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**VALUING THE FUTURES MARKET
PERFORMANCE GUARANTEE**

By

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Roger Craine

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Valuing the Futures Market Performance Guarantee

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Valuing the Futures Market Performance Guarantee

Abstract

This paper presents estimates of the market value of the Chicago Mercantile Exchange's exposure on the nearby S&P500 contract during the October 1987 market crash. The exchange clearinghouse guarantees the performance of all contracts. The performance guarantee is a credit guarantee. An underpriced guarantee exposes the exchange to risk and undermines the credibility of the clearinghouse's pledge to perform in the case of a default. The value of the underpricing is the market value of the exchange's exposure.

The main innovation in this paper is to extend the standard option valuation model of debt guarantees widely used in finance, eg, Merton (1974, 1977), to assign a price to the futures market performance guarantee and to use the price to evaluate the exchange's exposure on the S&P500 contract during the 1987 market crash. I find that the clearinghouse's exposure may have been as much \$1.4billion on October 19th. By the end of October the performance guarantee was fairly priced and the clearinghouse had no exposure.

Introduction

Traders in the futures market can enter a contract by posting a margin that is a fraction of the price. Margin traders are highly leveraged. At the beginning of October 1987 the maintenance margin on the nearby S&P500 futures contract was slightly less than 3% of the contract price. The futures market clearinghouses protect traders against default by their trading partner by guaranteeing the performance of all contracts. If a buyer, or seller, defaults the clearinghouse performs. The clearinghouse guarantee is a credit guarantee similar to the deposit insurance guarantee at banks. The futures markets' performance guarantee, however, is a private guarantee backed only by the credit of the exchange and claims the exchange can impose on its members.

On October 19, 1987 US futures and equity markets suffered the largest one day price decline in exchange history. On October 20 the Chicago Mercantile Exchange (CME) temporarily halted trading in the S&P500 futures contract. The Commodity Futures Trading Commission (the regulatory board for the futures markets) disclosed that fourteen Futures Clearing Merchants (FCM) became undersegregated (the FCM had less than the required cash in consumer accounts) and three firms were undercapitalized. In addition eleven firms, including six CME members, had margin calls to a single customer that exceeded their capital.¹ Rumors spread that a clearinghouse might fail.

Underpricing the performance guarantee exposes the exchange to risk and undermines the credibility

¹ See the Report of the Presidential Task Force on Market Mechanisms (1988, p VI-73.)

of the clearinghouse's pledge to perform if there is a default. The value of the underpricing is the market value of the exchange's exposure on that contract. I show that the value of a call (put) option with a strike price equal to the futures price minus (plus) the margin is an upper bound to the market value of the performance guarantee. This paper presents estimates of the market value of the exchange's exposure on the nearby S&P500 contract during the October 1987 market crash.

The same thorny problems that plague fairly pricing deposit insurance plague fairly pricing the performance guarantee. The private value of the performance guarantee varies inversely with the trader's net worth. If the exchange conditioned the performance premium on the trader's risk, it would have to verify and continuously monitor the value of each trader's portfolio (which includes private information assets and liabilities) to assess their risk. If the exchange charged the same premium to all traders it could lead to adverse selection and/or moral hazard problems.

The futures exchange, in contrast to the deposit insurance system, charges no explicit performance guarantee premium. The exchange implicitly prices the performance guarantee by requiring that traders post a margin and resettle at regular intervals. I define an adequate margin policy as a margin-settlement interval pair that makes the price of the options that bound the value of the guarantee economically insignificant. An adequate margin fairly prices the performance guarantee within an (economically insignificant) ϵ neighborhood.

I present estimates of the value of the bounding option on the December S&P500 futures contract for each day in October 1987. Price volatility set a record. The CME margin committee tripled the

margin on the S&P contract from \$5,000 to \$15,000 in a sequence of \$2,500 increases on October 16, 21, 27, and 28. On October 20 the Federal Reserve announced its readiness to serve as a source of liquidity. The estimate of the value of the performance guarantee and the exchange's exposure is conditional on the estimate of the volatility of returns. On the day of the crash, October 19, my estimate of the CME's maximum exposure on the nearby S&P500 contract ranges from an enormous \$1.4billion (based on the efficient Garman-Klass (1980) estimate of the daily variance of returns) to a modest \$33million (based on the variance of returns implied by traded options.) By the end of October the combination of a higher margin, a lower price, and reduced price volatility resulted in a fairly priced guarantee and no risk exposure for the CME. No FMCs defaulted, although some were slow in settling.

Many other empirical and theoretical articles examine futures market margin policy. Recent empirical work by Hardouvelis and Kim (1995b) documents that futures market margin committees increase margins in response to increased price volatility and possibly in anticipation of increased volatility. They (1995a) also find a causal negative influence from margin requirements to open interest and trading volume. Figlewski (1984), Gay, Hunter, and Kolb (1986), Kupiec (1993), and Warshawsky (1989), use estimates of the distribution of returns to calculate the probability that a price change will exceed the margin. The main innovation in this paper is to extend the standard option valuation model of debt guarantees widely used in finance, eg, Merton (1974, 1977), to assign a price to the futures market performance guarantee and to use the price to evaluate the exchange's exposure on the S&P500 contract during the 1987 market crash.

The paper is organized as follows: Section 1 derives the market value of a futures contract performance guarantee. Section 2 examines the exchange margin system, derives the value of the bounding options, and defines an adequate margin. Section 3 presents the estimates of the value of the bounding option.

Section 1: Value of the Performance Guarantee

The market value of the performance guarantee for the contract is the amount an agent would have to pay to insure his performance. I show the market value of the performance guarantee equals the value of a put option for a long position and the value of a call option for a short position.

The Contract

A futures contract commits an agent to purchase or sell an asset,² say A , at a fixed future date, T , at a price set today, say $F(t, T)$. Settlement--delivery and payment--takes place at the contract expiration date T . Delayed settlement introduces credit risk--one of the parties may default.

Assumptions

- (1) Perfect markets
- (2) Symmetric information
- (3) Spanning

Payoffs

The futures contract has three possible payoff states: (1) no default, or (2) the buyer (long position)

Contracts are written on assets, or commodities. I'll use assets that have no carrying costs and pay no dividends to keep the notation simple.

defaults, or (3) the seller (short position) defaults.

In the no default state the payoff to the buyer at maturity is the value of the asset minus the futures price, $A(T) - F(t, T)$. If the buyer defaults, then he gives the seller some compensation, say $B \geq 0$. If the seller defaults, then the buyer gets some compensation, say $S \geq 0$, from the seller.

The payoff to the buyer can be written as,

$$\begin{aligned}
 PFB(T) \equiv & \{A(T) - F(t, T)\} \\
 & + \max[0, \{F(t, T) - B(T)\} - A(T)] \\
 & - \max[0, A(T) - \{F(t, T) + S(T)\}].
 \end{aligned} \tag{1}$$

For example, when the buyer defaults the first line in equation (1) is negative, the second line is positive, and the third zero. Summing gives the payoff (cost) to the buyer when he defaults.

The payoff to the seller is the negative of the buyer's payoff, $PFS(T) = -PFB(T)$.

Replicating Portfolio

The buyer can replicate the payoff on the futures contract with a portfolio made up of a debt contract, a pair of options, and a spot market purchase.

The buyer (1) purchases the asset in the spot market and (2) sells a default free bond that pays the futures price, $D(t, T) = F(t, T)$, at maturity. He (3) purchases a put option written on the asset with

a striking price equal to the futures price minus the compensation he gives the seller if he defaults, $Y(T) = F(t, T) - B$, and (4) sells a call option with a strike price equal to the futures price plus the compensation he would receive if the seller defaulted, $X(T) = F(t, T) + S$.

The payoff on the buyer's replicating portfolio is,

$$\begin{aligned}
 RPB(T) &\equiv A(T) - D(t, T) \\
 &+ \max[0, Y(T) - A(T)] \\
 &- \max[0, A(T) - X(T)]; \quad D(t, T) = F(t, T)
 \end{aligned} \tag{2}$$

identical to the payoff on the futures contract.

The seller replicates the futures payoff by (1) selling the asset and (2) investing the proceeds at the risk free rate and (3) selling the put option and (4) buying the call. The payoff on the seller's replicating portfolio is the negative of the buyer's.

Equilibrium

Proposition 1: The equilibrium futures price discounted at the risk (default) free rate, rf , equals the value of the asset plus a risk premium (discount) equal to the difference between the value of a put option with striking price $Y(T) = F(t, T) - B$ and a call option with striking price $X(T) = F(t, T) + S$, ie,

$$e^{-r(T-t)}F(t, T) = A(t) + \{P(A(t), Y(T)) - C(A(t), X(T))\}$$

Here $P(A(t), Y(T))$ and $C(A(t), X(T))$ denote the price of the put and call option with striking prices Y and X .

Proof: The futures position is self financing (requires no current payment). No arbitrage profit opportunities requires that the self financing replicating portfolio has no value,

$$D(t) - A(t) + C(A(t), X(T)) - P(A(t), Y(T)) = 0. \quad (4)$$

Or, the current value of the debt equals the value of the asset plus the differential in option values.

Since the debt is a default free claim that pays the futures price at maturity,

$$D(t) = e^{-r(T-t)} D(t, T) = e^{-r(T-t)} F(t, T) \quad (5)$$

the current value of the debt is the futures price discounted at the default free rate.

Substituting the discounted futures price for the debt gives the result,

$$e^{-r(T-t)} F(t, T) = A(t) + P(A(t), Y(T)) - C(A(t), X(T)) . \quad (6)$$

With no default guarantee the equilibrium futures price contains a premium (discount) over the value of the asset that reflects the differential in value of the default risk between the buyer and seller. $\square$

A Performance Guarantee

The buyer or seller can guarantee their performance by delivering an option to their trading partner.

Exercising the option makes the payoff in the default state the same as the payoff in the no default state.

Proposition 2: The market value of the performance guarantee on a long futures position equals the value of a put option written on the underlying asset, A , with a striking price equal to the futures price minus the value of the buyer's compensation, $Y(T) = F(t, T) - B$. The market value of the performance

guarantee on a short futures position equals the value of a call option written on the underlying asset with a striking price equal to the futures price plus the value of the seller's compensation, $X(T) = F(t, T) + S$.

Proof: To guarantee his performance the buyer gives the seller a put option with the striking price $Y(T) = F(t, T) - B$. Now the seller gets the same payoff whether the buyer performs or defaults. The payoff to the seller when the buyer's performance is guaranteed is,

$$\begin{aligned}
 PGFS(T) \equiv & \{ F(t, T) - A(T) \} \\
 & - \max[0, \{F(t, T) - B(T)\} - A(T)] \\
 & + \max[0, A(T) - \{F(t, T) + S(T)\}] \\
 & \{ + \max[0, \{F(t, T) - B(T)\} - A(T)] \}.
 \end{aligned} \tag{7}$$

The first three lines of equation (7) give the payoff to the seller when there is no performance guarantee. If the buyer defaults the seller collects compensation, B (sum of the first three lines) from the buyer. The seller also exercises the option putting the asset to the option writer for $F(t, T) - B$. The payoff to the seller when the buyer defaults equals the payoff when the buyer performs.

The market value of the buyer's performance guarantee equals the value of the put option, $P(A(t), F(t, T) - B)$, he delivers to the seller.

The seller guarantees his performance by delivering a call option with the striking price, $X(T) = F(t, T) + S$, to the buyer. The market value of guaranteeing the seller's performance equals the value of the call option $C(A(t), F(t, T) + S)$. $\square$

Corollary: When the buyer's and seller's performance is guaranteed the equilibrium futures price discounted at the risk free rate equals the spot price.

$$e^{-r(T-t)}F(t, T) = A(t).$$

Proof: The corollary follows directly from Propositions 1 and 2. Adding the payoff on a put option (which the seller receives from the buyer) and subtracting the payoff on a call option (which the seller gives to the buyer) makes the payoff on the seller's replicating portfolio in any state the default free payoff.

Section 2: The Exchange Guarantee and the Margin System

In an economy with perfect markets and information agents would be indifferent between a futures contract, with or without a guarantee, or the replicating portfolio. Traders either (i) would arrange side payments and/or demonstrate that they have sufficient wealth to guarantee their performance, or (ii) the futures price would incorporate the relative default risk premium.

In an economy with transactions costs and/or private information it is costly to verify credit and/or arrange side payments. Forward contracts only trade between principals (mostly large banks) with high credit ratings and there is no active secondary trading. The futures exchange performance guarantee encourages trading among anonymous partners and promotes active secondary trading. Traders buy from or sell to the clearinghouse. The exchange clearinghouse is the trading partner in every transaction. As long as the exchange has sufficient resources traders know their partner (the clearinghouse) will perform.

Fairly Pricing the Exchange Performance Guarantee

The market value of the exchange's performance guarantee to the j^{th} trader is the value of the put $(P(A(t), F(t,T)-B(j)))$ or call option $(C(A(t), F(t,T)-S(j)))$ he would have to provide to insure his performance, ie, Proposition 2. The fair price of the guarantee is the market value of the option.

Fairly pricing the guarantee in an environment with transactions costs, however, is nontrivial. Conditioning the performance guarantee premium on a trader's risk means the exchange would have to verify and continuously monitor each trader's net worth to assess the value of the trader's default compensation. On the other hand, charging the same fixed premium to all traders would eliminate monitoring costs, but could lead to adverse selection problems.

The Margin System

The exchange uses a margin system to fairly price the guarantee and keep transactions costs low.

The margin system has two fundamental components: the margin (M) and periodic resettlement at intervals (n). Since futures contracts are not limited liability contracts the margin places an upper bound on the market value of the guarantee. Periodic resettlement shortens the maturity of the guarantee from the maturity of the futures contract to the resettlement interval.

The Margin

The exchange demands that traders post an initial margin of default free collateral³, $0 < M < F(t, T)$, when they enter a futures contract. The margin replaces costly credit checks and continuous monitoring to establish the value of the trader's default compensation. The margin insures that the exchange has an unencumbered claim to the minimum compensation, $M \leq B(j)$ or $S(j)$, if trader the defaults.

Value of the Bounding Options

The value of a put option with a strike price equal to the current futures price minus the margin, $y(T) = F(t, T) - M \geq Y(T, j) = F(t, T) - B(j)$,

$$p(A(t), y(T)) \geq P(A(t), Y(T, j))$$

$$y(T) = F(t, T) - M(T) \geq Y(T, j) = F(t, T) - B(j) \quad (9)$$

$$M \leq B(j), \forall j$$

is an upper bound to the value of the exchange's guarantee for a long position since the value of the put is decreasing in the strike price. The value of a call option with a strike price equal to the current futures price plus the margin,

$$c(A(t), x(T)) \geq C(A(t), X(T, j))$$

$$x(T) = F(t, T) + M(T) \leq X(T, j) = F(t, T) + S(T, j) \quad (10)$$

$$M \leq S(j), \forall j$$

is an upper bound to the market value of the exchange's guarantee for a short position since the value

The exchange accepts Treasury Bills or a line of credit at approved banks.

of the call option is decreasing in the strike price.

Mark to Market Accounting and Resettlement

The clearinghouse audits traders' positions at regular intervals, say n , and demands a variation margin to compensate for changes in the market value of their position. The variation margin for a short position equals the change in the price of the contract, ie,

$$MVS(t+\tau) = F(t+\tau, T) - F(t, T). \quad (11)$$

If the price of the futures contract goes up the short seller must add variation margin equal to the loss in market value. If the price of the futures contract goes down the short seller can withdraw the variation margin equal to the gain in market value. The variation margin for a long position is the negative of the variation margin for a short position, ie, $MVL(t+\tau) = -MVS(t+\tau)$.

If the trader does not meet the call for variation margin the clearinghouse suspends his trading privileges and liquidates the position. Resettlement makes the effective maturity of the exchange guarantee the audit interval, n , and it makes the underlying asset the futures contract.

Proposition 3

If the exchange clearinghouse's performance commitment is credible, then

(I) the value of a put option on the futures contract with a strike price equal to the current futures price minus the margin, $p(F(t+\tau, T), y(t+\tau), n)$; $y(t+\tau) = F(t+\tau, T) - M$, and maturity of the audit interval n , is an upper bound on the market value of the exchange performance guarantee on a long position, and,

(ii) the value of a call option on the futures contract with a strike price equal to the current futures price plus the margin, $c(F(t+\tau, T), x(t+\tau), n)$; $x(t+\tau) = F(t+\tau, T) + M$, and maturity equal to the audit interval n , is an upper bound on the market value of the exchange performance guarantee on a short position.

Proof:

Periodic resettlement through the variation margin means the strike price of the option that bounds the market value of the performance guarantee is independent of the initial contract date. The strike price can be expressed in terms of the current futures price and initial margin, ie,

$$\begin{aligned} y(t+\tau) &= F(t+\tau, T) + M \equiv F(t, T) + M + MVS(t+\tau), \text{ and} \\ x(t+\tau) &= F(t+\tau, T) - M \equiv F(t, T) - M - MVL(t+\tau). \end{aligned} \tag{12}$$

At the next audit, $t+\tau+n$, the payoff on the futures contract is:

$$\begin{aligned} &F(t+\tau+n, T) - F(t, T) \\ &\quad - \max[0, F(t+\tau+n, T) - \{F(t, T) - B(j)\}]; \\ &F(T, T) = A(T), \end{aligned} \tag{13}$$

the gain (loss) from the change in the price of the futures contract⁴ minus the payoff on a put option written on the futures contract with a strike price equal to the initial futures price minus the buyer's compensation, ie, $Y(t+\tau) = F(t, T) - B(j)$. Thus a put (call) option on the futures contract with a maturity of the audit interval and a strike price equal to the current futures price plus (minus) the initial margin bounds the market value of the exchange's performance guarantee. $\square$

In the terminal period when the contract expires the futures and spot price converge.

Adequate Margin Policy

The exchange implicitly prices the performance guarantee by choosing a margin and audit interval pair, (M, n) .

Definition

An adequate margin policy is a margin and audit interval pair (M, n) that makes the market value of the bounding options economically insignificant, $\epsilon \geq c(F(t+\tau), x(t+\tau))$, $p(F(t+\tau), y(t+\tau)) \geq 0$.

An adequate margin policy fairly prices the guarantee within an ϵ neighborhood for all traders, ie,

$$0 \leq c(F(t+\tau, T), x(t+\tau)) - C(F(t+\tau, T), X(t+\tau, j)) \leq \epsilon, \text{ and,}$$

$$0 \leq p(F(t+\tau, T), y(t+\tau)) - P(F(t+\tau, T), Y(t+\tau, j)) \leq \epsilon, \forall j.$$

As the value of the bounding options approaches zero the pricing error goes to zero.

A mispriced guarantee (inadequate margin) is suboptimal, but an adequate margin is not necessarily optimal. A very high margin would be adequate, but it would discourage trading. I use the robust no arbitrage profit condition, popular in finance, for valuation. I do not specify agents' tastes. An optimal policy is only defined in terms of an objective function. Baer, France, and Moser (1993, 1994), and Fenn and Kupiec (1993) specify agents' tastes. Their models that have well-defined optimal margin policies conditional on the specification.

Section 3: Valuing the Performance Guarantee in October 1987

As price volatility increased in October 1987 futures market margin committees responded by raising the margin. The CME tripled the margin on the S&P500 futures contract from \$5000 to \$15,000 in

a rapid sequence of \$2,500 increases on October 16, 21, 27, and 28. It was reported that some customers withdrew funds from their FCM because they feared a clearinghouse default.⁵ The CME vigorously denied that the clearinghouse was in any danger during October.

This section illustrates the underpricing of the performance guarantee and calculates value of the exchange's exposure using data⁶ on the S&P500 futures contract during the market crash. The S&P500 contract is a high volume contract traded on the CME that is popular with portfolio insurers and index arbitragers. At the beginning of October 1987 open interest in the nearby December futures was nearly 110,000 contracts worth approximately \$18billion. On so-called Black Monday, October 19, portfolio insurers sold 34,500 contracts worth about \$4billion.

Options on the S&P500 futures contract also trade on the CME. If the option contract specifications were rich enough the price of the traded options would reveal the bounding value of the guarantee. The CME required daily settlement⁷ in October of 1987 and the implicit bounding options have a maturity equal to the settlement interval, see Proposition 3. Traded options only expire once a month - October options expired on October 16 and November options on November 19.

Estimating the Value of the Performance Guarantee

⁵ Presidential Task Force, VI-74.

⁶ The price and volume data come from the Wall Street Journal. The CME supplied the margin data.

⁷ On some of the most volatile days the clearinghouse demanded intraday settlements.

Since the contract specifications on traded options are not rich enough to reveal the value of the performance guarantee I follow the standard procedure for an empirical evaluation and estimate a process for returns. The new empirical evidence in this paper comes from combining the estimates of the return process with the margin to evaluate Black's (1976) option pricing formula. A price is the natural economic metric. The option pricing formula maps the estimate of the return process and the margin into an estimate of the price of the option that bounds the value of the performance guarantee. The option price times the open interest gives an estimate of the clearinghouse's exposure.

The Returns Process

Figure 1 shows a histogram for the percentage daily price changes (logarithmic returns) during October. The distribution is skewed to the left with extremely fat tails. Price volatility in October 1987 set a record.

Insert Figure 1

A Jarque-Bera test, see line 1 of Table 1, rejects the null hypothesis that the sample of daily returns, denoted $R(t)/\sigma$, are drawn from a normal distribution (with a constant mean and variance.) Work by Hall, Brorsen, and Irwin (1989) indicates that a mixture of normal distributions describes returns on futures contracts better than a stable fat-tailed Paretian distribution.

I assume that the futures price follows geometric Brownian motion,

$$\frac{dF}{F} = \alpha dt + \sigma(t)dz; \quad (15)$$

Daily Return Frequencies

with volatility, $\sigma(t)$, that changes daily (ie, returns are drawn from a mixture of normals.⁸)

Estimates

Drift

The average daily return in October was -1%, but the estimate has a p-value of only .6. I assumed the instantaneous drift was zero.⁹ For my purposes estimates of the drift are not important since the option value is independent of the drift.

Variance

Estimates of variance are extremely important since the option price is a nonlinear function of the variance. I made two estimates of the variance. I backed out the variance implied by traded options. And I used the efficient Garman-Klass (G-K) range estimator. The specification of the G-K estimator is consistent with the specification of returns, equation 15, the option pricing formula, equation 19, and the one day settlement period at the CME. The maturity of traded options is longer than the maturity of the implicit options.

Implied Variance from Traded Options

Inverting the option pricing formula, equation 19, gives the implied variance, $\sigma^2_{\text{OPT}(n)}$, of the instantaneous return on the futures contract,

⁸ Hardouvelis and Kim (1995b) specify a mixture of normals with Poisson mixing variable.

⁹ With a short sample it is hard to precisely estimate the daily return since prices are noisy. Gay, Hunter, and Kolb p6 also find insignificant average daily price changes.

$$\sigma^2 OPT(n) = c^{-1}(\sigma^2, F, x, rf, n), \quad (16)$$

Here x denotes the strike price, rf the risk free rate (I used the treasury bill rate), and n the option maturity. The variance implied by traded options is the variance over the remaining life of the contract, n .

Table 2 in the appendix gives the implied variances from traded options expiring in October and November. The difference between the implied variances from October and November options during the first half of October does not indicate that the market anticipated the abrupt large increase in the price volatility that would be realized during the week of October 19. The average of the daily variances implied by November options during the first half of October, .0146%, is only slightly higher than the average of the daily variances implied by October options, .0131. The correlation between the daily variances implied by the options through the first half of October is .70. The variances implied by the traded October and November options do not seem to be very sensitive to the differences in the option maturities.

Garman-Klass Efficient Range Estimator

Garman and Klass (1980) derived an unbiased estimator of the daily variance that uses information on the log of the day's high (H), low (L), opening (O), and closing (C), prices. Their estimator is eight times more efficient than the "classical" estimate--squared difference of the log of the closing prices--if returns follow a diffusion process with no drift as specified in equation 15.

The Garman-Klass estimator is:

$$\sigma^2 GK(t) = 0.12 \frac{(O_1 - C_0)^2}{f} + (1 - 0.12) \frac{0.511(h-l)^2 - 0.019(c(h+l) - 2hl) - 0.383c^2}{1-f}$$

The first term on the left hand side is a classical estimate of the variance of the continuous process while the market was closed: the squared difference between the (log of the) opening price O_1 and the (log of the) previous day's close, C_0 . The second term estimates the intraday variance while the market is open. Here the lower case letters denote that prices are normalized by the opening price, eg, $h = H - O$. Weighting by the fraction of the day the market is closed, f , gives the variance for the day. The G-K estimator of the variance uses data from a 24hour¹⁰ window which equals the settlement interval.

Figure 2 plots the G-K estimates of the daily return variance and their asymptotic two-standard deviation confidence interval and the implied variance from traded options expiring in November. The lowest dashed line is the variance implied by November options.

INSERT FIGURE 2

The variances implied by November (or October) options always lie in or below the 95% confidence band for the G-K estimates. The solid line is the G-K estimates of the variance and upper dashed and lower dotted lines delineate the confidence band. The G-K estimates based on a 24hour window pick up the extreme realized price volatility on October 19, 20 and 22. The estimated G-K variances were 5.7% on the 19th, 11.3% on the 20th and 5.6% on the 22nd. The variances implied by November options, while very large by historical standards, are an order of magnitude (or two) smaller--.28%

¹⁰ For weekends the closing price is Friday's close.

Variance Estimates and Confidence Interval

on the 19th, .18% on the 20th, and .096% on the 22nd. Either the market did not anticipate the actual extreme price volatility, and/or the implied variances smooth the daily volatilities over a longer horizon. During the final week in October the market begins to settle down and the G-K estimates of the variances and variances implied by November options converge.

Tests for Normally Distributed Returns

Table 1 shows the results of Jarque-Bera tests for normality. I scaled returns by dividing the change in the log of the closing price (logarithmic return) by the estimate of the standard deviation for that day. The first row shows the rejection of a constant variance specification. The tails are way too fat and the distribution is badly skewed to the left.

TABLE 1: JARQUE-BERA TESTS FOR NORMALITY

Variable	Skewness	Kurtosis	JB Probability
$R(t)/\sigma$	-1.850	9.760	0.00
$R(t)/\sigma_{OPT}(n)$	0.076	4.294	0.44
$R(t)/\sigma_{GK}(t)$	0.737	3.318	0.32

Scaling returns by the standard deviation implied by traded options dramatically reduces the weight in the tails. Scaling returns by the G-K estimates of the standard deviations reduces the weight in the tails even more giving a distribution with tails only slightly fatter than a normal. Neither specification with time-varying variances (a mixture of normals) rejects the null of normality.

Evaluating the Option Pricing Formula

I use Black's (1976) option pricing formula for a call option¹¹ on a futures contract to value the option,

$$c = e^{-r_m}[FN(d_1) - xN(d_2)],$$

where,

$$d_1 = \frac{\ln(F/x) + (\sigma^2/2)n}{\sigma n} \quad (18)$$

$$d_2 = d_1 - \sigma n.$$

Here $N(\cdot)$ denotes the standardized cumulative normal distribution.

The futures price, the strike price, and the risk free interest rate are observable. The strike price equals the futures price plus the margin. I choose the hedging maintenance margin which is the lowest margin imposed (so I calculate the maximum value of the option.) I proxy the risk free rate with the Treasury Bill rate (since these are one to three day maturities the interest rate choice has no significant effect.)

Values of the Performance Guarantee

The estimates of the CME clearinghouse's exposure are extremely sensitive to the estimated variance of daily returns. If one uses the variance implied by traded options, then the performance guarantee is fairly priced except on the day of the crash and the clearinghouse has virtually no risk exposure, just as the CME claimed. On the other hand, if one uses the G-K estimates of daily variances, then

¹¹ The value of a call is slightly larger than a put when they have the same input variables.

there was substantial exposure.

Figure 3 plots the price of the bounding options on the nearby S&P500 futures contract as a fraction of the contract price. Table 3 in the Appendix shows the values as fractions of the contract price and gives the dollar values.

INSERT FIGURE 3

The lower line is the value of the of the bounding option based on the variances implied by November options. On the day of the crash, October 19, the value of the bounding option reached its maximum of \$223. On the 20th the value fell to \$100, and on the 26th it was \$28. For the rest of the month the value of the bounding option was less than \$10 (on a \$130,000-\$150,000 contract.) The estimates indicate very little exposure. On the day of the crash according to these estimates the clearinghouse's maximum exposure on the S&P500 contract was \$33million and on the following day about half that.

The G-K estimates paint a very different picture. The upper line shows the value of the bounding option as a function of the G-K variance estimate. The value of the bounding option exceeded \$400 for almost two weeks, from October 14th through the 26th. During this period the estimated exposure never fell below \$50million. On the Friday preceding the crash, October 16, the value of the bounding option jumped to \$10,000. On the 19th it hit \$10,500 and on the 21st \$8200¹². Translated to the maximum exposure for the exchange it implies an exposure of roughly \$1.5 billion on the 16th and 19th. By the end of the month, however, the combination of less price volatility,

¹² I used the G-K estimate of the variance, $\sigma^2(t+1)$, to value the option for t . So the large estimated variances on the 19th, 20th, and 22nd give high option values on the 16th (Friday preceding the crash), 19th and 21st.

GUARANTEE VALUE/FUTURES PRICE

higher margins, and lower prices reduces the value of the performance guarantee to an economically insignificant value.

Summary and Conclusion

This paper (i) extends the standard option valuation model of debt guarantees to value the futures market performance guarantee, (ii) defines an adequate margin as a margin that fairly prices the performance guarantee, and (iii) uses estimates of the returns process to estimate underpricing of the performance guarantee and the value of the exchange's exposure.

The value of the options that bound the value of the performance guarantee is conditional on estimates of the variance of the returns process. I estimated the variance of the returns process using the efficient Garman-Klass range estimator and I backed out the implied variance from traded options. The specification of the G-K estimator is consistent with the specification of the returns process, the option pricing formula, and the option maturity implied by the settlement interval. The G-K range estimator picks up the realized inter and intraday volatility. The estimates of the daily variance are large and change quickly from day to day. The average of the daily variances for the month is 1.24%. The average of the squared differences in closing prices (the sample variance of the interday price changes) is 50% smaller at 0.8%. The variances implied by traded options are much smaller and vary less from day to day. The average of the daily variances is 0.074%, an order of magnitude smaller than the variance of interday price changes.

The estimates of the bounding option based on the G-K estimates of the daily variance show that the

clearinghouse had considerable exposure--close to \$1.4billion on the 16th and 19th of October. The estimates of the bounding option using the implied variance from traded options show essentially no exposure except for the day of the crash.

By the end of the month the estimate of the value of the bounding option is economically insignificant for either estimate of the variance. No FMC defaulted during October although some were slow meeting their obligations.

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TABLE 1: Futures Prices

Date	Spot	Open	High	Low	Settle	Margin
1	\$163,665	\$163,250	\$165,900	\$163,025	\$165,850	\$5,000
2	\$164,035	\$165,500	\$166,500	\$165,100	\$165,675	\$5,000
5	\$164,040	\$165,550	\$165,900	\$164,150	\$165,400	\$5,000
6	\$159,610	\$164,700	\$164,850	\$159,875	\$159,925	\$5,000
7	\$159,270	\$159,700	\$161,025	\$158,850	\$160,325	\$5,000
8	\$157,080	\$160,300	\$160,600	\$156,850	\$157,900	\$5,000
9	\$155,535	\$157,600	\$158,525	\$155,800	\$156,100	\$5,000
12	\$154,695	\$155,475	\$156,850	\$154,200	\$155,800	\$5,000
13	\$157,260	\$157,050	\$158,500	\$156,125	\$157,825	\$5,000
14	\$152,615	\$156,100	\$156,725	\$152,400	\$152,500	\$5,000
15	\$149,040	\$151,650	\$153,800	\$148,975	\$149,125	\$5,000
16	\$141,350	\$150,250	\$150,500	\$138,500	\$141,125	\$7,500
19	\$112,420	\$132,000	\$134,500	\$99,000	\$100,750	\$7,500
20	\$118,415	\$110,500	\$121,000	\$90,500	\$108,125	\$7,500
21	\$129,190	\$121,000	\$129,750	\$126,235	\$129,125	\$10,000
22	\$124,125	\$101,000	\$125,250	\$97,500	\$122,250	\$10,000
23	\$124,110	\$120,000	\$126,500	\$117,000	\$120,500	\$10,000
26	\$113,835	\$114,000	\$118,500	\$109,000	\$110,125	\$10,000
27	\$116,595	\$121,000	\$121,000	\$111,500	\$114,300	\$12,500
28	\$116,640	\$110,000	\$117,000	\$109,000	\$115,625	\$15,000
29	\$122,385	\$118,500	\$124,550	\$117,500	\$122,850	\$15,000
30	\$125,895	\$126,500	\$130,000	\$126,000	\$129,675	\$15,000

TABLE 2: Variance Estimates

Date	G-K	std (G-K)	November	October
1	0.000144	0.000154	0.000130	0.000112
2	0.000094	0.000001	0.000129	0.000102
5	0.000150	0.000001	0.000127	0.000107
6	0.000437	0.000553	0.000145	0.000097
7	0.000227	0.000003	0.000131	0.000123
8	0.000537	0.000113	0.000144	0.000127
9	0.000315	0.000064	0.000156	0.000106
12	0.000381	0.000002	0.000161	0.000128
13	0.000285	0.000081	0.000151	0.000114
14	0.000551	0.000575	0.000179	0.000171
15	0.001093	0.000244	0.000193	0.000309
16	0.005603	0.001484	0.000252	0.000205
19	0.057303	0.055417	0.002797	0.000000
20	0.113356	0.002435	0.001827	0.000000
21	0.004356	0.015372	0.000857	0.000000
22	0.055461	0.001461	0.000958	0.000000
23	0.008116	0.000101	0.000954	0.000000
26	0.008736	0.003955	0.001667	0.000000
27	0.007568	0.000676	0.001616	0.000000
28	0.004283	0.000065	0.001501	0.000000
29	0.003227	0.001793	0.000606	0.000000
30	0.000801	0.001426	0.001573	0.000000

TABLE 3: Value of the Guarantee

Date	G-K	G-K/F	Novopt	Novopt/F
1	\$0.52	0.000003	\$2.90	0.000017
2	\$5.30	0.000032	\$2.80	0.000017
5	\$120.00	0.000750	\$2.50	0.000015
6	\$19.00	0.000120	\$3.40	0.000021
7	\$160.00	0.001000	\$2.20	0.000013
8	\$46.00	0.000290	\$3.00	0.000019
9	\$71.00	0.000450	\$3.80	0.000024
12	\$33.00	0.000210	\$4.30	0.000028
13	\$160.00	0.001000	\$3.60	0.000023
14	\$450.00	0.003000	\$5.60	0.000037
15	\$2,500.00	0.017000	\$6.30	0.000043
16	\$10,000.00	0.074000	\$0.35	0.000003
19	\$11,000.00	0.100000	\$220.00	0.002200
20	\$600.00	0.005500	\$100.00	0.000920
21	\$8,200.00	0.064000	\$6.90	0.000053
22	\$1,200.00	0.009900	\$7.10	0.000058
23	\$1,300.00	0.011000	\$6.10	0.000051
26	\$840.00	0.007600	\$28.00	0.000260
27	\$190.00	0.001700	\$7.60	0.000066
28	\$40.00	0.000340	\$1.10	0.000009
29	\$0.02	0.000000	\$0.00	0.000000
30	\$0.03	0.000000	\$4.80	0.000037