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Comparing Markov versus quantum dynamic models of changes in confidence during evidence monitoring

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Introduction

While monitoring evidence, a decision maker's confidence for each hypothesis changes across time. What are the fundamental dynamics that underlie these changes in confidence during evidence accumulation? There are two fundamentally different ways to understand the dynamics of confidence.

The "classical" way is to assume that the dynamics follow a Markov process, such as drift diffusion model (Pleskac & Busemeyer, 2010). According to this view, the decision maker's confidence state at any single moment is located at a specific point on some implicit strength of evidence scale. This confidence state changes moment by moment from one location to another, sketching out a trajectory like a random walk (with drift in the direction of the evidence). At the time point that an experimenter requests a judgment, the decision maker simply reads off and reports the pre-existing location of the state on the evidence scale.

A very different way is to assume that the dynamics follow a quantum type of process (Busemeyer et al., 2006). According to this view, the decision maker's confidence state at a moment is not located at any specific point on the evidence scale. Instead, at any moment, the state is represented by a wave dispersed across the scale. This wave of confidence flows across time as directed by the evidence. At the time point that an experimenter requests a judgment, the decision maker's indefinite state of confidence must be resolved, and the wave needs to "collapse" down into a specific location.

Previously, we empirically compared and tested these two models using an interference test (Kvam et al., 2015). In a choice-confidence condition, participants monitored evidence until making a choice at time t_1 and then they continued monitoring evidence until time t_2 at which point they rated confidence. In a confidence-only condition, participants did not make any choice at time t_1 (instead they pushed a prespecified button), and they only made a confidence rating at time t_2. According to a Markov model, the marginal distribution of confidence at time t_2 (pooled across choices at time t_1 for the choice-confidence condition), should be the same between the two conditions at time t_2. According to the quantum model, the choice at time t_1 produces a collapse that introduces interference such that the marginal distribution of confidence for the choice-confidence condition at time t_2 differs from the confidence-only condition. The results strongly favored the quantum model predictions.

The current paper presents another new test of the Markov versus quantum models for the dynamics of confidence. Once again participants monitored evidence until a confidence judgment was requested. In this experiment, two confidence ratings were made at a pair of time points. The experiment included three main conditions: (1) requests for confidence ratings at times t_1 and t_2, (2) requests for ratings at times t_2 and t_3, and (3) requests for ratings at times t_1 and t_3. Interference was tested by comparing the second rating at t_2 for condition 1 with the first rating at t_2 for condition 2. Once again the Markov model predicts no interference effect, whereas the the quantum model predicts an effect. For a proof, see Busemeyer & Bruza (2012), Chapter 8.

Method

A total of 11 (8 female) students from Michigan State University participated, who completed 2-3 sessions each, for about 800-1400 data points per person. Participants were paid \$10 per session plus a bonus depending on performance. On each trial, a random dot motion display was presented, and the participant had to infer the direction of motion from the jiggling display of dots (Ball & Sekuler, 1987). The coherence (proportion of dots systematically moving in one direction, e.g., right, rather than randomly) was .02, .04, .08, or .16. Confidence ratings were taken at 0.5 seconds and 1.5 seconds for condition 1, 1.5 and 2.5 seconds for condition 2, and .5 and 2.5 seconds for condition 3. Participants had to respond by moving the cursor (via joystick) across the edge of a semi-circular scale to rate the probability of moving from 0 (certain moving left) to 100 (certain moving right). Ratings for right-moving dots were used directly; but ratings favoring left-moving dots were rescored as (100 - rating).

Interference test results

Figure 1 shows a comparison of the distribution of confidence ratings, pooled across participants, at the lowest coherence level. Confidence decreased in the lowest range and increased in the higher ranges for the second confidence compared to first. This interference effect agrees with our previous finding Kvam et al. (2015). Although the interference effect appears small (because it is smeared across 100 levels), chi square tests of distribution differences (using 10 categories) indicated statistically significant differences at all coherence levels except the highest level (χ^2 statistic equals $227, p < .001$; $200, p < .001$, $168, p < .001$; $111, p = .43$ for coherence levels .02, .04,. 08, and .16 respectively).

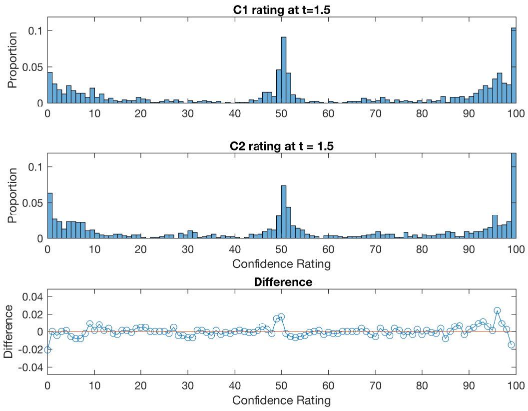

Figure 1: Results for lowest coherence level. Top panel shows the distribution of first rating at $t_1 = 1.5s$ for condition 2; middle panel shows the distribution of second rating at $t_2 = 1.5$ for condition 1; bottom panel shows the top minus middle.

Note also that the confidence ratings tend to cluster around low, medium, and high values with little use of other intermediate scale values. For this reason, we categorized the 101 point scale into three confidence levels (0-33 = low), (34 - 66 = medium), (67-100 = high) for the subsequent modeling.

Model comparison test results

A small but significant interference effect was observed, which is qualitatively consistent with the predictions of the quantum and inconsistent with the Markov model. However, it remains an open question whether the quantum versus the Markov model is better for quantitatively predicting the observed changes in confidence across time intervals. To evaluate quantitative predictions, both models were formed using 101 confidence states, which were then categorized into low, medium, high in the same manner as the data. Both models entail two free parameters to generate the predicted changes in confidence states across time: one corresponds to a mean "drift" rate that moves the distributions to the left or right depending on the sign and magnitude of the coherence, and the second is a "diffusion" parameter that spreads the distributions out across time. We used a generalization test criterion to compare the models (Busemeyer & Wang, 2000).

The models were fit to conditions 1 and 2 and the these same parameters were used to make predictions for condition 3. After categorizing the confidence ratings into low, medium, and high, each condition produces a 3×3 joint frequency table. First we conducted statistical tests to check whether the joint distribution for condition 3 differed significantly from conditions 1 and 2 (pooled across confidence levels). The chi-square tests produced a χ^2 statistic equal to $117, p < .02$ for condition 1 versus 3, and a χ^2 statistic equal to $192, p < .001$ for condition 2 versus 3.

The two parameters were fit separately for each participant and for each coherence level to conditions 1 and 2. Then these same parameters were used to predict the joint distribution for condition 3. The χ^2 difference in deviation (Markov - quantum) for each model and each confidence level are shown in Table 1. Positive differences favor the quantum model over the Markov model. In conclusion, the quantum model produced better quantitative predictions on generalization than the Markov model for all coherence levels except the highest.

Table 1: Generalization Test Results.

Coherence	Chi Diff
.02	207
.04	156
.08	166
.16	-114

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References

Ball, K., & Sekuler, R. (1987). Direction-specific improvement in motion discrimination. *Vision Research*, *27*(6), 953?965.

Busemeyer, J. R., & Bruza, P. D. (2012). *Quantum models of cognition and decision*. Cambirdge University Press.

Busemeyer, J. R., & Wang, Y.-M. (2000). Model comparisons and model selections based on generalization criterion methodology. *Journal of Mathematical Psychology*, *44*(1), 171–189.

Busemeyer, J. R., Wang, Z., & Townsend, J. (2006). Quantum dynamics of human decision making. *Journal of Mathematical Psychology*, *50*(3), 220-241.

Kvam, P. D., Pleskac, T. J., Yu, S., & Busemeyer, J. R. (2015). Interference effects of choice on confidence. *Proceedings of the National Academy of Science*, *112 (34)*, 10645-10650.

Pleskac, T. J., & Busemeyer, J. R. (2010). Two-stage dynamic signal detection: A theory of choice, decision time, and confidence. *Psychological Review*, *117 (3)*, 864-901.