

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

Characterizing artificial electromagnetic materials and their hybridization with fundamentally resonant magnetic materials

Permalink

<https://escholarship.org/uc/item/40w1n9m3>

Author

Gollub, Jonah Nathan

Publication Date

2008

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA, SAN DIEGO

**Characterizing Artificial Electromagnetic Materials and their
Hybridization with Fundamentally Resonant Magnetic Materials**

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Physics

by

Jonah Nathan Gollub

Committee in charge:

Professor Dimitri Basov, Chair
Professor David R. Smith, Co-Chair
Professor Michael Folger
Professor Lawrence Larson
Professor Paul Yu

2008

Copyright
Jonah Nathan Gollub, 2008
All rights reserved.

The dissertation of Jonah Nathan Gollub is approved, and it is acceptable in quality and form for publication on microfilm:

Co-Chair

Chair

University of California, San Diego

2008

To my parents

Bruce Gollub

and

Leah Morton

TABLE OF CONTENTS

Signature Page	iii
Dedication	iv
Table of Contents	v
List of Figures	vii
Acknowledgements	xiv
Vita and Publications	xvi
Abstract of the Dissertation	xviii
1 Introduction	1
2 Characterizing Disorder in Magnetic Split-Ring Resonator Metamaterials	11
2.1 Introduction	11
2.2 Experiment	12
2.3 Numerical Investigation	16
2.4 Conclusion	17
3 Magnetic Polaritons	18
3.1 Introduction	18
3.2 Derivation of Polariton Modes	22
3.2.1 Case 1: positive material $\epsilon_1, \mu_1 > 0$	24
3.2.2 Case 2: $\epsilon_1 < 0, \mu_1 > 0$	25
3.3 Fabricating a Metamaterial Structure to Support Surface Polaritons	26
3.4 Exciting Bound Surface Polaritons: OATR Reflection Coefficient .	29
3.5 Experimental Observation of Magnetic Surface Polaritons	33
3.6 Conclusion	38
4 Ferromagnetic Materials in Microstrip Structures	40
4.1 Introduction	41
4.2 Ferromagnetic Materials	42
4.3 The Schloemann Approximation	47
4.4 Numerical Simulations	49
4.5 Small Dimensional Microstrips	52
4.6 Microstrips with Embedded Ferromagnetic Layers	54
4.7 Experimental Results and Simulation	55
4.8 Conclusion	58

5	Planar Metamaterial Structures	60
5.1	Introduction	60
5.2	Complementary Structures	63
5.3	Tuneability of the CSRR Structure	70
5.4	Conclusion	73
6	Hybridization of Fundamental Magnetic Material Resonances with Arti- ficial Metamaterial Resonances	75
6.1	Introduction	76
6.2	Electromagnetic Response of a CSRR Waveguide Structure	76
6.3	Incorporation of a Resonant Magnetic Material into a CSRR Structure	80
6.4	Conclusion	86
7	Hybrid Resonant Phenomenon in a SRR/YIG Metamaterial Structure .	88
7.1	Introduction	88
7.2	Theory	90
7.3	Numerical Simulations	94
7.4	Conclusion	101
8	Concluding Remarks	102
A	Determination of Capacitance and Inductance Relations for Complemen- tary Structures	104
	Bibliography	109

LIST OF FIGURES

Figure 1.1: A diagram of the range of electromagnetic parameter space for natural materials. There are some natural materials that have a negative permittivity and permeability (as shown) but not simultaneously.	2
Figure 1.2: (a) The thin wire structure proposed by Pendry is shown and its characteristic dimensions. It exhibits a negative permittivity below the geometrically determined plasma frequency, ω_p . (b) The SRR structure proposed by Pendry is shown with its characteristic dimensions. It exhibits a negative permeability for a frequency range above the resonant frequency, ω , which is determined by the geometry of the structure. (c) A combined wire/SRR structure with simultaneous negative permittivity and permeability is shown. This structure was used by Smith et al. to demonstrate a negative index of refraction. It's characteristic index, n , is plotted	3
Figure 1.3: A metamaterial is effectively a second homogenization of matter. The effective constitutive parameters can be calculated through an inversion of the transmission and reflection coefficients (or related S-parameters) as discussed in the text.	6
Figure 1.4: Designing a metamaterial structure structure is an iterative process. A rough analytic guess is used to determine the initial geometry and then a full wave numerical simulation is performed on the structure to determine the associated scattering matrix. Effective constitutive parameters are then extracted from the scattering matrix data and the geometry is adjusted until the necessary electromagnetic properties are achieved.	8
Figure 2.1: The characteristic dimensions of the SRR structure are shown. The parameters $w1$ and $w2$ were varied in the samples to induce various amounts of disorder as pictured. The SRR used in the 0% SRR structure had dimensions $g = 0.22$ mm, $c = 0.08$ mm, $w1 = 2.25$ mm, $w2 = 0.45$ mm, and a unit cell size of 3.33 mm. Also shown is an illustration of the Bruggeman model for our structure in which the magnetic dipole moment of SRR unit cells are determined with respect to an average background permeability.	13
Figure 2.2: The experimental data is shown for transmission through disordered metamaterials of various linear distributions. Theoretical transmission for the sample is calculated using the Bruggeman mixing model to determine the material's effective permeability (and permittivity—not shown).	15

Figure 2.3: For a characteristic SRR structure in our experiment ($f_0 = 12.5$ GHz, $Q = 78$, $F = 0.14$), a numerical calculation of the width of negative region as a function of disorder introduced is shown for linear, Gaussian, and Cauchy distributions.	16
Figure 3.1: (a) A surface polariton travels along an infinite half-space boundary between two materials with permeability and permittivity indicated. A cutout shows the characteristic electric fields of an electric surface polariton. (b) The surface plasmon dispersion curve is plotted and relates frequency to propagating wavevector (k_x). The wavevector is to the right of the bounding dielectric light-line for frequencies below w_p	19
Figure 3.2: Range of possible electric (red) and magnetic (blue) polariton modes and their normalized energy flow densities are shown. Cases: (a) $\epsilon_1, \mu_1 > 0$ and (b) $\epsilon_1 < 0$ $\mu_1 > 0$ (figure credit: Yoon et al. [1]).	24
Figure 3.3: A metamaterial structure was constructed out of sheets of SRRs assembled into a 3D structure. The rings counterpose one another to satisfy the boundary conditions of the waveguide in which the experiment was performed. The characteristic dimensions of the SRRs are shown. The values for the sample considered in are: $w=2.14$ mm, $c=0.13$ mm, $d=0.35$ mm, and $g=0.47$ mm. Each unit cell is 2.5 mm x 2.5 mm x 2.5 mm.	27
Figure 3.4: (a) The real parts of the permeability $\mu_2(\omega)$ and permittivity $\epsilon_2(\omega)$ of the metamaterial are shown. They were derived from numerical simulations of split rings using Ansoft's HFSS software. (b) The associated range of magnetic polaritons supported by the metamaterial structure.	28
Figure 3.5: Numerical simulation of the transmission dispersion relation through the metamaterial done in Ansoft's HFSS software by modeling the structure of the split ring cells. The horizontal axis is the phase advance per unit cell or kd , where k is the wavenumber (2π divided by the free-space wavelength) and d is the SRR cell width.	29

Figure 3.6: (a) To excite surface polaritons, an Otto attenuated total reflection (OATR) technique is used. Radiation is incident on the boundary of a prism and medium of lesser index of refraction. The incident angle of the radiation is greater than the critical angle for total internal reflection resulting in an evanescent field decaying away from the prism. The evanescent field above the prism couples to surface polaritons on the metamaterial, which is close to but not touching the prism at a distance ‘ a ’. This coupling causes a dip in the reflected power from the prism boundary. A variation of the setup is the Kretschmann configuration where the prism is directly adjacent to the metamaterial and the surface polaritons are excited on the backside of the metamaterial. (b) The dispersion relation for magnetic surface polaritons is shown along with the coupling line of the prism	30
Figure 3.7: The experimental setup is shown inside the ARMs apparatus.	32
Figure 3.8: The normalized experimental and numerically simulated power transmitted through the metamaterial are shown versus frequency. The numerical simulation is done using Ansoft’s HFSS software by modeling, in the direction of propagation, a 10 cell thick split ring structure sandwiched between 1cm metal plates.	34
Figure 3.9: The simulated normalized reflected power for several metamaterial/prism separations, a , and the permeability of the metamaterial $\mu_2(\omega)$ are shown versus frequency. For incident radiation on the prism/air interface $\theta_{\text{inc}} > \theta_{\text{TIR}}$, yet there is not total reflection of the incident radiation. In the frequency range that $\mu(\omega) > 1$ the prism/metamaterial have closely matched indexes of refraction and the evanescent field in the gap provides a means of resonant light tunneling across the gap to the metamaterial. In the region that $\mu < -1$ the evanescent field couples to SPs. The evanescent field strength at the surface of the metamaterial falls off as the separation distance increases, resulting in decreased coupling to the above modes (i.e. increased reflected power).	35
Figure 3.10: The experimental data and numerical solution for the reflected power (normalized) as a function of the separation distance between the metamaterial and prism.	37
Figure 3.11: The experimental surface dispersion relation for several k_x values, the theoretical dispersion relation, and the light line $k_x = \omega/c$ are shown.	37

Figure 4.1: Diagram for the setup of the ferromagnetically embedded microstrip structure. Note that the ferromagnetic layer does not actually touch the waveport because HFSS does not allow a material with a negative permeability to be in direct contact with the waveports. An infinitesimal spacer of dielectric ($\sim 1/100000$ of the effective wavelength) is placed between the waveport and ferromagnetic layer to avoid simulation issues.	42
Figure 4.2: A plot of the magnetization of a ferromagnetic material versus applied field is shown. Above the saturation magnetization of a ferromagnetic material the electron spins and their associated magnetic moments are aligned.	44
Figure 4.3: The magnetic dipole moment, m , of an electron due to its spin is shown and its effective precession around the biasing field, H_0 . On the right, the magnetization of a thin ellipsoid (i.e. thin disk or film) is shown as a function of the applied field	45
Figure 4.4: A comparison between the attenuation derived from an HFSS simulation and that of the approximate analytic solution by Pucel et al. is shown for a microstrip with a dielectric height of $h = 4.5 \mu\text{m}$ and a top strip width of $W = 5 \mu\text{m}$ and thickness of $t = 2, 5$, and $10 \mu\text{m}$. The conductor is silver ($\sigma = 6.1 \times 10^7 \text{ S/m}$) and the dielectric is silicon dioxide ($\epsilon_r = 4$) with zero losses.	53
Figure 4.5: The simulated S_{21} attenuation is plotted for various positions of a ferromagnetic permalloy layer as measured from the ground plate. The dielectric is silicon dioxide ($\epsilon = 4$). The permalloy is $0.1 \mu\text{m}$ thick and has a width extending to the edges of the simulation ($100 \mu\text{m}$). The dimensions of the microstrip are $h = 3.1 \mu\text{m}$, $W = 5 \mu\text{m}$, and $t = 1.5 \mu\text{m}$. The parameters for permalloy are $\sigma = 1.6 \times 10^6 \text{ S/m}$, $M_0 = 0.8 \text{ kG}$, and linewidth of 0.08 kG . The applied field is set to 1 kG with a resulting resonance near 10 GHz	54
Figure 4.6: The simulated phase difference is plotted for two positions of a ferromagnetic permalloy layer: directly above the ground plate and at half-height of the microstrip. The conductor was copper, the dielectric was silicon dioxide ($\epsilon = 4$) and the permalloy layer was $0.1 \mu\text{m}$ thick. The dimensions of the microstrip were $h = 3.1 \mu\text{m}$, $W = 5 \mu\text{m}$, and $t = 1.5 \mu\text{m}$. The parameters for permalloy used in the simulation were $\sigma = 1.6 \times 10^6 \text{ S/m}$, $M_0 = 0.8 \text{ KG}$. The experiment was performed with a magnetic biasing field of $H_0 = 1 \text{ kOe}$; in the simulation $H_0 = 1.1 \text{ kOe}$ to fit the data. The linewidth parameter in the simulation, ΔH , was adjusted to fit the phase at resonance as given above. The simulated data was shifted down 30 degrees to match the experimental data.	56
Figure 4.7: The experimental and simulated attenuation corresponding to Fig. 4.6.	57

Figure 5.1: A circuit model of a transmission line is shown with characteristic per unit length lumped elements. L_R and C_R components characterizes conventional right hand (RH) transmission lines while L_L and C_L are representative of left-hand (LH) transmission lines. On the right, the characteristic dispersion diagram is shown for LH, RH, and RHLH transmission lines [2].	61
Figure 5.2: The dual split-ring resonator (SRR) and complementary splitting resonator (CSRR) metasurfaces are shown.	64
Figure 5.3: For the TEM incident wave shown, an SRR in the xz -plane is excited by the magnetic field. A CSRR in the yz -plane will be excited by the electric field. Note that the CSRR structure can only be excited if the TEM wave is incident either above or below the plate, but not both.	65
Figure 5.4: The numerical simulations of the magnetic field of a SRR structure and the electric field of a CSRR structure. A cross-section shows the fields are symmetric above the plane and symmetric, but in opposite directions, below the plane	66
Figure 5.5: The numerical simulations of the electric field of a SRR structure and the magnetic field of a CSRR near the gap. The fields are symmetric above the plane and symmetric, but in opposite directions, below the plane.	66
Figure 5.6: A comparison of the excitation of a CSRR structure in the parallel waveguide environment with respect to its orientation. The excitation is greater for the parallel excitation because the electric and magnetic dipole are excited simultaneous.	69
Figure 5.7: A fabricated negative index microstrip is shown that utilizes CSRR structures etched in the ground plane, for effective negative permittivity, and capacitive gaps in the top strip, for effective negative permittivity. The simulated S-parameters show a passband near 6 Ghz that signifies the negative index region.	69
Figure 5.8: A unit cell diagram is shown for a tunable double layer CSRR structure in a parallel waveguide transmission line. A tuneable dielectric can be place between the CSRR layers and biased with a voltage difference between the two layers. Also shown is S21(dB) (transmission) for a double layer CSRR structure separated by a 0.1 mm thick dielectric ($\epsilon_r = 4$). For comparison, a single CSRR Structure and a CSRR structure with the top plane replaced by a uniform ground plane are shown. The characteristic CSRR structure considered had a radius of 1.8 mm, notch width of 0.2 mm, neck width of 0.2 mm, and unit cell size of 3 mm.	72
Figure 5.9: The effect of changing the dielectric between the double CSRR structures with characteristic parameters given in Fig. 5.8 is shown.	73

Figure 5.10: A configuration for a tunable double CSRR structure implemented in the microstrip environment is shown. Numerical simulations show that for a liquid crystal whose molecules are aligned in the z -direction via a voltage bias between the two CSRR planes, the tuneability is up to 700 Mhz depending on the initial orientation of the liquid crystal molecules.	74
Figure 6.1: A diagram of a unit cell of the CSRR parallel plate waveguide is shown. When the CSRR is resonating, current flows along the edges of the structure and produces a magnetic field that is approximately perpendicular in the gap region. The extracted permeability and permittivity are shown for the frequency range including the first two resonances of the CSRR waveguide structure without the influence of the magnetic material.	77
Figure 6.2: The extracted permittivity for the 1st resonance is shown along with various fitting methods.	79
Figure 6.3: The parallel current sheets or <i>magnetic</i> capacitor approximation is shown for the gap region of the CSRR structure.	81
Figure 6.4: The left (line of equality) and right (1st/2nd hybrid resonance) side of Eq. 6.10 are plotted for $H_0 = 1.5$ kG and characteristic parameters stated in the text.	84
Figure 6.5: The dispersion curve for the 1st order hybrid mode of the CSRR/magnetic material waveguide is calculated both analytically and through numerical electromagnetic simulations as a function of the biasing field, H_0	85
Figure 6.6: The transmission results are shown for the HFSS electromagnetic simulation and the analytic prediction using effective medium theory.	86
Figure 6.7: The real part of the permeability is shown for the magnetic material for several magnetization values with the bias field set such that the antiresonance position matched the empty CSRR structure's resonance. The resulting hybrid permeability is also shown. .	87
Figure 7.1: (a) The SRR/YIG unit cell structure is shown. The characteristic dimensions of the structure are $d_3 = 2.4$ mm, $a_2 = 0.6$ mm, $a_3 = 0.04$ mm, $g = 0.4$ mm, substrate thickness of $s_2 = 0.2$ mm, and a unit cell size of 3 mm. (b) The extracted permittivity and permeability of the structure.	94
Figure 7.2: (a) The transmission simulation of the combined SRR/YIG structure, isolated YIG structure, and the analytic hybridization theory are shown for a biasing field of $H_0 = 3$ kG structure. (b) The extracted permeability over a range of biasing field, $H_0 = 1.7 - 4.5$ kG, is shown with steps of 0.1 kG.	95

Figure 7.3: Numerical simulations of the transmission for a one unit cell thick SRR/YIG metamaterial structure and the analytic theory are shown for a range of biasing fields. The position of the YIG resonance is noted by a vertical arrow and the splitting of the 1st and 2nd SRR resonances are noted for each bias value. The extracted permeability and permittivity of the numerical simulations are also shown.	97
Figure 7.4: (a) The characteristic dimensions of the SRR/YIG rod unit cell structure experimentally investigated by Yang et al. are shown with $d_1 = 2.2$ mm, $d_2 = 1.2$ mm, $c = 0.2$ mm, $g = 0.4$ mm, and $a_1 = 0.8$ mm, a substrate thickness of $s_2 = 0.9$ mm, and a unit cell size of 5 mm. (b) The numerically extracted permeability and permittivity of the structure are shown.	99
Figure 7.5: The numerically calculated transmission for the combined SRR/YIG structure investigated by Kang et al. is shown along with the analytically determined transmission determined through Eqs. 7.4-7.11. Also shown, is the numerically calculated transmission for the YIG rod structure only.	100
Figure A.1: Complementary planar structures and their field solutions. The structure, (a), and it's compliment, (b), can be broken into two boundary regions with complementary fields.	105
Figure A.2: The integration line along electric fields in boundary region 1 ($A \rightarrow B$) and boundary region 2 ($D \rightarrow C$) is shown in gray. . . .	107

ACKNOWLEDGEMENTS

This dissertation is the product of a sustained effort that would never have been possible without the continued and unfaltering support of my family and friends. In particular, I want to thank Marisea for her enduring support and deep friendship from afar.

When I first arrived in Durham, where I lived and worked on the research contained in this dissertation, I heard rumblings about the formation of a bike gang—not the ‘Hell’s Angels’ variety but the more socially responsible human powered version. Though intrigued, I never did find this mythical outfit. I did, however, come to depend on a gang of cyclist namely, Jack, Chris, Josh, Scott, Tom, Aloyse, Geraldine, David, and Jessie for friendship and diversion.

I also wish to thank my fellow graduate students: Soji, Vinh, Tong, Pae, Nathan, Tom, and Ruopeng for their extensive collaborations and camaraderie. It has truly been a pleasure to suffer the trials and tribulations of graduate school with these people.

Of course, I am also greatly indebted to my advisor David Smith. He once noted, in his understated manner, that by the ‘act of graduating’, “one is not the same person they are when they enter graduate school.” If I am a more proficient, more critical, and more effective investigator, it is largely due to the acumen I have developed under his guidance.

Much of the work described in this dissertation is the product of collaboration. Chapter 2 is in part a reprint of material as it appears in Jonah Gollub, Thomas Hand, Soji Sajuyigbe, Shawn Mendonca, Steve Cummer, and David R. Smith, “*Characterizing the effects of disorder in metamaterial structures*,” Appl. Phys. Lett. **91**, 162907 (2007). The dissertation author was the primary investigator of this paper.

Chapter 3 is in part a reprint of material as it appears in Jonah Gollub, David R. Smith, David C. Vier, Tim Perram and Jack J. Mock, “*Experimental characterization of magnetic surface plasmons on metamaterials with negative permeability*,” Phys. Rev. B **71**, 195402 (2005). The dissertation author was the primary investigator of this paper.

Chapter 4 is in part a reprint of material as it appears in Jonah Gollub, Bijoy Kuanr, Zbigniew Celinski, Robert Camley, and David R. Smith, “*Small Dimensional microstrips embedded with ferromagnetic layers: Numerical simulations and experimental results*,” **In Process:arXiv:0802.2599v1**. The dissertation author was the primary investigator of this paper.

Chapter 6 is in part a reprint of material as it appears in Jonah Gollub, David R. Smith, and Juan D. Baena, “*Hybrid resonant phenomenon in a metamaterial structure with integrated resonant magnetic material*,” **submitted Appl. Phys. Lett.** The dissertation author was the primary investigator of this paper.

Chapter 7 is in part a reprint of material as it appears in Jonah N. Gollub, Jessie Y. Chin, Tie Jun Cui, and David R. Smith, “*Hybrid resonant phenomenon in a SRR/YIG metamaterial structure*,” **submitted Opt. Express**. The dissertation author was the primary investigator of this paper.

VITA

1978	Born, Columbia, Missouri
2000	B. A. Reed College
2002–2005	Teaching assistant, Department of Physics, University of California San Diego
2004	M. A., University of California San Diego
2008	Ph. D., University of California San Diego

PUBLICATIONS

1. Jonah N. Gollub, David R. Smith, David C. Vier, Tim Perram and Jack J. Mock, “*Experimental characterization of magnetic surface plasmons on metamaterials with negative permeability*,” *Phys. Rev. B* **71**, 195402 (2005).
2. David R. Smith, Jonah Gollub, Jack J. Mock, Willie J. Padilla, and David Schurig, “*Calculation and measurement of bianisotropy in a split ring resonator metamaterial*,” *J. Appl. Phys.* **100**, 024507 (2006).
3. A. Degiron, D. R. Smith, J. J. Mock, B. J. Justice, and J. Gollub, “*Negative index and indefinite media waveguide couplers*,” *Appl. Phys. A* **87**, 321 (2007).
4. J. D. Baena, L. Jelinek, R. Marqués, J. J. Mock, J. Gollub, and D. R. Smith, “*Isotropic frequency selective surfaces made of cubic resonators*,” *Appl. Phys. Lett.* **91**, 191105 (2007).
5. Jonah Gollub, Thomas Hand, Soji Sajuyigbe, Shawn Mendonca, Steve Cumber, and David R. Smith, “*Characterizing the effects of disorder in metamaterial structures*,” *Phys. Rev. Lett.* **91**, 162907 (2007).
6. Jonah Gollub, Bijoy Kuanr, Zbigniew Celinski, Robert Camley, and David R. Smith, “*Small Dimensional microstrips embedded with ferromagnetic layers: Numerical simulations and experimental results*,” **In Process:arXiv:0802.2599v1**
7. Jonah Gollub, David R. Smith, and Juan D. Baena, “*Hybrid resonant phenomenon in a metamaterial structure with integrated resonant magnetic material*,” **submitted Appl. Phys. Lett.**
8. Ruopeng Liu, Xin Mi Yang, Jonah N. Gollub, Jack J. Mock, Tie Jun Cui, and David R. Smith, “*Gradient index waveguide structures using planar metamaterials*,” **submitted Appl. Phys. Lett.**

9. Thomas H. Hand, Jonah N. Gollub, Soji Sajuyigbe, David R. Smith, and Steven Cummer “*Characterization of Complementary Electric Field Coupled Resonator (CELC) Surfaces*,” **submitted Appl. Phys. Lett.**
10. Jonah N. Gollub, Jessie Y. Chin, Tie Jun Cui, and David R. Smith, “*Hybrid resonant phenomenon in a SRR/YIG metamaterial structure*,” **submitted Opt. Express**

ABSTRACT OF THE DISSERTATION

Characterizing Artificial Electromagnetic Materials and their Hybridization with Fundamentally Resonant Magnetic Materials

by

Jonah Nathan Gollub

Doctor of Philosophy in Physics

University of California San Diego, 2008

Professor Dimitri Basov, Chair

Professor David R. Smith, Co-Chair

The development of a new class of artificial materials, called metamaterials, can be harnessed to provide a wide range of electromagnetic properties beyond those of natural materials. These properties include positive and negative permeability and permittivity values, strong anisotropy, and the ability to construct spatially varying electromagnetic properties within the material. These unconventional electromagnetic materials have been used to demonstrate negative refraction and a cloaking structure among other interesting phenomena. To achieve their response, metamaterials employ periodic composite structures consisting of conductors and dielectrics that are shaped in various geometries which couple to incident electromagnetic waves. In this dissertation we investigate the introduction of fundamentally resonant materials into these geometrically resonant metamaterial structures. We are able to analytically characterize the unique hybridization that occurs. More specifically, we consider the integration of a resonant gyromagnetic materials into waveguide and bulk metamaterial structures. These hybrid structures are shown to provide a means of making strongly tunable metamaterial structures.

In this dissertation we initially investigate the physical nature of metamaterial structures. In Chapter 2, we consider the effect of geometrical disorder in the reso-

nant elements of a split-ring resonator (SRR) metamaterial and the resulting bulk constitutive parameters. In Chapter 3, we consider the potential for metamaterials to support novel surface modes. We experimentally demonstrate the excitation of magnetic surface polariton modes on a magnetic SRR metamaterial slab. In Chapter 4, as a precursor to considering how to integrate naturally magnetic materials into metamaterial structures, we consider ferromagnetic materials and their integration into small dimensional microstrip structures. We develop simulation techniques that accurately reproduce experimental results for these analytically intractable structures. In Chapter 5, we examine the metamaterial concept in the transmission line environment and consider the potential for constructing tunable metamaterial structures. Finally, in Chapter 6, we theoretically and numerically consider a metamaterial with incorporated naturally resonant magnetic material and demonstrate that it exhibits a unique hybrid resonance. In Chapter 7, we extend our analysis to a bulk hybrid metamaterial structure with integrated gyromagnetic material. In general, these hybrid structures show potential for extending the electromagnetic properties of metamaterials. They also provide a means of magnifying and decoupling the electromagnetic properties of the integrated resonant magnetic materials.

1

Introduction

A material's bulk electromagnetic attributes can ultimately be traced to the discrete field contribution of its constituent atoms and the thermodynamic laws that govern their composition within the material. But, when considering the interaction of matter with incident electromagnetic radiation, of which the wavelength is much larger than the inter-atomic spacing, it is sufficient to consider the spatial and time averaged response of the material. Indeed, this is the classical electrodynamic approach that was discovered long before the true atomic nature of matter was understood. In this approach, the material is viewed as a homogeneous medium with bulk electric permittivity (ϵ) and magnetic permeability (μ) parameters which relate the response of the material to applied electric and magnetic fields, respectively. Homogenization theory works surprisingly well for most wavelengths of the electromagnetic spectrum down to x-rays where it approaches the atomic length scale and field averaging finally breaks down. In fact, it is an interesting curiosity that thermodynamic considerations seem to fundamentally organize matter on a length scale not much larger than the atomic scale. One might conceive of imposing artificial structure upon natural matter at a larger scale, but still small with respect to the wavelength of incident radiation, in order to realize new electromagnetic properties. It is this perspective that has spurred the development of a new class of artificial materials called *metamaterials* that is the focus of this dissertation.

The thermodynamic organization of matter tends to fragment the electromag-

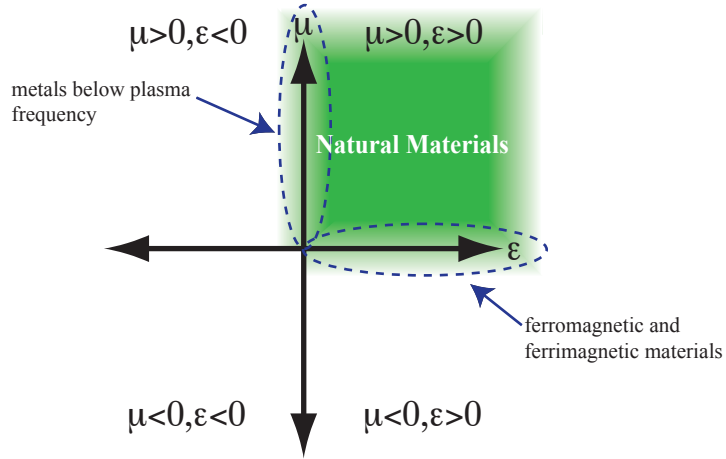


Figure 1.1: A diagram of the range of electromagnetic parameter space for natural materials. There are some natural materials that have a negative permittivity and permeability (as shown) but not simultaneously.

netic properties of natural materials into certain frequency ranges. Although, materials with a strong electric response can be found at most frequencies ranging from radio frequencies up to the ultraviolet, materials with a strong magnetic response are strikingly absent at frequencies above the low gigahertz regime. Further, if we consider the range of electromagnetic parameters for natural materials at all frequencies (sketched in Fig. 1.1), we notice an interesting omission; there are no materials which exhibit simultaneously negative permittivity and permeability. Veselago considered these so called negative index materials in a theoretical paper in 1968 and determined that they would exhibit novel and counterintuitive properties including a negative index of refraction and a group velocity that is antiparallel to the phase velocity [3]. At first glance negative index materials seem unphysical and it is true that the permittivity and permeability cannot be negative for static fields because it would violate conservation of energy [2]. However, negative permittivity or permeability values can result when an electromagnetic driving field oscillates with a frequency near a material's resonance just as an oscillating weight on a spring can be out of phase with its driving force. Near the resonance of an electric material, for example, the polarization is quite large due to the driving field dumping energy into the polarization over many cycles. As the frequency of

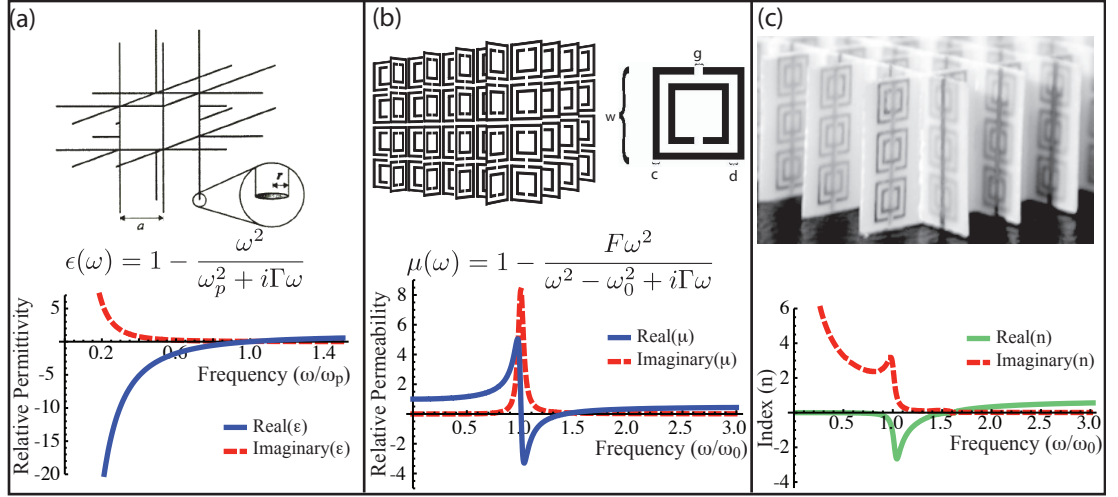


Figure 1.2: (a) The thin wire structure proposed by Pendry is shown and its characteristic dimensions. It exhibits a negative permittivity below the geometrically determined plasma frequency, ω_p . (b) The SRR structure proposed by Pendry is shown with its characteristic dimensions. It exhibits a negative permeability for a frequency range above the resonant frequency, ω , which is determined by the geometry of the structure. (c) A combined wire/SRR structure with simultaneous negative permittivity and permeability is shown. This structure was used by Smith et al. to demonstrate a negative index of refraction. Its characteristic index, n , is plotted

the driving field is swept through the resonance, and if the damping losses are not too large, the polarization flips from in-phase to out-of-phase with the driving field and the material exhibits a negative response [4]. Though Veselago realized there was no fundamental physical limitation preventing a material with simultaneously negative permittivity and permeability (often called a negative index material) from existing, he was unable to find any natural material with such attributes and he eventually abandoned this line of research. Fundamentally, the thermodynamic properties at the atomic scale, which govern the natural organization of matter, do not readily sustain these negative index properties.

Almost 30 years after Veselago's theoretical work, a new approach to designing "artificial" materials was developed which made it possible to realize these novel properties. John Pendry et al. [5] first proposed a thin wire mesh structure with a spacing much less than the wavelength of the radiation of interest, see Fig. 1.2(b). His initial motivation was to make a material with a moderately negative permit-

tivity in the microwave regime. By manipulating the geometry of a wire mesh he recognized it was possible to create a material with properties substantially different from its underlying constituent materials, i.e. metal and air. In the wire mesh, electrons are confined to move along the wire structure and the average charge density is effectively reduced compared to a bulk metal. Incident radiation cannot sense the discrete wire geometry but instead is influenced by the effective average charge density of the composite volume. In a sense, the structure can be thought of as a second homogenization of matter and hence sidesteps the thermodynamic constraints at the atomic level. Pendry derived that the wire mesh has a resonant permittivity with a characteristic Drude-Lorentz form given by,

$$\epsilon(\omega) \approx 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}, \quad (1.1)$$

where Γ is the loss factor and ω_p is the plasma frequency. Noticeably, this is the same resonant form metals possess, but the plasma frequency for metals is a function of the electron properties,

$$\omega_p = \frac{ne^2}{m_e}, \quad (1.2)$$

where n is the electron density, m_e is the electron mass, and e is the electron charge [6]. These values are effectively the same for most metals and ω_p is usually in the ultraviolet. Alternatively, for the wire structure we can define an effective “electron” density and “electron” mass, n_{eff} and m_{eff} , which are functions of the structure parameters, a , the wire spacing, and, r , the wire radius. The plasma frequency is then found to be,

$$\omega_p^2 = \frac{n_{\text{eff}}e^2}{m_{\text{eff}}} \approx \frac{2\pi c^2}{a^2 \ln(a/r)}. \quad (1.3)$$

Since the wire material has a permittivity that is determined by geometry, it can have a much lower plasma frequency than metals. For metals at microwave and lower frequencies, the permittivity is strongly negative, whereas the wire structure can be designed to have a more moderately negative permittivity in this spectrum. The resulting material provides access to interesting electric properties at frequencies that natural materials cannot provide.

Pendry et al. also suggested it was possible to design a magnetic structure with negative permeability at a frequency determined by geometrical parameters [7]. This was especially intriguing because, as mentioned previously, materials with a strong negative permeability are relatively uncommon; they include a few resonant ferromagnetic and antiferromagnetic systems [8, 9] that tend to have a response in the low gigahertz frequency range usually over a small bandwidth. The artificial structure suggested by Pendry was a split-ring resonator (SRR) structure composed of metal planar elements with subwavelength dimensions as illustrated in Fig. 1.2(b). A simple model for the SRR element is a driven “ LC ” circuit. A LC circuit has a characteristic inductance, L , and capacitance, C , that results in a resonance when the driving voltage has a frequency of $\omega_0 = 1/\sqrt{LC}$. Correspondingly, the SRR element is excited by an oscillating magnetic field perpendicular to the plane of the metallic ring. The driving field induces an electromotive force in the structure that drives current around the conducting ring with some associated inductance. A split in the ring generates an associated capacitance in the structure. Then, the resonant frequency of the SRR is, $\omega_0 = 1/\sqrt{L_g C_g}$, where the inductance, L_g , and capacitance, C_g , are determined by the geometry of the SRR structure. Though finite in size, when the individual SRRs are assembled into a composite structure, the electromagnetic response of the resulting material can effectively be treated as a uniform medium with a characteristic Drude-Lorentzian magnetic response given by [7],

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}, \quad (1.4)$$

where, $F \approx \pi(w/2)^2/a^2$ is the filling fraction, $\omega_0 = 1/\sqrt{L_g C_g} \approx 3/(\pi^2 \mu_0 r^2 \ln(\omega d(w/2)))$ is the resonant frequency, and $\Gamma \approx 2a\sigma/((w/2)\mu_0)$ is the damping factor [note: σ is the conductivity, a is the unit cell size, and the other geometrical quantities are defined in Fig. 1.2(b)]. For the SRR structure, the permeability is negative over a band of frequencies between ω_0 and $\omega_{mp} = \omega_0/\sqrt{1-F}$ (for small Γ).

Smith et al. made the remarkable discovery in 2000 [10], that the wire and SRR structures could be combined to form a composite material with simultaneously negative permittivity and permeability. Furthermore, experiments demonstrated

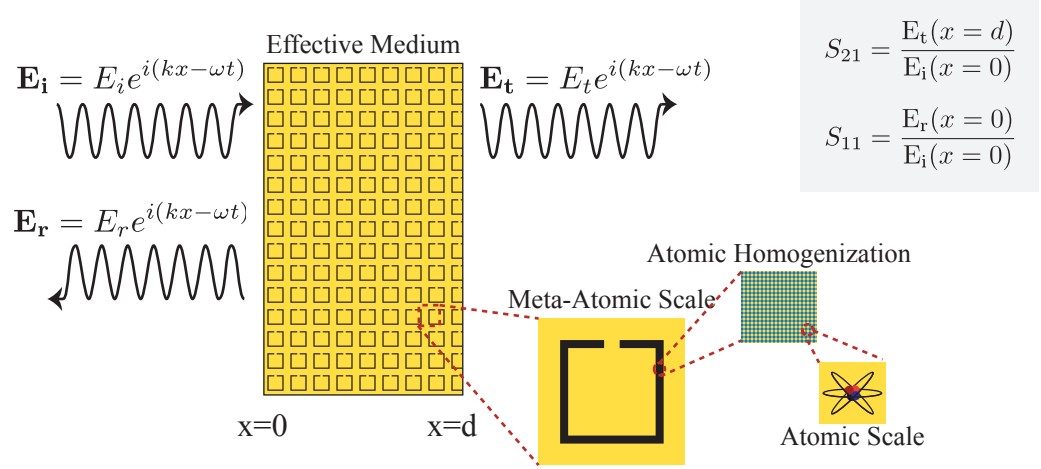


Figure 1.3: A metamaterial is effectively a second homogenization of matter. The effective constitutive parameters can be calculated through an inversion of the transmission and reflection coefficients (or related S-parameters) as discussed in the text.

that the material exhibited a negative index of refraction and anti-parallel phase and group velocity—hence, realizing Veselago’s theoretical negative index material [11]. In addition to demonstrating the unique material properties of a negative index material, Smith’s experiment validated the so called *metamaterial* approach in which periodic and resonant structural elements can be harnessed to provide a diverse range of electromagnetic properties. Just as for natural materials where constituent atomic resonances contribute to a bulk electric or magnetic polarizability per-unit-volume, artificially constructed *meta-atoms* can be tailored and arranged to manifest bulk electromagnetic properties (see Fig. 1.3). The best known resonant elements used in metamaterials are the wire and SRR structures, introduced above, but there are potentially an infinite number of geometries that can be considered.

The initial focus of metamaterials research has been largely concentrated on establishing the theoretical link between the properties of the meta-atom constituents and their resulting bulk electromagnetic properties. The geometrical complexity of metamaterial structures makes an exact theoretical analysis intractable, and therefore, numerical analysis has become an invaluable tool. In fact, the rapid development of metamaterials can be partially credited to the modern advancements

and proliferation of electromagnetic simulation software. The standard approach to determine the effective constitutive parameters of a metamaterial is to use an electromagnetic solver to perform a full-wave analysis of a unit cell of the metamaterial structure. The periodicity of the elements constituting a metamaterial structure make it only necessary to simulate one unit cell, with symmetric boundary conditions, in order to determine the equivalent properties of the full metamaterial structure. Full wave electromagnetic simulations of the unit cell structure are used to determine the transmission and reflection coefficients which in turn are mathematically inverted to extract the effective permeability and permittivity parameters. In this dissertation we extensively use Ansoft's HFSS finite element solver, a commercially available software, to perform our analysis.

The general procedure for analyzing a metamaterial structure is to first perform a HFSS simulation of the metamaterial's unit cell in order to determine the scattering matrix components, or S-parameters, which are related to the reflection and transmission coefficients as shown in Fig. 1.3. Then, the index, n , and impedance, z , are determined through a S-parameter inversion given by [12],

$$n = \frac{1}{k_x d} \left\{ \pm \cos^{-1} \left[\frac{1}{2S_{21}} (1 + S_{21}^2 - S_{11}^2) \right] + 2m\pi \right\}, \quad (1.5)$$

$$z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}}, \quad (1.6)$$

where m is an integer that is determined by continuity constraints across the frequency regime and d is the unit cell thickness [13]. Finally, the constitutive parameters are calculated through the relation $\epsilon = n/z$ and $\mu = nz$. To design a metamaterial with specific electromagnetic properties an iterative process is undertaken as shown in Fig. 1.4. An initial guess is made for the necessary geometry of the metamaterial (aided by analytic theory) and then the constitutive parameters are extracted from numerical simulations of the structure. The geometry is adjusted as necessary until the desired properties are obtained.

With design control of the metamaterial structure down to the meta-atom scale it is also possible to implement parameter gradients inside the material. This is a truly powerful tool because most natural materials have uniform properties inside the material. For example, modern optical design is largely a matter of manip-

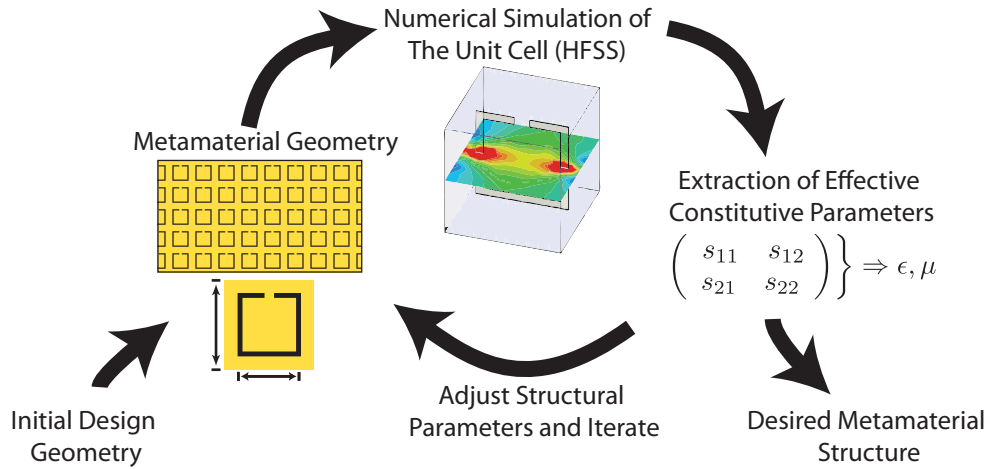


Figure 1.4: Designing a metamaterial structure is an iterative process. A rough analytic guess is used to determine the initial geometry and then a full wave numerical simulation is performed on the structure to determine the associated scattering matrix. Effective constitutive parameters are then extracted from the scattering matrix data and the geometry is adjusted until the necessary electromagnetic properties are achieved.

ulating interfaces between two materials—e.g. a concave surfaces for a focusing lens or a convex surfaces for a diverging lens. The ability to structurally design electromagnetic gradients in metamaterials has been harnessed to make a gradient index lens [14] which can have superior optical properties over regular lenses. In fact, the multiplicity of electromagnetic properties and spatial control of electromagnetic properties inside a metamaterial makes for an almost limitless range of applications. This fortunate conundrum has stimulated the development of a new field called transformation optics, in which one first defines the desired optical path of light in a material and then deduces the necessary material properties through a coordinate transformation. In 2007, this method was used to determine the necessary properties of a cloaking structure [15] in which light is smoothly redirected around an object completely concealing it from the electromagnetic radiation. Soon after this theoretical proposal, a cloaking structure was fabricated using a metamaterial structure with the necessary spatially varying constitutive parameters [16].

The concept of “properties through structural design” provides extraordinary

flexibility in engineering materials—whether the desired electromagnetic properties are positive, negative, or highly anisotropic. We can consider extending the properties of metamaterials even further by incorporating fundamentally resonant materials into these geometrically resonant structures. As we will explore, these hybrid metamaterials provide a means of creating tunable metamaterials and further have a unique physical response that is different from either of the contributing resonant responses (i.e. the resonant material or the resonant geometry). In addition, these hybrid structures provide a means of decoupling the resonant electromagnetic response of a fundamental resonant material from its less desirable electromagnetic properties through targeted integration of the material in the metamaterial structure.

The research in this dissertation was largely motivated by our interest in developing a physical understand of hybrid metamaterial structures. We focus in detail on the physical nature of these hybrid structures in the final two chapters. The research contained in the first half of this dissertation is linked by our effort to develop a foundational understanding of metamaterials and resonant magnetic materials. In Chapter 2, we consider the effects of disorder in the resonant elements of a SRR composed metamaterial structure and determine the resulting constitutive parameters. In Chapter 3, we explore the potential for metamaterials to support surface modes. The use of metamaterials can provide access to a full range of surface modes which Maxwell’s equations predict yet natural materials do not support. We demonstrate the flexibility of metamaterials by experimentally exciting surface polariton modes on a magnetic SRR metamaterial slab. In Chapter 4, we consider the use of ferromagnetic materials, which have a fundamental magnetic resonance, as a means of making tunable microstrip devices. We develop numerical simulation techniques that characterize these structures and agree extremely well with experimental results. In Chapter 5, we introduce the metamaterial concept in the transmission line environment and consider structures for constructing tunable metamaterials in this environment. Having established a principle understanding of metamaterials and resonant gyromagnetic materials through our initial research investigations, we turn our focus to considering hybrid resonant structures. Specif-

ically, we focused on integrating a resonant gyromagnetic material into a metamaterial structure. In Chapter 6, we consider a waveguide metamaterial structure with a characteristic resonant gyromagnetic material incorporated into the structure. Using numerical simulations, we demonstrate that this structure exhibits a unique hybrid resonance. We further apply an analytic model that accurately predicts the resulting hybridization. In Chapter 7, we extend our analysis to a bulk SRR structure with integrated magnetically resonant ferrite material. Using the same analytic model we developed for the waveguide structure, we are able to very accurately characterize the hybrid interaction of this structure. We further note the inadvertent observation of this hybrid interaction in a recently published experiment.

In general, this dissertation provides an aggregation of numerical and theoretical studies of hybrid structures that can be drawn upon to design new and advanced metamaterials.

2

Characterizing Disorder in Magnetic Split-Ring Resonator Metamaterials

In this chapter we investigate the effects of disorder on metamaterial samples composed of split ring resonators with randomly introduced variation in their geometrical dimensions. We demonstrate that disorder broadens the negative permeability band and introduces effective losses into the system. Transmission measurements on samples with varying degrees of disorder are found to be in excellent agreement with predictions based on standard homogenization theories.

2.1 Introduction

A class of artificial electromagnetic media, called metamaterials, has provided access to electromagnetic properties beyond those of natural materials [7, 10, 17]. These materials derive their unique properties from the collective response of many periodic subwavelength resonant structures. Metamaterial designs generally assume that these structures are identical, but in reality, fabrication error will introduce geometrical perturbations and cause some degree of deviation in their resonant properties. As has been shown in several prior studies [18, 19], a variation in the geometrical parameters of resonant metamaterial elements results in a

spreading of the oscillator strengths across a finite bandwidth, effectively reducing the strength of the collective resonance; increasing the apparent loss; and introducing other artifacts into the effective constitutive parameters. We demonstrate these effects experimentally in the microwave regime with a magnetic metamaterial consisting of split-ring resonators (SRRs) fabricated with varying amounts of disorder. We also explore a predictive model to determine the effective electromagnetic parameters of these structures from numerical simulations. Numerical calculations furthermore suggest that normal geometrical variations cannot vary more than 5.5% to maintain a negative permeability regime. This is easily satisfied with photolithographic methods in the microwave regime, but fabrication accuracy declines as metamaterial geometry is scaled down, and this becomes a potential limitation for samples being considered at terahertz and optical frequencies [20].

2.2 Experiment

To investigate the effect of the variation in metamaterials, we intentionally designed structural variations into SRR unit cells resonant at microwave frequencies and distributed these structures randomly throughout the media. Numerical simulations of unit cells, carried out using HFSS in a standard manner (Ansoft, a commercial finite-element solver) [12], demonstrated that varying the width of the outer and inner copper ring (while holding the unit cell size, gap size, and ring trace width constant), as shown in Fig. 2.1, excited a large range of resonant frequencies. Using these simulations as a guide, the structural parameters of a set of SRR structures were chosen to give an approximately linear distribution of resonant frequencies over 0%, $\pm 4\%$ (with 9 distinct cells), $\pm 14\%$ (with 21 distinct cells), $\pm 19\%$ (with 21 distinct cells), and $\pm 25\%$ (with 21 distinct cells) with a mean value of 12.5 GHz. The SRRs were designed so that their resonance occurred over a band of microwave frequencies consistent with our measurement equipment.

To determine effective constituent parameters for the fabricated samples, it was necessary to apply a mixing model. A common model is Maxwell-Garnett [21], in which a sufficiently dilute material is assumed such that a hierarchy of macroscopic

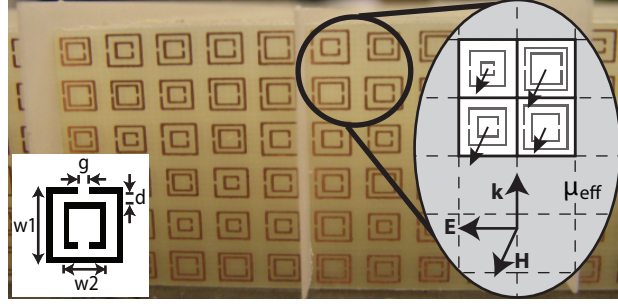


Figure 2.1: The characteristic dimensions of the SRR structure are shown. The parameters $w1$ and $w2$ were varied in the samples to induce various amounts of disorder as pictured. The SRR used in the 0% SRR structure had dimensions $g = 0.22$ mm, $c = 0.08$ mm, $w1 = 2.25$ mm, $w2 = 0.45$ mm, and a unit cell size of 3.33 mm. Also shown is an illustration of the Bruggeman model for our structure in which the magnetic dipole moment of SRR unit cells are determined with respect to an average background permeability.

and microscopic fields can be defined. But, the underlying assumption of diluteness becomes suspect when investigating metamaterials which are composed of closely packed and strongly resonant structures. An alternative model, which we apply here, is the Bruggeman mixing model [21, 22]. The Bruggeman model applies to mixtures of two or more media, and makes no distinction between species, such as host or inclusion. The Bruggeman model is thus conceptually closer to our experimental configuration. To determine the effective permeability of the metamaterial structure, we assign each unit cell its associated bulk metamaterial permeability. These “macroscopic” parameters are straightforwardly obtained with present numerical methods [12]. The magnetic dipole moment of each unit cell is then found with respect to a yet to be determined homogenous “background” effective permeability as illustrated in Fig. 2.1. The Bruggeman model assumes spheroid constituents, but in the quasi-static limit, where the unit cell is much smaller than the operating wavelength (as we are interested in here), the polarization of a cube is equivalent to that of a spheroid [23]. By self-consistency, the dipole moments (with respect to the background) must sum to zero and the material’s effective permeability can be determined. The general Bruggeman equation for the perme-

ability and similarly the permittivity are then

$$\sum_{i=1}^n f_i \frac{\mu_i - \mu_{\text{eff}}}{\mu_i + 2\mu_{\text{eff}}} = 0, \quad \sum_{i=1}^n f_i \frac{\epsilon_i - \epsilon_{\text{eff}}}{\epsilon_i + 2\epsilon_{\text{eff}}} = 0, \quad (2.1)$$

where μ_i and ϵ_i are the parameters of the constituent materials and f_i is the filling fraction of each element such that $\sum_i^n f_i = 1$.

To apply the Bruggeman model, it is necessary to extract the effective bulk parameters of each distinct SRR structure. The commercially available electromagnetic simulator *Ansoft HFSS* was used to determine the effective permeability and permittivity [12] for each unique SRR structure. These “homogenous” material parameters were applied to Eq. 2.1 and a numerical root finding algorithm was implemented in *Mathematica* to deduce the effective Bruggeman material parameters μ_{eff} and ϵ_{eff} . The retrieved μ_{eff} and ϵ_{eff} were then used in the Fresnel equations [12] to determine the theoretical transmission coefficient to which measured transmission was compared.

The SRR metamaterial structures were fabricated on FR4 sheets (250 μm thick with 17 μm copper thickness) using a standard optical lithographic method. A mask was designed consisting of strips of 3.33 mm SRR unit cells extending 6 cells wide and 45 cells long. *Mathematica* was used to generate the layout and to randomize the positions of the SRR unit cells on the mask. Upon completion of the photolithographic process, the strips of SRRs were cut out to dimensions 2 cm in width (6 unit cells) and 15 cm in height (45 unit cells). Supporting cross strips, constructed from bare FR4 substrate material, were used to assemble the strips into three-dimensional slabs as shown in Fig. 2.1. In total five samples were constructed with various amounts of disorder in the SRR structures.

A two horn antenna system was used to measure the transmission through each disordered metamaterial sample. Rexolite plano-convex lenses were placed in front of each antenna so that the exiting beam from the first horn was collimated and focused at 30 cm to a 7 cm diameter beam. The horn antennas were connected to a network analyzer (Agilent N5230A PNA-L) and the transmission between both the horns was calibrated. The random media samples were placed at the focal point of the lens and measured. The significant size of the sample (15 cm by 15 cm) ensured that the focused 7 cm microwave beam was fully incident on

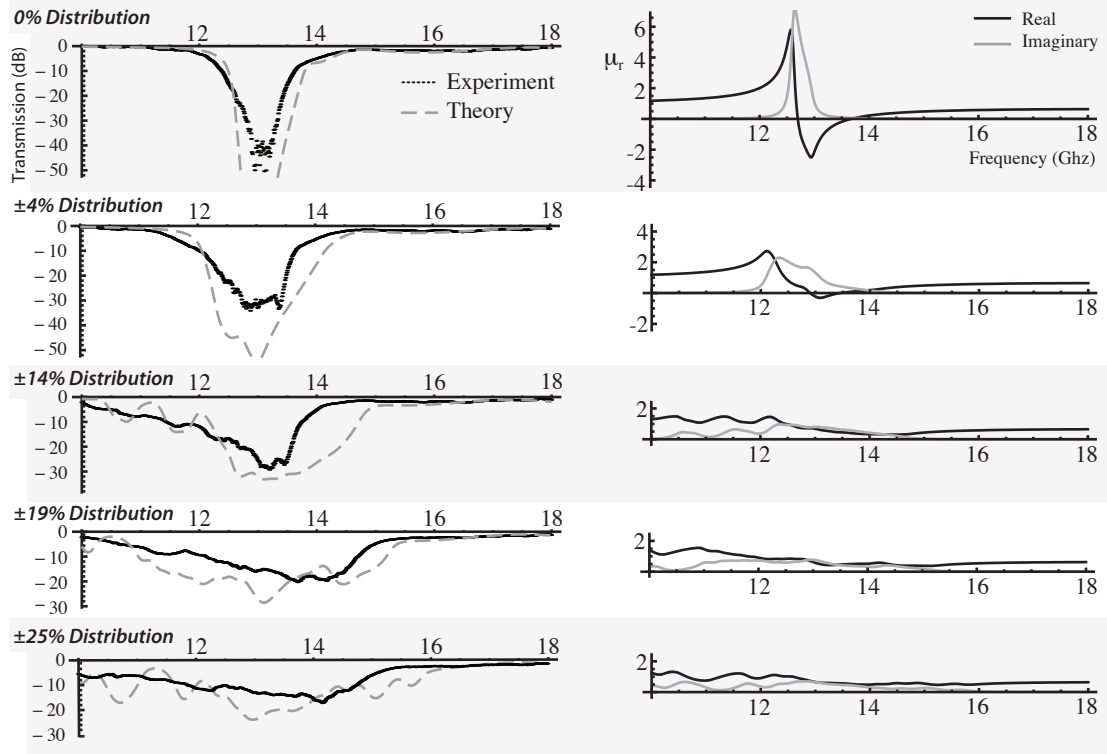


Figure 2.2: The experimental data is shown for transmission through disordered metamaterials of various linear distributions. Theoretical transmission for the sample is calculated using the Bruggeman mixing model to determine the material's effective permeability (and permittivity—not shown).

the sample. The focused beam illuminated approximately 1700 unit cells along its path through the sample, providing a well distributed measurement.

In Fig. 2.2 the results of the transmission experiment for the disordered metamaterials samples are plotted versus the theoretical transmission calculated using the Bruggeman effective permeability and permittivity parameters. Good correlation is seen for each of the disordered samples considered. As the distribution in the samples is increased, the stop band is widened and the response decreased, consistent with effective additional loss in the permeability of the material. The numerically determined Bruggeman effective permeability is also shown for each sample and predicts that the negative region should only exist for 0% and $\pm 4\%$ distributions. It's useful to note that a metamaterial can have a strong stop band without its parameters necessarily taking on negative values—simply due to impedance

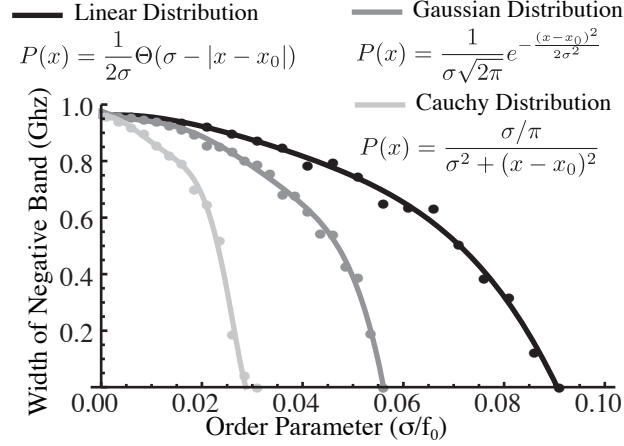


Figure 2.3: For a characteristic SRR structure in our experiment ($f_0 = 12.5$ GHz, $Q = 78$, $F = 0.14$), a numerical calculation of the width of negative region as a function of disorder introduced is shown for linear, Gaussian, and Cauchy distributions.

mismatch of its positive parameters with free space.

2.3 Numerical Investigation

We further performed a numerical investigation to determine the effect of disorder on the negative permeability band of our SRR structures for several distributions. For simplification, we assumed a Lorentz form for our constituent material permeabilities,

$$\mu_i = 1 - \frac{F\omega^2}{(\omega_{0j}^2 - \omega^2 + i(\omega\omega_0/Q))}. \quad (2.2)$$

The general parameters, $F = .14$ and $Q = 78$, were determined by fitting the Lorentz function to the numerically extracted permeability of our uniform SRR metamaterial with geometry listed in Fig. 2.1 ($\omega_0 = 12.5$ GHz). A 500 hundred element material was considered with linear (as in the experiment), Gaussian, and Cauchy distributions in the resonant frequencies, ω_0 , and the Bruggeman formula was solved with a numerical root finding method. The width of the negative band of the extracted permeability is shown in Fig. 2.3 as a function of the order parameter σ/f_0 where σ is a parameter of the probability functions as defined in the figure. For

the linear case, the numerical results are consistent with the extracted Bruggeman parameters for our samples. That is, the negative band does not exist for variation above $\pm 9\%$. One might also consider the Gaussian distribution that one would expect from normal fabrication error of a metamaterial. As is shown, the standard deviation cannot be greater than $\pm 5.5\%$. This cannot be translated into an exact constraint on geometrical variation since it depends on the specific parameter that is varied, but it can be noted that simulations show that geometrical variation translates into a slightly greater resonant frequency variation, so 5.5% can be used as conservative estimate on the allowable standard deviation of our geometrical structure in order to maintain a negative band. This is slightly larger than the theoretical predictions by Shadrivov et al. [18] who used a Cauchy distribution in their theory (we plot the Bruggeman parameters for a Cauchy distribution here as a comparison).

2.4 Conclusion

In summary, transmission through metamaterials composed of disordered SRR structures were measured and their spectra matched very closely with that predicted by a Bruggeman mixing model. Numerical modeling also suggest that tolerances for maintaining a negative band are about $\sigma = \pm 6\%$ for normal fabrication. This study of disorder also has implications for non-geometrically induced variability that might be found in tunable [24] and active [25] metamaterials due to integrated electrical components.

3

Magnetic Polaritons

The development of metamaterial structures has provided the means to engineer materials with a wide range of electromagnetic properties beyond those available with natural materials. This has led to the realization of novel wave propagation inside bulk metamaterial structures including wave propagation with anti-parallel phase and group velocity. Alternatively, we can consider the expanded range of surface polaritons (surface modes) that metamaterials may be able to sustain. In this chapter we first review the scope of surface polariton modes predicted by Maxwell's equations when given access to the wider range of electromagnetic parameter space afforded by metamaterial structures. We then experimentally demonstrate magnetic surface polaritons on a metamaterial structure. We also verify that the effective medium theory, which has been successfully applied to bulk metamaterial structures, is also applicable to surface mode phenomenon on metamaterial structures.

3.1 Introduction

Maxwell's equations predict the existence of propagating electromagnetic modes, often called surface polaritons (SPs), at the interface between two materials with oppositely signed permittivity, ϵ , or permeability, μ . SPs on materials with negative ϵ were first predicted and demonstrated in the 1950s [26], and have since been extensively studied in a variety of contexts [27]. The most commonly known

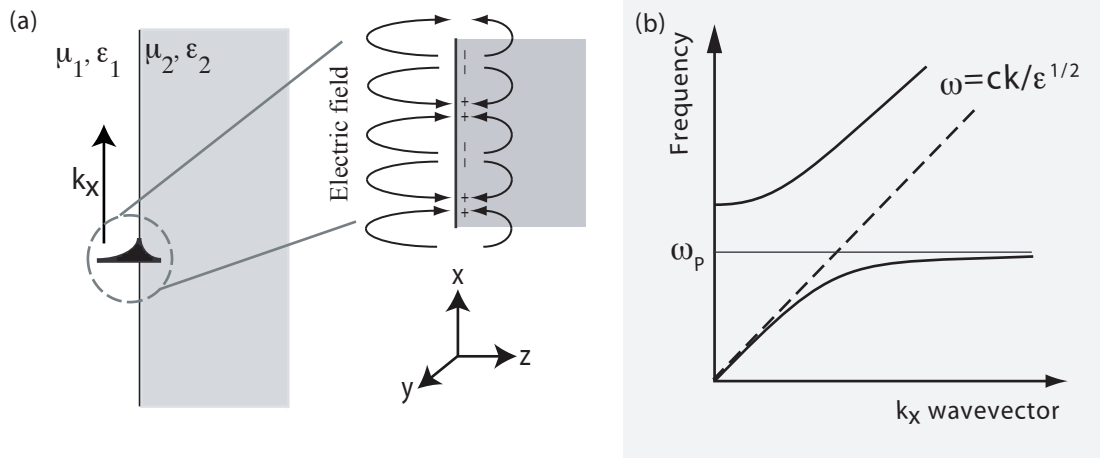


Figure 3.1: (a) A surface polariton travels along an infinite half-space boundary between two materials with permeability and permittivity indicated. A cutout shows the characteristic electric fields of an electric surface polariton. (b) The surface plasmon dispersion curve is plotted and relates frequency to propagating wavevector (k_x). The wavevector is to the right of the bounding dielectric lightline for frequencies below ω_p .

materials that support SPs are the noble metals—silver and gold (often called surface plasmons in this context). In these and in many other metals, the frequency dependent dielectric function is approximately free-electron like, characterized by a plasma frequency (usually in the UV), below which ϵ is negative. Because the plasma frequency for most metals is in the UV, SPs naturally tend to occur at wavelengths ranging from UV to visible.

The SP represents a coupling of the electromagnetic field with the conduction electrons in a metal. In a SP, charge density oscillations occur just along the interface of the metal and a second material whose $\epsilon > 0$. The resulting coupled electromagnetic state propagates along the interface of the metal as illustrated in Fig. 3.1(a). The electromagnetic fields of the SP mode perpendicular to the surface are evanescent and decay into the bounding materials. As a result, the mode is nonradiative as seen by mapping out the dispersion curve, shown in Fig. 3.1(b) (the wavevector is to the right of the lightline for frequencies $\omega < \omega_p$). The associated wavelength of the SP propagating mode is much smaller than its free space mode equivalent. This momentum mismatch prevents direct coupling between the sur-

face mode and a free propagating photon in the bounding materials. To overcome this mismatch, which must be done if you are interested in exciting these modes, it is necessary to couple using a prism, grating, or scattering event as is explained below.

Surface polaritons have long been appreciated for their ability to localize light to a scale not restricted by the wavelength. The resulting localized fields usually exhibit large enhancements over the incident field, making SPs useful probes of their local environment. The extreme sensitivity of the SP on gold films, for example, has been used as the basis for highly sensitive biochemical sensors [28, 29, 30]. These biochemical sensors exploit the sensitivity of surface modes to adjacent material parameters [31]. Biomolecular or chemical recognition elements are attached to a metal/solution boundary. When the element of interest binds to the surface, the local permittivity at the surface of the metal is altered. This small change in permittivity at the boundary strongly affects the propagation constant, which can then be monitored to determine the concentration of the substance.

With the recent advances in nanofabrication capabilities, SPs are being scrutinized as a means of manipulate light [32, 33, 34]. In this regard, SPs have been proposed as a method of scaling down optical devices to nanoscopic dimensions. The ratio of wavevector to frequency of surface modes is determined by the bounding material parameters and can be quite large (corresponding to a small wavelength), see Fig. 3.1(b). This suggest the possibility of converting light signals to surface modes on a photonic microchip, processing them on a subwavelength scale (beyond the diffraction limit of light) and then converting them back to light [35]. In this way, a variety of optical components (waveguides, switches, couplers, etc) could all be condensed down onto a microchip.

While there has been considerable analysis and application development for electric SP structures—those which require a material with negative ϵ , it is also possible for SPs to occur between two materials having opposite signs of μ . The difference between magnetic and electric SPs is that effective magnetic surface charges rather than electric surface charges participate in the coupled electromagnetic mode. In general, electric SPs will have p-polarized electromagnetic fields

(magnetic field perpendicular to propagation) while magnetic SPs will have s-polarized electromagnetic fields (electric field perpendicular to propagation). Naturally occurring systems such as resonant ferromagnets and antiferromagnets exhibit a band of negative μ just above the resonant frequency [8]. In fact, a convenient method for probing the properties of a negative μ frequency band is to excite magnetic SPs and characterize their dispersion [9]. Unfortunately for the prospect of using magnetic materials in potential applications directly, the frequency range over which the permeability is negative is considerably smaller than for available materials with negative permittivity.

SP modes can also occur on textured metallic surfaces, such as gratings, where their dispersion properties can be tuned by adjusting the surface patterning [36]. Textured conducting surfaces or structured conducting materials can drastically change the frequency range over which SP modes occur; in particular, the surface properties can predominate over the inherent material properties, such that SPs can be observed at microwave frequencies on suitably designed conducting gratings [35]. Since the surface properties of gratings or other structured materials can be significantly distinct from the underlying constituent materials, it might be expected that artificially patterned structures would support magnetic SPs. Over the past several years, there has been a renewed interest in the development of artificial materials to achieve electromagnetic responses that are difficult or impossible to achieve in conventional materials. A periodic array of conducting wires, for example, can be characterized via effective medium theory as having an ϵ with the same plasmonic form as silver, but with a plasma frequency determined by the wire radius and spacing [37]. Wire structures have been fabricated that have effective plasma frequencies in the microwave regime [5], and which have been shown to support electric SPs [38].

In 1999, it was suggested that an array of planar conducting, resonant elements, termed split ring resonators (SRRs), could behave as an artificial medium with a predominantly magnetic response to incident electromagnetic radiation [37]. The electromagnetic properties of the composite structure can be described by a frequency dependent μ similar in form to the Drude-Lorentz model, with a resonant

frequency ω_0 and a magnetic plasma frequency ω_{mp} where $\mu = 0$. If the SRRs are placed close enough together, and the losses in the conductors are reasonably low, then the effective oscillator strength of the SRR medium can be large enough that the effective μ is negative above ω_0 and below ω_{mp} . The negative μ response of the SRR medium was demonstrated in the context of a negative index metamaterial [10], in which the SRR medium was combined with a wire medium to produce a composite for which both ϵ and μ were negative over a range of microwave frequencies.

Our goal is to explore the excitation of SPs on a magnetic metamaterial composed of an SRR array. The SRR medium is designed such that the effective permeability μ is negative over a band of frequencies in the microwave regime. The occurrence of magnetic SPs at the interface of an artificially structured metamaterial, simply and correctly described by a Drude-Lorentz model, illustrates the utility of the metamaterials approach to realizing phenomena difficult to achieve in conventional materials. In studying the magnetic SPs here at microwave frequencies, we apply the same techniques used to excite SPs on thin metal films at optical wavelengths.

3.2 Derivation of Polariton Modes

To model the expected characteristics of the SPs, we first consider an infinite half-space of material in the region $z > 0$, see Fig. 3.1(a). The surface mode solutions to Maxwell's equations may be grouped into two different families, namely s and p polarized electromagnetic waves at the boundary of the two materials. The electric field for s -polarized waves lies parallel to the interface; the electric field for p -polarized waves lies in the plane of incidence. The specifics of these modes can be solved for by considering an infinite half space as shown in Fig. 3.1(a). We analyze the case of s -polarized fields (magnetic surface polaritons) in order to be consistent with the experiments presented later. The general s -polarized inhomogeneous propagating solution to the wave equation is

$$\mathbf{E}_j = (0, E_{y_j}, 0)e^{i(k_{x_j}x - \omega t)}e^{-k_{z_j}|z|} \quad (3.1)$$

$$\mathbf{H}_j = (H_{x_j}, 0, H_{z_j}) e^{i(k_{x_j} x - \omega t)} e^{-k_{z_j} |z|} \quad (3.2)$$

where ($j = 1, 2$) refers to the two bounding materials, respectively. Note that phase matching at the boundary requires $k_{x_1} = k_{x_2} \equiv k_x$. For simplicity, we define k_{z_j} to be positive and assign it's sign explicitly. From the scalar wave equation, $(\nabla^2 + \epsilon_j \mu_j (\omega/c)) E_y = 0$, the dispersion relation can be derived in each material to be [27],

$$k_x^2 = \epsilon_j \mu_j \left(\frac{\omega}{c} \right)^2 - k_{z_j}^2. \quad (3.3)$$

where j refers to one of the two bounding materials. At the interface, the tangential components of the electric and magnetic fields H_x and E_y must be continuous. This constraint yields a relation between the normal wavenumbers

$$\frac{k_{z_1}}{\mu_1} + \frac{k_{z_2}}{\mu_2} = 0. \quad (3.4)$$

The wavenumbers k_{z_1} and k_{z_2} were defined to be real and therefore $\mu_2/\mu_1 < 0$. Solving for k_x using Eq. 3.3 and Eq. 3.4 gives,

$$k_{x(\text{magnetic})} = \frac{\omega}{c} \sqrt{\frac{\mu_1 \mu_2 (\epsilon_1 \mu_2 - \epsilon_2 \mu_1)}{\mu_2^2 - \mu_1^2}}. \quad (3.5)$$

There is a dual relationship for p -polarized surface modes (electric surface polaritons) that is derived in a parallel method. The wavenumbers k_{z_1} and k_{z_2} are related by

$$\frac{k_{z_1}}{\epsilon_1} + \frac{k_{z_2}}{\epsilon_2} = 0, \quad (3.6)$$

and the propagating mode is

$$k_{x(\text{electric})} = \frac{\omega}{c} \sqrt{\frac{\epsilon_1 \epsilon_2 (\mu_1 \epsilon_2 - \mu_2 \epsilon_1)}{\epsilon_2^2 - \epsilon_1^2}}. \quad (3.7)$$

In order for a surface polaritons to be nonradiative, the propagation vector must be real and greater than the maximum photon wavevector of either bounding material ($k_x > \max(\sqrt{\epsilon_1 \mu_1} \omega/c, \sqrt{\epsilon_2 \mu_2} \omega/c)$).

We now derive the viable magnetic polariton modes, given the non-radiative constraint, as a function of the bounding materials' permittivity and permeability properties (note that the derivation of the electric modes is similarly determined).

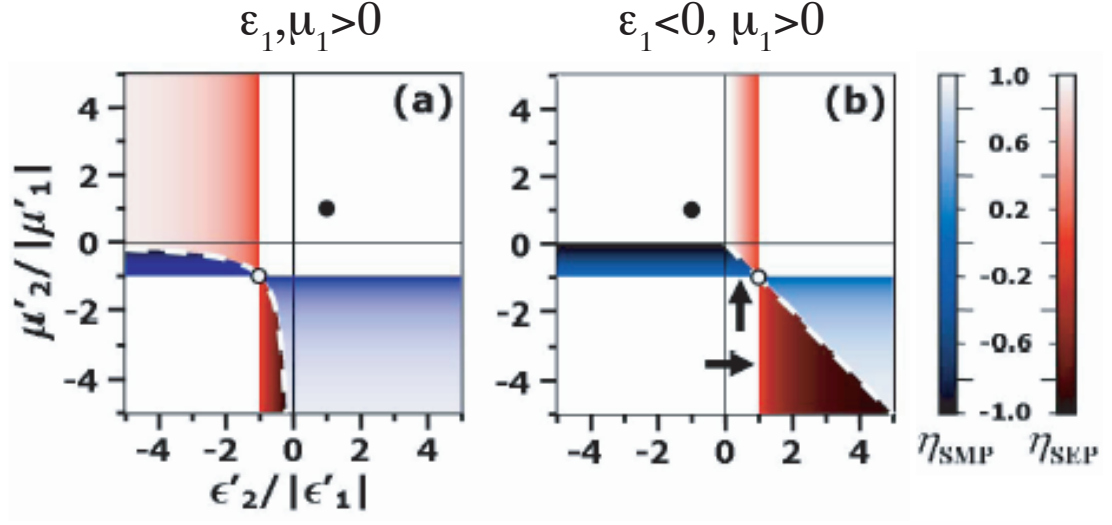


Figure 3.2: Range of possible electric (red) and magnetic (blue) polariton modes and their normalized energy flow densities are shown. Cases: (a) $\epsilon_1, \mu_1 > 0$ and (b) $\epsilon_1 < 0, \mu_1 > 0$ (figure credit: Yoon et al. [1]).

The analysis can be separated into two boundary types: case 1, one of the bounding materials has ϵ or μ both positive or both negative (it is assumed here $\epsilon_1, \mu_1 < 0$) or, case 2, one of the materials is cross negative (it is assumed here $\epsilon_1 < 0, \mu_1 > 0$).

3.2.1 Case 1: positive material $\epsilon_1, \mu_1 > 0$

The nonradiative constraint implies

$$k_x = \frac{\omega}{c} \sqrt{\frac{\mu_1 \mu_2 (\epsilon_1 \mu_2 - \epsilon_2 \mu_1)}{\mu_2^2 - \mu_1^2}} > \frac{\omega}{c} \sqrt{\epsilon_1 \mu_1}. \quad (3.8)$$

This inequality is solved by paying careful attention to the signs of the variables involved and noting that Eq. 3.4 requires $\frac{\mu_2}{\mu_1} < 0$. The possible modes are found to be:

sign: $\epsilon_1, \mu_1 > 0$		
case	$\epsilon_2 > 0$	$\epsilon_2 < 0$
$-1 < \frac{\mu_2}{ \mu_1 } < 0$	—	$< \frac{ \epsilon_1 }{\epsilon_2}$
$\frac{\mu_2}{ \mu_1 } < -1$	$< \frac{ \epsilon_1 }{\epsilon_2}$	$> \frac{ \epsilon_1 }{\epsilon_2}$

3.2.2 Case 2: $\epsilon_1 < 0, \mu_1 > 0$

In this case there is no propagation photon wavevector in medium 1 since the ϵ_1 and μ_1 are oppositely signed. Therefore, all surface polariton modes are nonradiative. The only constraint is that the propagating wavevector be real, that is, the radical in the solution for k_x (Eq. 3.5) must be positive,

$$\frac{\mu_1\mu_2(\epsilon_1\mu_2 - \epsilon_2\mu_1)}{\mu_2^2 - \mu_1^2} > 0. \quad (3.9)$$

Solving this inequality and invoking Eq. 3.4, provides the solutions:

sign: $\epsilon_1 < 0, \mu_1 > 0$		
case	$\epsilon_2 > 0$	$\epsilon_2 < 0$
$-1 < \frac{\mu_2}{ \mu_1 } < 0$	$< -\frac{\epsilon_2}{ \epsilon_1 }$	$< -\frac{\epsilon_2}{ \epsilon_1 }$
$\frac{\mu_2}{ \mu_1 } < -1$	$> -\frac{\epsilon_2}{ \epsilon_1 }$	—

Both the electric (red) and magnetic (blue) polaritons are shown in Fig. 3.2 [1]. One might hope to find combined electric/magnetic polaritons (which would allow one dimensional modes) but as shown in Fig. 3.2 there is no overlap between the two solutions.

The propagation direction of the surface mode can be obtained by calculating the real part of the Poynting vector,

$$\mathbf{S} = \frac{1}{2} \text{Re} \{ \mathbf{E} \times \mathbf{H}^* \}, \quad (3.10)$$

which gives the average energy flux density. Parameterizing the fields in Eqs. 3.1-3.2 in terms of E_y and solving gives,

$$(z < 0) \mathbf{S}_1 = \frac{c}{\omega} \frac{E_y^2}{2\mu_1} e^{2k_{z1}z} (k_x \hat{\mathbf{x}} - k_{z1} \hat{\mathbf{z}}), \quad (3.11)$$

$$(z > 0) \mathbf{S}_2 = \frac{c}{\omega} \frac{E_y^2}{2\mu_2} e^{-2k_{z2}z} (k_x \hat{\mathbf{x}} + k_{z2} \hat{\mathbf{z}}), \quad (3.12)$$

in regions 1 and 2, respectively. The decaying exponential in the z direction restricts the energy propagation along and near the surface. Just above and below the surface the relation between S_{x1} and S_{x2} is

$$S_{x1}(0^-) = \frac{\mu_2}{\mu_1} S_{x2}(0^+). \quad (3.13)$$

Again, Eq. 3.4 requires $\mu_2/\mu_1 < 0$, such that the power flows in opposite directions in the two regions. The “propagation” direction depends on the sum of the total power in each region. This is found by integrating the Poynting vector over a surface perpendicular to the interface,

$$P_{\text{tot}} = \int_{-\infty}^0 S_{z_1} dz + \int_0^{\infty} S_{z_2} dz = \frac{c}{\omega} \frac{k_x E_y^2}{4} \left[\frac{\mu_1 k_{z_1} + \mu_2 k_{z_2}}{\mu_1 \mu_2 k_{z_1} k_{z_2}} \right]. \quad (3.14)$$

The normalized energy flow, η is defined to be

$$\eta_{\text{magnetic}} = \frac{P_{x_1} + P_{x_2}}{|P_{x_1}| + |P_{x_2}|} = \frac{\mu_1 k_{z_1} + \mu_2 k_{z_2}}{|\mu_1 k_{z_1}| + |\mu_2 k_{z_2}|}. \quad (3.15)$$

This quantity is illustrated by the color density in Fig. 3.2 and provides insight into the direction of propagation. For example, consider the case $\epsilon_1, \mu_1 > 0$, then Eq. 3.15 simplifies to

$$\eta_{\text{magnetic}} = -\frac{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}{1 + \left(\frac{\mu_2}{\mu_1}\right)^2}. \quad (3.16)$$

Then for $-1 < \mu_2/|\mu_1| < 0$ the modes are back propagating, that is, the energy flow is negative and hence in the opposite direction of the phase velocity. The back propagating modes are also plotted for the other boundary cases in Fig. 3.2. It is interesting that in addition to existing on negative indexed materials, backpropagating modes can exist on cross negative materials (for example if $\epsilon_1 > 0$ $\mu_1 < 0$ and $\epsilon_2 < 0$ $\mu_2 > 0$).

Switching $\mu_1 \& \mu_2$ with $\epsilon_1 \& \epsilon_2$ in the above calculations provides the solution for electric polaritons as also shown in (red) in Fig. 3.2.

3.3 Fabricating a Metamaterial Structure to Support Surface Polaritons

We experimentally demonstrated that metamaterials can be tailored to support SPs [39]. We utilized a metamaterial structure composed of a collection of SRR resonators positioned periodically as shown in Fig. 3.3. An essential feature of a metamaterial is that the size and spacing of the SRR resonators is significantly smaller than the wavelengths of interest—on the order of 3 cm at X-band

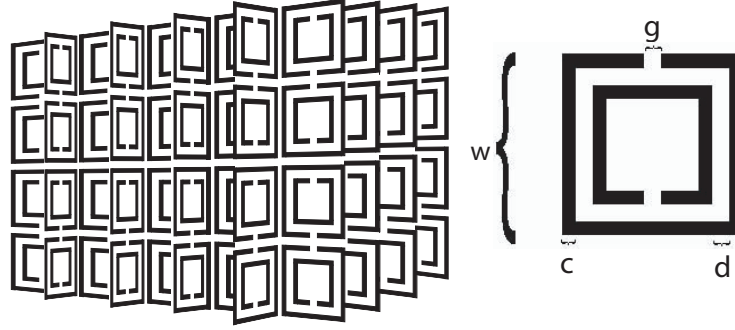


Figure 3.3: A metamaterial structure was constructed out of sheets of SRRs assembled into a 3D structure. The rings counterpose one another to satisfy the boundary conditions of the waveguide in which the experiment was performed. The characteristic dimensions of the SRRs are shown. The values for the sample considered in are: $w=2.14\text{mm}$, $c=0.13\text{mm}$, $d=0.35\text{mm}$, and $g=0.47\text{mm}$. Each unit cell is $2.5\text{mm} \times 2.5\text{mm} \times 2.5\text{mm}$.

frequencies (8–12 GHz). With this constraint fulfilled, the resulting electromagnetic response of the spatially inhomogeneous composite approximates that of a continuous material. The generic form of the permeability of the SRR medium has been found to have the resonant form [7]

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma\omega}, \quad (3.17)$$

indicating a band of frequencies between ω_0 and $\omega_{\text{mp}} = \omega_0/\sqrt{1-F}$ where μ is negative (for small Γ). We thus expect that in this frequency range the SRR medium will support magnetic SPs. We assume that the material has a isotropic frequency-dependent permeability, $\mu_2(\omega)$. While the generic form for the frequency-dependent permeability for the SRR medium, Eq. 3.17, has been shown to qualitatively describe the SRR medium, numerical results obtained from using this approximate expression are unlikely to be quantitatively correct for the fabricated SRR structure. Accurate values for the frequency dependent μ and ϵ can be obtained from numerical solutions of the electromagnetic modes corresponding to a single unit cell of the composite SRR structure.

The dimensions of the SRRs corresponding to the structure used in the measurements are indicated in Fig. 3.3. The structure, designed to be isotropic in a plane, consists of SRRs periodically repeated in two orthogonal directions. The

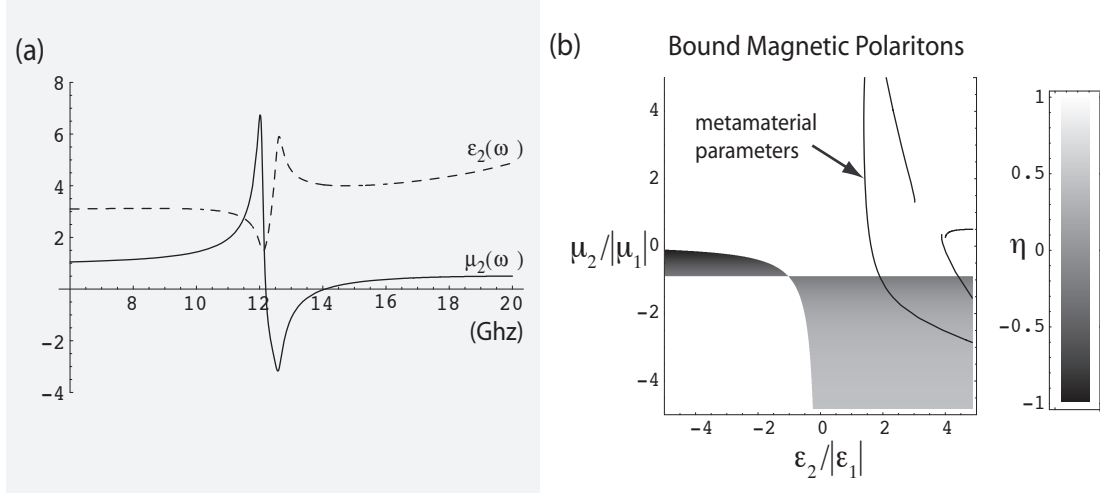


Figure 3.4: (a) The real parts of the permeability $\mu_2(\omega)$ and permittivity $\epsilon_2(\omega)$ of the metamaterial are shown. They were derived from numerical simulations of split rings using Ansoft's HFSS software. (b) The associated range of magnetic polaritons supported by the metamaterial structure.

SRRs are assumed to be deposited with a thickness of 0.017mm on a circuit board substrate with a dielectric constant of 4.07. All aspects of the geometry and material characteristics of the unit cell are included in the finite-element numerical simulations, performed using HFSS (Ansoft).

To determine the $\epsilon(\omega)$ and $\mu(\omega)$ functions, a unit cell of the metamaterial is simulated, with periodic boundary conditions applied in the directions perpendicular to propagation. The complex transmission and reflection coefficients (or scattering parameters) are then calculated, from which the values of ϵ and μ can be extracted using Eq. 1.5 and Eq. 1.6 [38, 40].

The result of this process yields the functions shown in Fig. 3.4. The theoretical dispersion curve for the SRR medium can also be calculated using the finite element numerical simulation. A dispersion diagram is shown in Fig. 3.5. The gap in transmission propagating modes over the frequency range 12-14 GHz corresponds to the region where $\mu_2(\omega)$ is negative. It is in this region that we expect to find propagating surface modes or SPs experimentally.

Having calculated the effective frequency dependent permeability $\mu_2(\omega)$ and permittivity $\epsilon_2(\omega)$, we can determine the theoretical SPs that can be excited at

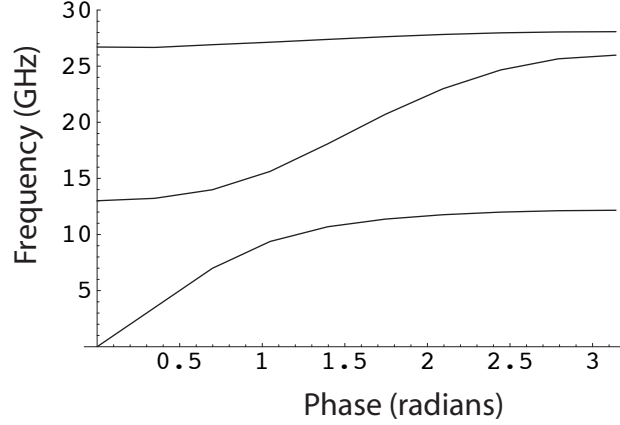


Figure 3.5: Numerical simulation of the transmission dispersion relation through the metamaterial done in Ansoft's HFSS software by modeling the structure of the split ring cells. The horizontal axis is the phase advance per unit cell or kd , where k is the wavenumber (2π divided by the free-space wavelength) and d is the SRR cell width.

the air/metamaterial interface. In the air region ($z < 0$), we have $\epsilon_1 \geq 1$ and $\mu_1 \geq 1$ such that Eq. 3.5 becomes,

$$k_x = \frac{\omega}{c} \sqrt{\frac{\mu_2(\omega) (\mu_2(\omega) - \epsilon_2(\omega))}{\mu_2(\omega)^2 - 1}}. \quad (3.18)$$

For $\epsilon_2 \approx 1$, inspection of Eq. 3.18 shows that surface polaritons are supported in the frequency range where $\mu_2(\omega) < -1$. This corresponds the region where the wavenumber is greater than its equivalent free space propagating mode, i.e. $k_x > \omega/c$. Using Eq. 3.18 and Section 3.2.1, we can plot out the range of surface polaritons supported by the designed metamaterials as a function of the metamaterials constitutive parameters as shown in Fig. 3.4(b).

3.4 Exciting Bound Surface Polaritons: OATR Reflection Coefficient

Experimentally the question arises of how to excite these nonradiative modes since they won't couple to free propagating modes in the bounding materials because of the momentum mismatch. In order to experimentally observe SPs, we

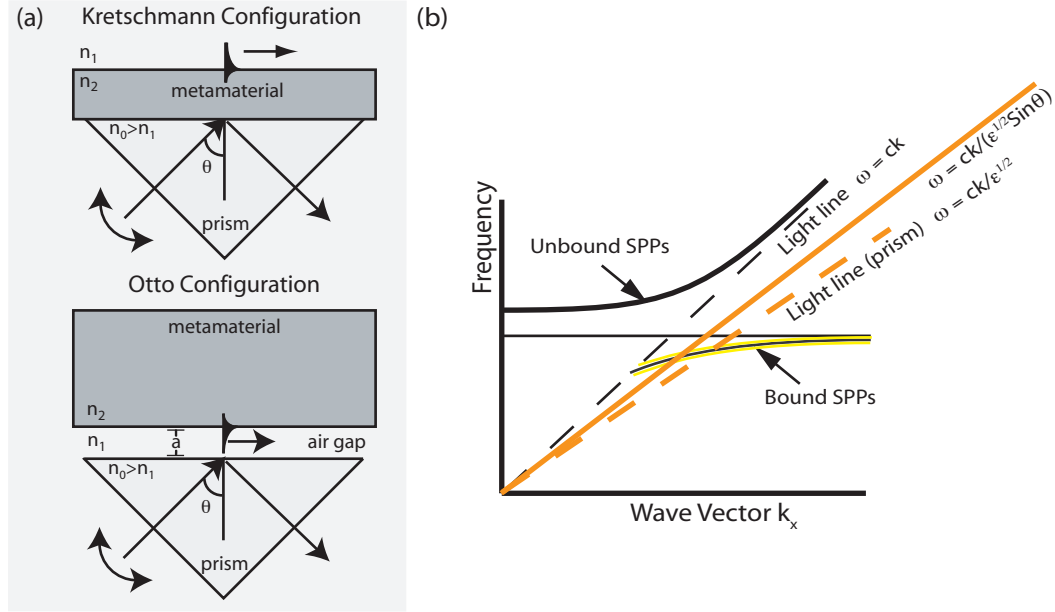


Figure 3.6: (a) To excite surface polaritons, an Otto attenuated total reflection (OATR) technique is used. Radiation is incident on the boundary of a prism and medium of lesser index of refraction. The incident angle of the radiation is greater than the critical angle for total internal reflection resulting in an evanescent field decaying away from the prism. The evanescent field above the prism couples to surface polaritons on the metamaterial, which is close to but not touching the prism at a distance ‘ a ’. This coupling causes a dip in the reflected power from the prism boundary. A variation of the setup is the Kretschmann configuration where the prism is directly adjacent to the metamaterial and the surface polaritons are excited on the backside of the metamaterial. (b) The dispersion relation for magnetic surface polaritons is shown along with the coupling line of the prism

employed the Otto attenuated total reflection (OATR) technique in order to couple incident radiation [41, 27], shown schematically in Fig. 3.6(a). A beam of radiation is incident from vacuum (index of n_1) through one face of a high index prism with index $n_0 \geq n_1$. The beam passes through the prism and is incident on a second interface at some angle. For angles greater than a critical angle, the beam undergoes total internal reflection at the second interface and exits the prism at the third interface. Above the reflection interface of the prism, a decaying evanescent wave exists, for which the parallel component of the wavevector $k_x > \omega/c$. When

the metamaterial is brought close to (but not touching) the prism, this evanescent wave can couple to a surface polariton mode. A minimum is observed in the power reflected from the prism/air interface at the frequency the parallel component of the wavevector of the incident wave matches the wavevector of the surface polariton. This method works well in the microwave regime where wavelengths are relatively long. For shorter wavelengths, such as in the optical regime, it is difficult to accurately space the prism and material since the prism must be within a distance of about a wavelength to get useful coupling. In this case an alternate method can be used called the Kretschmann configuration in which the material is deposited on a prism as a thin film. In this case, the evanescent field of the prism will extend through the material and excite surface polaritons on the backside of the material, also shown schematically in Fig. 3.6(a).

The analysis thus far presented has assumed that the material occupies an infinite half-space, whereas the metamaterial used in the experiments is of finite thickness. However, for slabs of sufficient thickness, the SPs on either side of the slab are essentially decoupled due to material losses so that the infinite half-space analysis applies.

To calculate the reflection coefficient for an OATR experimental setup, we assume an incoming s-polarized plane wave of the form $\mathbf{E} = E_{0R}e^{i(\mathbf{k}_0 \cdot \mathbf{r} - \omega t)}$. The general solution in each region is $E_{jR}e^{i(\mathbf{k}_j \cdot \mathbf{r} - \omega t)} + E_{jL}e^{i(-\mathbf{k}_j \cdot \mathbf{r} - \omega t)}$, where $j = 0, 1, 2$ (prism, gap, metamaterial respectively) refers to the region of interest. The wave equation for s-polarized waves is scalar and can be parameterized in terms of the y-component of the electric field in each region. Let ‘ a ’ be the separation distance between the prism and the metamaterial. Applying boundary conditions at $x = 0$ and $x = a$ leads to the following set of equations:

$$E_{y0R} + E_{y0L} = E_{y1R} + E_{y1L} \quad (3.19)$$

$$\frac{1}{\mu_0}k_{z0}(-E_{y0R} + E_{y0L}) = \frac{1}{\mu_1}k_{z1}(-E_{y1R} + E_{y1L}) \quad (3.20)$$

$$E_{y1R}e^{ik_{z1}a} + E_{y1L}e^{-ik_{z1}a} = E_{y2R}e^{ik_{z2}a} \quad (3.21)$$

$$\begin{aligned} \frac{1}{\mu_2}k_{z2}(-E_{y1R}e^{ik_{z2}a}) &= \frac{1}{\mu_1}k_{z1}(-E_{y1R}e^{-ik_{z1}a} \\ &+ E_{y1L}e^{-ik_{z1}a}). \end{aligned} \quad (3.22)$$

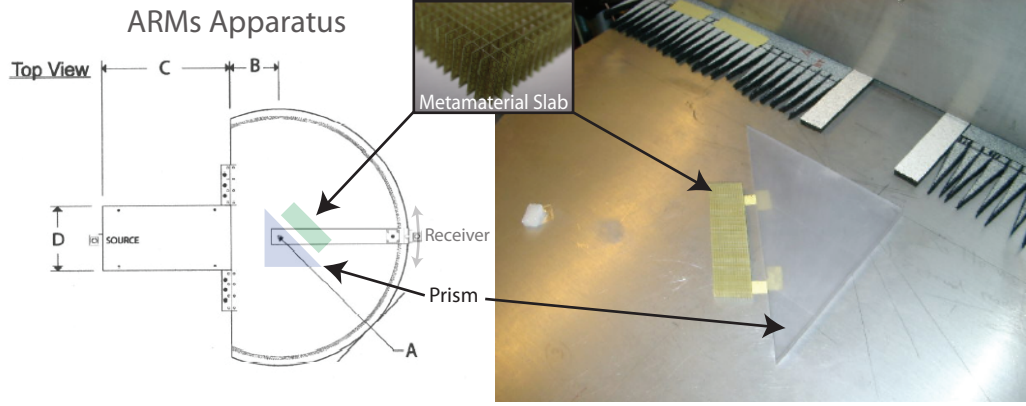


Figure 3.7: The experimental setup is shown inside the ARMs apparatus.

The wave number, k_{z_j} , follows from the dispersion relation (Eq. 3.3) and the continuity of k_x :

$$k_{z_0} = \frac{\omega}{c} \left(n_0^2 - n_0^2 \sin^2(\theta_0) \right)^{1/2} \quad (3.23)$$

$$k_{z_1} = \frac{\omega}{c} \left(n_1^2 - n_0^2 \sin^2(\theta_0) \right)^{1/2} \quad (3.24)$$

$$k_{z_2} = \frac{\omega}{c} \left(\mu_2(\omega)\epsilon_2(\omega) - n_0^2 \sin^2(\theta_0) \right)^{1/2}. \quad (3.25)$$

Given the $\mu_2(\omega)$ and $\epsilon_2(\omega)$ functions for the metamaterial and the index of refraction of the prism and dielectric, n_0 and n_1 , we have a set of equations that can be solved for the reflected electric field component, $E_{y_{0L}}$, in terms of the angular frequency ω , the separation distance a , and the incident angle θ_0 . We then obtain a numerical solution from these equations using Mathematica (Wolfram Research).

The reflectance, $R(\omega)$, or normalized reflected power from the surface of the prism is given by

$$R(\omega) \equiv \frac{\langle S_{0L} \rangle \cdot (-\hat{\mathbf{x}})}{\langle S_{0R} \rangle \cdot \hat{\mathbf{x}}} = \frac{|E_{y_{0L}}|^2}{|E_{y_{0R}}|^2}, \quad (3.26)$$

where S is the Poynting vector.

3.5 Experimental Observation of Magnetic Surface Polaritons

A convenient method for characterizing the scattering properties of a metamaterial sample is to place the sample in a planar waveguide scattering chamber. For a planar waveguide formed by two conducting sheets, the lowest TE mode has no cutoff frequency and is characterized by uniform fields; the electric field is polarized perpendicular to the plates and the magnetic field is polarized parallel to the plates. In effect, the lowest TE mode is equivalent to a free-space s-polarized plane wave. As long as the scattering elements that form the metamaterial sample possess reflection symmetry about the center plane between the upper and the lower waveguide plates, the equivalence of the waveguide modes to s-polarized free-space waves is maintained.

An angle resolved microwave spectrometer (ARMS) has been previously built to enable planar waveguide measurements on metamaterial samples [42] as shown in Fig. 3.7. Microwaves are generated and detected in the ARMS apparatus by an Agilent vector network analyzer (model 8722ES). Microwave energy is injected as an apertured beam via an extended arm connected to the flat side of a semi-circular planar waveguide chamber (plate separation of 1 cm). The metamaterial sample is placed at the center of the plates and scatters the incoming beam. The ARMS apparatus allows the scattered power to be detected around the waveguide periphery by means of a waveguide detector placed on a rotatable arm.

Because the ARMS apparatus allows s-polarized wave propagation solely in the plane of the waveguide, it is only necessary to construct a metamaterial for which the μ response is negative in two dimensions (the magnetic properties in the third dimension are not sensed by these waves). A metamaterial sample patterned in two rather than three dimensions significantly eases the sample fabrication burden.

The metamaterial constructed for this experiment consists of a 2D grid of SRRs cells as shown schematically in Fig. 3.3. A photolithography process was used to fabricate the SRR structure. The split ring design, also shown in Fig. 3.3, was drawn in Adobe Illustrator. A mask of the design was printed on a transparency

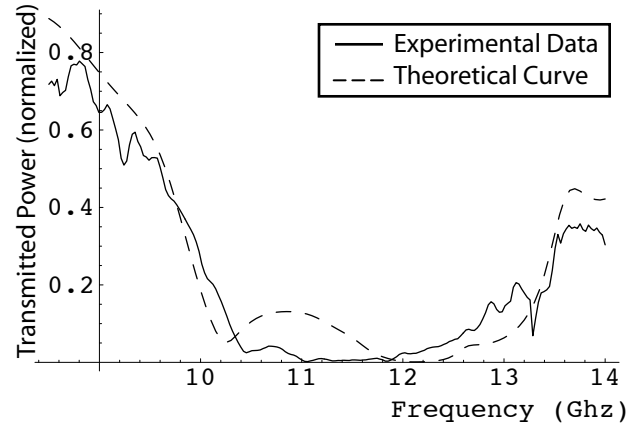


Figure 3.8: The normalized experimental and numerically simulated power transmitted through the metamaterial are shown versus frequency. The numerical simulation is done using Ansoft's HFSS software by modeling, in the direction of propagation, a 10 cell thick split ring structure sandwiched between 1cm metal plates.

with a 600 dpi laser printer. Single sided copper clad FR4 circuit board, concealed by a layer of photoresist, was covered by the mask and exposed to ultraviolet light for one minute, then put in a developer solution (Sodium Carbonate) and agitated. Next the board was placed in etching solution (Ferric Chloride) for one hour during which its orientation was altered every 15 minutes to insure even etching. Finally, the board was placed in a stripper solution (Potassium Hydroxide) for about 10 minutes. After drying, a ProtoMatC100/HF milling machine was used to cut the circuit board into strips, four cells in height and one hundred cells in length. Notches were cut between cells allowing the strips to be assembled together in a three dimensional (physical) structure. The pattern for the notches and the cutout were also drawn in Illustrator and processed using CircuitCAM and BoardMaster software (LPKF). For this experiment, the SRRs were assembled into a 13 x 3.5 x 1 cm rectangular slab.

To confirm that the simulated properties of the SRR metamaterial were consistent with those of the fabricated structure, a comparison was made between the simulated and measured transmitted power through the metamaterial slab as shown in Fig. 3.8. The measured power spectrum reveals a 3.6 GHz wide region of attenuation over the frequency range 10.4-14 GHz. The observed gap, over which μ

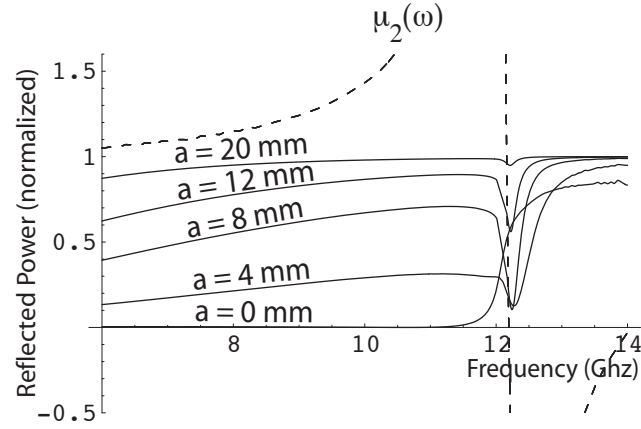


Figure 3.9: The simulated normalized reflected power for several metamaterial/prism separations, a , and the permeability of the metamaterial $\mu_2(\omega)$ are shown versus frequency. For incident radiation on the prism/air interface $\theta_{\text{inc}} > \theta_{\text{TIR}}$, yet there is not total reflection of the incident radiation. In the frequency range that $\mu(\omega) > 1$ the prism/metamaterial have closely matched indexes of refraction and the evanescent field in the gap provides a means of resonant light tunneling across the gap to the metamaterial. In the region that $\mu < -1$ the evanescent field couples to SPs. The evanescent field strength at the surface of the metamaterial falls off as the separation distance increases, resulting in decreased coupling to the above modes (i.e. increased reflected power).

is presumably negative, is in excellent agreement with the simulated transmission spectrum obtained from the numerical simulations. The agreement suggests that the simulations have accurately described the properties of the SRR metamaterial.

The recovered functional forms for $\mu_2(\omega)$ and $\epsilon_2(\omega)$ were used to solve numerically for the reflectance as described in Section 3.4. Fig. 3.9 shows the predicted SP resonance for an incident angle $\theta_0 = 45^\circ$ and a range of separation distances ‘ a ’ between the prism and metamaterial.

To characterize the SPs, an OATR experiment is performed. A 1 cm thick 45° by 45° right triangular prism constructed out of polycarbonate is used, for which the index of refraction of the prism is $n_0 = 1.63$. S-polarized electromagnetic radiation is directed normally onto one face of the prism, encountering the second surface of the prism at an angle of $\theta_0 = 45^\circ$. The reflected radiation then exits the third surface at normal incidence. The reflected power versus frequency is then measured by the vector analyzer, see Fig. 3.6, and used to normalize subsequent

data. When the metamaterial is brought close to the prism, radiation is coupled to SPs on the metamaterial as described above resulting in a dip in the power spectrum.

Fig. 3.10 shows the results of the OATR experiments for several separation distances between the metamaterial sample and the prism, along with the numerically computed reflection spectrum. There is a consistency between the data and the simulated solutions. The strength of the evanescent field is stronger at the metamaterial surface for the shorter separation distances, resulting in an increased coupling between the incident radiation and the SPs at small ‘ a ’. This can be seen in the data as the larger dip in the reflected power. The trend of increased coupling with shorter distance continues until the sample comes extremely close to the prism or touches it; in these cases, the sample either matches the prism so that transmission occurs, or acts to reflect the incident radiation entirely.

There is a discrepancy between the simulated and measured data for smaller separation distances. For a uniform material we would expect the characteristic reflected power to be that of a single metamaterial/prism interface when $a = 0$ as calculated in the theory plot. The experimental data does not reflect this, drifting away from the theory plot at lower frequencies. We suspect that this inconsistency is the result of the metamaterial not being truly homogenous and preventing a true boundary. Even at $a = 0$, the discrete size of the SRR cells makes the boundary between the prism/metamaterial poorly defined.

We can also calculate the dispersion relation for the SPs, i.e. the frequency versus resonant k_x . When we vary the incident angle of the radiation on the prism/air interface we change the k_x of the evanescent wave. The relation between the wave vector and incident angle is

$$k_x = n_0 \sin(\theta_0) \frac{\omega}{c}. \quad (3.27)$$

We map out the dispersion relation, k_x vs ω , by varying the incident angle over a range of values and measuring the resonant frequency of the SPs.

We must also account for the refraction of the radiation when the radiation enters the prism—it no longer enters at normal incidence. With another application

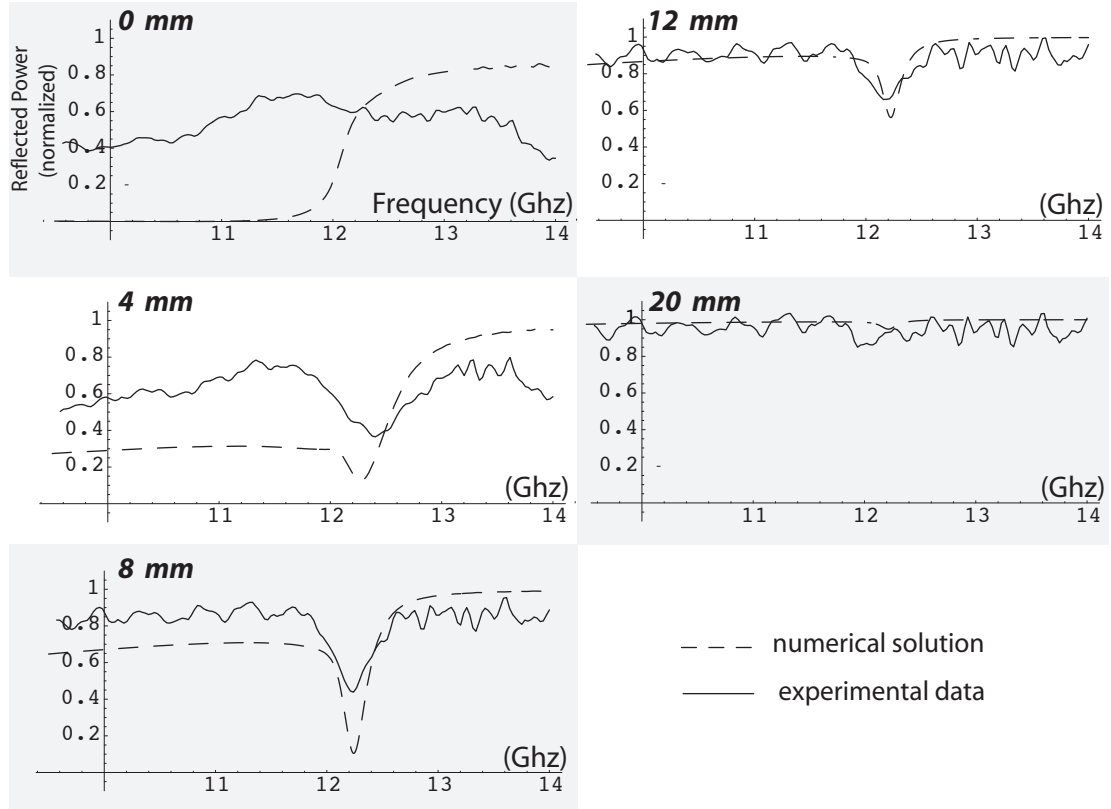


Figure 3.10: The experimental data and numerical solution for the reflected power (normalized) as a function of the separation distance between the metamaterial and prism.

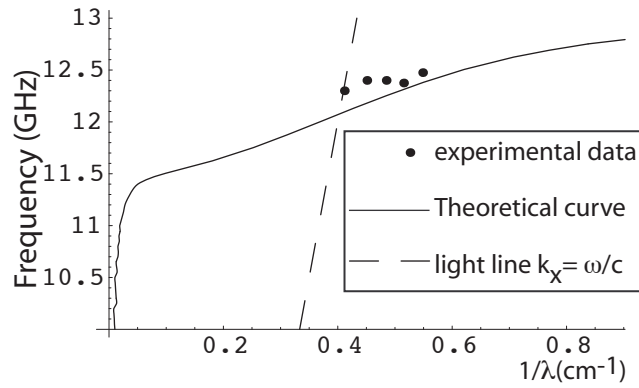


Figure 3.11: The experimental surface dispersion relation for several k_x values, the theoretical dispersion relation, and the light line $k_x = \omega/c$ are shown.

of Snell's Law we can write k_x as a function of the entering angle, θ_{ent} .

$$k_x = n_0 \sin \left[\frac{\pi}{4} + \sin^{-1} \left(\frac{\sin(\theta_{\text{ent}})}{n_0} \right) \right] \frac{\omega}{c} \quad (3.28)$$

where we have set the index of refraction of our boundary material to unity because it is air in our experiment.

The experimental data for several k_x values are shown in Fig. 3.11. The calculated theoretical curve, found using the HFSS data for $\mu(\omega)$ and $\epsilon(\omega)$, is also plotted along with the light line ($k_x = \omega/c$). We see that there is reasonable agreement with the theoretical curve.

3.6 Conclusion

The results of these experiments show that a structured metamaterial can support magnetic surface polaritons and that its properties are consistent with the composite being assigned averaged electromagnetic material parameters. The simplicity afforded by this description allows for a very convenient comparison between these magnetic SP modes and the electric SP modes that have been studied in depth at optical wavelengths over the past several decades. The metamaterial was constructed from individual SRR circuit elements; however, aside from the metamaterial boundary being somewhat ill-defined—an effect that becomes manifest when the metamaterial is closely coupled to the exciting prism—our results indicate that the measured surface properties are consistent with the composite being treated conceptually as an effective medium with homogenous electromagnetic properties. In particular, numerical simulations of the SRR elements provided anticipated values for the effective $\mu(\omega)$ and $\epsilon(\omega)$ functions, which were then used within the standard theoretical treatment of surface polaritons on a homogenous material. The resulting agreement between the models and measurements confirms the applicability of the metamaterial approach.

An alternative approach for calculating the various modes in an SRR medium has been suggested by Shamonina et al. [43], who treat the structure as a collection of periodically spaced coupled capacitive loops. their analysis has suggested that other modes may exist, including both propagating and surface modes. In our experiments we did not find evidence for such modes, indicating the metamaterials analysis correctly describes the SRR medium properties. Presumably, a method

of excitation could be found that would allow the investigation of these other predicted modes, to better determine when the artificial medium limit breaks down.

The good agreement found between the measured SRR structures and the analytical results assuming a continuous medium indicates that the surface mode structure of negative index metamaterials can now be probed. Combining the SRR metamaterial with a thin wire structure can produce a material that has frequency regions where both the permittivity and the permeability are negative. As discussed, the surface mode structure of such a material would support novel backward propagating modes [44, 45].

As noted earlier, our theoretical investigation was based on an infinite half-space of metamaterial and is valid to even relatively small widths of the metamaterial block (on the order of several wavelengths of the radiation). It would be of interest to investigate, both theoretically and experimentally, metamaterial strips that are thin enough that the evanescent field inside the metamaterial doesn't decay before it reaches the back surface of the metamaterial. In such a case we would expect to observe joint surface plasmon excitations on both the front and back surfaces.

4

Ferromagnetic Materials in Microstrip Structures

The development of ferrimagnetic ceramic materials, with resonant magnetic properties, has proven invaluable in building modern microwave devices. For example, ferrite materials are the basis of most microwave frequency circulators and isolators [46]. Recently, the technological trend in microwave devices is towards smaller scale and higher frequency applications. In order to realize these structures, magnetic materials are required with properties beyond what the established ferrite materials, such as yttrium iron garnet, can provide. Under consideration to fill this material void are high magnetic saturation materials that can be directly integrated into miniaturized microstrip structures. In this regard, there is a focus on using ferromagnetic materials which previously had been ignored due to their inherent conduction losses. If we limit the application of ferromagnetic materials to thin layers, less than their conduction skin depth (≈ 1 micron for Fe), then losses can be minimized. For micron scale devices, thin film ferromagnetic materials provide a sufficiently strong response.

In this chapter, we analyze small dimensional microstrips (i.e. microstrips with dimensions on the order of the skin depth of the conductor) with an embedded ferromagnetic thin layer. The ferromagnetic thin layer provides a tunable stop band and/or phase shift to the propagating microstrip mode. We first review the theory of ferromagnetic resonant materials for a thin film structure and then consider the

Schloemann approximation as a simple but accurate simplification to the theory that can be applied to thin film waveguide geometries. These microstrip structures are generally analytically intractable because of their complicated electromagnetic field distributions, but we use the Schloemann approximation to establish a fast technique for numerically characterizing them. We further demonstrate good correlation between our numerical analysis and experimental results.

A broader motivated for considering these structures, both analytically and numerically, is to develop an understanding of gyromagnetic materials such that we can consider integrating them into waveguide metamaterial structures as we proceed to do in Chapter 6.

4.1 Introduction

Microstrip structures are advantageous for their broad frequency range of operation, economical fabrication, and ease with which they can be integrated into microstrip based monolithic circuits [46]. We're interested in investigating the small dimensional microstrip structure, shown in Fig. 4.1. This structure operates as a notch filter. The filter, first proposed by Schoelmann et al., is formed by embedding a thin ferromagnetic layer at some position between the top and bottom conductors in a microstrip [47, 48, 49]. As we shall show, the thin ferromagnetic layer has a resonance governed by Eq. 4.14, $\omega = \mu_0\gamma\sqrt{H_0(H_0 + M_0)}$, where γ is the gyromagnetic ratio, H_0 is the applied static field, and M_0 is the saturation magnetization. For materials such as iron or permalloy (Py) with high magnetic saturation values, a modest magnetic biasing field of 2 kG gives a ferromagnetic resonance above 20 GHz. If we embedded a ferromagnetic layer in a microstrip, the layer will absorb power from the propagating quasi-TEM mode at the resonant frequency. By altering the intensity of the applied DC magnetic field, H_0 , one can tune the position of this absorption.

The magnitude of the insertion loss or phase shift is not possible to determine analytically for these small dimensional microstrips where the dimensions are much smaller than the propagation wavelength. There exists an analytic solution for the

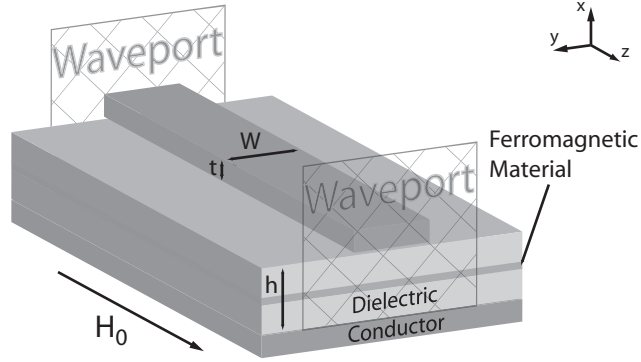


Figure 4.1: Diagram for the setup of the ferromagnetically embedded microstrip structure. Note that the ferromagnetic layer does not actually touch the waveport because HFSS does not allow a material with a negative permeability to be in direct contact with the waveports. An infinitesimal spacer of dielectric ($\sim 1/100000$ of the effective wavelength) is placed between the waveport and ferromagnetic layer to avoid simulation issues.

related parallel plate waveguide structure embedded with a ferromagnetic layer, by Camley et al. [50, 51], but numerical simulations are necessary to characterize the more complicated fields and current distributions present in microstrip structures.

4.2 Ferromagnetic Materials

We first investigate the permeability of a thin film gyromagnetic layer that is biased by some static magnetic field and excited by an incident TEM plane wave. This simple model can be solved analytically and provides some intuition about the nature of the ferromagnetic resonance in the microstrip environment [46].

Ferromagnetic materials derive their strong magnetic response from unpaired electrons in the orbitals of its constituent atoms. Each unpaired electron contributes a magnetic dipole moment as a result of its intrinsic quantum spin,

$$\mu_{\mathbf{m}} = -\gamma \mathbf{S}, \quad (4.1)$$

where $\gamma = e/m$ is the gyromagnetic ratio, e is the electron charge, and m is the electron mass (Note that the magnetic moment is in the opposite direction of the

spin). When a magnetic dipole is in a magnetic field, $\mathbf{B}$, there is a torque on the dipole moment given by

$$\tau = \mu_{\mathbf{m}} \times \mathbf{B}. \quad (4.2)$$

Since torque is the time derivative of the angular momentum, $\tau = d\mathbf{L}/dt$, we can use Eq. 4.1 to write the torque as a function of the dipole moment, $\tau = -(1/\gamma)(d\mu_{\mathbf{m}}/dt)$. Plugging this into Eq. 4.2 allows us to write down an equation of motion for the dipole moment,

$$\frac{d\mu_{\mathbf{m}}}{dt} = -\gamma\mu_{\mathbf{m}} \times \mathbf{B}. \quad (4.3)$$

We can generalize this equation to bulk materials by multiplying both sides of Eq. 4.3 by the dipoles per unit volume in the material. Then, the *magnetic dipole per unit volume* or magnetization, $\mathbf{M}$, is

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M} \times \mathbf{B}. \quad (4.4)$$

Solving this equation is complicated by the fact that the magnetic field strength, $\mathbf{B}$, depends on the applied external magnetic field and a demagnetization field which is a function of the geometry of the material. The applied magnetic field, $\mathbf{B}_0$, is a combination of a biasing static component $\mu_0 H_0$ in the z -direction plus a small dynamic component $\mu_0 h_x$ and $\mu_0 h_y$ in the x, y -direction representing the incoming plane wave (i.e. the signal component we are interested in modulating),

$$\mathbf{B}_0 = \mu_0(h_x\hat{\mathbf{x}} + h_y\hat{\mathbf{y}} + H_0\hat{\mathbf{z}}). \quad (4.5)$$

For this derivation we consider a biasing fields, $\mu_0 H_0$, above the saturation magnetization. The saturation magnetization defines the applied field magnitude above which all other microscopic biasing interactions of the individual dipole moments are overcome and the spins are aligned in the same direction, see Fig. 4.2. For small perturbations of the applied field about the axis of the biasing field, the magnetization is constant to first order and in the direction of the applied field,

$$\mathbf{M} = (m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + M_z\hat{\mathbf{z}}), \quad (4.6)$$

with m_x, m_y small, and M_z constant to first order.

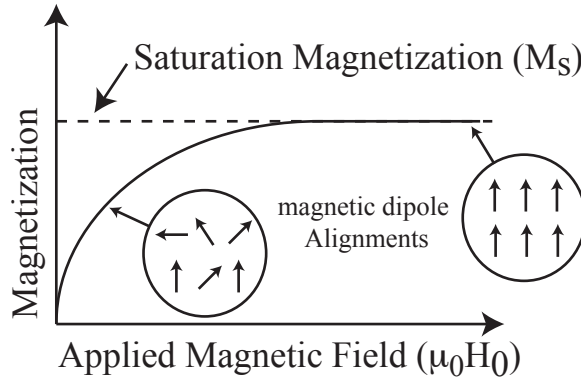


Figure 4.2: A plot of the magnetization of a ferromagnetic material versus applied field is shown. Above the saturation magnetization of a ferromagnetic material the electron spins and their associated magnetic moments are aligned.

The demagnetization field arises from magnetic ‘polarization’ in the material, That is, ‘effective’ magnetic monopoles are induced by the applied field at the opposite surfaces of the material. Of course these aren’t real monopoles, but are due to an offset of the net magnetic moment of the material. The offset induces a magnetic field, $-\overline{\overline{D}} \cdot \mathbf{M}$ in the opposite direction of the field, where the tensor $\overline{\overline{D}}$ is defined as the demagnetization factor and depends on the geometry of the magnetic structure. The Trace of D (the sum of the diagonal components) must equal 1. D can be a complicated factor to calculate but for an ellipsoid geometry with the applied fields along the principle axes it reduces to a diagonal matrix:

$$\begin{pmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix}. \quad (4.7)$$

The total magnetic field in the ellipsoid ferromagnetic material (ignoring anisotropic and exchange field contributions) can be written down as

$$\mathbf{B} = \mu_0[(h_x \hat{\mathbf{x}} + h_y \hat{\mathbf{y}} + H_0 \hat{\mathbf{z}}) - (D_x m_x \hat{\mathbf{x}} + D_y m_y \hat{\mathbf{y}} + D_z M_z \hat{\mathbf{z}})]. \quad (4.8)$$

In order to proceed further, we must choose a geometry of the sample in order to determine the demagnetization factor. Our goal is to understand the ferromagnetic resonance in a thin film geometry (which can be thought of as a flat ellipsoid

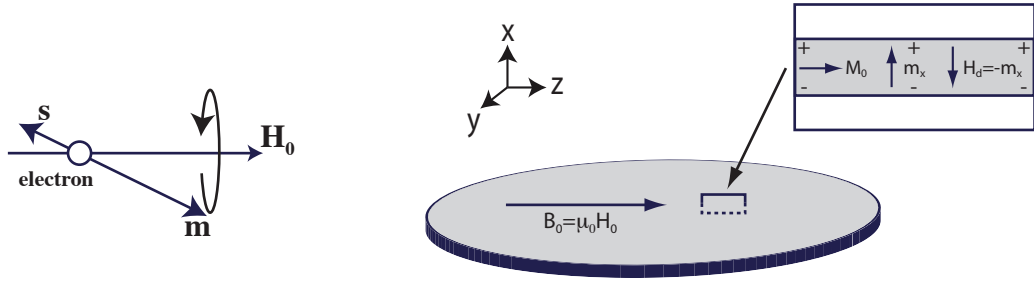


Figure 4.3: The magnetic dipole moment, m , of an electron due to its spin is shown and its effective precession around the biasing field, H_0 . On the right, the magnetization of a thin ellipsoid (i.e. thin disk or film) is shown as a function of the applied field

extending to infinite). We consider the case of the applied biasing field parallel to the thin film as shown in Fig. 4.3. In this case the static field μH_0 induces charge at the edge of the thin film which we assume to be very far away from where we are making measurements—the demagnetization field due to this charge is effectively zero. The only field that is going to induce a demagnetization field is the part of dynamic field that points perpendicular to the surface of the film (μh_x). The demagnetization factors are then $D_x = 1$, $D_y = D_z = 0$.

To solve, we write down Eq. 4.4 by components,

$$\frac{1}{\mu_0 \gamma} \frac{dm_x}{dt} = m_y H_0 - h_y M_z \quad (4.9)$$

$$\frac{1}{\mu_0 \gamma} \frac{dm_y}{dt} = -m_x (H_0 + M_z) + h_x M_z \quad (4.10)$$

$$\frac{1}{\mu_0 \gamma} \frac{dM_z}{dt} = m_x h_y - (h_x - m_x) m_y \approx 0 \text{ (to first order)}. \quad (4.11)$$

We set $M_z = M_0$ (to first order) and solve the first and second equation to get a differential equation:

$$\left[\frac{d^2}{dt^2} + \mu_0^2 \gamma^2 H_0 (H_0 + M_0)^2 \right] m_x = \mu_0^2 \gamma^2 M_0 H_0 h_x - \mu_0 \gamma M_0 \frac{dh_y}{dt}. \quad (4.12)$$

We assume an $e^{-i\omega t}$ dependence for m_x and h_y and solve to get

$$m_x = \frac{\omega_m}{\omega_0^2 - \omega^2} [\mu_0 \gamma H_0 h_x + i\omega h_y] \quad (4.13)$$

with the resonant frequency

$$\omega_0 = \mu_0 \gamma \sqrt{H_0(H_0 + M_0)}, \quad (4.14)$$

and $\omega_M = \mu_0 \gamma M_0$. Similarly we solve for m_y to get

$$m_y = \frac{\omega_m}{\omega_0^2 - \omega^2} [-i\omega h_x + \mu_0 \gamma (H_0 + M_0) h_y]. \quad (4.15)$$

The susceptibility tensor can then be written

$$\bar{\bar{\chi}}_{H_{0\parallel}} = \begin{pmatrix} \frac{\omega_m \mu_0 \gamma H_0}{\omega_0^2 - \omega^2} & i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & 0 \\ -i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & \frac{\omega_m \mu_0 \gamma (H_0 + M_0)}{\omega_0^2 - \omega^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (4.16)$$

and the permeability is

$$\bar{\bar{\mu}}_{H_{0\parallel}} = \mu_0 \begin{pmatrix} 1 + \frac{\omega_m \mu_0 \gamma H_0}{\omega_0^2 - \omega^2} & i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & 0 \\ -i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & 1 + \frac{\omega_m \mu_0 \gamma (H_0 + M_0)}{\omega_0^2 - \omega^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4.17)$$

The off diagonal points in the susceptibility tell us that the magnetization is not in the same direction as the applied field. In fact, the magnetization traces out an ellipse but lags behind the applied field. We can further include losses in the permeability by letting the resonant frequency become complex, $\omega_0 \rightarrow (\omega_0 - i\Gamma\omega)$ [It is common to define the damping factor in terms of the ferromagnetic linewidth, ΔH , as $\Gamma = \gamma \Delta H / (1.16\omega)$ [46]]. With the permeability derived above it is possible to determine the effect of the ferromagnetic resonance on a propagation TEM mode in or near the surface of the thin film.

The above analysis does not transfer directly to a microstrip structure because the field distribution of the propagating mode is more complicated. Also, we have not considered the effects of a conductive gyromagnetic material as we are principally interested in. Never-the-less, this model describes the correct resonant frequency, Eq. 4.14, observed in our microstrip structures. In order to calculate the magnitude of the interaction, we must turn to numerical simulations.

4.3 The Schloemann Approximation

The off-diagonal response of the permeability of a ferromagnetic material makes solving for even the simplest waveguide geometries intensive calculations. It has been done for a parallel plate waveguide geometry [51], but for more complicated structures, such as the microstrip structure we are interested in solving, the solution becomes intractable. We can, however, simplify our numerical model by implementing what is called the Schloemann approximation which we derive here. This simplification follows from the fact that the microstrip electromagnetic mode is quasi-TEM in nature and remains so even with implementation of the ferromagnetic layer. As a result, the permeability tensor can be approximated with a diagonal tensor as we proceed to show.

First, note that if we had proceeded with the derivation of the ferromagnetic resonance of Section 4.2 inside a bulk ferromagnetic material, i.e. ignoring boundary effects, the general permeability response is [46],

$$\bar{\bar{\mu}}_{H_{0\perp}} = \mu_0 \begin{pmatrix} 1 + \frac{\omega_m \omega_0}{\omega_0^2 - \omega^2} & i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & 0 \\ -i \frac{\omega_m \omega}{\omega_0^2 - \omega^2} & 1 + \frac{\omega_m \omega_0}{\omega_0^2 - \omega^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4.18)$$

Let us rewrite the tensor as

$$\bar{\bar{\mu}} = \mu_0 \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.19)$$

where the components μ_1 and μ_2 are defined by Eq. 4.18.

To derive the approximation we solve for the propagating wave vector in the ferromagnetic material. Starting with Maxwell's curl equations but neglecting the displacement current we get,

$$\left. \begin{aligned} \nabla \times \mathbf{E} &= -\frac{d}{dt} \bar{\bar{\mu}} \mathbf{H} \\ \nabla \times \mathbf{H} &= \mathbf{J}_f \end{aligned} \right\} \begin{aligned} \mathbf{k} \times \mathbf{E} &= \omega \bar{\bar{\mu}} \mathbf{H}, \\ i\mathbf{k} \times \mathbf{H} &= \sigma \mathbf{E}. \end{aligned} \quad (4.20)$$

Plugging the second equation into the first equation we get,

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{H}) = -i\sigma\omega \bar{\bar{\mu}} \mathbf{H}, \quad (4.21)$$

which simplifies to,

$$(k^2 \bar{I} - i\sigma\omega \bar{\mu} - \mathbf{k}\mathbf{k})\mathbf{H} = 0. \quad (4.22)$$

We see that Eq. 4.22 defines a matrix that can be written out explicitly (we have chosen the axis such that $k_y = 0$),

$$\begin{bmatrix} k^2 - i\sigma\omega\mu_1 & \sigma\omega\mu_2 & 0 \\ -\sigma\omega\mu_2 & k^2 - i\sigma\omega\mu_1 - k_y^2 & -k_y k_z \\ 0 & -k_y k_z & k^2 - i\sigma\omega\mu_0 - k_z^2 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = 0. \quad (4.23)$$

We wish to consider a thin ferromagnetic layer that is integrated into a parallel waveguide structure and excited with the waveguides fundamental TEM mode. The conductivity and small thickness of the ferromagnetic material in our microstrip structures ensures that the traveling electromagnetic mode remains largely TEM in nature. We can therefore make the approximation $k_z \gg k_x$ and hence the simplification that $k \rightarrow k_z$ and that the $k_x k_z$ terms are negligible. This decouples the potential modes in the ferromagnetic material and simplifies Eq. 4.23 to,

$$\begin{bmatrix} k_z^2 - i\sigma\omega\mu_1 & \sigma\omega\mu_2 & 0 \\ -\sigma\omega\mu_2 & -i\sigma\omega\mu_1 & 0 \\ 0 & 0 & k_x^2 - i\sigma\omega\mu_0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = 0. \quad (4.24)$$

To find the solutions for the wave vector of the propagating modes, we take the determinant and solve to get two solutions:

$$k_z = \sqrt{i\sigma\omega \frac{\mu_1^2 - \mu_2^2}{\mu_1}} = \sqrt{\frac{\sigma\omega}{2}} \sqrt{\frac{\mu_1^2 - \mu_2^2}{\mu_1}} (1 + i), \quad (4.25)$$

$$k_x = \sqrt{i\sigma\omega\mu_0} = \sqrt{\frac{\sigma\omega\mu_0}{2}} (1 + i). \quad (4.26)$$

The first solution is for a mode traveling in the z -direction (along the waveguide) while the second is for a mode traveling in the x -direction and is unphysical in our waveguide. From the solutions we see that the effective permeability for the quasi-TEM mode is,

$$\mu_v = (\mu_1^2 - \mu_2^2)/\mu_1. \quad (4.27)$$

This approximate permeability is known as the Schloemann permeability (sometimes called the Voigt Permeability). We can further define an effective skin

depth using this approximation as

$$\delta_{\text{fe}} = \sqrt{\frac{2}{\sigma\omega\mu_v}}, \quad (4.28)$$

where σ is the conductivity of the material. Finally, the Schloemann approximation for the permeability of the ferromagnetic material is,

$$\mu = \mu_0 \begin{pmatrix} \frac{\mu_1^2 - \mu_2^2}{\mu_1} & 0 & 0 \\ 0 & \frac{\mu_1^2 - \mu_2^2}{\mu_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4.29)$$

As mentioned, this approximation holds in the waveguide environment where the mode remains largely TEM in nature [48, 47]. Schloemann maintained that this should also hold for a microstrip structures whose modes are strongly TEM in nature over the central region of the microstrip structure where ferromagnetic layer is most strongly excited [47]. Incorporation of the Schloemann approximation greatly simplifies the complexity of the numerical simulations and, as we shall show, agrees with experimental results.

4.4 Numerical Simulations

To simulate microstrip structures, we used Ansoft's commercially available electromagnetic simulation software (HFSS) which is a finite element solver. Several software packages have been optimized for microstrip structures, including [52, 53], but HFSS allowed us the ability to easily integrate anisotropic materials without modification of the software. When the dimensions of the microstrip are significantly smaller than the propagation wavelength and on the scale of the skin depth of the conductors, the magnetic and electric fields vary strongly throughout the structure. Further complicating the field distribution is the introduction of a sub-micron layer of ferromagnetic material which is both conductive and strongly magnetic. HFSS has an efficient meshing algorithm which is crucial to minimizing the solution time necessary to solve these structures, but even with optimized meshing, densely meshed models are necessary to converge on accurate results.

Illustrations of the simulation and waveport setup are shown in Fig. 4.1. The width of the simulated volume was set to $20 * W$ and the height of the bounding volume was set to $10 * h$, where W is the width of the top strip and h is the height of the dielectric. These dimensions provided a large enough simulation volume such that the propagating fields are minimal near the boundaries and do not exit the model (as was checked by plotting the fields). The boundaries of the simulation volume were set to be radiative which absorb outgoing radiative modes, but as noted above, the power exiting these surfaces is insignificant.

The length of the microstrip simulated was set to minimize simulation volume but still provide accurate simulation results. HFSS is effective at decoupling the microstrip mode from any evanescent fields extending from waveport 1 to waveport 2. We were able to simulate a microstrip with a length of as little as $1/300$ of the effective microstrip wavelength (or .03 mm at 20 GHz) with results that accurately compared to within 5% of the results of a microstrip 100 times as long.

The process of meshing the simulated structure is done through a series of adaptive passes until the S-parameter solutions of the waveports change by less than .01% between passes. A discrete sweep, which calculates the fields of the model at each frequency requested, was performed over the frequency range of interest. The discrete frequency solution must be used for simulating these structures because of the frequency dispersion of the ferromagnetic material. The fast sweep option in HFSS is inaccurate because it interpolates results from calculations at a single frequency ignoring the frequency dependent properties of the ferromagnetic material. The ferromagnetic layer can have a permeability where the real part ranges from -100 to 100 near resonance. The ferromagnetic permeability was calculated in MATLAB using the Schloemann approximation, Eq. 4.27, as explained in Section 4.6, and then imported as a material parameter into HFSS.

The microstrip structure is excited in the simulation by waveports outlined on rectangular surfaces at both ends of the microstrip structure as shown in Fig. 4.1. HFSS uses the boundary conditions of the model touching the 2D waveport surface to determine a waveguide geometry (perfect electric boundaries are assumed on the edges of the waveport unless otherwise defined). From this geometry the

possible propagation modes are calculated and used to excite the structure. For the waveports touching the microstrip structure, as shown in Fig. 4.1, this equates to a rectangular coaxial waveguide (partially filled with a dielectric material) feeding the microstrip structure. Exciting the fundamental TEM mode of the waveport couples to the quasi-TEM mode of the microstrip. For these simulations we set the dimensions of the waveport to have a height of $6 * h$ and width of $5 * W$.

HFSS calculates the scattering matrix by exciting the fundamental TEM mode of the coaxial waveport normalized to the input impedance of the microstrip structure touching the waveport. At the opposite waveport HFSS decouples the incident quasi-TEM mode of the microstrip back into the fundamental TEM mode of the waveport, and again normalizes by the impedance of the microstrip structure. The normalization by the input and output impedance of the structure ensures that the scattering parameters of the microstrip structure are properly de-embedded from the two coaxial waveports. One technical issue in this excitation method is that the ferromagnetic material cannot touch the waveport. Ferromagnetic materials have negative permeability values which prevent a proper solution of the waveport modes and therefore are not allowed to touch the waveports by HFSS. In our simulations we placed an infinitesimal amount of dielectric material ($\sim 1/100000$ of the effective wavelength of interest) between the ferromagnetic material and the waveports to avoid this error. This spacing material is much smaller than the meshing elements and does not significantly affect the simulation.

Once the scattering parameters are derived for a short structure, it is possible to obtain those for a longer uniform structure (for comparison to experimental structures) using standard conversion formulas [46]. The conversion involves first converting the scattering parameters matrix, S , to a transmission matrix, A . The transmission matrix can then be decoupled into an eigenvector matrix, V , and a diagonal eigenvalue matrix, D .

$$A = VDV^{-1}. \quad (4.30)$$

The eigenvalue matrix, D , contains the propagation constant information, $e^{\pm\gamma}$, such that raising D to the n th power provides the phase shift due to the propagation delay through a device n times the length of the original device (where n does

not have to be an integer). To get the transmission matrix for a desired length, l , in terms of the original simulation length, l_s , we solve

$$A(l) = VD^{l/l_s}V^{-1}. \quad (4.31)$$

A standard conversion of the transmission matrix, $A(l)$, back to a scattering parameter matrix, $S(l)$, provides the final desired scattering matrix.

When experimental results are compared to simulation it is often the case that the constructed samples are not perfectly matched to the feed line that they are tested with, i.e. 50 Ohms. HFSS can be used to solve for the input impedance of the structure and any mismatch with a 50 Ohms line can be adjusted for in the translation $S \rightarrow S(l)$. These data manipulations were implemented using a Matlab script.

Matlab was also used extensively in running the simulations. A freeware ‘HFSS-MatLab Scripting-API’ toolbox written by V. C. Vijay was used to write scripts which produced the HFSS microstrip model and assigned material parameters. This script based approach allowed easy adjustment of simulation parameters.

4.5 Small Dimensional Microstrips

At microwave frequencies, the insertion loss of a microstrip is predominantly due to ohmic losses in the imperfect conductors. With knowledge of how the current is distributed over the conductors, one can integrate Joule’s law over the cross-section of the microstrip to determine the attenuation constant. This is easily done for a microstrip with a wide top strip, i.e. $W \gg h$ because the current is uniformly distributed over the bottom of the top strip and the top of the ground plate. The attenuation constant is found to be

$$\alpha_c \approx 8.68/(\sigma\delta Z_0 w) \text{ dB/cm} \quad (4.32)$$

where σ is the conductivity, $\delta = 1/\sqrt{\pi\mu\sigma f}$ is the skin depth, and Z_0 is the characteristic impedance of the structure [54]. But, as Pucel et al. demonstrate [54, 55], this approximation grossly overestimates the losses when $W/h < 2$ because the

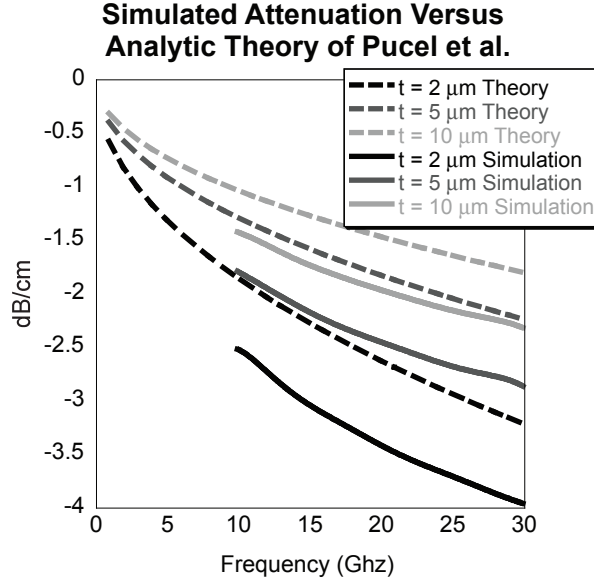


Figure 4.4: A comparison between the attenuation derived from an HFSS simulation and that of the approximate analytic solution by Pucel et al. is shown for a microstrip with a dielectric height of $h = 4.5 \mu\text{m}$ and a top strip width of $W = 5 \mu\text{m}$ and thickness of $t = 2, 5$, and $10 \mu\text{m}$. The conductor is silver ($\sigma = 6.1 \times 10^7 \text{ S/m}$) and the dielectric is silicon dioxide ($\epsilon_r = 4$) with zero losses.

current density over the top strip and ground plate are no longer uniform. In fact, because of the mathematical complexities involved [56] an exact analytical solution has never been found. Pucel et al. calculated an approximate analytic expression for the associated losses based on Wheeler's conformal mapping technique which is valid for small W/h values and conductors with a thickness of at least four skin depths. Beyond this limit, direct electromagnetic simulation may be the best method to determine losses.

As a precursor to simulating ferromagnetic microstrip structures we first verified our technique against Pucel's theory for small dimensional microstrip structures in the realm where W , h , and t are on the order of the skin depth. We simulated a microstrip structure as shown in Fig. 4.1, but without the ferromagnetic layer. The parameters were $W = 5 \mu\text{m}$, $h = 4.5 \mu\text{m}$, $t = 2 \mu\text{m}$ and with silver as the conductor and lossless silicon dioxide as the dielectric ($\epsilon_r = 4$). For reference the skin depth of silver is $0.4\text{-}0.7 \mu\text{m}$ between $10\text{-}30 \text{ GHz}$. The results are shown in Fig. 4.4 versus the theory of Pucel et al. and it can be seen that our simulations

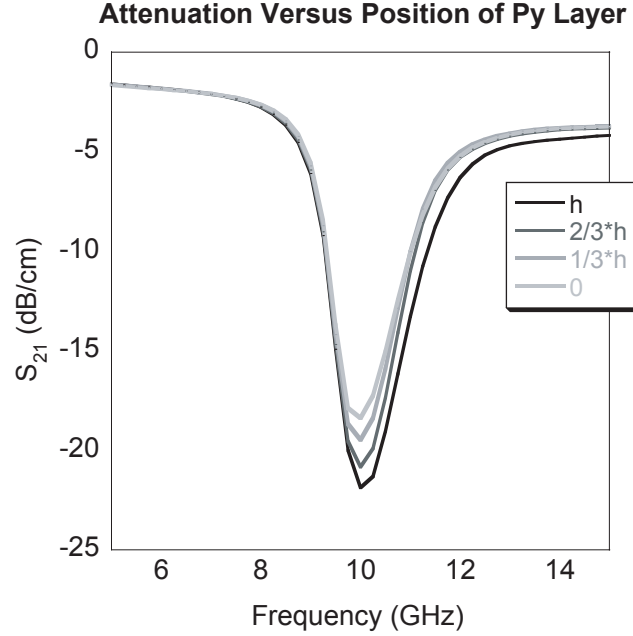


Figure 4.5: The simulated S_{21} attenuation is plotted for various positions of a ferromagnetic permalloy layer as measured from the ground plate. The dielectric is silicon dioxide₂ ($\epsilon = 4$). The permalloy is $0.1 \mu\text{m}$ thick and has a width extending to the edges of the simulation ($100 \mu\text{m}$). The dimensions of the microstrip are $h = 3.1 \mu\text{m}$, $W = 5 \mu\text{m}$, and $t = 1.5 \mu\text{m}$. The parameters for permalloy are $\sigma = 1.6 \times 10^6 \text{ S/m}$, $M_0 = 0.8 \text{ kG}$, and linewidth of 0.08 kG . The applied field is set to 1 kG with a resulting resonance near 10 GHz .

predict a loss of about 20% greater than their theory. This is in line with the higher losses shown in Pucel's own experiments [54]. It can also be seen that for both theory and simulation, the attenuation is very sensitive to the thickness of the top strip. Further simulations, not shown, found a much smaller variation in attenuation from changing the top strip width for $t > \delta$.

4.6 Microstrips with Embedded Ferromagnetic Layers

Using Eq. 4.29 the μ_{xx} and μ_{yy} components of the permeability were calculated in MATLAB and imported into HFSS as a data table of real values ($\text{Real}[\mu]$) and loss tangent values ($\text{Imag}[\mu]/\text{Real}[\mu]$).

We considered how the position of a ferromagnetic layer affects attenuation. For a microstrip, the magnetic field circulates around the top strip and hence provides a greater magnetic flux through a ferromagnetic layer that is closer to the top strip. Placement of a ferromagnetic layer directly under the top strip will result in the strongest magnetic excitation of the material and strongest attenuation at ferromagnetic resonance. This was verified by our simulations shown in Fig. 4.5, in which a thin layer of 100 nm permalloy layer was placed at various heights above the ground plate. It is shown that a layer directly below the top strip has a 4 dB/cm larger insertion loss than the one directly above the ground plate. Alternatively, for a parallel waveguide structure the attenuation is independent of layer position since the magnetic flux is constant for all positions. This was verified by simulation but is not shown here.

4.7 Experimental Results and Simulation

We further compared our simulations to experimental results. The filter structure was fabricated on a 0.5 mm silicon substrate. The structure was grown in a sputtering system with a background pressure maintained at $\sim 1 \times 10^{-7}$ Torr. After cleaning the silicon substrate in an ultrasonic bath, we annealed it to 300° C inside the vacuum chamber. All the depositions were done at room temperature. First a 5 nm thick Ta layer was deposited to achieve good adhesion of the grown structure to the substrate. This was followed by a 2 microns thick copper layer, which was used as the ground plane for our device. The next sequence of depositions, silicon dioxide (total thickness = 3 μm), Permalloy (100 nm), and finally 2 microns of copper, were made through a shadow mask. All depositions were carried out using magnetron sputtering. We patterned the film by photolithography and then dry etched to obtain the required strip widths and lengths for the devices. We used a variety of positions for the permalloy layer, as is discussed in the text.

The filter characterization was done using a vector network analyzer along with a micro-probe station in the frequency range from 100 MHz to 40 GHz and under static magnetic fields up to 4.5 kOe along the long axis of the microstrip line.

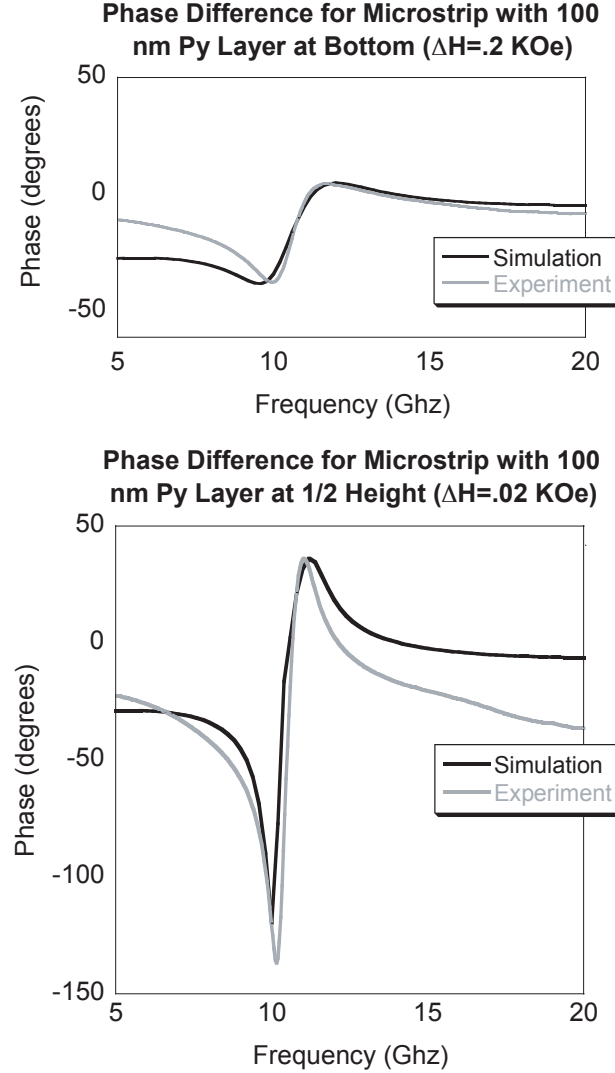


Figure 4.6: The simulated phase difference is plotted for two positions of a ferromagnetic permalloy layer: directly above the ground plate and at half-height of the microstrip. The conductor was copper, the dielectric was silicon dioxide ($\epsilon = 4$) and the permalloy layer was $0.1 \mu\text{m}$ thick. The dimensions of the microstrip were $h = 3.1 \mu\text{m}$, $W = 5 \mu\text{m}$, and $t = 1.5 \mu\text{m}$. The parameters for permalloy used in the simulation were $\sigma = 1.6 \times 10^6 \text{ S/m}$, $M_0 = 0.8 \text{ KG}$. The experiment was performed with a magnetic biasing field of $H_0 = 1 \text{ kOe}$; in the simulation $H_0 = 1.1 \text{ kOe}$ to fit the data. The linewidth parameter in the simulation, ΔH , was adjusted to fit the phase at resonance as given above. The simulated data was shifted down 30 degrees to match the experimental data.

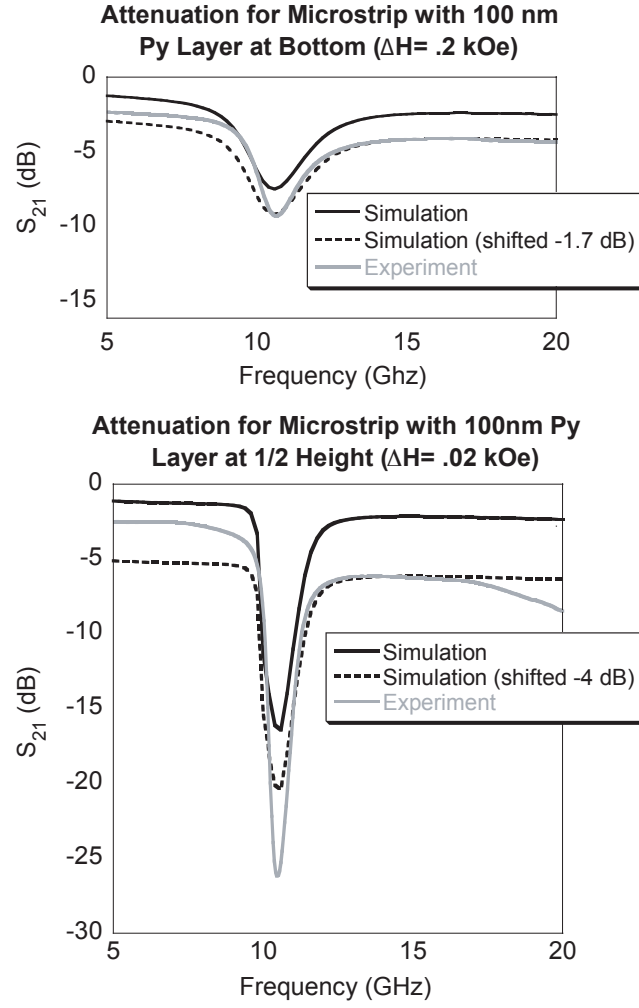


Figure 4.7: The experimental and simulated attenuation corresponding to Fig. 4.6.

Using a standardized copper coplaner structure (non-magnetic) obtained from the National Institute of Standards and Technology (NIST) at Boulder and NIST's Multical® calibration software, [57], a through-short-line (TRL) calibration was performed.

The microstrip operates in a TM mode, which ensures the ferromagnetic resonance condition because the RF magnetic field and the DC magnetic field are perpendicular to each other. The signal lines in our structures have a width (W) of $5 \mu\text{m}$, and a lengths (L) of 6 mm. The microstrips were generally designed to be close to a 50Ω characteristic impedance. For each filter the magnitude of S-parameters and phase advance were measured.

In the samples constructed, the magnetic quality of the permalloy film is dependent on the manufacturing process. A layer that is grown directly above the ground plate may have a substantially different linewidth compared to one grown at an intermediary position between the silicon dioxide layers. The linewidth of a permalloy film in the microstrip is difficult to measure directly but we can estimate it from simulations by using it as a fit parameter. In Fig. 4.6, the simulated versus experimental phase difference of S_{21} is plotted for two structures, one with the permalloy layer directly above the ground plate and another with the permalloy layer at the half-height position. The phase difference plot compares the phase difference between a microstrip with a 100 nm permalloy embedded layer and a microstrip with identical dimensions but no permalloy layer. In the simulations the linewidth parameter was varied until it fit the phase shift near resonance (note that the simulations were shifted down 30 degree to correlate with the experimental data at resonance). It can be seen from the fitted values of ΔH that the quality of the permalloy layer is much better for the half-height sample, with $\Delta H = 20$ Oe, than the bottom layer structure, with $\Delta H = 200$ Oe.

Using the same linewidth values we compared the insertion loss of the simulations to experimental data in Fig. 4.7. We see good agreement with the magnitude of loss due to ferromagnetic resonance for the two structures (dotted line). At frequencies far below the ferromagnetic resonance there is a nearly uniform overall offset of the experimental insertion loss from that of the simulation. We suspect that this difference is due to calibration issues in the experiment or possibly roughness in the conductor layers which have not been considered in the simulation.

4.8 Conclusion

We have demonstrated a useful method for characterizing ferromagnetic microstrip structures using the electromagnetic simulation software HFSS. These simulations provide good agreement with experimental results. Further, developing these numerical tools for simulating structures with integrated gyromagnetic materials allows us to now focus on the interesting resonant phenomena that occurs

when you combine fundamental material resonances with geometrically resonant metamaterials. We focus on this effect in the remaining chapters.

5

Planar Metamaterial Structures

The novel physical properties of metamaterials were first thoroughly demonstrated using bulk metamaterial structures, but shortly thereafter, Caloz and Itoh [58, 59] demonstrated that the metamaterial concept could also be applied in the transmission line environment. Transmission line structures, such as microstrip structures, support electromagnetic modes that travel along one or two dimension, and due to their energy confinement, have distinct advantages over bulk metamaterial structures.

In this chapter we introduce transmission line metamaterial structures and explore their properties. It is our goal to set the foundation for the ensuing discussion of hybrid metamaterials structures which incorporate resonant magnetic materials into planar metamaterial waveguide structures. With this objective, we first introduce metamaterial transmission line theory. We then characterize a new structure, the complementary split-ring resonator (CSRR), as a metamaterial structure for application in the transmission line environment. And finally, we demonstrate that the CSRR structure is a useful geometry for constructing tuneable metamaterials.

5.1 Introduction

A transmission line analysis can be applied to a wide variety of structures that support propagating electromagnetic modes. In the classical transmission line approach a structure, such as a microstrip, is assigned a per-unit-length shunt

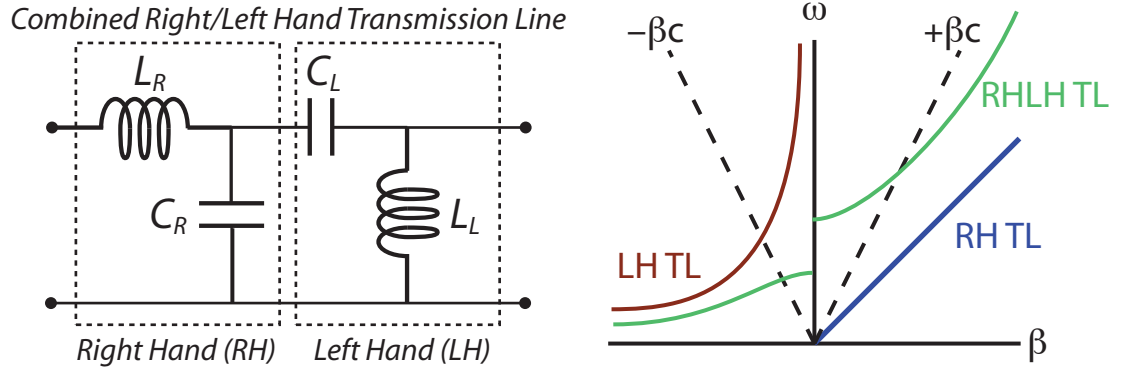


Figure 5.1: A circuit model of a transmission line is shown with characteristic per unit length lumped elements. L_R and C_R components characterizes conventional right hand (RH) transmission lines while L_L and C_L are representative of left-hand (LH) transmission lines. On the right, the characteristic dispersion diagram is shown for LH, RH, and RHLH transmission lines [2].

capacitance, C_R , and a series inductance, L_R , (we assume a loss-less line to simplify the analysis) and a lumped element “circuit” analysis is performed [46]. From this analysis general equations governing the propagation of electromagnetic modes are derived (the telegrapher equations),

$$\frac{d^2 V(z)}{dz^2} = -\gamma^2 V(z) = 0; \quad \frac{d^2 I(z)}{dz^2} = -\gamma^2 I(z) = 0. \quad (5.1)$$

The quantities $I(z)$ and $V(z)$ are the current and voltage as a function of position in the line and γ is the complex propagation constant. The complex propagation constant depends on the per unit length immittances (the impedance, Z , and admittance, Y) as,

$$\gamma = \alpha + j\beta = \sqrt{ZY}. \quad (5.2)$$

For a low loss transmission line, $\gamma \approx j\omega\sqrt{L_R C_R}$. The “effective” shunt capacitance and series inductance are fundamentally determined by the geometry of the waveguide and the electromagnetic properties of the material embedded in it. However, there is no reason we can’t consider the transmission line model literally. For example, a circuit structure with periodic lumped capacitor and inductor elements can support propagating electromagnetic modes.

Caloz and Itoh realized that it was possible to generate additional *artificial*

transmission line properties by including distributed elements in a microstrip structure [59]. If the periodic spacing of the elements are much less than the wavelength of interest, it is sufficient to treat the net effect of the discrete elements as a homogenous per unit length response. This parallels the methodology of treating bulk metamaterials as homogenous materials. Caloz and Itoh introduced periodic interdigital capacitors (to create a series capacitance C_L) and periodic vias (to create a shunt inductance L_L) in order to generate artificial electromagnetic properties in a microstrip structure, as illustrated in Fig. 5.1. These so called “left hand” (LH) properties cannot be obtained in a transmission lines using conventional “right hand” (RH) materials. The propagation constant, Eq. 5.2, for this artificial structure can be written in terms of the capacitance and inductance parameters as

$$\gamma = \sqrt{j\omega \underbrace{\left(L_R - \frac{1}{\omega^2 C_L}\right)}_{\mu} j\omega \underbrace{\left(C_R - \frac{1}{\omega^2 L_L}\right)}_{\epsilon}}. \quad (5.3)$$

Using these additional artificial transmission line components, Caloz and Itoh were able to create a negative index waveguide structures [59]. These Negative index metamaterial transmission lines share the same novel properties as bulk negative index materials including oppositely directed phase and group velocity for propagating waves. The dispersion curve for LH/RH transmission line structures is shown in Fig. 5.1. For a LH transmission line, the phase (slope of the line from the origin to the curve) is negative while the group velocity (tangent of curve) is always positive. This can be compared to the dispersion of a RH transmission line, also shown, which has both positive group velocity and phase velocity. Purely LH transmission lines are not physical because any transmission line structure will have inherent parasitic series inductance and shunt capacitance. The dispersion curve of a real metamaterial transmission line is always going to be a combined LHRH transmission line as shown Fig. 5.1. The LH band and RH band do not necessarily connect, but it is possible to pick parameters such that they do.

The lumped element transmission line analysis provides intuition on how to build a transmission line metamaterial structure and it is historically how this area was approached. However, the lumped element transmission line analysis

must correlate with a direct interpretation using Maxwell's equations. Through a comparison of the telegrapher equation with a full field analysis of a transverse electromagnetic mode in the structure [2], we can correlate the transmission line parameters (L_R , C_R , L_L , C_L) to the constitutive parameters (as noted in Eq. 5.3)

$$\mu = L_R - \frac{1}{\omega^2 C_L}, \quad (5.4)$$

$$\epsilon = C_R - \frac{1}{\omega^2 L_L}. \quad (5.5)$$

We can therefore use whichever conceptual framework is most convenient to analyze these transmission line structures. In this dissertation we focus on a direct field analysis of transmission line structures (corresponding to the analysis of bulk metamaterials structures). We approach the field in this way because we are interested in considering resonant geometrical elements, as opposed to Caloz and Itoh's distributed elements, as a method of making metamaterial transmission lines. These resonant structures have an electromagnetic response that is analogous to bulk metamaterial resonant elements and can be analyzed in the same fashion as we will demonstrate.

The resonant structures we consider have the advantage of being simply integrated into conventional planar circuit technology transmission lines such as microstrips and parallel waveguides. They also are easily scaled to work at frequencies up into the terahertz frequency range. We further demonstrate that these structures can be adapted to make tunable metamaterials in the transmission line environment.

5.2 Complementary Structures

Recent work has shown that another method for implementing the metamaterial concept in the transmission line environment is to use planar resonant structures. Martin et al. [60] demonstrated that SRRs can be excited directly by a coplanar waveguide structure when placed in close proximity to the coplanar strips. Further, when the SRRs were combined with periodic gaps in the coplanar strips, left handed transmission was observed. Such a planar structure is simpler to fab-

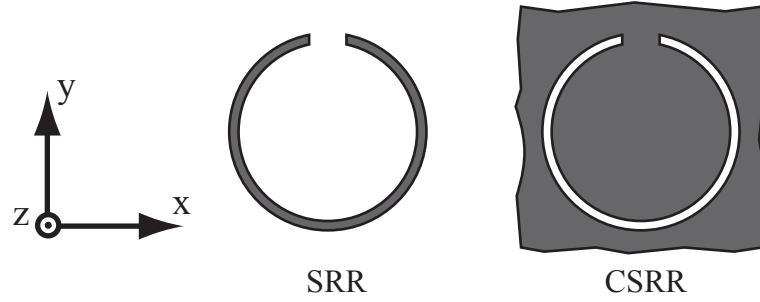


Figure 5.2: The dual split-ring resonator (SRR) and complementary split-ring resonator (CSRR) metasurfaces are shown.

ricate than the lumped element LH transmission line structures first proposed by Caloz and Itoh which require the construction of vias through the substrate. But, the disadvantage of integrating the SRR structure into transmission lines is that the magnetic flux of the coplanar line does not efficiently excite the SRR structures because of the geometry of the coplanar line.

A superior resonant planar particle for integration in the transmission line environment is known as the “complementary” split-ring resonator (CSRR) and was first proposed by Falcone et al. [61, 62]. The CSRR structure is the complementary metasurface of the SRR structure as shown in Fig. 5.2. The inspiration for the CSRR structure, and its nomenclature, comes from Babinet’s principle in diffraction theory [63] which relates the diffraction from one diffraction screen to that of its complementary diffraction screen.

Consider a SRR structure which has a magnetic moment that can be excited by a magnetic field perpendicular to the ring and a weak electric moment that can be excited by an electric field across the gap of the SRR. Babinet’s principle is the physical principle that the complementary CSRR structure will have a complementary excitation—the CSRR has a strong electric moment that is excited by an electric field perpendicular to the surface and a weak magnetic moment that can be excited by a magnetic field across the neck, see Fig. 5.3. This duality between the particles can be understood intuitively by considering the nature of Maxwell’s equations. Notice that in regions where there are no sources, i.e. charges or currents, Maxwell’s equations exhibit a high degree of symmetry,

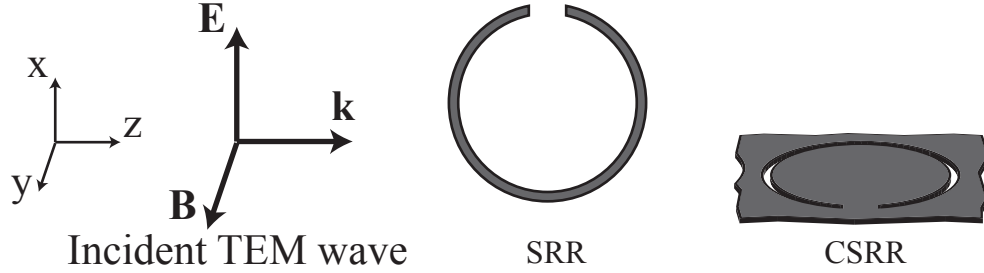


Figure 5.3: For the TEM incident wave shown, an SRR in the xz -plane is excited by the magnetic field. A CSRR in the yz -plane will be excited by the electric field. Note that the CSRR structure can only be excited if the TEM wave is incident either above or below the plate, but not both.

$$\left. \begin{aligned} \nabla \times \mathbf{E} &= -\mu \frac{d\mathbf{H}}{dt} \\ \nabla \times \mathbf{H} &= \epsilon \frac{d\mathbf{E}}{dt} \end{aligned} \right\} \begin{aligned} \nabla \times (\sqrt{\epsilon}\mathbf{E}) &= -\sqrt{\epsilon\mu} \frac{d}{dt}(\sqrt{\mu}\mathbf{H}) \\ \nabla \times (\sqrt{\mu}\mathbf{H}) &= \sqrt{\epsilon\mu} \frac{d}{dt}(\sqrt{\epsilon}\mathbf{E}). \end{aligned} \quad (5.6)$$

In this suggestive form, we notice that if we make the substitution

$$\mathbf{E}' \rightarrow \pm\alpha\sqrt{\frac{\mu}{\epsilon}}\mathbf{H}, \quad (5.7)$$

$$\mathbf{H}' \rightarrow \mp\alpha\sqrt{\frac{\epsilon}{\mu}}\mathbf{E}, \quad (5.8)$$

where α is a constant, we get back the original Maxwell equations. Hence, given a set of boundary conditions and their associated field solution (with no charges/current inside this region), we immediately know the field solution to a region with the electric and magnetic boundary conditions interchanged (to within a constant $\pm\alpha$).

We can observe this duality in numerical simulations of SRR and CSRR structures. Consider a SRR particle excited near resonance by a TM mode perpendicular to the structure and a CSRR structures being excited by a TE mode (incident along the top surface for reasons that will be explained shortly). As shown in Fig. 5.4, above the plane of the structures, the magnetic field of the SRR is symmetric to the electric field of the CSRR structure. Below the plane, the fields are symmetric but in opposite directions. Alternatively, we can compare the electric field in the SRR gap region to the magnetic field of the CSRR in the same region as in Fig. 5.5. We see again that the electric field of the SRR in the gap matches the magnetic field in the CSRR structure above and below the plain to within a negative sign.

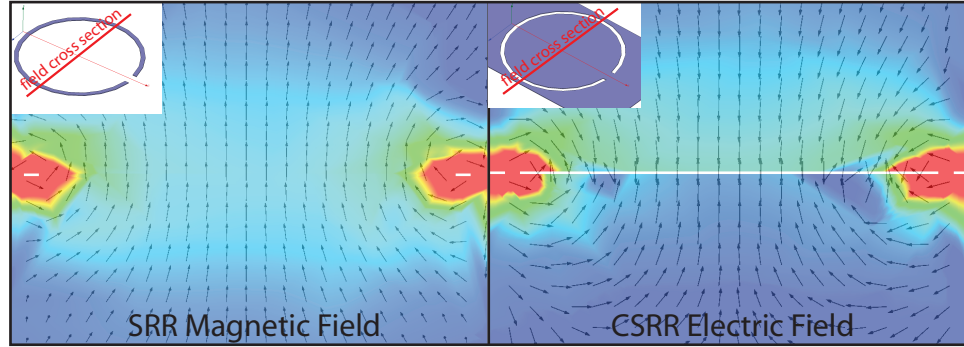


Figure 5.4: The numerical simulations of the magnetic field of a SRR structure and the electric field of a CSRR structure. A cross-section shows the fields are symmetric above the plane and symmetric, but in opposite directions, below the plane

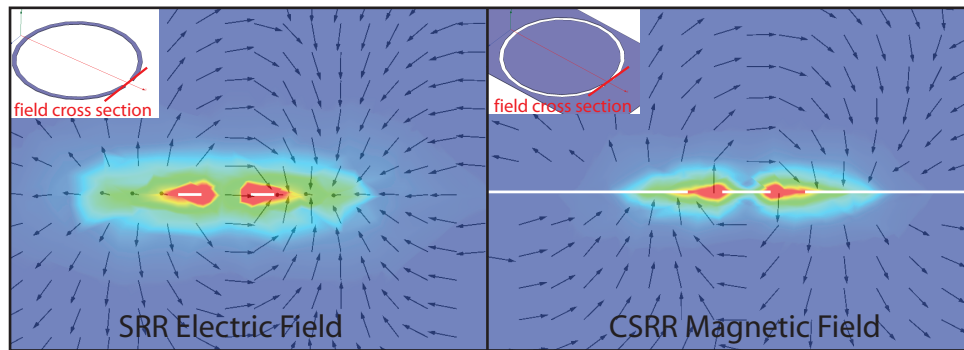


Figure 5.5: The numerical simulations of the electric field of a SRR structure and the magnetic field of a CSRR near the gap. The fields are symmetric above the plane and symmetric, but in opposite directions, below the plane.

The duality of planar structures has been tested experimentally by comparing the transmission for a single layer of SRR and CSRR structures for a variety of oblique TM and TE polarized incident waves by Beruete et al. [64]. The CSRR structure is excited most strongly by a TE wave incident at grazing angle because the electric field is directly perpendicular to the structure—the corollary of the SRR being most strongly excited by a perpendicular magnetic field. This is precisely the excitation a particle will experience in the transmission line environment with the TE mode traveling parallel to the surface. This is confirmed by numerical simulations in which a characteristic CSRR cut into a parallel plate waveguide is strongly excited by the incident TEM mode as shown in Fig. 5.6. Further the cross polarization of the CSRR structure can be seen. When the CSRR structure is parallel to the mode propagation, such that the mode excites the electric and magnetic moment simultaneously, it is more strongly excited then when the structure is oriented perpendicularly where only the electric moment is excited.

The symmetry in the fields between the structures can be exploited to determine the resonant properties of the CSRR structures. The SRR structure has already been well characterized in the literature [7] and we can use this duality in the fields to derive a functional relationship between the capacitance and inductance of complementary structures. In Appendix A, it is shown that for complementary metasurfaces the relationship between the capacitance and inductance is,

$$\frac{C}{L} = (\epsilon_1 + \epsilon_2) \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right), \quad (5.9)$$

where ϵ_i and μ_i are the permittivity and permeability above and below the metasurfaces. Generally, a planar structure has both an associated inductance and capacitance. Assuming, that the structure is small compared to the wavelength, so that the fields are generally decoupled, Eq. 5.9 allows us to quickly calculated C and L values from those of it's complimentary structure.

It is important to note that while the resonances have a dual relationship, the mechanisms of excitation are subtly different. The effective moments of the CSRR structure must be equal and opposite on either side of the CSRR screen in order to satisfy symmetry constraints placed on the fields—due to the current being constrained to move in the plane [63] (this is not true for the dual SRR structure

whose moment is in the same direction on either side of the plane). As such, if an incident TE mode is perpendicular to the CSRR particle but incident on both sides of the structure, the resonance will be inhibited. This makes the CSRR structure useless as a bulk metamaterial structure because the electromagnetic response averages to zero. But, if we limit our interest to incident modes on one side of the CSRR plane, precisely those modes found in transmission lines, the structure can be quite useful as a planar metamaterial structure. For such obliquely incident waves, duality of the SRR and CSRR structure allows us to surmise that in the homogenous limit, the permittivity of a CSRR structure has a Drude-lorentz form, i.e. the dual of the permeability for SRR structures[10]. Then, the permittivity of a CSRR structure is given by,

$$\epsilon(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\Gamma_c\omega}, \quad (5.10)$$

where F is the filling fraction, ω_0 is the resonant angular frequency, and Γ_c is the dissipation factor[7]. The resonant frequency ω_0 is related to the inductance, L , and capacitance, C , of the structure by

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (5.11)$$

As for bulk metamaterials, it is possible to extract the constitutive parameters for planar metamaterial structure from the transmission and reflection coefficients [12]. For a coplanar waveguide, microstrip, or any other transmission line structure whose propagating mode is quasi-TEM in nature, the constitutive parameters can be extracted using the inversion formula given in Eq. 1.5 and Eq. 1.6.

A sample microstrip structures was constructed as a demonstration of the implementation of the metamaterial concept in the transmission line environment using planar structures. The metamaterial structure was designed to exhibits negative index effects as shown in Fig. 5.7. The structure consisted of a microstrip with capacitive gaps along the top strip and CSRR particles etched into the ground plane using a photolithographic process. A passband is seen around 6 Ghz which signifies the negative index region. Numerical simulations using Ansoft's commercial software HFSS were used to design and verify the response as shown.

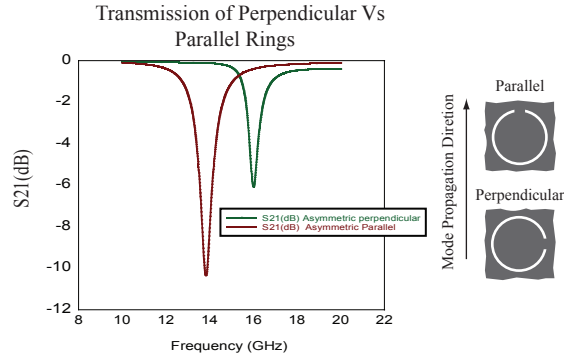


Figure 5.6: A comparison of the excitation of a CSRR structure in the parallel waveguide environment with respect to its orientation. The excitation is greater for the parallel excitation because the electric and magnetic dipole are excited simultaneously.

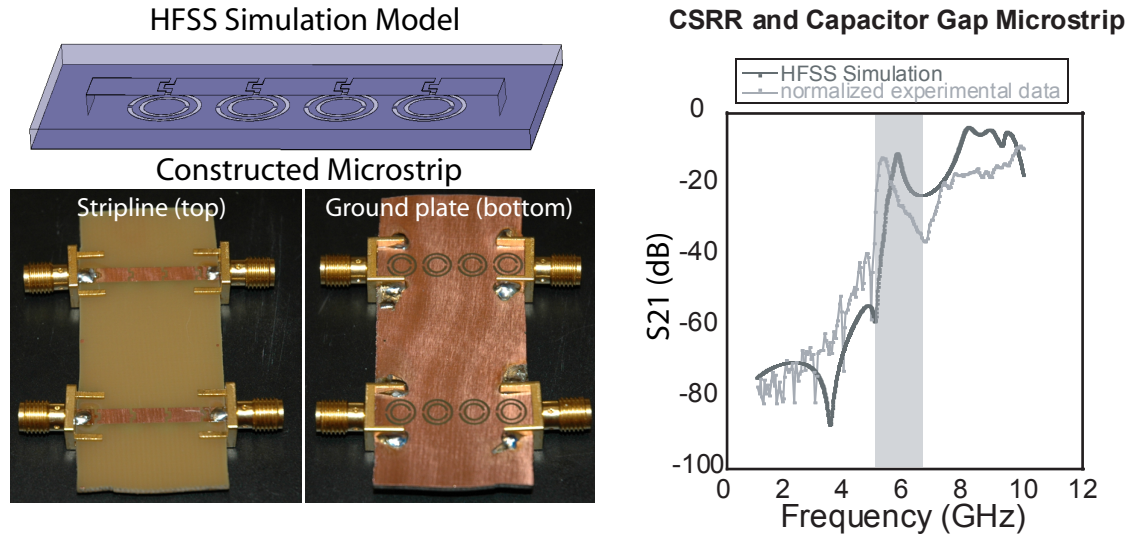


Figure 5.7: A fabricated negative index microstrip is shown that utilizes CSRR structures etched in the ground plane, for effective negative permittivity, and capacitive gaps in the top strip, for effective negative permittivity. The simulated S-parameters show a passband near 6 GHz that signifies the negative index region.

5.3 Tuneability of the CSRR Structure

A deficiency of the metamaterial approach is that it can provide a desired set of materials properties only over a narrow frequency band of operation. This physical constraint is a consequence of depending on resonant behavior to provide the metamaterials unique properties (any resonant system is necessarily dispersive [3]). This is not an intolerable limitation if we consider that it should be possible to make a tunable metamaterials that can be reconfigured to take on the properties of interest at a particular frequency of operation. To conceive of how this might be done, consider the classical metamaterial structures, SRRs and wires, first demonstrated by Smith. These structures depend on purely geometrical resonances, i.e. the material properties of the metamaterial have a constant response over the frequency range of interest. We can consider incorporating materials with tuneable electromagnetic properties into the metamaterial structures as a means of modulating the resonant response. Ferroelectric and gyromagnetic materials are good candidates for such integration. We first consider how ferroelectric materials can be used to make tuneable devices (we consider gyromagnetic materials in Chapter 6).

Ferroelectric materials have a nonlinear polarizability as function of applied electric field. This means a DC electric field can be used to bias the ferroelectric material to have a particular dielectric constant. An applied dynamic AC electric signal across the biased material will experience this modified dielectric constant (this is assuming the AC signal is significantly smaller than the DC field and or the mechanisms behind the biasing are slow acting). For a metamaterial, whose resonance is a function of the capacitance in the structure, tuning the dielectric constant in the vicinity of the structure provides a method of modulating the resonant response. This has been demonstrated with bulk SRR structures by Zhao et al. [65]. They positioned an array of SRR structures in a vat of nematic liquid crystal. Nematic liquid crystal consists of polar molecules that in their unperturbed state have an isotropic dielectric constant, ϵ_{iso} . When a biasing electric field is applied across the material, the polar molecules are aligned such that the material exhibits an increased dielectric constant in the direction of the field, $\epsilon_{\parallel}$, and

decreased dielectric constant in the other directions, $\epsilon_{\perp}$. By applying an electric field across the vat they were able to change the dielectric background of the SRRs and hence their structural capacitance resulting in a shift in the metamaterials resonance.

The actual electric field applied across the liquid crystal was on the order of 2 kilovolts per unit cell with a resulting 2% tuneability. This is a fairly extreme voltage value and highlights the low effectiveness of using liquid crystals pervasively throughout the unit cell. In fact, it is known that the local fields of the metamaterial structure are concentrated in certain regions of the unit cell and by concentrating the tuneable materials in these regions we get nearly the same tuneability as a wider application. For the SRR structure, the electric field is concentrated in the gap region and hence it is sufficient to tune the dielectric of the liquid crystal in this region to change its capacitance. For the SRRs in the Zhao experiment, providing a bias across the gap region would require only 100 volts of bias to get the same tuneability. Of course, designing a biasing mechanism solely across the gap in a 3D bulk metamaterial structure and fabricating it is not a simple task. Some progress has been made in this effort, for example Hand et al. [66] used ferroelectric lumped elements across the SRR gaps to bias bulk SRR structures.

In the transmission line environment we can conceive of tuneable metamaterial structures that rely on CSRR structures that are considerably easier to fabricate than their bulk metamaterial counterpart. For CSRRs, the local electric field is strongly concentrated near the notch region and changing the dielectric in this location can have a significant effect on the bulk properties of the system. One such system we have considered is the double ring structure shown in Fig. 5.8. As can be seen from numerical simulations of a characteristic structure, the resonance of two closely space CSRR structures is coupled and of a similar magnitude as a single CSRR structure. The advantage of the double layer structure is that a bias can be maintained between the two layers as a means of tuning a ferroelectric material in between the layers and therefore the capacitance of the CSRR structures. This will in turn alter the resonance of the CSRR structure. Because the biasing structure

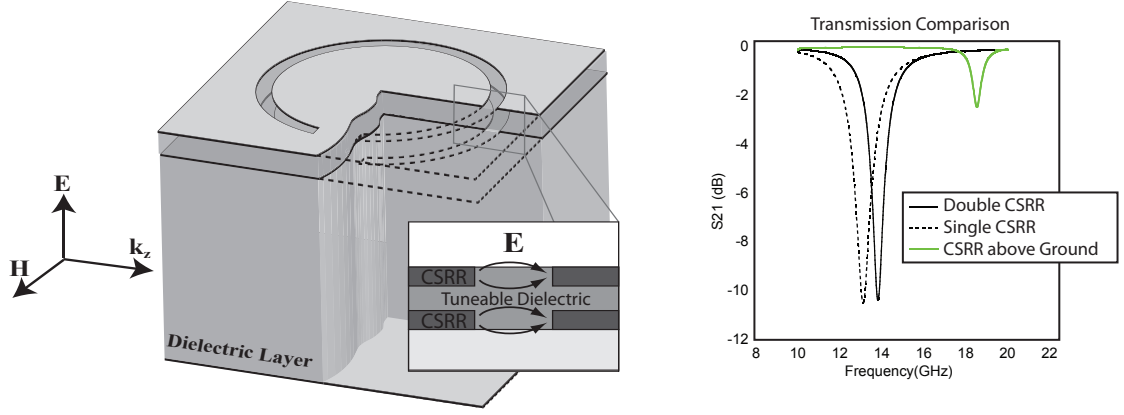


Figure 5.8: A unit cell diagram is shown for a tunable double layer CSRR structure in a parallel waveguide transmission line. A tuneable dielectric can be placed between the CSRR layers and biased with a voltage difference between the two layers. Also shown is $S_{21}(\text{dB})$ (transmission) for a double layer CSRR structure separated by a 0.1 mm thick dielectric ($\epsilon_r = 4$). For comparison, a single CSRR structure and a CSRR structure with the top plane replaced by a uniform ground plane are shown. The characteristic CSRR structure considered had a radius of 1.8 mm, notch width of 0.2 mm, neck width of 0.2 mm, and unit cell size of 3 mm.

is external to the parallel plate transmission line it is invisible to the propagating mode in the transmission line for frequency outside the resonance of the rings. By changing the dielectric of the sandwiched material we see that it is possible to tune the resonance significantly as shown in Fig. 5.9.

We can also consider a tunable double CSRR structures in a microstrip transmission line. The transmission line concentrates the mode and associated fields along the top strip of the structure. We obtain a strong interaction by inserting a double layer CSRR structures in this region as shown in Fig. 5.10. Since the electric field at resonance of the double layer structures is approximately parallel to the plates, we can take advantage of the anisotropy of the liquid crystal properties. If we initially orient the liquid crystal parallel to the microstrip (using some kind of surface coating as is often done with LCs devices) the capacitance in the gap will depend on $\epsilon_{\parallel}$. We can then use a controllable DC voltage bias between the two plates to reorient the LC alignment perpendicular to the gaps such that the capacitance is a function of $\epsilon_{\perp}$. The effective change in capacitance provides the tuning of the CSRR resonance. We investigated this design using a numerical

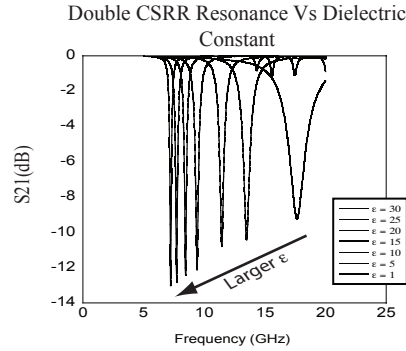


Figure 5.9: The effect of changing the dielectric between the double CSRR structures with characteristic parameters given in Fig. 5.8 is shown.

simulation of a microstrip constructed out of F4B (which we can easily fabricate in our lab) and Merck's MDA-03-2838 liquid crystal properties as characterized by Mueller et al.[67]. The results are shown in Fig. 5.10. We see that there is effectively 700 Mhz of tuneability if we can take advantage of the full anisotropy and about 200 Mhz if we can only take advantage of the isotropic to perpendicular LC transition, i.e if the liquid crystal is not biased in the parallel direction initially and the dielectric constant is some lesser value, ϵ_{iso} . Since liquid crystal materials are just starting to be designed for microwave applications it has proven difficult to obtain these prototype materials from the manufactures and we were unable to test this design. Never-the-less, it is hopeful that in the future this design can be used to implement the tuneable metamaterial concept.

The double layer rings might also be used with other ferroelectric materials such as barium strontium titanate (BST). Thin films of BST on the order of a micron can be manufactured to have strong tuneability when a voltage is applied across the material. One can conceive of making a tuneable double CSRR structure by scaling down its dimensions to complement the thin film nature of BST materials.

5.4 Conclusion

In general, complementary CSRR structures are effective metamaterial elements in the transmission line environment. They also provide a useful geometry

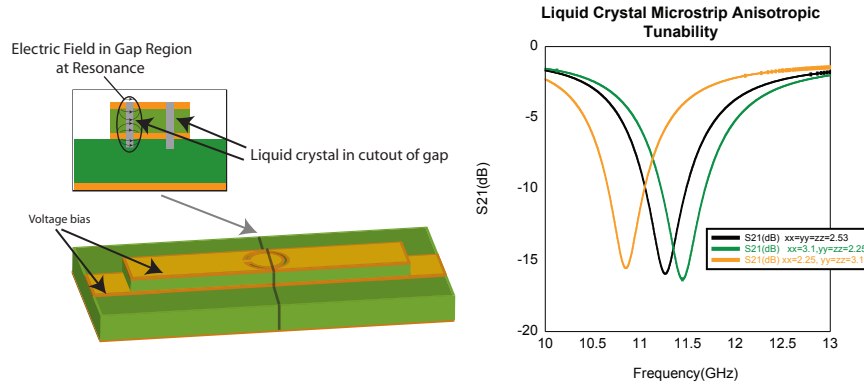


Figure 5.10: A configuration for a tunable double CSRR structure implemented in the microstrip environment is shown. Numerical simulations show that for a liquid crystal whose molecules are aligned in the z -direction via a voltage bias between the two CSRR planes, the tuneability is up to 700 Mhz depending on the initial orientation of the liquid crystal molecules.

for creating tunable material structures by strategically targeting the local fields of the CSRR structure with tunable materials. These tunable metamaterials can be conveniently implemented in the transmission line environment. Besides tunable materials, one might also consider integrating a material with a fundamental resonance into the CSRR structure. In the next chapter we focus on the interesting electromagnetic properties of a hybrid resonant structure.

6

Hybridization of Fundamental Magnetic Material Resonances with Artificial Metamaterial Resonances

Metamaterials derive the unique properties from geometrically derived resonances. The metamaterial structures considered to date have been constructed out of metallic and dielectric materials whose responses are constant over the frequency of interest. Recently, there has been interest in integrating tunable materials into metamaterial structures as discussed in Chapter 5. It is compelling to go one step further and consider integrating fundamentally resonant materials into metamaterial geometrical structures. In this chapter we investigate a tunable metamaterial structure that incorporates a resonant magnetic material that is itself frequency dispersive over the range of operation. The two dispersive systems, metamaterial structure and magnetic material, combine to exhibit a distinct hybrid resonance. We determine an analytical model for this system that demonstrates good agreement with numerical results.

6.1 Introduction

The unique electromagnetic properties of metamaterials are obtained by harnessing the resonant behavior of many periodic and subwavelength composite structures. Unfortunately, the resonant nature of metamaterials also ensures that they are frequency dispersive and limited to a narrow frequency band of operation. One method of bypassing this constraint is to design tunable metamaterials, which though still dispersive, can be tuned to have the desired properties over the frequency range of interest. A tunable metamaterial can be made by placing a material with a tunable electromagnetic parameter, permittivity or permeability, in a region of the metamaterial cell where the local fields are concentrated as discussed in Chapter 5. Tuning the local fields will in turn affect the bulk electromagnetic response of the metamaterial. If a tunable material is used, that itself is resonant, then a unique hybrid resonance occurs that is a unique blend of the material resonance and metamaterial resonance.

The most common resonant magnetic materials are the ferrimagnetic and ferromagnetic materials that have a characteristic gyromagnetic resonance (as discussed in Chapter 4). These materials provide tuneability and nonreciprocal properties that are the foundation of many modern microwave devices. With the recent advent of metamaterials, it is a timely question to ask what advanced metamaterial structures might be made by incorporating these gyromagnetic materials—or vice versa, how the unique properties of these magnetic materials might be extended by incorporating them into metamaterial structures.

6.2 Electromagnetic Response of a CSRR Waveguide Structure

Metamaterials with artificial magnetic properties have been constructed using SRR structures and exhibit an approximate magnetic Drude-Lorentz response that has been well documented in the literature [10]. As discussed in Chapter 5, a related resonant structure is called the complimentary split-ring resonator (CSRR)

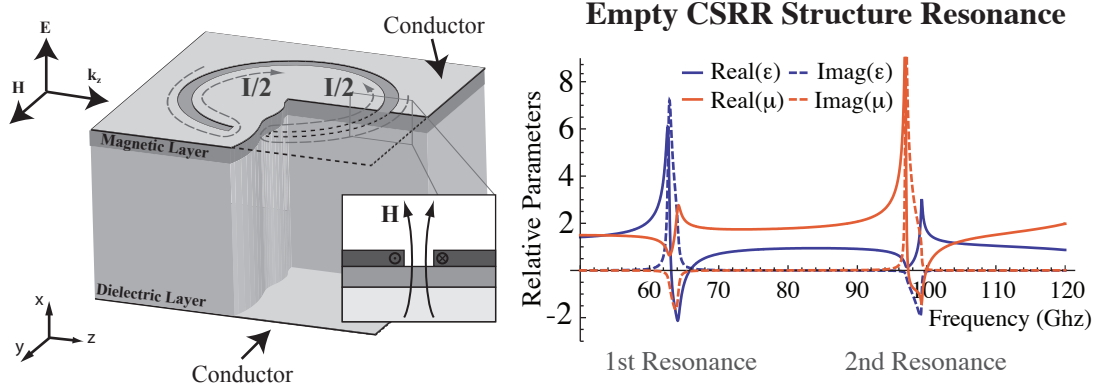


Figure 6.1: A diagram of a unit cell of the CSRR parallel plate waveguide is shown. When the CSRR is resonating, current flows along the edges of the structure and produces a magnetic field that is approximately perpendicular in the gap region. The extracted permeability and permittivity are shown for the frequency range including the first two resonances of the CSRR waveguide structure without the influence of the magnetic material.

and it has the “complementary” metasurface of a SRR (as shown in Fig. 6.1). The dual CSRR and SRR structures obey Babinet’s principle of equivalence with equal resonant responses [61, 62], but the CSRR is excited by a perpendicular electric field while the SRR structure is excited by a perpendicular magnetic field. It is possible to define effective bulk parameters for a waveguide metamaterial structure [68]. By Babinet’s equivalence, the CSRRs permittivity has a characteristic Drude-Lorentz response that is the dual of the SRR structure and is given by

$$\epsilon(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2 + i\omega\Gamma_c}, \quad (6.1)$$

where F is a constant, ω_0 is the resonant angular frequency, Γ_c is the dissipation factor, and ω is the angular frequency [7]. The resonant frequency ω_0 is related to the inductance, L , and capacitance, C , of the structure by

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (6.2)$$

The Drude-Lorentz constants can be determined approximately using analytical expressions as demonstrated by Baena et al. [62]. Alternatively, the constants can be determined by fitting the Drude-Lorentz form to the numerically simulated structure response as we now discuss.

A particular CSRR structure's bulk permittivity (and permeability) can be extracted from full-wave simulations of the CSRR structure. We utilize Ansoft's commercially available electromagnetic finite element solver (HFSS) to determine the scattering matrix of the structure. Then, an inversion of the scattering parameters, Eq. 1.5 and Eq. 1.6, are used to determine the constitutive parameters [12]. It is sufficient to simulate one CSRR unit cell, as shown in Fig. 6.1, and apply appropriate boundary conditions to reproduce the characteristic response of the periodic parallel plate waveguide structure. The simulated structure consisted of two parallel perfectly conducting plates and adjoining perpendicular perfect magnetic boundaries. The structure was excited through waveports on either sides of the waveguide with a fundamental TEM mode. The CSRR structure was cut into the top waveguide plate and an exterior volume above the structure was defined in order to accommodate electromagnetic fields emanating above the structure. Perfect magnetic boundaries were defined on the exterior volume's surfaces perpendicular to the propagation mode; perfect electric boundaries were defined on the other surfaces including the top of the extra boundary volume (note: the height of the extra volume is chosen such that on the boundary the electromagnetic fields decay to near zero—and hence its specification is irrelevant, but numerical convergence is faster by defining an electric boundary here).

For the hybrid resonance investigation, we choose geometrical parameters of the CSRR to give a resonance near 64 GHz. The unit cell was 1 mm, the ring radius was 0.4 mm, the neck was 0.15 mm wide, and the notch width was 0.035 mm. The spacing between the plates, which determines the strength of the CSRR resonance, was set to 0.4 mm. The top and bottom waveguide plates were 1 μm thick and had the conductivity of copper ($\sigma = 5.6 \times 10^7 \text{ S}$). For simplicity, the dielectric between the plates was set to have a dielectric constant of $\epsilon = 1$.

The structure was first simulated without any magnetic layer and the S-parameters calculated. A parameter extraction was performed on the S-parameters of the simulated structure to determine the effective permittivity and permeability as shown in Fig. 6.1. We expect the structure to behave as a Drude-Lorentz material in the homogenous limit, but at resonance, the wavelength of the traveling mode is on the

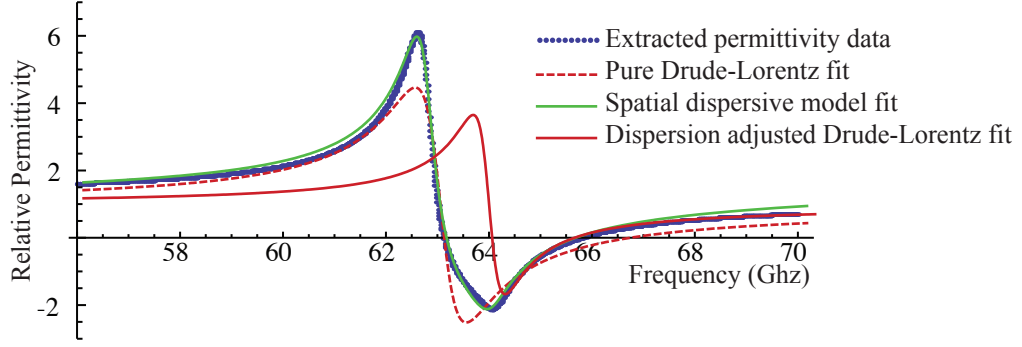


Figure 6.2: The extracted permittivity for the 1st resonance is shown along with various fitting methods.

order of the unit cell size which results in spatial dispersion effects that distort the “ideal” Drude-Lorentz form. In fact, a naive least-square fit of the extracted parameters using a Drude-Lorentz functional form will be unduly influenced by these spatial dispersion effects and result in a fit that is inaccurate. This can be seen in Fig. 6.1 where the pure Drude-Lorentz fit does not accurately match the data. In order to obtain an accurate fit, we applied a modified Drude-Lorentz model that incorporates spatial dispersion effects as proposed by Liu et al. [69] (we provide more details of this fitting method in Chapter 7). From this fitting method, the parameters of Eq. 6.1 were determined: the resonant frequency, $\omega_0 = 1/\sqrt{\epsilon_0\mu_0 g_{\text{geom}} f_{\text{geom}}}$; the constant, F ; and the loss tangent, Γ_c . We can plot the non-dispersion adjusted Drude-Lorentz model but with the resonant frequency determined by the Liu’s fitting method, as shown in Fig. 6.1. We see that the resonant position of this function is actually shifted to the right of the resonance, signifying the actual resonant position of the metamaterial structure. In the hybrid analytic model we will derive, it is important to accurately characterize the resonant frequency. In fact, it is sufficient to use this “resonant frequency adjusted” Drude-Lorentz model (ignoring dispersion effects). For simplicity, we use this model—even though it is not as accurate as the full dispersion adjusted Drude-Lorentz model, it provides very good agreement with simulated results.

6.3 Incorporation of a Resonant Magnetic Material into a CSRR Structure

We now consider the integration of a resonant magnetic material into the CSRR structure. We model our magnetic material after gyromagnetic materials which have a permeability tensor of the form,

$$\mu = \mu_0 \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (6.3)$$

where μ_1 and μ_2 are resonant functions of frequency as given by Eq. 4.18. In this study we consider a simplified “generic” resonant magnetic material which is isotropic and does not include the off-diagonal permeability components of the gyromagnetic tensor, in order to elucidate the underlying physics of the hybridization. The relative permeability is given by,

$$\mu_r(\omega) = \frac{(\xi M_s + \xi H_0 - i\omega\Gamma_m)^2 - \omega^2}{(\xi H_0 - i\omega\Gamma_m)(\xi M_s + \xi H_0 - i\omega\Gamma_m) - \omega^2} \quad (6.4)$$

where M_s is the magnetic saturation of the material, H_0 is the magnetic bias field, Γ_m is the damping constant, and $\xi = \gamma\mu_0$ is a constant with γ defined as the gyromagnetic ratio [48]. The function form of the permeability considered is based on the Schloemann permeability tensor which is a diagonal tensor whose components are a combination of the diagonal and off diagonal components of the gyromagnetic material as discussed in Chapter 4 [48]. As such, this investigation is suggestive of how ferrimagnetic or ferromagnetic materials can be used to make a tunable metamaterial structure but for actual gyromagnetic materials the interaction will have a more complicated dependence on the orientation of the biasing direction of the gyromagnetic material with respect to the geometry of the CSRR structure [70, 71]. We will consider the potential for integrating a real gyromagnetic material into the metamaterial structure after we consider the simplified magnetic interaction.

The inclusion of a thin layer of magnetic material below the CSRR structure as shown in Fig. 6.1 provides a means of influencing the the local fields of the

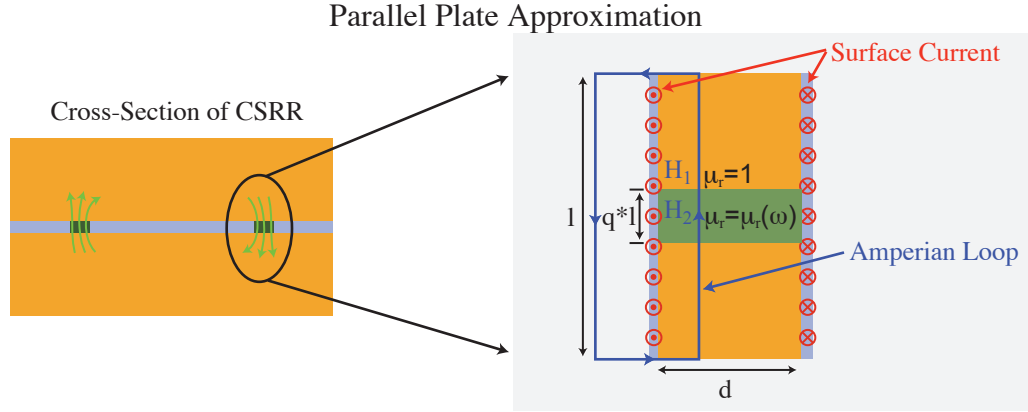


Figure 6.3: The parallel current sheets or *magnetic* capacitor approximation is shown for the gap region of the CSRR structure.

CSRR structure. The CSRR resonance and magnetic material resonance interact to produce a hybrid resonance. The mechanism of interaction can be understood by considering the dynamics of the current flow when the CSRR is resonating. An incident TEM wave in the waveguide drives current symmetrically in and out of the CSRR structure along the edges of the gap region as shown in Fig. 6.1. The current is concentrated on either side of the gap of the CSRR structure but flows in opposite directions creating a nearly perpendicular magnetic field inside the gap and the regions directly above and below the gap. Consequently, determining the inductance of the CSRR structure is analogous to analyzing the *magnetic* capacitor model of parallel current sheets filled with some volume fraction (to be determined) of frequency dependent magnetic material. Consider the structure in Fig. 6.3. The magnetic boundary condition at the interface between the magnetic material (H_2) and dielectric (H_1) is,

$$H_2 = \frac{1}{\mu_r(\omega)} H_1, \quad (6.5)$$

where $\mu_r(\omega)$ is the relative permeability of the magnetic material. Applying Amperé's law, we obtain the magnetic field in terms of the magnetic filling fraction q ,

$$\int H \cdot dl = I \Rightarrow B_2 = \frac{\mu_r(\omega)}{\mu_r(\omega)(1 - q) + q}. \quad (6.6)$$

If w is the length of the parallel sheets (not shown in the figure), d is the width

of the gap, and l is the height of the gap; we can solve for the inductance of the parallel current sheets configuration,

$$L = \frac{\Phi}{I} = \frac{\int B \cdot d\mathbf{A}}{I} = \frac{\mu_r(\omega)}{\mu_r(\omega)(1-q) + q} \left(\frac{wd}{l} \right). \quad (6.7)$$

We can apply the parallel plate approximation to the CSRR structure if we exchange the geometrical dependent term, wd/l , for a constant, g_{geom} with units of length that is specific to the geometry of the CSRR structure. The inductance of the CSRR structure is then,

$$L = \mu_0 \left(\frac{\mu_r(\omega)}{\mu_r(\omega)(1-q) + q} \right) g_{\text{geom}}. \quad (6.8)$$

If the parallel current approximation were exact, then $g_{\text{geom}} = (1/2)wd/l$ (the factor of 1/2 follows from the parallel current flow into the CSRR structure). In general, the geometrical function is more complex but it can be extracted from numerical simulations of the empty CSRR waveguide as we demonstrate below. The capacitance of the CSRR structure follows from the capacitance across the gap above and below the metal-surface. It is given by,

$$C = \epsilon_0 f_{\text{geom}}, \quad (6.9)$$

where again, f_{geom} is a constant with units of length that is determined by the geometry of the CSRR structure and can be extracted from numerical simulations.

Inserting Eq. 6.8 and Eq. 6.9 into Eq. 7.3 provides an equation that can be solved to determine the resonant frequency of the hybrid CSRR/magnetic structure,

$$\omega'_0 = \frac{1}{\sqrt{\frac{\mu_r(\omega'_0)}{\mu_r(\omega'_0)(1-q) + q}}} \frac{1}{\sqrt{\epsilon_0 \mu_0 f_{\text{geom}} g_{\text{geom}}}}. \quad (6.10)$$

We note that Eq. 6.10 reduces to the resonant frequency of the empty structure in the case $\mu_r(\omega)$ is of unity. As previously mentioned, the magnetic field generated in the gap region of the CSRR at resonance is predominately perpendicular (see Fig. 6.1) and interacts principally with the magnetic material through the x -component of the permeability. In fact, simulations (not shown here) have shown that variation of any of the other diagonal permeability components has no effect on the response of the structure. We can insert the permeability Eq. 6.4

into Eq. 6.10 and solve to get the new resonant frequency of the hybrid structure. Eq. 6.10 is transcendental in nature, as a result of the magnetic materials frequency dependence, but it can be solved straightforwardly using numerical methods.

In order to understand the dynamics of Eq. 6.10 it is instructive to plot the left and right side of the equation as a function of frequency, as shown in Fig. 6.4. Note that the solution to Eq. 6.10 is found at the intersection of these two lines, i.e. where the function is self-consistent. The characteristic phenomenon of the hybridization is seen to be a splitting of the CSRR's resonance into a lower, 1_a , and upper, 1_b , hybrid resonance which straddle the frequency at which the magnetic material's permeability is zero (above resonance—not the zero point at resonance). The hybridization is strongest when the natural resonance of the CSRR structure is near the zero permeability point of the magnetic material. If one increase the biasing field of the magnetic material, such that its zero permeability frequency advances through the CSRR's natural resonance, a characteristic growth and shift of the 1_a resonance is observed and then a subsequent decay and shift of the 1_b resonance is observed. Hence, tuning the magnetic material effectively tunes the response of the metamaterial structure. Because a CSRR structure inherently exhibits multiple resonance (the first two resonances of the CSRR structure are shown in Fig. 6.1) it is not only the fundamental resonance that is hybridized but also the higher order resonances. However, if these higher order resonances are far away from the magnetic material's zero permeability frequency they are highly damped. In practice this means that if the zero permeability frequency of the magnetic material is aligned with the fundamental CSRR structure resonance, then depending on losses in the system, residual higher order hybrid resonances $2_a, \dots, n_a$ might be found near the fundamental, 1_a , hybrid resonance as shown in Fig. 6.5(a). In contrast, the $1_b, 2_b, \dots, n_b$ resonances remain largely separated. This suggests that the 1_b resonance—which is strongly resonant, isolated from the other hybrid resonances, and strongly tunable—maybe the most useful for application.

Once the empty CSRR structure was characterized, numerical simulations incorporating the magnetic material were performed. In the interest of exploring

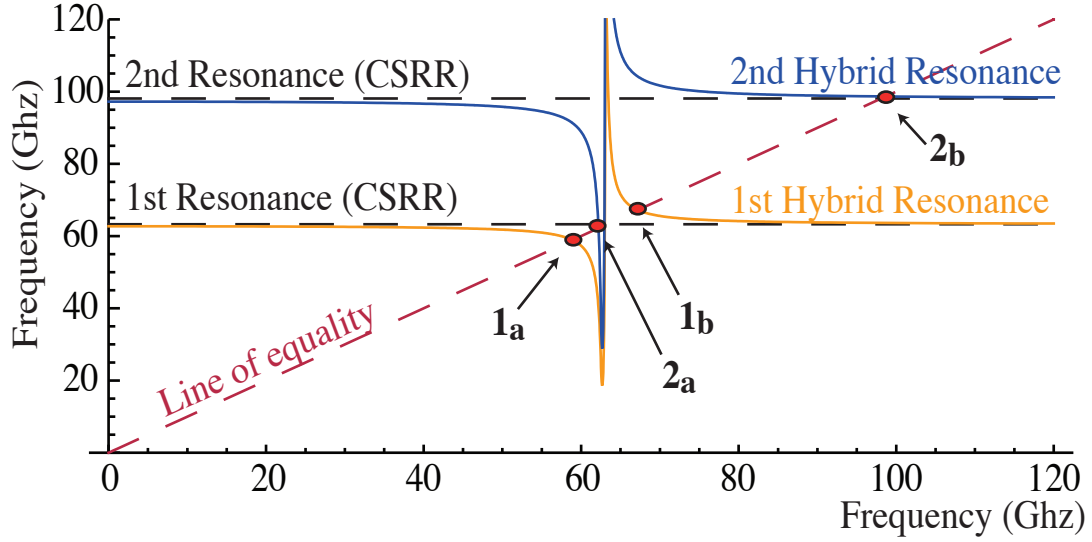


Figure 6.4: The left (line of equality) and right (1st/2nd hybrid resonance) side of Eq. 6.10 are plotted for $H_0 = 1.5$ kG and characteristic parameters stated in the text.

the potential of incorporating high frequency (40-70 GHz) thin film magnetic materials, such as hexagonal ferrites [72], we considered a $1 \mu\text{m}$ thick layer of magnetic material with parameters (Eq. 6.4): $\xi = 18 \text{ GHz kG}^{-1}$, $\Gamma_m = (0.70 \text{ GHz})/\omega$ and $\xi M_s = 380 \text{ GHz}$. The permeability was calculated in MATLAB and imported into HFSS as a data table of real values ($\text{Real}[\mu]$) and loss tangent values ($\text{Imag}[\mu]/\text{Real}[\mu]$). As previously mentioned, it is only the x -component of $\mu_r(\omega)$ that contributes to the interaction. Using the fitted parameters extracted from the empty structure we solved the analytic equation, Eq. 6.10, for the first hybrid mode, $1a/1b$, using a numerical root finding method implemented in MATHEMATICA. The magnetic filling fraction, q , was determined to have a value of 0.013 through comparison to the HFSS simulations results. A comparison of the analytic dispersion curve calculated from Eq. 6.10 versus the numerical simulations performed with HFSS is shown in Fig. 6.5 and demonstrates excellent agreement with variation of the bias field.

The transmission through the CSRR/magnetic material waveguide can be calculated analytically by assigning bulk electromagnetic parameters to the waveguide structure. The effective bulk permittivity of the waveguide structure can be calcu-

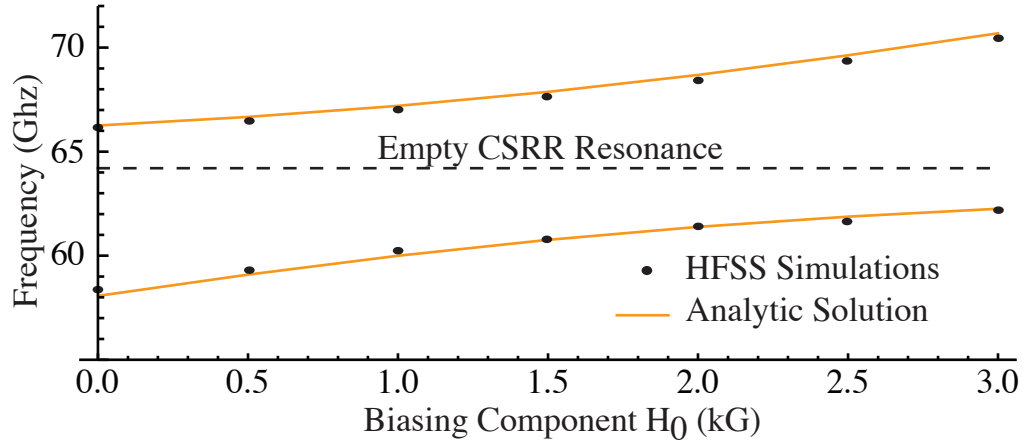


Figure 6.5: The dispersion curve for the 1st order hybrid mode of the CSRR/magnetic material waveguide is calculated both analytically and through numerical electromagnetic simulations as a function of the biasing field, H_0 .

lated by inputting Eq. 6.10 into Eq. 6.1 and then solving the Fresnel equations [12] to determine the transmission and reflection of the structure. This was done for the CSRR/magnetic waveguide structure and compared to HFSS simulations as shown in Fig. 6.6. In the HFSS simulations a splitting of the second order CSRR resonance can also be seen. In the analytic expression we were able to reproduce this by applying our technique to the first (electric) and second (magnetic) resonance of the CSRR structure. The overall agreement is seen to correlate well over a range of biasing values.

It is remarkable to note that the width and strength of the hybrid resonances is primarily a function of the CSRR's resonant properties. The properties of the magnetic material's resonance (and filling fraction q) predominantly determines the bandwidth over which the hybrid mode interaction exists. In Fig. 6.7 the permeability of the magnetic material, Eq. 6.4, and the associated effective permeability of the hybrid structure are plotted for various value of magnetization M_s . When the biasing field is chosen such that the zero point of the magnetic material correlates to the empty CSRR structure resonance, we see that the relative permeability strength of the hybrid structure is the same but that the splitting of the resonance is smaller for smaller values of M_s values. This suggest that it may be possible to harness narrowly resonant or weakly resonant magnetic materials

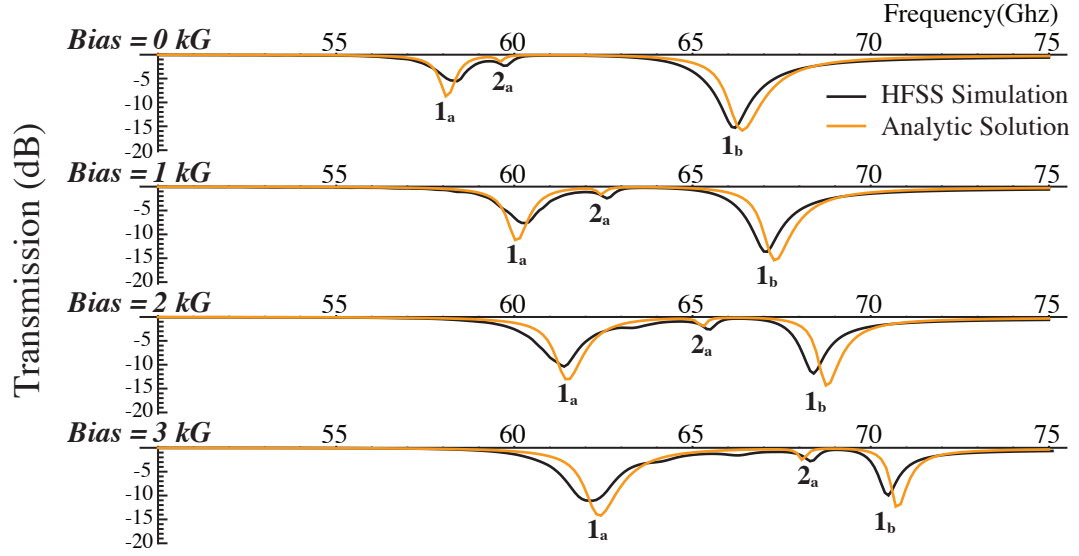


Figure 6.6: The transmission results are shown for the HFSS electromagnetic simulation and the analytic prediction using effective medium theory.

which cannot be utilized directly (though only over a narrow bandwidth). These potential materials include hexagonal ferrites and even antiferromagnetic materials such as MnF_2 [73].

6.4 Conclusion

In summary, the hybrid resonance that results from combining a resonant magnetic material and CSRR structure results in a unique hybrid resonance which can be harnessed to make tunable metamaterial structures. Gyromagnetic materials have a resonant permeability tensor form similar to that considered here but with the added complexity of off-diagonal resonant components. As a result, the resonant response of a hybrid structure with a true gyromagnetic material will have a more complicated dependence on the orientation of the biasing field with respect to the geometry of the CSRR structure [70, 71]. As we will demonstrate in the next chapter, if we limit ourselves to using thin layers then this analytic theory will also accurately describe the interaction of genuine gyromagnetic materials. Though gyromagnetic materials have been used directly to make tunable microwave devices [49, 51, 50], using them indirectly in metamaterial structures

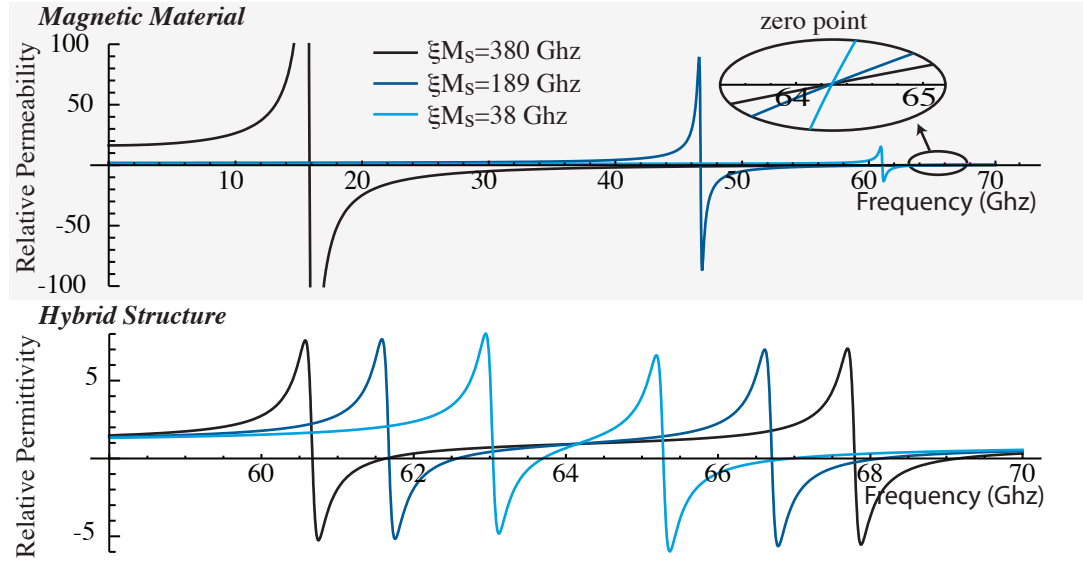


Figure 6.7: The real part of the permeability is shown for the magnetic material for several magnetization values with the bias field set such that the antiresonance position matched the empty CSRR structure's resonance. The resulting hybrid permeability is also shown.

has the potential advantage of increasing the range of effective material properties while at the same time reducing the amount of the magnetic material needed in the structure and hence the associated losses.

7

Hybrid Resonant Phenomenon in a SRR/YIG Metamaterial Structure

We now consider the hybrid effect in a bulk metamaterial structure. We numerically examine the hybridization of a SRR metamaterial resonance with the resonance of yttrium iron garnet (YIG) inclusions (the YIG material is simulated with its full tensorial permeability response). The predictive analytic model developed in Chapter 6 is applied to the SRR/YIG structure and provides an accurate characterization of this hybridization. We further expand the analytic model to fully include dispersive effects of the SRR structure.

In our analysis we consider an optimized design for the SRR/YIG structure that shows strong hybridization with a minimum amount of YIG material. We further scrutinize a structure experimentally considered by Kang et al. [74]. We suggest that the hybridization effect is present in their experimental data and present numerical simulations to support this assertion.

7.1 Introduction

As discussed in Chapter 6 the consequence of constructing metamaterials out of periodic and resonant structures is that they are necessarily dispersive. This

constrains metamaterial devices, in both the waveguide and bulk structures, to a small frequency bandwidth of operation. In order to overcome this limitation there is interest in creating tunable metamaterials which can be tuned to operate over the frequency range of interest. One method of creating bulk tunable metamaterials has been demonstrated by integrating lumped elements, such as a varactor diodes, into the gap regions of SRR structures [75, 76, 66]. Due to operational limitations of integrated devices, this solution is currently limited to the low gigahertz regime. Another method for creating tunability is to place a dielectric or magnetic material, that is itself tunable via some biasing technique, in the local fields of the resonant elements that compose the metamaterial structure. Tuning the natural material will indirectly tune the bulk electromagnetic properties of the metamaterial. This method has been successfully demonstrated at terahertz by manipulating the Schottky barrier of a semiconductor substrate [17] and at microwaves using nematic liquid crystal [65].

As an alternative to harnessing resonant geometrical elements to construct metamaterials, periodic naturally resonant material have been considered for metamaterial structures. It has been shown that periodic yttrium iron garnet (YIG) structures can be analyzed as an effective medium and that this metamaterial structure provides a tunable permeability via a magnetic biasing field [77]. YIG structures have also been combined with a wire mesh in order to create a negative index metamaterial structure [78]. As we suggested in Chapter 6 integrating resonant gyromagnetic materials into metamaterials [79, 74] can provide an indirect means of tuning geometrical metamaterial structures by influencing the local fields in a metamaterial unit cell. The tuning of a SRR structure has been experimentally demonstrated by Kang et al. [74] with a SRR/YIG rod structure. They suggested that in the weakly interacting limit, where the resonance of the YIG material is far from the resonance of the SRR structure, the interaction can be understood by considering the effective change in background permeability caused by the periodic YIG rods. But, they also note that in the strongly interacting region this understanding breaks down. In this region the combination of the resonant YIG material and the artificial magnetic resonance of the SRR structures interact to produce

a unique hybrid resonance distinct from either of the individual resonances. In this chapter we analyze the hybrid mode supported by a combined SRR and YIG metamaterial structure and provide an analytic understanding that is sustained by full-wave numerical simulations. We first consider a single ring SRR/YIG structure that demonstrates strong hybridization. We demonstrate that this structure has a broad range of tunability that can be harnessed to make interesting metamaterial devices. We also investigated the structure experimentally considered by Kang et al. [74]. We extend the characterization of this hybrid structure into the strongly interacting region and show that the hybrid theory proposed in Chapter 6 can be applied with excellent agreement. Integrating gyromagnetic materials is promising because there exist a large range of these compounds which can be harnessed to provide tunability from the low to high gigahertz frequency range.

7.2 Theory

The influence of a resonant gyromagnetic material has been theoretically and numerically considered in a waveguide metamaterial structure in Chapter 6 [79]. We develop a similar framework for the bulk SRR metamaterial structures considered here. Numerical simulations show that this hybrid theory is accurate over the full frequency regime (including the weakly interacting regions investigated by Kang et al.).

The standard metamaterial methodology for designing electromagnetic materials is to construct a structure composed of many periodic, resonant, and sub-wavelength elements. Through manipulation of the geometry of the individual metamaterial elements, the resonance of the bulk structure can be engineered to support the electromagnetic properties of interest (over a finite frequency range). To first order, the SRR structure has a resonant permeability with a characteristic Drude-Lorentz functional form [7]. In addition to the fundamental resonance, there are also higher order electromagnetic resonances associated with the metamaterial structure. For example, the second order resonance of the SRR structure is electric in nature. Higher order resonances are not usually of practical interest

for bulk metamaterial design because they are subject to strong spatial dispersion effects that result from the incident radiation having a wavelength that is sizable with respect to the unit cell size (i.e. the material acts less like a homogeneous material). We will see that the resulting hybrid resonances of the SRR/YIG material are a mix of both higher and lower order resonances in close proximity, and hence in order to obtain an accurate analytic theory, we must consider the higher order resonances in our analysis. Here, we consider just the first two metamaterial resonances which sufficiently predicts the hybridization that results when the YIG resonance is near the SRR's first order resonance.

We can modify the standard metamaterial Drude-Lorentz model to account for the spatial dispersion effects found in the higher order resonances (and to a lesser degree those found in the first order resonance) using a technique developed by Liu et al. [69]. For the SRR structure, we define the characteristic Drude-Lorentz permeability of the first order magnetic resonance (1), and similarly, the characteristic Drude-Lorentz permittivity that describes the second order electric resonance (2),

$$\bar{\mu}(\omega) = A_{(1)} \left(1 - \frac{F_{(1)}\omega^2}{\omega^2 - (\omega_{0_{SRR(1)}})^2 + i\omega\Gamma_{SRR(1)}} \right), \quad (7.1)$$

$$\bar{\epsilon}(\omega) = A_{(2)} \left(1 - \frac{F_{(2)}\omega^2}{\omega^2 - (\omega_{0_{SRR(2)}})^2 + i\omega\Gamma_{SRR(2)}} \right), \quad (7.2)$$

where ω is the angular frequency, $F_{(j)}$ and $A_{(j)}$ are constants, $\Gamma_{SRR(j)}$ is the dissipation factor, and $\omega_{0_{SRR(j)}}$ is the resonant angular frequency. The Drude-Lorentz resonant frequency, $\omega_{0_{SRR(j)}}$, is related to the local structural SRR capacitance, C , and inductance, L , by,

$$\omega_{0_{SRR}} = \frac{1}{\sqrt{LC}}. \quad (7.3)$$

The dispersive effects are implemented by defining a dispersion variable, $\theta(\omega)$, that is related to the Drude-Lorentz functions of Eq. 7.1 and Eq. 7.2 by,

$$\sin(\theta(\omega)/2) = \frac{k(\omega)d}{2} \sqrt{\bar{\mu}(\omega)\bar{\epsilon}(\omega)}, \quad (7.4)$$

where $k(\omega) = \omega/c$ is the free-space wavevector and d is the unit cell size of the

metamaterial. The effective wave impedance is then defined,

$$\eta = \begin{cases} \sqrt{\bar{\mu}(\omega)/\bar{\epsilon}(\omega)} \cos(\theta(\omega)/2), & \text{electric resonance} \\ \sqrt{\bar{\mu}(\omega)/\bar{\epsilon}(\omega)} \csc(\theta(\omega)/2), & \text{magnetic resonance} \end{cases} \quad (7.5)$$

and the effective index of refraction is given by,

$$n = \theta(\omega)/k(\omega)d. \quad (7.6)$$

The effective permittivity, ϵ_{eff} , and permeability, μ_{eff} , are then determined in the usual way, $\mu_{\text{eff}} = n(\omega)\eta(\omega)$ and $\epsilon_{\text{eff}} = n(\omega)/\eta(\omega)$. Given the Drude-Lorentz parameters ($F_{(j)}, A_{(j)}, \Gamma_{SRR(j)}, \omega_{0SRR(j)}$) and the unit cell size (d), we can use Eqs. 7.1-7.6 to accurately predict the effective electromagnetic response over a wide frequency range.

The inclusion of YIG material in a SRR structure affects the local electromagnetic response of the SRR structure. Determining the nature of this influence on the unit cell level, as we proceed to do, can be directly related to the bulk electromagnetic response of the metamaterial through the modified Drude-Lorentz model outlined above. We consider YIG material that is biased along the z -axis and above its saturation magnetization. Its resonant permeability is of the form,

$$\bar{\bar{\mu}}(\omega) = \mu_0 \begin{pmatrix} \mu_1(\omega) & i\mu_2(\omega) & 0 \\ -i\mu_2(\omega) & \mu_1(\omega) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (7.7)$$

where μ_1 and μ_2 are resonant functions of frequency given by

$$\mu_1(\omega) = \frac{\omega_{0YIG} \gamma \mu_0 M_s}{(\omega_{0YIG})^2 - \omega^2}, \quad (7.8)$$

$$\mu_2(\omega) = \frac{\omega \gamma \mu_0 M_s}{(\omega_{0YIG})^2 - \omega^2}, \quad (7.9)$$

with γ the gyromagnetic ratio and $\mu_0 M_s$ the saturation magnetization. The resonant frequency is given by $\omega_{0YIG} = (\gamma \mu_0 H_0 - i\omega \Gamma_{YIG})$ where H_0 is the DC magnetic bias field. The loss component, Γ_{YIG} , can be further written in terms of the linewidth of the resonance, ΔH , as $\Gamma_{YIG} = \gamma \mu_0 \Delta H / (2\omega)$. For our analytic analysis we ignore the off-diagonal components in Eq. 7.7 which only have a strong

influence at resonance. We show that this is a suitable approximation if we restrict our unit cells to include thin YIG sheets.

If we consider incident TEM radiation on the SRR metamaterial structure that is incident perpendicular to the biasing field, the incident wave largely interact with the x component of the YIG permeability tensor or $\mu_1(\omega)$ in Eq. 7.7. The resulting inductance of the SRR structure can be calculated by abstractly conceptualized a ‘magnetic capacitor’ with parallel current sheets sandwiching a volume containing some fraction of which is filled with YIG material. Considering this simple model developed in Chapter 6, the inductance of the SRR structure is found to be [79],

$$L = \mu_0 \left(\frac{\mu_1(\omega)}{\mu_1(\omega)(1-q) + q} \right) g_{\text{geom}}, \quad (7.10)$$

where the parameter q is a constant that depends on the ‘effective’ volume fraction of the YIG material (q must be determined through a fitting procedure), $\mu_1(\omega)$ is the relative permeability of the magnetic material, and g_{geom} is a constant with units of length that is determined by the geometry of the SRR structure. Alternatively, the capacitance of the SRR structure is constant and can be written as, $C = \epsilon_0 h_{\text{geom}}$, where h_{geom} is a constant with units of length that is determined by the geometry of the structure. If we plug the associated inductance and capacitance of the SRR structure into Eq. 7.3 we get an equation governing the new *hybrid* resonant frequencies of the SRR structure,

$$\omega'_{0\text{SRR}(j)} = \frac{1}{\sqrt{\frac{\mu_1(\omega)}{\mu_1(\omega)(1-q)+q}}} \omega_{0\text{SRR}(j)}. \quad (7.11)$$

where $\omega_{0\text{SRR}} = \mu_0 \epsilon_0 g_{\text{geom}} h_{\text{geom}} = 2\pi f_0$ is the resonant frequency of the SRR structure in the absence of the YIG material. One can replace $\omega \rightarrow \omega'_{0\text{SRR}}$ in Eq. 7.11 to obtain a transcendental equation that can be solved for the new hybrid resonant frequencies [79].

We can further determine the effective permeability, μ_{eff} , and permittivity, ϵ_{eff} , for the hybrid structure from the modified Drude-Lorentz model by replacing $\omega_{0\text{SRR}}$ in Eq. 7.1 and Eq. 7.2 with Eq. 7.11. To determine the Drude-Lorentz parameters for an actual structure is a two-step process. We first perform numerical simulations of the SRR structure and extract the effective constitutive parameters

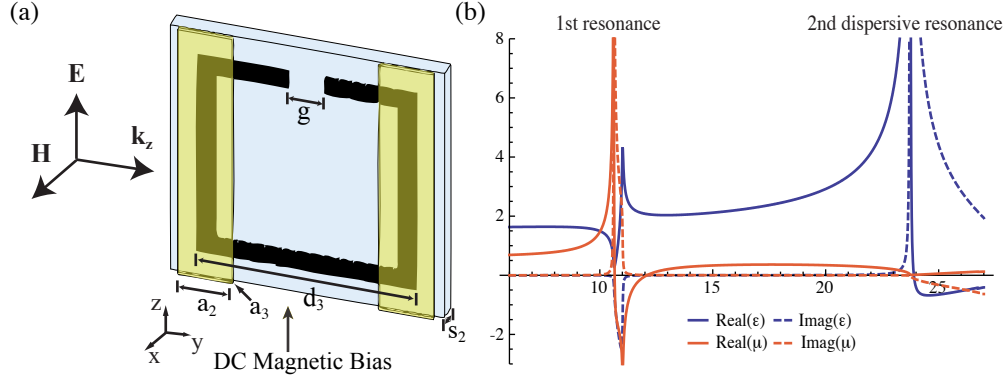


Figure 7.1: (a) The SRR/YIG unit cell structure is shown. The characteristic dimensions of the structure are $d_3 = 2.4$ mm, $a_2 = 0.6$ mm, $a_3 = 0.04$ mm, $g = 0.4$ mm, substrate thickness of $s_2 = 0.2$ mm, and a unit cell size of 3 mm. (b) The extracted permittivity and permeability of the structure.

without the magnetic response of the YIG material and use the least square fitting procedure outlined in Eqs. 7.1-7.6 to determine the “empty” Drude-Lorentz parameters. We can then integrate Eq. 7.11 into the Drude-Lorentz function and determined the magnetic filling fraction constant, q , through comparison of the analytic theory to the numerical simulations of the combined SRR/YIG structure at several magnetic biasing values. Once q is known, the analytic theory provides a predictive model that is valid over the full frequency range of interaction. Further, the fresnel equations can be solved to determine the analytic transmission and reflection of a SRR/YIG metamaterial structure [12].

7.3 Numerical Simulations

Numerical simulations of combined SRR/YIG structures were carried out using HFSS (Ansoft’s commercial finite-element solver) in order to investigate the resulting hybridization. HFSS supports the definition of gyromagnetic materials and calculates the full electromagnetic permeability response given by Eq. 7.7. The parameters of the YIG material were defined in these simulations to have a saturation magnetization $\mu_0 M_s = 1.7$ kGs, linewidth $\Delta H = 12$ Oe, and permittivity $\epsilon = 14.7$. A standard full wave analysis of a single unit cell of the SRR/YIG struc-

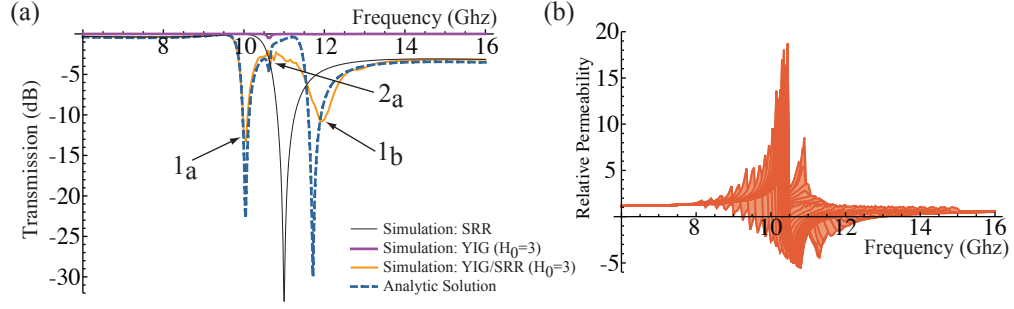


Figure 7.2: (a) The transmission simulation of the combined SRR/YIG structure, isolated YIG structure, and the analytic hybridization theory are shown for a biasing field of $H_0 = 3$ kG structure. (b) The extracted permeability over a range of biasing field, $H_0 = 1.7 - 4.5$ kG, is shown with steps of 0.1 kG.

ture, with appropriate boundary conditions, is sufficient to fully characterize the bulk SRR/YIG metamaterial structure's transmission and reflection coefficients.

We first considered the structure shown in Fig. 7.1 which consists of a single ring SRR structure with two very thin YIG strips covering the vertical sections of the split-ring. For this geometry the demagnetization fields, which can decrease the effective biasing of the material, are minimized because the YIG material is biased parallel to the strips [46]. First, a simulation of the structure with the YIG magnetic properties set to zero was performed. From the numerically determined transmission and reflection coefficients of the unit cell structure it is possible to compute the effective permittivity and permeability through an inversion technique [12]. The effective constitutive parameters of the empty structure were extracted over the frequency range including the two lowest order resonances as shown in Fig. 7.1(b). The resonances include a first order magnetic resonance at 10.6 GHz and a second order highly dispersive electric resonance at 23.7 GHz.

An advantage of using the single SRR geometry is the high frequency of the secondary resonance compared to that of the traditionally used double SRR structure. The hybridization effect splits both the fundamental resonance and any higher order resonances, but for the SRR resonances that are far from the resonant frequency of the YIG material, the splitting is highly damped. Hence, the single SRR structure supports 1st order, 2nd order, etc. hybridizations that are

largely isolated from one another. The disadvantage of using the single SRR structure is that it has a slightly weaker response and a higher fundamental resonant frequency than its equivalently sized double SRR counterpart. This limitation is easily compensated for by decreasing the unit cell size of the metamaterial and effectively increasing the filling fraction.

For the structure in Fig. 7.1, the Drude-Lorentz constants for the first resonance [$F_{(1)} = 0.228$, $\omega_{0_{SRR(1)}} = 2\pi(10.6)$ Ghz, $\Gamma_{SRR(1)} = 2\pi(0.00219)$ Ghz, $A_{(1)} = 1.13$] and the second order resonance [$F_{(2)} = 0.659$, $\omega_{0_{SRR(2)}} = 2\pi(23.7)$ Ghz, $\Gamma_{SRR(2)} = 2\pi(0.0347)$ Ghz, $A_{(2)} = 2.99$] were determined by a least square fit using Eqs. 7.1-7.6. The inclusion of Eq. 7.11 was then used in Eq. 7.1 and Eq. 7.2 to determine the effective hybridized permeability and permittivity of the structure as a function of the magnetic bias, H_0 , and magnetic filling fraction, q . The Fresnel equations were then used to determine the theoretical transmission of the SRR/YIG metamaterial structure. Through a fitting process of the analytic to the simulated transmission, the magnetic filling fraction was determined to be $q = 0.06$. The comparison of the HFFS calculated transmission and the analytic solution are shown in Fig. 7.2 for a bias field of $H_0 = 3$ kG which results in a YIG resonance that is near the natural SRR resonance. We see that there is good correlation between analytic solution and simulation in both the position of the resonant frequency and magnitude. The splitting of the fundamental mode is seen, 1_a and 1_b , and a small contribution from the splitting of the higher order resonance, 2_b , is also seen. The 1_b lower resonance deviates somewhat from the magnitude of the analytic theory and this may be the result of the off-diagonal components of the YIG permeability tensor having an effect near resonance. Also plotted is the simulation of the YIG strips by themselves (without the SRR structure) and it has an almost negligible effect on transmission. This confirms that the magnetic properties of the YIG material strongly interact with the local fields generated by the SRR structure in order to create the hybridization.

We can further investigate the hybridization over a range of biasing fields as shown in Fig. 7.3. It can be seen that when the YIG Resonance is far from the SRR resonance we see the weak tuning suggested by Kang et al. and as the YIG

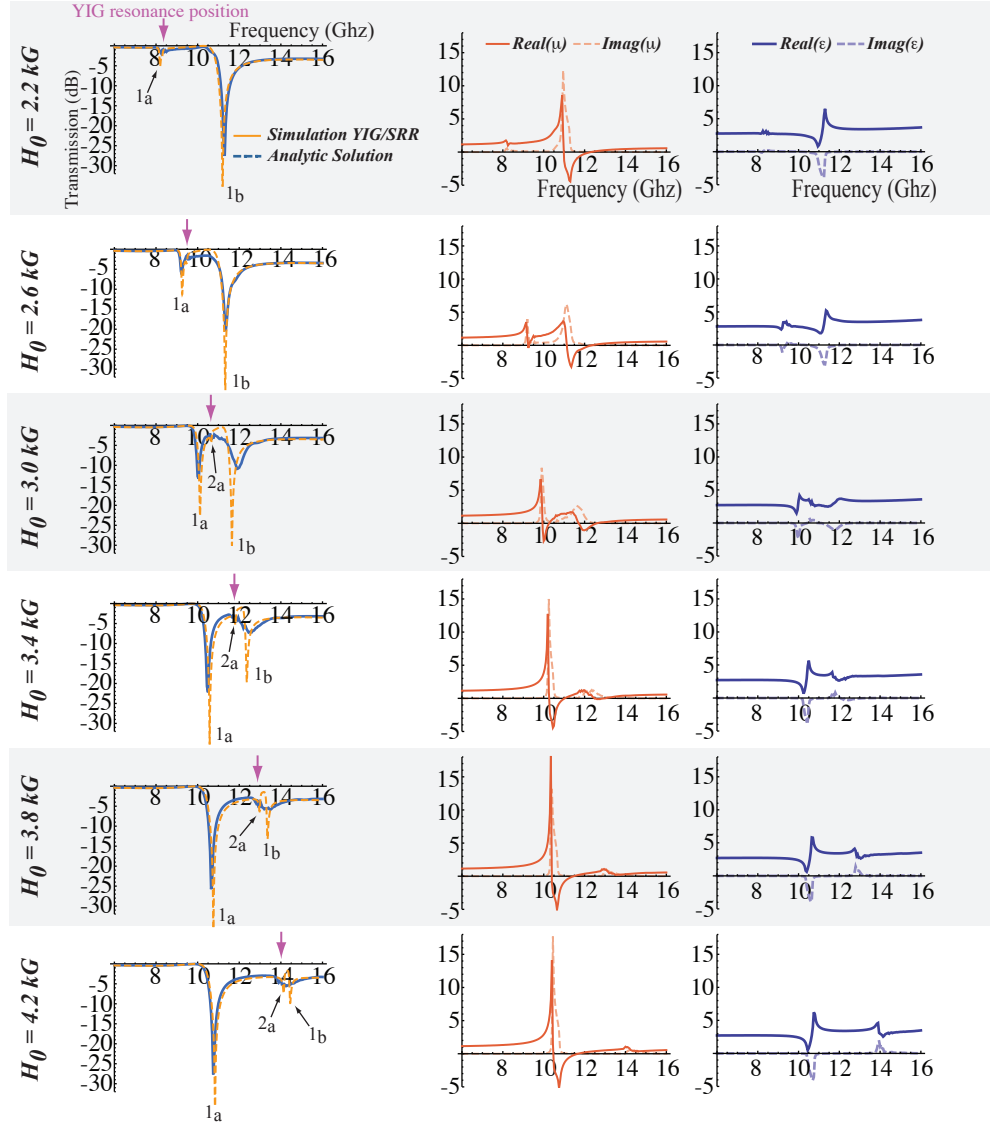


Figure 7.3: Numerical simulations of the transmission for a one unit cell thick SRR/YIG metamaterial structure and the analytic theory are shown for a range of biasing fields. The position of the YIG resonance is noted by a vertical arrow and the splitting of the 1st and 2nd SRR resonances are noted for each bias value. The extracted permeability and permittivity of the numerical simulations are also shown.

resonance approaches the original SRR resonance a strong hybridization of the two resonances develops. The original resonance, 1_b , of the SRR shifts to the right and a small resonance, 1_a , appears and grows to the left. Once the YIG resonance passes the natural SRR resonance, the 1_b resonance continues to decline and eventually disappears while the 1_a resonance reclaims the position of the natural SRR resonance. When YIG and SRR resonance overlap, the hybridization is greatest with two relatively equal resonances straddling the original SRR resonance position. Also shown in Fig. 7.3 are the extracted permeability and permittivity from the numerical simulations. In addition, the extracted permeability of the hybrid structure over the biasing range $H_0 = 1.7 - 4.5$ kG are plotted together on the same plot in Fig. 7.2(b) demonstrating the wide range of tunability that can be harnessed. We can also note the slight asymmetry in the resonant peaks and this may be the result of the off diagonal components in the permeability tensor of the YIG material which are not included in our analytic theory.

Next we numerically investigated the structure experimentally considered by Kang et al. [74] shown in Fig. 7.4(a). Kang demonstrated the tunability the SRR/YIG rod structure and provided an analysis that agreed in the weakly interacting realm. They also noted a complicated interaction occurring when the resonances are close together. We suggest that they observed the hybridization of the resonances, but for their structure, the hybridization is somewhat more intricate than the previous structure we considered.

A simulation of the structure with the YIG magnetic properties set to zero was performed to determine the effective constitutive parameters of the empty structure as shown in Fig. 7.4(b). The two lowest order resonances are shown—a first order magnetic resonance at 10 GHz and a second order highly dispersive electric resonance at 16.2 GHz. The Drude-Lorentz constants for the first resonance [$F_{(1)} = 0.0483$, $\omega_{0_{SRR(1)}} = 2\pi(9.97)$ GHz, $\Gamma_{SRR(1)} = 0.108$ GHz, $C_{(1)} = 1.22$] and the second order resonance [$F_{(2)} = 0.475$, $\omega_{0_{SRR(2)}} = 2\pi(16.2)$ GHz, $\Gamma_{SRR(2)} = 0.0461$ GHz, $C_{(2)} = 7.18$] were determined by a least square fit of Eqs. 7.1-7.6. As noted above, the double SRR structure used by Kang has a second order resonances that is very close to the fundamental resonance. Also, the large size of the unit

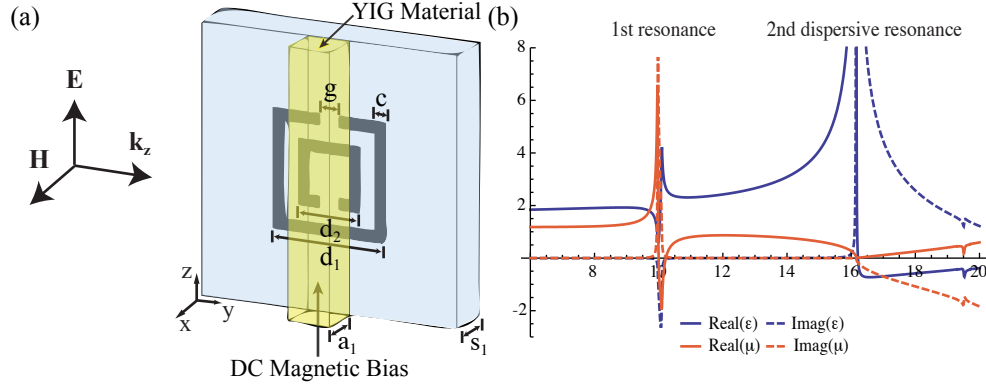


Figure 7.4: (a) The characteristic dimensions of the SRR/YIG rod unit cell structure experimentally investigated by Yang et al. are shown with $d_1 = 2.2$ mm, $d_2 = 1.2$ mm, $c = 0.2$ mm, $g = 0.4$ mm, and $a_1 = 0.8$ mm, a substrate thickness of $s_2 = 0.9$ mm, and a unit cell size of 5 mm. (b) The numerically extracted permeability and permittivity of the structure are shown.

cell size, 5 mm, with respect to the SRR structure structure, 2.2 mm, result in strong spatial dispersion effects. In addition, the significant thickness of the YIG rod used, 0.8 mm, suggest that the off-diagonal components of the permeability tensor of the YIG material will have a significant effect. Further, the significant volume the material takes up in the unit cell suggest that it will have strong direct interaction with incident radiation. Still, the theory developed previously, loosely characterizes the resulting hybridization.

In Fig. 7.5 the transmission as a function of several biasing fields is shown with the analytic solution overlaid. The transmission of the YIG rod in the unit cell by itself is also shown. It is interesting to note that the resulting stop band in the transmission is a combination of the hybrid resonance of the SRR and the pure YIG rod resonance. The YIG makes up a large enough fraction of the unit cell that it has a direct interaction with incident radiation. Further, the hybrid resonances are not well formed until the YIG resonance is near that of the SRR structural resonance ($H_0 = 3 - 3.8$ kG). This could potentially be the result of the greater influence of the off-diagonal permeability components in this geometry. We can also observe that the $2a$ resonance, which is due to the hybridization of the secondary SRR resonance, is much stronger because the secondary SRR resonance is closer to the fundamental resonance. This secondary hybridization appears to

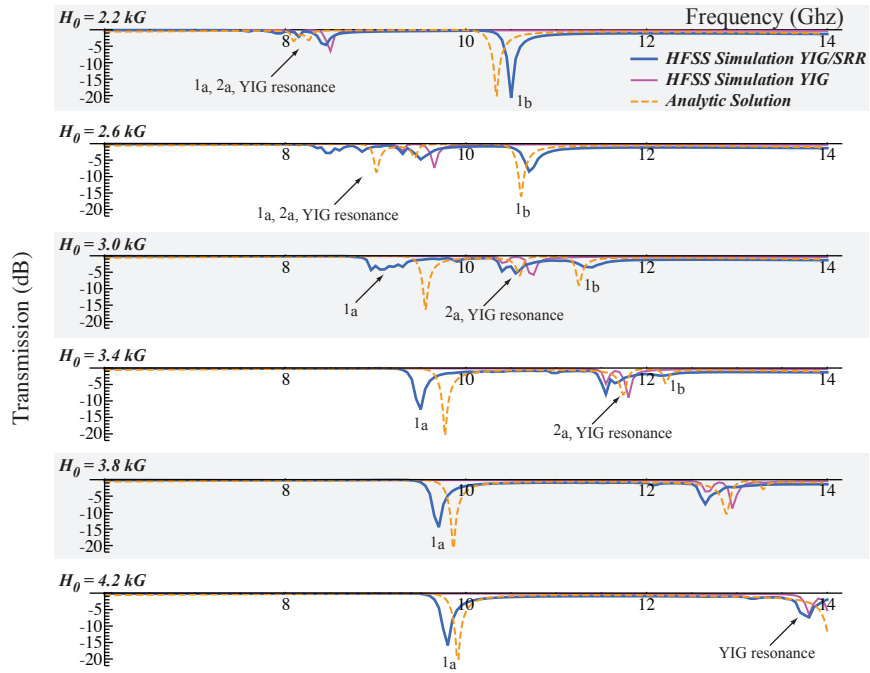


Figure 7.5: The numerically calculated transmission for the combined SRR/YIG structure investigated by Kang et al. is shown along with the analytically determined transmission determined through Eqs. 7.4-7.11. Also shown, is the numerically calculated transmission for the YIG rod structure only.

be mixed in with a direct response from the YIG material to incident radiation. This direct response appears now due to the significant volume fraction the YIG occupies in the unit cell.

Overall, we see that the tuning of the hybrid resonance is quite substantial. With a magnetic filling fraction of $q = 0.08$, the analytic theory roughly correlates with the three major hybrid resonances, though there is a slight offset at some frequencies. Kang et al. examined this structure experimentally. We suggest that the “interesting observation” noted by Kang et al. when the SRR and YIG resonances coincided is in fact the first experimental observation of the hybridization effect.

It is interesting to note that the effective magnetic filling fraction of the first structure considered, $q = 0.06$, (shown in Fig. 7.1) is approximately the same as Kang et al.’s structure, though it utilize 96% less YIG material. This shows the effectiveness of targeting the YIG material in the regions of strong local fields of the SRR structure, i.e. near the wire. For fabrication, the design of the optimized hybrid structure could be simplified by considering a single sheet of YIG material extending across the entire unit cell instead of the double stripped structure analyzed here. We consider the double stripped structure solely to demonstrate that targeting the material in the regions where the local fields of the SRR are strongest maximizes the interaction for the amount of YIG material used.

7.4 Conclusion

In summary, the experimental work of Kang et al. demonstrates the effectiveness of using YIG material to create tunable metamaterial structures. We provided a theory to characterize this tuning in the weak and strongly interacting region of the SRR/YIG interaction. By targeting the YIG material in regions of the metamaterial structure where the local fields are strongest we can extend the tuning properties of the YIG/SRR structure while minimizing losses due to the YIG material. This indirect manipulation can be harnessed to extend the bandwidth of bulk metamaterials and provide a means of making materials with reconfigurable electromagnetic properties.

Concluding Remarks

Truly it is an understatement to note that metamaterial research is progressing rapidly. In less than a decade, metamaterials have advanced from merely a theoretical concept to physical materials with complex electromagnetic properties far beyond natural materials. Fairly recently, metamaterials have even started to make their way into commercial products such as antennas. Metamaterials have been able to achieve their advanced properties through harnessing purely geometric resonances. It is clear that integrating additional fundamentally resonant materials into metamaterial design can further extend the electromagnetic properties of metamaterials. In this dissertation we've managed to characterize the unique resonant interaction that occurs in these hybrid structures. We've specifically considered metamaterial with integrated gyromagnetic materials, but the core insights of the analytical analysis should be applicable to any resonant material that is integrated into a metamaterial structure.

Hybrid metamaterial structures can provide a means of addressing some of the present limitations of the current metamaterial methodology such as bandwidth. The prospect of integrating resonant materials into metamaterials also provides a means of leveraging specific properties of the integrated material. The desired properties of a resonant material can be decoupled from its undesirable properties. For the ferrite material we integrated in the SRR structure, this meant that we gained the tunability of the ferrite material but used sufficiently small quantities that the high dielectric response of the material did not influence incident radiation.

This opens the possibility of utilizing other resonant materials in applications for which they had previously been discounted.

It now remains for this hybrid interaction effect to be experimentally demonstrated. As mentioned, there is some indirect evidence for this hybridization effect in the literature. We surmise that the construction of a hybrid waveguide metamaterials should prove very interesting for applications. These structures can provide a means of making better or cheaper microwave components such as phase shifters and filters that are desired by the current market demand for high frequency components. For bulk hybrid structures it would be interesting to demonstrate a tunable lens which should be achievable utilizing the structures we've considered. In general, the notion of hybrid metamaterials has exciting prospects for making reconfigurable electromagnetic materials and it will be interesting to see these advanced materials evolve.

Appendix A

Determination of Capacitance and Inductance Relations for Complementary Structures

In this appendix we proceed to derive a relationship between the inductance (L) and capacitance (C) of two complementary planar structures such as a the dual SRR and CSRR metasurfaces. Consider the flat wire carrying a current k and its compliment, a plane with a slot cut in it, as shown in Fig. A.1 (the complementary fields are denoted by a prime). Because, the complement of the fields only holds in source free regions, we divide the solution up into two regions: above the plane (boundary area 1) and below the plane (boundary area 2) of the structures. The effective boundary conditions along the interface of the structure, Fig. A.1(a), is a perfect magnetic boundary along the dashed line and perfect electric boundary along the flat metal wire. For the complementary structure, Fig. A.1(b), the boundaries are interchanged with a perfect magnetic boundary across the gap and perfect electric boundary elsewhere. In the two case we assume an arbitrary $\epsilon_{1,2}$ and $\mu_{1,2}$ above and below the structure as shown. The magnetic energy per unit volume in each region of the structure, Fig. A.1(a), is

$$U_{m,1} = \frac{1}{2}\mu_1 H_1^2, \quad (\text{A.1})$$

$$U_{m,2} = \frac{1}{2}\mu_2 H_2^2, \quad (\text{A.2})$$

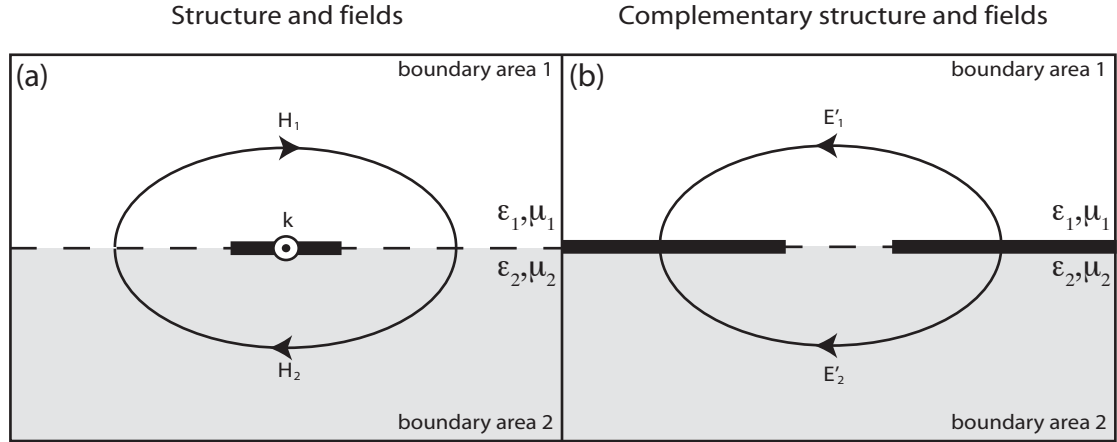


Figure A.1: Complementary planar structures and their field solutions. The structure, (a), and its complement, (b), can be broken into two boundary regions with complementary fields.

(A.3)

and the total energy is $U_m = U_{m,1} + U_{m,2}$. Similarly, we can write down the energy contained in the electric fields of the complementary structure Fig. A.1(b), and further, using equation Eq. 5.7 and Eq. 5.8, write it in terms of the H field of the original structure.

$$U_{e,1} = \frac{1}{2}\epsilon_1 E_1'^2 = \frac{1}{2}\epsilon_1 \left(\alpha_1 \sqrt{\frac{\mu_1}{\epsilon_1}} \right) H_1^2 = \alpha_1^2 U_{m,1}, \quad (\text{A.4})$$

$$U_{e,2} = \frac{1}{2}\epsilon_2 E_2'^2 = \frac{1}{2}\epsilon_2 \left(\alpha_2 \sqrt{\frac{\mu_2}{\epsilon_2}} \right) H_2^2 = \alpha_2^2 U_{m,2}. \quad (\text{A.5})$$

(A.6)

Of course, we must still eliminate the constants α_1 and α_2 and we apply boundary conditions between regions to do so. At the interface for the wire structure (a) (away from the current source) boundary conditions require

$$\mu_1 H_1 = \mu_2 H_2. \quad (\text{A.7})$$

Taking the ratio of Eq. A.1 and Eq. A.2 and using Eq. A.7 gives the ratio of the energy in the two regions

$$\frac{U_{m,1}}{U_{m,2}} = \frac{\mu_2}{\mu_1}. \quad (\text{A.8})$$

The total magnetic energy can be written $U_m = U_{m,1} + U_{m,2}$, which allows us to solve for the energy in the regions in terms of the total energy:

$$U_{m,1} = \left(1 + \frac{\mu_1}{\mu_2}\right)^{-1} U_m, \quad (\text{A.9})$$

$$U_{m,2} = \left(1 + \frac{\mu_2}{\mu_1}\right)^{-1} U_m. \quad (\text{A.10})$$

$$(\text{A.11})$$

With this relation we can now solve for the electric energy of the complementary structure in terms of the magnetic energy of the original structure using Eq. A.4 and Eq. A.5,

$$U_e = U_{e,1} + U_{e,2} = \alpha_1^2 U_{m,1} + \alpha_2^2 U_{m,2} = \left[\alpha_1^2 \frac{\mu_2}{\mu_1} + \alpha_2^2 \right] \left(1 + \frac{\mu_2}{\mu_1}\right)^{-1} U_m. \quad (\text{A.12})$$

The total electric field energy can alternatively be written in terms of the capacitance, C , and Voltage, V , as $1/2CV^2$ and the magnetic field energy of original structure can be written in terms of the inductance, L , and the current, I , as $1/2LI^2$. The above equation then simplifies to,

$$\frac{C'}{L} = \left[\alpha_1^2 \frac{\mu_2}{\mu_1} + \alpha_2^2 \right] \left(1 + \frac{\mu_2}{\mu_1}\right)^{-1} \frac{I^2}{V'^2}. \quad (\text{A.13})$$

This equation gives us the relationship between the capacitance of the complementary structure and the inductance of the original structure. To eliminate α_1 and α_2 , we can perform two separate line integrals of the electric fields through each region of the complementary structure as shown in Fig. A.2 and note that they should be equal.

$$\left. \begin{aligned} V'_{AB} &= -\int_A^B \mathbf{E}'_1 \cdot d\mathbf{l} \\ V'_{CD} &= -\int_C^D \mathbf{E}'_2 \cdot d\mathbf{l} \end{aligned} \right\} \frac{V'_{AB}}{V'_{CD}} = 1. \quad (\text{A.14})$$

If we apply Eq. 5.7 to each region we get,

$$\begin{aligned} V'_{AB} &= -\int_A^B \mathbf{E}'_1 \cdot d\mathbf{l} = \int_A^B \alpha_1 \sqrt{\frac{\mu_1}{\epsilon_1}} \mathbf{H}_1 \cdot d\mathbf{l}, \\ V'_{CD} &= -\int_C^D \mathbf{E}'_2 \cdot d\mathbf{l} = -\int_C^D \alpha_2 \sqrt{\frac{\mu_2}{\epsilon_2}} \mathbf{H}_2 \cdot d\mathbf{l}. \end{aligned} \quad (\text{A.15})$$

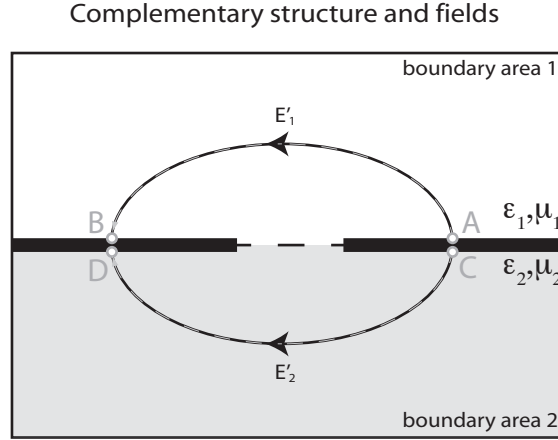


Figure A.2: The integration line along electric fields in boundary region 1 ($A \rightarrow B$) and boundary region 2 ($D \rightarrow C$) is shown in gray.

Note that Eq. A.15 becomes positive because we chose the negative solution for Eq. 5.7 since E'_1 points in the opposite direction as H_1 in boundary region one. We can now write,

$$\frac{V'_{AB}}{V'_{CD}} = 1 = -\frac{\alpha_1}{\alpha_2} \sqrt{\frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}} \frac{\int_A^B \mathbf{H}_1 \cdot d\mathbf{l}}{\int_C^D \mathbf{H}_2 \cdot d\mathbf{l}}, \quad (\text{A.16})$$

and solve for the ratio,

$$\frac{\alpha_2}{\alpha_1} = -\sqrt{\frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}} \frac{\int_A^B \mathbf{H}_1 \cdot d\mathbf{l}}{\int_C^D \mathbf{H}_2 \cdot d\mathbf{l}} = -\sqrt{\frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}} \frac{\mu_2}{\mu_1} = -\sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}, \quad (\text{A.17})$$

where we have used Eq. A.7 to eliminate the integral.

To eliminate the current, I , from Eq. A.13, we can solve for the line integral of $\mathbf{H}$ directly.

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_A^B \mathbf{H}_1 \cdot d\mathbf{l} - \int_C^D \mathbf{H}_2 \cdot d\mathbf{l} = \int_A^B \mathbf{H}_1 \cdot d\mathbf{l} - \int_C^D \left(\frac{\mu_1}{\mu_2} \mathbf{H}_1 \right) \cdot d\mathbf{l} \quad (\text{A.18})$$

$$= \int_A^B \mathbf{H}_1 \cdot d\mathbf{l} - \int_C^D \left(\frac{\mu_1}{\mu_2} \mathbf{H}_1 \right) \cdot d\mathbf{l} = \left(1 + \frac{\mu_1}{\mu_2} \right) \int_A^B \mathbf{H}_1 \cdot d\mathbf{l} = I \quad (\text{A.19})$$

So,

$$\int_A^B \mathbf{H}_1 \cdot d\mathbf{l} = \left(1 + \frac{\mu_1}{\mu_2} \right)^{-1} I. \quad (\text{A.20})$$

Alternatively for the compliment,

$$V'_{AB} = \int_A^B \mathbf{E}'_1 \cdot d\mathbf{l} = \alpha_1 \sqrt{\frac{\mu_1}{\epsilon_1}} \int_A^B \mathbf{H}_1 \cdot d\mathbf{l} = \alpha_1 \sqrt{\frac{\mu_1}{\epsilon_1}} \left(1 + \frac{\mu_1}{\mu_2} \right)^{-1} I. \quad (\text{A.21})$$

We can use Eq. A.21 and Eq. A.17 to simplify Eq. A.13,

$$\frac{C'}{L} = \left[\alpha_1^2 \frac{\mu_2}{\mu_1} + \alpha_2^2 \right] \left(1 + \frac{\mu_2}{\mu_1} \right)^{-1} \frac{I^2}{\left(\alpha_1 \sqrt{\frac{\mu_1}{\epsilon_1}} \left(1 + \frac{\mu_1}{\mu_2} \right)^{-1} I \right)^2} \quad (\text{A.22})$$

$$= (\epsilon_1 + \epsilon_2) \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right). \quad (\text{A.23})$$

We now have a relationship between the inductance of the structure and the capacitance of its complimentary structure. It's arbitrary what we call the original structure and what we call the complement. This result holds in the opposite direction so we might as well drop the prime and hold this as a general relationship for complementary planar structures:

$$\frac{C}{L} = (\epsilon_1 + \epsilon_2) \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right). \quad (\text{A.24})$$

Generally a planar structure will have both an associated inductance and capacitance. Assuming, that the structure is small compared to the wavelength, such that the fields are generally decoupled, the above analysis allows us to quickly calculated its C and L values from those of it's compliment.

Bibliography

- [1] Jaewoong Yoon, Seok Ho Song, Cha-Hwan Oh, and Pill-Soo Kim. Backpropagation modes of surface polaritons on a cross-negative interface. *Optics Express*, 13(2):417, January 2005.
- [2] Christophe Caloz and Tatsuo Itoh. *Electromagnetic Metamaterials: Transmission line Theory and Microwave Applications*. Wiley & Sons, Inc., 2006.
- [3] V. G. Veselago. The electrodynamics of substances with simultaneously negative values of ϵ and μ . *Sov. Phys. USPEKHI*, 10(4):509, 1968.
- [4] J. B. Pendry and D. R. Smith. Reversing light with negative refraction. *Phys. Today*, page 37, June 2004.
- [5] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart. Low frequency plasmons in thin-wire structures. *J. Conds. Mat.*, 10:4785–4809, 1998.
- [6] N. W. Ashcroft and N. D. Mermin. *Solid State Physics*. Saunders College Publishing, 1978.
- [7] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart. Magnetism from conductors and enhance nonlinear phenomena. *IEEE Trans. Microwave Theory Tech.*, 47:2075, 1999.
- [8] G. Suran, M. Naili, H. Niedoba, F. Machizaud, O. Acher, and D. Pain. Magnetic and structural properties of co-rich coFeZr amorphous thin films. *J. Magn. Mangn. Mater.*, 192:443–457, 1999.
- [9] M. R. F. Jensen, S. A. Feiven, T. J. Parker, and R. E. Camley. Experimental determination of magnetic polariton dispersion curves in FeF₂. *Phys. Rev. B*, 55(19):2745–2748, February 1997.
- [10] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz. A composite medium with simultaneously negative permeability and permittivity. *Phys. Rev. Lett.*, 84:4184, 2000.
- [11] R. A. Shelby, D. R. Smith, and S. Schultz. Experimental verification of a negative index of refraction. *Science*, 292:77, 2001.

- [12] D. R. Smith, S. Schultz, P. Markos, and C. M. Soukoulis. Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients. *Phys. Rev. B*, 65:195104, 2002.
- [13] Zudong Chen, Tomasz M. Grezegorezyk, Gae-Ian Wu, Jr Joe Pacheco, and Jun Au Kong. Robust method to retrieve the constitutive effective parameters of metamaterials. *Phys. Rev. E*, 70:016608, 2004.
- [14] D. R. Smith, J. J. Mock, A. F. Starr, and D. Schurig. Gradient index metamaterials. *Phys. Rev. E*, 71:036609, 2005.
- [15] J. B. Pendry, D. Schurig, and D. R. Smith. Controlling electromagnetic fields. *Science*, 312(5781):1780–1782, 2006.
- [16] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith. Metamaterial electromagnetic cloak at microwave frequencies. *Science*, 314(5801):977–980, 2006.
- [17] Hou-Tong Chen, Willie J. Padilla, Joshual M. O. Zide, Arthur C. Gossard, Anoinette J. Taylor, and Richard D. Averitt. Active terahertz metamaterial devices. *Nature*, 444:597, 2006.
- [18] I.V. Shadrivov. Scattering of electromagnetic waves in metamaterial superlattices. *Appl. Phys. Lett.*, 90:201919, 2007.
- [19] M.V. Gorkunov. Effect of microscopic disorder on magnetic properties of metamaterials. *Phys. Rev. E*, 73:056605, 2006.
- [20] T. J. Yen, W. J. Padilla, N. Fang, D. C. Vier, D. R. Smith, J. B. Pendry, D. N. Basov, and X. Zhang. Terahertz magnetic response from artificial materials. *Science*, 303:1494, 2006.
- [21] Ari Sihvola. *Electromagnetic mixing formulas and applications*. IEE, 1999.
- [22] R. Landauer. The electrical resistance of binary metallic mixtures. *J. Appl. Phys.*, 23:779, 1952.
- [23] Arthur. D. Yaghjian. Electric dyadic green’s function in the source region. *Proc. IEEE*, 68:248, 1980.
- [24] Shadrivov. Tunable split-ring resonators for nonlinear negative-index metamaterials. *Opt. Exp.*, 14:9344, 2006.
- [25] B.I. Popa. Active metamaterials. *Microwave Opt. Technol. Lett.*, 49:2574, 2007.
- [26] R. H. Ritchie. Plasma losses by fast electrons in thin films. *Phys. Rev.*, 106:874–881, 1957.

- [27] Heinz Raether. *Surface Plasmons*. Springer-Verlag, 1988.
- [28] Jiřrř Homola. Present and future of surface plasmon resonance biosensors. *Anal. Bioanal. Chem.*, 377(377):528–539, 2003.
- [29] R. L. Rich and D. G. Myszka. Survey of the 1999 surface plasmon resonance biosensor literature. *J. Mol. Recognit.*, 13:388–407, 2000.
- [30] Mathew A. Cooper. Optical biosensors in drug discover. *Nature Reviews: Drug Discover*, 1:515–528, July 2002.
- [31] J. Homola, S. S. Yee, and G. Gauglitz. Surface plasmons resonance sensors: review. *Sensors and Actuators B*, 54:3–15, 1999.
- [32] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff. Extraordinary optical transmission through sub-wavelength hole arrays. *Nature*, 391:667–669, 1998.
- [33] J. C. Weeber, A. Dereux, C Girard, J. R. Krenn, and J. P. Goudonnet. Plasmon polaritons of metallic nanowires for controlling submicron propagation of light. *Phys. Rev. B*, 60:9061–9068, 1999.
- [34] J. D. Kottmann, O. J. F. Martin, D. R. Smith, and S. Schultz. Scattering properties of nanoparticles with arbitrary shape. *Chem. Phys. Lett.*, 341, 2001.
- [35] W. L. Barnes, A. Dereux, and T. W. Ebbesen. Surface plasmon subwavelength optics. *Nature*, 424:824–830, August 2003.
- [36] A. P. Hibbins and J. R. Sambles. Grating-coupled surface plasmons at microwave frequencies. *J. of Appl. Phys.*, 86(4):1791–1795, August 1999.
- [37] J. B. Pendry, A. J. Holden, D. J. Robbins, and I. Young. Extremely low frequency plasmons in metallic mesostructures. *Phys. Rev. Lett.*, 76(25):4773, June 1996.
- [38] D.R. Smith, R. C. Vier, Willie Padilla, Syrus C. Nemat-Nasser, and S. Schultz. Loop-wire medium for investigating plasmons at microwave frequencies. *Appl. Phys. Lett.*, 75(10):1425–1427, September 1999.
- [39] Jonah N. Gollub, David R. Smith, David C. Vier, Tim Perram, and Jack J. Mock. Experimental characterization of magnetic surface plasmons on metamaterials with negative permeability. *Phys. Rev. B*, 71:195402, 2005.
- [40] C. G. Parazzoli, R. B. Gregor, K. Li, B. E. C. Koltenbah, and M. Tanielian. Experimental verification and simulation of negative index of refraction using snell’s law. *Phys. Rev. Lett.*, 90:107401, March 2003.

- [41] A. Otto. Excitation of nonradiative surface plasma waves in silver by the method of frustrated total reflection. *Zeitschrift für Physik*, 216:398, 1968.
- [42] A. F. Starr, P. M. Rye, J. J. Mock, and D. R. Smith. Angle resolved microwave spectrometer for metamaterial studies. *Rev. Sci. Instrum.*, 75(4):820–825, April 2004.
- [43] D. Shamonina, V. A. Kalinin, K. H. Ringhofer, and L. Solymar. Magnetoinductive waves in one, two, and three dimensions. *J. Appl. Phys.*, 92(10):6252–6261, November 2002.
- [44] R. Ruppin. Surface polaritons of a left-handed material slab. *J. of Phys.: Condensed Matter*, 13:1811–1819, 2001.
- [45] R. Ruppin. Surface polaritons of a left-handed material slab. *Phys. Lett. A*, 277:61–64, 2000.
- [46] David M. Pozar. *Microwave Engineering, 2nd ed.* John Wiley & Sons, Inc., 1998.
- [47] E. Schloemann, R. Tustison, J. Weissman, and H. J. Van Hook. Epitaxial fe films on gaas for hybrid semiconductor-magnetic memories. *J. Appl. Phys.*, 63:3140, 1988.
- [48] V. S. Liao, W. Stacey T. Wong, S. Ali, and E. Schloemann. Tunable band-stop filter based on epitaxial fe film on gaas. *IEEE MTT-S Digest*, page 957, 1991.
- [49] N. Cramer, D. Lucic, R. E. Camley, and Z. Celinski. High attenuation tunable microwave notch filters utilizing ferromagnetic resonance. *J. Appl. Phys.*, 87(9):6911, May 2000.
- [50] R. E. Camley and D. L. Mills. Theory of propagation in dielectric/magnetic film multilayer structures. *J. Appl. Phys.*, 82(6):3058, September 1997.
- [51] R. J. Astalos and R. E. Camley. Theory of high frequency magnetic tunable filter and phase shifter. *J. Appl. Phys.*, 83:3744, 1998.
- [52] G. Hebermehl, R. Schlundt, H. Zscheila, and W. Heinrich. Eigen mode solver for microwave transmission lines. *The International Journal for Computation and Mathematics in Electrical and Electronic Engineer*, 16:108, 1997.
- [53] G. Hebermehl, R. Schlundt, H. Zscheile, and W. Heinrich. Improved numerical methods for the simulation of microwave circuits. *Surveys on Mathematics for Industry*, 9:117, 1999.
- [54] R.A. Pucel, D.J. Masse, and C.P. Hartwig. Losses in microstrip. *Microwave Theory and Techniques, IEEE Transactions on*, 16(6):342–350, Jun 1968.

- [55] R.A. Pucel, D.J. Masse, and C.P. Hartwig. Correction to "losses in microstrip" (correspondence). *Microwave Theory and Techniques, IEEE Transactions on*, 16(12):1064–1064, Dec 1968.
- [56] J. D. Cockcroft. Skin effect in rectangular conductors at high frequencies. *Proc. Roy. Soc. (London)*, 122:533, 1929.
- [57] R. B. Marks. A multiline method of network analyzer calibration. *IEEE Transactions on Microwave Theory Techniques*, 39:1205–1215, July 1991.
- [58] Christopher Caloz and Tatsuo Itoh. Invited: Novel micorwave devices and structures based on the transmission line approach of meta-materials. *IEEE-MTT Int'l Symp.*, 1:TU2C–3, 2003.
- [59] C. Caloz and T. Itoh. Transmission line approach of left-handed (lh) materials and microstrip implementation of an artificial lh transmission line. *IEEE Antennas Propag. Mag.*, 52:1159, 2004.
- [60] F. Martin, J. Bonache, F. Falcone, M. Sorolla, and R. Marques. Split ring resonator-based left-handed coplanar waveguide. *Appl. Phys. Lett.*, 83:4652, 2003.
- [61] F. Falcone, T. Lopetegi, M. A. G. Laso, J. D. Baena, J. Bonache, M Beruete, R. Marqués, F. Martín, and M. Sorolla. Babinet principle applied to the design of metasurfaces and metamaterials. *Phys. Rev. Lett.*, 93(19):197401, November 2004.
- [62] Juan Domingo Baena, Jordi Bonache, Ferran martín, Ricardo Marqués Sillero, Francisco Falcone, Txema Lopetegi, Migul A. G. Laso, Joan García-García, Ignacio Gil, Maria Flores Protillo, and Mario Sorolla. Equivalent-circuit models for split-ring resonators and complementary split-ring resonators coupled to planar transmission lines. *IEEE MTT-S Digest*, 53, 2005.
- [63] John David Jackson. *Classical Electrodynamics*. Wiley & Sons, Inc., 1999.
- [64] M. Beruete, R. Marques, J. D. Baena, and M. Sorolla. Resonance and cross-polarization effects in conventioinal and complementary split ring resonators periodic screens. *IEEE Antennas and Propagation Society International Symposium*, 3A:797, 2005.
- [65] Qian Zhao, Lei Kang, Bo Du, Bo Li, and Ji Zhou. Electrically tunable negative permeability metamaterials based on nematic liquid crystals. *Appl. Phys. Lett.*, 90:11112, 2007.
- [66] Thomas Hand and Steve Cummer. Frequency tunable electromagnetic meta-material using ferroelectric loaded split rings. *J. of Appl. Phys.*, 2007.

- [67] Stefan Mueller, Andreas Penirschke, Christian Damm, Patrick Scheele, Michael Wittek, Carsten Weil, and Rolf Jakoby. Broad-band microwave characterization of liquid crystals using a temperature-controlled coaxial transmission line. *IEEE Trans. Micro. Theo. Tech.*, 53, 2005.
- [68] C. Caloz, C. C. Chang, and T. Itoh. Full-wave verification of the fundamental properties of left-handed materials (lhms) in waveguide configurations. *J. Appl. Phys.*, 90:5483, 2001.
- [69] Ruopeng Liu, Tie Jun Cui, Da Huang, Bo Zhao, and David R. Smith. Description and explanation of electromagnetic behavior in artificial metamaterials based on effective medium theory. *Phys. Rev. E*, 76:26606, 2007.
- [70] Ricardo Marques, Francisco Mesa, and Francisco Medina. Generalized quasi-tem approximation and telegrapher equations for nonreciprocal ferrite-loaded transmission lines. *IEEE Microwave and Guided Wave Letters*, 10:225, 2000.
- [71] Manuel Horno, Francisco L. Mesa, Francisco Medina, and Ricardo Marques. Quasi-tem analysis of multilayered, multiconductor coplanar structures with dielectric and magnetic anisotropy including substrate losses. *IEEE Trans. Microwave Theory Tech.*, 38:1059, 1990.
- [72] Howard L. Glass. Ferrite films for microwave and millimeter wave devices. *Proc. IEEE*, 76:151, 1988.
- [73] M. Lui, J. Drucker, A. R. King, J. P. Kotthaus, P. K. Hansma, and V. Jacarino. Observation of antiferromagnetic resonance in eptaxial films of mnfe_2 . *Phys. Rev. B*, 33:7720, 1986.
- [74] Lei Kang, Qian Zhao, Hongjie Zhao, and Ji Zhou. Magnetically tunable negative permeability metamaterial composed by split ring resonantors and ferrite rods. *Opt. Express*, 16:8825, 2008.
- [75] I. Gil, J. G. Garcia, J. Bonache, F. Martin, M. Sorolla, and R. Marques. Varactor-loaded split ring resonantors for tunable notch filters at microwave frequencies. *Electron. lett.*, 40:1347, 2004.
- [76] Sungjoon Lim, Christopher Caloz, and Tatsuo Itoh. Metamaterial-based electronically controlled transmission-line structure as a novel leaky-wave antenna with tunable radiation angle and beamwidth. *IEEE Trans. on Microw. Theory. Tech.*, 52:2678, 2004.
- [77] Vladimir B. Bregar. Effective-medium approach to the magnetic susceptibility of composites with ferromagnetic inclusions. *Phys. Rev. B*, 71:1098, 2005.

- [78] Y. He, P. He, S. D. Yoon, P. V. Parimi, F. J. Rachford, V. G. Harris, and C. Vittoria. Tunable negative index metamaterial using yttrium iron garnet. *J. Magn. Mater.*, 313:187, 2007.
- [79] Jonah N. Gollub, David R. Smith, and Juan D. Baena. Hybrid resonant phenomenon in a metamaterial structure with integrated resonant magnetic material. *Submitted Phys. Rev. Lett.*, 2008.