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What's the Option?

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Abstract: Global warming, alterations of ecosystems, and sunk investments all imply irreversible changes with uncertain future costs and benefits. Two concepts measure how this combination of uncertainty and irreversibility changes the value of preserving an ecosystem or postponing an investment. First, the environmental and resource economics literature developed the Arrow-Fisher-Hanemann-Henry quasi-option value. Second, the real options literature developed the Dixit-Pindyck option value. This paper clarifies the precise differences between the two approaches in a simple two period model. We explain that the quasi-option value captures the value of learning conditional on preservation, while the Dixit-Pindyck option value captures the net value of preservation under learning. We show how either of the two concepts alters the common net present value decision rule. We illustrate similarities, differences, and the decision rules in two instructive examples.

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1 Introduction

Climate change propels irreversible change to a new order of magnitude: global warming can annihilate ice sheets, coral reefs, the Amazon, and islands, and it can permanently alter ocean currents, ecosystems, and human habitats. On a smaller scale, man's altering of his environment has long included irreversible changes, including species extinction as well as the spreading of invasive species. Irreversible changes pose a challenge to conventional benefit cost analysis when the consequences of these changes are not fully understood and cannot be priced with certainty at the time of action. In the presence of irreversibility, postponement or omission of a project can be optimal even if the expected net present value is positive.

Arrow & Fisher (1974), Henry (1974) and Hanemann (1989) realized the shortcoming of conventional cost benefit analysis and developed the *Arrow-Fisher-Hanemann-Henry quasi-option value*, which attaches value to environmental preservation in the presence of irreversibility. This option value in environmental and resource economics developed independently from a related concept in the finance literature, until Dixit & Pindyck (1994) spread the work on real options analysis to a broad audience. Fisher (2000) tried to establish the equivalence between Dixit-Pindyck's option value and the quasi-option value. However, Mensink & Requate (2005) clarified that the two option values differ.

The current paper demonstrates that the quasi-option value captures the value of learning conditional on preservation, while the Dixit-Pindyck option value captures the (net) value of preservation under learning. We establish the general relation between the two option values, extending Mensink & Requate's (2005) result regarding their difference. Getting at the interaction between option values and decision rules, we show how each of the two option values changes the common net present value rule for project evaluation. Combing general results and instructive illustrations, the paper also sets out to introduce the theory of option value in environmental and resource economics to a broader audience.

2 Irreversibility and Learning

This section introduces the setting and the optimal decision anticipating learning under an irreversibility constraint. Subsequently, it defines different present values from undertaking or postponing the project in the presence or absence of learning. These present values will later help to define, interpret, and relate the option values.

2.1 The Setting

We need two periods for deriving our insights on the option and the quasi-option value. In the first period, the decision maker faces the discrete decision to either preserve or alter an ecosystem. We denote preservation in the first period by $x_1 = 0$ and alteration by $x_1 = 1$. If the decision maker preserves the ecosystem in the first period, he again has the option to either preserve the system ($x_2 = 0$) or to alter it ($x_2 = 1$) in the second period. However, if the decision maker decides to alter the system in the first period, then this change is irreversible ($x_1 = 1 \Rightarrow x_2 = 1$). In the first period, the decision maker is uncertain about the costs and benefits of his actions. By the beginning of the second period, this uncertainty has resolved. Alternatively, the decision variable x can specify a sunk investment determining uncertain future payoffs. In this case, the decision maker can either invest irreversibly in the first period ($x_1 = 1 \Rightarrow x_2 = 1$) or, if he does not invest in the first period, he can still do so in the second period.

The function $v(x_1, x_2, \tilde{\theta}) = u_1(x_1) + u_2(x_1, x_2, \tilde{\theta})$ characterizes welfare. The random variable $\tilde{\theta}$ represents the uncertain component of the problem, i.e., the value of the ecosystem, the value of the investment, the importance of particular climate functions, and/or the cost of global temperature increase. The true value of $\tilde{\theta}$ is revealed between period 1 and period 2. None of the subsequent analysis relies on the additive separability assumed in the welfare function.[1] Note that the welfare function allows stock effects to carry over

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[1] The reader can literally replace $u_1(x_1) + u_2(x_1, x_2, \tilde{\theta})$ with $v(x_1, x_2, \tilde{\theta})$ in the remaining

from one period to the next. In many applications, including the original definition by Dixit & Pindyck (1994), the utility functions are replaced by monetary payoffs to describe risk neutral firms. Similarly to Mensink & Requate (2005), we extend Dixit & Pindyck's (1994) option value to permit more general welfare effects.

A fully sophisticated decision maker anticipates in the first period that his decision in the second period will be based on better information. Once in the second period, he will $\max_{x_2 \in \{x_1, 1\}} u_2(x_1, x_2, \theta)$ for a given x_1 and a given realization $\tilde{\theta} = \theta$. The irreversibility constraint restricts his second period choice variable to the set $\{x_1, 1\}$: if $x_1 = 1$ the project has been carried out already and cannot be undone. Anticipating the second period action, the first period decision maker optimizes the expected payoff over x_1:

$$\max_{x_1 \in \{0,1\}} \mathbf{E} \max_{x_2 \in \{x_1, 1\}} v(x_1, x_2, \tilde{\theta}) = \max_{x_1 \in \{0,1\}} u_1(x_1) + \mathbf{E} \max_{x_2 \in \{x_1, 1\}} u_2(x_1, x_2, \tilde{\theta}). \quad (1)$$

In summary, the optimal decision first maximizes second period welfare for every possible realization of $\tilde{\theta}$ and x_1, and only then takes expectations and optimizes over the first period choice variable x_1.

2.2 Present Values

To define, interpret, and relate the different option values, we make use of a set of present values. They differ in the amount of information or sophistication that they incorporate. First, we denote the value of preserving the ecosystem in the first period to a decision maker who anticipates learning ("l") by

$$V^l(0) = u_1(0) + \mathbf{E} \max_{x_2 \in \{0,1\}} u_2(0, x_2, \tilde{\theta}).$$

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equations. The general form incorporates that currently experienced welfare can depend on the future welfare distribution.

We denote his value of developing the ecosystem in the first period by

$$V^l(1) = u_1(1) + \mathbf{E} \max_{x_2 \in \{1\}} u_2(1, x_2, \tilde{\theta}) = u_1(1) + \mathbf{E}\, u_2(1, 1, \tilde{\theta}).$$

Second, we spell out the present values for a decision maker who does not anticipate learning about the realization of the random variable before choosing x_2. Preserving the ecosystem in the first period then yields the present value

$$V^p(0) = u_1(0) + \max_{x_2 \in \{0,1\}} \mathbf{E}\, u_2(0, x_2, \tilde{\theta}),$$

while altering the system implies

$$V^p(1) = u_1(1) + \mathbf{E}\, u_2(1, 1, \tilde{\theta}).$$

In these definitions, the index "p" denotes the *p*ossibility of *p*ostponing the decision on ecosystem development (or investment).

Third, the decision maker considers only the possibility of developing (or investing) *n*ow or *n*ever ("n"). In this case we have

$$V^n(0) = u_1(0) + \mathbf{E}\, u_2(0, 0, \tilde{\theta}),$$

for the value of future welfare if not developing the valley (not investing), and

$$V^n(1) = u_1(1) + \mathbf{E}\, u_2(1, 1, \tilde{\theta}),$$

describing the value of developing the valley (investing) in the first period. Alternatively, we can think about the last setting as describing a *n*aive decision maker who uses the simplest form of a net present value calculation to decide whether to invest today or not to invest today.

We note the trivial relationship that $V^n(1) = V^p(1) = V^l(1)$ because a first period development implies that there is no more choice to be taken in the second period. Making use of the present values defined above, the

decision rule (1) tells us that altering the ecosystem or undertaking the investment in the present period is optimal if and only if

$$V^l(1) - V^l(0) > 0 \ . \tag{2}$$

3   The option values

This section introduces the Arrow-Fisher-Hanemann-Henry quasi-option value and the Dixit-Pindyck option value. We elaborate their interpretations and state some preliminary observations regarding their informational content.

3.1   Learning Conditional on Postponement: The Quasi-Option Value

The most common evaluation approach for investments, or ecosystem impacts, is the expected net present value (NPV) approach. It takes the difference between the expected aggregate costs and benefits in present value terms.[2] The approach stipulates that we carry out the project if the net present value is positive. In our notation, this decision rule suggests investing if, and only if,

$$NPV \equiv V^n(1) - V^n(0) \geq 0 \,.$$

Arrow & Fisher (1974), Henry (1974), and Hanemann (1989) point out that this naive decision rule does not optimize welfare in the case of irreversibility and learning. They show the existence of an additional value when preserving an ecosystem, or postponing an investment, a value not captured in the net present value calculation. This additional value received the name quasi-option value (QOV).

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[2]Making u_1 a money metric utility in the analyzed changes trivially translates the welfare units into consumption units, if desired.

In our setting, the quasi-option value is captured by the difference between the value from preserving the ecosystem under learning, and the value from preserving the ecosystem without anticipating new information:

$$QOV = V^l(0) - V^p(0) \ . \tag{3}$$

Our notation fleshes out that the quasi-option value specifies the *value of learning conditional on postponement*. We evaluate both present values in the defining equation (3) for a postponement of the project (i.e. zero). Hence, the QOV captures a difference that is conditional on postponement. The first term, $V^l(0)$, takes into account the anticipation of learning, while the second term, $V^p(0)$, only incorporates the option to postpone, but does not take into account that future decisions can incorporate new information. Observe that the quasi-option value is always (weakly) positive: the ability to (or anticipation of) learning cannot reduce the present value.

We make the following observation. Considering a postponement of the project, the full value of sophisticated decision making is given by the difference $V^l(0) - V^n(0)$. This full value of sophistication can be larger than the mere quasi-option value. We decompose this full value of sophistication conditional on not carrying out the project in the first period as follows

$$\underbrace{V^l(0) - V^n(0)}_{\text{full sophistication}} = \underbrace{V^l(0) - V^p(0)}_{\text{quasi-option value (QOV)}} + \underbrace{V^p(0) - V^n(0)}_{\text{simple option value (SOV)}} \ . \tag{4}$$

The last term will feature prominently in the modified decision rules and the relationship between the quasi-option value and the Dixit-Pindyck option value. We named it the simple option value. It captures the value of the option to postpone the project without incorporating new information. We can paraphrase it as the value of the option to carry out the project in the second period, conditional on not carrying out the project in the first period, in the absence of information flow. The simple option value, like the other option values, is always (weakly) positive: the option to carry out the project in the second period cannot reduce welfare.

3.2   Postponement under Learning:
The Dixit-Pindyck Option Value

A different concept of option value is defined by Dixit & Pindyck (1994), who develop the similarity between investment projects and financial options. This perspective is known as the real options approach. Translated into our slightly more general welfare setting, Dixit & Pindyck (1994, 97) define the value of the option to postpone the project as

$$DPOV \equiv \max\{V^l(1), V^l(0)\} - \max\{V^n(1), V^n(0)\}. \tag{5}$$

The Dixit-Pindyck option value (DPOV) is the maximal value that can be derived from the option to invest now or later (incorporating learning) less the maximal value that can be derived from the possibility to invest now or never. Also, the Dixit-Pindyck option value is always positive.[3]

In the following, we show that the DPOV captures the net value of project postponement conditional on learning. For this purpose, we express the DPOV in terms of the present values defined in section 2.2. The following notation from the option value literature proves useful:

$$[x]_+ = \max\{x, 0\} \quad \text{and} \quad [x]_- = \min\{x, 0\}.$$

This notation allows us to rewrite the Dixit-Pindyck option value as

$$DPOV = V^l(1) + \left[V^l(0) - V^l(1)\right]_+ - \left\{V^n(1) + [V^n(0) - V^n(1)]_+\right\}$$
$$= \left[V^l(0) - V^l(1)\right]_+ - [V^n(0) - V^n(1)]_+. \tag{6}$$

Limiting the value to its positive domain, the $[\cdot]_+$ operation turns a value into an option value: the option to postpone cannot reduce (expected) welfare because it is only an option, not an obligation. The heart of the DPOV is the first term in the equation (6). It states the option value of postponement, given learning in an irreversible environment.

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[3]We know that $V^l(1) = V^n(1)$ and $V^l(0) \geq V^n(0)$.

The second term in equation (6) characterizes the value of the option never to invest. The option to never invest has a positive value if and only if the net present value of the project is negative. The DPOV subtracts this value from the value of the option to postpone under learning. In consequence, the DPOV captures only that part of the option value from postponing that exceeds a potential net present value loss. We can flesh out this intuition by rewriting equation (6) as

$$DPOV = \left[V^l(0) - V^l(1)\right]_+ + [NPV]_- \ . \tag{7}$$

If the net present value is positive, the DPOV characterizes the value of the option to postpone under learning $\left[V^l(0) - V^l(1)\right]_+$. If the net present value is negative, we subtract the expected loss $-[NPV]_-$ from the value of the postponement option, and only capture the collectible value of postponement in the DPOV.[4] Therefore, the DPOV is a *net* value from postponement conditional on learning.

4 Relating Option Values and Decisions

Sections 3.1 and 3.2 defined and explained the different content of the Arrow-Fisher-Hanemann-Henry quasi-option value and the Dixit-Pindyck option value. The first captures the value of learning conditional on postponement, while the second captures the value of postponement under learning. We now proceed to show how these two net present values alter the common net present value decision rule. Subsequently, we state the general relationship between the two different option values, extending a result by Mensink & Requate (2005).

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[4]Note that the first term is always weakly greater than the absolute of $[NPV]_-$.

4.1 Decisions on Project Execution versus Postponement

How do the insights provided by Arrow, Fisher, Hanemann, Henry, Dixit, and Pindyck change the simple, most common decision rule of cost benefit analysis? We now state how either of the two option values modifies the simple net present value rule stating that a project is to be carried out if and only if $NPV > 0$. First, we show how Arrow, Fisher, Hanemann, and Henry's insights on the quasi-option value interact with the net present value in an optimal decision rule.

Proposition 1 (Quasi-Option Value): *The sophisticated decision maker who anticipates learning is*

    i) strictly better off undertaking the project in the present if, and only if,

$$NPV > QOV + SOV \ , \tag{8}$$

    ii) strictly better off postponing the project if, and only if,

$$QOV + SOV > NPV \ ,$$

    iii) and indifferent to the timing otherwise.

A necessary condition for undertaking a project in the present is now that the net present value NPV exceeds the quasi-option value QOV. A necessary and sufficient condition for optimality of present action is that the net present value exceeds the sum of the quasi-option value and the simple option value SOV. The quasi-option value only captures the value of learning conditional on postponement; the complete right hand side of equation (8) captures the full value of sophistication conditional on postponing the project. This full value is composed of the value of learning as well as the mere value of having the option to carry out the project in the second period (as opposed to a now or never decision). The simple option value captures that the opportunity cost of dismissing the project today is not the NPV itself, but rather the

cost of delaying the NPV (without learning). If the (net) payoff from a delayed project and the value of learning by delaying the project exceed the net present value, then the policy maker should postpone the project.

Second, we show how Dixit Pindyck's option value qualifies the net present value rule.

Proposition 2 (Dixit-Pindyck Option Value): *The sophisticated decision maker who anticipates learning is*

    i) better off undertaking the project in the present if

$$NPV > 0 \ and \ DPOV = 0 \ ,$$

    ii) strictly better off postponing the project if

$$DPOV > 0 \ ,$$

    iii) and better off never undertaking the project otherwise.

Using the Dixit-Pindyck option value, a sufficient condition for undertaking the project in the present is that the net present value is strictly positive *and* the $DPOV$ is zero. Whenever the $DPOV$ is strictly positive, the policy maker is better off postponing the project.

We point out two implications of Proposition 2 that easily slips the attention. First, when the Dixit-Pindyck option value is zero, both can be strictly welfare increasing: undertaking the project in the present, and never undertaking the project, i.e., postponing it forever (statement ii is if, not only if). Second, even if the $DPOV$ is zero and the NPV is strictly positive, the policy maker might be as well off from postponing the project as from undertaking the project in the present (statement i is a weakly rather than a strictly better off). This happens in the situation where the value of full sophistication under postponement equals the net present value.

The intuition why the cases in Proposition 2 differ from those in Proposition 1 is the following. Whenever there is an additional value to waiting, then $QOV + SOV > 0$. Whenever this value is larger than the value of undertaking the project in the present, then $QOV + SOV > NPV$. So, quasi-option

and simple option value measure the immediate value from waiting. If, and only if, this value is greater than the value of not waiting, the policy maker should postpone the project. In contrast, the Dixit-Pindyck option value only measures the value from postponement that is due to sophistication *and* collectible. If the value of waiting is strictly dominated by the value of undertaking the project $QOV + SOV < NPV$, the Dixit-Pindyck option value is zero. But also if $QOV + SOV > NPV$, the Dixit-Pindyck option value can be zero if postponing, or better discarding, the project is optimal even for the naive net present value maximize $NPV < 0$.

4.2   The Relationship Between QOV and DPOV

The quasi-option value measures the value of learning under postponement. The Dixit-Pindyck option value measures the collectible net benefit from postponement under learning. The following proposition states the precise relationship between these distinct option values.

Proposition 3: *The Dixit-Pindyck option value (DPOV) and the Arrow-Fisher-Hanemann-Henry quasi-option (QOV) value relate as*

$$DPOV = QOV + SOV - [NPV]_+ - \left[V^l(0) - V^l(1)\right]_- . \qquad (9)$$

We first assume that the value of postponement under learning is positive, implying both $\left[V^l(0) - V^l(1)\right]_- = 0$ and $DPOV > 0$. In addition, let the net present value of the project be zero. Then, the Dixit-Pindyck option value measures jointly $QOV + SOV$, the value from learning conditional on postponement and the value from a postponed implementation in the absence of learning (as opposed to no implementation at all). If the only value of a postponed project stems from learning ($SOV = 0$), then the Dixit-Pindyck and the quasi-option value coincide. See section 5.1 for an example. More interesting is the case where the net present value of the project is strictly positive. Then, the Dixit-Pindyck option value only measures the surplus

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that postponement creates over direct implementation, and we have to subtract the NPV of the project from the value achieved under postponement $QOV + SOV$. If the net present value is negative, then the $DPOV$ does not add this loss from implementing the project in the present to the value of postponement, but only measures the surplus from postponement over dismissing the project.

Second, we assume that the value of postponement under learning is negative, implying $\left[V^l(0) - V^l(1)\right]_- < 0$. Then, even under learning, the best option is to carry out the project straight away (assuming $NPV > 0$). The quasi-option value and the simple option value can still both be positive: conditional on not undertaking the project today, there can still be a positive value from learning and from carrying out the project tomorrow. These values, however, are not large enough to make postponement worthwhile; they don't outweigh the benefit from carrying out the project straight away, $QOV + SOV - NPV < 0$. Then, the option of postponement will be dismissed and the Dixit-Pindyck option value is zero rather than negative: the formula in equation (9) subtracts the negative value of postponement under learning.

We briefly relate equation (9) to a closely related result by Mensink & Requate (2005). The authors derive the relationship $DPOV = QOV + PPV$, where PPV stands for pure postponement value. In our setting, we find $PPV = V^p(0) - V^n(1) = SOV - NPV$. The pure postponement value directly captures the net surplus, in the absence of learning, from carrying out the project in the second period as opposed to carrying it out in the present. As the authors note, the PPV can also be negative and, thus, is not an option value. To derive this most interesting special case of the general relationship (9), Mensink & Requate (2005) assume that the first maximum in equation (5), defining the $DPOV$, is taken on by $V^l(0)$, and that the second maximum in equation (5) is taken on by $V^n(1)$. These assumptions navigates elegantly around the option nature of the DPOV, which introduces

the modification in the general relationship (5).[5]

5 Examples

This section illustrates the different option values and their applications. The first example illustrates how the Arrow-Fisher-Hanemann-Henry quasi-option value makes postponement worthwhile. It also serves as an example where QOV and DPOV coincide. The second example illustrates the case where QOV and DPOV differ.

5.1 Coinciding QOV and DPOV

We assume two possible states of the world. In the state $\tilde{\theta} = \overline{\theta}$, preservation of an ecosystem has a high payoff, e.g. because it performs important climate regulating functions, or because its biodiversity inspired important pharmaceuticals. If $\tilde{\theta} = \underline{\theta}$, preservation has a low payoff. Similarly, an irreversible investment could either result in a successful new technology or a flop. When the project is not carried out, welfare is zero independent of the period and the state of the world: $u_1(0) = u_2(0, 0, \overline{\theta}) = u_2(0, 0, \underline{\theta}) = 0$. If the policy maker decides to undertake the project in the present, he pays an immediate cost, yielding $u_1(1) = -1$. In the high payoff state, he faces the welfare $u_2(1, 1, \overline{\theta}) = 2$; in the low payoff state, he faces the welfare $u_2(1, 1, \overline{\theta}) = 0$. If

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[5]In comparing our setting to Mensink & Requate (2005), note that the NPV abbreviation in their setting does not truly measure the net present value of the project, which is the expected present value difference between carrying out and dismissing the project. Instead, the authors define the NPV to be the maximum of either the absolute welfare when presently carrying out the project, or the absolute welfare when dismissing the project for all time. In consequence, if e.g., the welfare in the baseline without the project is larger than zero, then their NPV is also strictly positive, no matter what the project costs or pays. This peculiarity is of no further relevance in their paper, as they do not analyze how the option values alter the net present value decision rule, but could be potentially confusing if the reader tries to relate their formulas to the corresponding special case of our setting.

the policy maker undertakes the project in the second period, he discounts both upfront cost and future benefits at the discount factor β, implying $u_2(0, 1, \bar{\theta}) = \beta(-1 + 2) = \beta$ and $u_2(0, 1, \underline{\theta}) = -\beta$.[6]

The example implies an expected net present value $NPV = 0$, as well as a simple option value $SOV = 0$: Without learning, also a postponed implementation of the project has a zero expected net value.[7] Without the anticipation of learning, the decision maker would be indifferent between carrying out or postponing the project. However, Arrow-Fisher-Hanemann-Henry teach us that we have to take into account our ability to learn when postponing the project. Their quasi-option value, $QOV = \frac{1}{2}\beta$, captures the value of learning under postponement and, here, tells the policy maker that postponement of the project is strictly better than carrying out the project in the present: $QOV + SOV > NPV$.

The Dixit-Pindyck option value gives us directly the total value of postponement, in a setting with learning. We calculate $DPOV = \frac{1}{2}\beta$ and learn once more that postponement of the project increases welfare. In this example, the value of postponing the project is entirely due to the value of learning. Therefore, Dixit and Pindyck's option value coincides with the quasi-option value of Arrow-Fisher-Hanemann-Henry.

We close the first example with a slight variation. We reduce the welfare in the high payoff state from 2 to $\frac{3}{2}$, creating a negative $NPV = -\frac{1}{4}$.[8] The trivial finding is that $SOV = 0 > NPV = -\frac{1}{2}$ now implies that the quasi-option value insight is no longer needed to realize that dismissing the project in the first period is welfare increasing. Obviously, given $NPV < 0$, the

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[6]Note that u_2 already discounts second period welfare to the present. Carrying out the project in the present, we think of $w^+ = u_2(1, 1, \tilde{\theta})$ as the infinite stream of welfare changes following the upfront investment $w^- = u_1(1)$, both measured in present value welfare units. Undertaking the project only in the second period, the policy maker in the second period faces the welfare change of $u_2(0, 1, \tilde{\theta}) = w^- + w^+$, and these values are discounted to first period welfare using the discount factor β.

[7]We find $V^n(1) = V^p(1) = V^l(1) = 0$, $V^n(0) = V^p(0) = 0$, and $V^l(0) = \frac{1}{2}\beta$.

[8]The reduction of welfare in the high payoff state implies $u_2(1, 1, \bar{\theta}) = \frac{3}{2}$ and simultaneously $u_2(0, 1, \bar{\theta}) = \beta(-1 + \frac{3}{2}) = \frac{1}{2}\beta$.

decision maker does not even have to realize the possibility of postponement in order to take the right decision. More interestingly, we note that the QOV and the DPOV continue to coincide. The value of learning under postponement is now $QOV = \frac{1}{4}\beta$. The gross value of postponement, in a setting with learning, is $V^l(0) - V^l(1) = \frac{1+\beta}{4}$. It is larger than the mere value of learning conditional on postponement captured by the QOV: even without learning and/or any sophistication, the policy maker gains $-NPV = \frac{1}{4}$ from not carrying out the project. However, the $DPOV$ only captures the collectible net value of postponement, i.e., the value exceeding the option to entirely dismiss the project. Subtracting the loss from undertaking the project in the present from the gain of postponement yields $DPOV = V^l(0) - V^l(1) + [NPV]_- = \frac{1}{4}\beta$.

5.2 Distinct QOV and DPOV

We modify the example in section 5.1 to yield a positive NPV. Let the payoff in the high state $\bar{\theta}$ be $u_2(1,1,\bar{\theta}) = 4$ instead of 2 as above. To be consistent, we also increase the payoff from the high realization when carrying out the project in the second period: $u_2(0,1,\bar{\theta}) = \beta(-1+4) = 3\beta$. We find $NPV = 1$ and $SOV = \beta$.[9] Here, the SOV is simply the discounted expected value from carrying out the project in the second period. We assume a strictly positive discount rate ($\beta < 1$). Neglecting the value of learning under postponement, the policy maker would now strictly prefer to undertake the project in the present: $SOV < NPV$. However, given the policy maker has read Arrow-Fisher-Hanemann-Henry, he finds that postponement comes with an additional value from learning: $QOV = \frac{1}{2}\beta$. As long as his discount factor satisfies $\beta > \frac{2}{3}$, he can increase welfare by postponing the project because $SOV + QOV > NPV$.

How does a policy maker address the project if he read Dixit and Pindyck instead of Arrow-Fisher-Hanemann-Henry? He calculates $DPOV = [\frac{3}{2}\beta - 1]_+$.

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[9]We calculate $V^n(1) = V^p(1) = V^l(1) = 1$, $V^n(0) = 0$, $V^p(0) = \beta$, and $V^l(0) = \frac{3}{2}\beta$.

Realizing that $DPOV > 0$ as long as $\beta > \frac{2}{3}$, the Dixit-Pindyck reader draws the same conclusion as the policy maker of the previous paragraph: the value of postponement, in a setting with learning, is positive and postponement increases his welfare.

In the current example, the Dixit-Pindyck option value differs from the Arrow-Fisher-Hanemann-Henry quasi-option value. Our change with respect to section 5.1 has driven two wedges between these two option values. First, the simple option value SOV is now positive: the option to carry out the project in the second period, if it is postponed, is now valuable even without learning. Collecting the total value from postponement, the $DPOV$ picks up the SOV as well. Second, undertaking the project in the first period now has a strictly positive payoff. The $DPOV$ only measures the collectible value of the option to postpone. It therefore subtracts the value from carrying out the project right away ($NPV = 1$) from the gross value of postponement ($QOV + SOV = \frac{3}{2}\beta$).

Finally, assuming $\beta < \frac{2}{3}$ introduces a third wedge, $[V^l(0) - V^l(1)]_- = \frac{3}{2}\beta - 1 < 0$, between the DPOV and the QOV, implying that all terms in equation (9) contribute non-trivially. The value of postponement $V^l(0) - V^l(1)$ becomes negative. Full sophistication *conditional* on postponement still has the positive value $QOV + SOV$. However, this value no longer outweighs the benefits NPV from carrying out the project right away. Then, the option to postpone is not exercised, and we have to subtract the negative $[V^l(0) - V^l(1)]_-$ from $QOV + SOV - NPV$ to keep Dixit-Pindyck's option value at zero (as opposed to a strictly negative net value of a postponement obligation).

6   Conclusion

We have explored the differences and similarities between Arrow, Fisher, Hanemann, and Henry's concept of quasi-option value and the Dixit-Pindyck option value in a simple, but general, two period welfare model. The Dixit-

Pindyck option value captures the collectible net value from postponing a project under learning. The Arrow-Fisher-Hanemann-Henry quasi-option value captures the value of learning conditional on the project's postponement. We showed how these values differ in general, and how they relate in examples.

We explained how these option value concepts change the common net present value decision rule. Using Dixit-Pindyck's option value, we first have to check the option value's sign. If it is positive, we postpone the project. If it is zero, we use the usual expected net present value decision rule: we undertake the project if the NPV is positive, and dismiss it if the NPV is negative. Using the quasi-option value instead, we compare its magnitude directly to the NPV. If the quasi-option value dominates the NPV, we increase welfare by postponing the project. If, however, the quasi-option value does not dominate the NPV, we have to calculate an additional quantity, which we called the simple option value. This additional quantity captures the value of postponing the project as opposed to dismissing it, ignoring informational changes. Only if the NPV dominates the sum of this simple option value and the quasi-option value is it optimal to carry out the project in the present.

As a mere decision rule, the Dixit-Pindyck option value is likely to be the most straightforward approach, capturing directly the net welfare gain from a sophisticated postponement. If the option values should also convey information about the trade-offs at stake, then the quasi-option value approach, in combination with the simple option value, is likely more useful: it pinpoints directly the gain from learning as well as the opportunity cost from waiting without learning.

Appendix

Proof of Proposition 1: Using $V^n(1) = V^p(1) = V^l(1)$ and equation (4) we find

$$V^l(0) - V^l(1) = V^l(0) - V^n(0) + V^n(0) - V^l(1)$$

$$= QOV + SOV - NPV . \tag{10}$$

The proposition follows immediately from combining equation (10) with the optimality rule (2). $\qquad\square$

Proof of Proposition 2:

i) Given $NPV > 0$, equation (7) implies $DPOV = \left[V^l(0) - V^l(1)\right]_+ = 0$ $\Rightarrow V^l(1) \geq V^l(0)$. Thus, the sophisticated decision maker gains the highest expected welfare from carrying out the project in the current period. Note that in the subcase, where $V^n(1) = V^l(1) = V^l(0) > V^n(0)$, the decision maker is indifferent. As we expect from Proposition 1, in this subcase, the value of full sophistication under postponement $V^l(0) - V^n(0)$ equals the net present value $V^n(1) - V^n(0)$.

i) If the Dixit-Pindyck option value is strictly positive we know from equation (7) that $\left[V^l(0) - V^l(1)\right]_+ > 0$ and, thus, equation (2) is satisfied, implying optimality of postponing the investment project.

iii) The remaining case is $DPOV = 0$ and $NPV \leq 0$. From equations (6) and (7) we then find

$$\left[V^l(0) - V^l(1)\right]_+ = [V^n(0) - V^n(1)]_+ = -[NPV]_- \geq 0 . \tag{11}$$

Let us first assume a strict inequality. Then $V^l(0) = V^n(0) > V^l(1) = V^n(1)$. It follows immediately that postponement is optimal. It also follows from $V^l(0) = V^n(0)$ that in all states of the world the choice $x_2 = 0$ is (weakly) maximizing welfare. Thus it is optimal never to carry out the project. Now let us assume that equation (11) holds with equality. If $V^l(0) = V^l(1)$, then everything goes through as above (because we know that $V^n(0) - V^n(1) \geq 0$

from $NPV \leq 0$). The remaining case where $V^l(0) < V^l(1)$ would imply $V^l(0) < V^l(1) = V^n(1) \leq V^n(0)$. But this statement contradicts the tautology $V^l(0) \geq V^n(0)$ and therefore the case is the empty set. □

Proof of Proposition 3: Using (7), we find

$$V^l(0) - V^l(1) = DPOV - [NPV]_- + \left[V^l(0) - V^l(1)\right]_- . \tag{12}$$

Equating the right hand sides of equations (10) and (12), we obtain

$$DPOV = QOV + SOV - NPV + [NPV]_- - \left[V^l(0) - V^l(1)\right]_- .$$

Employing the relation $[NPV]_- - NPV = -[NPV]_+$ leads to the equation stated in the proposition. □

References

Arrow, K. J. & Fisher, A. C. (1974), 'Environmental preservation, uncertainty, and irreversibility', *Quarterly Journal of Economics* **88**, 312–9.

Dixit, A. K. & Pindyck, R. S. (1994), *Investment under Uncertainty*, Princeton University Press, Princeton.

Fisher, A. C. (2000), 'Investment under uncertainty and option value in environmental economics', *Resource and Energy Economics* **22**, 197–204.

Hanemann, M. W. (1989), 'Information and the concept of option value', *Journal of Environmental Economics and Management* **16**, 23–37.

Henry, C. (1974), 'Investment decisions under uncertainty: The 'irreversibility effect'', *American Economic Review* **64**(6), 1006–1012.

Mensink, P. & Requate, T. (2005), 'The dixit-pindyck and the arrow-fisher-hanemann-henry option values are not equivalent: a note on fisher (2000)', *Resource and Energy Economics* **27**, 83–88.