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Author
Swiatecki, W.J.

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W.J. Swiatecki

March 1985

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Friction in Nuclear Dynamics

W.J. Swiatecki

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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FRICTION IN NUCLEAR DYNAMICS*

W.J. Swiatecki

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

The problem of dissipation in nuclear dynamics is related to the breaking down of nuclear symmetries and the transition from ordered to chaotic nucleonic motions. In the two extreme idealizations of the perfectly Ordered Regime and the fully Chaotic Regime, the nucleus should behave as an elastic solid or an overdamped fluid, respectively. In the intermediate regime a complicated visco-elastic behaviour is expected. The discussion is illustrated by a simple estimate of the frequency of the giant quadrupole resonance in the Ordered Regime and by applications of the wall and window dissipation formulae in the Chaotic Regime.

1. INTRODUCTION

In the context of nuclear physics, the concepts of friction, dissipation, viscosity have come into widespread use only in the last 10-15 years, but early precursors can be found in the literature. Fig. 1 reproduces the last paragraph from a 1940 paper by H.A. Kramers, entitled "Brownian Motion in a Field of Force and the Diffusion Model of Chemical Reactions."[1]

Apart from its intricate logic, this passage is remarkable in that it introduces into nuclear physics, perhaps for the first time, concepts that are currently central in discussions of nuclear dynamics: viscosity, friction, superfluidity, plasticity and crystallization.

As we know today, all these concepts have a place in the interpretation of nuclear phenomena, but their relations are quite intricate and, even now, are not fully understood. No wonder the logic in Kramers' concluding paragraph is so contorted! [Niels Bohr is said to have advocated the rule: never talk more clearly than you think. ("Bohr's regel om aldrig tale klarere end man taenker."[2]) In the paragraph I quoted, Kramers seems to have followed closely that rule.]

Another early paper, in which nuclear viscosity is discussed, is the 1953 article by D.L. Hill and J.A. Wheeler on "Nuclear Constitution and the Interpretation of Fission Phenomena."[3] (It is a very clearly written paper--I wonder if this caused Niels Bohr, who is cited as co-author of an

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earlier draft, to withdraw his name from the final version!)

Fig. 2 reproduces parts of a section from this paper, entitled "Nucleus as Quantum Fluid."

Physica VII, no 4 April 1940

BROWNIAN MOTION IN A FIELD OF FORCE AND THE DIFFUSION MODEL OF CHEMICAL REACTIONS

by H. A. KRAMERS

Leiden

Still it is not uninteresting to consider the question of the coefficient of viscosity of nuclear matter somewhat more closely. Even if a nucleus in its normal state behaved as a perfectly hard, non plastic crystal, there is no reason to exclude the possibility that the excited nucleus possesses a finite coefficient of internal friction. In view of the surprising properties of He II it is even dangerous to assert that this coefficient cannot be extremely small; this assumption would not necessarily contradict Bohr's asssumption that a single neutron impinging on a nucleus is in first instance captured. This assumption is, however, not well reconcilable with the idea that nuclear matter should behave as a perfectly hard crystal, i.e. a certain amount of plasticity is anyhow to be expected.

3 Jan. 1940

Received January 29th 1940.

FIGURE 1
Kramers' 1940 remarks on nuclear viscosity[1]

Nucleus as Quantum Fluid

We have encountered in this discussion some of the properties of an unusual idealized quantum fluid. It is considered to be completely transparent internally with respect to motion of the constituent particles, and to receive disturbances solely by way of surface deformations.

It is capable of collective oscillations, but it is the wall which organizes these disturbances, not nucleon to nucleon interactions. Oscillations experience a damping, but the mechanism of the damping is unlike that encountered in ordinary liquids.

The wave function of the particle to come out is spread over the whole nucleus and has energy pumped into it by Doppler effect; it is not concentrated near a part of the surface before emission.

Altogether one is dealing with a most interesting new form of matter.

FIGURE 2
Hill and Wheeler's view of a nucleus[3]

As is well known, one can find in the text of Hill and Wheeler's paper the seeds of almost every important idea in nuclear physics. (The few exceptions not appearing in the text can be found in the figure captions[4].) The idea that I would like to single out in the present context is that the breaking down of nuclear symmetries should introduce a *viscosity* into the dynamics of nuclear shape evolutions. (Hill and Wheeler's "shape-dependent viscosity"). Since the presence or absence of symmetries is also decisive for whether the nucleonic motions inside the nucleus are ordered or chaotic, the problem may be stated as the effect of an Order-to-Chaos transition on nuclear dynamics.

2. ORDER AND CHAOS IN DYNAMICAL SYSTEMS

The study of Order-to-Chaos transitions in dynamical systems in general has seen a tremendous development in the past 2-3 decades[5] and these developments have begun to make their way into nuclear physics as well[6,7]. The research in the field of Order-to-Chaos transition impinges on an incredibly large number of topics, including pure mathematics as well as plasma, solid-state and accelerator physics, hydrodynamics, astronomy, chemical kinetics, biology, physiology and ecology[8,9,10]. This is not really surprising: after all, the description of almost any system in nature--once one goes beyond the simplest idealization--involves equations coupling two or more degrees of freedom by means of non-linear terms. And it has been a great discovery of the past decades that in most cases like that, the resulting dynamics is not just either ordered or chaotic--as one might have believed at the turn of the century--but that there is an incredibly rich spectrum of intermediate behaviours. This spectrum is so rich, in fact, that comparisons with organic systems come naturally to mind.

Let me flash before you a few figures which--even if I don't explain them properly--give a taste of what is involved.

Fig. 3 represents the results of a computer study of the problem of a ball bouncing between two parallel walls, whose distance apart oscillates in a periodic way. (The problem was examined in 1949 by Fermi as an analogue to a possible acceleration mechanism for cosmic rays, in which charged particles

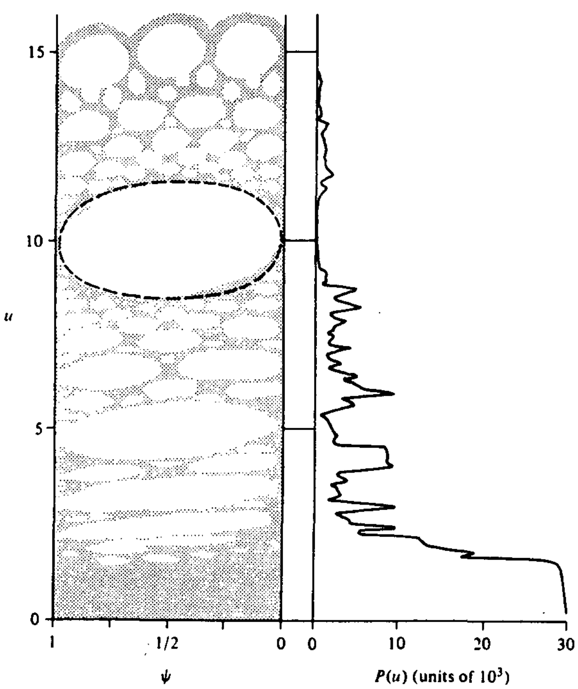

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FIGURE 3
Phase space and velocity spectrum of a ball bouncing between oscillating walls[10]

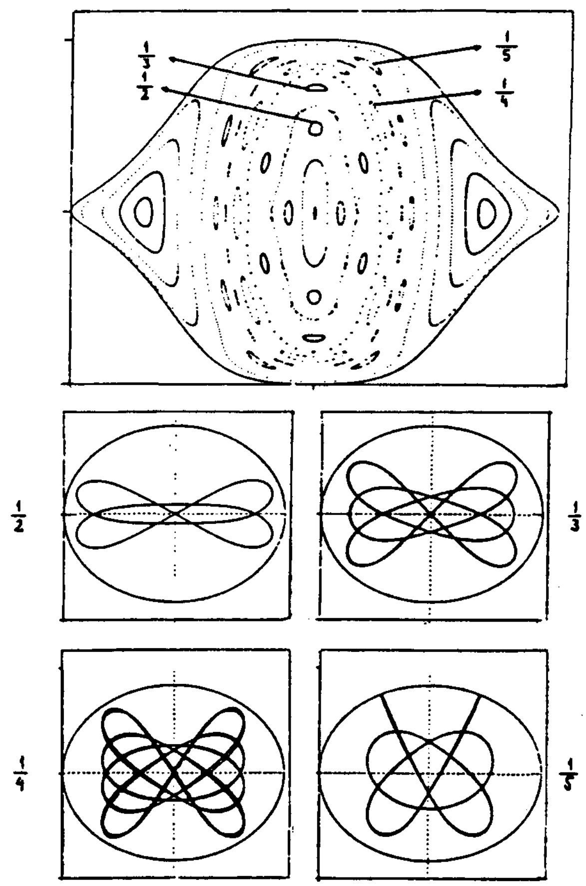

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FIGURE 4
Poincaré section through the phase space and four regular orbits in the Perey-Pilt potential with ratio of axes 1.2[7]

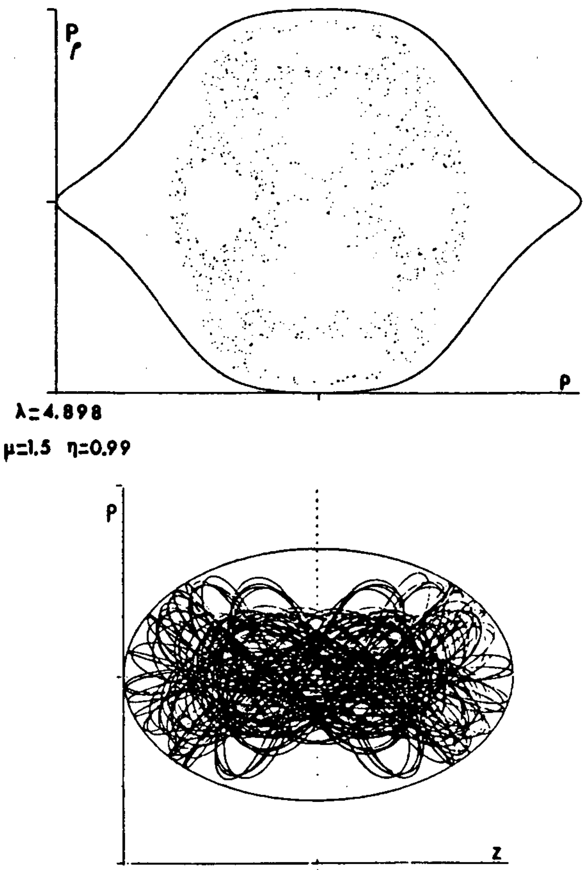

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FIGURE 5
Chaotic phase space and orbit in Perey-Pilt potential with ratio of axes 1.5[7]

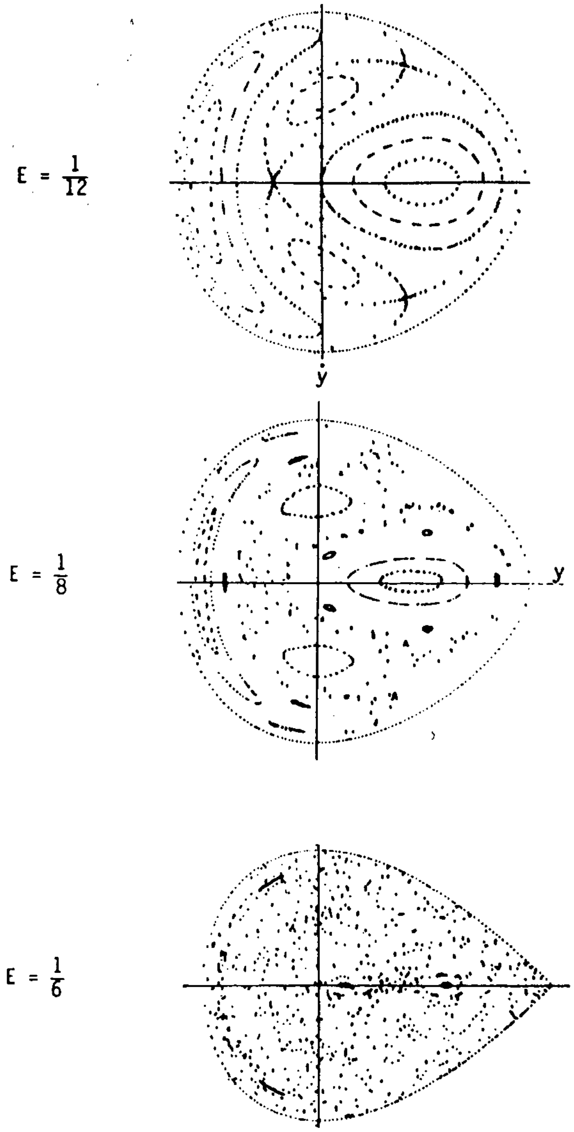

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FIGURE 6
Ordered, intermediate and chaotic phase space for Hénon-Heiles trajectories at three energies[9]

gain energy by colliding with moving magnetic field structures. This wall-bouncing or Doppler acceleration mechanism is also somewhat related to the so-called "wall formula" for nuclear dissipation, to the "escape width" of nuclear giant quadrupole resonances and to the "Fermi jet" mechanism of pre-equilibrium emissions of nucleons in nucleus-nucleus collisions.) Fig. 3 is a look at the two-dimensional phase space of the particle bouncing between the walls. (The ordinate u is the particle's velocity and c is related to its position.) In the computer studies, low velocity test particles were injected between the walls and you can see how the oscillating wall pumped energy into the particles. Thus the right hand side of the figure is the velocity spectrum of the particles after many bounces. For low velocities, the spectrum is pretty featureless, but for higher velocities weird structures appear unexpectedly, as seen particularly clearly in the phase space on the left.

Fig. 4 is taken from a recent numerical study of the classical orbits in the Perey-Pilt potential, a potential resembling a diffuse nuclear Woods-Saxon well. The size of the potential is appropriate to a nucleus with mass number A = 16 and the shape is a spheroid with a ratio of axes equal to 1.2. The lower part of the figure shows some regular two-dimensional periodic orbits in such a potential and the upper part shows a section through the three-dimensional phase space of all possible orbits with the given energy (the so-called Poincaré section). Unless you already know how to read such Poincaré plots, you are not supposed to understand the figure--the only message I want to convey is that there are some very definite structures in the phase space, associated with the existence of regular symmetric orbits in ordinary space. Now contrast Fig. 4 with Fig. 5, which shows a trajectory with the same energy as before (i.e. within 1% of the dissociation energy in the Perey-Pilt potential) but for a ratio of axes of 1.5 instead of 1.2. Both the trajectory and the Poincaré section show that order has given place to chaos at some critical value of the deformation between 1.2 and 1.5. Fig. 6 shows similar plots in the so-called Hénon-Heiles problem (a mass point in a two-dimensional anharmonic oscillator well). This problem was studied originally in connection with the oscillations of stars about the galactic plane. The three parts of Fig. 6 are again Poincaré sections through the (three-dimensional) phase space of the particle's motion, for three values of the (dimensionless) particle energy E. As this energy is increased from E = 1/12 to E = 1/6, one can again see order giving place to chaos (in a fairly abrupt way).

Fig. 7 is taken from a numerical study of the phase space for orbits in intersecting storage rings for particle accelerators. The next case, Fig. 8, is a real beauty. The top also refers to intersecting storage rings, and the bottom is a three-dimensional representation of the structure of the phase space for a system like the Hénon-Heiles problem. Note in particular the last sentence in the original captions: "A magnification about such an elliptic orbit yields the same picture all over, *etc. ad infinitum*"!

The next two pictures illustrate the occurrence of such infinite regressions in a related problem of pure mathematics, concerned with repeated applications of a mapping or transformation such as $z \to \lambda z\,(1 - z)$, where z is a (complex) number and λ a (complex) parameter[8,11]. Fig. 9 shows a structure that appears in the complex λ plane. Fig. 10 is a related

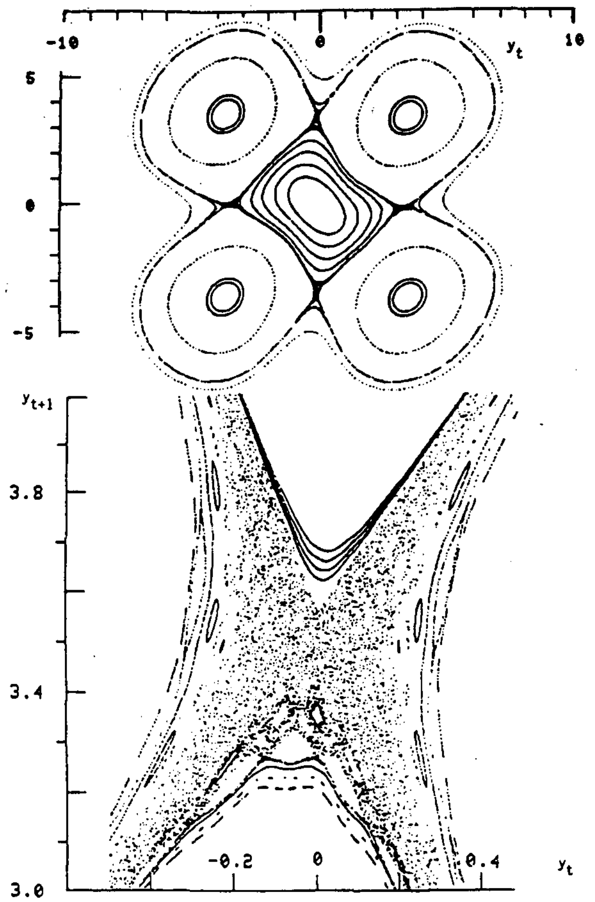

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FIGURE 7
Phase plot for Intersecting Storage Rings[9]

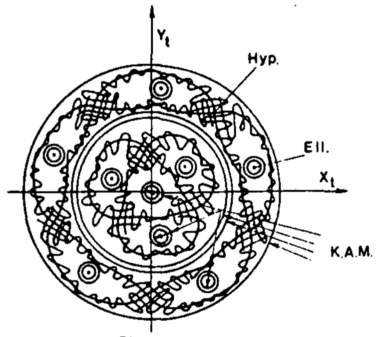

Figure 10
Composite Phase Plot near the elliptic origin, for the intersecting storage rings (2.39/29) (after [22,15,2]). The rational tori of Fig. 8 have broken up into disjunct elliptic and hyperbolic orbits, cf. Fig. 9. From the latter now emanate the wild separatrices (sketch) and chaotic regions discussed in section 2.4. Concentric with the origin are irrational K.A.M. tori which do survive, containing the majority of all orbits in this picture. Magnifications about the elliptic orbits yield pictures similar to this Fig. 10, etc.! Of that *infinite hierarchy* only the next K.A.M. tori are sketched.

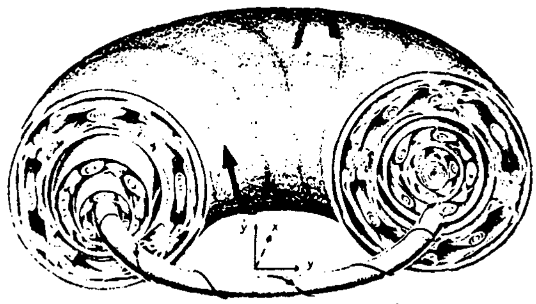

Figure 11
Nested K.A.M. tori in the 3-dimensional $x, y, \dot{y}$ phase space of a (reduced) Hamiltonian system like the Hénon-Heiles system (2.8-10); see figs. 10 and 4 (taken from [16]). Note the smooth regular K.A.M. tori about the center and about the elliptic orbits which, together with the chaotic regions, populate the gaps between the K.A.M. tori A magnification about such an elliptic orbit yields the same picture all over, *etc. ad infinitum.*

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FIGURE 8
Structures in phase space[9]

structure.

All this was just to give a hint of the baroque goings on in phase space when order gives place to chaos in dynamical systems. Note also the diversity of the applications of this research. (I attended recently a conference on Non-linear Systems and Dynamics. One of the lectures was entitled "Fish Gotta Swim" and dealt with how a dogfish swims. Apparently, one gets useful insights into this problem by modeling the spinal chord of the dogfish as some 70 coupled non-linear oscillators and studying the periodic solutions that one finds numerically using a computer.)

But back to the nucleus. As I hinted already, the nucleus is a system where some of the richness of the order-to-chaos transition may be explored under novel and sometimes unique conditions, in particular where quantal and dissipative effects are concerned. So let me sketch the three principal phenomena where dissipation or friction make an appearance in nuclear physics--fission, nucleus-nucleus collisions and giant resonances--and point out the relation to the order-to-chaos transition.

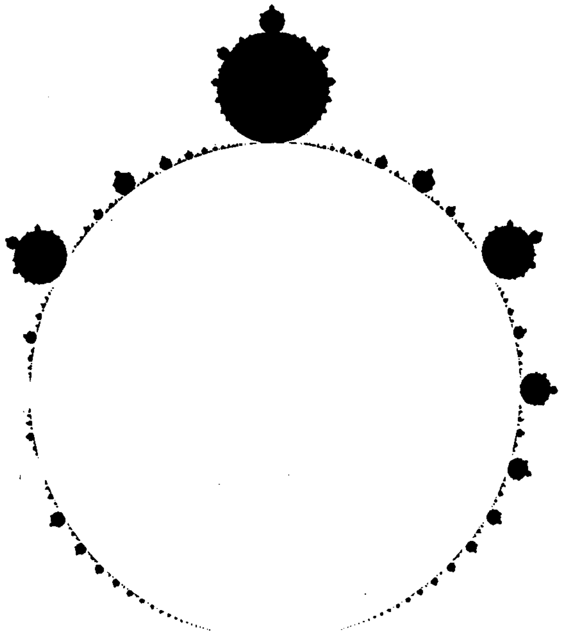

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FIGURE 9
Structures in the complex λ plane associated with the iteration $z \to \lambda z(1 - z)$, from[11]

Mandelbrot: Fractal Aspects of Iteration of $z \to \lambda z(1 - z)$

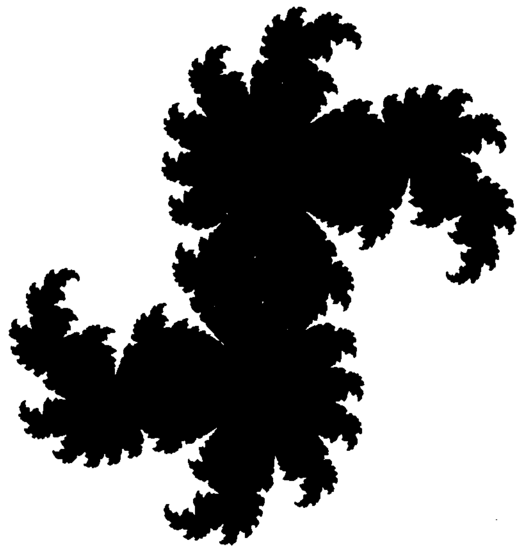

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FIGURE 10
Structures associated with the iteration $z \to \lambda z(1 - z)$, from[11]

3. FRICTION IN NUCLEAR DYNAMICS

In the case of fission it is clear that the dynamical evolution of the shapes of a fissioning nucleus will depend on whether the nuclear fluid is viscous or not. Think of the division of a charged liquid drop: the fission will look drastically different depending on whether you have a drop of honey or a drop of mercury. In particular, the kinetic energy of the separating fragments would be much less for honey than for mercury because of the viscous drag between the two separating pieces. On the experimental side, fission-fragment energies have been studied systematically and in great detail for some 45 years. The theory of fission is just as old but, in the early days, the question of viscosity was, quite mistakenly, underemphasized. Fig. 11 shows some sketches made by Niels Bohr during a conversation about fission, on October 7, 1950, his 65th birthday. Alongside is an example of a calculation from the sixties[12] of the fission of an idealized non-viscous liquid drop. How strongly the fragment energies could be affected by viscosity is shown in Fig. 12, which illustrates more recent calculations with and without viscosity[13,14]. The viscosity used to calculate the lower solid curve was of the ordinary kind, familiar from conventional hydrodynamics. (From the comparison with the experimental data you might want to conclude that nuclei must be only slightly viscous--but this is almost certainly wrong. As I will

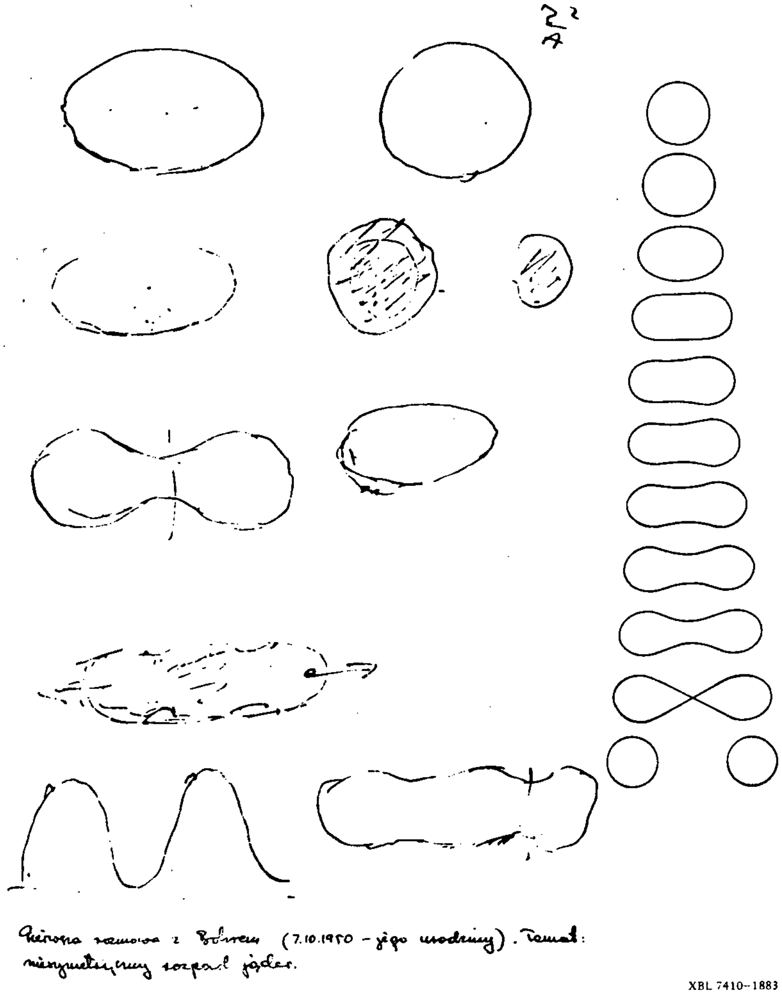

FIGURE 11
Niels Bohr's sketches illustrating fission, and a calculation of the fission of an idealized, non-viscous ^{237}Np nucleus, from[12]

argue later, nuclei are very viscous, but the viscosity is not of the ordinary hydrodynamic kind).

In the case of the collision of two nuclei, one may again expect very different behaviour depending on whether the nuclei are viscous or not. For example, in somewhat peripheral collisions, the relative kinetic energy of the nuclei will be reduced rapidly if there is a lot of friction between the two pieces. Moreover, it should be more difficult to make two nuclei fuse into one if there is a large viscosity resisting the flow. It was, in fact, the study of so-called deep-inelastic collisions between nuclei some 15 years ago that stimulated the development of dissipative theories of nuclear dynamics[15]. In such collisions, illustrated in Fig. 13, the two nuclei are brought to relative rest without fusing (and almost without changing their masses). This suggests at once a large friction between the two pieces and a large viscosity resisting the flow of nuclear matter.

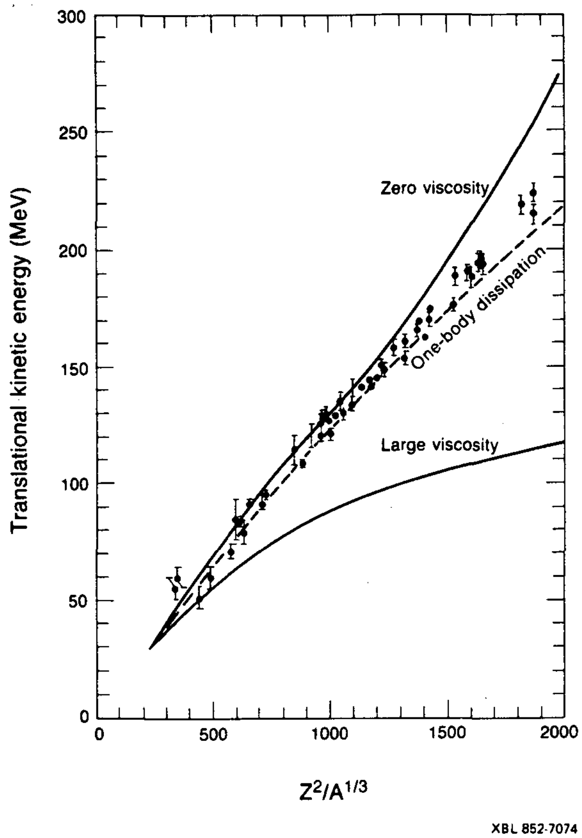

FIGURE 12
Fission-fragment energies compared with idealized calculations with no viscosity, large (ordinary) viscosity and "one-body" dissipation[13,14]

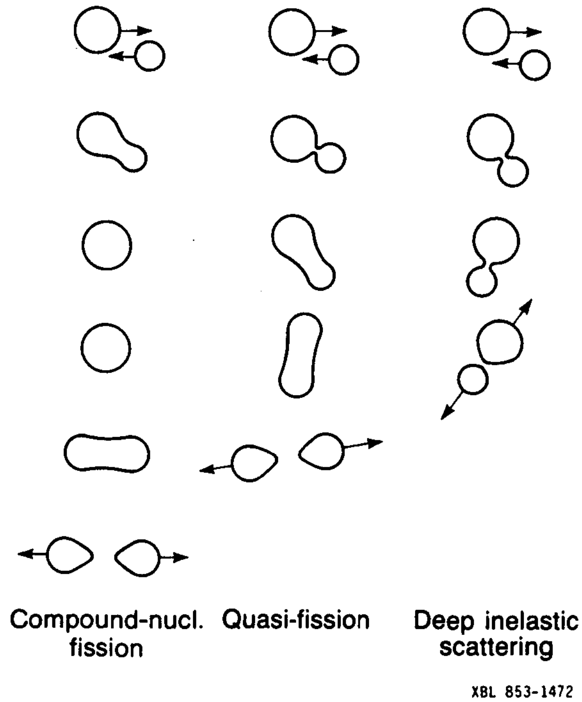

FIGURE 13
Three types of nucleus-nucleus reactions

In addition to fission and nucleus-nucleus collisions, evidence for friction in nuclear physics comes from damping widths of so-called giant resonances[16,17]. These are resonances of a collective nature, of which the three most familiar examples are the giant dipole vibration of all the neutrons against all the protons, the spherically symmetric compressive monopole mode and the giant quadrupole mode, where the nucleus is alternately stretched and flattened along an axis of symmetry. Fig. 14 shows the systematics of the giant quadrupole resonance energies for nuclei in the periodic table and Fig. 15 shows the observed widths of these resonances, testifying to the presence of dissipative effects.

There exists today a wealth of experimental data on fission, nuclear collisions and giant resonances, all bearing on the problem of dissipation. As regards the interpretations of these experiments, one has available a whole spectrum of theories, ranging from microscopic, quantal descriptions using the Time-Dependent Hartree-Fock method, to macroscopic, hydrodynamic theories ignoring altogether the nucleonic degrees of freedom. I can do little more than list the main groupings of these theories, mentioning their relevance to

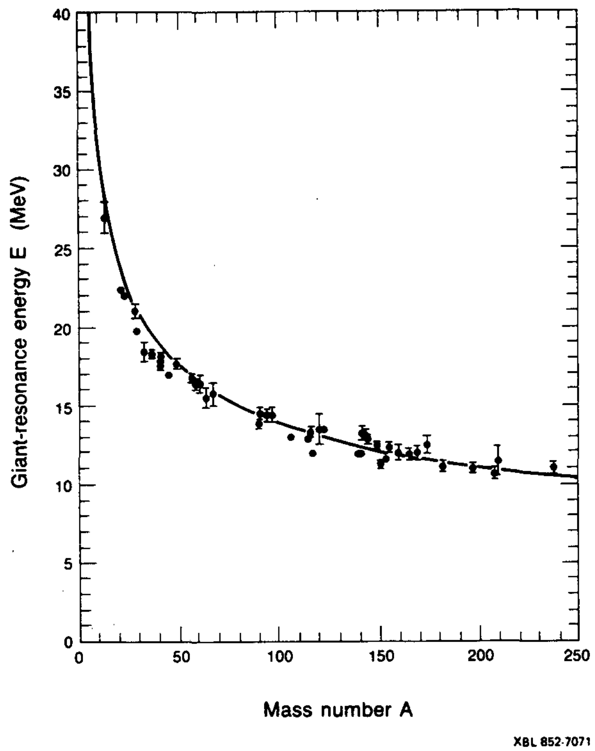

FIGURE 14
Giant quadrupole resonance energies compared with a theoretical estimate[38]

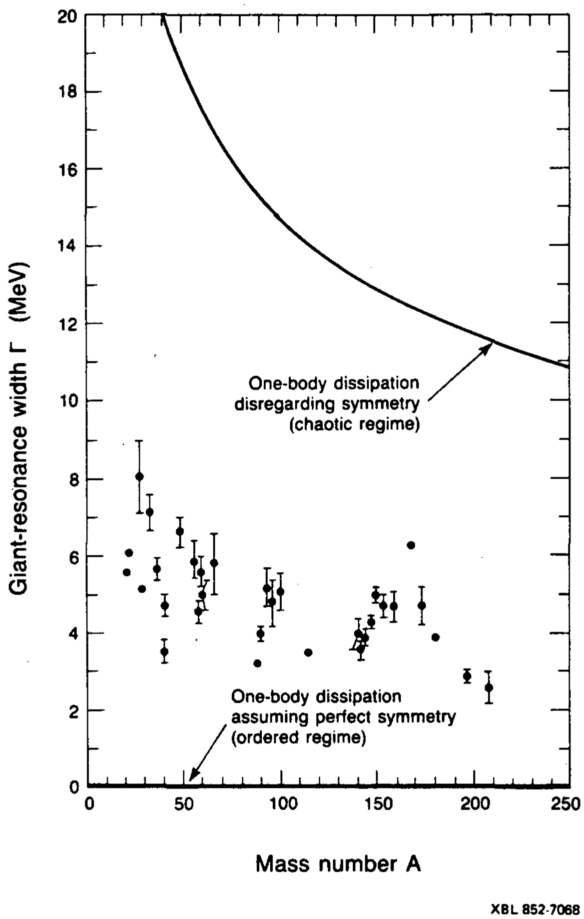

FIGURE 15
Widths of giant quadrupole resonances compared with two limiting cases[38]

the question of dissipation. After that, I will come back to the relation of dissipation to the Order-to-Chaos transition.

4. GROUPING OF THEORIES

I have divided the theories into two main classes, microscopic and macroscopic. The latter is further subdivided into statistical and hydrodynamic (Fig. 16). There are, of course, mixed and intermediate situations that are not easy to classify.

The most detailed are microscopic theories, in which all the nucleonic degrees of freedom are treated explicitly. At very high energies they are often of the type of Monte Carlo cascade calculations[18], or even brute-force computer solutions of the A-body problem of interacting classical mass points[19]. At low energies, the self-consistent Time-Dependent Hartree-Fock method has seen very active development in the past ten years[20]. Being microscopic, these theories do not need the concepts of friction or viscosity in their formulation and they are in this sense outside the scope of this

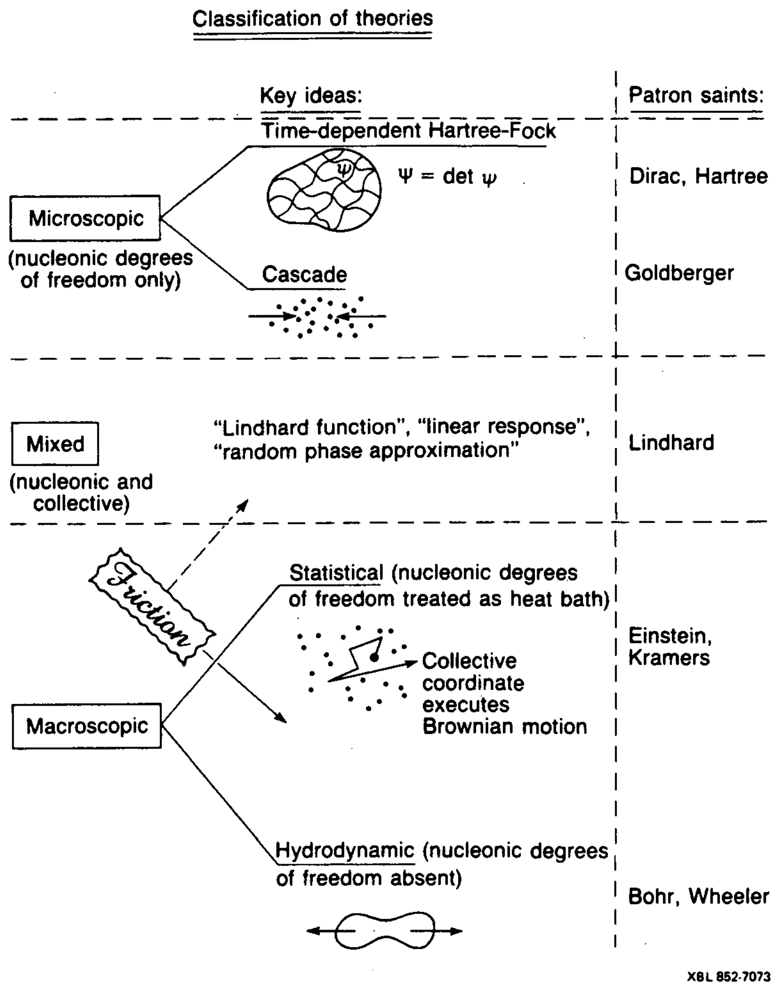

FIGURE 16
Classification of nuclear theories

talk. On the other hand, the results of the microscopic calculations do, in general, display viscous features to a greater or lesser extent and they could serve as a testing ground for macroscopic approximations. But, on two accounts, there is need for caution in using most existing TDHF calculations as a benchmark when the specific question of dissipation is at issue. First, the standard TDHF treatment is based on disregarding altogether residual interactions between nucleons. Second, many TDHF calculations introduce into the problem various symmetries to make the calculation more tractable. Both these approximations tend to favour the preservation of regularities in the nucleonic motions and so delay the development of chaos. Since, as we shall see, the transition from order to chaos is intimately related to dissipation, it is important to keep in mind these approximations of many currently available TDHF calculations.

In macroscopic theories, a set of collective coordinates is introduced from the beginning and their time development is described by differential (or integro-dfiferential) equations with or without a fluctuating (Langevin) term. In the former case one has a type of Brownian motion in the space of the collective coordinates, with the microscopic coordinates treated as a heat bath[21]. In the latter case one ends up with hydrodynamic-type collective theories, in which only the average or most probable time development of the system is described. This is in contrast to the statistical theories, where the average drift of the coordinates, as well as their fluctuations about the average, are included in the description.

The concept of friction or viscosity enters directly as input into macroscopic theories of the hydrodynamical kind. In the statistical theories there appears in addition a diffusion coefficient describing the width of the fluctuations. Under certain conditions (near equilibrium) it is related to the friction coefficient by the Einstein relation[21].

Here are a few illustrations of the type of results obtained with the different approaches. Fig. 17 shows a microscopic classical treatment of a high-energy collision between Niobium nuclei[19]. Fig. 18 shows an example of a TDHF study of a collision between ^{16}O and ^{40}Ca at 315 MeV[20]. Fig. 19

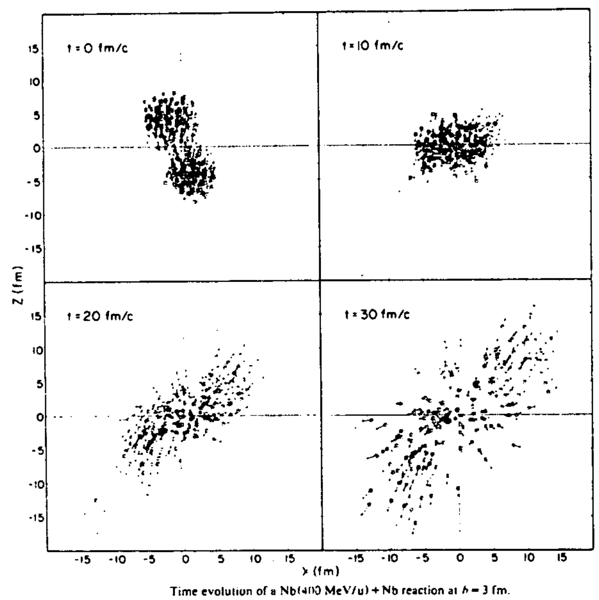

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FIGURE 17
Computer simulation of a high-energy collision[19]

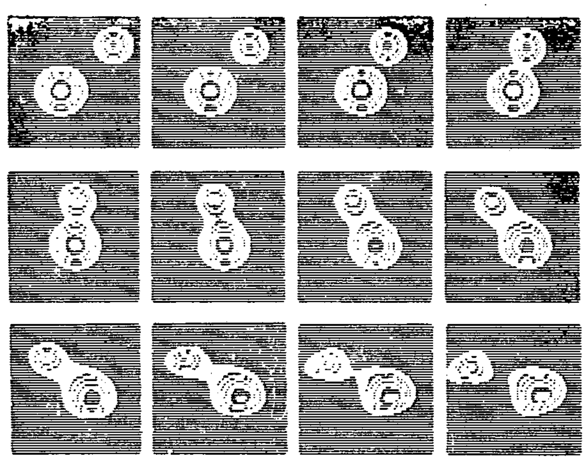

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FIGURE 18
A collision between ^{16}O and ^{40}Ca at E_{lab} = 315 MeV and $\ell = 80\hbar$, according to a TDHF calculation[20]

shows the angle-energy correlation for a deep-inelastic collision in which a statistical spreading due to fluctuations is included[22]. There are many such calculations in the literature, dealing with collisions between different pairs of nuclei, with damping of giant resonances and with fission. I have included a number of references to recent articles in which the state of this

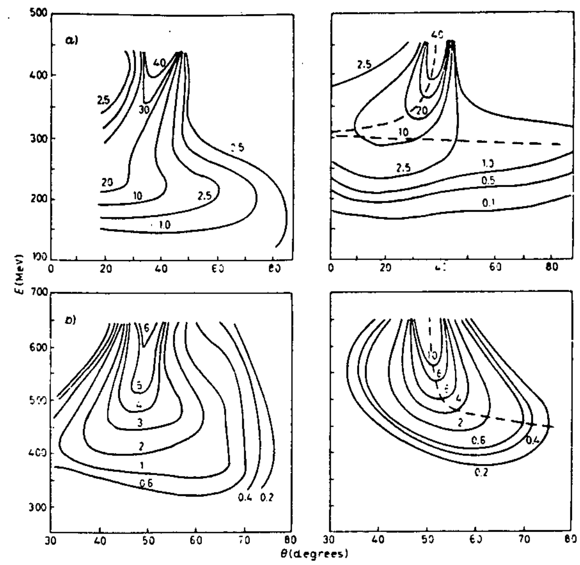

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FIGURE 19
Angle-energy correlations (Wilczynski plots) for the reaction $^{86}K + ^{166}Er$ at E_{lab} = 703 MeV (upper) and E_{lab} = 1130 MeV (lower).
Left: measurements, right: calculation[22]

research is reviewed[14-17,20-30]. The general situation is that the degree of correspondence between the calculations and experimental data is uneven and often depends on how many adjustable parameters one is willing to introduce in the theories.

Rather than attempting to describe in more detail the current situation, I would like to focus on one theme that seems to be emerging from the multitude of experimental and theoretical studies, and which I take as the heading for the next section.

5. THE NATURE OF NUCLEAR DYNAMICS IS DETERMINED BY SYMMETRIES

I have tried to summarize the idea in a single diagram (Fig. 20). In this figure I have focused on two limiting cases, the Chaotic Regime and the Ordered Regime. In the former the nuclear shape is supposed to have no symmetries and the nucleons move chaotically inside the irregular mean-field potential. There are no good quantum numbers for the single-particle motions (except for the particle energy in the case when the mean field is static). In the opposite extreme, the nuclear potential is dominated by symmetries, the nucleonic motions are regular orbits and there are other constants of the motion and other quantum numbers besides the energy.

The inset pictures in Fig. 20 illustrate qualitatively the energy spectrum of the system, plotted as a function of deformation. The lowest curve is the

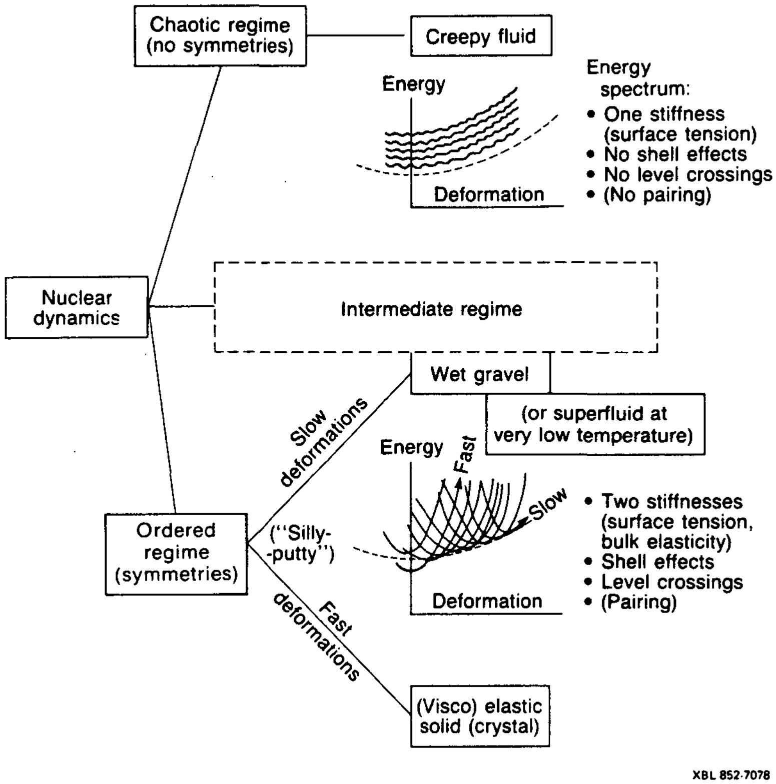


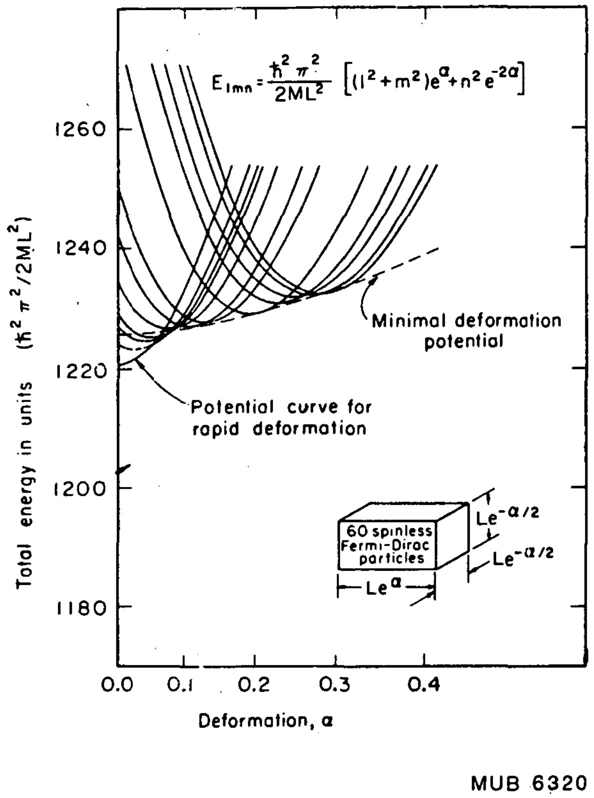
FIGURE 20
Symmetries and nuclear dynamics

ground state and its behaviour as a function of deformation corresponds simply to the dependence of the potential energy of a nucleus on shape. In the Chaotic Regime this ground state is drawn with a gentle positive curvature, corresponding to the increasing surface energy of the deforming nucleus. It is well known that, in the absence of shell effects (and there are no shell effects in the Chaotic Regime), a surface energy is an excellent approximation to the deformability for all but the lightest nuclei. (The wiggles in the figure are meant to hint at the quantal nature of the particles: this may introduce a small-scale structure with a characteristic length of the order of the wavelengths $\lambda\!\!\!^{-}$ of the nucleons in the nucleus.) The other curves in the Chaotic Regime are the excitated states of the nucleus. In the mean-field approximation they are obtained by rearranging the single particles from the set of eigenvalues corresponding to the ground state, to some other set with higher energy. Since the individual-particle states are assumed to have no good quantum numbers besides the energy, their eigenvalues will not, in general, cross as a function of deformation. It follows that also the energy levels of the nucleus, when approximated as a sum of individual-particle eigenvalues, will almost never cross. Consequently, the excited states of the system have to follow the general trend of the ground state, as shown in the figure. (We are looking apart from the few good quantum numbers for the system as a whole, such as the total angular momentum.)

To get an idea about the energy spectrum in the Ordered Regime, let us take a highly schematic model of a nucleus, namely A independent particles in a very symmetric potential, such that the three-dimensional motion of each particle is *separable* into three one-dimensional motions. A spheroidal, infinitely deep, sharp potential well would be a good choice, but it turns out that an even simpler potential, the Hill-Wheeler box, is just as good for purposes of illustration and is easier to interpret. The lower inset in Fig. 20 is a qualitative sketch of the energy spectrum, and Fig. 21 is a quantitative illustration. In this example the box has x,y,z dimensions $L\exp(-\alpha/2)$, $L\exp(-\alpha/2)$, $L\exp\alpha$, so that α is a measure of a volume-preserving quadrupole-type stretching deformation. The eigenvalue ε_i of each particle in the box is proportional to $(\ell^2 + m^2)\exp\alpha + n^2\exp(-2\alpha)$, where ℓ,m,n are the numbers of half-wavelengths along the three axes of the box. Each parabola-like curve in Fig. 21 is a plot of the total energy, $\Sigma\,\varepsilon_i$, in the case where the quantum numbers ℓ,m,n for each particle are kept fixed. In other words, it is the energy of the system when all the wave functions are stretched (in the quadrupole mode) while keeping the volume and the nodal structures intact. The dashed curve in Fig. 21 indicates the trend of the envelope of the parabola-like curves. Thus it corresponds to the deformation energy of the system in the case when, at each deformation, the particles *are* allowed to readjust their quantum numbers (and thus their nodal structures) so as to minimize the total energy. The envelope is consequently the ground-state deformation energy and the set of higher-lying curves at each deformation gives the single-particle excitations of the system. In contrast to the Chaotic Regime, these energy levels do cross, a consequence of the separability of the equations of motion and the presence of additional quantum numbers for the individual particles.

Note also that in Fig. 21 the energy of the cubic shape (with $\alpha = 0$) sags below the dashed curve corresponding to the envelope. This is because the case illustrated refers to 60 eigenvalues (or 240 nucleons in quadruply occupied orbits) and 60 happens to be a closed shell for the cubic shape. (Several degenerate eigenvalues ε_i have just been filled at this particle number.) This shell effect gives the cubic shape special stability.

In addition to the Chaotic and completely Ordered limiting cases, there will be an Intermediate Regime, where the system is neither chaotic nor fully separable. The deviations from a fully separable situation may be associated with three different physical features: a) the mean field may have irregularities that spoil the separability, b) the mean field has a finite



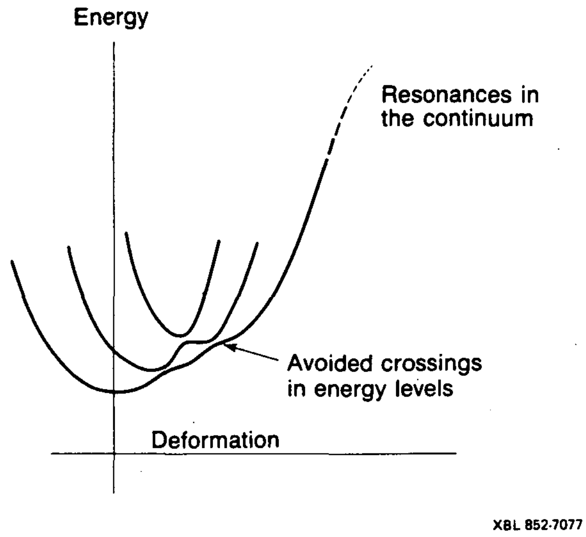


FIGURE 21
The energy of a Hill-Wheeler box as a function of a stretching deformation α. From [37]

FIGURE 22
Schematic energy spectrum in the Intermediate Regime

depth, so that particles are sometimes ejected into the continuum and c) there are residual interactions between the particles, which break down the mean-field approximation in the first place. Fig. 22 illustrates schematically the effect of these perturbations on the energy spectrum of a nucleus. Instead of crossings between parabola-like levels, there will be near-crossings and, at higher energies, some of the parabolas will peter out into resonances corresponding to particles ejected into the continuum.

Note that in the Ordered Regime there are two characteristic stiffnesses in the problem. The following noteworthy features of these stiffnesses may be demonstrated. The soft stiffness of the envelope is proportional to the surface area of the system and acts, therefore, like a surface energy of a fluid, just as in the Chaotic Regime. The large stiffness is proportional to the *volume* of the system (or to the number of nucleons) and acts, therefore, like an *elastic energy* of a solid body. Whether a nucleus will respond to a deformation as a fluid or as an elastic body will depend on the speed of the deformation. For very slow deformations the system will tend to stay close to the ground state and behave like a fluid; for very fast deformations it will tend to follow a parabola-like stiffness and behave like an elastic solid. In general, neither limit will be followed exactly and the system will get derailed into neighbouring excited levels. This means that collective energy of deformation will be partly converted into excitation. Thus, in the case of the nearly ordered regime, damping may be expected to be a sensitive function of the residual interactions, of the speed of the deformation and of deviations of the mean field from the symmetries that produced separability. The deviations from separability are themselves functions of the amplitude of the deformation, so one may expect a very complicated behaviour, where both the stiffness and the damping are complicated functions of several variables. In addition, for a sufficiently fast deformation, particles may be ejected directly into the continuum[31-36].

In the Chaotic Regime the situation ought to be simpler, since (looking apart from the wiggles) there is only one stiffness and there are no symmetries and shell effects to worry about. As I will discuss in the next section, one finds that as regards large-scale deformations, the nucleus should behave as a very viscous fluid, with a type of viscosity quite different from that found in ordinary liquids. So the qualitative picture is this: an elastic behaviour in the perfectly Ordered Regime, a complicated visco-elastic behaviour in the Intermediate Regime and a simple viscous behaviour in the Chaotic Regime (except, perhaps, for small-amplitude motions).

6. QUANTITATIVE ESTIMATES

Let us start with the Ordered Regime. In the case of A quantized particles in the Hill-Wheeler box, it is easy to derive elementary expressions for the volume energy, the surface energy and the elastic energy. Thus

$$\text{Volume energy} = (\text{energy per particle})A$$
$$= (\tfrac{3}{5}\,\varepsilon)A = \tfrac{3}{5}(\tfrac{1}{2}\,mv^2)A = \tfrac{3}{10}\,Mv^2 \quad . \tag{1}$$

Here ε is the Fermi energy (the energy of the highest occupied eigenvalue ε_i), v is the corresponding Fermi velocity, m is the nucleon mass and M is the total mass of the nucleus. Similarly,

$$\text{Surface energy} = (\text{Surface Area})\gamma \quad , \tag{2}$$

where

$$\gamma = \frac{1}{40}\left(\frac{9\pi}{4}\right)^{1/3}(\text{particle density})^{2/3}\varepsilon \quad . \tag{3}$$

(Subtleties arising in the evaluation of γ are discussed in[37].)

$$\text{Elastic energy} = (\text{Volume energy})\alpha^2$$
$$= \frac{1}{2}\left(\frac{3}{5}\,Mv^2\right)\alpha^2 \qquad . \tag{4}$$

The numerical value of the specific surface energy given by eq. (3) is not relevant for nuclear applications--it would be different for a more realistic, diffuse surface. But as regards the elastic stiffness, there are two remarkable features of eq. (4). First, it would hold for other types of potential wells, provided the wave functions were stretched in a quadrupole mode without changes in the nodal structures[38]. (But only in special cases would such stretched wave functions continue to be eigensolutions in the stretched mean field.) Second, the formula is *identical* for a gas of *classical* independent particles in a box (with the same velocity distribution as the Fermi gas). Thus a classical gas of independent, long-mean-free-path particles (a Knudsen gas) in a box with perfectly reflecting walls would respond to (volume-preserving) deformations not as a gas at all, but as an elastic solid! Moreover, its elastic stiffness would be quantitatively identical with the stiffness of a *quantized* gas. This sounds unexpected, since in the quantized case the stiffness reflects a property of specifically quantal objects--the wave functions--to stretchings at fixed nodal structures. This puzzle is closely related to the fact--not widely recognized, for some strange reason--that the response of an ideal classical gas to *compression* is exactly the same as the response of a quantized Fermi gas. In other words, that the ideal gas law for point particles

$$pV = NkT \qquad , \tag{5}$$

where p = pressure, V = volume, N = number of particles, k = Boltzmann constant, T = temperature, is equivalent to the relation between energy E and density ρ for a (degenerate) Fermi gas:

$$\frac{E}{E_0} = \left(\frac{\rho}{\rho_0}\right)^{2/3} \qquad , \tag{6}$$

where E_0 is the energy at the density ρ_0. Equation (6) is most easily derived by remembering that each momentum component of a quantized particle scales inversely as the wavelength, or directly with the cube root of the density. Hence the energy scales with the two-thirds power of the density. But since one invokes here *wave* mechanics, how can the resulting equation be equivalent to the classical gas law?

Well, it is. Rewrite eq. (5) as

$$
\begin{aligned}
pV &= \frac{2}{3} N(3 \cdot \frac{1}{2} kT) \\
&= \frac{2}{3} N[3(\text{energy per translational degree of freedom})] \\
&= \frac{2}{3} N(\text{energy per particle}) = \frac{2}{3} E \quad . \qquad (7)
\end{aligned}
$$

If now the gas is allowed to increase its volume adiabatically by δV, the work done will be $p\delta V$, so its energy will decrease by that amount:

$$p\delta V = -\delta E \quad . \qquad (8)$$

Dividing eq. (8) by eq. (7) we find

$$\frac{\delta V}{V} = -\frac{3}{2}\frac{\delta E}{E} \quad , \qquad (9)$$

$$\therefore \frac{E}{E_0} = \left(\frac{V}{V_0}\right)^{-2/3} = \left(\frac{\rho}{\rho_0}\right)^{2/3} \quad , \qquad (10)$$

an equation identical with the Fermi gas relation.

This miracle was vividly commented on by Ehrenfest in 1911 and 1916 (in the related context of the unexpected survival of Wien's displacement law of black-body radiation through what Ehrenfest called the "quantum conflagration"[39]). As shown by Ehrenfest, the explanation lies in the adiabatic invariance of the action integral $\int pdq$ in *classical* mechanics. This ensures that the process of (semi-classical) quantization, which demands that such integrals be multiples of Planck's constant, brings in nothing new as regards the response of a system to adiabatic changes of parameters: the integrals are *already* constants, in classical mechanics, as regards slow variations of parameters.

The relevance of this in the present context is that, in the Ordered Regime, where the particle motions are separable (as in the case of the Hill-Wheeler box) quantal and classical results can be very similar or even identical. In particular, the unexpected *elastic* response of a classical or quantized gas to volume-preserving deformations of the box are a consequence of the separability of the problem. Thus, the volume-preserving three-dimensional gas is equivalent to three one-dimensional gases (one for each of the separate motions). Since the volume of each gas is *not* conserved (the edge-lengths of the box are not constant, even though their product is) each gas taken separately is being compressed and decompressed during the deformation. It responds, therefore, in an elastic way. It then follows that the sum of the three elastic energies leads to an elastic response of the whole system.

Estimates of the frequency of elastic quadrupole oscillations of a nucleus may be found in[38] and references[10-30] quoted in that paper. In our present schematic model, let us combine the stiffness coefficient from eq. (4) with the inertia M_{eff} against quadrupole oscillations, as given by assuming irrotational flow. (It turns out that this is justified for quadrupole stretchings of wave functions with frozen nodal structures[40].) Thus we write the kinetic energy for quadrupole oscillations of a nucleus of mass M and radius R as

$$\text{K.E.} = \frac{1}{2} M_{eff}\dot{\alpha}^2 \quad , \tag{11}$$

$$\text{with } M_{eff} = \frac{3}{10} MR^2 \quad . \tag{12}$$

From eqs. (4) and (12) the frequency of the giant quadrupole mode follows as

$$\omega = \sqrt{\frac{3}{5} Mv^2 \Big/ \frac{3}{10} MR^2} = \sqrt{2}\ v/R \quad . \tag{13}$$

Planck's constant does not appear in this formula, testifying to its classical origin. As is appropriate in a symposium on semi-classical methods, $\hbar$ only appears at the last moment, when the collective quadrupole oscillation itself is quantized, to give a resonance energy

$$\hbar\omega = \sqrt{2}\ \hbar v/R \quad . \tag{14}$$

The result may be readily rewritten in a more obscure way, which hides its essentially classical content, by expressing the Fermi velocity v in terms of the particle density, which in turn is written in terms of the nuclear radius constant r_o. This gives[38]

$$\hbar\omega = \frac{(9\pi)^{1/3}}{\sqrt{2}} \frac{\hbar^2}{mr_o^2} A^{-1/3} \quad . \tag{15}$$

Using the nominal value r_o = 1.18 fm and taking for m the nuclear mass unit, 931.5 MeV/c^2, one finds[38]

$$\hbar\omega = 64.7\ A^{-1/3}\text{MeV} \tag{16}$$

This is the solid curve in Fig. 14. To what extent the very close correspondence of the (parameterless) theory with experiment is due to accidental cancellations of substantial corrections is a somewhat open question. From the derivation of eq. (16) one would be tempted to conclude that giant quadrupole resonances are specific to symmetric systems close to ideal separability, that one is witnessing directly the elastic response of a set of separable single-particle motions and that, owing to the adiabatic invariance of the action integral, it does not matter whether these motions are quantized or not. However, some caution is in order, since such elastic resonance behaviour might conceivably occur also in the Chaotic Regime if the amplitude of the oscillations were small enough to fit into one of the wiggles in the upper inset of Fig. 20. Now the maximum amplitude of the stretching oscillator α in its first excited state is

$$\alpha_{max} = \sqrt{3\hbar/\sqrt{(\text{Stiffness constant})M_{eff}}} \quad , \tag{17}$$

which gives for the maximum amplitude, ΔR_{max}, of the oscillating major axis of the nucleus, the formula

$$\Delta R_{max} = R\alpha_{max} = (8/9\pi)^{1/6}(150)^{1/4}A^{-1/3}r_o = 3.346\ A^{-1/3}\text{fm} \quad . \tag{18}$$

The reduced wavelength $\lambda\!\!\!^{-}$ of the fastest particle in a nucleus is related to r_o by

$$\lambda\!\!\!^{-} = (8/9\pi)^{1/3}r_o = 0.7747\ \text{fm} \quad , \tag{19}$$

and this might be a typical length dimension for the wiggles. Thus, for typical nuclei, there is no order of magnitude distinction between ΔR_{max} and $\lambda\!\!\!^{-}$. This seems to leave somewhat open the question of the possible existence of giant quadrupole resonances also in the Chaotic Regime. Still, one would certainly expect the resonances to be more clearly developed and to have smaller damping widths for systems close to the Ordered Regime.

Concerning the question of widths, many different estimates are available. According to the 1983 review article by Bertsch, Bortignon and Broglia[16], the relation of theory to experiment is quite intricate, but detailed microscopic calculations can often reproduce the observed widths to within a factor of two, or better. As I mentioned before, the damping *is* expected to be sensitive to details of nuclear structure, and no simple average treatment may be easy. But it *would* be nice to have a simple quantitative formula for the average trend of the points in Fig. 15. I will comment on this further after discussing the damping mechanism in the simpler Chaotic Regime.

In this regime two elementary dissipation formulae have been derived to describe the rate of energy flow from collective to single-particle degrees of freedom: the wall formula and the window formula[41-44]. The former is relevant for convex nuclear shapes (without a neck), the latter for shapes with a strong constriction, as in the late stages of fission or the initial stages of a nucleus-nucleus collision. The wall formula states that the rate of energy dissipation is given by

$$-\frac{dE}{dt} = \rho\bar{v}\oint(\dot{n} - D)^2 d\sigma \quad , \tag{20}$$

where ρ is the nuclear mass density, $\bar{v}$ is the average nucleonic speed (equal to $\frac{3}{4}v$) and the integral is taken over the nuclear surface, whose elements $d\sigma$ have normal velocities $\dot{n}$. (The quantity D is zero in the absence of overall translations and rotations; otherwise it gives the "drift" of the particles about to impinge on the surface element $d\sigma$ and is determined--under certain assumptions--by the requirement of linear and angular momentum conservation.) The wall-and-window formula states that for two nuclei interacting through a small neck or window of area $\Delta\sigma$, the rate of energy dissipation is given by two terms like eq. (20), one for each fragment, plus a window contribution of the form

$$-\frac{dE}{dt} = \frac{1}{4}\rho\bar{v}\Delta\sigma(u_t^2 + 2u_r^2) + \frac{16}{9}\frac{\rho\bar{v}}{\Delta\sigma}\dot{V}_1^2 \quad . \tag{21}$$

In the above, u_t and u_r are the tangential and radial components of the relative velocity of the two fragments and $\dot{V}_1$ is the rate of change of the volume of one of the fragments (equal to minus the rate of change of the other, since the total volume is assumed constant).

The physical content of the wall formula is illustrated in Fig. 23. A gas of particles bombarding a stationary element of surface (a piston[45]) exerts a pressure $\frac{1}{3}\rho\overline{v^2}$, but if the piston is moving away from the gas with speed $\dot{n}$, the pressure is reduced by a correction, which turns out to be $\rho\bar{v}\dot{n}$. If one now writes down the expression for the work done by the gas on a deforming container (by integrating pressure times displacement over the surface) one finds that the leading term does no work (if the container deforms at constant volume) but that the correction term leads to a one-sided flow of energy from the collective wall motion into the gas, i.e. to dissipation of energy.

An essential condition for the validity of the wall formula is the assumption that, at each instant, the surface elements of the container are bombarded by particles whose velocity distribution is that of a chaotic isotropic gas, unaware of the details of the state of motion of the surface (or its time history). This is the assumption of *randomization* of the particle motions, which is expected to be valid in the Chaotic Regime, but which is obviously not valid in the Ordered Regime. (As is readily verified, the leading term in the pressure does <u>not</u> cancel when integrated over the surface of, for example, a deforming Hill-Wheeler box. It is precisely this leading term, made up of the pressures of three one-dimensional gases, that produces the *elastic* response of a gas in the Ordered Regime.)

In Fig. 24 the validity of the wall formula, originally derived for a classical gas, is tested against numerical computer studies of both classical and quantal particles in a time-dependent potential well. The well is oscillating in a hexadecapole mode around the spherical shape. The solid curve is the wall-formula prediction for the excitation energy pumped into the gas during one period of the oscillation of a sharp-walled container. The triangles show the result of actually following, by means of a computer, the time history of a swarm of classical point particles and keeping track of their energy as a function of time (rather like the case of the Fermi acceleration mechanism shown in Fig. 3). The dot-dashed curve is the result of repeating the calculation for 112 *quantized* particles (56 doubly-filled neutron wave functions) by numerically solving the time-dependent Schrödinger equation. In this case the container was an almost sharp Woods-Saxon well (with a diffuseness about one seventh of a realistic diffuseness). The dashed curve is a similar quantal calculation for a well with a realistic diffuseness. (The reason for the decreased dissipation in the latter case has not been settled.)

The effect of symmetries on the dissipation is illustrated in Fig. 25. Here, a sharp-walled container is oscillating in a quadrupole ($n = 2$) or an $n = 6$ mode, with relative amplitudes chosen so that the wall-formula prediction for the dissipation is the same for both (the solid curve). The numerical study for $n = 6$, shown by the dotted curve, is close to the theoretical curve, presumably because the short-wavelength ripples in the $n = 6$ mode of oscillation produced adequate randomization of the particle motions. By contrast, the $n = 2$ case, with its approximately spheroidal symmetry (leading to separability) shows only about half the dissipation at the end of one cycle. One can also see a *bump* in the dissipation curve, where energy was stored elastically in the particles before being given back

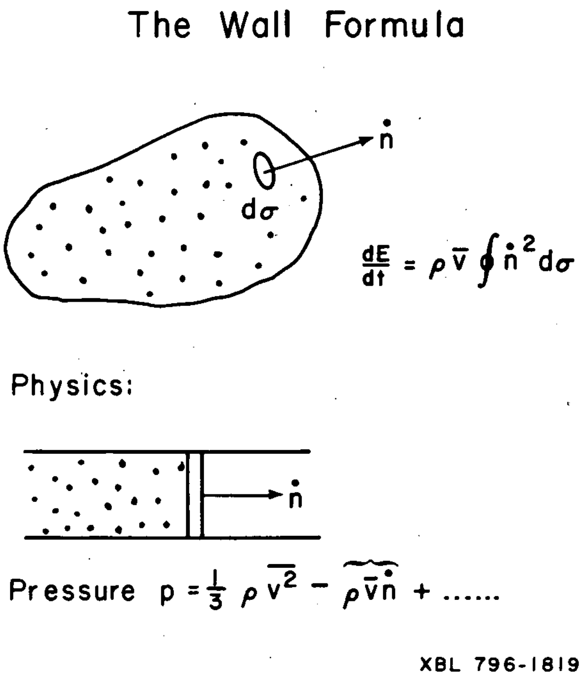


FIGURE 23
The physics of the wall formula

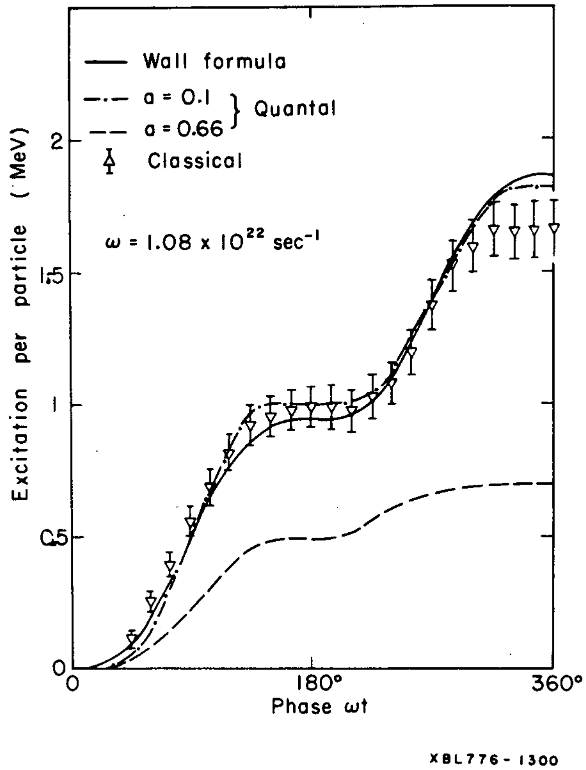


FIGURE 24
Tests of the wall formula against classical and quantal microscopic calculations[41]

in the second half of the oscillation. This is just what one expects when the particle motions are separable, as indicated by the dashed curve, referring to the oscillations of a Hill-Wheeler box. A similar elastic behaviour was observed in the numerical studies when the oscillation was made *exactly* spheroidal. (Note that a finite-amplitude deformation specified by a Legendre polynomial with n = 2 is only *approximately* spheroidal.)

The absolute magnitude of the dissipation predicted by the wall formula is very large. One finds that the characteristic damping time in which a typical kinetic energy of collective nuclear deformation would be dissipated is, in order of magnitude, given by the factor $R/\bar{v}$. This, as we saw, is the same order of magnitude as the period of the giant quadrupole oscillation. In fact, if one applied the wall formula to estimate the width of the giant quadrupole oscillation, forgetting that this is precisely the Ordered Regime where the wall formula is <u>not</u> supposed to be valid, one would find for the width Γ the result $\Gamma/\hbar = 3v/2R$. This is the solid curve in Fig. 15. (Note that in ref. 38, 1980 item, third equation on p. 401, the exponent of 9π should be 1/3 and not 1/2.) This curve is, indeed, close to the value of the resonance energy itself (Fig. 14) and a factor of about 3 larger than the experimental widths. If, on the contrary, one assumed perfect separability, the predicted width would be zero. The experimental widths are between the two limits, as might be expected.

The wall formula might be more nearly applicable to very deformed shapes, such as occur in fission, especially at excitation energies above 20 Mev or so, where shell effects are also washed out. The dashed curve in Fig. 12 shows fission-fragment energies in a dynamical calculation in which the wall formula was used.

To be more precise, the wall formula was made to go over (smoothly) into the wall-and-window formula as the nucleus necked-in and divided. This brings us to the question of the physical content of the window formula, eq. (21).

The essence of the first term in eq. (21) is illustrated in Fig. 26. It is simply that particles exchanged back and forth between two containers in relative motion act as a brake and tend to bring the containers to relative rest. The last term in eq. (21) describes the dissipation associated with the net flow of nucleons from one fragment to the other. (There is an analogy with the Joule-Thompson throttling experiment in thermodynamics.) Without going into the derivation of the wall and the wall-and-window formulae, let me just note that the idealization of a long-mean-free path gas in a volume-preserving container is used consistently, and this shows up in the structure of the formulae: they both contain only one physical property of the nuclear Fermi gas, the "flux factor" $\rho\bar{v}$. It follows that the basic time unit for the dissipation of the kinetic energy of relative motion by the window formula is again the short time $R/\bar{v}$, except that it is modified by the geometric

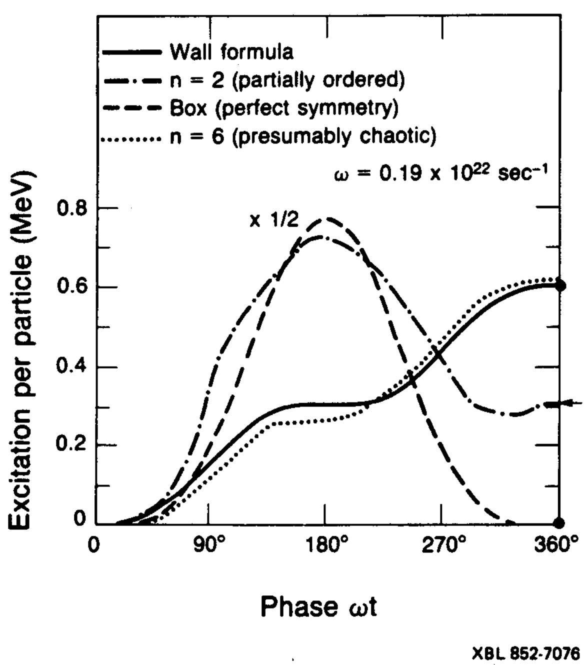


FIGURE 25
Effect of symmetries on the one-body dissipation[41]

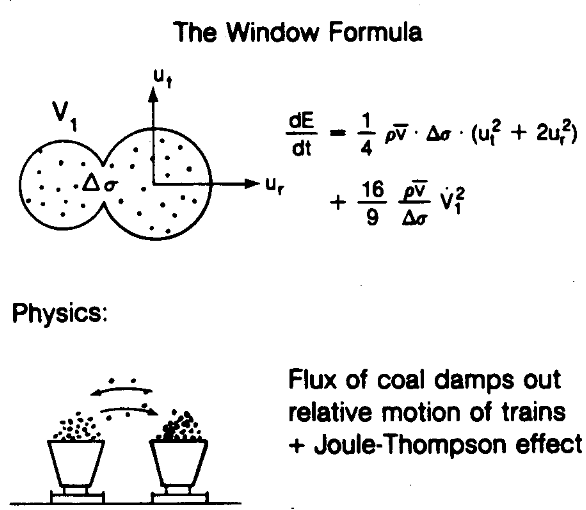


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FIGURE 26
The physics of the window formula

factor involving the window area $\Delta\sigma$ in the first part of eq. (21). Because the window area appears in the *denominator* in the second term, the window formula predicts also an inhibition of the mass flow between the fragments for small $\Delta\sigma$. These features are what is needed to account qualitatively for deep inelastic collisions of nuclei (i.e. sticking with little mass transfer).

Several studies are in progress to test the degree of quantitative correspondence between experiments and theories incorporating the wall and window formulae[46,47,48]. In the general case of an off-center collision between two unequal nuclei (where experimental data are most comprehensive), the theoretical problem is complicated because of the presence of the asymmetry degree of freedom and of angular momentum, which calls for at least three *more* (angular) degrees of freedom. The simplest case to analyze is a central collision of two equal nuclei--the inverse process to the symmetric fission illustrated in Fig. 12.

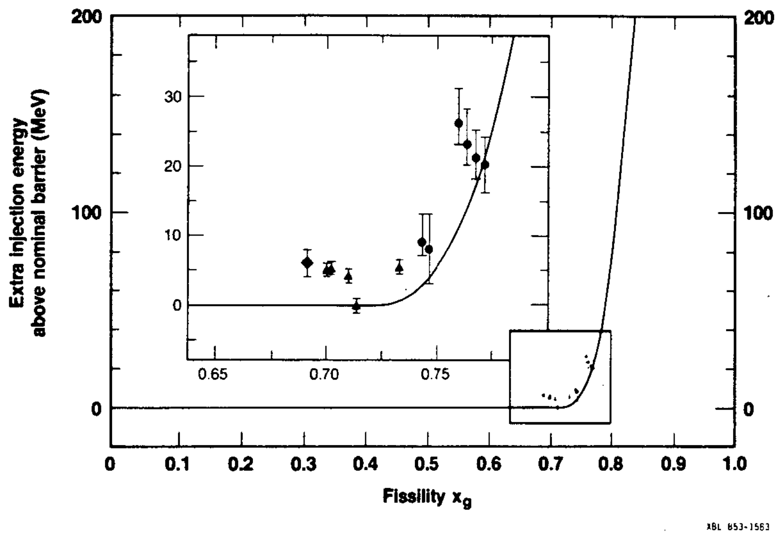


FIGURE 27

Extra injection energy above a nominal Coulomb barrier, necessary to make two (nearly) equal nuclei fuse into a compound nucleus[49]. The geometric mean fissility x_g is related to the conventional fissility x by $x_g = x/\frac{1}{2}(K^2 + K + K^{-1} + K^{-2})^{1/2}$, where K = (ratio of target and projectile masses)$^{1/3}$. This factor allows (approximately) for the slight inequality of the masses. The curve was calculated with a model incorporating the wall and window formulae[46]

Instead of the kinetic energy of the fission fragments, one now tries to predict the opposite: the collision energy necessary to make two equal nuclei fuse into a compound nucleus. In particular, how this energy varies as one goes up the periodic table. Of course, up to a certain critical size of the colliding nuclei, this energy is simply the energy necessary to bring the nuclei in contact, i.e. the Coulomb or contact energy. From then on the nuclear forces take care of the rest of the fusion process. But beyond a critical size, the electric forces are effectively stronger than the nuclear forces and the system will re-separate without fusing, unless it is given an extra push over the Coulomb barrier. This extra injection energy will be a function of the excess of the electric repulsion over the nuclear attraction or, in other words, a function of the excess of the fissility parameter x over some threshold value x_{th}. (The fissility parameter is proportional to the dimensionless ratio of the electrostatic energy to the nuclear surface energy.) The absolute value of the extra push (for a given system) may be expected to be a sensitive function of the dissipation, and its experimental determination provides a test of this aspect of a theory. The curve in Fig. 27 shows the extra energy calculated using the wall and window formulae, incorporated in a liquid-drop type model similar to the one used to calculate the fission fragment energies. The relevant experiments on nearly symmetric systems are, unfortunately, very few and not easy to interpret[49]. As a result, it is far from clear at the moment how close is the correspondence between experiment and the simple dissipation theory based on the Chaotic Regime dynamics.

7. QUANTAL CHAOS

I described above some simple results that one may derive in the extreme limiting cases of complete separability and complete chaos: elastic response in one case and strongly damped motion in the other. Cases of actual interest are often in the Intermediate Regime, where the situation is much less clear and progress much more difficult. In the classical studies of the transition from order to chaos, a useful tool is the examination of the phase space, in particular of the Poincaré sections. Such plots are not directly available in the case of quantal systems. One way of distinguishing order from chaos in such systems is to display the statistics of the level spacings in the energy spectrum. In the Chaotic Regime, energy levels must avoid crossings, and consequently the frequency of small spacings (in particular zero spacings) will be suppressed. On the other hand, levels with different quantum numbers don't care about one another and can cross if they like. In the case of complete lack of correlations between levels, a Poisson distribution of spacings is expected. In the case of a chaotic spectrum, a theoretical model resulting from diagonalizing a matrix with random matrix elements is proving very successful (the so-called Gaussian Orthogonal Ensemble, or GOE)[50]. Numerical studies of the transitions from one type of spectrum to another are being pursued for various simple potentials[50]. Three of those are shown in Fig. 28. Fig. 29 shows some level-spacing statistics for the stadium potential. In the lower part only levels with the same symmetry are included,

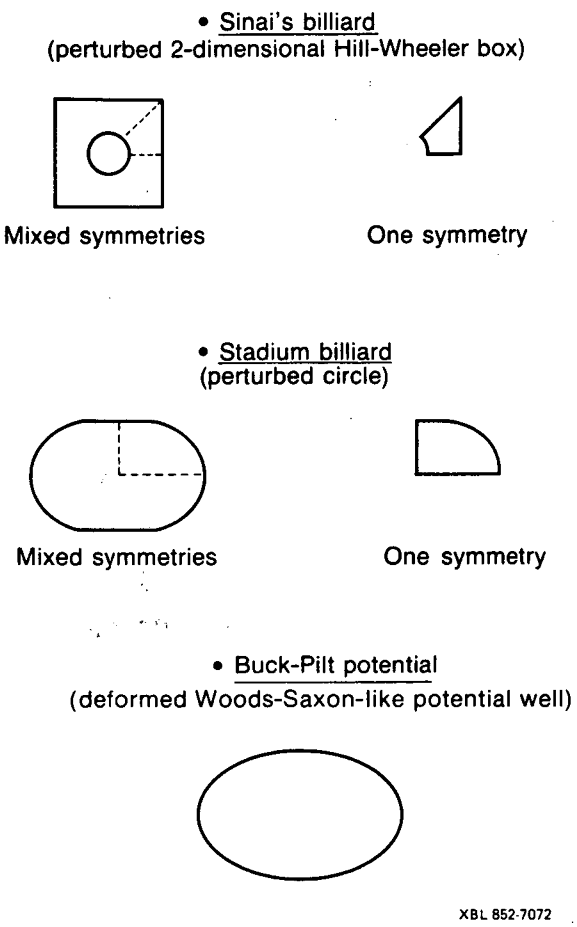


FIGURE 28
Several simple potentials for studying transition to quantum chaos

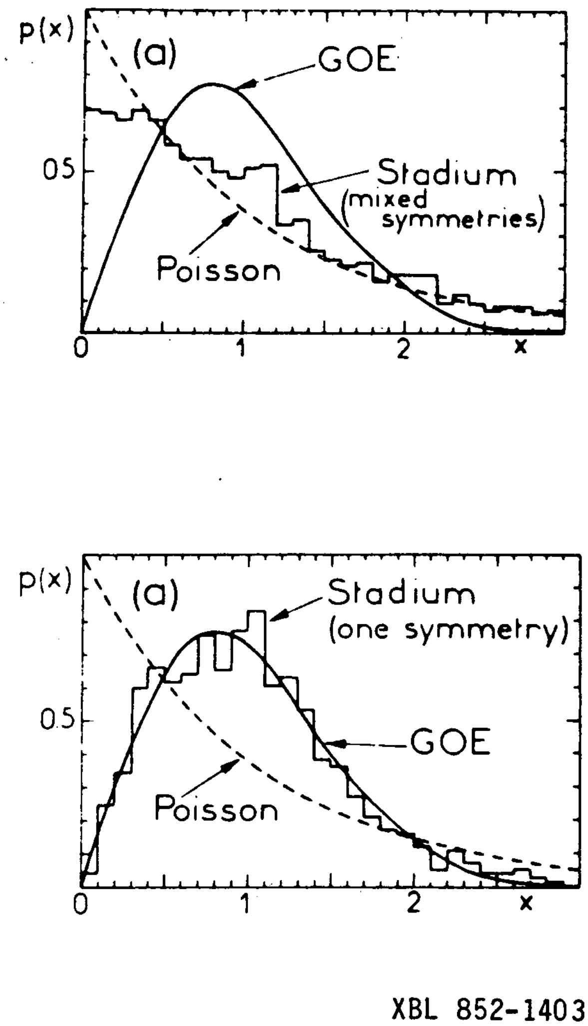


FIGURE 29
Statistics of level spacings in the stadium potential[50]

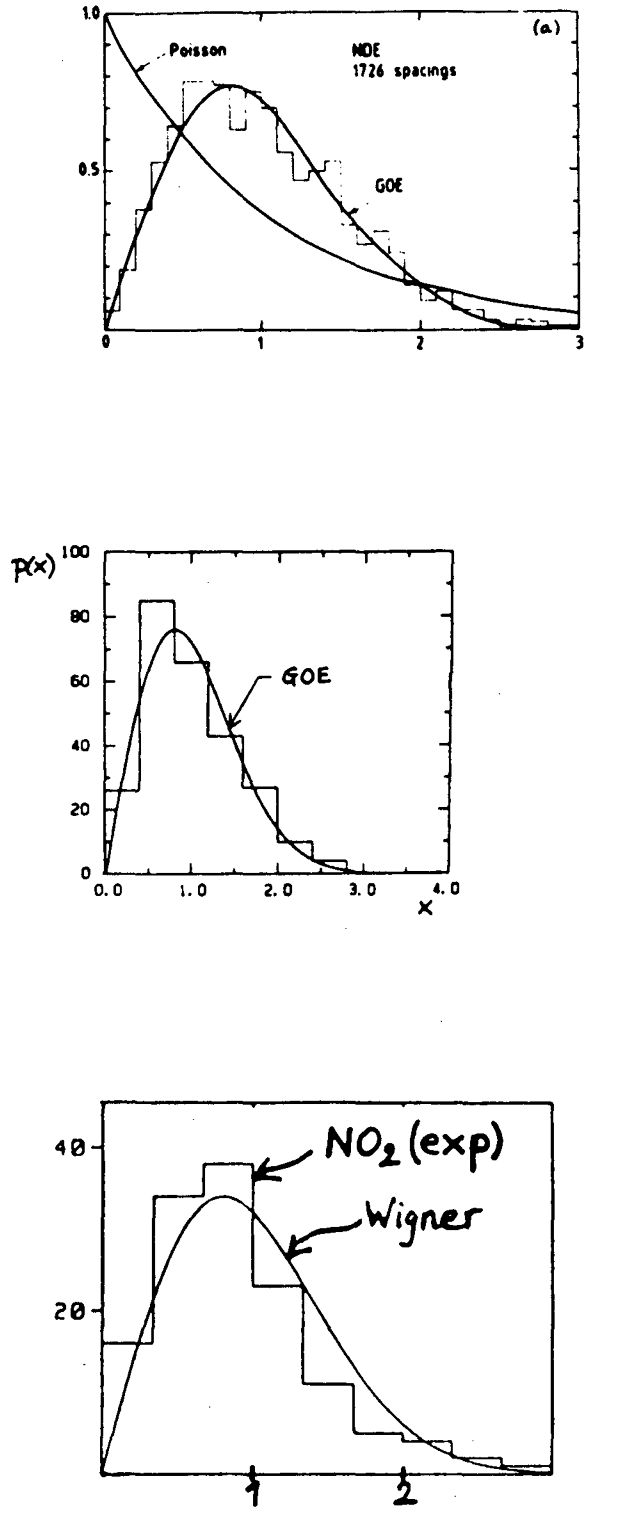
and the agreement with the GOE model is very good. In the upper part levels with several symmetries are present, many of the levels are allowed to cross, and the level spacings tend towards a Poisson distribution.

Fig. 30 shows some level spacing statistics taken from experimental spectra: nuclear, atomic and molecular[50]. The levels were selected to have all the same symmetries and follow fairly well the GOE predictions. Note that it is in the nuclear case that the most unambiguous test of the theory is possible at the present time.

These are only a few examples of current studies of the transition from order to chaos in quantal systems. Despite rapid progress being made, the field is much less advanced than in the classical case. The nuclear system, where order and chaos are tied to the problem of dissipation in collective dynamics, can be expected to contribute to this progress in the future.

8. RELATION TO BROADER PROBLEMS

Let me add one remark about the wider-context in which friction problems



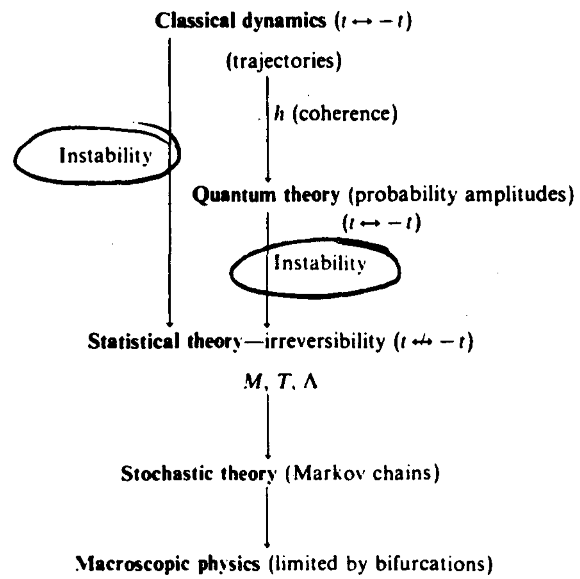


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FIGURE 30
Statistics of level spacings taken from nuclear, atomic and molecular spectra[50]

FIGURE 31
Prigogine's levels of description of the physical world[51]

are discussed: the question of irreversibility in general and the role of irreversibility in quantum mechanics, in particular. The recent studies of order-to-chaos transitions, which have revealed the fantastic intricacies and instabilities associated with this transition, have stimulated attempts at new approaches to these perennial problems. An interesting example is Prigogine's book "From Being to Becoming"[51] in which even the meaning of time receives a new interpretation. Fig. 31 shows a diagram taken from that book, in which Prigogine illustrates his discussion of the relation between classical dynamics, quantum theory, statistical mechanics and macroscopic physics. The "Instability" in this picture, which I have circled, refers to the transition from order to chaos. I hesitated to quote from Prigogine's book, because there are several things in it that I do not understand. However, I do share the belief that the relation between classical and quantum mechanics needs further clarification and that the transition from order to chaos may be an important element in this problem. A related question is whether it is not, in fact, misleading to continue juxtaposing classical and quantum mechanics, since "classical mechanics" is only a formal abstraction--invalid in practice--and there is in nature only one mechanics, namely quantum mechanics. From this point of view, the problem of measurement in quantum mechanics, with its references to large-scale, irreversible, classical measuring apparatus, should be restated as the clarification of questions arising when very large and very small *quantal* systems interact. But once the problem is stated in this way, one feels the need to generalize it immediately to the discussion of interacting systems of any relative sizes. Could it be that the standard exposition of quantum mechanics relies too heavily on the special case of interacting systems of hugely different sizes, with one of them actually taken all the way to the invalid classical limit? Could it be that the more general case continues to await a proper formulation?

The possible role of nuclear physics in this wider context will probably be modest and indirect: a nucleus is a nice quantal system of moderate size, which exhibits both ordered and chaotic behaviour. Its study will certainly contribute to the technical understanding of the transition from order to chaos in quantal systems in general. But to what extent this might contribute to the clarification of the broader problem that I hinted at, remains to be seen.

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LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720