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### Publication Date

1965-11-22

UCRL-16533

# University of California Ernest O. Lawrence Radiation Laboratory

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UCRL-16533

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory  
Berkeley, California

AEC Contract No. W-7405-eng-48

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ABSTRACT

We consider the effect of coupling between photon-meson channels and strong-interaction channels on the electromagnetic mass-splitting of the  $\rho$ -meson multiplet. The result is that no drastic effect occurs, but rather a further small increase in the mass difference of earlier calculations (namely,  $M(\rho^0) - M(\rho^+) \approx 13 \text{ MeV}$ ).

## INTRODUCTION

An estimate of the electromagnetic mass-splitting of the vector meson,<sup>1,2</sup> which we denote by  $\delta(\rho) \equiv M^2(\rho^+) - M^2(\rho^0)$ , can be obtained by examining electromagnetic perturbations<sup>2</sup> of the S-matrix dynamical equations for the  $\rho$  (i.e., "bootstrap" equations in popular parlance). In this note we consider an additional perturbation not previously included: the coupling of Compton-scattering and meson-photoproduction scattering channels to the basic strong-interaction channel. For the  $\rho$  we assume the simplest dynamical model which might be meaningful; the  $\rho$  is described as a  $\pi\pi$  scattering resonance.

Surprisingly, the result is that the  $\pi\pi \leftrightarrow \pi\gamma$  channel has no effect, but other photoproduction channels tend to increase the magnitude of the predicted mass difference [which was  $M(\rho^0) - M(\rho^+) \approx 13$  MeV in references 1,2] by an estimated 1 or 2 MeV.

## 1. GENERAL FEATURES

The structure of the scattering T-matrix is illustrated in Fig. 1. We now assume that this partial wave amplitude has been obtained in the usual form  $ND^{-1}$  ( $N, D$  are matrices) as discussed in reference 2. We will write  $N = \bar{N} + n$ , where  $\bar{N}$  is only the original non-EM matrix (here  $\bar{N}_{ij} = \delta_{i=1} \delta_{j=1} \times N_{\pi\pi}$ ) and  $n$  is due to the new EM channels. It should be noted that if  $N_{\pi\pi}$  is assumed due to a "force" represented by a single pole, and if we subtract  $D$  at this pole,<sup>2</sup> then we obtain for  $i, j \neq 1$

$$n_{ij} = b_{ij}, \quad (1)$$

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where  $b_{ij}$  is a function possessing the exact left-hand cut of  $t_{ij}$ . Here we take  $b_{ij}$  equal to the Born amplitude for particle exchanges in the  $i \rightarrow j$  process. Thus,  $b_{ij}$  does not include the direct channel  $\rho$  pole, which is a consequence of the coupling to the  $\pi\pi$  channel. It is similarly found that if  $i = 1$ ,  $j \neq 1$ , then (1) is still true. However, if  $j = 1$ ,  $i \neq 1$ , then

$$n_{ij} = b_{ij} + \text{small correction of } O(e^2) \quad (2)$$

which is a consequence of the integral equation<sup>2</sup> for  $N$ . By definition,  $b_{11} = 0 = n_{11}$ . We shall henceforth use the approximation  $n_{ij} = b_{ij}$ ; the above discussion indicates that this is better justified than the usual determinantal approximation for  $N$ .

We next consider the structure of  $D$ , where

$$D \equiv 1 - \int (\text{phase space, etc.}) \times N. \quad (3)$$

Clearly,  $D_{1j}$  and  $D_{j1}$ ,  $j \neq 1$ , are of order  $e$  [we use the notation  $O(e)$  hereafter]. Note that  $D_{1j} \neq D_{j1}$  because of different ranges of integration, but the two quantities will have the same sign. We also observe that

$$D_{jj, j \neq 1} = 1 - O(e^2). \quad (4)$$

The location of the  $\rho$  resonant state is given by

$$\det |\text{Re } D| = 0. \quad (5)$$

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Comparing this statement for the two-charge states  $(+,0)$  of the  $\rho$ , we obtain

$$\begin{aligned} \Delta \equiv & \det \{ \text{Re } D[s = M^2(\rho^+) , \text{ all parameters of } \rho^+ \text{ channel}] \} \\ & - \det \{ \text{Re } D[s = M^2(\rho^0) , \text{ all parameters of } \rho^0 \text{ channel}] \} = 0, \end{aligned} \quad (6)$$

where  $s$  = square of CM energy. With the above properties of  $D$ , and using

$$D_{\pi\pi}(s = M_\rho^2) = 0 + O(e^2), \quad (5a)$$

we obtain, from (6), keeping only terms of order  $e^2$ ,

$$\begin{aligned} \Delta \equiv & (\delta D_{\pi\pi} \text{ as in reference 2, ignoring other channels of the present study}) \\ & - \sum_{i \neq 1} D_{1i} D_{i1} (\rho^+ \text{ channel} - \rho^0 \text{ channel}) \\ \equiv & 0. \end{aligned} \quad (7)$$

If we define  $\mu$  = pion mass, and

$$X = \left. \frac{\partial D_{\pi\pi}}{\partial s} \right|_{M^2(\rho)} + \frac{\partial D_{\pi\pi}}{\partial [M^2(\text{exchanged } \rho)]}, \quad (8a)$$

then from reference 2 we obtain

$$\delta D_{\pi\pi} \equiv X \left[ \frac{M^2}{2\mu^2} \delta(\mu) + \delta(\rho) \right] + F, \quad (8b)$$



where  $F$  is due to photon-exchange forces and is positive. We ignore small coupling shifts here.<sup>2</sup> Thus

$$\delta(\rho) + \frac{M^2}{2\mu^2} \delta(\mu) = \chi^{-1} \left[ -F + \sum_{i=1} D_{11} D_{11} \begin{matrix} + \\ 0 \end{matrix} \right]. \quad (9)$$

## 2. DETAILS OF "PHOTON-CHANNEL" EFFECTS

In this section we consider in detail the effects of the transitions  $\pi\pi \rightarrow (\pi\gamma, \eta\gamma, \rho\gamma)$ , i.e., "non-dubious" two-body photon channels available with appropriate quantum numbers, as well as channels involving scalar and high-spin mesons which are currently not well known experimentally.

First we consider the  $\pi\pi \rightarrow \pi\gamma$  transition. We are only interested in exchange forces, which mean here the exchange of a  $G = +$  entity which then couples with a  $G = -$  pion and a photon. Now, we can think of separate isovector and isoscalar photon couplings; they have the properties  $[I = 0, G = -]$  and  $[I = 1, G = +]$ . [This fact is familiar from discussions of nucleon form factors dominated by  $\gamma \rightarrow \omega$  (isoscalar) and  $\gamma \rightarrow \rho$  (isovector) transitions.] In the present case, then, only  $I = 0$  coupling is permitted, and this gives equal exchange forces in each charge channel (i.e.,  $+$  or  $0$ ). Thus we obtain the unexpected result that the  $\pi\pi - \pi\gamma$  transitions contribute no  $O(e^2)$  effect. For the dubious reader, this argument is concretely illustrated in Fig. 2, for the case of  $\rho$  exchange.

We next consider  $\pi\pi \rightarrow \eta\gamma$ . It is trivial to remark that this occurs only in the  $\rho^0$  channel. Similarly,  $C$  invariance only permits

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$\gamma\rho$  in the  $\rho^+$  channel. We thus find photoproduction contributions to the bracket in Eq. (9) to be as follows:

$$-(F > 0) + (\pi\pi \rightarrow \gamma\rho; > 0) - (\pi\pi \rightarrow \gamma\eta; > 0) . \quad (10)$$

In addition, if there is an isoscalar,  $G = +$ , scalar particle (the so-called  $\sigma$ ), then it contributes another negative term to Eq. (10). Similarly, the  $\chi^0$  (959 MeV) contributes a negative term,  $\gamma\phi$  and  $\gamma\omega$  are C forbidden, and  $\gamma f^0$  contributes a negative term. Because the  $A_2$  has<sup>4</sup>  $G = -$ , the same argument as for  $\pi\gamma$  implies no  $\gamma A_2$  contribution. It would thus appear that negative contributions probably are preponderant<sup>5</sup> in (10).

In reference 2, it is shown that  $\chi > 0$  and  $M(\rho^0) - M(\rho^+) \approx 13$  MeV. The present considerations therefore imply a mass difference that is even larger,<sup>6</sup> by as much as 1 or 2 MeV. The significance of this study is therefore mainly in the verification that no drastic effect is expected from the meson-photon channels. Nevertheless, the effects are comparable in magnitude to those of photon exchanges in the strong-interaction channels<sup>7</sup> (but are of the same sign, it would appear from the above discussion).

#### ACKNOWLEDGMENTS

The author thanks Dr. Boris Kayser for pointing out the relevance of photon-meson channels and Dr. Fred Gilman for a conversation on the subject.

He also thanks Dr. David Judd for his hospitality in the Theoretical Group at the Lawrence Radiation Laboratory.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

## FOOTNOTES AND REFERENCES

1. S. Bose, Phys. Rev. 140, B349 (1965).
2. D. S. Beder, UCRL-16465, submitted to Nuovo Cimento.
3. F. Gilman, Ph.D. Thesis, Princeton (1965) (unpublished).
4. A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz and Matt Roos, UCRL-8030, Part I (August 1965).
5. It should be noted that the  $f^0$  couples quite strongly to  $\pi\pi$ , and furthermore there are more independent helicity states of  $f^0\gamma$  than of  $\rho\gamma$ , each contributing to Eq. (9). Thus, despite the higher  $f^0\gamma$  threshold, one justifiably anticipates considerable cancellation between  $\rho\gamma$  and  $f^0\gamma$  terms. Also, if there is a scalar meson (" $\sigma$ "), one can estimate the Born amplitude for  $\pi\pi \rightarrow \sigma\gamma$ . (This might also represent almost-resonant S-wave  $\pi\pi\gamma$  states.) It turns out that the  $\sigma\gamma$  amplitude is appreciably larger than the  $\rho\gamma$  amplitude, further substantiating our claim that negative terms predominate in Eq. (9).
6. One can estimate that

$$D_{1j} \approx \frac{1}{G^2(\text{strong})} \times \left| D_{\pi\pi} - 1 \right|$$

$$\approx \frac{1}{G^2(\text{strong})}, \text{ at } s = M^2(\rho).$$

Therefore the corrections are of order  $\frac{(eG)^2}{G^4} \approx \frac{e^2}{G^2}$

$$\approx 0.006.$$

## 6. (Cont.)

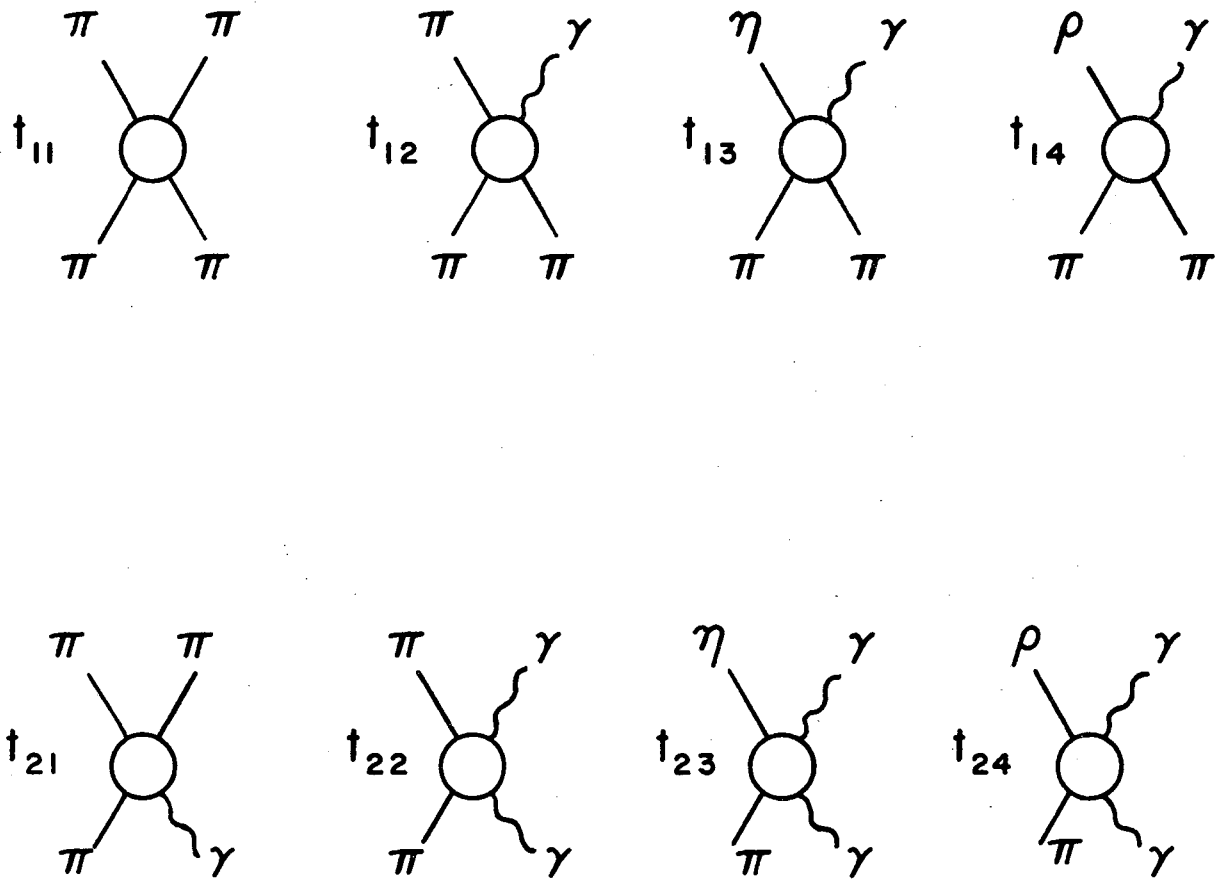
With the numbers of reference 2, this translates into an additional 1 or 2 MeV mass shift at most.

7. In reference 2, photon-exchange contributed about 2 MeV to the 13 MeV mass difference.

## FIGURE CAPTIONS

Fig. 1. T-matrix including "X- $\gamma$ " channels.

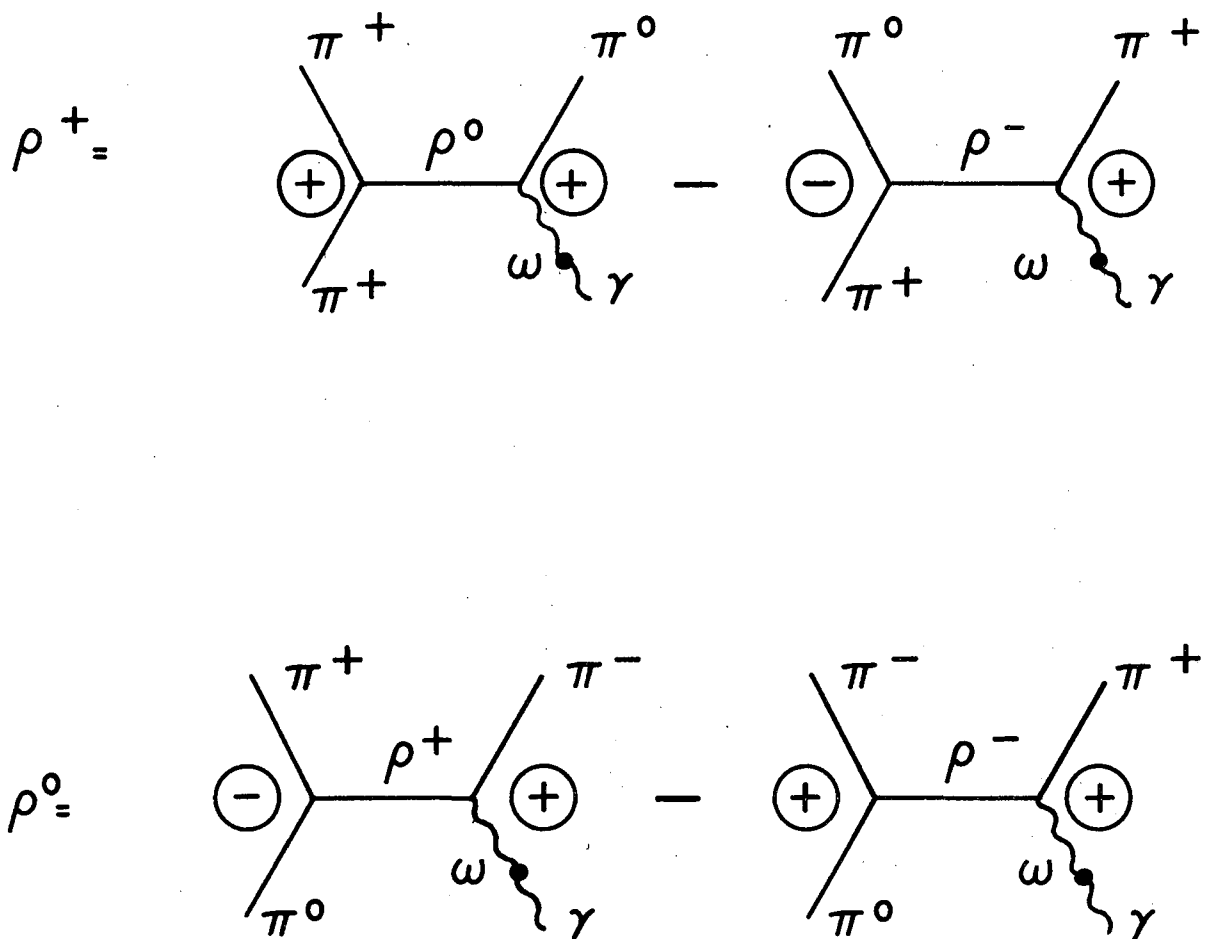
Fig. 2.  $\pi\pi \rightarrow \pi\gamma$  transitions via  $\rho$  exchange. The  $\gamma$  vertices are dominated by virtual vector states and the  $\pm$  signs indicate the signs of the relevant coupling constants.



etc.

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Fig. 1



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Fig. 2



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