Problem-Solving Strategy Selection in Relation to Formal Schooling

Mennat-Allah Saleh (mennat-allah.saleh@student.guc.edu.eg)
Department of Media Engineering and Technology
German University in Cairo
New Cairo City -Main Entrance Al Tagamoa Al Khames, Egypt

Christian Sturm (christian.sturm@guc.edu.eg)
Department of Media Engineering and Technology
German University in Cairo
New Cairo City -Main Entrance Al Tagamoa Al Khames, Egypt

Abstract

A study of the literacy-generated cognitive cultural gap was carried out on subjects of different literacy background ranging from illiterate individuals to university students in different majors. The characteristics that aid literate and illiterate people in solving mathematical problems efficiently were identified and analyzed. A field research was carried out in the field of algorithmic problem solving and in the reasoning domain, followed by constructing a software cognitive model to represent the findings. Findings showed that in both domains cognitive ability did not improve with level of literacy, rather the formality of the problem solving strategy selected demonstrating a link between these two domains.

Keywords: Cognitive Psychology, Cognitive Modeling, Problem Solving, Literacy, Deductive Reasoning.

Introduction

The interaction with illiterate people is a common experience in Egypt. More than one third of the adult population is not able to read and write (UNICEF, 2012). One could conclude that due to these every-day encounters, illiterates would be appreciated as they represent an important part of Egyptian's workforce and are preservers of Egyptian's rich cultural heritage. The Egyptian society, however, marks clearly its division not only based on economic power. Throughout the country, illiteracy is commonly associated with a lack of mental and cognitive capabilities that leads to a tremendous depreciation of this group by the literate part of the society (Hollingshead, 1975). Given the nature of this stereotype, the area of problem-solving strategies has been chosen for comparison. The motivation of the research is to explore this cultural gap and find ways to bridge it.

Several approaches to cultural differences in problem-solving have been explored in the past. Tedre et al. have shown how computation is done across different cultures and looked at the education of computer science students (Tedre, Sutinen, Kahkonen, & Kommers, 2003). Gerdes looked in a similar way at the cultural differences on math and mathematical problem solving (Gerdes, 2005). Several researches suggest that human reasoning is based on building personalized mental models; hence using mental models in attempting to formalize and represent reasoning is a valid approach (Knauff, Mulack, Kassubek, Salih, & Greenlee, 2002). In addition to formulating the models, the field of cognitive psychology also draws attention to the basic thinking principles that are present in all individuals, but are used differently across cultures. It was used in combination with cross cultural psychology, which is the study concerned with the thinking principles that are generated from cultural differences (Adler & Gielen, 2001). This field has been used by many researches to study how education affects cognitive behaviors, memory, problem solving and logical reasoning (Segall, Dasen, Berry, & Poortinga, 1999).

Based on this previous work and the given circumstances in Egypt, it was decided to look at reasoning and algorithmic problem solving tasks together with the mental models created during the process for both literate and illiterate subjects including different educational domains.

Problem Space

Domain Selection

It was hypothesized that the level of a person's formal schooling is related to the level of formality regarding their approach to the selection of problem solving strategies.The functionally illiterate subjects in this context are compared to the formal schooling subjects as having a lower level of schooling formality (Kosmidis, Zafiri, & Politimou, 2011). Hence, domains that need a formal and strategic approach were selected for this research. The goal was to identify and quantify problem-solving strategies easily. Numeric problem solving, algorithmic problem solving and reasoning meet this requirement. As the relationship between literacy and numeracy has been widely studied in this context already, the two latter domains were selected.

Algorithm Domain

The main focus while testing the algorithmic domain was to identify the subjects strategies in conjunction with the level and type of education while solving a procedural algorithmic problem. The subjects were asked for a self-report on their strategies. This report was contrasted with the steps they actually took in solving the problem. The Towers of Hanoi problem was used to test this domain due to its strong mathematical basis and the possibility to easily adapt it to different cultural contexts. This was specifically important for the illiterate subjects that are not used to any kind of formal testing situations encountered in the lab. The most important characteristic of the Towers of Hanoi consists in the variety of correct strategies to solve it. Hence, this did not limit the subjects options to only one correct way, but would allow them to select the strategy they are most comfortable with based on their personal literacy backgrounds (Gunzelmann & Anderson, 2001). The expected problem solving strategies were: analogy, divide and conquer, mean-ends analysis, trial and error, random strategy, research and working backwards (Chiew & Wang, 2004). In addition to the problem solving strategy, the mental model created by the solver during the test was of special interest, too. It was expected that solvers would have one of the following mental models:

- Formal Representation: Using logic and formal mathematical representation for the problem.
- Previously-Prepared Mental Model: Solver solves the entire problem in their mind before solving on board.
- On-the-go Mental Model: Subjects build an image of the disks in their mind and use it to solve the problem virtually before physically, a few steps at a time.
- No Representation

A dynamic, additional element in form of a random disk was introduced by the test facilitator during the test in order to be able to distinguish between subjects that would apply a previously prepared model and subjects that would create the models on-the-go. The subject was asked to proceed with the solution using the additional disk. This procedure provided the opportunity to check, in addition, the degree of agility of the subjects strategy as well as their understanding of the problem. The point in time as well as the type of disk added was determined by the facilitator during the test based on the subject's performance.

Reasoning Domain

Reasoning was used as a problem solving domain in this research based on the previous work of Tulviste et al. (1978). They have shown that reasoning is a skill that improves strongly with the level of literacy (Tulviste, Riikliku, & Toimetised, 1978). Several types of reasoning were considered for this study. Syllogistic deductive reasoning was selected as studies have shown that it is a skill that comes with formal schooling and evolves to be used in everyday life on different types of problems. In addition, the mental proof theory shows that syllogistic deductive reasoning is approached by solvers using one of the following techniques (Knauff et al., 2002; Rips, 1994): spatial reasoning, visual reasoning or reasoning using formal logic, entailing a variety of strategies with different levels of formality that relates to the level of the subjects formal schooling. The Zebra puzzle was suggested as a research test question (Stangroom, 2010). The approach needed to solve the problem, however, requires that the subjects use a formalized written schedule. This would have forced them to select one strategy over the others for correctness instead of their own personal preference. Moreover, it would have meant that illiterate subjects would have never been able to solve it as they are not able to use pen and paper for their solution. Therfore, a smaller version of the puzzle with only 3 instead of 5 variables was created as stated below:

A street has 3 houses, each house has a different color, and in each house the owner has a different nationality and owns a different pet. Given these following clues, what is the color of the fish's house?

- The cat lives in the center house.
- The green house is on the left.
- The French lives in the blue house.
- The German owns a dragon.
- The Egyptian lives in the red house directly to the right of the dragon.

Subsequently, the subjects were asked to answer the following question: what is the color of the fish's house?

The spatial memory is used for solving the Towers of Hanoi Problem while deductive syllogistic reasoning tasks tend to make use the verbal memory (Handley, Capon, Copp, & Harper, 2002). Studies have shown that the performance of both types of memory is independent from each other. Measuring the performance, however, was not the main objective to measure in this study but rather the strategy selection. This selection, as mentioned before, is hypothesized to depend on the subjects formal schooling independent from the type of memories. Therefore, it is expected that the subjects show a similar selection of strategies for both domains chosen here.

Hypothesis and Test Design

General Hypothesis

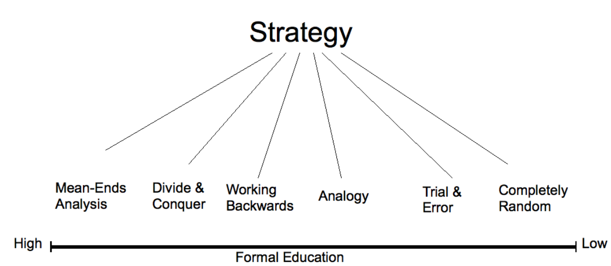

Figure 1: Towers of Hanoi strategy selection tree

It was hypothesized that the strategy selected for the Towers of Hanoi would be based on the formality of the subject's education as shown in figure 1. In addition, it was hypothesized that the subjects may move between different strategies when solving the problem. This shift, however, would be minimal as the general tendency is expected to be clear and stable.

Test Design

The following profiles were defined for the test subjects in order to test the hypotheses mentioned above:

- 20 computer science university students, who received training in formal logic.
- 20 Illiterate subjects, who are now in their early stages of literacy classes. The short class-room experience equipped them with the necessary preparation and tolerance towards participating in the experiments.
- 20 applied arts university students, who have only received formal schooling but no training in formal logic. They have, however, strong spatial reasoning skills.

The content of the test has been adapted to Egyptian illiterate. The deductive reasoning problem became the narration of a story replacing nationalities with common names and pets with farm animals. The Towers of Hanoi problem was contextualized by representing the disks with water buckets of different sizes and the pegs with floor tiles. The explanation of the problem was based on a story to justify the reason for moving the buckets.

During the test, the subjects' age, study-major (if applicable) and duration in literacy class (if applicable) were captured. This data defined the independent variables as follows: Level of literacy, type of education and training in formal logic. In addition, the subjects were asked questions regarding their approach, mental representations and changes in strategies. The strategy was classified as either: none, on-the-go or prepared beforehand. The following dependent variables were recorded: completion of task (y/n), time for task completion, number of attempts in deductive reasoning, approach to deductive reasoning, Towers of Hanoi strategy selection, Towers of Hanoi strategy change, Towers of Hanoi adaptation to dynamic disk addition.

Subject-Specific Hypothesis

Figure 2 and tables 1 and 2 represent the three hypotheses of each subject background.

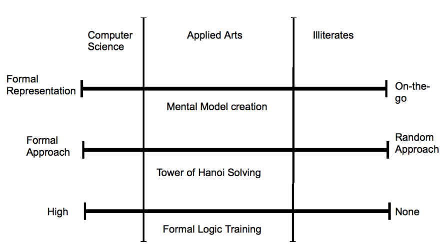

Figure 2: Towers of Hanoi Hypothesis

Subject	Mental Representation
Computer Science	Numerical and Formal
Applied Arts	Visual model
Illiterates	Visual model or none

Table 1: Towers of Hanoi Mental Model Hypothesis

Subject	Approach	Memory
Computer Science	Formal Logic	Verbal Memory
Applied Arts	Spatial Reasoning	Spatial Memory
Illiterates	Intuition	N/A

Table 2: Deductive Reasoning Hypothesis

Test Results and Discussions

Pretest

A pretest was conducted using the above test design on engineering students who have studied the Constraint Programming course, applied arts students and Illiterate subjects. No modifications were recommended for applied arts and illiterate tests. . All of the engineering students, however, were already familiar with the solution to the Towers of Hanoi problem. Therefore, another control group with a lower level of formal training was added. This new group consisted of computer science students in their sophomore year. The results recorded for the deductive reasoning question were observed by asking the subject questions about their strategy and deducing their approach from the paper they used in solving. After the test, it was concluded that there is no observed difference when using spatial or visual reasoning and that both strategies may have different mental representations, but they use the same inference technique.

The Main Test Results

The Towers of Hanoi problem was analyzed in two steps: the observation and the inference. The observed results are the strategies recognized by the facilitator. They were recorded because it was difficult for some of the subjects to properly explain their own approach. They included:

- The Random strategy was recorded when the subject showed no learning pattern and moved the disks haphazardly often reaching no correct solution.
- The Trial and Error strategy was recorded when the subject first started moving disks randomly and then appeared to recognize incorrect moves and avoided repeated them, this was often observed when the board reaches a similar state to a previously encountered stage and the subject seemed to recall their previous error.
- The Wrong Towers strategy was recorded when the subject used the right pattern of movements but choose the wrong destination for the movement of the first disk.
- The Correct strategy was recorded when the subject used the right pattern of movements from the first move without any errors.

The inferred results were reported by the subjects' own explanation of their strategy. They were used in addition to the observed strategies for the final analysis and included:

- Random strategy was recorded when the subject says that they did not understand the game and they just moved the disks in any fashion attempting to reach a solution.

- Trial and Error strategy was recorded when the subject says that at first they started without knowing how to proceed, but with time they understood the game and learned from their errors.

- Pattern Recognition occurs when the subject identifies patterns. A pattern is a group of disks stacked in a certain way on the board. The pattern recognized by subjects were either the 3-disk pattern, disks stacked on top of each other or the flat pattern. The flat pattern means that three disks are placed on the board, while each disk is placed on a different tower. The subjects would always follow a learned set of moves upon identifying any of the patterns.

- Mean Ends Analysis was recorded when the subject says they solved the problem by breaking it down and attempting to move the largest disk to the destination first, which required moving all the disk stacks above it to the intermediate tower and so forth. This strategy was recorded when the subjects identified the recursive nature of the problem.

The following are the observed and inferred results for each group of test subjects:

- Computer science students trained in constraint programming were almost equally divided between using the formal approach (12) and spatial reasoning (8) for the deductive reasoning problem as opposed to the original hypothesis of a high tendency for formal approach. Those who choose a formal approach tended to use mean-ends analysis to solve the Towers of Hanoi before and after adding the dynamic disk. The ones that used spatial reasoning were divided between trial and error and mean-ends analysis before adding the disk and continued to use their selected strategy after addition of the disk.

- Engineering subjects tended to choose spatial reasoning (17) over formal approach (3) for solving the deductive reasoning problem, as opposed to the hypothesis that they would have a tendency for a formalized approach. For the Towers of Hanoi, most of them used mean-ends analysis before disk addition. Only a few used pattern recognition, too. The majority that used mean-ends continued after the disk addition with mean-ends, whilst only a few applied Trial and Error. Those who selected pattern recognition were equally divided upon the four strategies after disk addition.

- Applied Arts subjects mostly selected spatial reasoning (18) as hypothesized. A surprising 10%, however, approached the deductive reasoning problem with a formal strategy (2). The applied arts subjects were the most diversified in their strategy selection showing that their education did not seem to limit their approach. They were equally divided upon all strategies before and after disk addition except for the unselected random strategy.

- As hypothesized, 100% of the illiterate subjects (20) used spatial reasoning to solve the deductive reasoning problem. The subjects were evenly divided between all the four strategies of the Towers of Hanoi before disk addition. This contradicted the initial belief that all illiterates would approach the problem in a random way. Upon dynamic disk addition, most of the subjects would either continue using their current strategy or use a strategy that is one degree less formal according to the hierarchy shown in figure 3. The results clearly indicate that illiterates were affected by the disk addition process.

Statistical Results Analysis

The different strategies were numbered according to the pyramid shown in figure 3. In order to determine the significance of the results, the Kruskal Wallis test was used.

Subject	N	Min	Max	Median
Constraint Programming	20	4.0	2.0	4.0
Arts	20	2.0	4.0	2.0
Engineering	20	3.0	4.0	4.0
Illiterates	20	1.0	4.0	2.0

*p < .01

Table 3: The statistical analysis of all groups before addition
H=12.65*

Subject	N	Min	Max	Median
Constraint Programming	20	4.0	2.0	4.0
Arts	20	2.0	4.0	2.0
Engineering	20	2.0	4.0	4.0
Illiterates	20	2.0	4.0	2.0

*p < .01

Table 4: The statistical analysis of all groups after addition
H=12.30*

Both tests were statistically significant at a 1% level of significance. Therefore, we can state that the literacy and formal schooling background significantly affects the selection of problem-solving strategies when solving the Towers of Hanoi problem. In addition and in line with the initial assumption, there was no correlation found between the task performance and the level of formality of the selected strategy (p =.08). This means that the use of formal methods did not lead to a faster performance in comparison to using spatial reasoning. Finally, the effect of literacy on the type of mental model created in the Towers of Hanoi problem was tested. The results showed no statistical significance (p=.94). This shows that all four subject domains created spatial, beforehand or no mental models depending on a different parameter other than their educational backgrounds. It also showed that strategy selection in the Towers of Hanoi problem is not entirely dependent on the mental model created as opposed to the original hypothesis, since the selection of the strategy proved to be dependent on the subject's educational background.

The original hypothesis of the tree of problem selection strategies was not confirmed. It was rather observed that the strategy selection was done using the pyramid shown in figure 3. The strategies are ranked according their proximity to the most optimum Towers of Hanoi algorithm with one being the least optimum. It was observed that subjects would start at a level of the above pyramid depending on their level and type of formal education, their working background and their understanding of the problem and rules. Subjects go up the pyramid but never down. No subject was observed to move 2 levels up the pyramid, they only move up 1 level per game or none at all. Once an additional disk was introduced, however, most subjects would move down the pyramid considering the new situation a new problem. Some of the subjects did not even capitalize on the previous knowledge they obtained before the addition of the new disk.

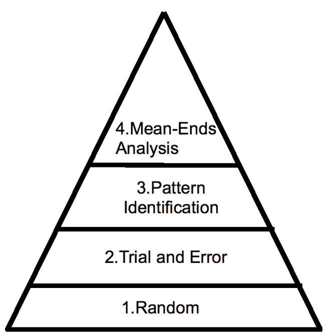

Figure 3: Towers of Hanoi Observations

Software Model

As a first step towards possible future prediction of cognitive behaviors of subject profiles, a software model was used to represent the strategies observed. It was implemented to formalize and represent the test results and experiment with new strategies. It was also implemented as a guidance for predicting problem solving strategy selection and execution based on a given subject profile. Having such a tool can help educational institutes in understanding the mindset of their target groups as well as guide them through their cognitive development needs which will assist in curricula formation.Four agents were created, one for each subject background and the agents navigated between the strategies implemented below according to the percentage of subjects within that background recorded to use that strategy. The model was created on Java as its object-oriented nature facilitated the implementation and its high-level characteristic allowed enough abstraction to represent only the details examined during the tests.

Modeling Reasoning

The reasoning problem strategies were modeled separately for formal representation and spatial reasoning.

Formal reasoning was modeled using Constraint Programming. Constraint Programming (CP) is a recent paradigm based on artificial intelligence and declarative programming (Rossi, Beek, & Walsh, 2006). Its advantage over other programming methodologies is its abstraction. The houses are represented as a data domain from 1 to 3 and the pets, nationalities and colors as sets of variables that associate with these domains. By placing the puzzle restrictions above on the representation, the CP solver finds the house that has no pet and allocates the fish to it.

The spatial reasoning model was simulated the way the solvers described their approach. A list of possible streets was kept in the subject's brain at a time. Each street had its unique configuration of houses. Upon presenting the subject with an additional clue from the puzzle, the subject attempts to add this to his current streets list. A clue can either be added to an existing street, if it can be merged with one, delete an existing street, if it presents a contradiction showing the initial street was not correct, or create a new street, if that clue cannot be represented sufficiently within the existing context. At the end of the puzzle, the subjects have eliminated all incorrect streets and are left with one street.

Modeling Towers of Hanoi

The following four strategies are modeled separately:

- Random Strategy: Each move entails moving a randomly selected disk to a random tower.
- Trial and Error: The trial and error strategy implementation was divided into two modules: representing memory and using learning, and representing how the subject learned from their movements. Only the first module was implemented and the second was left as future work. The approach represented the subject's memory as a pre-set number of movements that they have executed. The memory starts as initially empty. Before undergoing any movement, the agent checks if the current game uses a configuration that the subject has already learnt, in this case it switches to the pattern identification strategy below. If the game represents a pattern that the subject did not learn yet, it marks the current move. The subject selects a disk using a random generator in this case. After the second module mentioned above will be implemented, however, it will be constricted by learning. Once a move is executed, it is saved into the subject's memory. If there is no space in the memory, the oldest move is deleted. Now, the agent checks upon the last sequence of moves starting from the marked one. If they have solved the pattern correctly up to the pattern threshold, then the subject learned the pattern. A pattern threshold is the initial number of difficult moves the subject needs to do to understand a pattern. The rest of the moves then come easily as described and observed during tests.

- Pattern Identification: The 3-pattern and flat pattern as described in Test Results and Discussions section are implemented. The first disk destination is selected randomly.

- Mean-Ends Analysis: The algorithm moves all disks smaller than the *n*th disk to the intermediate tower. The *n*th disk is then moved to the destination tower. Finally, all the disks at the intermediate tower are moved on top of the *n*th disk at the destination using the same fashion.

Conclusion

This research has tested the hypothesis that *cognitive ability improves with literacy* is a common misconception. It is rather the case that the selection of given strategies is affected by the literacy background.. The hypothesis was proven correct in both the domain of reasoning and algorithmic problems. The relationship between these two domains was also examined in this research. It was hypothesized that individuals who select a formal problem solving strategy in one of them would select a formal one in the other. This formality would be directly traced back to their level of formal schooling. The hypothesis was confirmed. It was shown that, despite the fact that deductive reasoning is located in a different part of the working memory than algorithmic problem solving and that both cognitive processes are completely independent, the level of formal schooling equally impacts both.

A software model was based on the findings of the research and represented agents from the following literacy backgrounds: Illiterates, Computer Science, Constraint Programming and Applied Arts students. The agents used the different cognitive approaches of each subject domain to solve the Towers of Hanoi problem and a deductive syllogistic reasoning problem adapted from the Zebra puzzle.

Future Work

The software model can be enhanced by adding the second element of the Trial and Error learning system using reinforcement learning technique. It may also be broadened after adapting more tests to present a general thinking pattern in the domains of algorithmic problem solving and reasoning by the agents. This would mean that the model, given any newly introduced problem within these two domains, would be able to simulate the behavior of a given subject's profile. Finally, the model may be built on a pre-existing cognitive framework which will improve its accuracy and make it more representative of the human brain.

This model can be used to enhance the understanding of the different cognitive profiles in the Egyptian society. It can be used by Education Scientists to understand the cognitive gap generated by the literacy cultural gap and make use of the country's "cultural capital" as referred to by Pierre Bourdieu (Bourdieu, 1984). This capitalization can best be practiced by analyzing the reason behind these cognitive gaps and adapting the school, university as well as literacy classes curricula to make use of this information. In addition, the newly improved model can be used to test how students would think and adapt to different problems presented to them to predict their understanding and cognitive behavior.

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