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Recent Work

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Deterministic stabilization of eight-way 2D diffractive beam combining using pattern recognition.

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We demonstrate a new method for controlling diffractive, high power beam combination, sensing phase errors by analyzing the intensity pattern of uncombined side beams at the output. A square array of eight beams is combined with $<0.3\%$ stability and $84.6\%$ efficiency. As channel count is increased, so does the usable information, enabling scaling to large channel counts without significant slowing of control loop response time, an advantage over single-input algorithms.

2. PRINCIPLE

Figure 1 shows the beam paths into and out of the diffractive combiner. The $N = 8$ input beams are aligned in a $3 \times 3$ array in free space with equal power. The first element, DOE1, applies pulse front tilt to each beam, compensating the tilt introduced by the combining element. The second element, DOE2, is a diffractive splitter operated in reverse. Since each input beam is, by itself, split into eight output beams which overlap with adjacent beams, there will be 25 outputs for 8 inputs. Ideally, all but the central beam are minimized, but they won’t be zero, and can provide information. In order to theoretically determine the 25-spot pattern given a set of input beam phases, we need to know the relative phases of the outputs from one input beam. In general, these output phases will not all be zero, will be asymmetric about the axes of the combiner [6], and are exactly the input beam phases needed to achieve efficient combination. This set of phases is a characteristic of the combining element, and can be found experimentally by correlating the intensities of output spots contributed to by only two input beams with the relative phase between those input beams. With that information, we can predict the output spot pattern given any set of amplitudes and phases of the inputs.

![Fig. 1. 2D coherent combining using two DOEs.](image-url)
A. System modeling

As a beam splitter, the DOE diffracts the incident light into many output beams. At the far field after the DOE, the periodic grating pattern translates to sampling of its 2D Fourier transform, which defines the DOE transmission transfer function. The beam size and (Gaussian) spatial beam profile acts as a window function. As a result, amplitude and phase diffracted beams are experimentally measurable, denoted as a complex array \(d(x, y)\), where \(x, y\) is location index relative to the 0 order beam at the center. In our case, first orders of \(d(x, y)\) will be \(d_{3,3}\):

\[
d(x, y) = D_{xy}e^{i\theta_{xy}}, \quad d_{3,3} = \begin{pmatrix} 1 & e^{i\theta_{0,1}} & e^{i\theta_{1,1}} \\ e^{i\theta_{-1,0}} & 0 & e^{i\theta_{1,0}} \\ e^{i\theta_{-1,-1}} & e^{i\theta_{0,-1}} & e^{i\theta_{-1,-1}} \end{pmatrix}
\]

The phases are defined with respect to the top-left element. In our case, the DOE is built with a specification of equal amplitude of all \(\pm 1\) order beams in a \(M \times M\) (where \(M = 3\)) array, with the 0 order (center) mostly suppressed. The input beam array should have the same intensity pattern as \(d_{3,3}\) and with phases \(\phi_{xy}\) must be found:

\[
b(x, y) = A_{xy}e^{i\phi_{xy}}, \quad x, y \in [-1, 0, 1]
\]

Fig. 1 shows the optical paths and diffracted beam array. At the combining DOE, an input beam at \((m, n)\), which is \((\frac{M-1}{2} - m, \frac{M-1}{2} - n)\) away from center, is injected at an angle such that its diffracted beams are spatially shifted by \((m - \frac{M-1}{2}, n - \frac{M-1}{2})\) with respect to 0 order location at the far field output plane. Then, the diffracted beam intensity just depends on the incident beams’ interference condition, and is distributed as a function of output diffraction. Superposition of all beams’ diffraction patterns creates a complex matrix \(s_{out}\) at detection plane, whose intensity is measurable by a camera or photodiode array. This diffraction and superposition process can be seen as 2D convolution between the incident beam complex array \(b(x, y)\) and the DOE transfer function complex array \(d\), except the indices are mirrored in both dimensions due to the optical paths crossing at the combiner. The following discussion flips \(s_{out}\) back as \(s\) for convenience, which can be done either optically or numerically in experiment.

\[
s(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} b(x, y) d(x - m, y - n) = b(x, y) * * d(x, y)
\]  

While the indices cover all possible orders of diffraction, only a few will be non-negligible. In practice, even with the highest efficiency in combining, the un-combined beams will never be zero and can be used for phase detection.

Optimal combining requires that all incident beams are constructively interfered at the zero order diffracted beam located at \((0, 0)\). Selecting the first incident beam as phase reference, \(\angle b(-1, 1) = 0\), the optimal incident beam phase condition (operation point) for coherent diffractive combining can be defined as:

\[
\phi_{x,y} = -\theta_{x,y}
\]

This means that the optimal input beam phases will be the same as the combining element’s (DOE2’s) phases when operated as a splitter, except that the indices are reflected around the x and y axes.

3. DETERMINISTIC STABILIZATION IN EXPERIMENT

From Eq. 1, a \(M \times M\) incident beam array going through a matched DOE would create \((2M - 1)^2\) diffracted beams. The desired condition of coherent combining is to have maximum amplitude at combined (center) beam with minimum at all other locations. Because the beam paths are perturbed by environment, one has to actively correct optical phase in real-time, in order to achieve and maintain optimal stabilization.

A. Identification of diffracted beam phases

The first step is to identify \(d(x, y)\) by measuring \(|s(x, y)|\) from the interference of diffracted beams due to two adjacent incident beams at different locations.

For two incident beams \(b(x, y)\) and \(b(m, n)\), the intensity of interference between their diffracted beams at \(x + m, y + n\) directly reveals the phase difference:

\[
s(x + m, y + n) = A_{xy}e^{i(\phi_{xy} + \theta_{mn})} + A_{mn}e^{i(\phi_{mn} + \theta_{xy})}
\]

\[
|s(x + m, y + n)|^2 = 2A_{xy}A_{mn}\cos(\Delta \phi - \Delta \theta) + A_{xy}^2 + A_{mn}^2
\]

where \(\Delta \phi = \phi_{xy} - \phi_{mn}\), and \(\Delta \theta = \theta_{xy} - \theta_{mn}\). For equal amplitudes \(A_{xy} = A_{mn} = 1\), we have

\[
|s(x + m, y + n)| = 2\cos\left(\frac{\Delta \phi - \Delta \theta}{2}\right)
\]  

Fig. 2 shows the measured and theoretical amplitude cross-correlations between two overlapping elements in the diffracted beam array when only two incident beams \(b(-1, 1)\) and \(b(-1, 0)\) are passed through, taken from 100 video frames (with 10 frames per second rate) from the far field camera, as phases are allowed to drift. Each blue dot represents one video frame. It is clear that the diffracted beams have ±90° or ±45° phase relationships, where the signs can be determined by directions of change, which are indicated by orange and green markers from two adjacent frames. There are six such cross-correlations between output beams due to two inputs (only two are shown).

Fig. 2. Diffracted beam amplitude cross correlations when only \(b(-1, 1)\) and \(b(-1, 0)\) are input to DOE2.

By analyzing different combinations of two beam interference, we conclude that the measured first order DOE transfer function is as in Eq. 3.

\[
d_{3,3} = \begin{pmatrix} 1 & i & 1 \\ 1 & 0 & -i \\ -1 & 1 & 1 \end{pmatrix}
\]

Simulation confirms this estimation is consistent with measurement, as shown in Fig. 2. It is possible to include higher order diffracted beam phases analysis, but only first orders are considered.
B. Feedback using reduced-set pattern

It is essential for coherent stabilization to measure the sign of phase errors, which is typically unavailable from intensity detectors such as a camera. However, under certain DOE characteristic phase array conditions, a reduced-set of the diffracted beam intensities can work as phase detectors. We have identified such a condition, and developed a feedback control method by stabilizing the intensities of these points.

Looking at the number of contributors to the diffracted beam array, one can always find 8 points where there are only 2 beams interfering, typically next to corners in \( s(x, y) \):

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1
\end{pmatrix} \ast
\begin{pmatrix}
1 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1
\end{pmatrix} =
\begin{pmatrix}
1 & 2 & 3 & 2 & 1 \\
2 & 2 & 4 & 2 & 2 \\
3 & 4 & 8 & 4 & 3 \\
2 & 2 & 4 & 2 & 2 \\
1 & 2 & 3 & 2 & 1
\end{pmatrix}
\]

From interference Eq. 2, we know that the intensity is a cosine function of phase error, and would provide a “slope” for feedback if \( \Delta \phi - \Delta \theta \neq 0 \) at the optimal combining phase condition.

\[
\frac{\partial |s(x + m, y + n)|}{\partial \Delta \theta} = - \sin \frac{\Delta \phi - \Delta \theta}{2} \neq 0
\]

Therefore the condition for direct feedback using the reduced-set of two beam interference is:

\[
\begin{align*}
\phi_{x,y} &= -\theta_{x,-y} & \forall x, y \in [-M+1, \cdots, M-1] \\
\phi_{x,y} - \phi_{0,0} &= -\theta_{x,y} - \theta_{0,0} & |x + m| + |y + n| = 2M - 3
\end{align*}
\]

This translates to the requirement that the characteristic phase of DOE2 is asymmetric. Fortunately, a high-efficiency DOE splitter/combiner tends to have this characteristic [6]. In general, for a \( M \times M \) shape DOE:

\[
\theta_{m,n} + \theta_{-m,-n} - \theta_{x,y} - \theta_{-x,-y} & |x + m| + |y + n| = 2M - 3
\]

In other words, the adjacent summations of diagonal elements in DOE2’s characteristic phase array must be different. Under this condition, there is a unique mapping between the diffracted beam intensity array \( |s(x, y)|^2 \) and the incident beam phase \( \angle b(x, y) \), which makes it possible to have deterministic phase feedback from pattern recognition.

Simulations using \( d_{3 \times 3} \) shows that the diffracted beam intensity patterns are asymmetric for positive and negative phase errors, suggesting that error magnitude and direction can be derived, as shown in Fig. 3.

![Fig. 3. Diffracted beam pattern with ideal phases, and for upper left corner input beam phase perturbed by minus and plus 90 degrees.](image)

For balanced and normalized amplitudes, the reduced-set intensity array \( \tilde{I} \) is a function of beam phase difference array \( \Delta \phi \):

\[
\tilde{I} = 2(\cos(\Delta \phi - \Delta \theta) + 1)
\]

\[
\frac{\partial \tilde{I}}{\partial \Delta \phi} = -2 \sin(\Delta \phi - \Delta \theta)
\]

The diffracted beam pattern variation \( d\tilde{I} \) is then proportional to the phase difference error array \( d\Delta \phi \) times the sign of slope:

\[
d\tilde{I} \propto d\Delta \phi \cdot \text{sgn}\left( \frac{\partial \tilde{I}}{\partial \Delta \phi} \right)
\]

where for our measured \( d_{3 \times 3} \), around operating point we have:

\[
\text{sgn}\left( \frac{\partial \tilde{I}}{\partial \Delta \phi} \right) (x, y) = \begin{pmatrix} - & + & + \\ + & - & - \end{pmatrix}
\]

In order to map from 8 elements of a phase difference error array to 7 phase shifter voltages for feedback, we use linear least squares to solve this overdetermined system. In practice, because only phase difference matters, we move the 8th phase shifter to the average amount of phase error, so the system locking range is maximized. Given a set point of \( \tilde{I}_0 \), one can apply a many-input-many-output proportional-integral controller in software to stabilize the reduced-set intensities to achieve and maintain the optimal combined state, which is defined and tunable by \( \tilde{I}_0 \).

C. Experiment Setup and result

![Fig. 4. Theoretical intensities of eight beams of the "reduced-set" versus phase error, and combined beam intensity versus phase error.](image)

![Fig. 5. Experiment setup. DAC: digital-to-analog converter. FPGA: field-programmable gate array.](image)

The experimental setup is shown in Fig. 5. A single frequency CW laser at 1060 nm is split into eight fiber channels, each of which is electronically phase controlled. The power of 8 input beams are balanced \( \sim 15 \mu W \) each, and the total input power
variation is \( \sim 0.3\% \) RMS. The output beams are formed into a two-dimensional array and input to the diffractive combiner. The output spot intensities are recorded on the far field camera (8 bit, 800 × 800 pixels), and analyzed in software to derive the phase errors. The correction signals drive piezoelectric fiber phase shifters in the fiber array. The feedback control bandwidth is limited by far field camera frame rate, which is about 16.7 Hz. A separate measurement is made of the central, combined spot using a power meter. All measurements are sampled synchronously at 16.7 Hz.

![Simulated Ideal Phase](image1)

![Measured Random Phase](image2)

![Measured Combined Condition](image3)

**Fig. 6.** Simulated and measured diffracted beam intensities

We compared the measured results with simulation using ideal amplitudes and phase condition as shown in Fig. 6. The diffracted beam intensities are measured by a far field camera in the free-running condition and controlled combining condition, and normalized to a calibrated power meter reading, to correct for saturation of the camera by the combined beam. Despite variations in the relative beam powers, camera nonlinearity and calibration inaccuracy, the system model approximates the experimental data. Including higher orders in the DOE model would help improve the theoretical combining efficiency, higher than our first order estimation. However, because the first order response dominates, higher order diffracted beams won't change the monotonic phase detection slope at the reduced-set locations.

![Far field 25 beam intensities](image4)

![Reduced-set output beam intensities](image5)

![Normalized combined beam power](image6)

![Input beam phase corrections](image7)

**Fig. 7.** Measurements from one closed loop data set.

Fig. 7 shows the resulting intensity values for the measured spots and the central spot, when the control loop is closed. We normalized the output power measurement to a simultaneously acquired total input power measurement, to find the variation due to combining stability only. The combined beam power is stable to within 0.22% RMS as measured by a power meter after an iris, which is independent of the control loop. The combining efficiency \( \eta \) – defined as the ratio of the central combined beam power to the total power from all 25 diffracted beams – is measured to be 84.6% using the power meter, by inserting or removing the iris to block/unblock uncombined beams. This is close to the 90.7% intrinsic efficiency of DOE2, measured by configuring it as a beam splitter.

The experiment described here uses a CW source for convenience. We have demonstrated that an identical combiner works for 120 fs pulses with high efficiency [3]. In practice, the power and delay variation for each beam must be made stable (about 10% relative power and 10% of pulse width [3, 7]) in order to achieve high efficiency. Also, the pointing stability must be good enough to give a single interference fringe across the beam for the combiner to work at all. These requirements can be met by an optical setup with good mechanical stability and thermal control, plus a slow control loop for coarse delay. We intend to combine high energy chirped pulses to be compressed to \( \sim 100\) fs, allowing for about three waves of phase error before loss becomes significant due to delay variation. While the loop is closed, the accumulated phase error of each beam can be tracked by phase shifter control voltages. For larger phase drift beyond the phase shifter range (in our case \( \sim 8\pi \)), it is possible to record the automatic resetting of the phase shifters’ drive function after multiple \( 2\pi \) excursion, and compensate with a coarse delay using a separate control loop.

We have shown that, for a small number of beams, the higher order side beams can be used to derive phase error information sufficient to stabilize the combination. This method is deterministic, does not require careful calibrations, is tolerant to (currently) uncontrolled errors including amplitude unbalance, and is potentially scalable. Since phase corrections are derived from a single image, it locks quickly and robustly, recovering from an unlocked condition in a single operation. Our output stability result is achieved even without the output power being in the loop. For larger numbers of input beams \( M > 3 \), there would still be 8 reduced-set points where only two beam interfere, and locking their intensities would stabilize the phase difference between corner input beams and the beams next to them, so that the system control dimensionality could be reduced; additional “layers” of stabilization could be applied to other beams. This will be a subject of future work.

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**REFERENCES**

FULL REFERENCES


