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Dissipative Effects of Bubbles and Particles in Shear Flows

Chemical reactors, air lubrication systems, and the aeration of the oceans rely, either in part or in whole, on the interaction of bubbles and their surrounding liquid. Even though bubbly mixtures have been studied at both the macroscopic and bubble level, the dissipation field associated with an individual bubble in a shear flow has not been thoroughly investigated. Exploring the nature of this phenomenon is critical not only when examining the effect a bubble has on the dissipation in a bulk shear flow but also when a microbubble interacts with turbulent eddies near the Kolmogorov length scale. In order to further our understanding of this behavior, this study investigated these interactions both analytically and experimentally. From an analytical perspective, expressions were developed to model the dissipation associated with the creeping flow fields in and around a fluid particle immersed in a linear shear flow. Experimentally, tests were conducted using a simple test setup that corroborated the general findings of the theoretical investigation. Both the analytical and experimental results indicate that the presence of bubbles in a shear flow causes elevated dissipation of kinetic energy. [DOI: 10.1115/1.4035946]

Introduction

Bubbles have a significant impact on the flow field within both the surrounding fluid and the bubble itself. These altered flow fields can affect other properties of the mixture [1] and the processes that result from the associated fluid motion. Since bubbles may affect almost every aspect of a fluid flow, establishing how they interact with their surrounding liquid is critical to understanding their impact on natural and engineering processes. Drag reduction is one aspect of this interaction and it has been studied in numerous ways. From the electrolysis of water on the surface of a submerged axisymmetric body [2] to flow through water tunnels [3,4] and pipes [5], these and other efforts have shown that introducing bubbles between a surface and an adjacent bulk flow leads to generally reduced skin friction. This is not only true for hydrogen [2] and air [3–5] but also for bubbles created from various species of gasses [6].

One of the technologies to benefit from these studies has been air lubrication systems which reduce the drag on watercraft by introducing bubbles along the wetted portions of a hull. Use of these systems has increased vessel efficiencies anywhere from 3% to 12% [7–10]. The mechanisms behind drag reduction have also been the focus of numerous studies. These and similar investigations have found that bubbles and particles modify turbulence in the surrounding fluid [11–14] and the shear stresses applied to solid boundaries [15]. They have also identified the distribution of bubbles in the boundary layer [16], impeded flow at the wall [17], the mixture density [18–20], turbulence modification [20], and the reduction of turbulent energy [21] as factors contributing to the reduction of drag. Numerical studies have also indicated that microbubbles reduce drag by forcing turbulent vortical structures away from the wall [22,23].

Although it is one of several factors, modifying turbulence may play an important role in reducing the drag applied to a solid boundary. Even though efforts have been undertaken to understand the mixing [24–26] environments that may be experienced by a microbubble, in the end, the impact of a bubble on the local

dissipation of turbulent kinetic energy (TKE) has not been fully investigated. Consequently, this study focused on determining the dissipation field in and near a bubble subjected to a creeping linear shear flow; the type of flow expected to be experienced by a microbubble located in the smallest of turbulent structures. Once the dissipation equations have been developed, the significant features of this dissipation field are discussed.

Background

There are several factors that influence the size and shape of bubbles. In a bubbly flow, the Weber number (We), which captures inertia and surface tension effects [27], is commonly used to classify bubble behavior. Among others, Chanson [28] and Qian et al. [29] have examined the breakup of bubbles in various flow fields and related the Weber number to bubble sizes and distributions in those flows. Chanson [28] proposed a modified Weber number to include the influence of the magnitude of the velocity gradient. In Chanson's formulation of the Weber number, it will have a larger value if either the bubble is relatively large or the velocity gradient is relatively extreme, and it will have a smaller value if the surface tension is significant. Based on the influence of these various factors, high Weber numbers indicate conditions where bubbles are likely to breakup into smaller ones, and, conversely, bubbles would be expected to remain intact in low Weber number flows.

For the work considered here, it can be understood that, because of its small size, a microbubble immersed in a Kolmogorov eddy will correspond to a low Weber number flow. As a result, it is unlikely the bubble will be broken-up by the eddy, eliminating the complexity associated with such an event. Beyond the case of a microbubble in a turbulent flow field, this work may also be applied to other low Weber number cases. This includes cases with larger bubbles where the magnitude of the velocity gradient is low enough to ensure the bubble remains intact.

In addition to bubble integrity, determining the shape of the bubble is also important to make the analysis tractable. Clift et al. [30] indicate that three dimensionless parameters are primarily responsible for determining the shape and size of a bubble: the Eotvos number (Eo , which is the same as the Bond number [31]), the Reynolds number (Re), and the Morton number (M). Even though Clift et al. use these numbers to investigate a bubble as it

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moves through a static liquid, and therefore introduce the buoyant force as another factor influencing the behavior of a bubble, they can provide some insight into the shape of the bubble. For the particular case of a microbubble, these numbers indicate that surface tension forces will dominate for very small bubbles, and this will cause small bubbles to assume a spherical shape provided that they are not exposed to a flow with severe velocity gradients. This is also supported at low values of the Weber number, which will generally be small for small bubbles and low-velocity gradients. Additionally, even though this conclusion is less accurate for larger bubbles, the assumption that larger bubbles are essentially spherical is reasonable provided that gravitational effects are neglected and the velocity gradient is taken to be sufficiently small.

In order to ensure the values for these numbers remain low enough to make certain the bubble remains spherical and ensure the flow field inertia effects are negligible, this effort focused on a bubble immersed in a creeping linear shear flow. Additionally, to eliminate the complexity associated with buoyancy, gravitational effects were also neglected.

These assumptions are consistent with those of similar investigations. Both Stokes flow and the flow fields determined by Hadamard and Rybczynski, for uniform flow around a fluid particle [32], were found for creeping flow fields. In addition to the creeping flow assumption, Cox et al. [33] and Leal [34] also assumed that gravitational effects could be neglected. Cox et al. and Cherry et al. [35] examined the flow around a solid particle immersed in a linear creeping shear flow (which development was reprised by Leal), and Leal presented the equations for the shape of a fluid particle located in this same type of flow field. Central to the development of his equations, Leal used harmonic vector functions to describe plausible flow fields both in and outside the bubble.

Although the focus of Leal's work was to determine the shape of a fluid particle suspended in a uniform creeping shear flow, a similar approach can be used to develop flow field equations that will lay the foundation for developing the dissipation equations that are the focus of this effort. The difference between these two analyses, beyond their eventual goals, lies in the boundary conditions used to capture the physical behavior at the interface. In the Theoretical Investigation section that follows, we slightly modify Leal's solution for the flow field and follow with a detailed dissipation analysis which is the main thrust of this paper.

Theoretical Investigation

Flow Field Equations. Using the nondimensional form of the general equations from Leal's development [34] as the starting point for this analysis, the general flow field equation in a Cartesian coordinate system for the substance surrounding the fluid particle (otherwise referred to as the continuous or outer phase) is

$$\mathbf{u} = \begin{bmatrix} y \\ 0 \\ 0 \end{bmatrix} + \left(\frac{c_1}{2r^5} + \frac{c_3}{r^7} \right) \begin{bmatrix} x^2y \\ xy^2 \\ xyz \end{bmatrix} - \left(\frac{2c_3}{5r^5} \right) \begin{bmatrix} y/2 \\ x/2 \\ 0 \end{bmatrix} + \frac{c_4}{r^3} \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix} \quad (1)$$

and for the fluid particle (otherwise known as the discrete or inner phase) it is

$$\hat{\mathbf{u}} = \left(d_3 + \frac{5}{21}r^2d_2 \right) \begin{bmatrix} y/2 \\ x/2 \\ 0 \end{bmatrix} + d_4 \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix} - \frac{2}{21}d_2 \begin{bmatrix} x^2y \\ xy^2 \\ xyz \end{bmatrix} \quad (2)$$

In these equations, $\mathbf{u}$ and $\hat{\mathbf{u}}$ are the continuous and discrete phase velocity fields, respectively. Here, r is the magnitude of a position vector in the flow field; c_1 , c_3 , c_4 , d_2 , d_3 , and d_4 are constants whose values can be found from imposed constraints; and x , y , and z are the coordinates defined by a Cartesian system located at

Fig. 1 Definition of the shear flow relative to the coordinate axes as well as other aspects of the spatial geometry associated with the flow field. Note: this figure and figures or elements of Figs. 2, 3, and 7 were created using PARAVIEW, version 4.3.1 64-bit.

the center of the bubble and oriented as shown in Fig. 1. These equations have been normalized, as appropriate, using the bubble diameter and the undisturbed velocity gradient as scales.

Additionally, these equations only account for a shear flow around the bubble. It is assumed that since the fluids are flowing slowly, the bubble will simply be carried along with any bulk translational or rotational motion associated with the continuous phase. Consequently, from the perspective of this study, only a shear flow in the continuous phase will have any meaningful impact on the bubble.

Transforming these equations into a spherical coordinate system results in

$$\mathbf{u} = \begin{bmatrix} \left(1 + \frac{c_1}{2r^3} + \frac{3c_3}{5r^5} \right) r (\sin \theta)^2 \sin \phi \cos \phi \\ \left(1 - \frac{2c_3}{5r^5} \right) r \sin \theta \cos \theta \sin \phi \cos \phi \\ \left\{ \left(\frac{1}{2} - \frac{1c_3}{5r^5} \right) \cos(2\phi) - \left(\frac{1}{2} + \frac{c_4}{r^3} \right) \right\} r \sin \theta \end{bmatrix} \quad (3)$$

and

$$\hat{\mathbf{u}} = \begin{bmatrix} \left(d_3 + \frac{3}{21}r^2d_2 \right) r (\sin \theta)^2 \sin \phi \cos \phi \\ \left(d_3 + \frac{5}{21}r^2d_2 \right) r \sin \theta \cos \theta \sin \phi \cos \phi \\ \left\{ \frac{1}{2} \left(d_3 - \frac{5}{21}r^2d_2 \right) \cos(2\phi) - d_4 \right\} r \sin \theta \end{bmatrix} \quad (4)$$

again, for the continuous and discrete phases, respectively, and where the velocity vectors capture the r , θ , and ϕ directions, in that order. Additionally, it must be noted that ϕ is the longitudinal angle, measured in the x y plane from the x -axis of the rectangular system, and θ is the latitudinal angle, measured from the z -axis of the rectangular system about the ϕ -axis of the spherical system, again as shown in Fig. 1.

In order to ensure the velocity components are consistent at the surface of the particle (i.e., where the normalized radius $r = 1$), the tangential velocity components must be the same in both the continuous and discrete phases. The velocity components in the θ direction yield

$$1 - \frac{2}{5}c_3 = d_3 + \frac{5}{21}d_2 \quad (5)$$

and the ϕ components yield the following independent expression:

$$\frac{1}{2} + c_4 = d_4 \quad (6)$$

Since it is assumed the bubble remains spherical and also assuming its size does not change, the radial velocity components in both phases must be zero. This leads to

$$1 + \frac{3}{5}c_3 + \frac{1}{2}c_1 = 0 \quad (7)$$

and

$$d_3 + \frac{3}{21}d_2 = 0 \quad (8)$$

Additionally, in the case considered here, the tangential stresses must be consistent across the bubble surface. Applying the general forms of the tangential stress components in a spherical coordinate system from Potter and Wiggert [36], the r - θ stress components for the continuous, $\tau_{r\theta}$, and discrete, $\hat{\tau}_{r\theta}$, phases can be expressed as

$$\tau_{r\theta} = \mu \left(2 + \frac{c_1}{r^3} + \frac{16c_3}{5r^5} \right) \sin \theta \cos \theta \sin \phi \cos \phi \quad (9)$$

and

$$\hat{\tau}_{r\theta} = \hat{\mu} \left(2d_3 + \frac{16}{21}r^2d_2 \right) \sin \theta \cos \theta \sin \phi \cos \phi \quad (10)$$

where μ and $\hat{\mu}$ are the continuous and discrete phase dynamic viscosities, respectively. Additionally, the r - ϕ stress components for the continuous, $\tau_{r\phi}$, and discrete, $\hat{\tau}_{r\phi}$, phases can be expressed as

$$\tau_{r\phi} = \mu \left[\left(1 + \frac{c_1}{2r^3} + \frac{8c_3}{5r^5} \right) \cos(2\phi) + \frac{3c_4}{r^3} \right] \sin \theta \quad (11)$$

and

$$\hat{\tau}_{r\phi} = \hat{\mu} \left[\left(d_3 + \frac{8}{21}r^2d_2 \right) \cos(2\phi) \right] \sin \theta \quad (12)$$

Then, equating and simplifying the r - θ stress components, again, at $r=1$ yields

$$1 + \frac{8}{5}c_3 + \frac{1}{2}c_1 = \lambda \left(d_3 + \frac{8}{21}d_2 \right) \quad (13)$$

and the only independent expression from equating the r - ϕ stress components is

$$3c_4 = 0 \quad (14)$$

where λ , the viscosity ratio, is the ratio of $\hat{\mu}$ to μ .

Solving these equations simultaneously, the constants were determined to be

$$c_1 = -\frac{2+5\lambda}{1+\lambda} \quad (15)$$

$$c_3 = \frac{5}{2} \frac{\lambda}{1+\lambda} \quad (16)$$

$$c_4 = 0 \quad (17)$$

$$d_2 = \frac{21}{2} \frac{1}{1+\lambda} \quad (18)$$

$$d_3 = -\frac{3}{2} \frac{1}{1+\lambda} \quad (19)$$

and

$$d_4 = \frac{1}{2} \quad (20)$$

Although these constants are similar in form to those presented by Leal (or identical in the case of c_4 and d_4), the main difference between these two sets of constants is the absence of the shape perturbation terms that appear in Leal's constants. Whereas Leal's effort was focused on the distortion of a spherical bubble, again, the goal of this effort is to determine the effect a spherical bubble has on the dissipation in both phases.

This yields the following flow field equations for continuous phase (in Cartesian coordinates):

$$\begin{aligned} u_1 &= y - \frac{1}{2 \cdot r^5 \cdot (1+\lambda)} \cdot \left(\left(2 + 5\lambda - \frac{5\lambda}{r^2} \right) \cdot x^2y + \lambda y \right) \\ u_2 &= -\frac{1}{2 \cdot r^5 \cdot (1+\lambda)} \cdot \left(\left(2 + 5\lambda - \frac{5\lambda}{r^2} \right) \cdot xy^2 + \lambda x \right) \\ u_3 &= -\frac{1}{2 \cdot r^5 \cdot (1+\lambda)} \cdot \left(2 + 5\lambda - \frac{5\lambda}{r^2} \right) \cdot xyz \end{aligned} \quad (21)$$

For the discrete phase, these equations are

$$\begin{aligned} \hat{u}_1 &= \frac{1}{1+\lambda} \cdot \left((5 \cdot r^2 - 3) \frac{y}{4} - x^2y \right) + \frac{y}{2} \\ \hat{u}_2 &= \frac{1}{1+\lambda} \cdot \left((5 \cdot r^2 - 3) \frac{x}{4} - xy^2 \right) - \frac{x}{2} \\ \hat{u}_3 &= -\frac{1}{1+\lambda} \cdot xyz \end{aligned} \quad (22)$$

where u_1 , u_2 , and u_3 are the velocity components in the x , y , and z directions, respectively. Again, it is important to note that these equations apply to a case of creeping linear shear flow around a spherical fluid particle with a normalized radius of unity.

Flow Field Results. Figure 2 shows the flow fields in the gradient plane. As can be seen in this figure, the fluid in the particle primarily rotates in response to the disturbed shear flow in the surrounding fluid. As expected, at the relatively low viscosity shown in the figure, the inner fluid deviates from pure rotational motion most dramatically at the particle's equator (where the relative motion between the particle and the undisturbed shear flow would ideally be zero). As can also be seen, a very noticeable sympathetic bulge in a circulation region around the particle is created in the outer fluid at this same location. This bulge allows for a smooth transition of the shear flow streamlines around the particle. It is important to note that for cases where the particle viscosity approaches infinity (i.e., the particle is a solid object), the particle simply rotates, as expected, and the equatorial circulation regions around the particle become even more pronounced. These general flow field features are very similar to those shown in the experimental results from Cox et al. [33] for a solid spherical particle (i.e., λ approaches infinity) in Couette flow.

Dissipation Equations. Once the flow field equations were established, we proceeded to the main objective of this study which is the development of the dissipation equations. Among other phenomena, the differential form of the energy equation

captures the viscous losses within a flowing fluid. Isolating these viscous terms and applying them to a Newtonian fluid flowing in an incompressible fashion, the dissipation in both phases can be found using the dissipation function [37]

$$\Phi = \mu [2(\varepsilon_{rr}^2 + \varepsilon_{\theta\theta}^2 + \varepsilon_{\phi\phi}^2) + \varepsilon_{\theta\phi}^2 + \varepsilon_{\theta r}^2 + \varepsilon_{r\phi}^2] \quad (23)$$

where Φ is the dissipation associated with the viscous losses in an incompressible flow and ε_{rr} , $\varepsilon_{\theta\theta}$, $\varepsilon_{\phi\phi}$, $\varepsilon_{\theta\phi}$, $\varepsilon_{\theta r}$, and $\varepsilon_{r\phi}$ are the

fluid strains. The form of the dissipation function presented in Eq. (23) neglects the effects of the bulk viscosity. This is appropriate for the slow moving incompressible flow considered in this case.

Then, normalizing this dissipation by the dissipation in the undisturbed flow field and substituting the strains found from the spherical form of the flow field equations, the dissipation functions for the discrete and continuous fluids were determined to be

$$\Phi = \left[\begin{aligned} & \left(\frac{\sin(\theta)\cos(2\phi)\{5\lambda + (r-1)[2(\lambda+1)r^2(r^2+r+1) - 3\lambda(r+1)]\}}{2r^5(\lambda+1)} \right)^2 \\ & + 2 \left(\frac{\sin(2\phi)[-2r^5(\lambda+1) + (3\lambda - (5\lambda+2)r^2)(\sin(\theta))^2 + 4\lambda - 2\lambda(\cos(\theta))^2]}{4r^5(\lambda+1)} \right)^2 \\ & + \frac{(\sin(2\phi))^2 \left\{ 2[(2r^5 + 2\lambda(r^5-1))(\cos(\theta))^2 + (5\lambda(1-r^2) - 2r^2)(\sin(\theta))^2]^2 \right.}{16r^{10}(\lambda+1)^2} \\ & \quad \left. + 8(\sin(\theta))^4[(5\lambda+2)r^2 - 6\lambda + (\lambda+1)r^5]^2 \right. \\ & \quad \left. + (\sin(2\theta))^2[8\lambda - (5\lambda+2)r^2 + 2(\lambda+1)r^5]^2 \right\}}{16r^{10}(\lambda+1)^2} \\ & + \frac{(\cos(\theta))^2(\cos(2\phi))^2((\lambda+1)r^5 - \lambda)^2}{r^{10}(\lambda+1)^2} \end{aligned} \right] \quad (24)$$

$$\hat{\Phi} = \lambda \left[\frac{3r^4(\sin(2\phi))^2(\sin(\theta))^4 + [39r^4 - 18r^2](\sin(\theta))^2 + 25r^4 - 30r^2 + 9}{4(\lambda+1)^2} \right] \quad (25)$$

where Φ and $\hat{\Phi}$ are the dissipation functions for the continuous and discrete phases, respectively. From this point forward, except when discussing the overall dissipation below, the term “dissipation” will be used to refer to the normalized dissipation.

Fig. 2 Streamlines associated with the analytical solution for the flow fields around and inside a bubble. Axes are normalized by the bubble radius, a .

Dissipation Results

Continuous Phase

Obstruction pattern. These dissipation equations are for a creeping linear shear flow around a fluid-filled unit sphere. The sphere in Fig. 3(a) shows the dissipation associated with the liquid at the surface of the bubble (when $\lambda = 0$). From the scale shown, it can be seen that the greatest dissipation occurs 45 deg between the axial and gradient poles. This also can be seen in Figs. 4(a)–4(c), which are surface plots of the continuous phase dissipation in the xy -plane. As can be seen in this figure and in Fig. 5(a), four dissipation peaks are located at the surface of the bubble. Each of these peaks has a “tail” of tapering elevated dissipation that emanates radially from the bubble. At the axial and gradient poles themselves, troughs of suppressed dissipation are formed and taper radially away from the bubble. Additional examination of Fig. 3(a) reveals that there are regions of reduced dissipation located at the transverse poles. Because of this reduced dissipation at the poles and along the great circles connecting the poles, it turns out that the continuous phase dissipation is suppressed across the entire xz - and yz -planes for this case.

Since there is no friction at the interface when $\lambda = 0$, the fluid motion within the bubble has no impact on the fluid motion in the continuous phase, and the bubble simply acts as an obstruction in the shear flow. Consequently, the dissipation features discussed above will be called the “obstruction” pattern. For this pattern, the elevated dissipation near the bubble is primarily driven by high normal strains that develop in the continuous phase as the liquid deforms to move around the bubble. The regions of reduced dissipation in the xy -plane correspond to areas of reduced shear strain.

Fig. 3 Continuous phase dissipation field at the bubble surface for (a) $\lambda = 0$, (b) $\lambda = 10^6$. Note: scales are the same for both sub-plots.

Rotation pattern. Even though the main focus of this effort involves bubbles interacting with shear flows, greater insight into the present study can be gained by also investigating solid spherical particles and their effects on the dissipation patterns in a linear shear flow. A solid particle can be modeled as a fluid particle that has a very high dynamic viscosity. When this is done, much as in the case of an inviscid bubble, as can be seen in Figs. 4(d)–4(f), the flow field immediately adjacent to a solid particle has four dissipation peaks in the xy -plane separated by an equal number of troughs. Several important differences exist between these two cases, however. First, as can be seen in this figure, the peaks and troughs associated with the flow around a solid particle are rotated by 45 deg in the xy -plane from their locations when the particle is inviscid. This, then, places these regions of elevated dissipation directly on the axial and gradient poles. Second, unlike the long tapering tails in the inviscid case, peaks and troughs associated with a solid particle rapidly transition to a circular region of constant and slightly elevated dissipation. This circular region is located in the xy -plane, centered on the particle, and is just outside the solid particle's peaks and troughs (at a normalized radius of approximately 1.175 and a normalized dissipation of 1.552 in the case of a solid particle). Third, this constant dissipation region transitions back into a region of four shorter peaks (which are more like hills) and four shallower troughs (which are more like valleys). These hills and valleys are offset by 45 deg from the peaks and troughs located at the surface of the solid particle. This, then, places the hills directly on the long tails of the dissipation peaks associated with an inviscid bubble.

The inboard and outboard regions in the xy -plane that are separated by the circular region of constant dissipation make for a more complex and interesting dissipation pattern than the one associated with the inviscid bubble. Even though this dissipation pattern is fairly complex, the physical reasoning behind it is rather simple. The flow field associated with the inboard region is driven primarily by the rotation of the particle. As the particle rotates, it forces the surrounding liquid to rotate as well. Since the liquid at the particle's equator would normally be stationary for a nonrotating particle, as it is with an inviscid bubble, the rotational motion of the particle creates shear strains in the equatorial liquid.

These strains are particularly high at the axial poles and again, as seen in Fig. 3(b), they lead to high dissipation levels at these locations. The fluid motion induced at the equator acts to retard the rotational motion of the particle. Consequently, the angular velocity of the particle is only half of what it would be if the linear

velocities at the particle's gradient poles were to match those in the undisturbed shear flow (e.g., per Trevelyan and Mason [38], the dimensionless angular velocity of the particle is 1/2 rather than 1). As a result, elevated shear strains, and the associated elevated dissipation levels, are developed at the gradient poles as the fluid at these poles helps propel the rotational motion of the particle. In this way, the fluid motion at the gradient poles (which contributes to the forces that sustain the particle's rotational motion) is coupled to the fluid at the axial poles (which react those forces out) through the particle itself.

This rotating fluid, whose motion constitutes what we have termed the "rotation" pattern, creates an envelope around the particle. In the xy -plane, the edge of this envelope is the circular region of constant dissipation. Beyond this envelope, the shear strains that dominated the dissipation near the particle give way to normal strains that dominate the dissipation in the outboard region as this outboard fluid deforms to move around this now effectively larger "bubble." As a result, the dissipation pattern in the outboard fluid takes on the same form as the obstruction pattern that was associated with an inviscid bubble. Clearly, though, since the rotational motion of the inboard region helps the outboard fluid move around this larger obstruction, and, since the outboard fluid also now has a larger region to adjust to this obstruction, the dissipation intensities are lower in the outboard fluid but remain elevated over a wider area in these regions.

Dissipation pattern evolution. It can thus be seen that the dissipation behavior of the continuous phase evolves as the dynamic viscosity of the bubble is increased. As shown in Fig. 6, this is most easily seen on a polar plot of the dissipation in the xy -plane. Instead of focusing on the overall dissipation distribution, however, this figure captures the radial variation of the dissipation by plotting the radial and transverse values on rectangular axes and then looking directly down the transverse axis (note that the bubble is to the left of the dissipation axis). In this way, both the minimum and maximum dissipation values at any given radial value can be readily determined. Note that, for simplicity, this discussion is limited to trends in the xy -plane. Even though there are interesting trends at the transverse poles (and at other locations in the flow fields), their impact on the bubble's overall effect on the dissipation are not of primary importance.

This figure also shows the radial dissipation distribution for various viscosity ratios. For a point of reference, Fig. 6(a) again shows the case where the fluid in the bubble is inviscid. As before,

Fig. 4 Dissipation field in the xy -plane for the continuous phase flow field around the bubble. For an inviscid bubble [$\lambda = 0$], the following views are (a) top oriented, (b) bottom oriented, and (c) side. For a solid particle [$\lambda = 10^6$], the following views are (d) top oriented, (e) bottom oriented, and (f) side. Scales are the same as Fig. 3.

the dissipation peaks are immediately adjacent to the bubble, and they have long tails that taper radially. Because it is inviscid, the fluid in the bubble has no effect on the fluid motion in the continuous phase other than to simply obstruct the shear flow. Again, the dissipation in this case is primarily driven by the normal strains, and the shear strains at the bubble surface in the xy -plane are nonexistent. Consequently, there is no rotational through-bubble coupling between regions in the continuous phase.

For Fig. 6(b), the viscosity ratio is 0.1. Consequently, the fluid in the bubble will start to move in response to the continuous phase flow field. Since the material in the bubble is trapped by the surrounding fluid, the only viable flow patterns result in circulation of the fluid within the bubble itself. Because of the no-slip condition at the bubble surface, this circulation inside the bubble creates a weak rotation in the fluid surrounding it. This rotation

makes it slightly easier for the fluid in the continuous phase to move around the bubble. As a result, the normal strain intensity is diminished slightly with a corresponding decrease in the maximum dissipation. Because the fluid surrounding the bubble starts to rotate, though, small but finite shear strains start to develop at the axial and gradient poles in this particular case indicating that the strains at these locations are becoming loosely coupled.

Figure 6(c) corresponds to a viscosity ratio of 0.5. The same trends observed when λ was 0.1 are present in this case (reduced normal strains and increased shear strains and gradient plane coupling), and they are becoming more pronounced. In this case, the better organized rotation of the bubble and the associated rotation in the fluid have made it easy enough for the fluid to move around the bubble that the location of the peak dissipation moves away from the surface of the bubble, and its magnitude continues to

Fig. 5 Top view of the dissipation field in the xy -plane of the continuous phase around the bubble for (a) $\lambda = 0$ and (b) $\lambda = 10^6$

Fig. 6 XY-plane dissipation field in the continuous phase for increasing viscosity ratios represented here in a polar system $[r-\phi]$. The dissipation is projected down the ϕ -axis for (a) $\lambda = 0$, (b) $\lambda = 0.1$, (c) $\lambda = 0.5$, (d) $\lambda = 1.058$, (e) $\lambda = 2$, and (f) $\lambda = 10^6$. Scales are the same as Fig. 3. Note: the bubble is immediately to the left of the dissipation axis.

drop. Even though the rotational motion in both phases is starting to have an observable effect on the dissipation pattern, the obstruction dissipation pattern still dominates.

As the bubble viscosity is increased and normal strains continue to drop while the shear strains (and the continuous phase coupling) increase, a point is reached at which the effect at the bubble's surface from the shear strains is the same as that from the normal strains. This happens at a viscosity ratio of approximately 1.058, and this case is shown in Fig. 6(d). At this viscosity ratio,

neither the dissipative effects associated with the fluid rotation nor that associated with the fluid motion around the bubble dominate at the bubble surface, thus indicating a transition in the dominant dissipation mode near the bubble.

As expected, when the viscosity ratio is increased to two, as shown in Fig. 6(e), the dissipation peaks associated with the obstruction pattern have moved further away from the bubble, and the peaks associated with the rotation dissipation pattern start to develop at the bubble surface. Separating these two patterns is the

circular region of constant dissipation (shown as a single point in Fig. 6(e)). As the viscosity is increased to the point at which the particle is essentially solid ($\lambda = 10^6$ as shown in Fig. 6(f)), the circular region of constant dissipation has moved further out from the bubble, the obstruction dissipation peaks are at their lowest level, these peaks are located at a normalized radius of approximately 1.4, and the rotation pattern dissipation peaks are at their highest level (indicating complete coupling of the fluid at the axial and gradient poles through this now solid particle).

As indicated by the discussion earlier, the rotation and obstruction regions can be viewed from a different perspective. Since the limiting form of the obstruction pattern occurs when the bubble viscosity is zero, indicating that no-force coupling exists between the phases, the enhanced dissipation associated with this pattern results from the continuous phase material experiencing elevated strain intensities within itself. The rotation pattern, on the other hand, can only occur when the two phases are coupled to each other.

Isodissipation circles. As mentioned earlier, when the viscosity ratio is greater than or equal to a specific threshold value, an isodissipation curve in the form of a circle exists in the continuous phase's gradient plane. This circle moves further away from the bubble as the viscosity ratio is increased. An expression relating the radius of this circle to the viscosity ratio can be found from Eq. (24)

$$20(5\lambda + 2)(\lambda + 1)r^7 - 140\lambda(\lambda + 1)r^5 + 4(5\lambda + 2)^2r^4 - 40\lambda(5\lambda + 2)r^2 + 90\lambda^2 = 0 \quad (26)$$

For the purposes of this effort, this expression can be considered the envelope for the inner region that contains the rotation pattern. Additionally, this expression can be used to show that the isodissipation circle is located on the surface of the bubble (i.e., $r = 1$) when $\lambda = \sqrt{28/25}$. This, of course, is also the lower viscosity ratio limit for Eq. (26). Consequently, the discrete phase must be slightly more viscous than the continuous phase, i.e., slightly more coupling between the fluid at the gradient and axial poles is required than can be provided by two fluids that have the same viscosity, in order for the obstruction and rotation dissipations to be equal at the surface of the bubble.

Though less of a concern for this research, there is also an isodissipation circle that forms in the equatorial plane. The equation for the radius of this circle, as a function of the viscosity ratio, is

$$r = \sqrt{\frac{10\lambda}{5\lambda + 2}} \quad (27)$$

The circle is located on the bubble's surface when, consequently, the lower viscosity ratio limit for this equation is, $\lambda = 0.4$, and the corresponding normalized dissipation is $\Phi = 25/49$. For a solid particle, the normalized radius of this circle is $r = \sqrt{2}$, and the corresponding normalized dissipation is $\Phi = (33 - 8\sqrt{2})/32$.

Continuous phase dissipation discussion. Numerical simulations have predicted and experimental results [39] have shown similar continuous phase behavior. Wang et al. [40], Yeo et al. [41], and Goa et al. [42] have all noted elevated levels of dissipation when small particles and/or bubbles are present in turbulent flow fields. In particular, Wang et al. noted that the dissipation was highest for a solid particle within 0.4 radii of its interface with the continuous phase, which, as it happens, corresponds almost identically with the obstruction peaks for a solid particle in the gradient plane (again, as shown by that peak at a normalized radius of 1.4 in Fig. 6(f)). It is also important to note that the particles in these investigations ranged in size from the Kolmogorov to the Taylor length scales [42]. This, then, means that the particles investigated in these numerical simulations were interacting with diverse features in the flow field, not simply with a creeping

linear shear flow. However, these investigators reached similar conclusions about the interaction between the discrete and continuous phases. Consequently, even though this theoretical development only applies to a case with fairly restrictive assumptions, it can be seen that the general trends identified by this effort may apply to a much broader set of interphase interactions.

As an aside, it is interesting to note that all of the xy -plane dissipation pattern features discussed previously (or their parent features) are present in the continuous phase flow field equations at all viscosity ratios. They are simply inside the bubble boundary at low-viscosity ratios until they expand radially outward to finally join the continuous phase's physical domain as the viscosity ratio is increased.

Discrete Phase

Discrete phase patterns. In addition to viscous losses in the continuous phase, the bubble itself will also dissipate kinetic energy. As it turns out, the dissipation pattern in the bubble is more consistent than the pattern associated with the continuous phase. This pattern is so consistent that regardless of the viscosity ratio, the maximum values for all six normal and shear strain components at the bubble's surface are simply scalar multiples of each other. This is very different from the continuous phase trends at the interface in which the intensity of the shear strains generally moves opposite the normal strain intensities.

It can be seen in Fig. 7 that in all cases, the greatest dissipation associated with the bubble is along the gradient great circle. The relatively uniform dissipation in this region results from the fact that the locations at which the shear strains reach their maximum values are offset by 45 deg from the points on the gradient great circle where the normal strains are at their maximum. This has a tendency to even out the dissipation along the gradient great circle, which results in the relatively uniform dissipation pattern shown. Additionally, the minimum dissipation on the surface of the bubble is at the transverse poles. Since these poles are on what is essentially the rotational axis of the bubble, the strains in this region are fairly small as the bubble fluid on the surface primarily rotates about the transverse poles.

Figure 8 shows the magnitude of the bubble dissipation in the xy -plane. In addition to the surface dissipation, which is represented by the high values at the rim of the "bowl," in all cases, there is an inner core of elevated dissipation within the bubble, as indicated by the bump in the bottom of the bowl. The dissipation in the core is relatively low when compared to the maximum surface dissipation, and it is separated from the surface dissipation region by a region of reduced dissipation, which is typically near zero. The dissipation associated with this inner core is primarily driven by the distorted fluid motion that results from coupling the fluid motion in the continuous phase at the axial and gradient poles.

Viscosity ratio. Even though the bubble dissipation pattern remains fairly consistent, as can be seen in Fig. 9(a), the overall normalized dissipation in the bubble, as represented by integrating the dissipation across the bubble's volume and normalizing it to the background dissipation in the same volume, varies considerably depending on the viscosity ratio. As expected, when the bubble is inviscid or solid (i.e., the viscosity ratio is either zero or very large, respectively), then no energy is dissipated by the bubble. This dissipation then increases as the viscosity ratio moves away from these extremes, and it reaches its maximum value when the continuous and discrete phase viscosities are the same. A similar curve can be created for the continuous phase.

As can be seen in Fig. 9(b), the overall normalized dissipation in the continuous phase (integrating from the bubble surface out to 5 bubble radii) starts at a relatively moderate value when the bubble is inviscid. It then drops as the bubble viscosity is increased and rotation is developed in the continuous phase. After above a viscosity ratio of approximately two-thirds, the increased coupling between the fluid at the axial and gradient poles starts to dominate and this dissipation starts to rise. This trend continues

Fig. 7 Discrete phase dissipation field at the bubble surface for (a) $\lambda = 0.1$ (note relatively high maximum dissipation for a low value of λ) and (b) $\lambda = 1$. The scales are different for the subplots but the dissipation patterns are nearly identical. Note: the dissipation magnitude between these cases changed by a factor of approximately three.

Fig. 8 Dissipation field in the xy -plane for the discrete phase flow field for (a) $\lambda = 0.1$ and (b) $\lambda = 1$

until this dissipation reaches a maximum value when the particle is essentially solid.

Figure 9(c) represents the combined overall dissipation of the discrete and continuous phases. As can be seen from this figure, even though there is a relatively complex relationship between the viscosity ratio and the overall dissipation for each phase individually, the combined overall dissipation increases monotonically with increasing bubble viscosity. Consequently, bubbles of all viscosities will lead to increased overall dissipation, and the level of this dissipation increases with increasing viscosity ratio.

General Observations

Dissipation discussion. As a result, it can be seen that microbubbles (and even microparticles) will accelerate the dissipation of TKE. Again, considering that this theoretical development did not explicitly assume a particular bubble size or range of sizes, though, these results then apply to any essentially spherical bubble in a creeping linear shear flow. Consequently, this analysis also applies to larger spherical bubbles (again, as determined by the values of the Eotvos, Reynolds, Morton, and/or Weber numbers, as appropriate) subjected to a bulk linear shear flow in a laminar flow field.

Additionally, it must be noted that even though the dissipation model developed as a part of this effort applies to the specific case defined by the underlying assumptions, the qualitative reasoning

associated with this research may be applied more broadly to bubbles of different geometries immersed in a general shear flow. Starting by relaxing the requirement that the bubbles remains purely spherical, it is indicated by Leal [34] that a bubble in creeping linear shear flow will take on an oblong shape with the longest dimension of the bubble located 45 deg from the undisturbed gradient vector. Next, as the undisturbed velocity gradient becomes more extreme, including moving out of the creeping flow regime, it could reasonably be expected that the distortion of the bubble would become more severe, possibly taking on shapes beyond those that could be described as being a disturbed sphere. This would be especially true if a relatively large bubble is considered or if the general shear flow is allowed to include features that are decidedly nonlinear.

Even though it would be relatively difficult to make rigorous statements about the fluid motion associated with the rotation pattern, or inner, region under these conditions, given its proximity to a possibly distorted bubble, it is distinctly possible that the bubble will still develop a pocket of continuous phase fluid whose motion is primarily driven by the interaction between the two phases. Because the combined size of the distorted bubble and its attendant pocket of surrounding fluid would, in all likelihood, be larger than the rotational pattern for the case of a purely spherical bubble investigated here (assuming the quantity of discrete phase material remains the same), the obstruction pattern that would

Fig. 9 Normalized dissipation volume integral as a function of viscosity ratio for r/a between (a) 0 and 1 (bubble only), (b) 1 and 5 (liquid only), and (c) 0 and 5 (both bubble and liquid)

develop around this pocket would be located further from the center of the bubble, again, when compared to the same pattern for a spherical bubble. Consequently, the obstruction pattern for a distorted bubble would almost certainly still exist, but it would affect a larger volume of continuous phase material than the corresponding pattern for the simplified case that was the focus of this research.

Experimental Investigation

Simple experiments were run to demonstrate the trends captured by the theoretical development. For each test, a working fluid was released via a ball valve from an L-shaped upper

Fig. 10 Diagram of experimental setup. Not to scale.

reservoir, made from 1.25 in. (3.175 cm) plastic pipe, and was directed in such a way that it induced a rotational motion as it entered a working fluid in a lower reservoir. This configuration is shown in Fig. 10. The working fluid was either water, which was used as the control fluid, or carbonated water, which was used as the test fluid. In this test setup, a needle was suspended from the rim of the lower reservoir by a string. The needle was properly positioned so that it would be in contact with the combined fluid in the lower reservoir throughout the test. The string's motion was captured on video for each test and the time was measured for the string to return to the same line of pixels established from one of the pretest images in the video. This time was then normalized to a time scale defined by $t^* = D_{LR}/\sqrt{2gh}$, where D_{LR} is the diameter of the lower reservoir (taken as approximately 0.5 m), g is the acceleration due to gravity, and h is the height of the fluid's center of gravity in the upper reservoir (taken as approximately 1 m). Based on the pipe diameter, the Reynold's number for these tests was approximately 1.4×10^5 .

As can be seen in Fig. 11, tests indicated that on average the carbonated water decreased the time taken to dissipate the motion of the fluid in the lower reservoir by 38%, compared to the case where only water was used in both reservoirs. Even though the use of carbonated water in these simple tests did not allow for control over the bubble size or distribution, which precludes direct verification of the dissipation equations previously presented, these results clearly demonstrate the general conclusion reached by the theoretical part of this effort: bubbles enhance the dissipation of energy in shearing flows.

Conclusions

The study reprises a flow field solution for a spherical bubble in linear shear flow. However, this solution goes beyond the case of a gas bubble in a liquid to include flows involving liquid particles in gasses such as in mists and fogs. Next, from this flow field solution, the analytical expression for the dissipation distribution was derived for the creeping linear shear flow about a unit sphere filled with an immiscible fluid. The presented results have shown how bubbles enhance the dissipation of energy. They indicate that this enhanced dissipation will occur in small scale flows down to and including the Kolmogorov length scale. As a result, under the appropriate conditions, bubbles will not only enhance the dissipation of kinetic energy in bulk shear flows but also of TKE in turbulent flows. The results also show that there is a distinct pattern associated with the dissipation field and that the elevated dissipation created within and outside the bubble is not uniform. In fact, regardless of the viscosity ratio, a disproportionate amount of the elevated dissipation occurs very close to the bubble itself, i.e., within one radius. The general findings from the theoretical results were confirmed by a sequence of simple tests, and, if the

Fig. 11 Experimental results plotted on a dimensionless time scale. The working fluids for the lower reservoir are indicated on the horizontal axis and those in the upper reservoir are indicated by the legend. The error bars at the top of each column indicate the standard deviation associated with each case.

appropriate resources are forthcoming, performing a battery of precisely controlled tests will be part of future efforts.

With this in mind, and taking a brief opportunity to return to the original inspiration for this study: drag reduction, it seems unlikely that simply introducing any form of bubbles near a solid surface would, in and of itself, tend to reduce the skin friction applied to that surface. Consequently, there are almost certainly some other factors at play that contribute to the positive effect bubbles have on pipe flow [43] or an air lubrication system has on a vessel. Marie [44] and Foeth [45] attribute the drag reducing properties of a bubbly mixture to an increase in the thickness of the boundary layer. Whether bubble-induced drag reduction results from an increased boundary layer thickness or from some other interaction or combination of interactions, clearly bubbles have a complicated relationship with their surrounding liquid, in some cases even reducing dissipation of TKE [46]. This, then, should act as a cautionary note for the designers of air lubrication systems and any other systems that rely on the physics of bubbly mixtures: bubbles interact with their surrounding liquid in a very complex fashion, and, as was seen by Foeth [45], Foeth et al. [47], and Kato and Kodama [48], the act of introducing bubbles along a vessel's hull may not, in and of itself, produce the desired effect. To be useful, bubbles and bubble fields may need to be "tuned" to their particular application, and this tuning process may not be a trivial matter.

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