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Particle beam self-modulation instability in tapered and inhomogeneous plasma

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The particle beam self-modulation instability in tapered and inhomogeneous plasmas is analyzed via an evolution equation for the beam radius. For a sufficiently fast taper the instability is suppressed, and the condition for growth suppression is derived. The form of the taper to phase lock a trailing witness bunch in the plasma wave driven by a self-modulated beam is determined, which can increase the energy gain by several orders of magnitude. Growth of the instability places stringent constraints on the initial background plasma density fluctuations.

Plasma-based accelerators have attracted considerable attention owing to the ultrahigh field gradients sustainable in an electron plasma wave. The electric field amplitude of the electron plasma wave (space-charge oscillation) can be on the order of $E_0 = cm_e\omega_p/e$, or $E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$, where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency, n_0 is the ambient electron number density, m_e and e are the electron rest mass and charge, respectively, and c is the speed of light in vacuum. Plasma waves with relativistic phase velocities may be excited by the ponderomotive force of an intense laser[1] or the space-charge force of a charged particle beam, i.e., a plasma wakefield accelerator (PWFA).[2–4]

It has been proposed to drive a PWFA with a highly relativistic proton beam, such as those available at CERN (European Organization for Nuclear Research).[5,6] Plasma wave excitation requires a drive beam density profile with frequency components at the plasma frequency, i.e., a beam density longitudinal scale length on the order of $\lambda_p = 2\pi/k_p = 2\pi c/\omega_p$, or $\lambda_p[\mu\mathrm{m}] = 3.3 \times 10^{10}/\sqrt{n[\mathrm{cm}^{-3}]}$. Compact, high-gradient accelerators require high plasma density, and therefore short drive beams. For example, the peak accelerating field driven by a narrow ($k_p\sigma_r < 1$) Gaussian beam (with $k_p\sigma_z = 1$) in the linear regime is $E_z[\mathrm{GV/m}] \approx 2\ln(\sigma_z/\sigma_r)(N_b/10^{10})(100\ \mu\mathrm{m}/\sigma_z)^2$, where σ_r is the rms radius, σ_z is the rms length, and N_b is the number of particles. Hence a high gradient ($\gtrsim 1$ GV/m) PWFA requires short beams (e.g., $\sigma_z \lesssim 100\ \mu$m).

Generating short proton beams (or proton beams with spatial structure at λ_p) is challenging, and it has been proposed to rely on a beam-plasma instability to modulate the beam at λ_p, driving a large amplitude plasma wave.[7,8] The beam self-modulation occurs through the beam-driven plasma wakefield creating periodic regions of focusing and defocusing, modulating the beam density at λ_p, driving a larger plasma density modulation that further focuses the beam periodically. The growth rate for the beam self-modulation instability was calculated in the long-beam regime (i.e., $k_p\sigma_z \gg 1$) in Ref. 9, and shown to be comparable to that of the hose instability.[10]

The phase velocity v_p of the plasma wave can limit the energy gain in a plasma accelerator. For example, if $v_p < c$, a highly relativistic electron will outrun the plasma wave and phase slip from an accelerating to a decelerating phase region of the plasma wave. This limits the length over which the electron will gain energy to a dephasing length. For a plasma accelerator driven by a short ($< \lambda_p$) laser pulse, v_p can be relatively low and dephasing can limit the energy gain.[11] For a PWFA driven by a short ($< \lambda_p$) highly-relativistic beam, v_p

can be sufficiently high so that dephasing will not limit the energy gain.

Recently it was shown that the phase velocity of the plasma wave excited by a beam undergoing self-modulation is significantly less than the velocity of the drive beam.[9,12] This result indicates that the energy gain of a PWFA driven by a self-modulated beam in a uniform plasma will be severely limited by dephasing, and that, in the long-beam regime, dephasing is reached in less than four e foldings, independent of beam-plasma parameters.[9] For a PWFA in the linear regime, a witness bunch that propagates more than a dephasing length will not only move from an accelerating to a decelerating region, losing energy, but also from a focusing to a defocusing region, scattering the beam transversely.

To compensate for the slow wake phase velocity of the self-modulated PWFA, it is natural to consider tapering the plasma density, i.e., increasing the plasma density to reduce λ_p, thereby increasing the phase velocity and phase locking the witness bunch in the plasma wave.[13,14] Tapering will increase the efficiency of a self-modulated PWFA provided the taper is sufficiently slow so that decoherence effects do not suppress the instability.

In this work, we analyze the beam self-modulation instability including a spatial variation in the plasma density. The evolution equation of the beam radius in a tapered plasma is solved numerically, and the exponential growth of the instability is calculated in the limit of a slow taper. It is shown that, for a sufficiently fast taper, the instability is suppressed, and the condition for growth suppression is derived. The functional form of the taper (background plasma density profile) to phase lock a relativistic beam in the plasma wave driven by the self-modulated beam is determined. It is shown that growth of the instability places stringent constraints on the initial background plasma density fluctuations.

The wake generated by a particle beam moving through an initially neutral plasma can be calculated using the cold plasma fluid and Maxwell equations. We will assume that the plasma density is varying along the direction of beam propagation $n_0(z)$, with a variation slow compared to the plasma wavelength $|\partial_z n_0| \ll k_p n_0$. Here we consider a wake driven by a relativistic beam, $\gamma = (1-\beta_b^2)^{-1/2} \gg 1$ where $v_b = c\beta_b \simeq c$ is the beam velocity, consisting of particles with charge $\pm e$ (e.g., proton or electron) and mass M_b. In the linear wake regime, the envelope equation for the beam radius $r_b(\zeta, z)$ at any beam slice $\zeta = z - \beta_b ct$ is[9]

$$\frac{d^2 r_b}{dz^2} - \frac{\epsilon_n^2}{\gamma^2 r_b^3} = -\frac{4k_b^2 r_{b0}^2 I_2(k_p r_b)}{\gamma r_b} \int_\infty^\zeta d\zeta' \sin[k_p(\zeta-\zeta')] f(\zeta') K_1(k_p r_b(\zeta'))/r_b(\zeta'), \qquad (1)$$

where $k_b^2 = 4\pi n_{b0} e^2/M_b c^2$ is the plasma wavenumber of the beam, $k_p^2 = 4\pi e^2 n_b(z)/m_e$ is

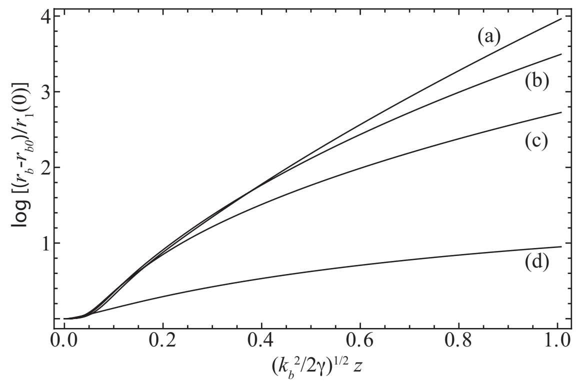

FIG. 1. Growth of beam radius modulation (at $k_{p0}|\zeta| = 410$) versus normalized distance $(k_b/\sqrt{2\gamma})z$ from the numerical solution of Eq. (1) for a proton beam propagating through a linearly tapered plasma density $k_p^2 = k_{p0}^2(1 + z/L_t)$, with (a) uniform plasma $L_t^{-1} = 0$, (b) $k_{p0}L_t = 2 \times 10^5$, (c) $k_{p0}L_t = 10^5$, and (d) $k_{p0}L_t = 10^4$. The beam-plasma parameters are $n_0 = 10^{15}$ cm^{-3}, $\gamma = 480$, $n_{b0}/n_0 = 0.008$, and $k_{p0}r_{b0} = 1$.

the tapered plasma wavenumber, ϵ_n is the normalized beam emittance, and I_m and K_m are modified Bessel functions. The right-hand side of Eq. (1) is the focusing force (at a given ζ) of the transverse plasma wakefield excited by the beam. For simplicity, Eq. (1) has assumed a transversely-flat-top beam density profile $n_b = n_{b0}(r_{b0}/r_b)^2\Theta(r_b - r)f(\zeta)$, where n_{b0} is the initial peak density, r_{b0} is the initial beam radius, f is the longitudinal distribution, and Θ is the Heaviside function. Note that this form of the beam density profile conserves charge during beam radius modulations, in contrast to that considered in Ref. 12.

Figure 1 shows the numerical solution to Eq. (1) for the beam radius modulation amplitude versus normalized propagation distance $k_bz/(2\gamma)^{1/2}$ at $k_p(0)\zeta = -410$ (with the beam head at $\zeta = 0$). Figure 1 assumes a proton beam propagating through a linearly tapered plasma density $n_0(z) = n_0(0)(1 + z/L_t)$, for several values of L_t, with $n_0(0) = 10^{15}$ cm^{-3}, $\gamma = 480$, $n_{b0}/n_0(0) = 0.008$, and $k_{p0}r_{b0} = 1$. These beam parameters are near that of the 450 GeV CERN Super Proton Synchrotron (SPS) beam. Figure 1 shows that for sufficiently fast taper, the instability growth will be suppressed.

The linear growth rate can be calculated by assuming a slow taper and perturbing Eq. (1) about the long-beam equilibrium radius r_0 satisfying $\epsilon_n^2 k_p = 4\gamma k_b^2 r_0^3 K_1(k_pr_0)I_2(k_pr_0)$. Assuming $r_{b0} = r_0$, $r_b = r_0 + r_1$ with $|r_1/r_0| \ll 1$, and a narrow beam $k_pr_0 \ll 1$, Eq. (1) yields the evolution of the beam radius perturbation[9]

$$\left(\frac{\partial^2}{\partial \hat{z}^2} + 4\hat{k}_b^2\right) r_1 = 2\hat{k}_p \hat{k}_b^2 \int_\infty^\zeta d\hat{\zeta}' \sin[\hat{k}_p(\hat{\zeta} - \hat{\zeta}')] r_1(\hat{\zeta}'), \tag{2}$$

with the normalized variables $\hat{\zeta} = k_{p0}\zeta$, $\hat{z} = k_{p0}z$, $\hat{k}_p = k_p(z)/k_{p0}$, and

$$\hat{k}_b^2 = \frac{k_b^2}{2\gamma k_{p0}^2} = \frac{n_{b0} m_e}{2\gamma n_0 M_b}, \tag{3}$$

with $k_{p0} = k_p(0)$. In the narrow beam regime, the equilibrium radius satisfies $r_0 = [2\epsilon_n^2/k_b^2\gamma]^{1/4}$. Applying the plasma wave operator to Eq. (2) yields

$$\left\{\left[\partial_{\hat{\zeta}}^2 + \hat{k}_p^2(\hat{z})\right]\left(\partial_{\hat{z}}^2 + 4\hat{k}_b^2\right) - 2\hat{k}_p^2(\hat{z})\hat{k}_b^2\right\} r_1 = 0. \tag{4}$$

For a long bunch $\hat{\zeta} \gg \hat{k}_b \hat{z}$, the self-modulation instability is in the strongly-coupled regime where the growth length is short compared to $\gamma^{1/2} k_b^{-1}$. In this regime we may consider a slowly varying envelope $r_1 = \hat{r}\exp(i\hat{k}_p\hat{\zeta})/2 + \text{c.c.}$ such that $|\partial_{\hat{\zeta}}\hat{r}| \ll \hat{k}_p|\hat{r}|$ and $|\partial_{\hat{z}}\hat{r}| \gg 2\hat{k}_b|\hat{r}|$. From Eq. (4) we see that the taper will strongly perturb the instability when $|\partial_{\hat{z}}\hat{r}| \sim |\hat{\zeta}(\partial_{\hat{z}}\hat{k}_p)\hat{r}|$ or $|\partial_{\hat{\zeta}}\hat{r}| \sim |(\hat{k}_p^2 - 1)\hat{r}|$.

Assuming the taper is sufficiently slow such that $|\partial_{\hat{z}}\hat{r}| \gg |\hat{\zeta}(\partial_{\hat{z}}\hat{k}_p)\hat{r}|$, the solution to Eq. (4) may be approximated in the strongly-coupled regime as $r_1 \sim \exp(i\hat{k}_p\hat{\zeta} + N)$, where the exponentiation of the fastest growing mode is

$$N = \frac{3}{4}(i + \sqrt{3})\left[2\hat{k}_b^2|\hat{\zeta}|\left(\int_0^{\hat{z}} \hat{k}_p^{1/2} d\hat{z}\right)^2\right]^{1/3}. \tag{5}$$

The number of e foldings of the wave amplitude growth is $\Re(N)$. For example if we consider a linear density taper $n_0(z) = n_0(0)(1 + z/L_t)$, the number of e foldings of growth is $\Re(N) \simeq (3\sqrt{3}/4)[2|\hat{\zeta}|\hat{k}_b^2\hat{z}^2(1 + z/2L_t)]^{1/3}$.

If the rate of taper is too large then decoherence effects will suppress the instability growth. From Eq. (4), the growth of the instability will not be affected by the taper provided $|\hat{r}^{-1}\partial_{\hat{z}}\hat{r}| \gg |\hat{\zeta}(\partial_{\hat{z}}\hat{k}_p)|$. With $\hat{r} \sim \exp(N)$, this condition may be expressed as

$$|\partial_{\hat{z}}\hat{k}_p| \ll \left[\hat{k}_b^2/(\hat{\zeta}^2\hat{z})\right]^{1/3}. \tag{6}$$

For example, with a linear taper in plasma density $\hat{k}_p^2 = 1 + z/L_t$, growth will be suppressed behind a beam of length L_b if $L_t \lesssim \hat{k}_b^{-2/3} L_b^{2/3} z^{1/3}$. Suppression of growth at large z for fixed L_t is evident in Fig. 1.

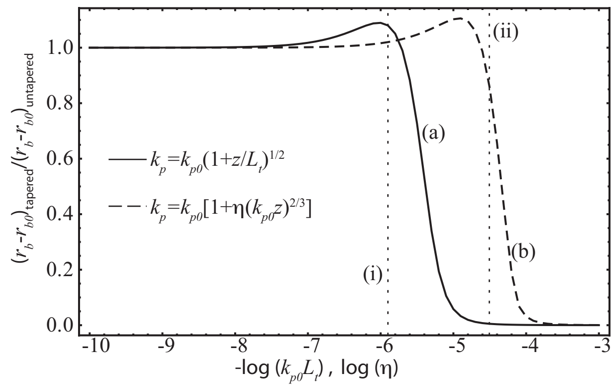

FIG. 2. Growth of the modulation at $k_{p0}|\zeta| = 410$ and $k_{p0}z = 15000$ ($\hat{k}_b\hat{z} = 1$), normalized to untapered growth, versus the taper scale length for the same beam-plasma parameters as Fig. 1. (a) Solid curve is a linear taper $\hat{k}_p = (1 + z/L_t)^{1/2}$, and (b) dashed curve is for $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$. Vertical dotted lines are given by Eq. (6) for (i) linear taper and (ii) $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$.

Figure 2 shows the numerical solution for the growth of the modulation at $|k_{p0}\zeta| = 410$ after propagating $k_{p0}z = 15000$ ($\hat{k}_b\hat{z} = 1$) versus the taper length L_t for a linear taper (solid curve). Here the beam-plasma parameters are the same as in Fig. 1. Equation (6) predicts growth suppression for $(k_{p0}L_t)^{-1} \gtrsim 10^{-6}$ [vertical dotted line (i) in Fig. 2], in reasonable agreement with the numerical solution of Eq. (1). Figure 2 also shows the numerical solution for the growth at $|k_{p0}\zeta| = 410$ after $k_{p0}z = 15000$ for a taper of the form $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$ (dashed curve). For this taper, Eq. (6) predicts the instability suppression for $\eta \gtrsim (\hat{k}_b/\hat{\zeta}^2)^{1/3}$, e.g., $(\hat{k}_b/\hat{\zeta}^2)^{1/3} \simeq 3 \times 10^{-5}$ [vertical dotted line (ii) in Fig. 2]. As shown below this form of the taper ($\propto z^{2/3}$) can phase lock a witness bunch in the wake of a self-modulating drive beam.

For sufficiently slow tapers satisfying Eq. (6), the phase shift of the plasma wave is given by the imaginary component of Eq. (5) such that $r_1 \sim \exp(i\psi)$ with

$$\psi = \varphi - k_p\zeta - \Im(N), \tag{7}$$

and φ is a constant. For a short ($< \lambda_p/2$) witness beam injected behind the drive beam at an initial position ζ_i with $|\zeta_i| > L_b$, the exponentiation is $N = N(|\zeta| = L_b, z)$. Note that, for a taper to be sufficiently slow, as to satisfy Eq. (6) and not suppress the instability, yet large enough to phase lock a witness bunch, requires injecting the beam well behind the drive beam $|\zeta_i| \gg L_b$.

For large energy gains in the plasma wave it is desirable for the witness bunch to remain at a constant phase in the plasma wave (e.g., near the peak of the accelerating field), such that $\psi = \psi(z=0) = \varphi - \hat{\zeta}_i$. Here we assume the witness bunch velocity is relativistic, such that ζ_i is constant. For the witness bunch to remain at a constant phase, Eq. (7) implies, provided the taper is slow, the tapered plasma is approximately

$$\hat{k}_p(z) \simeq 1 + \frac{3}{4|\hat{\zeta}_i|}\left[2\hat{k}_b^2(k_{p0}L_b)\right]^{1/3}\hat{z}^{2/3}. \tag{8}$$

Equation (8) indicates that the optimal taper scale length is on the order of the growth length reduced by the number of plasma periods behind the head of the drive beam that the witness bunch is injected, and that we expect the form of the taper to be $\hat{k}_p = 1+\eta\hat{z}^{2/3}$. Note that the condition for a slow taper Eq. (6) can be satisfied for the taper Eq. (8) only for $|\zeta_i| \gg L_b$. For $|\zeta_i| \gtrsim L_b$ (i.e., injection just behind the drive beam), we expect $\eta \gtrsim (3/4|\hat{\zeta}_i|)[2\hat{k}_b^2(k_{p0}L_b)]^{1/3}$ to compensate for the reduced instability growth owing to the tapered density.

Figure 3(a) shows the normalized wake phase velocity $2\gamma^2(\beta_p - 1)$ in a linearly tapered plasma with $\hat{k}_p^2 = 1 + z/L_t$ for a witness bunch injected at $|\hat{\zeta}_i| = 657$ (behind the drive beam with $k_{p0}L_b = 600$), for several taper scale lengths and the beam-plasma parameters of Fig. 1. As shown in Fig. 3(a), a linear taper is a poor choice to phase lock a witness bunch, as the phase velocity continues to grow as the beam propagates, and the witness bunch will slip in phase with respect to the accelerating field.

Figure 3(b) shows the normalized wake phase velocity $2\gamma^2(\beta_p - 1)$ using a taper with the form $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$, with $\eta = 3.5\times10^{-5}$, at phase positions $|\hat{\zeta}_i| = 600$ (tail of drive beam), $|\hat{\zeta}_i| = 657$, and $|\hat{\zeta}_i| = 700$. Note that η is somewhat larger than that predicted by Eq. (8) to compensate for the reduced instability growth owing to the density taper. For a witness bunch injected at $|\hat{\zeta}_i| = 657$ behind the head of the drive beam, the phase velocity has been increased to $\beta_p \simeq 1$ in the asymptotic regime of the instability. For $|\hat{\zeta}_i| > 657$ (further behind the beam) the phase velocity is super-luminal, and for $|\hat{\zeta}_i| < 657$ the wake phase velocity is sub-luminal.

Using a taper of the form $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$ a short relativistic witness bunch injected at a specific phase location behind the drive beam can be phase locked to the plasma wave [cf. Fig. 3(b)]. For example, consider a wake driven by a 450 GeV proton beam ($\gamma = 480$), with $r_0 = 0.2$ mm, $L_b = 10$ cm, and 10^{11} particles (i.e., near the parameters of the SPS).

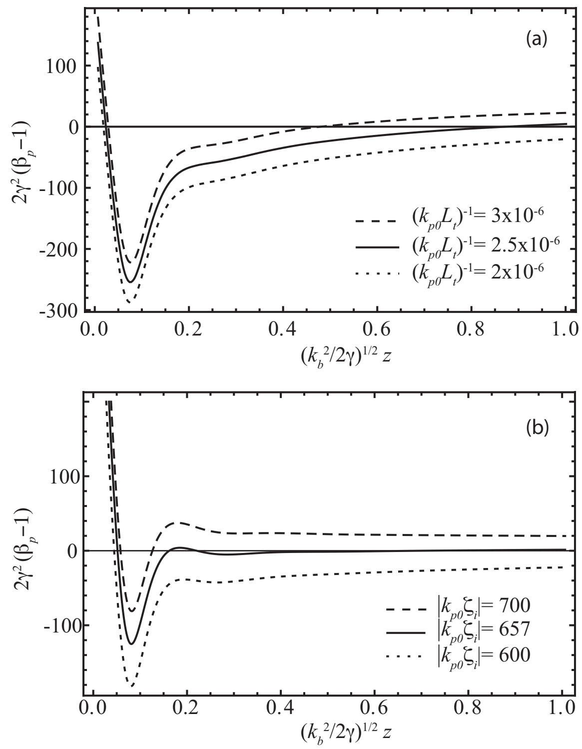

FIG. 3. Normalized wake phase velocity $2\gamma^2(\beta_p - 1)$ versus $\hat{k}_b\hat{z}$ for (a) a linear taper with a taper with scale lengths $(k_{p0}L_t)^{-1} = 2\times10^{-6}$, 2.5×10^{-6}, and 3×10^{-6} at $|\hat{\zeta}_i| = 657$, and (b) a taper of the form $\hat{k}_p = 1+\eta\hat{z}^{2/3}$, with $\eta = 3.5\times10^{-5}$. The beam-plasma parameters are the same as Fig. 1.

Operating at $n_0(0) = 10^{15}$ cm^{-3}, corresponds to $n_{b0}/n_0(0) = 0.008$, $E_0 = 3$ GV/m, $k_{p0}r_0 = 1$, and $k_{p0}L_b = 600$. If we consider a witness electron beam injected behind the proton beam at $|\hat{\zeta}_i| = 657$, in a tapered plasma $\hat{k}_p = 1 + \eta\hat{z}^{2/3}$ with $\eta = 3.5\times10^{-5}$, and assuming an instability seed amplitude of $E_{z,\text{seed}}/E_0 \simeq 10^{-5}$, then the energy gained by the witness bunch is $m_ec^2\Delta\gamma \simeq 2.2$ GeV after propagating $\hat{z} = 26400$ (i.e., 4.4 m). For longer propagation the instability enters a nonlinear regime where Eq. (1) is no longer valid. In a homogeneous plasma the energy gain is 0.01 MeV after a dephasing length, and the witness bunch slips through ≈ 1.5 plasma periods with respect to the wake over $\hat{z} = 26400$.

In general, the self-modulation instability will be suppressed for sufficiently large density variations (Δn_f large) and sufficiently fast variations (k_f large), as can be seen by considering an inhomogeneous density of the form $n_0(z) = n_0(0)[1 + \Delta n_f \sin(k_f z)]$. Figure 4 shows the instability growth normalized to the initial perturbation $\log[\hat{r}/\hat{r}(0)]$, with $|\hat{\zeta}| = 410$, $\hat{z} = 12000$ ($\hat{k}_b\hat{z} = 0.8$), and the beam-plasma parameters of Fig. 1. When the scale length of

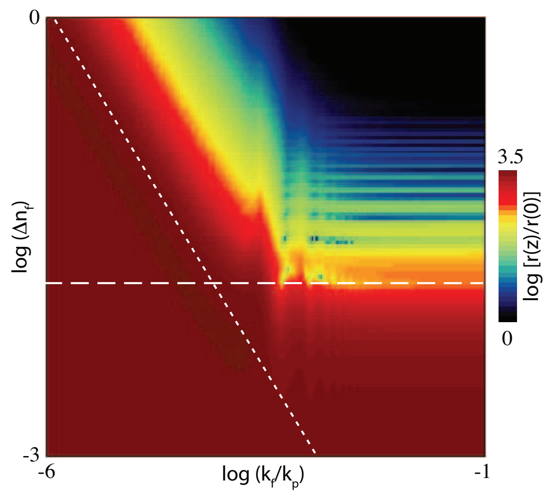

FIG. 4. (Color) Growth of the modulation at $k_{p0}|\zeta| = 410$ after propagating $k_{p0}z = 12000$ ($\hat{k}_b\hat{z} = 0.8$), for the same beam-plasma parameters as Fig. 1, assuming an inhomogeneous plasma density $n_0(z)/n_0(0) = 1 + \Delta n_f \sin(k_f z)$. The dotted line is Eq. (6), and the dashed horizontal line is Eq. (9).

the fluctuations is long compared to the propagation distance ($k_f z \ll 1$), then $\hat{k}_p(z) \simeq 1 + \Delta n_f k_f z/2$, i.e., a linear taper, and Eq. (6) predicts that the instability will be suppressed for $\Delta n_f k_f \gtrsim \hat{k}_b^{2/3}(z\zeta^2)^{-1/3}$ (dotted line in Fig. 4). If the scale length of the density fluctuations is short, then decoherence between plasma oscillations at different plasma frequencies may occur between the head and tail of the beam suppressing the instability (if the instability growth is not sufficiently fast). Equation (4) indicates the inhomogeneity will not affect the growth if $|(\hat{k}_p^2 - 1)\hat{r}| \ll |\partial_{\hat{\zeta}}\hat{r}|$, which yields the condition

$$\Delta n_f \ll (\hat{k}_b z/\zeta)^{2/3}. \tag{9}$$

Figure 4 shows the approximate condition for growth suppression $\Delta n_f \gtrsim (\hat{k}_b z/\zeta)^{2/3}$ (dashed horizontal line). For the parameters of Fig. 4, suppression of growth occurs when $\Delta n_f \gtrsim 0.01$ for fluctuation scale lengths $k_f^{-1} \lesssim 2 \times 10^3 k_{p0}^{-1}$, i.e., for background plasma fluctuations >1% over <2 m at $n_0(0) = 10^{15}$ cm^{-3}.

In this work we have shown that tapering can be an effective method to compensate for the reduced phase velocity of the self-modulated beam-driven plasma wave.[9,12] For sufficiently fast taper, decoherence effects suppress the instability. A general condition for suppression of the growth via an initial plasma density inhomogeneity was derived, Eq. (6). A taper that scaled with propagation distance as $\propto z^{2/3}$ was shown to phase lock a relativistic witness bunch behind a drive beam in the asymptotic regime of the instability, increasing the energy gain of a witness bunch by several orders of magnitude. It was also shown, Eq. (9), that small density fluctuations ($\sim$1% for the parameters of Fig. 4) can suppress the instability. Hence achieving the required plasma density homogeneity will be a significant technical challenge for future self-modulated PWFA experiments.

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