Lawrence Berkeley National Laboratory

Recent Work

Title

AN ANALYTICAL EXPRESSION FOR THE EFFECTIVE SOLID ANGLE. OF A RECTANGULAR COLLIMATOR FOR THE PASSAGE OF FRAGMENTS FROM BREAKUP PROJECTILES

Permalink

<https://escholarship.org/uc/item/42w3s8dz>

Authors

Shotter, A.C.

Bice, A.N.

Cerny, J.

Publication Date

1980-08-01

9/23

LBL-11375 c.2
Preprint

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE

BERKELEY LABORATORY

SEP 26 1980

LIBRARY AND
DOCUMENTS SECTION

Submitted to Nuclear Instruments and Methods

AN ANALYTICAL EXPRESSION FOR THE EFFECTIVE SOLID ANGLE OF A
RECTANGULAR COLLIMATOR FOR THE PASSAGE OF FRAGMENTS FROM
BREAKUP PROJECTILES

A. C. Shotter, A. N. Bice and Joseph Cerny

August 1980

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782

LBL-11375
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

AN ANALYTICAL EXPRESSION FOR THE EFFECTIVE SOLID ANGLE
OF A RECTANGULAR COLLIMATOR FOR THE PASSAGE OF FRAGMENTS
FROM BREAKUP PROJECTILES*

A. C. Shotter

Physics Department
Edinburgh University

A. N. Bice and Joseph Cerny

Department of Chemistry and
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

July 1980

Abstract:

An analytical expression is given for the effective solid angle of a rectangular collimator that is used as the defining aperture of a detection system for recording an excited projectile which breaks up into two fragments. Values given by the expression are compared to values from a calculation based on Monte Carlo methods.

*This work was supported by the Nuclear Physics and Nuclear Sciences Divisions of the U.S. Department of Energy under contract No. W-7405-ENG-48.

1. Introduction

In certain experimental investigations the outgoing nuclei from a nuclear reaction can be excited to energies above particle breakup thresholds; these nuclei can then break up, usually into two fragments. More often than not, the best way to record such events is to detect in coincidence the breakup fragments arriving in separate counters. However, it is sometimes experimentally expedient to record the two fragments in the same detector system as for example in the ($^{12}\text{C}, ^8\text{Be}$) reaction.¹ If the detector system is a $\Delta E \times E$ particle telescope then these events may be identified by their special mass identification signals; for the above example the two α 's from ^8Be would be recorded as a ^7Li . If such a procedure is followed then it is important for cross section estimates to determine the effective solid angle of the collimator. In this paper an almost general analytical expression is derived for the effective solid angle of a rectangular collimator in terms of the laboratory kinetic energy of the projectile before breakup and the center-of-mass kinetic energy ϵ between the breakup fragments.

2. Detection Efficiency

The breakup kinematics for the situation considered is shown in fig. 1. The center-of-mass velocity of the decaying projectile is denoted by v_c . The velocities of the two fragments in the center-of-mass frame are v_1, v_2 ; their masses are m_1, m_2 . The total laboratory energy of the projectile is denoted by E and the center-of-mass energy ϵ . Conservation of momentum requires $m_1 v_1 = m_2 v_2$. In the following, it is assumed that the breakup fragments are emitted isotropically in the center-of-mass system. The locus of the vectors v_1, v_2 are therefore spheres with radii v_1, v_2 as indicated in the figure.

The geometrical location of the collimator with respect to the source of particles is shown in fig. 2. The source of particles is point S which is separated from the collimator center by a distance d . The collimator is a rectangle with sides $(2a) \times (2b)$; the plane of the collimator is perpendicular to the vector $\overline{SO}$.

The situation that occurs when a projectile is emitted from S such that the vector V_c intercepts the collimator at x, y is shown in fig. 3. The radius of the breakup sphere corresponding to $v_1 (\geq v_2)$ at the collimator plane is given by $r = v_1 d / v_c$, where $r = d \left(\frac{m_2}{m_1} \frac{\epsilon}{E} \right)^{1/2}$. The locus of the vector v_1 may be anywhere on this sphere. However, if the particle m_1 is to pass through the collimator then the vector v_1 must be located on that part of the sphere corresponding to the projection of the

rectangle $2x \times 2y$ onto the sphere. (This assumes the projectile rays are parallel to $\overline{SO}$.) If this is so, and $v_1 \geq v_2$ then both particles m_1 and m_2 will enter the collimator. The probability that this will occur is $(A(x,y)/2\pi r^2)$ where A is the projected surface area onto the sphere. The total of such events is then obtained by integration of this probability for all directions of v_c over the collimator area ($2a \times 2b$).

The area A may be found by integration to be:

$$A(x,y) = 4r \left(y \tan^{-1} \left(\frac{x}{(r^2 - x^2 - y^2)^{1/2}} \right) + x \tan^{-1} \left(\frac{y}{(r^2 - x^2 - y^2)^{1/2}} \right) - r \tan^{-1} \left(\frac{yx}{r(r^2 - x^2 - y^2)^{1/2}} \right) \right) \quad (1)$$

provided $r^2 \geq x^2 + y^2$.

The probability $\frac{A(x,y)}{2\pi r^2}$ should then be integrated over x and y to yield the total probability P that any projectile with vector v_c , located anywhere over the rectangle, will be recorded.

This probability is given by:

$$P(a,b) = \frac{1}{4ab} \left\{ \int_0^{2a} \int_0^{2b} \frac{A(x,y)}{2\pi r^2} dx dy \right\}.$$

Integration of $A(x,y)$ yields the following expression for $P(a,b)$:

$$\begin{aligned}
 P(a,b) = & \frac{1}{\pi\alpha\beta} \left(\beta(1+\alpha^2)\tan^{-1} \left[\frac{\beta}{(1-\alpha^2-\beta^2)^{1/2}} \right] \right. \\
 & + \alpha(1+\beta^2)\tan^{-1} \left[\frac{\alpha}{(1-\alpha^2-\beta^2)^{1/2}} \right] \\
 & + (1-\alpha^2-\beta^2)^{1/2} \left(\frac{\alpha^2+\beta^2+2}{3} \right) \\
 & - 2\alpha\beta \tan^{-1} \left[\frac{\alpha\beta}{(1-\alpha^2-\beta^2)^{1/2}} \right] - (1-\alpha^2)^{1/2} \left(\frac{2+\alpha^2}{3} \right) \\
 & \left. - (1-\beta^2)^{1/2} \left(\frac{2+\beta^2}{3} \right) - \alpha \sin^{-1}(\alpha) - \beta \sin^{-1} \beta + 2/3 \right) \quad (2)
 \end{aligned}$$

where $\alpha = (a/r)$, $\beta = (b/r)$.

3. Results

As a check to the accuracy of this expression, numerical calculations were also undertaken. These calculations simulated breakup events and tested whether any particular pair of breakup fragments passed through the collimator. It was again assumed that the breakup fragments were emitted isotropically with respect to the rest frame of the decaying projectile. Typically, for each $P(a,b)$ calculated, $\sim 10^6$ breakup events were sampled. In fig. 4 a comparison between the results from the two methods is shown. It is seen there is reasonable agreement between them for different collimator configurations. The small disagreement between the two methods probably arises from the finite sampling of the Monte Carlo calculation.

The analytical expression, eq. (1), is only valid under the condition $r^2 \geq a^2 + b^2$. For situations outside this limit an analytical expression may still be computed for $P(a,b)$ but the expression becomes more complicated than given by eq. (2) and will change for different relative sizes of a , b , r . However, the condition $r^2 \geq a^2 + b^2$ is sufficiently general to cover most experimental situations.

The analytical expression eq. (2) can be compared with the numerical calculations of Menchaca-Rocha² for the specific case of ^8Be breakup. Such a comparison indicates that the two calculations are in agreement within a few percent.

Footnotes and References

- 1) G. J. Wozniak, H. L. Harney, K. H. Wilcox, and Joseph Cerny,
Phys. Rev. Lett. 28 (1972) 1278.
- 2) A. Menchaca-Rocha, Nucl. Instr. and Methods 141 (1974) 425.

Figure Captions

Fig. 1 Kinematic breakup cone.

Fig. 2. Geometrical arrangement of the collimator. The center of the collimator is point 0; the half-width and half-height are a and b , respectively.

Fig. 3. The breakup sphere at radius r .

Fig. 4. The analytical function $P(a,b)$ for a normalized breakup sphere of projected radius 1.0, for a square collimator ($a = b$) and a rectangular collimator $b = 2a$. The circles denote values from a Monte Carlo numerical calculation for comparison.

$$E_{\text{LAB}} = 1/2 (m_1 + m_2) V_c^2; \epsilon = 1/2 m_1 v_1^2 + 1/2 m_2 v_2^2$$

$$m_1 v_1 = m_2 v_2$$

XBL-807-1531

Fig. 1

XBL-807-1529

Fig. 2

XBL-807-1530

Fig. 3

XBL-807-1532

Fig. 4

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720