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Standard model and supersymmetric flavor puzzles at the CERN Large Hadron Collider

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I. THE FLAVOR PUZZLES

The standard model (SM) of particle physics suffers from problems in both its gauge and flavor sectors. In the gauge sector, the Large Hadron Collider (LHC) is expected to shed light on the hierarchy problem, and the potential for discovering the Higgs boson and the mechanism of electroweak symmetry breaking is largely responsible for the recent flood of data from events. We outline many flavor measurements that are then possible and describe their prospects for resolving both the standard model and new physics flavor puzzles at the Large Hadron Collider.

Can the Large Hadron Collider explain the masses and mixings of the known fermions? A promising possibility is that these masses and mixings are determined by flavor symmetries that also govern new particles that will appear at the LHC. We consider well-motivated examples in supersymmetry with both gravity and gauge mediation. Contrary to spreading belief, new physics need not be minimally flavor violating. We build nonminimally flavor violating models that successfully explain all known lepton masses and mixings, but span a wide range in their predictions for slepton flavor violation. In natural and favorable cases, these models have metastable sleptons and are characterized by fully reconstructible events. We outline many flavor measurements that are then possible and describe their prospects for resolving both the standard model and new physics flavor puzzles at the Large Hadron Collider.

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The standard model (SM) of particle physics suffers from problems in both its gauge and flavor sectors. In the gauge sector, the Large Hadron Collider (LHC) is expected to shed light on the hierarchy problem, and the potential for discovering the Higgs boson and the mechanism of electroweak symmetry breaking is largely responsible for the keen anticipation for LHC data in the coming years.

In contrast, the LHC’s prospects for explaining the SM flavor puzzle, that is, the observed masses and mixings of the SM quarks and leptons, are far less well known. Unlike the gauge sector puzzles, the SM flavor puzzle is not necessarily connected to the weak scale. In addition, data already stringently constrain most of the SM flavor parameters, and it is not clear that more data will fix the problem. As a case in point, the recent flood of data from neutrinos, far from pointing the way to a compelling theory of flavor, has instead served mainly to eliminate previous models, multiply the number of possible explanations, and suggest that the origin of flavor is to be found at very high energy scales.

What is often overlooked, however, is that new weak-scale physics may shed light on the SM flavor puzzle in a very different way. In many well-motivated extensions of the SM, an understanding of the flavor structure of new states can impose additional constraints on the same set of theoretical parameters that govern the SM Yukawa couplings. This is qualitatively different from the case of neutrinos, in that new observables provide new constraints without introducing new degrees of freedom.

A prominent example, and one we will consider in detail here, is the case of supersymmetry. For example, if SM flavor is determined by the existence of an approximate horizontal symmetry (as in the Froggatt-Nielsen mechanism [1]), then the SM particles and their superpartners transform under the symmetry in the same way. The same set of horizontal charges will then be further constrained if existing flavor-changing neutral currents (FCNC) measurements are augmented by flavor measurements at the LHC. From this viewpoint, supersymmetry provides a simple, representative example of new physics in which new particles and the SM fermions behave identically under any underlying flavor theory. This feature is shared by other ideas for new physics, including, for example, many models with extra dimensions, where the flavor parameters of the SM fermions and their Kaluza-Klein excitations are related.

Of course, new physics also brings with it the new physics flavor puzzle: If there is new physics at the TeV scale, why does it not contribute to FCNC processes at much higher rates than currently observed [2]? In the case of supersymmetry, for example, if the supersymmetric flavor parameters are generic, then loop diagrams involving gauginos and sfermions induce FCNC processes, such as $K\rightarrow\bar{K}$ mixing and $\mu\rightarrow e\gamma$, at levels that are orders of magnitude above the experimentally allowed ranges. There are essentially three mechanisms to suppress the supersymmetric contributions to such processes:

(i) **Decoupling.**—The sfermion mass scale can be very high. Such scenarios may be probed at the LHC through nondecoupling effects, for example, through the superoblique parameters [3]. However, as we are primarily interested here in the possibility of direct flavor measurements, we do not consider such scenarios further.

(ii) **Degeneracy.**—The sfermion masses can be approximately degenerate, leading to Glashow-Iliopoulos-Maiani effects.

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1. If the flavor symmetry is an $R$-symmetry, the transformation properties of the SM particles and their superpartners are not the same, but still they are related.
Maiani–like suppression. Such degeneracy could be the result of gauge-mediated supersymmetry breaking (GMSB) or of another type of flavor-blind mediation of supersymmetry breaking.

(iii) Alignment.—The sfermion mixings, that is, the flavor-changing gaugino-sfermion-fermion couplings, can be suppressed. Such alignment could be the result of an approximate horizontal symmetry.

Measurements of the supersymmetric flavor parameters—the particle masses and their flavor decomposition (mixing angles)—will shed light on the issue of how the new physics flavor puzzle is solved. In addition, as we will show below, there are many possible flavor models that currently explain all available data, but which differ in their resolution of the new physics flavor puzzle. Determining how the new physics flavor puzzle is resolved may therefore also play a key role in leading us toward the correct solution to the SM flavor puzzle.

In this work, we focus on the slepton sector of supersymmetry. We expect that the slepton sector is better suited for flavor measurements at the LHC than the squark sector. Many of the theoretical issues that we raise also apply, however, to the squark sector. Roughly speaking, the experimental upper bounds on the rates of lepton flavor-changing processes (e.g., $\mu \to e\gamma$) give allowed ranges in the $(\Delta m^2_{ij}, K_{ij})$ planes. Here, $\Delta m^2_{ij}$ stands for the mass-squared splitting between the corresponding slepton generations, and $K_{ij}$ is the relevant mixing angle. (Precise definitions are given below.) We argue that viable and natural models can lead to many different points in the allowed range. In the future, by combining the information from low and high energy flavor measurements, we may be able to narrow the allowed range considerably, select a specific supersymmetric model, and eventually find the solution to both the new physics and SM flavor puzzles.

To demonstrate our point, we will analyze simple gauge-gravity “hybrid” models. (Other examples are possible, too.) These are minimal GMSB models with a high messenger scale, such that gravity-mediated contributions cannot be neglected. Since the GMSB scalar masses are universal, the gravity-mediated contributions dictate the mixings, even if their overall size is quite small, and so we will invoke horizontal symmetries to adequately suppress the mixings. The models we present are consistent with all known lepton masses and mixings, and they satisfy all FCNC and rare decay constraints. At the same time, they span a wide range of predictions for slepton flavor violation and are amenable to many LHC flavor studies, providing a useful starting point for our purposes.

The flavor problems associated with supersymmetry, and more generally with TeV scale new physics, have led to the spreading belief in the flavor physics community that minimal flavor violation (MFV) is perhaps inescapable.

The models we present demonstrate that this is far from being true. One of the general conclusions of this work is that the question of whether new physics is MFV or non-MFV is open, and it is therefore of great interest to determine the LHC’s potential for distinguishing between models that exhibit MFV and models that do not.

II. PHENOMENOLOGICAL CONSTRAINTS

Rare flavor-changing charged lepton decays constrain a combination of supersymmetric parameters. It is a reasonable approximation, for our purposes, to think of the constrained quantities as

$$\delta^M_{ij} \sim \frac{\Delta m^2_{Mji}}{m^2_M} K^M_{ij},$$

where $M = L, R$ specifies left-handed sleptons $\tilde{E}_L$ or right-handed sleptons $\tilde{E}_R$, $i, j = 1, 2, 3$ are generation indices, and

$$\hat{m}_M = \frac{(m_{\tilde{E}_L i} + m_{\tilde{E}_L j})}{2}, \quad \Delta \hat{m}^2_{Mji} = m^2_{\tilde{E}_{Lj}} - m^2_{\tilde{E}_{Li}}.$$

The matrix $K^M$ is the mixing matrix of electroweak gaugino couplings. In other words, $g^a K^M_{ij}$ is the coupling strength of the $\lambda^a$ gaugino $(\alpha = 1, 2$ for the $b$-ino, $W$-ino) to the lepton $E_{Lji}$ and the slepton $\tilde{E}_{Mji}$. We ignore here the constraints on the left-right block in the slepton mass-squared matrix, which are, however, satisfied in our models.

In Ref. [5], the experimental bounds

$$B(\mu \to e\gamma) \leq 1.2 \times 10^{-11},$$

$$B(\tau \to e\gamma) \leq 1.1 \times 10^{-7}, \quad B(\tau \to \mu\gamma) \leq 6.8 \times 10^{-8}$$

were used to derive the following constraints:

$$\delta^L_{12} \leq 6 \times 10^{-4}, \quad \delta^L_{13} \leq 0.09, \quad \delta^M_{13} \leq 0.15, \quad \delta^M_{23} \leq 0.12.$$  

These results assumed a universal scalar mass $m_0 < 380$ GeV, a unified gaugino mass $M_{1/2} < 160$ GeV, and $5 < \tan\beta < 15$. We are interested in a more generic framework that does not assume a priori universality or unification. Nevertheless, we will require that our models satisfy the constraints of Eq. (4), because the numerical details are not significant for our purposes. In particular, they can be modified in a straightforward way to meet stronger (or milder) constraints. Our models also reproduce the present ranges of the leptonic mixing angles:

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2The LHC has the potential to test MFV also in the context of other extensions of the SM [4].

3The upper bound on $\tau \to \mu\gamma$ has since been improved [6] to $B(\tau \to \mu\gamma) \leq 4.5 \times 10^{-8}$. 
\[
\sin^2 \theta_{12} = 0.31 \pm 0.02, \quad \sin^2 \theta_{13} = 0^{+0.008}_{-0.013}, \\
\sin^2 \theta_{23} = 0.47 \pm 0.07.
\]

Our models employ a small symmetry breaking parameter, \( \lambda \sim 0.1\sim0.2 \). The various physical parameters are suppressed by powers of \( \lambda \), with unknown \( O(1) \) coefficients. We thus interpret the constraints of Eq. (4), the measurements of Eq. (5), and the information on lepton masses in terms of \( \lambda \) as follows:

\[
\delta_{12}^L \leq \lambda^4, \quad \delta_{12}^R \leq \lambda^2, \quad \delta_{13}^M \leq \lambda, \quad \delta_{23}^M \leq \lambda, \\
\sin \theta_{ij} \sim 1, \quad m_1/m_2 \sim 1, \quad m_2/m_3 \sim 1, \\
m_e/m_\mu \sim \lambda^2, \quad m_\mu/m_\tau \sim \lambda^2.
\]

Note that we assume here that \( \sin \theta_{13} \) is accidentally, rather than parametrically, suppressed. If the experimental upper bound on \( \sin \theta_{13} \) improves significantly, the models would have to be modified.

## III. MINIMAL FLAVOR VIOLATION

The supersymmetric flavor puzzle is solved if the mediation of supersymmetry breaking obeys the principle of MFV [7]. In MFV models, in the absence of the SM Yukawa couplings (possibly extended to allow for neutrino masses), the leptonic sector has a global \( SU(3)_C \times SU(3)_E \) symmetry. The \( SU(3)_C \) acts on the three \( SU(2) \)-doublet lepton supermultiplets, and the \( SU(3)_E \) acts on the three charged \( SU(2) \)-singlet lepton supermultiplets. The symmetry is broken by the charged lepton Yukawa couplings, which constitute spurions transforming as \((3, \bar{3})\) under the global symmetry. There could be additional spurions related to neutrino masses [8]. For the sake of simplicity, we assume that these additional spurions can be neglected. Indeed, they are expected to be negligible whenever the scale of mediation of supersymmetry breaking is much lower than the scale of lepton number breaking (the seesaw scale). In any case, the possible effects of such spurions can be included in a straightforward way. Our conclusions for right-handed sleptons are hardly affected by such spurions.

Within MFV, we can choose to work in a basis where the \( Y_E \) spurion is diagonal,

\[
Y_E = \text{diag}(y_e, y_\mu, y_\tau).
\]

If the spurions related to the neutrino masses are negligible, the soft slepton mass-squared terms have the form

\[
M_{\tilde{E}\mu}^2 = \overline{m}_M^2 \begin{pmatrix} 1 + a_M y_e^2 & 0 & 0 \\ 0 & 1 + a_M y_\mu^2 & 0 \\ 0 & 0 & 1 + a_M y_\tau^2 \end{pmatrix},
\]

where the dimensionless coefficients \( a_M \) are \( \leq 1 \). The spectrum and flavor decomposition of sleptons in MFV models therefore have the following properties:

1. The spectrum has three-fold degeneracy to a good approximation. The fractional mass splitting of the third generation is of order \( y_e^2 \). For \( \tan \beta \sim m_1/m_2 \), \( \tilde{\tau}_{L,R} \) is split from \( \tilde{e}_{L,R} \) and \( \tilde{\mu}_{L,R} \) by roughly 10%. For smaller \( \tan \beta \), the splitting scales down as \( \tan^2 \beta \).
2. The first two generations are degenerate to an excellent approximation, as their fractional mass splitting is of order \( y_\mu^2 \).
3. There is no mixing.

## IV. NONMINIMAL FLAVOR VIOLATION

In this section we argue that there is much room for flavor physics that is far from MFV. To do so, we present explicit models that are both natural (i.e., small couplings are related to approximate symmetries) and viable, and yet violate some or all of the MFV predictions listed above in a significant way. These models are based on balancing two types of contributions to the slepton mass matrices, in the spirit of Ref. [9]: a gauge-mediated contribution, which is MFV, and a gravity-mediated contribution that is non-MFV. We assume, however, that the gravity-mediated contribution is subject to an approximate horizontal symmetry and can thus exhibit approximate alignment [10]. Thus, the supersymmetric flavor problem is solved by a combination of degeneracy and alignment. We construct models that saturate the upper bound on the \( \delta_{12}^L \) parameter, but the models can be easily modified to give contributions that are well below the present bound.

We further require that the horizontal symmetry accounts for the observed flavor features of leptons. Specifically, we require that the symmetry gives anarchical neutrino mass matrices (with neither hierarchy nor degeneracy in the neutrino masses), \( O(1) \) leptonic mixings, and hierarchical charged lepton masses. Again, our models can be modified in a straightforward way to provide parametric suppressions that are different from these choices. Thus, both the approximate alignment of the sleptons and the structure of the lepton masses and mixings are dictated by the same symmetry and the same horizontal charges. As explained in Sec. I, this is exactly the sort of scenario in which experimental information on slepton mixing will contribute to our understanding of the SM flavor puzzle.

Let us be more specific now. Most of our models are high-scale GMSB models such that the soft masses are dominated by the GMSB contributions, with a smaller component of gravity-mediated masses. It is convenient to parameterize the ratio between the gravity-mediated and the gauge-mediated contribution by \( x \):

\[
M_{\tilde{E}_L}^2 = \overline{m}_L^2 1 + m \overline{m}_E 1 + x \overline{m}_L^2 X_L, \\
M_{\tilde{E}_R}^2 = \overline{m}_R^2 1 + m \overline{m}_E 1 + x \overline{m}_R^2 X_R.
\]

The GMSB contributions are universal, but the gravity-mediated contributions result in potentially large mixings. We invoke horizontal (Abelian) symmetries to suppress these mixings to acceptable levels, in the spirit of align-
We can estimate the messenger scale \(M_m\) as a function of \(x\),

\[
M_m \sim \sqrt{N_m \cdot \frac{\alpha_2}{\pi}} M_p.
\]

Examining Eq. (9), we note that it can lead to degeneracy, \(\Delta \tilde{m}_M^2 / \tilde{m}_M^2 \sim x\) for small \(x\), and to alignment, \(K_{ij}^M \sim \max \{(X_M)_{ij}, (V^E_M)_{ij}\}, \) where \(V^E_M V^{E\dagger}_M = \text{diag}(m_e, m_\mu, m_\tau)\). We learn that

\[
\delta_{ij}^M \sim x \times \max \{(X_M)_{ij}, (V^E_M)_{ij}\}.
\]

\(^4\)Strictly speaking, we would get a somewhat different estimate if we used \(\tilde{m}_m^2\) instead of \(\tilde{m}_L^2\), since the former involves \(\alpha_1\) and the latter involves \(\alpha_2\), with different numerical coefficients. However, at the high messenger scales we are considering, these two couplings are not very different, and we are neglecting \(O(1)\) coefficients throughout anyway. Furthermore, the only part of our analysis that is sensitive to the messenger scale is the NLSP slepton lifetime. Regardless of these \(O(1)\) differences, the NLSP will always decay outside the detector in our models.

Thus, if the horizontal symmetry produces strong alignment (small mixing angles), then the degeneracy can be mild \((x \sim 1)\), but if the supersymmetric mixing angles are large (similar to the leptonic mixing angles \(\theta_{12}\) and \(\theta_{23}\)), then the degeneracy must be strong \((x \ll 1)\). We present four different models that demonstrate the various possibilities, from strong alignment/no degeneracy to large mixing/small splittings.

We assume a horizontal \(U(1) \times U(1)\) symmetry, where each of the \(U(1)\)'s is broken by a spurion of corresponding charge \(-1\) and size \(\lambda \sim 0.2\). [Our results would remain the same if the first \(U(1)\) were broken by two spurions of charges \(\pm 1\).] For each model, we give the horizontal charges of the left-handed lepton doublet \(L_i\) and antilepton singlet \(\bar{E}_i\) supermultiplets (setting the Higgs charges to zero), the lepton mass matrices, and the gravity-mediated contribution to the slepton mass-squared matrices (omitting coefficients of order one in each entry), the parametric suppression (i.e., the \(\lambda\)-dependence) of the resulting mixing angles, and the maximum allowed value of \(x\), which gives the level of degeneracy.

### A. Small mixing, no degeneracy

The first model has pure gravity mediation, supplemented by the horizontal symmetry. More generally, there can be small gauge-mediated contributions, but they are at most comparable to the gravity-mediated ones.

The horizontal \(U(1) \times U(1)\) charges are

\[
L_1(4, 0), L_2(2, 2), L_3(0, 4); \quad \bar{E}_1(1, 0), \bar{E}_2(1, -2), \bar{E}_3(0, -3).
\]

The resulting lepton mass matrices have the following structure:\(^5\)

\[
M_\nu \sim \frac{(\phi_\nu)^2}{\lambda^8} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\]

\[
M_E \sim (\phi_\nu)\lambda \begin{pmatrix} \lambda^4 & 0 & 0 \\ \lambda^4 & \lambda^2 & 0 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}.
\]

The \(X_M\) matrices have the following structure:

\[
X_L \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^8 \\ \lambda^4 & 1 & \lambda^4 \\ \lambda^8 & \lambda^4 & 1 \end{pmatrix}, \quad X_R \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}.
\]

The parametric suppression of the mixing angles is given by

\(^5\)The zeros in these mass matrices follow from holomorphy. The vanishing entries would require powers of \(\lambda^\dagger\) to form \(U(1) \times U(1)\)-invariant combinations but, since the superpotential is holomorphic, \(\lambda^\dagger\) cannot appear.
The resulting lepton mass matrices have the following structure:

\[
K_{12}^L \sim \lambda^4, K_{13}^L \sim \lambda^8, K_{23}^L \sim \lambda^4; \\
K_{12}^R \sim \lambda^2, K_{13}^R \sim \lambda^4, K_{23}^R \sim \lambda^2.
\] (19)

There is no degeneracy:

\[
x \simeq 1,
\] (20)
as is the case for

\[
M_m \simeq 0.1M_P.
\] (21)

Given this high messenger scale, and the fact that the gravity- and gauge-mediated contributions are comparable, we will think of this model as a pure gravity-mediation model with no GMSB contribution.

The flavor suppression in this model comes entirely from the smallness of the supersymmetric mixing angles. In other words, the alignment of the charged slepton mass eigenstates with the charged lepton mass eigenstates is precise enough to satisfy all phenomenological constraints. No degeneracy is required.

### B. Large 2–3 mixing, small 1–2 mixing, \(\mathcal{O}(0.01)\) degeneracy

This model is a gauge-gravity hybrid model. The suppression in the 2–3 sector is provided by degeneracy, while in the 1–2 sector it comes mainly from alignment.

The horizontal \(U(1) \times U(1)\) charges are

\[
L_1(2, 0), L_2(0, 2), L_3(0, 2); \\
\bar{E}_1(1, 2), \bar{E}_2(2, -1), \bar{E}_3(0, -1).
\] (22)

The resulting lepton mass matrices have the following structure:

\[
M_\nu \sim \frac{\langle \phi_d \rangle^2}{M} \lambda^2 \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix},
\]

\[
M_E \sim \langle \phi_d \rangle \lambda \begin{pmatrix}
\lambda^4 & 0 & 0 \\
\lambda^4 & \lambda^2 & 1 \\
\lambda^4 & \lambda^2 & 1
\end{pmatrix}.
\] (23)

The \(X_M\) matrices have the following structure:

\[
X_L \sim \begin{pmatrix}
1 & \lambda^2 & \lambda^4 \\
\lambda^2 & 1 & \lambda^4 \\
\lambda^4 & 1 & \lambda^4
\end{pmatrix},
\]

\[
X_R \sim \begin{pmatrix}
1 & \lambda^2 & \lambda^4 \\
\lambda^2 & 1 & \lambda^4 \\
\lambda^4 & \lambda^2 & 1
\end{pmatrix}.
\] (24)

The parametric suppression of the mixing angles is given by

\[
K_{12}^L \sim \lambda^2, K_{13}^L \sim \lambda^2, K_{23}^L \sim 1; \\
K_{12}^R \sim \lambda^2, K_{13}^R \sim \lambda^4, K_{23}^R \sim \lambda^2.
\] (25)

The required level of degeneracy is dictated by the \(\delta_{12}^L\) constraint:

\[
x \sim 0.02,
\] (32)
leading to

\[
M_m \sim 10^{-4}M_P.
\] (33)

### C. Large 2–3 mixing, mildly small 1–2 mixing, \(\mathcal{O}(0.02)\) degeneracy

This is another gauge-gravity hybrid model. The horizontal \(U(1) \times U(1)\) charges are

\[
L_1(1, 0), L_2(0, 1), L_3(0, 1); \\
\bar{E}_1(2, 1), \bar{E}_2(2, -1), \bar{E}_3(0, -1).
\] (28)

The resulting lepton mass matrices have the following structure:

\[
M_\nu \sim \langle \phi_d \rangle^2 \lambda^2 \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix},
\]

\[
M_E \sim \langle \phi_d \rangle \lambda \begin{pmatrix}
\lambda^4 & 0 & 0 \\
\lambda^4 & \lambda^2 & 1 \\
\lambda^4 & \lambda^2 & 1
\end{pmatrix}.
\] (29)

The \(X_M\) matrices have the following structure:

\[
X_L \sim \begin{pmatrix}
1 & \lambda^2 & \lambda^4 \\
\lambda^2 & 1 & \lambda^4 \\
\lambda^4 & 1 & \lambda^4
\end{pmatrix},
\]

\[
X_R \sim \begin{pmatrix}
1 & \lambda^2 & \lambda^4 \\
\lambda^2 & 1 & \lambda^4 \\
\lambda^4 & \lambda^2 & 1
\end{pmatrix}.
\] (30)

The parametric suppression of the mixing angles is given by

\[
K_{12}^L \sim \lambda^2, K_{13}^L \sim \lambda^2, K_{23}^L \sim 1; \\
K_{12}^R \sim \lambda^2, K_{13}^R \sim \lambda^4, K_{23}^R \sim \lambda^2.
\] (31)

The level of degeneracy required for satisfying the \(\delta_{12}^L\) constraint is rather mild:

\[
x \sim 0.1,
\] (26)
leading to

\[
M_m \sim 10^{-3}M_P.
\] (27)

### D. Strong degeneracy, large mixing

We can also take a single horizontal \(U(1)\) and assign all left-handed lepton doublets the same horizontal charge:

\[
L_1(2), L_2(2), L_3(2); \\
\bar{E}_1(4), \bar{E}_2(2), \bar{E}_3(0).
\] (34)

The resulting lepton mass matrices have the following structure:
\[ M_p \sim \frac{(\phi_n)^2}{M} \lambda^4 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \]
\[ M_E \sim (\phi_d) \lambda^2 \begin{pmatrix} \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \]

The \( X_L \) matrix is anarchical:
\[ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & \lambda^2 & \lambda^4 \\ \lambda^2 & 1 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}. \]

The parametric suppression of the mixing angles is given by
\[ K^{L}_{ij} \sim 1, \quad K^{L}_{13} \sim 1, \quad K^{L}_{23} \sim 1; \]
\[ K^{R}_{ij} \sim \lambda^2, \quad K^{R}_{13} \sim \lambda^4, \quad K^{R}_{23} \sim \lambda^2. \]

The required suppression must come entirely from degeneracy:
\[ \chi \sim 0.001, \]
leading to
\[ M_m \sim 10^{-4} M_p. \]

E. Features of the \( \bar{E}_R \) sector

A summary of the spectra and the mixings in the \( \bar{E}_R \) sector in MFV and in our non-MFV models is presented in Table I. We conclude:

1. Measurements of upper bounds on mass splitting at any level will be informative.
2. If mass splitting is established and \( \tan \beta \) is not large, that will clearly signal non-MFV.

(3) Mixing effects are small, \( \mathcal{O}(\lambda^2) \), with or without MFV. If one could be sensitive to \( \tilde{e}_R-\tilde{\mu}_R \) mixing \( (\tilde{\mu}_R-\tilde{\tau}_R \) mixing) of order a (few) percent, that would clearly distinguish MFV from non-MFV.

We stress that while we did not make an effort to vary the level of mixing in the \( \bar{E}_R \) sector in our models, we expect this mixing to be a small model independently. The reason is that in models of Abelian horizontal symmetries, we have a generic upper bound on the mixing [11,12]:
\[ K_{ij}^{R} \leq \left( \frac{m_{\ell_i}/m_{\ell_j}}{|U_{ij}|} \right), \]
where \( U \) is the lepton mixing matrix. Given our assumptions that the lepton mixings have no \( \lambda \)-suppression while the charged lepton masses have a \( \lambda^2 \)-hierarchy, then all our non-MFV models satisfy the naïve upper bound on the slepton mixing. In particular, \( K_{ij} \) cannot be order 1.

F. Features of the \( \bar{E}_L \) sector

We summarize our results in Table II. We give the mass splittings and the flavor decompositions of the mass eigenstates for the MFV framework and for each of our four non-MFV examples. We draw the following conclusions:

(1) Mass measurements with an accuracy of \( \mathcal{O}(0.1-0.001) \) (which, for 200 GeV sleptons, translates into mass resolutions of \( 20-0.2 \) GeV) will provide valuable information.

(2) Of course, it will be possible to probe the flavor decomposition only if the mass eigenstates can be separated. In this case,

(i) it would be helpful to learn whether the mass eigenstate that decays dominantly into tau leptons (muons) decays to muons (tau leptons) at a comparable rate or a much smaller rate.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Delta \tilde{m}<em>{R</em>{21}}/\tilde{m}_R )</th>
<th>( \Delta \tilde{m}<em>{R</em>{32}}/\tilde{m}_R )</th>
<th>( \bar{E}_{R1} )</th>
<th>( \bar{E}_{R2} )</th>
<th>( \bar{E}_{R3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MFV</td>
<td>( 10^{-6} \tan^2 \beta )</td>
<td>( 10^{-4} \tan^2 \beta )</td>
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A, B, C, D | \( \approx 1 \) | \( \approx 1 \) | (1, 0.01, 0.001) | (0.01, 1, 0.1) | (0.001, 0.1, 1) |
(ii) it would be informative to measure (or put an upper bound on) the $\mu$ and/or $\tau$ branching ratio of the mass eigenstate that decays dominantly to electrons.

V. THE LSP AND NLSP

Although our models differ in their flavor structures, they share the feature of a gravitino LSP. In addition, for moderate to large $N_m$, the next-to-lightest supersymmetric particle (NLSP) is a charged slepton [13]. The reason for that is, in our framework,

$$m_{3/2} \sim \sqrt{x} m_{\text{slepton}}.$$  \hspace{1cm} (41)

Thus, the only possible exception is model A, where $x \sim 1$, and the slepton and gravitino masses are consequently comparable. But even in this model it is possible that the gravitino is “accidentally” lighter than the charged sleptons.

The decay width for slepton decay to gravitino is independent of the slepton’s flavor and chirality composition, and is given by

$$\Gamma(\tilde{\ell}_M \rightarrow l \tilde{G}) = \frac{1}{48 \pi M_{\tilde{p}}} \frac{m^5_{\tilde{M}}}{m^4_{\tilde{G}}} \left[1 - \frac{m^2_{\tilde{G}}}{m^2_{\tilde{M}}}ight]^3.$$  \hspace{1cm} (42)

For $m_{\tilde{G}} \ll m_{\tilde{M}}$, the slepton lifetime is

$$\tau \approx 16 \text{ hours} \left[\frac{m_{\tilde{G}}}{\text{GeV}}\right]^2 \left[\frac{100 \text{ GeV}}{m_{\tilde{M}}}\right].$$  \hspace{1cm} (43)

For all of the models considered here, then, the slepton is effectively stable for collider experiments. Supersymmetric events are fully reconstructible, and, at least in principle, the final state jets and leptons can be combined to form the intermediate supersymmetric particles all the way up the cascade decay chains.

This scenario differs from the usual supersymmetric scenario, in which supersymmetric events are characterized by two missing neutralinos. Slepion flavor violation at future colliders in missing energy scenarios has been the subject of many studies. (See, e.g., Refs. [14–19].) These results will be improved markedly in the scenario discussed here, where supersymmetric events are fully reconstructible. In particular, final state leptons may be identified as originating from interactions with left- or right-handed sleptons. Thus, the LHC may be able to determine the slepton-lepton-gaugino mixing angles in both left- and right-handed sectors independently, providing extremely incisive tests of all of the flavor models presented above.

In addition, for the high-scale gauge-mediation models discussed here, the extremely long lifetime of the slepton implies that it may be possible to trap and collect sleptons and observe their decays [20–22]. Collider measurements of the slepton mass and a determination of the slepton lifetime determines the gravitino mass. (In fact, a measurement of the lepton energy in slepton decays provides another measurement of the gravitino mass, or alternatively, a check that the outgoing particle couples with gravitational strength [23,24].) The gravitino mass is a measure of the $F$-term relevant for gravity mediation, and comparison with the $F$-term relevant for gauge mediation will provide useful insights into supersymmetry breaking.

Of course, the observation of NLSP decay also provides flavor information. Comparison of the decay rate of the NLSP to $e\tilde{G}$, $\mu\tilde{G}$, and $\tau\tilde{G}$ provides a direct measurement of the flavor composition of the NLSP. Depending on the number of NLSP decays observed, this may provide precise constraints on slepton flavor violation [25], which will supplement the other flavor information derived directly from collider experiments.

VI. SLEPTON FLAVOR AT THE LHC

We have demonstrated that measurements of (or bounds on) mass splittings and mixings in the slepton sector will be very valuable for our understanding of flavor and of supersymmetry breaking. Can these goals be achieved at the LHC? We postpone a detailed study of this question to future work [26] and confine ourselves here to brief comments on the opportunities and limitations of the ATLAS and CMS experiments for lepton flavor violation (LFV) studies.

There have been several suggestions in the literature for signal channels at future colliders in which to look for lepton flavor violation [14–17,27,28], and a few works that study whether such signals will actually be measurable at the LHC [18,19,29]. There are, however, a number of reasons to revisit this question now. The ATLAS and CMS collaborations have recently made great progress by (1) tying their detector simulations more closely to the as-built geometries and material budgets, (2) replacing back-of-the-envelope estimates of reconstruction performance (such as vertexing resolutions and charged lepton and jet resolutions and efficiencies) with models based on full simulations and algorithms approaching those that will be used for the final detector, (3) modeling the trigger response, and (4) generating a wider range and greater number of background samples than were available a few years ago. With regard to background samples, the latest next-to-leading-order parton shower-matching event generators [30–32] have yielded more accurate production of multiparton SM processes.

Such advances are of great relevance for LFV studies. For example, before particle misidentification is taken into account, the multilepton signatures generic in many LFV studies will have low SM backgrounds. However, in reality, jets misidentified as leptons may create significant backgrounds, given the gigantic QCD cross section at the LHC. Proper modeling of jet misidentification is therefore highly
relevant for all LFV studies, and particularly those that use tau leptons.

Most previous studies of LFV have considered missing energy signals. As noted in Sec. V, however, in the hybrid gauge-gravity-mediation scenarios considered here, long-lived charged sleptons are at least as likely, and it is possible, at least in principle, to reconstruct the four-momenta and invariant masses of all particles in each event on an event-by-event basis \cite{33–36}. Of course, it is essential to determine whether preselection cuts may be devised to isolate a relatively pure sample of such events, and to assess the precision with which masses and mass splittings may be measured in such scenarios. Such studies will require adequate modeling of backgrounds and lepton and jet reconstruction, as well as trigger efficiencies for slow metastable sleptons \cite{37}.

Finally, it is also of great interest to investigate the experimental sensitivity to mass splittings in situations where flavor mixing is small, and when the final supersymmetric particles are not seen by the detector, as may be the case in our model A \cite{26}. Since the direct reconstruction of the previous paragraph is no longer possible, it may be necessary to use one or more of the indirect techniques for placing constraints on supersymmetric events with incomplete information which have been suggested in the past \cite{38–46}. We plan to check whether they are applicable to our framework. For our purposes, the principal idea is to plot invariant mass spectra of dilepton pairs, and look for an upper limit (a cutoff) in their distributions \cite{38}. For example, a difference in the end-point positions of the $m_{\tau^+\tau^-}$ and $m_{\mu^+\mu^-}$ distributions constrains the mass-squared differences of the selectron and the smuon, even though the absolute value of either mass would be almost unconstrained.

\section*{VII. DISCUSSION}

It is convenient to think about the issue of supersymmetric lepton flavor in the $(\Delta m^2_{ij}, K_{ij})$ plane, i.e., the mass-squared splitting between slepton generations vs the mixing among them.

From the experimental point of view, we can probe this plane in at least two independent ways. Upper bounds on rare charged lepton decays, such as $\mu \rightarrow e\gamma$, give an upper bound on the combination $\Delta m^2_{ij} \times K_{ij}$, which corresponds to a curve in the plane. The lower left region below this curve is allowed. If future experiments actually measure these rates, they will confine models to the corresponding curve.

Direct observations of sleptons at the ATLAS and CMS experiments can provide upper bounds on either or both of the mass splitting and mixing. Under favorable circumstances, both can be measured. It is thus not inconceivable that some combination of lepton flavor precision measurements and direct measurements at the LHC will eventually allow only a small region in the $(\Delta m^2_{ij}, K_{ij})$ plane.

From the theoretical point of view, the supersymmetric “model space” is rich enough that it can (naturally) lead to almost any point in the plane. GMSB contributions to the soft breaking terms lead to degeneracies that can be as small as the corresponding lepton mass squared (e.g., $m^2_{\mu}$ between sleptons of the first and second generations). The larger the gravity-mediated contribution, the larger the splitting would be, and it can reach order one if these contributions are comparable to or larger than the gauge-mediated ones.

Horizontal symmetries can lead to alignment. Depending on the symmetry and on the horizontal charges, the mixing in the left-handed sector can be as large as the corresponding lepton mixing, or much smaller. The mixing in the right-handed sector can be as large as the ratio between the lepton mass ratio and the mixing angle, i.e., $K^R_{ij} \leq (m_{\ell_i}/m_{\ell_j})/U_{ij}$ (here $U$ is the mixing matrix in the lepton sector). Either mixing can also be much smaller than these upper bounds.

Thus, the ratio between gauge- and gravity-mediated contributions allows us to move along the $\Delta m^2_{ij}$ axis, while the horizontal charge assignments allow us to move along the $K_{ij}$ axis. Conversely, if experiments determine the actual values of $\Delta m^2_{ij}$ and $K_{ij}$, we will gain information on both the ratio between gauge- and gravity-mediated contributions and the approximate horizontal symmetry that determines the flavor structure.

The information on $\Delta m^2$ probes for us the mechanism by which supersymmetry breaking is mediated. In particular, it can determine the mass scale of the messenger fields. It is important in this context, however, to have complimentary information about the scale of $m_{3/2}$ in order to test whether the $F$-term that is relevant for gauge mediation is the same as the one that is relevant for gravity mediation (or smaller). This information can be extracted from the lifetime of the NLSP. (Recall that in our framework the gravitino is the LSP.)

The information on $K^M_{ij}$, on the other hand, can give us guidance about the way in which the SM flavor puzzle is solved. For example, if an approximate horizontal symmetry $H$ is at work in structuring the SM Yukawa matrices, then measurements of $K^M_{ij}$ give additional new information about this symmetry as well as the charge assignments $H(\Phi)$ of the various fields. In particular, while the size of the Yukawa couplings depends on charge differences between left-handed and right-handed fields, such as $H(L_i) - H(E_i)$, those of the slepton mass-squared matrices depend on independent combinations, such as $H(L_i) - H(L_j)$ and $H(E_i) - H(E_j)$. In the example models discussed here, then, flavor information from the LHC sheds light on the same flavor parameters that determine the SM fermion masses, yielding additional constraints without additional.
degrees of freedom. This information is therefore likely to exclude many flavor models, and, in some cases, would provide stringent constraints that could lead us to compelling resolutions of both the SM and new physics flavor puzzles.

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APPENDIX: IS PURE ALIGNMENT VIABLE?

Model A in Sec. IV employs only alignment to solve the supersymmetric lepton flavor problem. One may ask whether a situation where slepton masses are within reach of the LHC and there is no degeneracy between them is worth thinking about in view of constraints on the squark sector. In this appendix, we would like to make two points:

(1) Combining the experimental constraints from $K$–$ar{K}$ mixing and $D$–$ar{D}$ mixing requires some level of degeneracy in the squark sector [2, 47, 48].

(2) Yet, it is possible to have models where the dominant contributions to the soft breaking terms are gravity-mediated and there is no degeneracy in the slepton sector.

To make the first point, we focus our attention on the contributions of the first two generations of squark doublets to $K$–$ar{K}$ mixing and to $D$–$ar{D}$ mixing. The mass-squared matrices for these squarks have the following form:

$$M_{U_L}^2 = m_{Q_L}^2 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) m_Z^2 \cos 2\beta + M_U M_U^\dagger,$$

$$M_{D_L}^2 = m_{Q_L}^2 - \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_W \right) m_Z^2 \cos 2\beta + M_D M_D^\dagger.$$  \hfill (A1)

Here, $m_{Q_L}^2$ is the $2 \times 2$ Hermitian matrix of soft supersymmetry-breaking terms. It does not break SU(2)$_L$ and consequently it is common to $M_{U_L}^2$ and $M_{D_L}^2$. The contribution that is proportional to $m_Z^2$ does break SU(2)$_L$, but it respects the flavor SU(2)$_Q$ symmetry. Finally, the contributions that are proportional to the quark mass matrices $M_U$ and $M_D$ break both SU(2)$_L$ and SU(2)$_Q$. Assuming that the mass scale of squark doublets is in the range of 300 GeV to 1 TeV, the contributions from $m_{Q_L}^2$ dominate over the second terms by order 10–100 and over the third terms by order $10^5$ – $10^6$. This situation leads to the following consequences:

(1) The average squark mass $m_{Q_L}$ is the same for left-handed up and down squarks to an accuracy that is better than $O(0.1)$.

(2) The mass-squared difference between the first two squark-doublet generations $\Delta m_{Q_L}^2$ is the same in the up and down sectors to an accuracy that is better than $O(10^{-2})$.

(3) If the splitting between the diagonal elements of $m_{Q_L}$ is larger than $O(m_Z^2)$, then the mixing angles between the first two left-handed quark-squark generation fulfill, to an accuracy that is better than $O(10^{-2})$, the following relation:

$$\sin \theta_{u_L} - \sin \theta_{d_L} = \sin \theta_c,$$  \hfill (A2)

where $\sin \theta_c = 0.23$ is the Cabibbo angle.

Thus, the constraints from $K$–$ar{K}$ mixing (assuming that the relevant phase in the supersymmetric mixing matrix is $\approx 0.1$) and from $D$–$ar{D}$ mixing read as follows (we take the gluino mass to be comparable to the squark mass):

$$\frac{1}{m_{Q_L}} \frac{\Delta m_{Q_L}^2}{m_{Q_L}^2} \sin \theta_{d_L} \leq 0.01,$$

$$\frac{1}{m_{Q_L}} \frac{\Delta m_{Q_L}^2}{m_{Q_L}^2} \sin \theta_{u_L} \leq 0.10.$$  \hfill (A3)

If we assume that these squarks are within the reach of the LHC, $m_{Q_L} \leq 1$ TeV, and that there is no degeneracy at all between the first two generations of squark doublets, $\Delta m_{Q_L}^2/m_{Q_L}^2 \sim 1$, then the supersymmetric flavor suppression must come entirely from alignment,

$$|\sin \theta_{u_L}| \leq 0.01, \hspace{1cm} |\sin \theta_{d_L}| \leq 0.1.$$  \hfill (A4)

Such a situation is, however, inconsistent with the constraint of Eq. (A2). We conclude that if the first two generations of squark doublets are lighter than TeV, they must be approximately degenerate. The minimal level of degeneracy can be derived by setting $m_{Q_L} = 1$ TeV, $|\sin \theta_{d_L}| = 0.02$, and $|\sin \theta_{u_L}| = 0.21$:

$$\frac{m_{Q_L}^2 - m_{Q_L}^2}{m_{Q_L}^2 + m_{Q_L}^2} \approx 0.12.$$  \hfill (A5)

Hence our first point above: we know from experiment that there must be some level of degeneracy in the squark sector.\footnote{Our argument holds barring accidental, fine-tuned cancellations between various independent supersymmetric contributions to the mixing, or between the supersymmetric and SM contributions.}
The second point has to do with the quantitative strength of the bound given in Eq. (A5). Even if the squark spectrum is entirely nondegenerate at a high scale, where supersymmetry breaking is mediated to the supersymmetric SM, it is expected to be approximately degenerate at low energy. The reason is that renormalization group evolution (RGE) introduces a universal contribution to the squark mass-squared that is of order $7m^2_w$. Thus, if at the high scale the squark masses are comparable to the gluino masses (and, indeed, they are in the GMSB framework), then squark degeneracy at the level of 10% is unavoidable at low energy. Since this is the presently required level of degeneracy, it is quite possible that gravity-mediated contributions are comparable to (or even larger than) the gauge-mediated ones [49]. In this context, future experimental information on the size of squark mass splittings will be very significant, as it can distinguish between RGE and flavor-blind mediation as the source of squark degeneracy.

The large universal contribution to squark masses is related to the large QCD coupling. Leptons, however, have no strong interactions. Since they experience only flavor-blind mediation as the source of squark degeneracy.

The conclusion is that there cannot be a direct, model-independent argument that suppression of supersymmetric lepton flavor violation cannot come entirely from alignment. An indirect argument may arise in the future, if the $D$-$\bar{D}$ mixing constraint becomes significantly stronger, thus requiring squark degeneracy that is substantially stronger than that given in Eq. (A5). In such a case, when it will become unlikely that the degeneracy is a consequence of RGE only, a flavor-blind mechanism to mediate supersymmetry breaking will be required, suggesting that sleptons are also quasidegenerate.