat{H}_L)|^2). \quad (6)$$

These two terms are not symmetric under the twin defined originally in the LRTH model

$$\begin{aligned} H_L &\leftrightarrow H_R \\ \hat{H}_L &\leftrightarrow \hat{H}_R, \end{aligned} \quad (7)$$

where the gauge and matter fields transform as

$$\begin{aligned} A_{L\mu}^a T_L^a &\to A_{R\mu}^a T_R^a \\ A_{B-L} &\to A_{B-L} \\ Q_L &\to Q_R^c, \end{aligned} \quad (8)$$

in two-component Weyl notation. However, one can define an alternative twin parity

$$\begin{aligned} H_L &\leftrightarrow H_R \\ \hat{H}_L &\leftrightarrow \tau_2 \hat{H}_R^*, \\ A_{L\mu}^a T_L^a &\to A_{R\mu}^a T_R^a \\ Q_L &\to Q_R^c, \end{aligned} \quad (9)$$

It can be shown explicitly that the quartic terms given in eq. (6) preserve the Z_2 symmetry given in eq. (9), which is as powerful as the original twin parity in protecting the Higgs mass from receiving quadratically divergent corrections. All interactions in this model except the $U(1)_{B-L}$ gauge interaction and the new quartic potential we introduced in eq. (6) preserve both of the parities given above. The quartic potential breaks the first parity and the $U(1)_{B-L}$ breaks the second.

Since every term in this extended LRTH model breaks no more than one parity defined in eqs. (7,8) and eq. (9), quadratically divergent masses of the PNGB can only be generated when both parities are broken collectively. The quadratically divergent contributions to the PNGB masses are generally expected to arise at two loop. However, a more detailed analysis shows that two-loop contributions are also absent, and that contributions begin at three loops. We have thus succeeded in constructing a tree level quartic without generating a large mass term for the Higgs.

B. SU(4) invariant top Yukawa interaction

Since precision measurements prefer a light Higgs, $m_h < 200$ GeV [16], a tree level quartic by itself is not as useful as one might hope in addressing the LEP paradox. In order to have a complete solution to the

problem, a further suppression of Higgs mass parameter is desirable. An obvious way to achieve this is to extend the top sector to include a $U(4)$ invariant Yukawa and terms that only break the global symmetry softly. Then, the Higgs potential will receive only a finite contribution from the top sector [6].

The top sector in the original LRTH model contains $Q_{L,R}$ and $T_{L,R}$ charged under $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as

$$Q_L = (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/3) \quad Q_R = (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{2}}, -1/3)$$
$$T_L = (\mathbf{3}, \mathbf{1}, \mathbf{1}, 4/3) \quad T_R = (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, -4/3), \quad (10)$$

where we are using two-component Weyl notation. The gauge invariant top Yukawa terms can then be written down as

$$y(H_R^\dagger \tau_2 Q_R T_L + H_L^T \tau_2 Q_L T_R). \quad (11)$$

Without introducing any more extra fields, all other quarks and charged leptons can get their masses from non-renormalizable operators like

$$\frac{y_u}{\Lambda}(H_R^\dagger \tau_2 Q_R)(H_L^T \tau_2 Q_L) + \frac{y_d}{\Lambda}(H_R^T Q_R)(H_L^\dagger Q_L) \quad (12)$$

Due to the smallness of the Yukawa couplings, these non-renormalizable operators will not affect our discussion later of fine tuning. Even after we have modified the Higgs sector by adding a new quartic term eq. (6), the charges of the Higgses and their vevs remain the same as that were defined originally and thus these operators remain valid to give masses to light fermions. We will ignore these operators for the rest of this paper.

Notice that neither Q_R nor Q_R^c, the complex conjugate of Q_R, can be combined with Q_L to form an SU(4) multiplet due to the different charges under the gauge or Lorentz groups. To complete the SU(4) representation, we need to introduce two extra vector-like quarks

$$\Phi_R = (\mathbf{3}, \mathbf{1}, \mathbf{2}, 1/3) \quad \Phi_L = (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1}, -1/3)$$
$$\bar{\Phi}_R = (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{2}, -1/3) \quad \bar{\Phi}_L = (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/3).$$

Q_L and Φ_R form a $\mathbf{4}$ representation of SU(4) and similarly for Φ_L and Q_R. The top Yukawa term then becomes

$$\begin{aligned}\mathcal{L}_{top} =&\ y(H_L^T \tau_2 \Phi_L + H_R^T \tau_2 Q_R)T_L \\ &+ (H_L^T \tau_2 Q_L + H_R^T \tau_2 \Phi_R)T_R + h.c.. \quad (13)\end{aligned}$$

We also add the following soft masses to decouple the extra vector-like quarks,

$$M_R \bar{\Phi}_R \Phi_R + M_L \bar{\Phi}_L \Phi_L + M_0 T_L T_R + h.c.. \quad (14)$$

For simplicity, we set $M_0 = 0$ in the analysis below.

III. RADIATIVE CORRECTIONS AND EW SYMMETRY BREAKING

In this section we determine the radiative corrections to the pseudo-Goldstone mass and verify that electroweak symmetry is indeed broken by a light Higgs. In particular, we will compute the CW potential [13] for the light fields, as given by

$$V = \pm \sum_i \frac{1}{64\pi^2} M_i^4 (\ln \frac{\Lambda^2}{M_i^2} + \frac{3}{2}), \quad (15)$$

where the sum is over all degrees of freedom. The sign is positive for fermions and negative for bosons. At one loop the Yukawa couplings, gauge couplings and Higgs self-couplings all contribute separately to the sum, simplifying the calculation. For simplicity, we will work in the context of a model where the symmetry breaking pattern is realized linearly, by the terms

$$\eta(|H|^2 - f^2)^2 + \hat{\eta}(|\hat{H}|^2 - \hat{f}^2)^2. \quad (16)$$

We begin by considering the loop contributions from the self-couplings of the scalar fields. Obviously, there can be no η or η^2 contribution to the potential of Goldstone bosons since all vertices in the relevant diagrams preserve $SU(4)$. Hence, these diagrams will only correct η, a free parameter. Also, to one-loop, the diagrams with one mismatched quartic and one SU(4) invariant quartic ($\eta\lambda$ contribution) will only generate corrections to η and λ, both free parameters. This can be understood by the observation that

$$\lambda(|(H_R^T \tau_2 \hat{H}_R)|^2 + |(H_L^\dagger \hat{H}_L)|^2) \quad (17)$$
$$= \lambda|H_L^\dagger \hat{H}_L + H_R^\dagger i\tau_2 \hat{H}_R^*|^2 + \lambda|H_L^\dagger \hat{H}_L - H_R^\dagger i\tau_2 \hat{H}_R^*|^2.$$

The first operator is invariant under an SU(4), which is also preserved by η, if we arrange $\hat{H} = (\hat{H}_L, i\tau_2\hat{H}_R)$. The same holds for the second if we arrange $\hat{H} = (\hat{H}_L, -i\tau_2\hat{H}_R)$. At one loop, the four-point diagrams that include the SU(4) invariant quartic can only include one of these operators and thus are invariant under the corresponding SU(4). Hence, the combination of the operators above will only correct the tree level parameters η and λ. Therefore, when computing the one loop radiative corrections to quartic terms in the Higgs potential, we can ignore the SU(4) invariant term given in eq. (16).

The effective potential may however contain operators of higher dimensionality involving η arising at one loop, but these operators will make only a finite contribution to the potential of the pseudo-Goldstone bosons. We will therefore neglect this contribution in our analysis. As mentioned in the previous section, new quadratic contributions could arise from the combination of the quartics above and the $U(1)$ gauge coupling at the three loop level, which we will also ignore.

The vev that preserves $U(1)_{EM}$ can be written as

$$\langle H \rangle = f \begin{pmatrix} 0 \\ i\sin x \\ 0 \\ \cos x \end{pmatrix} \quad \langle \hat{H} \rangle = \hat{f} \begin{pmatrix} 0 \\ i\sin \hat{x} \\ 0 \\ \cos \hat{x} \end{pmatrix} \quad (18)$$

Expanding the tree level Higgs potential given in eq. (6) and keeping only the mass terms we find

$$\lambda \ \{ |\hat{f} \cos \hat{x} H_{R1} - f \cos x \hat{H}_{R1}|^2$$
$$+ \ |f \sin x \hat{H}_{L2} - \hat{f} \sin \hat{x} H_{L2}^*|^2$$
$$+ \ f\hat{f} \sin x \sin \hat{x} (H_L^\dagger \hat{H}_L + h.c.) \}. \tag{19}$$

For the right-handed fields, obviously three of them are massless and the last one has mass squared $\lambda(\hat{f}^2 \cos^2 \hat{x} + f^2 \cos^2 x)$. For the left-handed fields, the eigenvalues are $\pm \lambda f\hat{f} \sin x \sin \hat{x}$, $\lambda f^2 \sin^2 x$, $\lambda \hat{f}^2 \sin^2 \hat{x}$ and

$$\frac{1}{2} \lambda (\hat{f}^2 \sin^2 \hat{x} + f^2 \sin^2 x)$$
$$\pm \sqrt{f^4 \sin^4 x + \hat{f}^4 \sin^4 \hat{x} + 14 \hat{f}^2 f^2 \sin^2 x \sin^2 \hat{x}}.$$

It is now clear how the quadratically divergent mass terms for the pseudo-Goldstone bosons vanish. The quadratic terms in the one-loop CW potential are proportional to $\sum_i M_i^2$. From the masses given above, the trace is not zero but independent of x and $\hat{x}$, which are the two Higgs fields.

We now turn our attention to contributions arising from the top Yukawa coupling. The masses of fermions in the top quark sector are given by

$$\frac{1}{2}(f^2 + M^2 \pm \sqrt{(f^2 + M^2)^2 - 4M^2 f^2 \sin^2 x})$$
$$\frac{1}{2}(f^2 + M^2 \pm \sqrt{(f^2 + M^2)^2 - 4M^2 f^2 \cos^2 x}), \tag{20}$$

where we have imposed a left-right symmetry to the soft masses, so $M_L = M_R = M$. Again, the sum of M_i^2 is independent of x.

Finally, we turn our attention to the gauge sector. The masses of the gauge bosons are

$$m_{W_H}^2 \ = \ \frac{g_2^2}{2}(f^2 + \hat{f}^2) - m_W^2$$
$$m_{Z_H}^2 \ \approx \ \frac{g_1^2 + g_2^2}{2}(f^2 + \hat{f}^2) - \frac{2g_1^2 + g_2^2}{g_1^2 + g_2^2} m_W^2. \tag{21}$$

To quadratic order, the CW potential is

$$V_2^{(1)} = v^2(V_a + V_b \cos^2 \beta), \tag{22}$$

where

$$V_a \ = \ \frac{1}{32\pi^2}$$
$$\{ \frac{3}{2} g_2^4 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{WH}^2} + 1)$$
$$+ 3 \frac{2g_1^2 + g_2^2}{4} g_2^2 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{ZH}^2} + 1)$$
$$+ 2\lambda^2 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{\lambda(f^2 + \hat{f}^2)} + 1) \}, \tag{23}$$

$$V_b \ = \ \frac{1}{32\pi^2} 12 y^2 \frac{M^2}{y^2 f^2 - M^2}$$
$$(y^2 f^2 \ln \frac{y^2 f^2 + M^2}{y^2 f^2} - M^2 \ln \frac{y^2 f^2 + M^2}{M^2}) \tag{24}$$

and

$$v \sin \beta = f \sin x, \qquad v \cos \beta = \hat{f} \sin \hat{x}. \tag{25}$$

To align the direction of the electro-weak symmetry breaking, we add the following soft mass terms

$$V^{(0)} \ = \ m^2 H_L^\dagger H_L + \hat{m}^2 \hat{H}_L^\dagger \hat{H}_L$$
$$+ \ \mu^2 (H^\dagger \hat{H} + h.c.). \tag{26}$$

Together with the SU(4) breaking quartic term given in eq. (6), the tree level potential is given by

$$V^{(0)} \ = \ \lambda v^4 \cos^2 \beta \sin^2 \beta$$
$$+ \ v^2 (m^2 \sin^2 \beta + \hat{m}^2 \cos^2 \beta + 2\mu^2 \sin \beta \cos \beta)$$
$$+ \ 2\mu^2 f\hat{f}\sqrt{A} \tag{27}$$

where $A = (1 - \frac{v^2}{f^2} \sin^2 \beta)(1 - \frac{v^2}{\hat{f}^2} \cos^2 \beta)$.

We now minimize the potential $V = V^{(0)} + V_2^{(1)}$ to find v and $\sin \beta$. The potential V has the form

$$V \ = \ v^2(a + b \sin^2 \beta + 2\mu^2 \cos \beta \sin \beta)$$
$$+ \ \lambda v^4 \cos^2 \beta \sin^2 \beta, \tag{28}$$

where

$$a \ = \ \hat{m}^2 - \mu^2 \frac{f}{\hat{f}} + V_a$$
$$b \ = \ m^2 - \hat{m}^2 - \frac{\mu^2}{f\hat{f}}(\hat{f}^2 - f^2) + V_b. \tag{29}$$

After minimization, we find

$$\sin^2 \beta \ = \ \frac{a}{2a + b}$$
$$v^2 \ = \ -\frac{2a + b}{\lambda}(1 + \frac{\mu^2}{\sqrt{a(a+b)}}). \tag{30}$$

The fine tuning is about 13% for $\hat{f} = 2.0$ TeV and about 18% for $\hat{f} = 1.6$ TeV, with the feature that λ is much less than 1. Unfortunately, this is not significantly better than the original twin model. Notice that a mass squared is generated at loop level proportional to $\hat{f}^2 \lambda^2$ (See eq. (23)). Since $\hat{f}$ must be greater than 1.6 TeV to evade the bound from direct Z' and W' gauge boson searches [17], the $\hat{f}^2 \lambda^2$ contribution to the mass squared could be large if we push λ too high, which will tend to increase fine tuning. Thus, a small λ is preferred. However, with a smaller λ, we should account for the one-loop contribution to the quartic, since it may no longer be negligible. The largest loop contribution to the quartic is from the top Yukawa and is given by

$$V_4^{(1)} = \lambda_t v^4 \sin^4 \beta, \tag{31}$$

where

$$\lambda_t \ = \ \frac{3}{16\pi^2} y^4 \frac{M^4}{m_T^4} \left\{ \ln \frac{m_T^2}{m_t^2} - \frac{1}{2} \right. \tag{32}$$
$$+ \ \left. (\frac{m_T^2}{2M^2 - m_T^2})^3 \ln \frac{M^2}{m_T^2 - M^2} - 2(\frac{m_T^2}{2M^2 - m_T^2})^2 \right\}$$

and

$$m_T^2 \;=\; M^2 + y^2 f^2, \qquad m_t^2 = \frac{M^2}{m_T^2} y^2 v^2 \sin^2 \beta. \quad (33)$$

After adding eq. (31) to eq. (28) and repeating the analysis above, we find that a fine tuning of about 30% is easily achieved. Selected points are shown in table (I).

$\Lambda_{(\mathrm{TeV})}$	$f_{(\mathrm{GeV})}$	$\hat{f}_{(\mathrm{TeV})}$	$M_{L,R}_{(\mathrm{TeV})}$	$m_{h\,(\mathrm{GeV})}$	$\sin^2\beta$	Tuning
10	800	1.6	4	150/233	0.54	0.30 (y)
10	800	3.5	4	150/236	0.54	0.10 $(\hat{f})$
20	1600	1.6	4	163/213	0.66	0.11 (M)
10	800	1.6	10	147/266	0.51	0.19 (y)
5	800	1.6	4	150/233	0.54	0.30 (y)
5	800	3.5	4	150/236	0.54	0.16 $(\hat{f})$
10	1600	1.6	4	163/213	0.66	0.11 (M)

TABLE I: A summary of the Higgs mass and fine tuning, $\partial \log M_Z^2 / \partial \log f^2$, for sample points of parameter space. The two values of m_h correspond to the masses of the two neutral Higges. The most fine tuned parameter at each point is shown in the parenthesis. At these points, the other parameters are $\mu^2 = -(150 \text{ GeV})^2$, $\lambda = 0.5$ and $y = \sqrt{2}$.

IV. MIRROR MODEL

As far as addressing the little hierarchy problem, the mirror twin Higgs model with a tree level quartic [8] provides an improvement over the original mirror model. However, as shown in section II, in this theory the mirror photon is necessarily massive. As a consequence, this theory has difficulty in explaining the absence of a mirror electron relic density. In the absence of a massless mirror photon, electrons cannot efficiently annihilate to photons. We now show that using the same mechanism that was discussed in the previous section, this difficulty can be avoided.

The gauge group in the mirror model is $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$ which is a subgroup of the global U(4) symmetry. The scalar fields are H and $\hat{H}$ which have the same charge under the gauge group. The top sector is just the SM top Yukawa plus its twin counter part.

$$\mathcal{L}_{top} \;=\; y(H_L^T \tau_2 Q_L t_R + H_R^T \tau_2 Q_R t_L) \qquad (34)$$

We now calculate the CW potential in this model. The masses of heavy gauge bosons are

$$m_{W_H}^2 \;=\; \frac{g_2^2}{2}(f^2 + \hat{f}^2) - m_W^2$$

$$m_{Z_H}^2 \;=\; \frac{g_1^2 + g_2^2}{2}(f^2 + \hat{f}^2) - \frac{g_1^2 + g_2^2}{g_2^2} m_W^2. \quad (35)$$

For the top sector, up to finite terms which do not significantly alter the fine tuning, we can just take $M = \Lambda$ to produce the results that correspond to the non-SU(4) invariant top sector. For the Higgs potential, we add the same tree level potential as given in eq. (6). The CW potential due to this tree level potential is exactly the same as that obtained in our previous analysis on the left-right model. To quadratic order, the potential is

$$V_2^{(1)} \;=\; \frac{v^2}{32\pi^2}$$
$$\{ \; \frac{3}{2} g_2^4 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{WH}^2} + 1) \qquad (36)$$
$$+ \; \frac{3}{4}(g_1^2 + g_2^2)^2 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{m_{ZH}^2} + 1)$$
$$+ \; 2\lambda^2 (f^2 + \hat{f}^2)(\ln \frac{\Lambda^2}{\lambda(f^2 + \hat{f}^2)} + 1)$$
$$- \; 12 y^4 f^2 \sin^2 \beta (\ln \frac{\Lambda^2}{y^2 f^2} + 1)\}.$$

The one-loop quartic from the top sector is

$$V_4^{(1)} \;=\; \frac{3}{16\pi^2} y^4 [\ln \frac{\Lambda^2}{m_t^2} + \ln \frac{\Lambda^2}{m_T^2} + \frac{3}{2}]$$

where

$$m_T^2 \;=\; y^2 f^2 \quad , \quad m_t^2 = y^2 v^2 \sin^2 \beta. \qquad (37)$$

We then analyze the effective potential given by $V = V^{(0)} + V_2^{(1)} + V_4^{(1)}$ as in the previous section. The fine tuning for this model is shown in table (II). We see that the results represent an improvement over the mirror model. We expect that further enhancement may be obtained by making the top Yukawa coupling SU(4) invariant as in [6], but we leave this for further work.

$\Lambda_{(\mathrm{TeV})}$	$f = \hat{f}_{(\mathrm{GeV})}$	λ	$m_{h\,(\mathrm{GeV})}$	Tuning
10	800	0.5	178/213	0.16 (y)
10	800	1	183/213	0.21 (y)

TABLE II: A summary of the Higgs mass and fine tuning, $\partial \log v^2 / \partial \log f^2$, for sample points of parameter space. The two values of m_h correspond to the masses of the two neutral Higges. The most fine tuned parameter at each point is shown in the parenthesis. At these points, the other parameters are $\mu^2 = -(150 \text{ GeV})^2$, $y = 1.2$ and $\sin^2 \beta = 0.69$.

V. CONCLUSION

We have constructed a twin Higgs model based on left-right symmetry with an order one tree level quartic for the light Higgs. The structure of the electroweak symmetry breaking is similar to that of two Higgs doublet model. We analyzed the model and showed electroweak

symmetry breaking can happen naturally. For $\hat{f} = 1.6$ TeV, which is the lower bound from the direct searches on heavy gauge bosons, the fine tuning is found to be about 30% for $\Lambda = 10$ TeV. The bound on $\hat{f}$ gets stronger if we also require the left-right symmetry on the first two generation quarks. The K_0-$\bar{K}_0$ mixing puts a very strong constraint on the mass of W_H which require $\hat{f} > 3.5$ TeV [18]. In that case, the fine tuning is found to be about 10%.

The phenomenology of the model introduced in section II and III is not significantly different from that of the original left-right twin Higgs model[19, 20, 21]. The extra quarks introduced to complete the SU(4) multiplet could have masses of about 4 TeV which is difficult to observe at the LHC. Among these extra quarks there are some with electric charge $Q = -1/3$. These new down-type fermions in the model might have sizable contributions to the $D^0 - \bar{D}^0$ mixing depending on their masses[22]. The current experimental bound can be used to put a bound on the parameter M in the model. Another difference is that the parity we introduced to make the $\hat{h}_L$ stable, under which $\hat{H}_L$ is odd and all other fields are even, is here softly broken by the term $H_L^\dagger \hat{H}_L$ in eq. (26). Hence, $\hat{H}_L$ is no longer a dark matter candidate and

will be produced and decay just like all other scalars in the model. The phenomenology of the scalar sector of the original LRTH model has been studied in ref. [19, 21]. Most of these studies have focused on the scalars in the 'right-handed' H_R and $\hat{H}_R$ since all other scalars in the 'left handed' sector other than the SM Higgs do not interact directly with fermions. In both of our new models, for the same reason that $\hat{H}_L$ is no longer stable, all scalars in the 'left-handed' sector can interact with fermions and will behave just like the scalars of two Higgs doublet model. This new phenomenon, probably in combination with some others, may be used to test the tree level quartic coupling introduced in these twin Higgs models. We leave these studies for future work.

In summary we have shown how to incorporate a tree level quartic into the left-right twin Higgs model, leading to a substantial improvement in fine-tuning. We have further applied this mechanism to the mirror twin Higgs model and established that the fine tuning is about 20% for a 10 TeV cutoff scale.

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