Title

Self-Interacting Dark Matter

Authors

Carlson, E.D.
Machacek, M.E.
Hall, L.J.

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Self-interacting Dark Matter

Eric D. Carlson

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Marie E. Machacek

Department of Physics
Northeastern University
Boston, MA 02115

Lawrence J. Hall

Department of Physics
University of California
and
Theoretical Physics Group
Physics Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, CA 94720

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DISCLAIMER

A new type of dark matter is considered. As with cold dark matter, when the dark matter is non relativistic it does not couple to the electron-photon plasma, so its entropy per comoving volume is fixed. The new feature is that there is a cosmological era where number changing reactions keep the dark matter in chemical equilibrium so that the chemical potential vanishes. This has several interesting consequences: during this era the dark matter cannibalizes its rest mass to keep warm, its temperature dropping only logarithmically with scale. The energy density and density perturbations of the dark matter also have unusual scale dependences, modified by logarithms compared to usual cold dark matter. We have done a general study of such self-interacting dark matter to identify the interesting ranges for its mass, coupling and entropy. There are two consequences of this scheme which are particularly noteworthy. The unusual evolution of energy density and density perturbations allows for the possibility of significantly decreasing the predicted anisotropies in the cosmic microwave background. This dark matter allows a completely new scheme for processing density perturbations. The theory introduces a new cosmological mass scale: the Jeans mass when the number changing processes decouple. Perturbations on this supercluster scale are the first to go non-linear. Unlike hot dark matter, density perturbations on galactic scales are not negligible.

1. Introduction

One of the long-standing problems of cosmology and astrophysics is the existence of dark matter in the universe. Galactic rotation curves as well as other dynamical arguments indicate that non-luminous matter may be the dominant component of the universe [1]. Inflation, as well as theoretical bias, suggests that the universe may be critically closed, that is, Ω is very close to unity [2].

Numerous dark matter candidates have been proposed. These candidates can be classified by how their final relic density is determined. The relic density is determined by the strength of interactions at the time when the temperature of the universe drops below the mass of the relic particles. For example, hot dark matter is any candidate whose interactions are so weak that the particles deplete neither their comoving number nor their comoving entropy when the temperature drops below their mass. Cold dark matter, in contrast, is assumed to have annihilation processes which deplete the number of dark matter particles and have their entropy (and energy) transferred into ordinary particles. The final abundance of cold dark matter particles can be determined either by the size of their non-relativistic annihilation cross section, or by their cosmological asymmetry. Most cold dark matter candidates lose thermal contact with the electron-photon plasma at an early stage. This is to be contrasted with baryons which remain thermally coupled down to the era of hydrogen formation. All these scenarios, hot dark matter, cold dark matter, and baryonic dark matter, are difficult to reconcile with observation [3]. Certainly it is worthwhile to consider alternatives in case all of these eventually prove inadequate.

We will classify dark matter candidates according to whether their comoving number, N, and entropy, S, changes during the era when the temperature drops beneath their mass, M. There are four possibilities: each of N and S can change or remain fixed during this era. Hot dark matter is the case when both N and S are fixed, while cold dark matter is the case when they both change. To our knowledge the other two possibilities have escaped attention. The first new possibility is that during this era there is no annihilation, so N is fixed, but scatterings with the photon plasma allow the dark matter entropy S to vary. This could lead to very interesting phenomena, such as an increase in n_B/s_γ subsequent to nucleosynthesis, allowing an evasion of the usual nucleosynthesis constraint on Ω_B. We will consider this case in future work. In this paper we will only study the second new possibility: as the temperature drops beneath M, there are processes which change N, the number of dark matter particles, but the dark matter sector is not able to exchange heat with the photon plasma so S is fixed. Having S fixed means not only that the dark matter is not scattering from ordinary particles, but the processes which change N do not involve ordinary particles either. There is an era when the dark matter particles are forced to have zero chemical potential, while their comoving entropy, S, is fixed. We concentrate on this unusual era as it involves an evolution of temperature, density and density perturbations that differs from either hot or cold dark matter. We call such matter self-interacting dark matter, and find that it is very simple to realize in particle physics.

Let us illustrate the idea with an example. Suppose that at high temperatures, there exists a grand unified theory which breaks to the standard model, but with a left-over non-abelian gauge group to which no light fermions (including the standard model fermions) couple. The gauge coupling will renormalize and become strong at some low energy scale, at which point the theory will no longer have freely propagating massless gluon-like states but instead will have numerous glueball states. These glueballs will interact strongly with themselves, but can only interact with or decay to ordinary particles through loops of GUT-scale particles. If we trace the behavior of this component of the energy density of the universe from early times to the present, we find that the gauge bosons behave as a relativistic gas, just like the rest of the universe, until confinement sets in, at which point the entropy of the gauge particles is converted into entropy of the glueball states. As the universe continues to expand, these glueballs interact strongly with themselves, reducing their comoving number density by number changing processes like $3 \rightarrow 2$ and $4 \rightarrow 2$. However, the entropy in these particles cannot transfer to ordinary matter, because no interactions are of sufficient strength to transfer anything from the dark sector to the ordinary sector. As we shall see in the next section, this leads to a very unusual scale-time-temperature relationship. Eventually even the strong interactions become insufficient to deplete glueball number further, and the glueballs cease to eliminate themselves. Ultimately, they can be used to make galaxies and large-scale structure.

We will make a distinction in this paper between thermal and chemical equilibrium. When we speak of *thermal* equilibrium, what we mean is that the

dark matter has a well defined temperature, and the occupation number of momentum states is determined only in terms of the density and temperature of the particles, or alternatively, the chemical potential and the temperature. Thermal equilibrium can be maintained by elastic collisions between pairs of dark matter particles, which presumably are much faster than $3 \rightarrow 2$ or $4 \rightarrow 2$ processes. In contrast, *chemical* equilibrium means that number changing processes are also fast, which assures that the chemical potential is zero, and consequently the occupation numbers can be written in terms of only one parameter, the temperature.

Because we are introducing a new category of dark matter, rather than a specific candidate, we have attempted to keep our discussion as general as possible. For this reason, we have avoided using a detailed glueball-like model in favor of a scalar model with arbitrary couplings and effective interactions. We have then performed calculations in as general a way as possible, to maximize the utility of our work. In section 2, we discuss the unusual scale-time-temperature relations in a model independent way. In section 3 we work out all relevant cosmological constraints on a model. In section 4 we use specific toy models to demonstrate how decoupling can be calculated. In section 5 we consider astrophysical limits on radiative couplings in a specific model. Section 6 discusses the most incomplete section of our work, the growth of density perturbations in a self-interacting dark matter dominated universe. Section 7 discusses the related question of fluctuations in the cosmic microwave background radiation in the context of self-interacting dark matter. We then conclude with a summary of our work and a discussion of what remains to be done.

2. Conservation of Entropy and the Scale-Time-Temperature Relations

Because the self-interactions of the dark matter are assumed strong at the time when the dark matter becomes non-relativistic, but the couplings to ordinary matter are assumed weak, the dark matter will have a temperature T' that is different from the photon temperature T. We will consistently use primes to denote properties of the dark matter, such as T', m', g' and ρ', n', s' for the temperature, mass, degrees of freedom and energy, number, entropy densities, respectively. Unprimed symbols represent the same quantities for ordinary matter. Because the interactions within each sector are faster than expansion, but there is no interaction between the two sectors, comoving entropy density will be conserved within each sector. Hence the ratio of the entropy densities will simply be a constant ξ:

$$\xi = \frac{s}{s'} \, . \tag{2.1}$$

If the dark and visible sector are assumed to have been in equilibrium at some early time, this ratio will just be the ratio of the effective number of degrees of freedom at that time; subsequently, this ratio can change only due to out-of-equilibrium processes such as a first order phase transition. Normally, these processes would produce neither enormous nor tiny values for ξ, but, for complete generality, we will not constrain ξ in any way.

We can use the conservation of the comoving entropy density to calculate the temperature-scale relationship of the dark matter at all times. For example, when the dark matter is relativistic, the entropy density is just proportional to T'^3. Since the comoving volume of the universe is proportional to a^3, where a is the scale factor of the universe, the product of these must be constant, so during this era we have

$$T' \propto a^{-1}. \tag{2.2}$$

When the dark matter becomes non-relativistic, the mass density is given by $\rho' = m'n'$, and the pressure is smaller, so that the number density and entropy density are just given by

$$n' = g' \left(\frac{m'T'}{2\pi}\right)^{3/2} e^{-m'/T'} \, , \tag{2.3}$$

and

$$s' = \frac{m'n'}{T'} = \frac{m'}{T'} \left(\frac{m'T'}{2\pi}\right)^{3/2} g' e^{-m'/T'} \, . \tag{2.4}$$

Note that the chemical potential μ is zero because the number changing reactions give chemical equilibrium. This is the essential difference compared

with the cold dark matter scenario, where μ rapidly increases after freezeout. Entropy conservation in a comoving volume then yields

$$\frac{(2\pi)^{3/2} s' a^3}{g' m'^3} = a^3 \left(\frac{T'}{m'}\right)^{1/2} e^{-m'/T'} \equiv \overline{a}^3 \, , \tag{2.5}$$

where $\overline{a}$ is a constant. For $T' < m'$, this has the approximate solution

$$\frac{T'}{m'} \simeq \frac{1}{3\ln(a/\overline{a})} \, . \tag{2.6}$$

Thus, the temperature falls only logarithmically during this era, in contrast to linearly or quadratically as expected for ordinary relativistic or non-relativistic matter.

How can we understand such a slow temperature decrease? Consider the effect of a small increase in the size of the universe on self-interacting dark matter. Because the dark matter is non-relativistic, the expansion of the universe will (ignoring self-interactions) cause a quadratic decrease in the temperature of the dark matter. However, lower temperatures would demand, according to (2.3), a substantial decrease in the number density of particles. Hence number changing processes like $3 \to 2$ or $4 \to 2$ will tend to deplete the number of dark matter particles. But these processes take non-relativistic particles in and produce (fewer) relativistic particles out, so that the outgoing particles have much more kinetic energy than the mean $\frac{3}{2}T'$. Hence subsequent $2 \to 2$ processes will transfer the kinetic energy of these few particles to all the dark matter, increasing the temperature. So as the universe expands, the dark matter cannibalizes itself to keep warm.

The mass density ρ' can be most easily calculated by noting that $\rho' = s'T'$, and since we know how both s' and T' depend on a, we know that

$$\rho' \propto \frac{1}{a^3 \ln(a/\overline{a})} \, . \tag{2.7}$$

This represents a slightly more rapid decrease in the energy density than for ordinary non-relativistic matter.

Eventually, the density drops so low that interactions among the dark matter are too slow to further deplete their number density. We will call this process decoupling, and we will denote the decoupling temperature of the dark matter by T'_d, and the photon temperature at that time T_d. After decoupling, the temperature and mass density of the dark matter must fall like a^{-2} and a^{-3} respectively. Thus self-interacting dark matter behaves much like ordinary cold dark matter after it falls out of chemical equilibrium, except for the fact that it is not collisionless.

For ordinary matter, the entropy s is given by the expression

$$s = \frac{2\pi^2}{45} g_{\text{eff}} T^3 , \tag{2.8}$$

where g_{eff} counts the number of effective relativistic degrees of freedom that contribute to the entropy. The photon temperature-scale relationship remains

$$T \propto a^{-1} \tag{2.9}$$

for all times after recombination as required for a black body spectrum. These relationships are sketched in Figure 1.

Because the ratio of entropies in the two sectors is a constant, we can calculate the mass density of dark matter at the time of decoupling. Combining (2.1) with $\rho' = s'T'$, we find

$$\frac{\rho'_d}{s_d} = \frac{T'_d}{\xi} . \tag{2.10}$$

After decoupling, both ρ' and s scale like a^{-3}, so that their ratio remains constant. Hence their ratio now is given by

$$\frac{\rho'_{\text{now}}}{s_{\text{now}}} = \frac{T'_d}{\xi} . \tag{2.11}$$

The entropy density today is

$$s_{\text{now}} = \frac{2\pi^2}{45} g_{\text{eff}} T_\gamma^3 , \tag{2.12}$$

where $g_{\text{eff}} = 3.91$ and $T_\gamma = 2.735^\circ$K are the present effective entropy degrees of freedom and photon temperature. The mass density can be replaced in favor of Ω, the ratio of dark matter density to closure density, which is given by

$$\Omega H^2 = \frac{8\pi}{3} G \rho'_{\text{now}} , \tag{2.13}$$

where G is the gravitational constant, and $H = 100h\,\text{km/sec/Mpc}$ is the Hubble constant today, with $0.4 < h < 1$. Combining these equations, we can find the dark matter decoupling temperature:

$$T'_d = \frac{135 H^2 \Omega \xi}{16\pi^3 G\, g_{\text{eff}} T_\gamma^3} = (3.6\,\text{eV}) h^2 \Omega \xi . \tag{2.14}$$

Thus the decoupling dark matter temperature can be determined if we know the entropy ratio and present density. This relationship was obtained without knowing the couplings, mass, or number of degrees of freedom of the dark matter.

There are two general decoupling scenarios for self-interacting dark matter that determine how it affects Einstein's equations and thus the time evolution of the universe. We will compare these scenarios to that of ordinary cold dark matter that would dominate the energy density of the universe at photon temperature

$$T_1 = \frac{45 H^2 \Omega}{4\pi^3 G\, g_* T_\gamma^3} = 5.59 h^2 \Omega\,\text{eV} \tag{2.15}$$

where $g_* = 3.36$ is the number of effective relativistic energy density degrees of freedom of ordinary matter at decoupling ($g_* \neq g_{\text{eff}}$ simply because $T_\nu \neq T$ at this time). Let T_m, T_e, and T_d represent the photon temperature when self-interacting dark matter becomes nonrelativistic, begins to dominate the energy density of the universe, and decouples, falling out of chemical equilibrium, respectively (note $T_m > T_e$). Early decoupling (Case I) is characterized by $T_e < T_d$ $(< T_m)$, so that dark matter dominates the universe only after decoupling. After T_e, the relation between temperature, density, and scale factor matches that of cold dark matter, so to get to the same density, we must have $T_e = T_1$. Because of its similarity to the cold dark matter scenario, this case is not particularly interesting.

The more interesting case (Case II) is late decoupling of dark matter characterized by $T_e > T_d$, so that dark matter dominates the energy density of the universe while particle number changing interactions are still strong enough to maintain chemical equilibrium. Because the dark matter density is falling more quickly between these two temperatures than in the cold dark matter scenario, we can show that $T_e > T_1 > T_d$ (as we shall see in the next section, nucleosynthesis bounds will assure that initially there is more density in ordinary matter than in dark matter, so that $T_m > T_e$). Einstein's equations in the region $(T_e > T > T_d)$ imply

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G s' m' \left(\frac{T'}{m'}\right). \tag{2.16}$$

Substituting equation (2.5) for s' and (2.6) for T'/m', we find the approximate scale temperature relationship:

$$\frac{a}{\bar{a}} \simeq \left(\frac{t}{\tau}\right)^{2/3} \left[\ln\left(\frac{t}{\tau}\right)\right]^{-1/3} . \tag{2.17}$$

where

$$\tau = \sqrt{\frac{2\sqrt{2\pi}}{3Gg'm'^4}} . \tag{2.18}$$

Due to the logarithmic suppression, the universe expands slightly more slowly during this era than expected for cold dark matter. As discussed further in Section VI, this may enhance the growth of matter perturbations over that expected for cold dark matter.

3. Cosmological Limits

One of the most stringent cosmological constraints on the introduction of new light, long-lived particle species comes from the excellent agreement between the predictions of the standard model and the measured primordial abundances of deuterium, helium and lithium. The analysis has traditionally been used to limit the number of light weakly interacting neutrino species in particle physics models and is often stated in terms of the number of neutrinos N_ν allowed by the data. Although recent authors [4] suggest that this bound may be as strong as $\Delta N_\nu = N_\nu - 3 \leq 0.3$, we will use the somewhat conservative limit

$$\Delta N_\nu \leq 1.0 . \tag{3.1}$$

Primordial nucleosynthesis, in fact, constrains any new particle species through its contribution to the effective number of density degrees of freedom at the time of nucleosynthesis. A standard light neutrino in thermal equilibrium with photons at nucleosynthesis would contribute 1.75 to the effective degrees of freedom. Thus, restating the neutrino limits in terms of the contribution of any other light species [3]

$$(1.75)\Delta N_\nu \geq g'_{\rm eff} \left(\frac{T'}{T}\right)^4 , \tag{3.2}$$

where $g'_{\rm eff} = g'$ for bosonic dark matter and $g'_{\rm eff} = \frac{7}{8} g'$ for fermionic dark matter. The ratio of temperatures can be simply reexpressed in terms of the entropy density at the time of nucleosynthesis ($T \sim 1\,\mathrm{MeV}$)

$$\xi = \frac{s}{s'} = \frac{g_{\rm eff} T^3}{g'_{\rm eff} T'^3} , \tag{3.3}$$

where $g_{\rm eff} = 10.75$ is the number of effective degrees of freedom in conventional matter at this time. If we define $\xi' = g'^{1/4}\xi$, this leads to a constraint on ξ' at the time of nucleosynthesis. The nucleosynthesis bound for bosonic self interacting dark matter becomes

$$\xi' \geq 7.1\,(\Delta N_\nu)^{-3/4} . \tag{3.4}$$

For fermions the constraint can be improved by a factor of $(\frac{8}{7})^{1/4}$, which we consider negligible. This leads to the constraint $\xi' > 7.1$ and $\xi' > 17$ for $\Delta N_\nu = 1$ and $\Delta N_\nu = 0.3$, respectively.

Another cosmological constraint is derived from galaxy formation if we assume that self interacting dark matter comprises galactic halos. If that is the case then dark matter must have cooled sufficiently so that galaxies can be composed of dark matter with thermal velocities typical of measured galactic rotation velocities, $v/c = \beta \sim 10^{-3}$. The average kinetic energy of material in the galaxy gives a rough estimate of the temperature of dark matter forming the galaxy. It is difficult to see how this temperature could decrease once galaxies have formed, so it is reasonable to assume that the dark matter temperature at galaxy formation is constrained by

$$T'_{GF} \lesssim \frac{1}{2} m' \beta^2 . \tag{3.5}$$

The temperature of dark matter during the epoch of galaxy formation can be rewritten in terms of the dark matter and photon decoupling temperatures and the redshift Z_{GF} of the epoch of galaxy formation by using the temperature-scale relationships after decoupling and the definition of redshift $1 + Z_{GF} = a_{\rm now}/a_{GF}$. We find

$$T'_{GF} = T'_d \left(\frac{T_{GF}}{T_d}\right)^2 = T'_d \left(\frac{T_\gamma}{T_d}\right)^2 (Z_{GF}+1)^2 \,. \tag{3.6}$$

Recent studies of quasars at redshifts $Z > 4$ indicate that even at these redshifts heavy elements are already present in these objects with solar abundances [5]. Thus the epoch of galaxy formation must have occurred even earlier. Using $Z_{GF} \sim 5$, and $\beta \sim 10^{-3}$, we find a conservative bound

$$\frac{T_d}{T_1} \gtrsim \frac{0.36}{h^2\Omega}\sqrt{\frac{T'_d}{m'}} \tag{3.7}$$

if self-interacting dark matter is to be associated with dark matter in galactic halos.

4. Decoupling Conditions

We now turn to the question of decoupling; that is, when precisely do number changing processes first fail to maintain chemical equilibrium? As the universe expands, the number of particles N' in a comoving volume must (slowly) decrease. Provided the number changing processes occur at a rate Γ that is faster than the changes required by the expansion of the universe, chemical equilibrium is maintained; that is, so long as

$$\Gamma > \frac{\dot{N}'}{N'}\bigg|_{\mu'=0} \,. \tag{4.1}$$

Up until decoupling, entropy conservation allows this expression to be restated in terms of the rate of change of the dark matter temperature T' since

$$s'a^3 = \frac{m'N'}{T'} = \text{constant} \tag{4.2}$$

implies

$$\frac{\dot{N}'}{N'} = \frac{\dot{T}'}{T'} \,. \tag{4.3}$$

By using equation (2.6), this can be rewritten

$$\Gamma_d \sim \frac{3T'_d H_d}{m'}, \tag{4.4}$$

where H_d is the Hubble constant at the time of decoupling. This differs somewhat from the usual condition $\Gamma \sim H$ assumed for cold dark matter.

To illustrate these ideas, we will consider two possible realizations of self-interacting dark matter, one in the form of a scalar field ϕ_s whose leading particle number changing interactions can be represented by an effective Lagrangian

$$L_s = \frac{f_s\phi_s^5}{5!m'} \,, \tag{4.5}$$

and the other in the form of a pseudoscalar field ϕ_p with its leading particle number changing interactions given by an effective Lagrangian

$$L_p = \frac{f_p\phi_p^6}{6!m'^2} \,. \tag{4.6}$$

These theories are not to be regarded as fundamental or realistic, but only as illustrative. For example, if we have a theory of glueballs, ϕ_p or ϕ_s can be thought of as the lowest energy glueball, whichever happens to be lighter. We can then calculate the rate of annihilation for an average particle to be

$$\Gamma(3\to 2) = \frac{\sqrt{5} f_s^2 n'^2}{2304\pi m'^5} \,, \tag{4.7}$$

for the scalar case, and

$$\Gamma(4\to 2) = \frac{\sqrt{3} f_p^2 n'^3}{6144\pi m'^8}, \tag{4.8}$$

for the pseudoscalar case, respectively. In each case we can define a coupling λ such that

$$\Gamma(I\to 2) = \lambda m' \left(\frac{n'}{m'^3}\right)^{I-1} , \tag{4.9}$$

where $I = 3(4)$ in the scalar (pseudoscalar) case. For some models, the decay rates Γ may be momentum dependent, in which case λ effectively has powers of T'/m' in it. In this case, we will simply define λ to be $\lambda(T'_d)$. The decoupling condition then becomes

$$\lambda m' \left(\frac{n'_d}{m'^3}\right)^{I-1} = \frac{3T'_d H_d}{m'} . \tag{4.10}$$

The dark matter particle mass and both its temperature and the photon temperature at decoupling for early or late decoupling scenarios can be expressed in terms of two free model parameters, λ, the strength of the effective self-coupling of the dark matter, and ξ, the ratio of the entropy carried in the ordinary sector to that in the dark matter sector. In general, the procedure will be to use Einstein's equations to eliminate H from the decoupling condition and determine an expression for the number density n'_d at decoupling. We can then find m'/T'_d as a function of the model parameters. The mass m' is easily extracted using (2.14). Finally the entropy ratio between the two sectors at decoupling is used to relate the photon and dark matter temperatures.

For Case I, early decoupling, Einstein's equations are still radiation dominated. Thus,

$$H^2 = \frac{8\pi G}{3}\left(\frac{\pi^2 g_* T_d^4}{30}\right) . \tag{4.11}$$

The results of the above procedure for the scalar and pseudoscalar models are

$$\begin{aligned}\frac{m'}{T'_d} + 2\ln\left(\frac{m'}{T'_d}\right) &= \frac{3}{4}\ln\lambda_s - \frac{5}{4}\ln\xi - \frac{3}{4}\ln\left\{\frac{135\sqrt{2\pi}H^2\Omega g_*^{1/2}}{\sqrt{40G}g'^{4/3}g_{\rm eff}T_\gamma^3}\left(\frac{45}{2\pi^2 g_{\rm eff}}\right)^{2/3}\right\}\\ &= \frac{3}{4}\ln\lambda'_s - \frac{5}{4}\ln\xi' + 43.39 ,\end{aligned} \tag{4.12}$$

for the scalar model ($I = 3$), and

$$\begin{aligned}\frac{m'}{T'_d} + \frac{25}{14}\ln\left(\frac{m'}{T'_d}\right) &= \frac{3}{7}\ln\lambda_p - \frac{5}{7}\ln\xi - \frac{3}{7}\ln\left\{\frac{135(2\pi)^2H^2\Omega g_*^{1/2}}{\sqrt{40G}g'^{7/3}g_{\rm eff}T_\gamma^3}\left(\frac{45}{2\pi^2 g_{\rm eff}}\right)^{2/3}\right\}\\ &= \frac{3}{7}\ln\lambda'_p - \frac{5}{7}\ln\xi' + 23.61 ,\end{aligned} \tag{4.13}$$

for the pseudoscalar model ($I = 4$), where

$$\lambda'_s \equiv \frac{\lambda_s g'^{7/4}}{h^2\Omega} \qquad \text{and} \qquad \lambda'_p \equiv \frac{\lambda_p g'^{11/4}}{h^2\Omega} .$$

Notice that in each example m'/T'_d has only a weak logarithmic dependence on the model parameters ξ', λ'. Thus $m' \propto \xi$, the entropy ratio between the sectors. We can then determine the photon decoupling temperature using

$$\begin{aligned}\ln\left(\frac{T_d}{T_1}\right) &= \frac{1}{3}\ln\left(\frac{1215 g_*^3}{(2\pi)^{7/2}32 g_{\rm eff}^4}\right) + \frac{5}{6}\ln\left(\frac{m'}{T'_d}\right) - \frac{m'}{3T'_d} + \frac{4}{3}\ln\xi'\\ &= -1.54 + \frac{5}{6}\ln\left(\frac{m'}{T'_d}\right) - \frac{m'}{3T'_d} + \frac{4}{3}\ln\xi'\end{aligned} \tag{4.14}$$

where the λ'_j dependence occurs only implicitly through the weak model dependence of m'/T'_d. Since dark matter begins to dominate the energy density of the universe in the early decoupling scenario at $T_e = T_1$, the scenario assumptions break down when $T_d < T_1$, below which the conditions of Case II, late decoupling, must be applied.

For Case II dark matter dominates Einstein's equations before decoupling. When equations (2.14), (4.10), and (2.3)are combined with Einstein's equation $H^2 = \frac{8\pi}{3}G\rho'$, we again determine m'/T'_d for each example model. For the scalar model ($I = 3$),

$$\begin{aligned}\frac{m'}{T'_d} + \frac{3}{2}\ln\left(\frac{m'}{T'_d}\right) &= -\frac{2}{3}\ln\left(\frac{135\sqrt{3}H^2\Omega\xi}{4\pi^2\sqrt{2\pi G}g{\rm eff}g'^{3/2}T_\gamma^3\lambda_s}\right) - \frac{3}{2}\ln(2\pi)\\ &= \frac{2}{3}\ln\left(\frac{\lambda'_s}{\xi'}\right) + 38.06 .\end{aligned} \tag{4.15}$$

The pseudoscalar model ($I = 4$) has the factor $\frac{2}{3}$ replaced by $\frac{2}{5}$ to give the numerical result

$$\frac{m'}{T'_d} + \frac{3}{2}\ln\left(\frac{m'}{T'_d}\right) = \frac{2}{5}\ln\left(\frac{\lambda'_p}{\xi'}\right) + 21.74 . \tag{4.16}$$

Again m'/T'_d is nearly constant, depending only logarithmically on ξ' and λ'_j. ξ' is bounded above if we demand we are in case II, and it is bounded below by nucleosynthesis constraints, so that the range of masses expected for late decoupling dark matter are $\sim 1-30$ keV, virtually independent of the details of a particular model. The photon temperature at decoupling is once again given by (4.14).

We complete this discussion by determining the temperature T_e at which self-interacting dark matter begins to dominate the energy density in this scenario and compare it to T_1, the temperature when cold dark matter would have dominated. The ratio of entropy carried in the ordinary and dark matter sectors is valid for all times. Evaluating the ratio of entropies at dark matter-radiation equality gives

$$\frac{\xi\rho'}{T'} = \xi s' = s = \frac{2\pi^2}{45} g_{\rm eff} T^3 = \frac{4 g_{\rm eff}\rho}{3 g_* T} = \frac{\xi T_1 \rho}{T_d' T} . \tag{4.17}$$

By imposing the condition at equality that $\rho_e' = \rho_e$, we find

$$T_e' = \frac{T_d' T_e}{T_1}. \tag{4.18}$$

If we eliminate s and s' in favor of T and T' from $\xi s' = s$, and then use (4.18) to eliminate T', we can find the temperature when dark matter first dominates, which is given by

$$\frac{m' T_1}{T_d' T_e} - \frac{5}{2}\ln\left(\frac{m' T_1}{T_d' T_e}\right) = 4\ln\xi' + \ln\left(\frac{1215 g_*^3}{(2\pi)^{\frac{7}{2}} 32 g_{\rm eff}^4}\right) = 4\ln\xi' - 4.61 . \tag{4.19}$$

Of course, this equation makes sense only when $T_d < T_1$, which will assure $T_e > T_1$. For $T_d > T_1$, this formula is invalid and T_e is simply given by $T_e = T_1$. Representative values of T_d, T_e, and m' can all be seen in Figure 2. The cosmological bounds given by equations (3.4) and (3.7) for nucleosynthesis and galaxy formation constrain the allowable range of ξ' and λ_j'. The allowed regions of model parameter space for the late decoupling scenario are graphed in Figure 3 for the scalar and pseudoscalar models, respectively.

5. Astrophysical Limits on Radiative Couplings

Self-interacting dark matter is envisioned to interact with or decay to ordinary matter only through loops of GUT-scale particles (and, of course gravity). If these GUT-scale particles in turn couple to photons with ordinary strength, the resulting effective electromagnetic coupling could contribute to stellar evolution [6] through the Primakoff effect or to the diffuse extragalactic background radiation (DEBRA) through its rare, two photon radiative decays. It is the latter contribution to DEBRA that places the strongest constraint on the GUT-scale masses and effective couplings in the models we have considered here and also offers the hope that as the measurements of DEBRA are improved, this dark matter might be detected.

In our scalar (pseudoscalar) examples, we model the effective photon coupling through the triangle diagram, where heavy GUT-scale masses run around the loop. While there may be several multiplets of fermions in the loop, we assume that their mass splittings are small compared to the GUT-scale mass and represent their masses by a single average GUT-scale mass M_j. We also define an effective coupling for fermions in the loop to be

$$\bar{g}_f = \sum g_f q_f^2 , \tag{5.1}$$

where g_f is the dark matter-fermion coupling and q_f is the fermion charge. The two photon decay rate for the triangle graph is well known [7]

$$\Gamma(\phi_{s(p)} \to 2\gamma) = \left(\frac{\alpha}{4\pi}\right)^2 \frac{m'^3}{4\pi}\left(\frac{\bar{g}_f}{M_f}\right)^2 \left\{1 + O\left(\frac{m'^2}{M_f^2}\right) + \cdots\right\} . \tag{5.2}$$

The decay photons are produced isotropically with energy $E_\gamma = \frac{1}{2}m'$ at emission. However, after emission the photons are redshifted by the expansion of the universe and, since decays occur at different times, are redshifted by different amounts. Thus the line spectrum produced at the time of dark matter decay evolves into a broad spectrum of energies E_0 observed today. The typical keV masses expected for self-interacting dark matter lead us to expect the dominant contribution of this spectrum to lie in the far ultraviolet–soft x-ray region accessible to current satellites. The expected flux per unit energy interval observable today is [8]

$$I_E\left(\frac{\text{keV}}{\text{sr}\cdot\text{cm}^2\cdot\text{s}\cdot\text{keV}}\right)=\frac{\rho_0'\,\Gamma(\phi\to 2\gamma)}{2\pi m' H_0(1+Z)\sqrt{1+\Omega Z}}\,, \tag{5.3}$$

where $1+Z=m'/2E_0$ is the redshift of the epoch of emission and E_0 the energy observed today. This represents a rising spectrum with a sharp cutoff at $E_0=m'/2$. Since experimental observations in the far ultraviolet–soft x-ray region find a slowly falling power law that can not be fully explained by known resolved point sources [9], the strictest bound on the dark matter contribution to DEBRA comes from the upper limit of the spectrum, $E_0=\frac{1}{2}m'$, arising from the most recent decays ($Z=0$). Using equation (2.13) for the present dark matter density and equation (5.2) in equation (5.3), we find the spectrum has a peak value

$$I_E=\frac{3H_0\Omega m'^2}{64\pi^3 G}\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{\bar{g}_f}{M_f}\right)^2\,, \tag{5.4}$$

Since the contribution from self-interacting dark matter I_E must not exceed the measured flux, I_{meas} we find a bound.

$$\left(\frac{M_f}{\bar{g}_f}\right)^2\geq\frac{3H_0\Omega m'^2}{64\pi^3 G I_{\text{meas}}}\left(\frac{\alpha}{4\pi}\right)^2\,. \tag{5.5}$$

For example, for $m'\sim 1$ keV, a mass typical for dark matter in the pseudoscalar model, $I_{\text{meas}}(E_0=0.5\text{keV})\sim 10\,\text{keV/sr/cm}^2\text{/sec}$ so that the expected GUT-scale mass and coupling from equation 5.4 is

$$\frac{M_f}{\bar{g}_f}\geq 6\times 10^{12}\sqrt{h\Omega}\,\text{GeV}\,. \tag{5.6}$$

The scalar model increases this bound by roughly an order of magnitude. In either case, these limits permit perfectly reasonable GUT-scale masses. Further, equation (5.2) produces a lifetime for the dark matter $\tau\geq 3\times 10^{19}$ years, more than a billion times the age of the universe, reaffirming our original assumption that this dark matter would be long-lived and entropy processes between dark and ordinary matter negligible.

6. Qualitative Growth of Density Perturbations

Undoubtedly the real test of a dark matter model is whether it can successfully predict structure formation in the early universe. Cold dark matter seems to produce relatively too much structure on small scales, whereas hot dark matter completely washes out structure on small scales. As we have seen in previous sections, self-interacting dark matter has properties in some ways intermediate between hot and cold dark matter, and hence may be ideal for structure formation. However, this subject is complex, and we cannot yet draw any definite conclusions.

We adopt the standard view that structure formation in the universe resulted from initially small density fluctuations $\delta\rho/\rho\ll 1$ that grew in size to eventually form the structure we see. When the density fluctuations are small, in the linear regime, the growth of these fluctuations can be treated perturbatively. However, when the fluctuations become nonlinear, $\delta\rho/\rho\sim 1$, they must be treated numerically. In this qualitative treatment of matter perturbations, we will consider only the linear regime, assuming instantaneous structure formation when $\delta\rho/\rho=1$.

If we assume a near uniform initial matter distribution, the linear perturbation equations can be well approximated by an ideal fluid with pressure and density only slightly perturbed from the mean. We will assume that dark matter dominates and consider, in this first approximation, only a one component fluid with density

$$\rho(x)=\rho_0+\int d^3k\,\delta\rho(k)\,e^{ik\cdot x}\,, \tag{6.1}$$

where ρ_0 is the average density of dark matter and $\delta\rho(k)$ is the Fourier mode of wave number k for the deviation from that average. This one component will be only the dominant component, whether it is radiation (as it is early on) or dark matter (at temperatures below T_e). The spectrum of fluctuations of wave number k is denoted by $\delta_k\equiv\delta\rho(k)/\rho_0$. A particular fluctuation mode will enter the horizon when its wavelength $\lambda=\lambda_H\sim H^{-1}$ after which microphysical processes can affect its evolution. For wavelengths $\lambda<\lambda_J$, the Jean's wavelength

$$\lambda_J=\left(\frac{\pi\beta_s^2}{G(\rho+P)}\right)^{1/2}\,, \tag{6.2}$$

with β_s the adiabatic sound speed in the dark matter fluid, the perturbations behave as acoustic waves with amplitude [10]

$$|\delta_k| \propto \left(\frac{\rho + P}{\beta_s \rho^2 a^4}\right)^{1/2} . \tag{6.3}$$

For wavelengths $\lambda > \lambda_J$ the perturbations can grow. One can argue from the condition for uniform Hubble flow [3] or verify explicitly for small amplitude perturbations that the expected growth behaves as

$$\delta\rho \propto \frac{1}{a^2} . \tag{6.4}$$

The mass M associated with a given fluctuation mode can be defined as the mass contained within a sphere of radius $\lambda/2$. Allowing for the evolution of the wavelength and density to the current epoch,

$$M_J = \frac{H_0^2 \Omega (\lambda_J T)^3}{16 G T_\gamma^3}, \tag{6.5}$$

where T_γ is the current photon temperature of the universe and λ_J and T are the Jean's wavelength and photon temperature evaluated in the earlier epoch. Note that the product $\lambda_J T$ is unaffected by the expansion of the universe.

For our discussion of the evolution of density fluctuations in self-interacting dark matter, we will assume an inflation-motivated flat spectrum of density perturbations with $\delta_k \propto k^{-3/2}$ at horizon crossing. Density fluctuations will only be able to grow after the universe cools to the temperature T_e, the temperature at which dark matter begins to dominate, since before that time the Jean's mass is greater than the horizon mass and all modes entering the horizon remain acoustic. For the early decoupling scenario, $T_e = T_1$, the temperature at which cold dark matter would dominate. After T_1 this dark matter, except for collisions, behaves like ordinary cold dark matter so that we do not expect perturbation growth to be significantly different from that of ordinary cold dark matter.

For the late decoupling scenario, the case is much more interesting. Self-interacting dark matter not only dominates earlier than in the cold dark matter case, but also alters the evolution of density perturbations in favor of more rapid growth during the era $T_e > T > T_d$ when chemical equilibrium is maintained. Using equations (6.4) and (2.7), growing modes are expected to evolve as

$$\frac{\delta\rho}{\rho} \propto a \ln(a/\overline{a}) \tag{6.6}$$

during this era. This growth is less rapid than that of hot dark matter where one expects $\delta\rho/\rho \propto a^2$, yet logarithmically enhanced over cold dark matter, where one expects $\delta\rho/\rho \propto a$.

Consider the qualitative evolution of the spectrum of density fluctuations after horizon crossing in the late decoupling scenario. Again we divide the evolutionary history into three distinct eras: era 1 for $T > T_e$ where ordinary relativistic matter dominates, era 2 for $T_e > T > T_d$ where dark matter dominates while maintaining chemical equilibrium through particle number changing interactions, and era 3 for $T_d > T$ where dark matter dominates after chemical equilibrium is lost. The Jean's mass evaluated in each era determines the critical division between acoustic and growing modes. During era 1 using $\beta_s^2 = \frac{1}{3}$ and $\rho + P = \frac{2\pi^2}{45} g_* T^4$ in equation (6.2) gives a Jean's mass of

$$M_J = \frac{H_0^2 \Omega}{16 G^{5/2} T_\gamma^3 T^3} \left(\frac{15}{2\pi g_*}\right)^{3/2} = 3\times10^{18} M_\odot h^2 \Omega \left(\frac{\text{eV}}{T}\right)^3 \tag{6.7}$$

valid for $T_e < T$. Since the Jean's mass is about equal to the horizon mass all the modes crossing the horizon propagate as acoustic waves. Equation (6.3) then assures that the amplitude of acoustic waves remain constant throughout this era.

During era 2 dark matter now controls the sound velocity, density and pressure. Thus at T_e we expect a discontinuous change in the Jean's mass. Using the defining relation $\beta_s^2 \equiv (\partial P'/\partial\rho')_{\text{adiabatic}}$ for the speed of sound and $P' = \rho' T'/m'$ with ρ'/m' given by equation (2.3), we find

$$\beta_s^2 \sim \frac{T'}{m'} . \tag{6.8}$$

Since $\rho' + P' = s'T' = sT'/\xi$, we can substitute $s = \frac{2\pi^2}{45} g_{\rm eff} T^3$ to obtain the Jean's wavelength

$$\lambda_J = \left(\frac{45\xi}{2\pi g_{\rm eff} G m' T^3}\right)^{1/2} \tag{6.9}$$

and the Jean's mass

$$\begin{aligned} M_J &= \frac{H_0^2 \Omega}{16 G^{5/2} T_\gamma^3} \left(\frac{45\xi}{2\pi g_{\rm eff} m' T}\right)^{3/2} \\ &= 1.9\times10^{18} M_\odot \left(\frac{\rm eV}{T}\right)^{3/2} \left(\frac{m'}{T'_d}\right)^{-3/2} (h^2\Omega)^{-1/2} , \end{aligned} \tag{6.10}$$

for $T_e > T > T_d$. Since $m'/T'_d \sim 10$, and $T \sim$ eV, this is, indeed, somewhat smaller. Notice that M_J grows with decreasing temperature in this era, reaching its maximum value at decoupling. As indicated in equation (6.6), growing modes with $M > M_J$ grow logarithmically more rapidly with scale factor than those in a universe dominated by cold dark matter. The propagation of acoustic waves, $M < M_J$ is also modified. Writing $\rho' a^3 = s'T' a^3 \propto T' \propto 1/\ln(a/\overline{a})$, we can see that acoustic waves are slightly damped by an amount

$$|\delta_k| \propto \frac{[\ln(a/\overline{a})]^{3/4}}{a^{1/2}} . \tag{6.11}$$

During era 3, when chemical equilibrium is lost, density perturbations behave much the same as in the cold dark matter case. The sound velocity is $\beta_s^2 = 5T'/3m'$ and the density is $\rho' = s'T'_d = sT'_d/\xi \propto a^{-3}$. Thus acoustic mode amplitudes are constant and growing modes grow as $\delta\rho/\rho \propto a$. The Jean's wavelength and Jean's mass are then given by

$$\lambda_J = \left(\frac{50}{\pi g_* G T T_1 T_d^2}\right)^{1/2} \left(\frac{m'}{T'_d}\right)^{-1/2} \tag{6.12}$$

and

$$\begin{aligned} M_J &= \frac{H_0^2 \Omega}{16 G^{5/2} T_\gamma^3} \left(\frac{50}{\pi g_*}\right)^{3/2} \left(\frac{T}{T_d^2 T_1}\right)^{3/2} \left(\frac{m'}{T'_d}\right)^{-3/2} \\ &= 4\times10^{18} M_\odot (h^2\Omega)^{-1/2} \left(\frac{T\cdot{\rm eV}}{T_d^2}\right)^{3/2} \left(\frac{m'}{T'_d}\right)^{-3/2} . \end{aligned} \tag{6.13}$$

Again the maximum Jean's mass occurs at $T = T_d$. Since we expect the transition between era 2 and era 3 to be smooth, we do not expect a physical discontinuity in the Jean's mass at the decoupling temperature. Thus the Jean's mass at decoupling in these models introduces a physical scale

$$(M_J)_{\rm max} \sim 3\times10^{18} M_\odot \, (h^2\Omega)^{-1/2} \left(\frac{\rm eV}{T_d}\right)^{3/2} \left(\frac{m'}{T'_d}\right)^{-3/2} \tag{6.14}$$

Characteristic values are $m'/T'_d \sim$ 10-30, $h^2\Omega \lesssim 1$ while T_d, for late decoupling, is constrained to be less than about $6(h^2\Omega)$ eV, so that this mass is no smaller than about $10^{15} M_\odot$. This is of the order of supercluster scale. However, unlike hot dark matter, smaller scales also have structure.

7. The Cosmic Microwave Background

One of the most interesting features of self interacting dark matter is that $\delta T/T$, the temperature fluctuations in the cosmic microwave background, may be modestly reduced compared with regular cold dark matter (CDM).

An important feature of CDM is that, between T_1 and T_{LS},the temperature of photon last scatter, $(\delta\rho/\rho)_{CDM} \propto a$. On the other hand, electromagnetic interactions prevent the growth of baryonic perturbations at temperatures above T_{LS}. After the electromagnetic processes decouple, the baryon perturbations rapidly increase, driven by the gravitational effects of the dominant CDM perturbations. Thus the $\delta T/T$ of the background radiation is suppressed compared to a baryon dominated universe by the growth factor of CDM perturbations between T_1 and T_{LS}:

$$F_{CDM} \simeq \frac{a_{LS}}{a_1} = \frac{T_1}{T_{LS}} \simeq 21.5 \; \Omega h^2. \tag{7.1}$$

The corresponding growth factor for the self-interacting dark matter is

$$F_{SIDM} \simeq \left(\frac{a_d}{a_e} \frac{\ln a_d/\overline{a}}{\ln a_e/\overline{a}}\right) \left(\frac{a_{LS}}{a_d}\right) , \tag{7.2}$$

where we assumed $T_e > T_d > T_{LS}$. It is larger than F_{CDM} for two reasons: the era of self-interacting dark matter domination begins at a higher temperature than does the CDM dominated era, and during this era the dark matter perturbations grow faster than in CDM. These factors turn out to be the same, specifically

$$\frac{F_{SIDM}}{F_{CDM}} = \frac{a_1}{a_e}\frac{\ln a_d/\overline{a}}{\ln a_e/\overline{a}} = \frac{T_e}{T_1}\frac{T'_e}{T'_d} = \left(\frac{T_e}{T_1}\right)^2 , \tag{7.3}$$

where we have used equations (2.6), (2.9), and (4.18) to simplify this expression. As we increase the ratio T_e/T_1, we will have an easier time making galaxies from SIDM without running afoul of small measured $\delta T/T$ than we will from standard CDM. Hence a large ratio is desirable.

Unfortunately, T_e/T_1 cannot be made arbitrarily large in this scenario. The combination of the nucleosynthesis lower bound (3.4) on ξ', and the galaxy formation bound (3.7), together with some reasonable upper bound on the current density like $\Omega h^2 < 1$, gives rise, through equations (4.14) and (4.19) on an upper bound for this ratio. Saturating all possible bounds by setting $\xi' = 7.1$, $\Omega h^2 = 1$, $m'/T'_d = 17.8$, and $T_d = 0.48\,\text{eV}$ (which can be attained in the scalar model if $\lambda'_s = 3\times10^{-10}$ or in the pseudoscalar model if $\lambda'_p = 20$), we find that we can attain an equality temperature of $T_e = 11.6\,\text{eV}$, resulting in a maximum increase in density perturbations (or, equivalently, a decrease in temperature fluctuations) of about a factor of 4.3. We note that the decoupling temperature $T_d = 0.48\,\text{eV}$ is a little bigger than the last scattering temperature $T_{LS} \approx 0.3\,\text{eV}$, justifying our assumption $T_d > T_{LS}$.

The most important constraint restricting the ratio T_e^2/T_1^2 is the galaxy formation constraint (3.7), so that if this can be relaxed even higher suppressions are possible. Somewhat smaller suppressions occur for $h^2\Omega < 1$ and a wide range of ξ', so a factor of two for T_e^2/T_1^2 is quite conceivable. Although this is a modest suppression, if observations exclude CDM it would be important to do numerical work to obtain this factor more accurately.

8. Conclusion

The properties of self-interacting dark matter are surprisingly different from any other dark matter considered in the past. For example, the temperature of the dark matter falls only logarithmically as a function of scale

factor prior to freeze out, rather than as a power law. The linearized growth of density perturbations also has unusual properties. It remains to be seen if self-interacting dark matter is successful at predicting large scale structure in the universe.

Much work still remains to be done in this subject. Most important is the study of the growth of density perturbations and their effect on the cosmic microwave background radiation. In addition, realistic models need to be studied, like the glueball model discussed in the introduction. Such models may have additional complications that have not been considered here. For example, the application of the nucleosynthesis bounds in a glueball model would have to be applied with care, because the confinement phase transition could result in a change in the number of degrees of freedom and an increase in entropy of the dark sector.

We feel that self-interacting dark matter is an interesting and unusual dark matter candidate which deserves further study.

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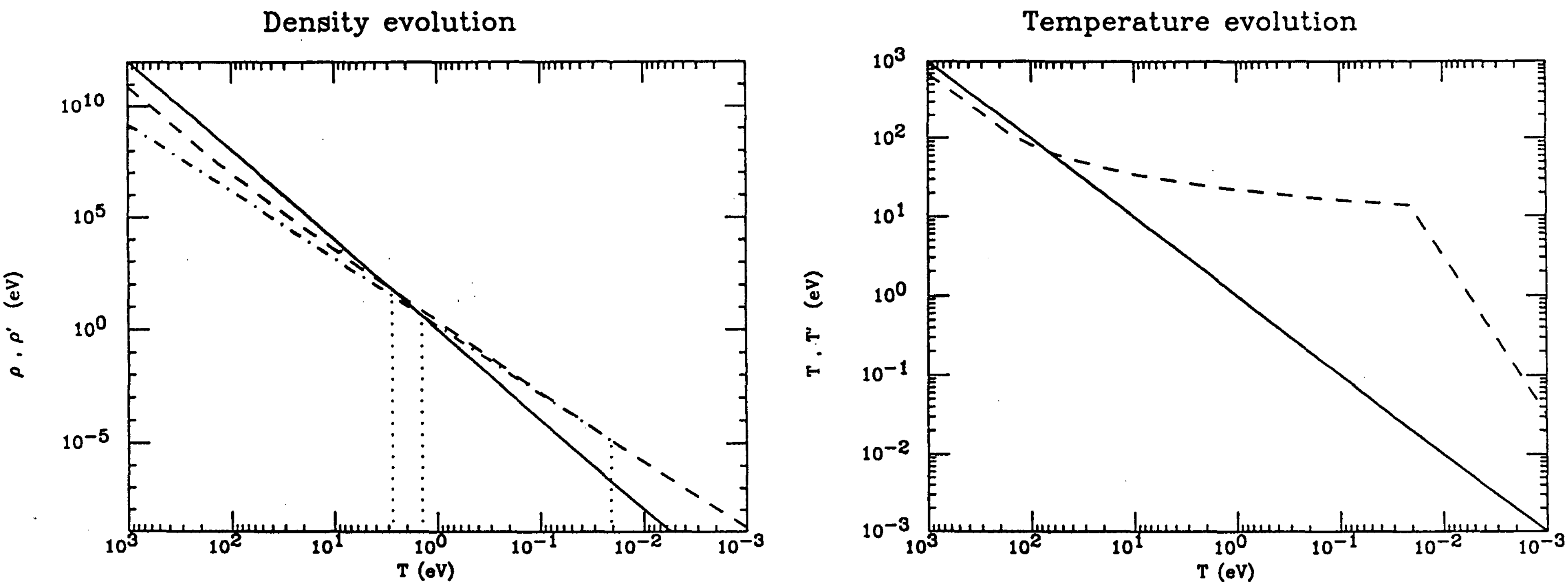
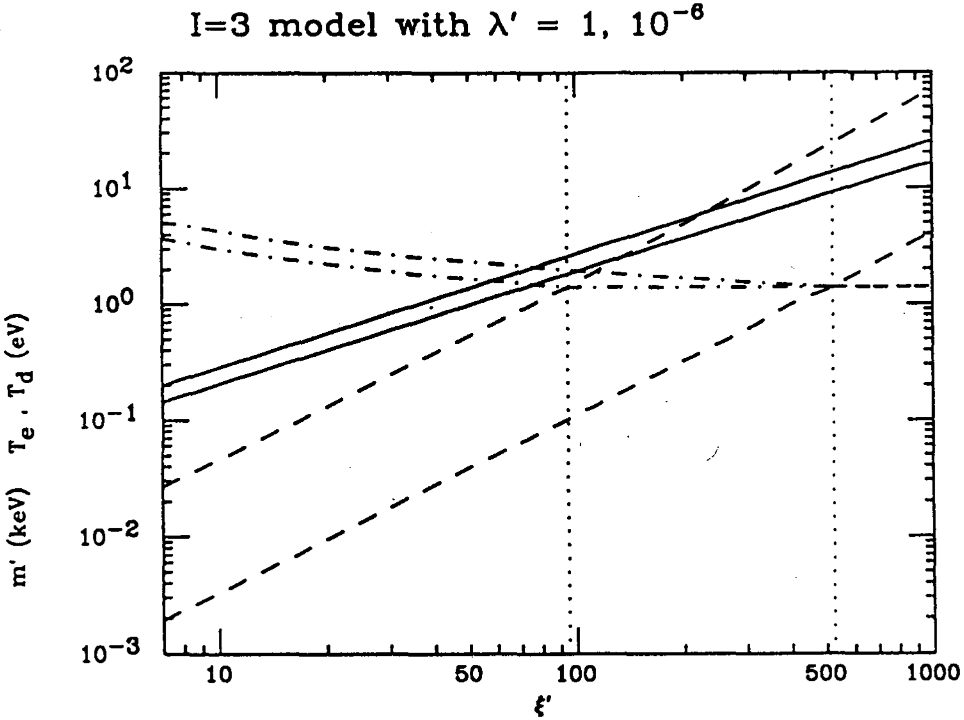


Figure 1. The temperature and mass density evolution for characteristic values, using $\Omega h^2 = 0.25$. The solid and dashed lines display the evolution of the radiation and dark matter, respectively. The sudden change in the slope of the temperature graph occurs at the temperature T_d when the number changing interactions freeze out. In the density evolution graph, the dot–dash line indicates the evolution of cold dark matter with the same final density as self-interacting dark matter. The vertical dots correspond to the temperature T_e, T_1, and T_d from left to right.



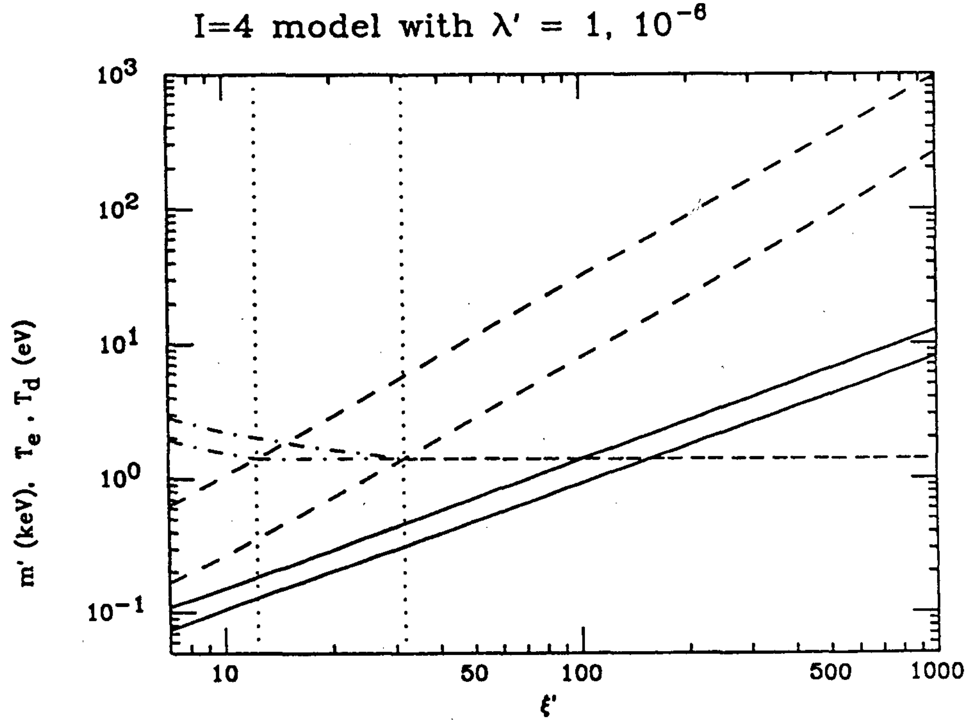


Figure 2. The relationship between ξ' and m' (solid), T_d (dashes), and T_e (dot–dash) for $I = 3$ or $I = 4$ for couplings $\lambda' = 1$ (lower curve for T_d, upper curve for m' and T_e) and for $\lambda' = 10^{-6}$. The vertical dots correspond to the boundary between case I (to the right of the dots) and case II (to the left of the dots) for the two values of λ'. This boundary value is where the curves for T_e and T_d come together at the value $T_1 - 1.4$eV (for $\Omega h^2 = 0.25$). Note that m' is in keV, while T_e and T_d are in eV.

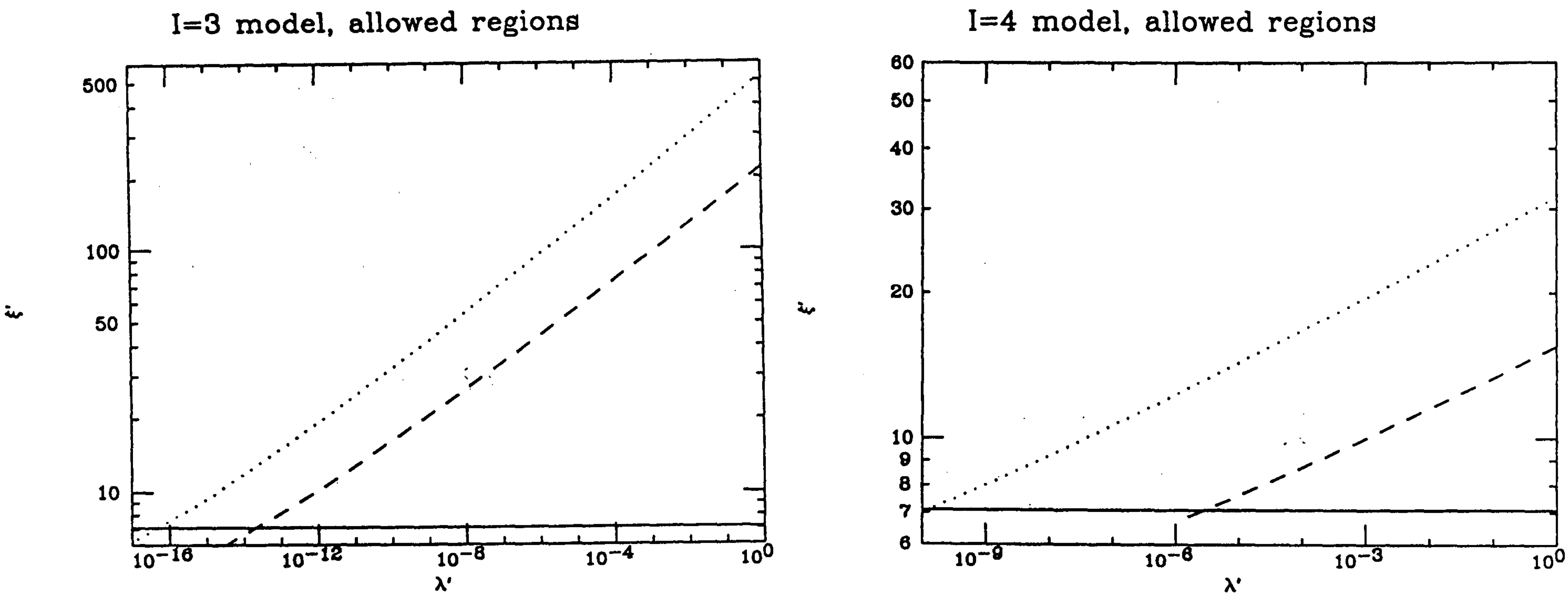


Figure 3. The allowed region in (λ', ξ') parameter space for $I = 3$ and $I = 4$. The solid curve is the nucleosynthesis bound, and the dashed curve is the galaxy formation bound (for $\Omega h^2 = 0.25$). The regions above the two curves are allowed. The dotted line is the boundary between case I (above the line) and case II (below the line).