

Theory of plasma injection into a magnetic field

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Analytic and self-consistent solutions for the propagation of a low-beta, large-gyroradius plasma across a transverse magnetic field are derived. It is shown that if $\omega_{pi}^2/\Omega_i^2 \gg (M/m)^{1/2}$, the beam can propagate into the field by means of an $\mathbf{E} \times \mathbf{B}$ plasma drift. The cross-field velocity of the plasma in this case is found to be very nearly equal to the beam injection velocity u_0 . The theory is discussed with reference to recent proof-of-principle experiments on cross-field propagation.

I. INTRODUCTION

The objective of the present investigation is to provide a theoretical basis for the injection of pulsed, neutralized ion beams into toroidal fields for the purpose of driving currents¹ or of supplementary tokamak heating.² Cross-field injection experiments can be divided into the following two categories: (i) those in which the beam radius R is much greater than an ion gyroradius a_i , and (ii) those in which $R \lesssim a_i$. Condition (i) is usually applicable to beams injected from plasma guns, while condition (ii) is usually applicable to high-energy neutralized ion beams injected from ion diodes. We shall call a plasma beam satisfying the condition $R \gg a_i$ a small-gyroradius beam; correspondingly, a beam for which $R \lesssim a_i$ will be called a large-gyroradius beam.

In the case that $R \gg a_i$, the beam can be considered as a one-dimensional slab of plasma which is incident upon the magnetic field. This is because the space charge layers at the beam boundaries are now separated by a distance $2R$ much larger than the characteristic scale length a_i in the longitudinal direction. Hence, the resultant polarization electric field E_y in the transverse direction would give rise to a potential energy difference $q\varphi = qE_y(2R)$ across the beam much larger than the initial longitudinal beam particle energy $\frac{1}{2}Mu_0^2$. In this case, we must then require $E_y = 0$, which is the usual boundary condition for the one-dimensional case. We have previously studied an equilibrium based on a generalization of the Ferraro-Rosenbluth sheath,³ and a stability analysis of this equilibrium has now been completed.⁴ Such an investigation is important because of the many reported observations of Rayleigh-Taylor instabilities in cross-field injection experiments⁵⁻⁷.

We consider only the case of $R \lesssim a_i$ in the present study so that space charge polarization is induced on the edges of a bounded plasma column as it passes through a magnetic field.⁸ The polarized space charge layers create a self-consistent electric field which makes it possible for the plasma to propagate across the field by means of an $\mathbf{E} \times \mathbf{B}$ plasma drift (Fig. 1). A few simple relations which describe the conditions for this motion to take place can be derived. First, the beam must have more than sufficient energy to set up the required electric field. If the kinetic energy density of the plasma is $\frac{1}{2}n_0Mu_0^2$, we have

$$\frac{1}{2}n_0Mu_0^2 \gg E_y^2/8\pi, \quad (1)$$

where n_0 and u_0 are the initial density and velocity of the

plasma. If we further require that the plasma should drift across the field lines with its initial forward velocity u_0 , it is seen that

$$E_y = u_0 B_0/c. \quad (2)$$

In this relation, B_0 represents a constant magnetic field applied in a direction transverse to the beam propagation, and along the z axis (cf., Fig. 1). Throughout our paper we will be assuming a low-beta plasma so that the magnetic field due to induced plasma currents will be taken to be negligible. Substituting the relation for the electric field E_y from Eq. (2) into Eq. (1) we obtain

$$\omega_{pi}^2 \gg \Omega_i^2, \quad (3)$$

where ω_{pi} is the ion plasma frequency and Ω_i is the ion gyrofrequency. Equation (3) is often written in terms of the plasma dielectric constant $\epsilon = 1 + \omega_{pi}^2/\Omega_i^2$ as $\epsilon \gg 1$. It was first derived by Schmidt⁸ in his study of plasma motion across a curved magnetic field. His analysis was based on an adiabatic magnetic field gradient, so that the guiding center approximation could be used. We have obtained more quantitative results⁴ using an explicit magnetic field distribution in the adiabatic case. In the case we shall be concerned with in this paper, the motion is nonadiabatic, so that we must consider the more general two-fluid equations. However, the basic polarization scheme of the "plasma capacitor" model of Schmidt is still applicable.

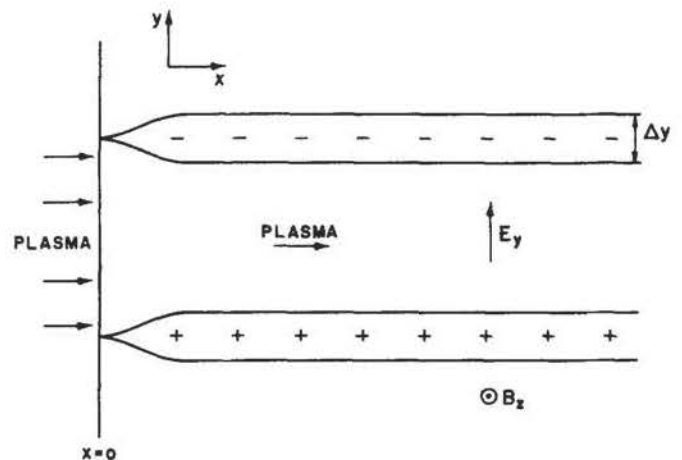


FIG. 1. Schematic drawing of the polarization of a bounded plasma beam in a transverse magnetic field B_z . The resultant electric field E_y allows the beam to propagate through the magnetic field by means of an $\mathbf{E} \times \mathbf{B}$ plasma drift.

In this scheme, shown in Fig. 1, the incident plasma particles are deflected by the magnetic field. The electrons are shifted upward by a distance y_e and the ions are shifted downward by a distance $y_i \ll y_e$. The resulting charge layers have a width $\Delta y = y_e - y_i$ which, in general, is a function of the longitudinal distance x . Far from the $x = 0$ plane, we can treat the charge layer configuration as a plasma capacitor with an electric field

$$E_y = 4\pi n_0 q \Delta y. \quad (4)$$

A necessary condition for this scheme to be valid is that $\Delta y \ll R$. Solving for Δy from Eq. (4), and using Eq. (2), we can derive from this inequality a lower limit on the beam radius R for our model to be valid

$$R \gg a_i / \epsilon. \quad (5)$$

An upper limit can be derived by requiring that the maximum potential difference that can develop across the beam current should not exceed the initial energy of the ions.⁹ Hence, $qE_y(2R) < \frac{1}{2} M u_0^2$, or from Eq. (2),

$$R < \frac{1}{4} a_i. \quad (6)$$

If the beam radius is initially larger than this quantity, it is possible that the beam will break up into smaller beams which satisfy this relation.⁹

Finally, we shall derive a simple expression for the expected plasma velocity through the field by means of an energy balance argument. The initial kinetic energy of the beam goes into the kinetic energy of motion and the energy needed to produce the electric field E_y ,

$$\frac{1}{2} n_0 M u_0^2 = \frac{1}{2} n M u_x^2 + E_y^2 / 8\pi.$$

In addition, because the plasma drifts across the field we have that $u_x = cE_y/B_0$. The continuity equation $mu_x = n_0 u_0$ then enables us to write the above relation as a quadratic equation in the variable u_x . The only physical root of this equation is

$$u_x = u_0(1 - 1/\epsilon). \quad (7)$$

A more detailed derivation of Eq. (7) can be derived directly from the model equations we shall be presenting in Sec. II. Analytic solutions to these equations are also derived in this section. In Sec. III we examine the validity of the charge-neutral approximation in our model, and in the last section we discuss our theory with respect to recent cross-field injection experiments.

II. BASIC EQUATIONS

Assume that at $t = 0$ plasma is injected from $x < 0$ into the applied magnetic field B_0 in the region $x > 0$. The equations of motion for each species are

$$\frac{\partial \mathbf{v}_j}{\partial t} + (\mathbf{v}_j \cdot \nabla) \mathbf{v}_j = \frac{q_j}{m_j} \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_j \times \mathbf{B} \right), \quad (8)$$

where $\mathbf{v}_j = (v_{jx}, v_{jy}, 0)$ is the velocity vector, and $j = (e, i)$ is the species index which can denote either electrons or ions. In Eqs. (8), \mathbf{E} and \mathbf{B} represent the usual electric and magnetic fields, and we shall write $m_i = M$ and $m_e = m$ for the mass of the individual species. The continuity equation is

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0. \quad (9)$$

All physical variables, such as velocity and density, etc., will be taken to be a function of the longitudinal space coordinate x and the time t only. In our model equations, we shall assume plasma quasi-neutrality^{3,10} so that we may set the ion density equal to the particle density, $n_i = n_e$. This implies from Eq. (9) that $v_{ix} = v_{ex}$. Defining the new variables

$$n \equiv n_i = n_e, \quad (10a)$$

$$u_x \equiv v_{ex} = v_{ix}, \quad (10b)$$

$$u_y \equiv v_{ey} - v_{iy}, \quad (10c)$$

Eqs. (8) and (9) can be written

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} = -\Omega_e u_y, \quad (11a)$$

$$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} = \Omega_e u_x - \frac{q}{m} E_y, \quad (11b)$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (n u_x) = 0, \quad (11c)$$

where terms of order m/M (the electron-to-ion mass ratio) have been neglected with respect to unity. The longitudinal electric field E_x is expressible as

$$E_x = -(1/c) u_y B_0, \quad (12)$$

which can be obtained directly from Eqs. (8) and (9). The corresponding relation for the polarization electric field is given as

$$E_y = 4\pi n q \Delta y. \quad (13)$$

We note that the total time derivative of the charge-layer thickness Δy is just the relative velocity u_y , between the electrons and ions, hence,

$$u_y = \left(\frac{d}{dt} \right) \Delta y. \quad (14)$$

Equations (11)–(14) are the basic equations for our model. They are most easily solved by reformulating them in terms of Lagrangian variables.¹¹ This is equivalent to following a particular fluid element whose path is given by $x = x(x_0, t)$, instead of the usual Eulerian approach of balancing conserved fluid quantities in a fixed "control volume" of space. The parameter x_0 identifies a particular fluid element, and is usually chosen to be the initial value of $x(t)$ at $t = 0$. In their Lagrangian form, Eqs. (11) can be written in terms of the derivative (d/dt) following a fluid element:

$$\left(\frac{\partial u_x}{\partial t} \right)_{x_0} = \frac{du_x}{dt} = -\Omega_e u_y, \quad (15a)$$

$$\left(\frac{\partial u_y}{\partial t} \right)_{x_0} = \frac{du_y}{dt} = \Omega_e u_x - \frac{q}{m} E_y, \quad (15b)$$

$$n \left(\frac{\partial x}{\partial x_0} \right)_t = n_0. \quad (15c)$$

The partial derivative appearing in Eq. (15c) is just the Jacobian of the transformation between the independent variables x_0 and x . To describe a uniform beam incident on a region containing a constant magnetic field $B_x = B_0$ for $x > 0$, the initial conditions are developed as follows: For $t < -x_0/u_0$,

$$x = x_0 + u_0 t, \quad u = u_0, \quad n = n_0,$$

$-\infty < x_0 < 0$; this describes a uniform beam for $x < 0$.

For $t = 0$, $x = x_0$, $u_x = u_0$. We assume that for the incident beam $u_y = 0$ so that from Eq. (15a), $du_x/dt = 0$. For $t - x_0/u_0$ the solution $x = x(x_0, t)$, $u_x = u_x(x_0, t)$, $u_y = u_y(x_0, t)$ satisfies Eqs. (15a) and (15b).

Combining Eqs. (14) and (15a), we can eliminate the velocity u_y , and solve for the charge-layer thickness Δy in terms of the longitudinal velocity u_x . The result is

$$\Delta y = (u_0 - u_x) / \Omega_i. \quad (16)$$

We also take the time derivative of Eq. (15a) and substituting for du_y/dt from Eq. (15a), we obtain

$$\frac{d^2 u_x}{dt^2} + \Omega_h^2 u_x = \frac{q}{m} \Omega_i E_y, \quad (17)$$

where $\Omega_h^2 = \Omega_i \Omega_e$ is the hybrid gyrofrequency.

The charge layer thickness Δy from Eq. (16) can be substituted into the electric field relation Eq. (4), to give

$$\frac{\partial^2 u}{\partial \tau^2} + \left(1 + \frac{n}{n_0} (\epsilon - 1)\right) u = \frac{n}{n_0} (\epsilon - 1), \quad (18)$$

$$\frac{n_0}{n} = \frac{\partial \bar{x}}{\partial x_0}, \quad u = \frac{\partial \bar{x}}{\partial \tau}.$$

The dimensionless quantities $\tau = \Omega_h t$, $u = u_x/u_0$, $\bar{x} = x/a_h$, $\bar{x}_0 = x_0/a_h$, and $\epsilon = 1 + \omega_{pi}^2/\Omega_i^2$ have been introduced. The quantity a_h is the hybrid gyroradius $a_h = (a_i a_e)^{1/2}$. The initial conditions for $\tau = -\bar{x}_0$ are $\bar{x} = 0$, $u = 1$, and $\partial u/\partial \tau = 0$.

Equation (18) can be solved for $\epsilon \gg 1$ by first assuming $n/n_0 \cong 1$. The solution is

$$u = 1 - (1/\epsilon)[1 - \cos\sqrt{\epsilon}(\tau + \bar{x}_0)], \quad (19)$$

$$\bar{x} = (1 - 1/\epsilon)(\tau + \bar{x}_0) + (1/\epsilon^{3/2}) \sin\sqrt{\epsilon}(\tau + \bar{x}_0),$$

$$\frac{n_0}{n} = \frac{\partial \bar{x}}{\partial x_0} = 1 - \frac{1}{\epsilon} [1 + \cos\sqrt{\epsilon}(\tau + \bar{x}_0)]. \quad (19)$$

Iteration can be carried out by substituting $n/n_0 = 1 + O(1/\epsilon)$ into Eq. (18). Equation (19) is thus the first iteration and the correction term is of order $1/\epsilon^2$.

In terms of the usual physical variables our solution becomes

$$u_x \cong u_0 \left\{ 1 - \frac{1}{\epsilon} \left[1 - \cos\sqrt{\epsilon} \Omega_h \left(t + \frac{x_0}{u_0} \right) \right] \right\} + O\left(\frac{1}{\epsilon^2}\right), \quad (20a)$$

and

$$x = u_0 \left\{ \left(t + \frac{x_0}{u_0} \right) \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon^{3/2} \Omega_h} \sin \left[\sqrt{\epsilon} \Omega_h \left(t + \frac{x_0}{u_0} \right) \right] \right\} + O\left(\frac{1}{\epsilon^2}\right). \quad (20b)$$

The oscillatory behavior of the plasma velocity in Eq. (20a) is due to the competing effects of the polarization electric and applied magnetic fields. The frequency of the oscillations, $\sqrt{\epsilon} \Omega_h$, is just the electron plasma frequency ω_{pe} for $\epsilon \gg 1$. Note that our solutions are well-behaved even for small values of ϵ [$\leq (M/m)^{1/2}$] although the charge-neutral approximation we have assumed is not valid in this parameter range (see Sec. III). For a value of ϵ equal to unity (i.e., a plasma of very low density), the resulting charge is unable to generate an electric field sufficient to allow propagation. The plasma

stops on a scale length equal to the hybrid gyroradius a_h (as expected from previous results^{3,4}) and returns to the injection plane. Because the limit $\epsilon \rightarrow 1$ does not satisfy the assumption of local charge neutrality, we must exercise care in the strict interpretation of this result.

The analytic solution of Eq. (20a) agrees with the simple back-of-the-envelope calculation for the mean plasma velocity u_x presented in the Introduction. Averaging out the fast time oscillations in Eq. (20a), we obtain

$$\bar{u}_x = u_0(1 - 1/\epsilon) \quad (21)$$

for the mean plasma velocity \bar{u}_x . This relation is identical to Eq. (7), which was derived by the simple energy balance argument discussed earlier.

We shall now discuss the limit when the plasma dielectric constant ϵ is much larger than unity. In this case, it can be seen from Eq. (20a) that the plasma will traverse the field with a velocity very nearly equal to its injection velocity u_0 . A steady-state model for cross-field propagation has been discussed by Sinelnikov and Rutkevich,¹² who obtained a solution to their model equations by assuming that the velocity of the plasma through the field did not differ significantly from its injection value u_0 . We shall show that our solution reduces to theirs in the limit of large ϵ . In this limit, Eq. (20b) becomes

$$x \cong u_0(t + x_0/u_0).$$

Substituting this into Eq. (20a) for the time t , we obtain a relation between the velocity u_x and the distance x ,

$$u_x = u_0 \{ 1 - (1/\epsilon) [1 - \cos\sqrt{\epsilon}(x/a_h)] \}, \quad (22)$$

which is just the steady-state solution derived by Sinelnikov and Rutkevich in the so-called "slow-deceleration approximation."

This solution is plotted in Fig. 2. It should be noted that the solution of Eq. (18) on which Eq. (22) is based is only valid for $\epsilon \gg 1$. In the next section we shall see that there is a further restriction to $\epsilon \gg (M/m)^{1/2}$ so that the large oscillations for $\epsilon = 10$ and the nonpropagation for $\epsilon = 1$ are outside the domain of validity of the model.

Finally, we shall discuss the effects of collisions in our model. Because it is apparent that the oscillatory

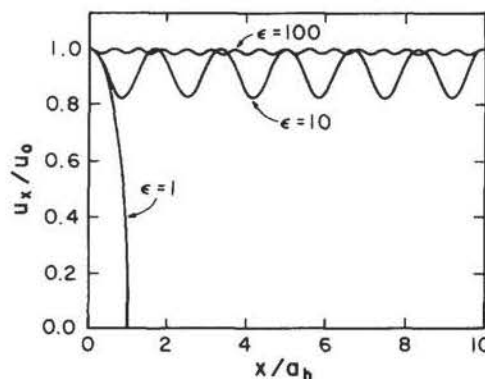


FIG. 2. Plots of longitudinal velocity u_x/u_0 versus distance x/a_h into the field for three different values of the plasma dielectric constant ϵ .

behavior of the plasma cannot sustain itself indefinitely, we can take into account the expected damping of these oscillations by introducing a collisional term in our equations. This has been done⁴ by using the Langevin description for the collision frequency, so that a term proportional to $\nu(v_i - v_e)$ is added to Eqs. (8). The result is that the plasma oscillations damp out after a few hybrid gyroperiods, and the motion of the plasma is steady $\mathbf{E} \times \mathbf{B}$ drift motion with a velocity u_x given by Eq. (21). The equations can be solved analytically, and the details can be found elsewhere.⁴

III. VALIDITY OF CHARGE-NEUTRAL APPROXIMATION

We will now consider the validity of the charge-neutral approximation in our model. From Poisson's equation, the net fractional charge density in the plasma is given by

$$\frac{n_i - n_e}{n_0} = \frac{1}{4\pi n_0 q} \frac{\partial E_x}{\partial x}.$$

From the relation $E_x = -(1/c)u_y B_0$ in Eq. (12), this expression can be written

$$\frac{n_i - n_e}{n_0} = -\frac{B_0}{4\pi n_0 q c} \frac{\partial u_y}{\partial x}.$$

We consider the solution to our model equations in the steady-state Eulerian representation for $\epsilon \gg 1$. Then, the velocity $u_y(x, t)$ can be expressed as a function of the coordinate x only, and is independent of the time t . From Eq. (11b) we can write

$$\frac{n_i - n_e}{n_0} = -\frac{\Omega_h^2}{\omega_{pi}^2} \left(1 - \frac{cE_y/B_0}{u_x} \right). \quad (23)$$

In the steady-state, the density is given by the continuity equation $n(x) = n_0 u_0 / u_x(x)$. From this relation we can solve for the electric field E_y using Eqs. (13) and (16). Equation (23) then becomes

$$\frac{n_i - n_e}{n_0} = -\frac{\Omega_h^2}{\omega_{pi}^2} \left(1 - \frac{\omega_{pi}^2}{\Omega_i^2} \frac{u_0(u_0 - u_x)}{u_x^2} \right).$$

We are considering the parameter range where $\epsilon \gg 1$, which describes relatively undisturbed plasma motion across the magnetic field. Using Eq. (22) we can then expand the expression $(u_0 - u_x)/u_x$ in powers of $(1/\epsilon)$, keeping terms to first order. The result is

$$\frac{n_i - n_e}{n_0} = -\frac{\Omega_h^2}{\omega_{pi}^2} \left[1 - \frac{\omega_{pi}^2}{\epsilon \Omega_i^2} \left(1 - \cos \sqrt{\epsilon} \frac{x}{a_h} \right) \right].$$

Taking the mean value of this expression, we can average out the fast spatial oscillations to obtain

$$\frac{\delta n}{n_0} = -\frac{\Omega_h^2}{\omega_{pi}^2} \left(1 - \frac{\omega_{pi}^2}{\epsilon \Omega_i^2} \right), \quad (24)$$

where $\delta n = \langle n_i - n_e \rangle$. The assumption of charge neutrality requires the condition

$$|\delta n / n_0| \ll 1,$$

and from Eq. (24) we can write this inequality as

$$(\Omega_h^2 / \omega_{pi}^2) (1 - \omega_{pi}^2 / \epsilon \Omega_i^2) \ll 1.$$

If we use the definition $\epsilon = 1 + \omega_{pi}^2 / \Omega_i^2$, this relation sim-

plifies to

$$(1/\epsilon^2)(M/m) \ll 1.$$

Hence, the necessary condition for charge-neutrality in our model is

$$\epsilon \gg (M/m)^{1/2}. \quad (25)$$

If this condition is not satisfied, we expect longitudinal space-charge polarization effects to be dominant. We shall discuss this condition in relation to current experiments on cross-field injection in Sec. IV. Space charge effects in the transverse y direction have been discussed elsewhere.⁴ This was done by solving for the surface charge density σ and iterating our solution by recalculating the resultant electric fields. In this case, by requiring plasma quasi-neutrality, we can rederive the condition obtained in Eq. (5) by more pedestrian means. This is another example of the similarity in results between the formal and informal methods of approach, and can be understood with a simple example. If the charge layer thickness Δy is not small with respect to the beam radius R , there will exist finite regions (in each charge layer) containing an imbalance of charge. Within those regions, therefore, the assumption of plasma quasi-neutrality breaks down.

IV. DISCUSSION

In Table I, the results of a few recent large-gyroradius experiments in order of increasing dielectric constant ϵ are detailed. The first entry, a neutralized ion beam experiment by Wessel and Robertson,¹³ studied plasma injection into a toroidal magnetic field. The value of ϵ in this case was measured to be between 25 and 50. According to the theory we have just presented, the plasma cannot remain quasi-neutral unless $\epsilon \gg (M/m)^{1/2} = 43$. A region of positive space charge is expected to form that will reflect the beam. In the Wessel and Robertson experiment, where this condition was not satisfied, the ions were seen to separate from the electrons, producing a positive space-charge region near the field boundary. Such a longitudinal separation of charge is in accordance with the non-neutral behavior expected for ϵ

TABLE I. Results and parameters of some large-gyroradius experiments. Unless otherwise noted, the data are taken at 2 kG.

Experiment	Beam type	R/a_i	ϵ	Result
Wessel and Robertson (Refs. 7, 13)	ion beam	1.5	26	no propagation; boundary layer formed
Kamada <i>et al.</i> (Ref. 14)	ion beam	0.1	208	$\mathbf{E} \times \mathbf{B}$ drift
Ishizuka and Robertson (Ref. 15)	ion beam	1.5	300	$\mathbf{E} \times \mathbf{B}$ drift
Wickham and Robertson (Ref. 16)	potassium plasma	0.7	1.3×10^6 (@ 750 G)	$\mathbf{E} \times \mathbf{B}$ drift

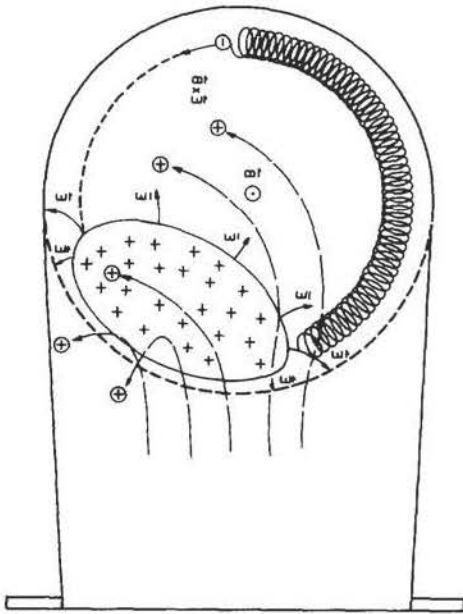


FIG. 3. Schematic drawing of the positive space-charge region formed near the toroidal field boundary in the Wessel and Robertson experiment.¹³ Such longitudinal charge separation is expected when the plasma dielectric constant does not satisfy the charge-neutrality condition in Eq. (25).

$< (M/m)^{1/2}$. A schematic drawing of the particle orbits in and around this space-charge region is given in Fig. 3.

A similar experiment (using a linear pyrex tube instead of a tokamak as the vacuum vessel) was conducted by Kamada *et al.*¹⁴ In this case, a value of $\epsilon \geq 208$ was found sufficient to allow for cross-field plasma motion by means of an $\mathbf{E} \times \mathbf{B}$ plasma drift. Further experimental work at the University of California at Irvine using a similar drift tube and an improved ion diode was then begun, and the results of this experiment, obtained by Ishizuka and Robertson,¹⁵ demonstrated $\mathbf{E} \times \mathbf{B}$ plasma motion for values of $\epsilon \geq 300$. A related experiment using a potassium plasma source in a Q machine has also been reported by Wickham and Robertson.¹⁶ In this case the plasma motion across the field could be observed visually. The value of the dielectric constant for this experiment was $\epsilon = 1.3 \times 10^6 \gg (M/m)^{1/2}$.

An explanation for the higher threshold $\epsilon \gg (M/m)^{1/2}$ required for cross-field propagation in our study as opposed to the adiabatic condition $\epsilon \gg 1$ derived by Schmidt is that in our case the plasma beam enters the applied magnetic field from a region of no field. For the beam to propagate, transverse charge separation must occur before a virtual anode is formed. The scale length for the formation of a virtual anode³ is $a_h = u_0 / \Omega_h$ where $\Omega_h^2 = \Omega_i \Omega_e$. The scale length for transverse charge separation is $a_i / \epsilon = u_0 / \Omega_i \epsilon$. The condition $a_i / \epsilon \ll a_h$ is the same as the condition $\epsilon \gg (M/m)^{1/2}$.

An important limitation of the present theory is its inability to describe the formation of the polarization charge-layers. Indeed, the formation of these charge layers was specifically assumed by the use of the relation, Eq. (13). A more complete theory would necessi-

tate the consideration of a truly bounded beam with densities and velocities which are functions of both x and y . Such a two-dimensional treatment would, of course, be difficult to do analytically, but numerical integration or particle code simulations may be of some use in this case.

It should be mentioned that because we are considering low-beta plasmas only, there also exists an upper limit on the values that the dielectric constant ϵ can have. From the relation $\beta = (u_0^2/c^2)\epsilon \ll 1$, we have the condition that $\epsilon \ll (u_0^2/c^2)^{-1}$. The parameter range $(M/m)^{1/2} \ll \epsilon \ll (u_0^2/c^2)^{-1}$ is easily satisfied in modern ion beam experiments.

Finally, we recall that our theory is good only for large-gyroradius beams such that $R \lesssim a_i$. If $R \gg a_i$, as it is in experiments involving plasma guns,⁵⁻⁷ cross-field propagation probably takes place because of a Rayleigh-Taylor instability. The measurements of electric polarization and plasma transport across a magnetic field in this case are not very reproducible. This is physically different from the case $R \lesssim a_i$ and will be treated in a different publication.

In conclusion, the experimental data for the large-gyroradius, cross-field injection experiments support the theoretical model developed here for $\mathbf{E} \times \mathbf{B}$ drift motion across field lines. The necessary condition for this propagation to take place is given by $\epsilon \gg (M/m)^{1/2}$. If this condition is not satisfied, longitudinal space-charge effects dominate. As mentioned, such non-neutral effects were seen in the Wessel and Robertson ion beam experiment at the University of California at Irvine. To ensure propagation of the plasma, shorting of the polarization electric field^{17,18} in the experiments must be carefully avoided.

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