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MAXIMAL ANALYTICITY OF THE SECOND DEGREE

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April 23, 1963

MAXIMAL ANALYTICITY OF THE SECOND DEGREE*

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The meaning of what I shall here call "maximal analyticity of the first degree" for the S-matrix is rapidly becoming clarified; recent papers by Stapp,¹ Polkinghorne,² Zwanziger,³ Gunson,⁴ and Olive⁵ strongly suggest that a satisfactory set of axioms is going to be achieved. Crudely speaking, the picture emerging is that, given all the energy-momentum poles of the S-matrix, the analytic continuation of the unitarity condition completely determines the remaining singularity structure in the complex energy-momentum variables. All further singularities appear to be branch points whose location is prescribed in terms of the pole positions by suitable generalization of the Landau rules,⁶ the discontinuities across the associated cuts being given by generalization of the Cutkosky rules.⁷ All poles have an equivalent status with respect to first-degree analyticity, whether they appear on the physical sheet (stable particles), near the physical region but on an unphysical sheet (unstable particles); or so far from the physical region that a particle interpretation is difficult. No distinction is made between "elementary" or "composite" particles, and correspondingly it is expected that

weak as well as strong interactions are encompassed. Now, in collaboration with Frautschi the author has attempted to formulate a more potent analyticity assumption to be applied only to strong interactions.⁸ Physically this "second-degree analyticity" corresponds to the notion that all strongly interacting particles are dynamical composites of one another, associated with poles whose positions and residues cannot arbitrarily be assigned but which are consequences of self-consistency in an analytic S-matrix (sometimes called the "bootstrap" mechanism). In other words we want the minimum number of strong poles consistent with the analytically continued unitarity condition, as well as the minimum number of other singularities. Our first attempt at a formal statement of second-degree analyticity was to assert that all strong energy-momentum poles are Regge poles. The purpose of this Letter is to discuss a different formulation that may well be equivalent to the foregoing, but which is more powerful in its immediate consequences and perhaps more attractive esthetically. The possibility of an alternative formulation of second-degree analyticity was suggested to the author by private discussions with S. Mandelstam and E. Squires.

In the new formulation the detailed nature of singularities in the angular-momentum plane does not arise. One starts with the assumption that first-degree analyticity (in linear momentum) implies for every energy a unique continuation in total angular momentum J for $\text{Re} J$ sufficiently large. Froissart⁹ and Gribov¹⁰ have found such a continuation for two-particle amplitudes satisfying the Mandelstam representation, and it is plausible from classical limit considerations that an analyticity domain

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for large J should exist in general. One also must assume that S-matrix elements can be continued from this region of large J to all physical J values ($J = 0, 1/2, 1, \dots$), but the S-matrix need not be meromorphic in the entire right-half J -plane. Isolated branch points, in particular, can be tolerated. The new formulation of second-degree maximal analyticity, then, is the assumption that, at all physical J values, the actual S-matrix elements (for zero lepton number) coincide with this analytic continuation from large J . (An exception may be necessary for elements with the quantum numbers of the photon, but as discussed below, the S-matrix has never really been defined in a way that properly accommodates photons.)

Froissart's work¹¹ makes it plausible from first-degree analyticity alone that for $J = 3/2, 2, 5/2, \dots$ the actual S-matrix must coincide with its continuation from large J if unitarity is not to be violated at high energies in crossed channels. However, no arguments have been given that, without some new assumption, such a coincidence is required for $J = 0, 1/2$, and 1 . We are suggesting that the coincidence may be taken as the defining characteristic of strong interactions, and that most if not all the observed manifestations of strong interactions derive therefrom. In other words, it is conjectured that the only S-matrix (not equal to the unit matrix) possessing analyticity of both first and second degree is the actually observed strong-interaction S-matrix. The principle of maximum strength discussed in earlier papers we believe to be valid and extremely useful, but probably redundant in its content.⁸

An immediately useful consequence of the new formulation of second-degree analyticity is the unambiguous elimination of arbitrary parameters not usually associated with poles, such as the so-called $\pi\pi$ coupling

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constant. If the $J = 0$ $\pi\pi$ amplitude is to be connected by analytic continuation of $J = 2, 4, 6, \dots$, the determining equations for $J = 0$ clearly cannot contain the distinguishing parameter that has been inserted often in the past.¹² Actually it seems probable that the insertion of such a parameter must produce somewhere (perhaps far from the physical region) a pole of non-Regge type, in which case the parameter also could be eliminated by the earlier form of second-degree analyticity that specifies Regge poles only. The new formulation, however, leaves no uncertainty about this question.

In understanding that the new version of second-degree analyticity implies the earlier one, it is important to distinguish between singularities in the J -plane whose position is energy-independent and a failure of the S -matrix to be equal to its analytic continuation at some particular physical J .¹³ For example, a spin- $1/2$ "elementary-particle" pole in the energy variable does not correspond to a fixed singularity in J , but rather to a discrepancy between the S -matrix and its analytic continuation at the point $J = 1/2$. Such a discrepancy is explicitly forbidden by our new formulation, one of whose consequences being that all energy-poles are required to have a continuation in J . That strong-pole positions and residues are not arbitrary thus follows from either formulation.

I conclude with a remark concerning why, in the attempt to formulate a comprehensive theory of microscopic phenomena in terms of the analytic S -matrix, one should expect electromagnetism to play an entirely different role from the strong interactions. The suggestion is that electromagnetic

interactions may be essential, not only in practice but in principle, in defining the trajectories through which initial and final particle momenta are determined. The space-time continuum is an unobservable and unnecessary concept, and in S-matrix theory one deals entirely with the energy-momentum continuum, which is physically observable (in an infinite universe). Nevertheless, to define the concept of momentum (or velocity) one must introduce space-time in the large--or in other words -- approximate classical trajectories. No systematic attempt has been made yet to trace the connection between macroscopic and microscopic worlds through the S-matrix, although a promising beginning has been reported by Goldberger and Watson.¹⁴ Nevertheless, it is extremely difficult even to imagine a sensible connection if there existed only short-range interactions. Something very much like the Coulomb force, and therefore the zero-mass photon, appears essential.

A possibly related circumstance is that an acceptable definition of the S-matrix to include photons has never been given, the conventional framework completely ignoring the infrared complication associated with the zero photon mass.¹⁵ It is only the smallness of the fine-structure constant that allows us approximately to treat certain photon problems within the established framework. The above two circumstances lead me to believe that electromagnetism ultimately will play more of a kinematical than a dynamical role in S-matrix theory. In other words the photon, with its zero mass, will be built into the defining fabric rather than emerging as just another pole from bootstrap conditions. In such an event, questions of the type

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discussed above for strong poles will be found meaningless for the photon. Such questions may not be meaningless for leptons, but I have not even an opinion to offer concerning the raison d'être of these particles.

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