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ABSTRACT

We consider the effect of coupling between photon-meson channels and strong-interaction channels on the electromagnetic mass-splitting of the ρ-meson multiplet. The result is that no drastic effect occurs, but rather a further small increase in the mass difference of earlier calculations (namely, $M(\rho^0) - M(\rho^+) \approx 13$ MeV).

INTRODUCTION

An estimate of the electromagnetic mass-splitting of the vector meson,[1,2] which we denote by $\delta(\rho) \equiv M^2(\rho^+) - M^2(\rho^0)$, can be obtained by examining electromagnetic perturbations[2] of the S-matrix dynamical equations for the ρ (i.e., "bootstrap" equations in popular parlance). In this note we consider an additional perturbation not previously included: the coupling of Compton-scattering and meson-photoproduction scattering channels to the basic strong-interaction channel. For the ρ we assume the simplest dynamical model which might be meaningful; the ρ is described as a π-π scattering resonance.

Surprisingly, the result is that the $\pi\pi \leftrightarrow \pi\gamma$ channel has no effect, but other photoproduction channels tend to increase the magnitude of the predicted mass difference [which was $M(\rho^0) - M(\rho^+) \approx 13$ MeV in references 1,2] by an estimated 1 or 2 MeV.

1. GENERAL FEATURES

The structure of the scattering T-matrix is illustrated in Fig. 1. We now assume that this partial wave amplitude has been obtained in the usual form ND^{-1} (N,D are matrices) as discussed in reference 2. We will write $N = \overline{N} + n$, where $\overline{N}$ is only the original non-EM matrix (here $\overline{N}_{ij} = \delta_{i=1}, \quad \delta_{j=1} \times N_{\pi\pi}$) and n is due to the new EM channels. It should be noted that if $N_{\pi\pi}$ is assumed due to a "force" represented by a single pole, and if we subtract D at this pole,[2] then we obtain for $i,j \neq 1$

$$n_{ij} = b_{ij} , \tag{1}$$

where b_{ij} is a function possessing the exact left-hand cut of t_{ij}. Here we take b_{ij} equal to the Born amplitude for particle exchanges in the $i \to j$ process. Thus, b_{ij} does <u>not</u> include the direct channel ρ pole, which is a consequence of the coupling to the $\pi\pi$ channel. It is similarly found that if $i = 1$, $j \neq 1$, then (1) is still true. However, if $j = 1$, $i \neq 1$, then

$$n_{ij} = b_{ij} + \text{small correction of } O(e^2) \qquad (2)$$

which is a consequence of the integral equation[2] for N. By definition, $b_{11} = 0 = n_{11}$. We shall henceforth use the approximation $n_{ij} = b_{ij}$; the above discussion indicates that this is better justified than the usual determinantal approximation for N.

We next consider the structure of D, where

$$D \equiv 1 - \int (\text{phase space, etc.}) \times N . \qquad (3)$$

Clearly, D_{1j} and D_{j1}, $j \neq 1$, are of order e [we use the notation O(e) hereafter]. Note that $D_{1j} \neq D_{j1}$ because of different ranges of integration, but the two quantities will have the same sign. We also observe that

$$D_{jj,j\neq 1} = 1 - O(e^2) . \qquad (4)$$

The location of the ρ resonant state is given by

$$\det |\mathrm{Re}\, D| = 0 . \qquad (5)$$

Comparing this statement for the two-charge states $(+,o)$ of the ρ, we obtain

$$\Delta \equiv \det \{\mathrm{Re}\, D[s = M^2(\rho^+)\ ,\ \text{all parameters of } \rho^+ \text{ channel}]\} - \det\{\mathrm{Re}\, D[D(s = M^2(\rho^o),\ \text{all parameters of } \rho^o \text{ channel}]\} = 0\ , \tag{6}$$

where s = square of CM energy. With the above properties of D, and using

$$D_{\pi\pi}(s = M_\rho{}^2) = 0 + O(e^2)\ , \tag{5a}$$

we obtain, from (6), keeping only terms of order e^2,

$$\Delta \equiv (\delta D_{\pi\pi} \text{ as in reference 2, ignoring other channels of the present study}) - \sum_{i \neq 1} D_{1i}\, D_{i1}\, (\rho^+ \text{ channel} - \rho^o \text{ channel}) \equiv 0\ . \tag{7}$$

If we define μ = pion mass, and

$$X = \left.\frac{\partial D_{\pi\pi}}{\partial s}\right|_{M^2(\rho)} + \frac{\partial D_{\pi\pi}}{\partial[M^2(\text{exchanged } \rho)]}\ , \tag{8a}$$

then from reference 2 we obtain

$$\delta D_{\pi\pi} \equiv X\left[\frac{M^2}{2\mu^2}\ \delta(\mu) + \delta(\rho)\right] + F\ , \tag{8b}$$

where F is due to photon-exchange forces and is positive. We ignore small coupling shifts here.[2] Thus

$$\delta(\rho) + \frac{M^2}{2\mu^2}\delta(\mu) = \chi^{-1}\left[-F + \sum_{i=1} D_{1i}\, D_{i1}\Big|_o^+\right] . \qquad (9)$$

2. DETAILS OF "PHOTON-CHANNEL" EFFECTS

In this section we consider in detail the effects of the transitions $\pi\pi \rightarrow (\pi\gamma,\ \eta\gamma,\ \rho\gamma)$, i.e., "non-dubious" two-body photon channels available with appropriate quantum numbers, as well as channels involving scalar and high-spin mesons which are currently not well known experimentally.

First we consider the $\pi\pi \rightarrow \pi\gamma$ transition. We are only interested in exchange forces, which mean here the exchange of a $G = +$ entity which then couples with a $G = -$pion and a photon. Now, we can think of separate isovector and isoscalar photon couplings; they have the properties $[I = 0,\ G = -]$ and $[I = 1,\ G = +]$. [This fact is familiar from discussions of nucleon form factors dominated by $\gamma \rightarrow \omega$ (isoscalar) and $\gamma \rightarrow \rho$ (isovector) transitions.] In the present case, then, only $I = 0$ coupling is permitted, and this gives equal exchange forces in each charge channel (i.e., + or o). Thus we obtain the unexpected result that the $\pi\pi - \pi\gamma$ transitions contribute no $O(e^2)$ effect. For the dubious reader, this argument is concretely illustrated in Fig. 2, for the case of ρ exchange.

We next consider $\pi\pi \rightarrow \eta\gamma$. It is trivial to remark that this occurs only in the ρ^o channel. Similarly, C invariance only permits

$\gamma\rho$ in the ρ^+ channel. We thus find photoproduction contributions to the bracket in Eq. (9) to be as follows:

$$-(F > 0) + (\pi\pi \to \gamma\rho; > 0) - (\pi\pi \to \gamma\eta; > 0) \, . \tag{10}$$

In addition, if there is an isoscalar, $G = +$, scalar particle (the so-called σ), then it contributes another negative term to Eq. (10). Similarly, the X^0 (959 MeV) contributes a negative term, $\gamma\phi$ and $\gamma\omega$ are C forbidden, and γf^0 contributes a negative term. Because the A_2 has[4] $G = -$, the same argument as for $\pi\gamma$ implies no γA_2 contribution. It would thus appear that negative contributions probably are preponderant[5] in (10).

In reference 2, it is shown that $X > 0$ and $M(\rho^0) - M(\rho^+) \approx 13$ MeV. The present considerations therefore imply a mass difference that is even larger,[6] by as much as 1 or 2 MeV. The significance of this study is therefore mainly in the verification that no drastic effect is expected from the meson-photon channels. Nevertheless, the effects are comparable in magnitude to those of photon exchanges in the strong-interaction channels[7] (but are of the same sign, it would appear from the above discussion).

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FOOTNOTES AND REFERENCES

1. S. Bose, Phys. Rev. 140, B349 (1965).

2. D. S. Beder, UCRL-16465, submitted to Nuovo Cimento.

3. F. Gilman, Ph.D. Thesis, Princeton (1965) (unpublished).

4. A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz and Matt Roos, UCRL-8030, Part I (August 1965).

5. It should be noted that the f^0 couples quite strongly to $\pi\pi$, and furthermore there are more independent helicity states of $f^0\gamma$ than of $\rho\gamma$, each contributing to Eq. (9). Thus, despite the higher $f^0\gamma$ threshold, one justifiably anticipates considerable cancellation between $\rho\gamma$ and $f^0\gamma$ terms. Also, if there is a scalar meson ("σ"), one can estimate the Born amplitude for $\pi\pi \to \sigma\gamma$. (This might also represent almost-resonant S-wave $\pi\pi\gamma$ states.) It turns out that the $\sigma\gamma$ amplitude is appreciably larger than the $\rho\gamma$ amplitude, further substantiating our claim that negative terms predominate in Eq. (9).

6. One can estimate that

$$D_{ij} \approx \frac{1}{G^2(\text{strong})} \times \left| D_{\pi\pi} - 1 \right|$$

$$\approx \frac{1}{G^2(\text{strong})}, \text{ at } \quad s = M^2(\rho) \; .$$

Therefore the corrections are of order $\dfrac{(eG)^2}{G^4} \approx \dfrac{e^2}{G^2}$

$$\approx 0.006 \; .$$

6. (Cont.)

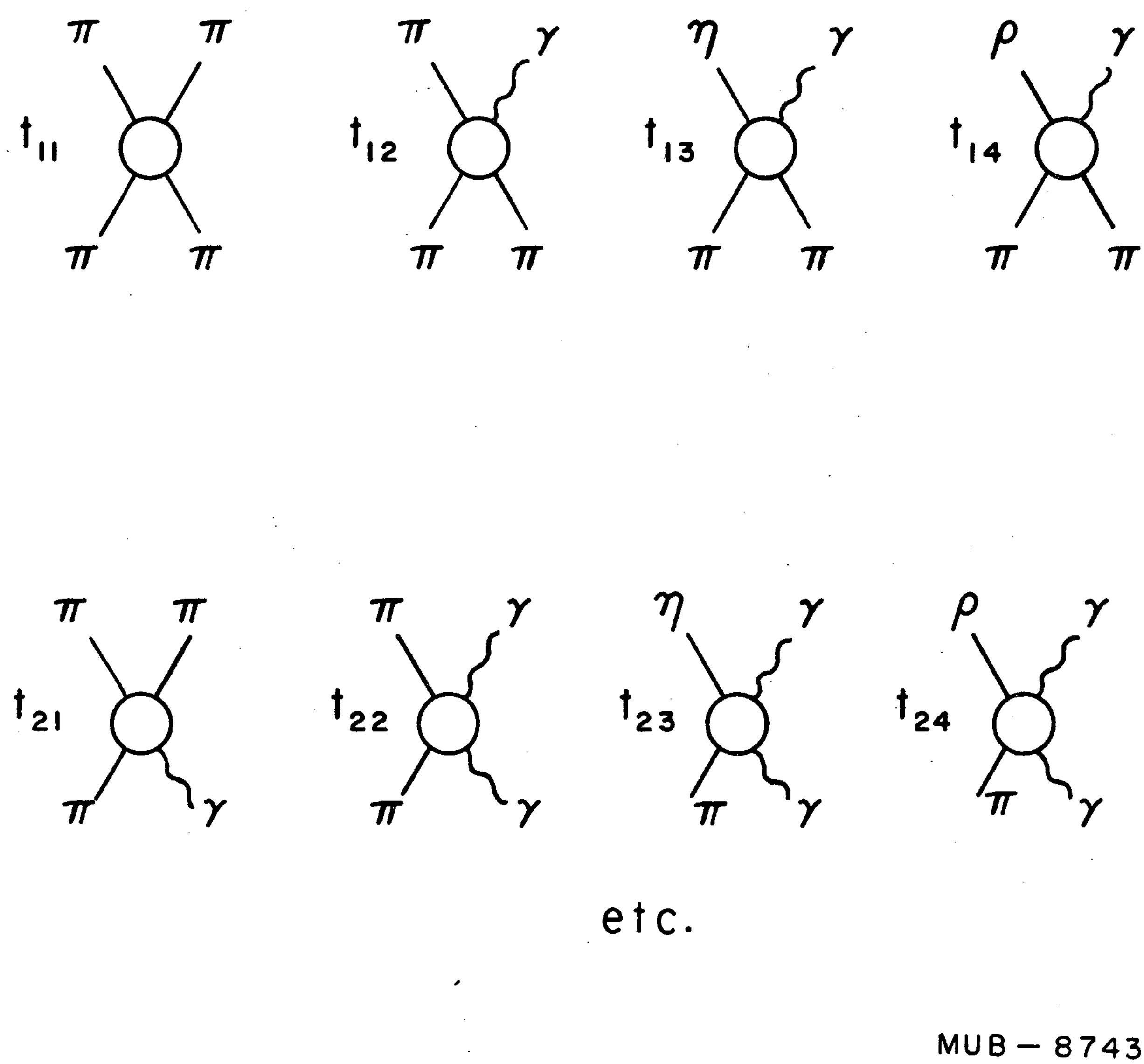
With the numbers of reference 2, this translates into an additional 1 or 2 MeV mass shift at most.

7. In reference 2, photon-exchange contributed about 2 MeV to the 13 MeV mass difference.

FIGURE CAPTIONS

Fig. 1. T-matrix including "X-γ" channels.

Fig. 2. $\pi\pi \to \pi\gamma$ transitions via ρ exchange. The γ vertices are dominated by virtual vector states and the $\pm$ signs indicate the signs of the relevant coupling constants.



π π t_{11} π π
π γ t_{12} π π
η γ t_{13} π π
ρ γ t_{14} π π
π π t_{21} π γ
π γ t_{22} π γ
η γ t_{23} π γ
ρ γ t_{24} π γ
etc.
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Fig. 1

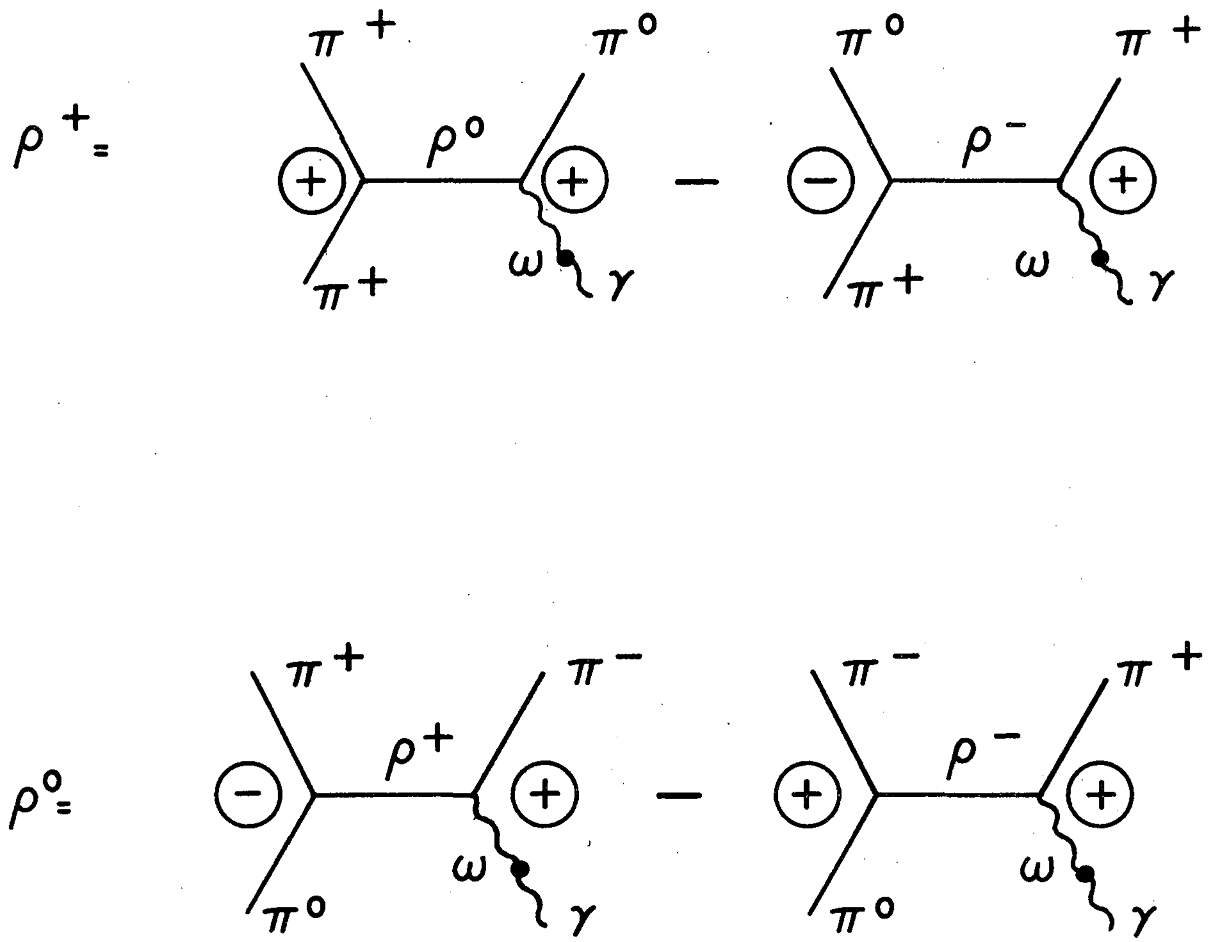

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Fig. 2