Title
Application of a Pulsed, RF-driven, Multicusp Source for Low Energy Plasma Immersion Ion Implantation

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RESONANCES THAT OVERLAP

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ABSTRACT

When a final state can be reached by the production of resonances in different subsystems, the usual approximation is to add them. We show that the "exact" treatment introduces correction terms which are quite small.

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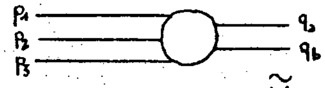
I. COMPETING CHANNELS

Let $T(p_1\, p_2\, p_3\, ,\, q_a\, q_b)$ be the amplitude for the process



$a + b \longrightarrow 1 + 2 + 3$ and $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{M}_3$ the elastic scattering amplitudes for the pairs (23), (31), (12). The discontinuity of T with respect to $s_i = (p_j + p_k)^2$ round the threshold $s_i^T = (m_j + m_k)^2$ will be noted $[T]_{s_i}$. $\tilde{M}_i(p_j p_k p_j' p_k')_{s_i - i\varepsilon}$ is the value of $\tilde{M}_i$ at $s_i - i\varepsilon$, sometimes written $\tilde{M}_i(-)$.

Our purpose is to solve the set of 3 equations

$$(1) \quad [T]_{s_i} = \int \tilde{M}_i(p_j\, p_k\, p_j'\, p_k')_{s_i - i\varepsilon}\, T(p_j'\, p_k'\, p_i\, ,\, q_a\, q_b)\, \frac{d^3 p_j'}{2E_j}\, \frac{d^3 p_k'}{2E_k}\, \delta_4(p_j + p_k - p_j' - p_k')$$

We now define $M_i(p_i\, p_j\, p_k\, ,\, p_i'\, p_j'\, p_k') = 2E_i\, \delta(\vec{p}_i - \vec{p}_i')\, \tilde{M}_i(p_j\, p_k\, p_j'\, p_k')$

and the two and three particle phase space integration :

$$\tilde{*} = \int \frac{d^3 p_1}{2E_1}\, \frac{d^3 p_2}{2E_2}\, \delta_4(P - (p_1 + p_2)) \qquad * = \int \frac{d^3 p_1}{2E_1}\, \frac{d^3 p_2}{2E_2}\, \frac{d^3 p_3}{2E_3}\, \delta_4(P - (p_1 + p_2 + p_3))$$

so that the final aspect of (1) is $[T]_{s_i} = M_i(-) * T$.

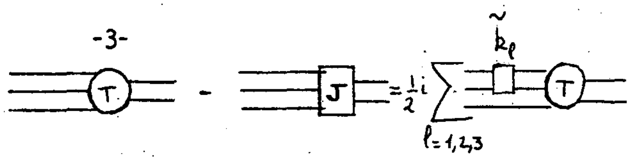
II. THE K MATRIX INTEGRAL

1. Obtaining a J With no Branch Points

P. Graves Morris has shown [1] that one can define a function J related to T by an integral operator, such that J has no discontinuity at the two particle thresholds. Introducing the standard 2 body k-matrices: $\tilde{M}_i - \tilde{k}_i = \frac{1}{2} i\, \tilde{k}_i\, \tilde{*}\, \tilde{M}_i$ (2)

$$\left.\begin{aligned} &T(p_1 p_2 p_3, q_a q_b) - J(p_1 p_2 p_3, q_a q_b) = \tfrac{1}{2} i \int \frac{d^3 p_1'}{2E_1'}\, \frac{d^3 p_2'}{2E_2'}\, \frac{d^3 p_3'}{2E_3'}\, \delta_4(P - (p_1' + p_2' + p_3'))\, x \ldots \\ &\ldots \left[2E_1\, \delta(\vec{p}_1 - \vec{p}_1')\, \tilde{k}_1(p_2 p_3 p_2' p_3') + 2E_2\, \delta(\vec{p}_2 - \vec{p}_2')\, \tilde{k}_2(p_1 p_3 p_1' p_3') + 2E_3\, \delta(\vec{p}_3 - \vec{p}_3')\, \tilde{k}_3(p_1 p_2 p_1' p_2')\right] \ldots \\ &\ldots T(p_1' p_2' p_3', q_a q_b) \end{aligned}\right\} (3)$$

Or pictorially



Conversely if we choose for J any function of the momenta free from discontinuity in the three subenergies, this is enough to insure that T obeys (1). To prove this one can use a consequence of (3) :

$$\left(1-\left(\tfrac{1}{2} i k_1\right)^2\right) * \left(T(+)-T(-)-i\, M_1(-)*T(+)\right) = \left(1-\tfrac{1}{2} i k_1\right) * \left(\phi_1(+)-\phi_1(-)\right)$$

where $\phi_1 = J - k_2 * T - k_3 * T$

2. Making the kernel smooth

We shall take the driving term J to be $J = \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3$ with $\mathcal{J}_\ell = \frac{1}{\Delta_\ell} k_\ell * J_\ell$. The Δ_ℓ factors ($\Delta_\ell = q_\ell / 4\sqrt{s_\ell}$) cancel the threshold behaviour of the phase space integrals. Equation (3) is, however, rather singular when $\tilde{k}_\ell$ are pole terms, i.e. when we deal with resonances. We shall recast it into an equivalent form with a smooth kernel.

It is natural to seek solutions of (3) of the same form as J : namely $T = \sum_i \frac{1}{\Delta_i} M_i * T_i$ so that

$$\frac{1}{\Delta_1} M_1 * T_1 + \frac{1}{\Delta_2} M_2 * T_2 + \frac{1}{\Delta_3} M_3 * T_3 - (\mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3) = \frac{1}{2} i \left(k_1 + k_2 + k_3\right) * \left(\frac{1}{\Delta_1} M_1 * T_1 + \frac{1}{\Delta_2} M_2 * T_2 + \frac{1}{\Delta_3} M_3 * T_3\right)$$

substituting (2) yields

$$-(\mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3) = -\sum_j \frac{1}{\Delta_j} M_j * T_j + \frac{1}{2} i k_1 * \left(\frac{1}{\Delta_2} M_2 * T_2 + \frac{1}{\Delta_3} M_3 * T_3\right) + \dots$$
$$+ \frac{1}{2} i k_2 * \left(\frac{1}{\Delta_1} M_1 * T_1 + \frac{1}{\Delta_3} M_3 * T_3\right) + \frac{1}{2} i k_3 * \left(\frac{1}{\Delta_1} M_1 * T_1 + \frac{1}{\Delta_2} M_2 * T_2\right)$$

When J has the previous form $\frac{1}{\Delta_\ell} k_\ell * \mathcal{J}_\ell$ one solution is

immediately suggested:

$$\left.\begin{aligned} T_1 - J_1 &= \tfrac{1}{2} i \Delta_1 \left(\tfrac{1}{\Delta_2} M_2 * T_2 + \tfrac{1}{\Delta_3} M_3 * T_3\right) \\ T_2 - J_2 &= \tfrac{1}{2} i \Delta_2 \left(\tfrac{1}{\Delta_1} M_1 * T_1 + \tfrac{1}{\Delta_3} M_3 * T_3\right) \\ T_3 - J_3 &= \tfrac{1}{2} i \Delta_3 \left(\tfrac{1}{\Delta_1} M_1 * T_1 + \tfrac{1}{\Delta_2} M_2 * T_2\right) \end{aligned}\right\} (4)$$

The new kernel is very convenient for computation purposes, and is reminiscent, formally, of the full solution to our problem in the non-relativistic case.*

III. CONNECTION WITH THE ISOBAR MODEL

We shall see later on that the right-hand side of (4) is usually small. At order zero : $T_k = J_k$ which is the isobar approximation if the partial waves of J_k have just the minimum variation required by analyticity at thresholds (centrifugal barriers).

At first order: $T_j = J_j + \frac{1}{2} i \Delta_j \sum_{k \neq j} \left(\frac{1}{\Delta_k} M_k * J_k\right)$

1. Partial Waves

The effect of the correction terms are best seen in a partial wave analysis. As usual (Ref. [2]) we define $\tau^{JL\ell}$ such that**

$$T_i = \sum_{JL\ell MNm} \tau_i^{JL\ell} \left(\frac{Q_i}{Q_r}\right)^L \left(\frac{q_i}{q_r}\right)^{-\ell} \langle LN\ell m | JM\rangle\, Y_L^N(\Omega_i)\, Y_\ell^m(\omega_i) \langle JM | in\rangle$$

which is the spherical harmonics expansion for subsystem i. $|in\rangle$ is the incoming state, and other factors are explained in the

* While this work was in progress, a considerable simplification of the non-relativistic case has been achieved in a pre-print of R.T. Cahill (Australia National University of Canberra)

** The unexpected $\left(\frac{q_i}{q_r}\right)^{-\ell}$ stems from the later multiplication by $\tilde{M}_i$ or $\tilde{k}_i$ which behave like $q_i^{2\ell}$.

appendix. When $\mathcal{J}_i$ is written in the same way with reduced amplitudes $\gamma_i^{JL\ell}$, the quantities τ, γ are free from kinematical singularities and obey

$$\tau_i^{JL\ell}(s)-\gamma_i^{JL\ell}(s) = \frac{i}{2}\,\frac{q_i}{4\sqrt{s_i}}\left(\frac{q_i}{q_r}\right)^{\ell}\left(\frac{Q_i}{Q_r}\right)^{L}\sum_{j\neq i}\int ds_j \langle i|j\rangle_{J^P}\left(\frac{Q_j}{Q_r}\right)^{L_j}\left(\frac{q_j}{q_r}\right)^{\ell_j} I_j(s_j)\,\tau_j^{JL_j\ell_j}(s_j)$$

where $I_j(s_j) = \Gamma/\left(M_r - \sqrt{s_j} - \frac{i}{2}\Gamma\left(\frac{q_j}{q_r}\right)^{2\ell+1}\right)$ in the resonance approximation. Within our conventions, recouplings should be as follows : $\langle i|j\rangle_{J^P} = \langle JML_2\ell_2S_2|JML_1\ell_1S_1\rangle =$

$$\frac{64\pi^2}{2J+1}\delta_{JJ'}\delta_{MM'}\left[\frac{\sqrt{s_1 s_2}}{Q_1 q_1 Q_2 q_2}\right]\sum_N \langle JNL_2\ell_2S_2|\Uparrow S_1S_2\rangle \times \langle \Downarrow S_1S_2|JNL_1\ell_1S_1\rangle$$

Only Clebsch Gordan coefficients and spherical harmonics appear in the last two scalar products, defined with the z axis normal to the Dalitz plot. As $\tau_i^{JL\ell}$ will be multiplied by $\tilde{M}_i^{\ell}$, only states with $\tilde{M}_i^{\ell} \neq 0$ need to be considered.

2. Two Examples

a) The $A_1(1070)$ $J^P = 1^+$

On Fig. 1a) we plot the amount of correction from the $A_1 \to \sigma\pi$ branching in system 2 upon the $A_1 \to \rho\pi$ mode in system 1 assuming $\gamma_\rho = \gamma_\sigma = 1$. Fig. 2a) shows the effect of this correction on contour lines of the Dalitz plot if we take for granted the results of D.V. Brockway [3] from a partial wave analysis ($\gamma_\rho = 1$, $\gamma_\sigma = 2i$)*.
We observe a small modification in the overlap region.

* We stress that we only use these results for illustrative purposes, without worrying about the difficulties of the partial wave analysis.

b) $\bar{p}n \to \pi^+\pi^-\pi^-$ at 2.5 GeV in the center of mass

Approximate starting values were found from a first study by Ron Huesman (LBL), with production of $\rho(750)$, $f(1250)$, $\sigma(750)$. Here again, as can be seen from Figs. 1b), 2b), corrections lead to very small effects. Fig. 3 shows the experimental distribution (Ref. [4]) for comparison.

IV. EXTENSION TO HIGHER ENERGIES

It is an experimental fact that most events have at least one small associated mass, which justifies taking as ansatz $J = \sum_{j=1,2,3}\gamma_j(t_j)(1+\xi_j e^{-i\pi\alpha_j(t_j)})s_j^{\alpha_j(t_j)}\mathcal{T}_j(s_j)$ in equation (4). The derivation is no more valid, since other discontinuities in s_j are now present, associated with inelastic channels, but the error thus made is likely to be even smaller than the overlap correction.

ACKNOWLEDGMENTS

Ron Huesman kindly provided us with his first fits. We also thank G.C. Fox for challenging us to solve this problem, Professor A.H. Rosenfeld for his constant interest, and Professor H. Stapp who straightened the errors, discussed the manuscript, and encouraged us on the way.

APPENDIX

We recall our notations : Q_i, q_i are the lengths of the momentum $\vec{Q}_i$ of (jk) in the center of mass, and of its decay momentum $\vec{q}_i$ respectively. Ω_i, ω_i are the directions of $\vec{Q}_i$, $\vec{q}_i$.

$$\langle p_1 p_2 p_3 | J M L_i \ell_i s_i \rangle = \frac{4W}{Q_i} \frac{4\sqrt{s_i}}{q_i} \delta(s_i - (p_j + p_k)^2) \sum_{Nm} \langle JM | L_i N \ell_i m \rangle Y_{L_i}^{N}(\Omega_i) Y_{\ell_i}^{m}(\omega_i)$$

$$\langle J M L_i \ell_i s_i | J M L'_i \ell'_i s'_i \rangle = \frac{4W}{Q_i} \frac{4\sqrt{s_i}}{q_i} \delta(s_i - s'_i) \delta_{JJ'} \delta_{MM'} \quad \text{so that}$$

$$\langle p_1 p_2 p_3 | T | in \rangle = \sum_{JML\ell} \langle p_1 p_2 p_3 | J M L \ell s_i \rangle \frac{Q_i q_i}{16 W \sqrt{s_i}} \langle J M L \ell s_i | T | in \rangle$$

Furthermore, we use in the partial wave expansion factors Q_r, q_r inside "centrifugal barriers". These terms were chosen so that in average, the kinematical factor would be of the order of unity.

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2. P. Moussa and R. Stora, Methods of Subnuclear Physics (1966).
3. D.V. Brockway, PhD. Thesis, unpublished (University of Illinois).
4. Bettini et al., N.C. 1A, N2, p. 333 (1971).

FIGURE CAPTIONS

Fig. 1 : Ratio of the overlap corrections to the amplitudes assuming all amplitudes are equal to unity.
a) Effect of σ on ρ amplitude b) effect of ρ on f^0 amplitude

Fig. 2 : Contour lines of the density in the Dalitz plot associated to $A_1 \to \pi^+ \pi^+ \pi^-$ at 1.07 GeV
a) without correction. b) with the overlap correction.

Fig. 3 : Contour lines of the density in the Dalitz plot for the reaction $\bar{p} n \to \pi^+ \pi^- \pi^-$ at 2.15 GeV a) without correction. b) with the overlap correction.

Fig. 4 : Experimental distribution of the events in $\bar{p} n \to \pi^+ \pi^- \pi^-$ at 2.15 GeV.

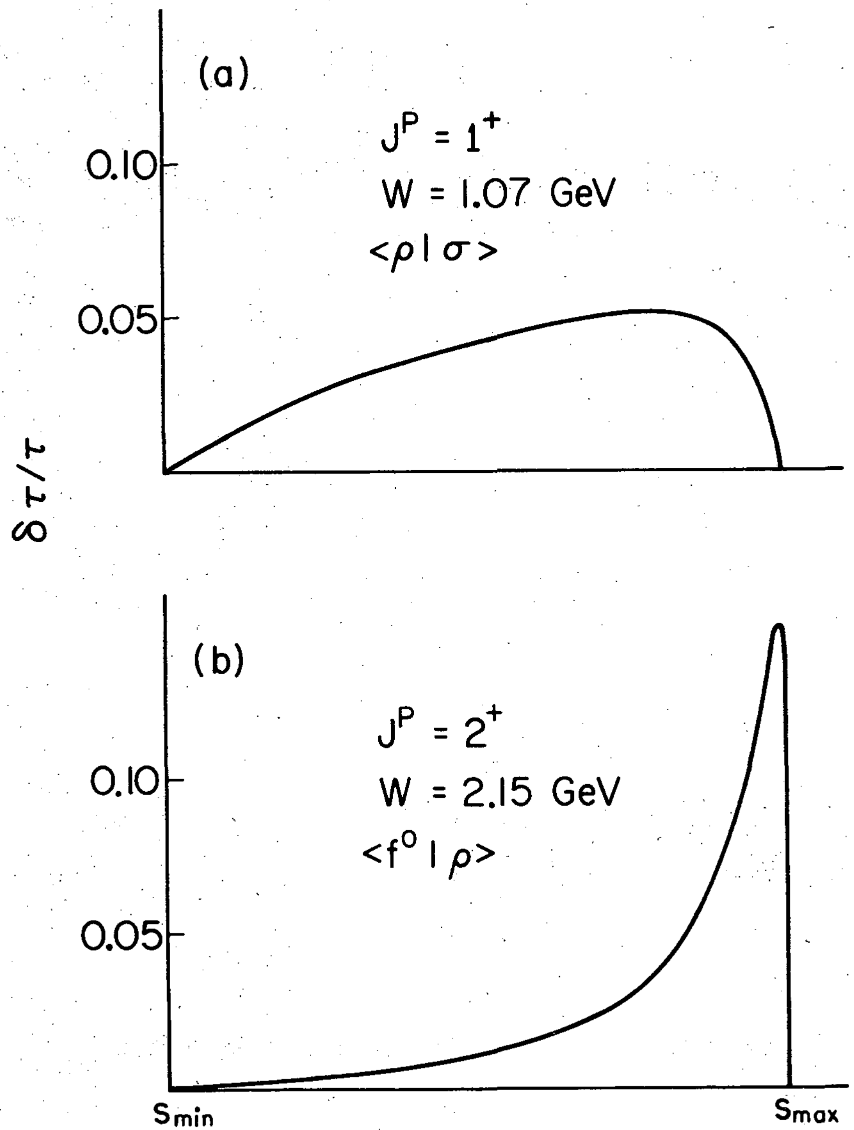

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Fig. 1

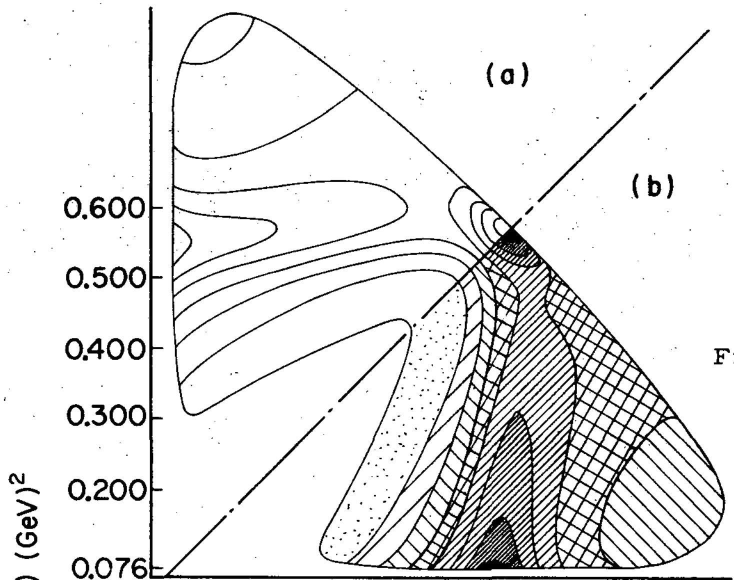

Fig. 2

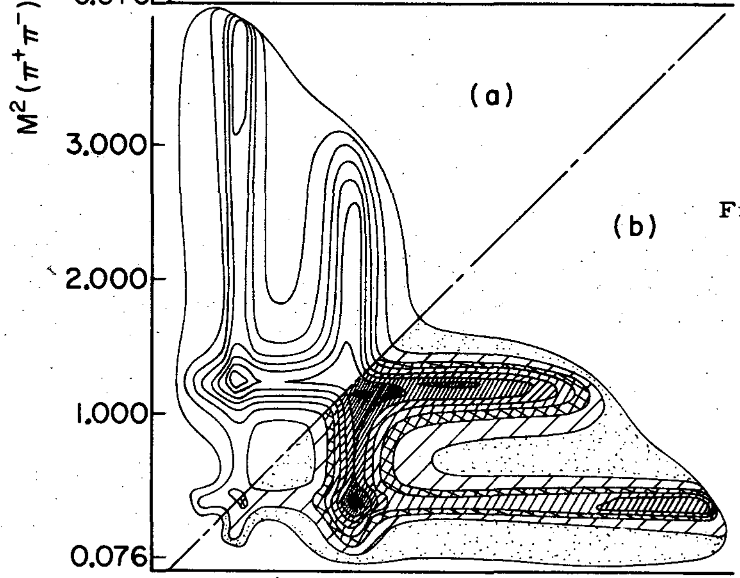

Fig. 3

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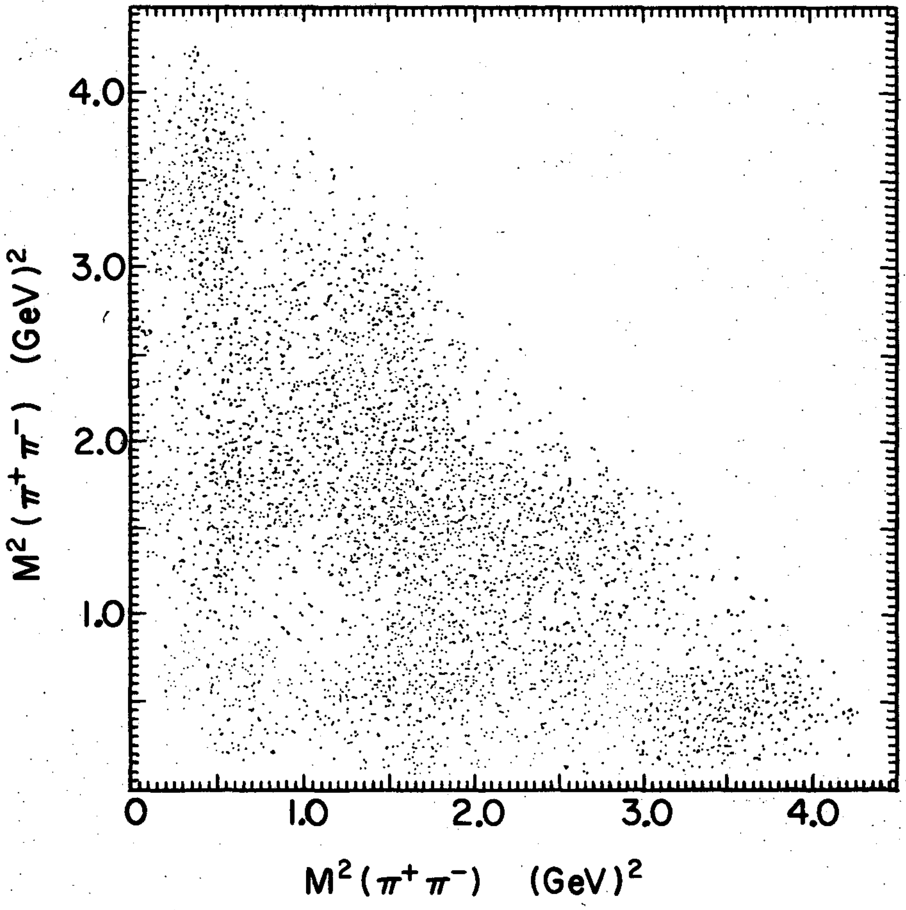

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Fig. 4

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