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Publication Date

1965-12-06

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Submitted to Physical Review Letters

UCRL-16560

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

PARAMETERS OF THE $K_0 \bar{K}_0$ SYSTEM

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December 6, 1965

Parameters of the $K_0 \bar{K}_0$ System*

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When no conservation law is assumed--P, C, CP, T, or CPT--

R. Sachs¹ has shown that the time dependence of a $K_0 \bar{K}_0$ mixed state can still be described with the help of a short-lived state $|K_S\rangle$ and a long-lived state $|K_L\rangle$:

$$\left. \begin{aligned} |K_S\rangle &= p_S |K_0\rangle + q_S |\bar{K}_0\rangle \\ |K_L\rangle &= p_L |K_0\rangle - q_L |\bar{K}_0\rangle \end{aligned} \right\} (1)$$

Several authors¹⁻⁴ have set limits for the parameters p_S , q_S , p_L , and q_L under different assumptions concerning the decay processes--either CPT invariance, the $\Delta I = \frac{1}{2}$ rule, or excluding the possibility of accidental cancellation. To our present knowledge, it has not been pointed out yet how well all these parameters are known from already performed experiments, from the properties of unitarity, and from an assumption of negligible C violations in strong interactions only.

Of the eight real parameters that define the four complex parameters p_S , q_S , p_L , and q_L , five can be chosen by convention if no conservation law is assumed. We take the following conventions:

$$\left. \begin{aligned} p_S &= \cos \alpha_S e^{i \theta/2} & p_L &= \cos \alpha_L e^{-i \theta/2} \\ q_S &= \sin \alpha_S e^{-i \theta/2} & q_L &= \sin \alpha_L e^{i \theta/2} \end{aligned} \right\} (2)$$

The following restrictions can be imposed:

$$\begin{aligned} 0 &\leq \alpha_S \leq \pi/2 \\ 0 &\leq \alpha_L \leq \pi/2 \\ -\pi/2 &\leq \theta \leq \pi/2. \end{aligned} \quad (3)$$

Without assuming any invariance in the decay process, we determine α_S , α_L , and θ from known experimental results.⁵⁻¹¹ Our result is given in Eq. 20.

If $|j\rangle$ is a particular state into which the K mesons can decay, we call Γ_j^S (Γ_j^L) the rate of decay of a pure $|K_S\rangle$ ($|K_L\rangle$) state into $|j\rangle$ and

$$\begin{aligned} \kappa &= \langle K_S | K_L \rangle = p_S^* p_L - q_S^* q_L \\ &= \cos(\alpha_S + \alpha_L) \cos \theta - i \cos(\alpha_S - \alpha_L) \sin \theta. \end{aligned} \quad (4)$$

Unitarity gives the relation¹²

$$|\kappa| \leq \frac{2 \sum_j (\Gamma_j^S \Gamma_j^L)^{1/2}}{|\tau_S^{-1} + \tau_L^{-1} - 2i \Delta m|}, \quad (5)$$

where⁵

$$\left. \begin{aligned} \tau_S &= |K_S\rangle \text{ lifetime} = (0.88 \pm 0.04) 10^{-10} \text{ sec,} \\ \tau_L &= |K_L\rangle \text{ lifetime} = (5.8 \pm 0.6) 10^{-8} \text{ sec,} \\ \text{and} \\ \Delta m &= \text{mass difference between } |K_S\rangle \text{ and } |K_L\rangle = (0.55 \pm 0.1) \tau_S^{-1}. \end{aligned} \right\} (6)$$

From the inequality (5) and using a Schwarz inequality and the relations

$$\sum_j \Gamma_j^S = \tau_S^{-1} \quad \text{and} \quad \sum_j \Gamma_j^L = \tau_L^{-1}, \quad \text{we get the relation}^{13}$$

$$\left. \begin{aligned} |\kappa| &\leq \frac{2 (\tau_S/\tau_L)^{1/2}}{|1 + (\tau_S/\tau_L) - 2i \Delta m \tau_S|} \approx 5 \times 10^{-2} \\ |\kappa|^2 &\leq 3 \times 10^{-3}. \end{aligned} \right\} (7)$$

Three experiments have measured the short-life decay rate of K_0 into $\pi^+\pi^-$.⁶ According to our description and convention, those results are a measurement of the quantity

$$\frac{\Gamma_{\pi^+\pi^-}^S \sin^2 \alpha_L}{1 - |\kappa|^2} = (0.36 \pm 0.01) \tau_S^{-1}. \quad (8)$$

Several experiments have measured the branching ratio of $|K_S\rangle$ into $2\pi^0$ to $|K_S\rangle$ into $\pi^+\pi^-$:^{7, 8}

$$\Gamma_{2\pi^0}^S / \Gamma_{\pi^+\pi^-}^S = 0.46 \pm 0.02. \quad (9)$$

Two experiments have measured the ratio R of the reactions $\bar{p}p \rightarrow K_0 + (\text{system of particles with negative strangeness})$ and $\bar{p}p \rightarrow \bar{K}_0 + (\text{system of particles with positive strangeness})$ when the K_0 or the \bar{K}_0 decays into two charged pions.⁹ We assume that C violations in strong interactions are small compared to the accuracy of the experiments; therefore equal amounts of K_0 and \bar{K}_0 are produced. Considering that the ratio R is about 1 and that the state $|K_L\rangle$ decays into $\pi^+\pi^-$ no more than 0.2% of the time,¹⁰ while the state $|K_S\rangle$ decays into $\pi^+\pi^-$ with a branching ratio of at least 0.36 [from relations (7) and (8)], we conclude that only the short-lived state $|K_S\rangle$ contributed sensibly to the $\pi^+\pi^-$ decay, whenever a K_0 or a \bar{K}_0 was produced initially. Therefore R is a measurement of $\tan^2 \alpha_L$. Combining both experiments, we get

$$\tan^2 \alpha_L = 1.0 \pm 0.03. \quad (10)$$

Moreover, combining (7), (8), (9), (10) we get

$$\text{and } \left. \begin{aligned} \Gamma_{\pi^+\pi^-}^S &= (0.72 \pm 0.023) \tau_S^{-1} \\ \Gamma_{2\pi^0}^S &= (0.33 \pm 0.018) \tau_S^{-1} \end{aligned} \right\} (11)$$

We also can get the total rate for decay into two pions, $\Gamma_{\pi^+\pi^-}^S + \Gamma_{2\pi^0}^S$, by using references 6 and 7, and combine it with the direct measurement of $\Gamma_{\pi^+\pi^-}^S + \Gamma_{2\pi^0}^S$ given in reference 8 [using also relations (7) and (10):

$$\Gamma_{\pi^+\pi^-}^S + \Gamma_{2\pi^0}^S = (1.034 \pm 0.03) \tau_S^{-1}. \quad (12)$$

Because the sum of all decay rates of the $|K_S\rangle$ state should add up to τ_S^{-1} , by using Eq. (12) one can set an upper limit to the decay rates into all final states that are not two pions:

$$\Gamma_{\text{not } 2\pi}^S < 0.03 \tau_S^{-1}. \quad (13)$$

We can now turn to the $|K_L\rangle$ decay rates. Christenson et al.¹⁰ have measured

$$\Gamma_{\pi^+\pi^-}^L \cong 2 \times 10^{-3} \tau_L^{-1}. \quad (14)$$

An upper limit for the $2\pi^0$ decay mode of the $|K_L\rangle$ state,

$$\Gamma_{2\pi^0}^L < 8 \times 10^{-3} \tau_L^{-1}, \quad (15)$$

has been set.¹¹ Therefore most of the $|K_L\rangle$ state decays into states that are not two pions:

$$\Gamma_{\text{not } 2\pi}^L \cong \tau_L^{-1}. \quad (16)$$

Substituting (11), (13), (14), (15) and (16) in Eq. (5), we get

$$|\kappa| \leq \frac{2 [(\Gamma_{\pi^+\pi^-}^S - \Gamma_{\pi^+\pi^-}^L)^{1/2} + (\Gamma_{2\pi^0}^S \Gamma_{2\pi^0}^L)^{1/2} + (\Gamma_{\text{not } 2\pi}^S \Gamma_{\text{not } 2\pi}^L)^{1/2}]}{|\tau_S^{-1} + \tau_L^{-1} - 2i \Delta m|} < (1.3 \pm 0.45) \times 10^{-2}. \quad (17)$$

Relation (4) implies that

$$|\kappa|^2 = \cos^2(\alpha_S + \alpha_L) \cos^2 \theta + \cos^2(\alpha_S - \alpha_L) \sin^2 \theta, \quad (18)$$

In (18) $\cos(\alpha_S - \alpha_L)$ cannot be close to zero because of relation (10) and the restrictions (3). Therefore (17) and (18) imply that $\alpha_S + \alpha_L \cong \pi/2$, and $\theta \cong 0$. More precisely, we have

$$[\alpha_S + \alpha_L - (\pi/2)]^2 + \theta^2 < 1.7 \times 10^{-4}. \quad (19)$$

The value given in Eq. (19) can be increased by a factor of 2 if we stretch the error by one more standard deviation.

Relation (19) represents a condition for both $\alpha_S + \alpha_L$ and θ . Using (19) and (10), we have

$$\left. \begin{aligned} \alpha_S &= \pi/4 \pm 0.015, \\ \alpha_L &= \pi/4 \pm 0.008, \\ \theta &= 0 \pm 0.013. \end{aligned} \right\} \quad (20)$$

and

These values show that our parameters are determined without assumption of a conservation law in weak interactions. They are close to their theoretical value if CP were conserved, i. e.,

$$\left. \begin{aligned} \alpha_S &= \alpha_L = \pi/4 \\ \theta &= 0. \end{aligned} \right\} \quad (21)$$

and

Relation (20) shows that the measurement of $|K_L\rangle$ decay rates¹⁴ (made under the assumption that $p_S, q_S, p_L,$ and q_L are close to $1/\sqrt{2}$) are valid. We could now split the $\Gamma_{\text{not } 2\pi}$ into several categories, but this would not improve our result (20) sensibly because of the uncertainties about the $|K_S\rangle$ decay rates in the already performed experiments¹⁵ and about possible unknown modes of decay of $|K_L\rangle$.¹⁶

The errors quoted in (20) result from quadratic combination of experimental errors and the condition of unitarity (5). Because the latter is an absolute limit and should not be interpreted as a Gaussian error, the

probabilities of situations corresponding to several standard deviations in the errors in α_S and θ are somewhat less than those expected for a Gaussian distribution.

Since this work was performed, we have seen a preprint by Bell and Steinberger¹⁷ in which our unitarity condition (5) is derived. They obtain a somewhat better constraint than our relation (17) for $|\kappa|$ under reasonable assumptions about a few decay rates.

I am indebted to Prof. L. W. Alvarez for his support, Prof. F. S. Crawford, Jr. for enlightening discussions, and Dr. A. Barbaro-Galtieri for making available some experimental data used in this paper.

FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U. S. Atomic Energy Commission.

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12. If all $|j\rangle$ represent a complete orthogonal set of final states and \mathcal{M} is the transition matrix, then we have

$$\Gamma_j^S = |\langle j | \mathcal{M} | K_S \rangle|^2,$$

$$\Gamma_j^L = |\langle j | \mathcal{M} | K_L \rangle|^2,$$

and

$$\begin{aligned} (\tau_S^{-1}/2 + \tau_L^{-1}/2 - i \Delta m) \times (p_S^* p_L - q_S^* q_L) &= \zeta \\ &= \sum_j \langle j | \mathcal{M} | K_S \rangle^* \langle j | \mathcal{M} | K_L \rangle = \sum_j (\Gamma_j^S \Gamma_j^L)^{1/2} e^{i\phi_j}, \end{aligned}$$

where ϕ_j is the phase difference between $\langle j | \mathcal{M} | K_L \rangle$ and $\langle j | \mathcal{M} | K_S \rangle$.

Therefore we can write

$$|\zeta| \leq \sum_j (\Gamma_j^S \Gamma_j^L)^{1/2}.$$

If j is used to describe several states of decay of the orthogonal set, the above inequality is still valid as a result of the Schwarz inequality.

13. Inequality (7) is weaker than inequality (5); it has already been reported in reference 2.

14. The data considered here for branching ratios is the world compilation quoted in reference 5.
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16. When we sum up all known decay rates of $|K_L\rangle$, also taking into account measurements of some of the ratios of the decay rates, and compare the result to the value of τ_L^{-1} determined by exponential decay, the experimental accuracies allow us to write only the inequality

$$\Gamma_{\text{unknown}}^L < 0.08 \tau_L^{-1}$$

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