A MODEL OF PUBLIC SECTOR GROWTH

by

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INTRODUCTION

In 1971, State government spending exceeded revenue from all sources for the first time in a decade, with a resultant deficit of $1.6 billion.[1] This deficit appeared despite an average of 112% increase in the state and local per capita tax burden over the past decade.[2] Similarly, the recently published Brookings report shows that over the next two years, if the average federal tax rate or overall tax structure remains unchanged, the federal government will not be able to cover its already established spending commitments from expected full employment revenues.[3] The report suggests that by 1975 the full employment deficit should rise to $17 billion, more than double the estimated full employment deficit of $8.1 billion for 1972. Moreover, the prospect of continuing and growing federal debts appears at a time when war expenditures have actually dropped from their all time highs in the late sixties. All this suggests that the present rapid growth in government spending is hardly an aberration, but reflects rather deeper structural conditions that have pushed, and will continue to push, spending commitments far above available revenue sources.

In this paper we would like to examine some of the structural conditions that lie behind the growing spending commitments of all levels of governments. We particularly want to analyze those factors that might cause government spending to rise more rapidly than overall GNP. In the process we hope to provide some insight into the debt and fiscal implications of such growth, and also to provide a greater understanding of the relationship between public sector growth and the overall stage of economic development.

It is clear that in an economy of full employment, an increased flow of resources to the public sector will result either in increased taxes, or, in the absence of rising taxes, increasing budget deficits and a consequent mix of rising interest rates and an increasing rate of inflation. But still we require an explanation for the increased flow of resources to the public sector. One explanation has been outlined in the now popular "Baumol Model"[4] of a service economy. In this model, William Baumol shows that "balanced growth" between the service and goods sectors, i.e., equal rates of growth in output between the two, combined with wage rate equalization between service and goods workers, and a lower rate of productivity growth in the former, must result in an increasing flow of real resources to the service sector. The same argument can be made for the relationship between the private and public sectors of the economy, showing that balanced growth between the two sectors must result in a rising tax rate on the private economy. In short, along the "full employment path" of growth, public sector expenditures must grow faster than private sector income if public sector output is to keep pace with private sector output.

Nevertheless, the Baumol Model does not provide an entirely satisfactory framework for the discussion of the growth in government spending and the rising tax rate. For, while in a fully employed economy, government spending competes with private sector spending, in an economy of less than full employment government spending creates additional private income. Thus government might be able to meet its commitments to balanced growth out of the additional income created by its own spending. In other words, its growth need not imply a subtraction of resources from the private sector. Moreover, government spending policy, at least at the federal level, might be less motivated by the need to maintain balanced growth between the two sectors, and more by the desire to insure full employment. This suggests that any fiscal crisis or government spending rising more rapidly than GNP, might not be a function of a balanced growth policy, but rather of increasing employment stagnation within the private sector of the economy.

In effect, we wish to present a more complete analysis of the growth of government spending. To do this, we want to chart the change in the ratio of government spending to GNP (G/GNP) by first examining the growth in government spending in an economy of unemployed resources, and then studying the interaction of a full employment maintenance policy and balanced growth spending policy. In this way we hope to develop a more complete analysis of public sector growth.

To develop our analysis we have divided our paper into four sections. In Section I we assume that government spending is based solely on a full employment maintenance policy. We then chart the course of G/GNP in the face of differing rates of divergence between the "warranted" rate of growth of the economy, the rate determined by its full supply capacities, and the actual rate. In Section II we examine the interaction between a full employment spending policy and a balanced growth constraint. In Section III we examine some of the determinants of a possible balanced growth requirement. In Section IV we examine the impact of the balanced growth constraint when the government must compete with the private sector for resources. Finally, in the summary we consider some policy implications of our analysis.

By way of summary, we can present our central conclusions here. Accepting the Baumol assumptions of the equalization of wage rates between the public and private sectors, and a lower rate of growth of productivity in the former, we find:

1) No matter how great the divergence between the actual and warranted rate of growth of the private sector, the steady state ratio of G/GNP that insures full employment remains unchanged and is given by 1/multiplier. Similarly, the steady state ratio of government to private sector employment is also invariant with respect to different degrees of divergence between the warranted and actual growth rate and is given by $r/f(m-r)$, where f is the wage share in the private sector, r the proportion of government spending that directly supports government worker salaries, and m the multiplier.

2) Comparison of our empirical predictions of the steady state ratios with the actual ratios suggests that the economy is within an order of magnitude range of its steady state values,

the latter calculated on the basis of estimates of the present values of m, r and f.

3) Government spending policy based solely on full employment maintenance results in imbalanced growth, with private sector output growing more rapidly than the public sector output.

4) If balanced growth is essential to maintain the private rate of growth of the economy, then a full employment spending policy will generate a higher steady state G/GNP ratio through a drop in the multiplier effect of government spending. Empirical estimates suggest that the multiplier might at present be falling at a rate of 1.8% if the balanced growth constraint were operative.

5) In an economy with a multiplier of one, the balanced growth constraint will force the public sector to compete with the private sector for resources, <u>even</u> in the presence of unemployed resources. As a consequence, the tax rate on the private sector rises without limit.

Most of the more tedious of the mathematical manipulations are in the Appendix. The interested reader should consult them when necessary.

SECTION I

In this section we want to explore the fiscal implications of a government full employment spending policy in the face of continuing employment stagnation.[5] We can simulate the latter by assuming that the warranted rate of growth, the rate of growth based on the full supply capacities of the economy, diverges from the actual rate of growth, the rate of growth determined by the growth rates of the different factors of demand (investment, consumption, etc.). We will not explain the roots of such a possible divergence here.[6] Rather, we simply assume the possibility of such a divergence and then explore the fiscal consequences of full employment policy in the light of this divergence. Let us assume initially that the actual and warranted rates are equal and are given by

1) $k + n$

where k is the rate of growth of the productivity of labor and n is the rate of growth of supply of labor. Now let us assume that, due to technological advances, the same rate of investment growth generates a higher rate of growth of productivity k' so that the warranted rate of growth is then given by

2) $k' + n$

Now assume, in a dynamic version of the Keynesian demand problem, that the <u>rate of growth</u> of demand is constrained to its previous level of $(k + n)$.

Then we assume that employers adjust to this constrained rate of growth of demand by hiring labor at a slower rate than before so that the rate of growth of demand will once again match the rate of growth of supply. In other words, in contrast to the static short run Keynesian problem suppliers do not adjust to overproduction by cutting back on supply, but rather by cutting back on the rate of growth of supply through slowing down the rate of new hiring [e.g. the process of attrition, or "silent firings"]. In other words the rate of growth of supply will now be given by $(k' + n')$ where

$$3) \quad k' + n' = k + n$$

or

$$4) \quad k' - k = n - n' \qquad n' < n$$

Now let us call time period zero, $t = 0$, the point when k rises to k'. Let $L(0)$ be the labor force at $t = 0$. Then at time t, the available labor force will be $L(0)e^{nt}$ while the employed labor force will be $L(0)e^{n't}$ so that the employment gap will be

$$5) \quad L(0)\,(e^{nt} - e^{n't}) = L(0)e^{nt}(1 - e^{(n' - n)t})$$

Thus government expenditure must take up the employment slack given in (5). Now let real government expenditure to insure full employment at time t be given as $G(t)$. In addition let the total income multiplier of government expenditure be given as m so that an expenditure of $G(t)$ income will create total additional income of $mG(t)$. In other words, of the total income created, $G(t)$ will be the result of direct government expenditure, while $(m-1)G(t)$ will be the multiplier effect.

Now we must determine how much employment will be created by $G(t)$ expenditure. That is, we want to convert our income multiplier into an employment multiplier. Let us make the following assumptions.

a) Of the $G(t)$ income spent by the government, $rG(t)$ is spent by increasing the number of government employees, while $(1-r)G(t)$ is spent by purchasing output produced in the private sector. While the theoretical upper limit of r is one, the practical limit will be less than one, since government production itself requires the purchase of materials and capital from the private sector. So we write $r < 1$

b) The wage rate in the private sector is directly proportional to productivity in the private sector or $PW(t) = fae^{k't}$, $PW(t)$ = the wage rate in the private sector, a = productivity at time $t = 0$, and $f < 1$ ($f < 1$ since the total value added of per labor input, $ae^{k't}$, must be greater than the wage rate).

c) The wages of employees employed directly by the government are equal to the wages of private sector employees.[7]

With these assumptions, the total amount of employment created with an expenditure of $G(t)$ will be

6) $rG(t)/fae^{k't} + (G(t)/ae^{k't})(m-r)$[8]

The first expression of (6) represents the number of new government employees created. Note that we divide the total income spent for the new government employees by the wage rate of government employees, since employee income exhausts the total value added created by direct government expenditure for its own payrolls. The second expression represents the combined effect of government purchases from the private sector and the multiplier effect of all government spending on the private sector. We divide this second expression by productivity rather than the wage rate since the total value added in the private sector is greater than the total wage payments.

Since (6) represents the total employment created with an expenditure of G(t), we can equate expression (6) with the right hand side of (5) and solve for the necessary G(t) required to maintain full employment at any time t. Solving in this fashion, we have

$$7)\quad G(t) = L(0)ae^{(n + k')t}\ (1 - e^{(n' - n)t}) \ / \ B$$

where B is given as

$$8)\quad B = r(1 - f)/f + m$$

Now we can begin to derive the G/GNP ratio implied by a government expenditure policy based on the maintenance of full employment. Private sector output at time t exclusive of the multiplier effect and direct government purchases will be

$$9)\quad aL(t)e^{k't} = L(0)ae^{(n' + k')t}$$

Let us label this expression as 'Y'(t). Then total private sector output inclusive of all multiplier effects will be 'Y'(t) + (m - 1)G(t). Thus the ratio of G/GNP will be

$$10)\quad G(t)/'Y'(t) + (m - 1)G(t) + G(t) \text{ or}$$

$$11)\quad G(t)/'Y'(t) + mG(t) \text{ or}$$

$$12)\quad 1/ \frac{'Y'(t)}{G(t)} \quad + m$$

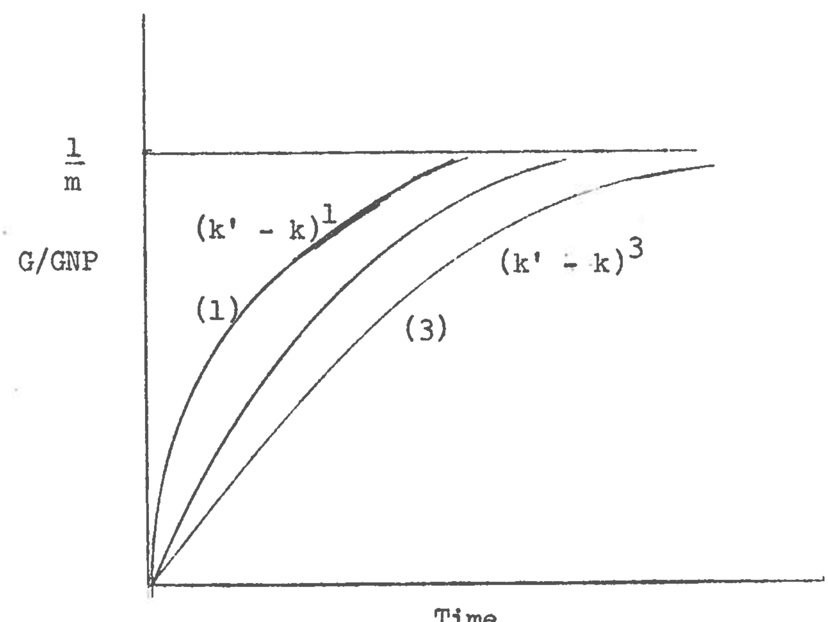
Now a little arithmetic will show that

$$13)\quad 'Y'(t)/G(t) = B'/e^{(k' - k)t} - 1 \text{ so that clearly}$$

$$14)\quad \lim_{t \to \infty} \ 'Y'(t)/G(t) = 0 \quad \text{so that} \lim_{t \to \infty} \ G(t)/GNP(t) = 1/m$$

But note from (12) and (13) that the greater the divergence between the warranted rate and actual growth rate, the greater will be the transitional G/GNP ratio. However, no matter how great the divergence, the steady state or equilibrium ratio is the same.

These results are summarized in Figure 1. The intuition behind our result is quite simple. The greater the divergence between the actual and warranted rate, the greater must be the level of government spending, but the greater will be the corresponding multiplier effect on private income. The two effects cancel out in the limit, so that the equilibrium G/GNP ratio is determined by the multiplier power of a dollar of government spending alone. In short, government spending commitments that grow faster than GNP cannot in the long run be attributed to employment stagnation alone, however great is the degree of stagnation -- i.e. the degree of divergence between the warranted and actual growth rates of the private economy.



$(1)(k' - k)^1 > (k' - k)^3$

FIGURE 1[9]

In a similar manner we can calculate the equilibrium or steady state ratio of government to private sector employees. The total number of government employees GL(t) will be given as

$$15)\quad GL(t) = rG(t)/fae^{k't}$$

while total private sector employment inclusive of the multiplier effect will be given by

$$16)\quad PL(t) = G(t)\ (m - r)/ae^{k't} + {}'Y'(t)/ae^{k't}$$

so that we have

$$17)\quad GL(t)/PL(t) = \frac{rG(t)}{fae^{k't}} \Big/ \frac{G(t)(m - r) + {}'Y'(t)}{ae^{k't}}$$

Dividing numerator and denominator by G(t) and making the appropriate cancellations we have

$$18)\quad \frac{r}{f} \Big/ (m - r) + \frac{'Y'(t)}{G(t)} = \frac{r}{f(m - r)} \qquad \lim t \to \infty$$

The same logic developed with respect to the G/GNP ratio holds for the ratio of government to private sector employees. The steady state ratio is constant, but the greater the divergence between the warranted and actual growth rates the greater will be the ratio in the transition to the steady state.

Substituting some plausible numbers in our formulas enables us to derive some empirical estimates of our steady state ratios. Some calculation and estimation suggest that in 1968, a recent full employment year, r was about .55[10], and f about .75[11]. In addition the multiplier effect due to consumption is about two[12] so the total multiplier will probably range from between two and three. (If the consumption multiplier is two, then additional multiplier effects such as the propensity to invest must sum to 1/6 if the multiplier is to rise to 3.) We then have the following estimates:

m	G/GNP	GL/PL
2	.50	.51
2.5	.40	.38
3	.33	.29

In 1968, actual G/GNP was .21 and GL/PL was .27[13]. These figures seem within the proper range for m between 2 and 3. In other words the actual figures seem consistent with our notion that the determinants of G/GNP and GL/PL are given by a full employment spending policy in the face of some degree of employment stagnation. The economy would thus seem to be moving along its steady state path toward its equilibrium values of G/GNP and GL/PL, the latter determined by the present values of m, r, and f.[14]

Let us summarize. We have seen that long term employment stagnation in the form of a divergence between the warranted and actual growth rates can lead to a rising G/GNP ratio. In additon, we have seen that the greater the divergence the more rapidly will the G/GNP ratio rise to its steady state value. Moreover, it is possible that within the transition period to the steady state, an economy might experience a "technological shock" that raises k' but again leaves the rate of growth of demand unchanged. In such a case there will be an acceleration in the rate of the rise of G/GNP ratio to its steady value (e.g. as we move from curve three to curve one in Figure 1). However, we also saw that no matter how great the divergence between the actual and warranted rates, the G/GNP ratio must converge to the steady state value given by 1/m.

SECTION II

In this section we want to determine the impact of a full employment spending policy on the relative rates of growth of the public and private sector output. To do this we first construct a simple production function for government. If we let GY(t) be government production and PY(t) be private sector production inclusive of all multiplier effects, then we wish to determine the course of the ratio GY(t)/PY(t). First let us write

19) $GY(t) = GL(t) \int e^{dt}$

where d is the rate of growth of productivity in government sector production. In addition we assume that $d < k'$[15]. But we know from (6) that

20) $GL(t) = rG(t)/fae^{k't}$ so substituting (20) into (19)

we have

21) $GY(t) = rG(t) \int e^{(d - k')t}/fa$

Then with some manipulation we find that

22) $GY(t)/PY(t) = 1/\frac{fa}{rz} \left(\frac{Be^{(k' - d)t}}{e^{(k' - k)t} - 1} + (m - r)e^{(k' - d)t} \right)$

23) $GY(t)/PY(t) \cong (rz/fa)/BE^{(k - d)} + (m - r)e^{(k' - d)t}$

Thus we see that as long as $k > d$, the limit of GY (t)/PY(t) will be zero.

In other words, a government spending policy based on the maintenance of full employment alone results in imbalanced growth between the private and public sectors, i.e. in a slower rate of growth of public sector output.

The logic of our argument is again very simple: a full employment spending policy implies a convergence to a constant G/GNP ratio. But since productivity in the public sector is less than that of the private sector, a constant equilibrium G/GNP ratio, i.e. a constant proportion of real income generated by the public sector must result in a slower rate of growth of public sector output.[16]

There is, of course, nothing sacrosanct about a balanced growth requirement. There may be periods of economic development in which private sector output must and does grow faster than public sector output. We discuss this in more detail in the next section. However, let us assume for the moment that the balanced growth constraint does operate on the process of economic development, i.e. that the level of sustainable private sector output and its rate of growth is related to the level of public sector output and its growth rate.

Since we assume that government spending is determined by a full employment maintenance policy, the ratio of $GY(t)/PY(t)$ will still be given as in (23). Assuming for simplicity of expression that $k = d$, we can write the balanced growth constraint as

$$24) \quad GY(t)/PY(t) = e^{(d - k')t} / \frac{fa}{rz}(m - r) \geq V$$

where V is a constant, or the "social infrastructure" ratio.

We write the balanced growth constraint as an inequality rather than an equality since we assume that the constraint is best expressed as the minimum amount of government expenditure required to sustain some level of private sector output. In other words, some level of private sector output will require minimum levels of health, education, federal research outlays, transportation expenditures, and other infrastructural expenditures, if that level of private sector output is to be maintained. More than the minimum can be spent. But beyond that minimum such public expenditures are better considered as consumption items, rather than as factors of production that enter indirectly into private sector output.

Now, as is clear, the numerator in (24) will be falling so that if the inequality is to be maintained, the denominator at some point must also fall. Now the denominator can fall either through a rise in r, or through a fall in m. The impact of a rise in r is easy to understand. By increasing the proportion of government expenditure that goes directly for the hiring of government workers, the greater will be the proportion of government production to multiplier induced private production implied in any fixed amount of government spending. In this way, if r rises (toward its limit of some fraction less than one), the balanced growth constraint will not be violated. The increasing proportion of public to private sector production as a result of the continuous rise in r will offset the more rapid rate of private sector output growth due to the prouctivity differential between the two sectors.

But how are we to interpret the fall in m? If the government pursues a full employment policy it will spend G(t) as given in (7). But due to the productivity differentials between the public and private sectors any given amount of G(t) expenditure will support a continuously declining amount of new private income. Or, to put it another way, because public sector income or value added grows faster than public sector output (due again to the productivity differential), the amount of new private income supported by a given amount of new government expenditure must fall. In effect, the multiplier must fall.

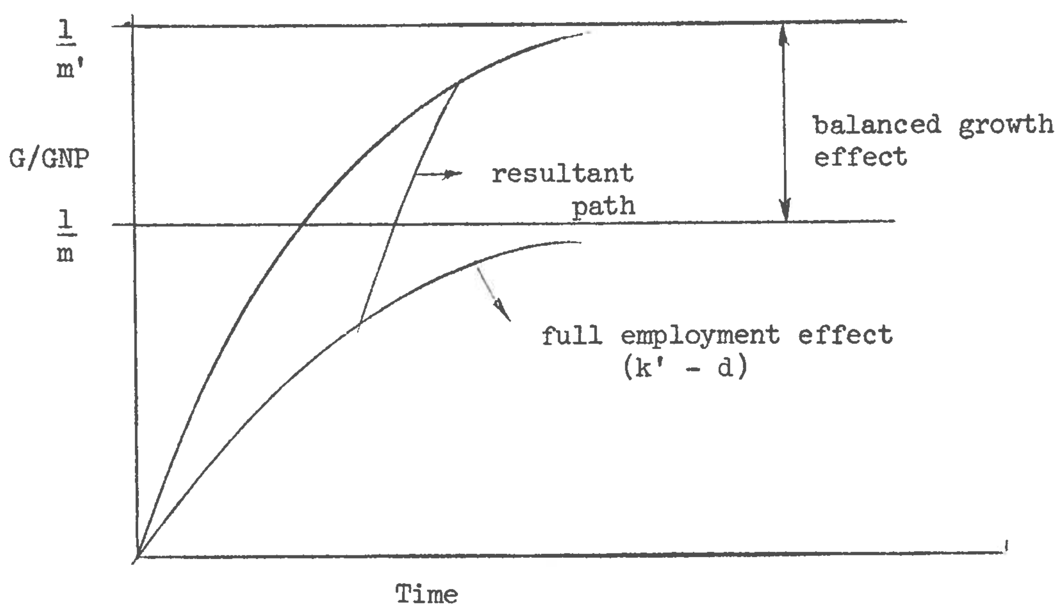
But how will this fall in the multiplier preserve the balanced growth constraint? (24) is a result of assuming that the government pursues a full employment policy. If the multiplier falls, then in order to maintain full employment, the government will have to proportionately increase G(t) in order to increase employment by the correct amount. With r unchanged this means that there will be a proportionately greater amount of public sector production entailed in increasing employment by some given amount. Thus the balanced growth constraint will be preserved, as the falling multiplier forces an increasing reliance on direct government expenditure and thus government production to fill the employment gap. Note then that a full employment policy can effectively function as a balanced growth policy through the fall in the multiplier. There need not be any "conscious attention" to balanced growth, only to full employment.[17]

Which factor, the rise in r or the fall in m, is most likely, in practice, to preserve the balanced growth constraint? If r is to be the key factor, the government decision-makers must pursue not only a full employment policy but must also have a conscious knowledge of the balanced growth constraint. They must be able to increase r, and thus public sector production, in just the correct amount required to support the desired increase in multiplier induced private sector production so that full employment will be maintained. In other words, they must pursue both a balanced growth and full employment policy simultaneously.

But as we have just seen, where m is the key factor, the balanced growth constraint operates through the full employment policy and "behind the backs" of the decision-makers. In effect, the decision-makers need to pursue only a full employment policy, and the balanced growth constraint will be maintained.

This suggests that in a "Keynesian Era" in which full employment is the central economic goal, and the informational tools required to construct a balanced growth policy are deficient or lacking, the balanced growth constraint will be expressed through a falling multiplier in the context of a full employment spending policy. Note then that the multiplier is not purely demand determined, but is also supply determined, its upper limit being given by V.

Returning now to (14) we can see that a falling multiplier must lead to a rise in the steady state ratio of G/GNP (toward its limit of one). Thus where the balanced growth constraint operates through a full employment policy, the steady state ratio of G/GNP rises toward one. This result is demonstrated in Figure 2.[18]



Note: m>m'* graph for constant (k' - d)

FIGURE 2

In Appendix IV we suggest that the multiplier might at present be falling at a rate of 1.8% per year to maintain the balanced growth constraint.

Our analysis might explain the recent appearance of larger than expected full employment deficits.

Thus we can imagine that an economy is in a stage of economic development in which the balanced growth constraint is not operative. In this period its G/GNP and GL/PL values will be rising toward their steady state values. Assume then that at some point due to the greater complexity of economic development, the balanced growth constraint becomes operative. Returning to equation (24) we assume, however, that given the initial values of the relevant constants, the balanced growth constraint can be maintained without any change in m. However, as t increases, GY(t)/PY(t) must at some point fall below V, at which point the multiplier must fall.

Now let us assume that the government undertakes deficit financing to insure full employment, leaving the tax rate constant. Now, as we have seen, with m constant, G/GNP will rise at a decreasing rate to its steady state value. With the tax rate constant, this implies that the rate of deficit financing, i.e. the rate of new debt contracted to GNP (D/GNP) will also rise at a decreasing rate toward its steady state value (given by 1/m - Tax rate). In short, with m constant, the economy will approach a steady state rate of deficit financing. Now, if we assume that government decision-makers develop their fiscal plans (e.g. some expected rate of deficit financing) on the basis of a constant m, that is, an m consistent with past experience, then a falling m, due to the appearance and operation of the balanced growth constraint, will generate a higher than planned rate of deficit financing, if full employment is to be maintained. In other words, the full employment deficit will rise above its originally planned level.[19]

Recent fiscal history may in part reflect these developments. The appearance of larger than expected full employment deficits and the growing indebtedness of state and local governments may all be signs of the appearance of the balanced growth constraint within the process of economic development. This argument is summarized in Figure 3:

(Note: T is the tax rate. At time t, the balanced growth constraint becomes operative; at time t' the multiplier falls.)

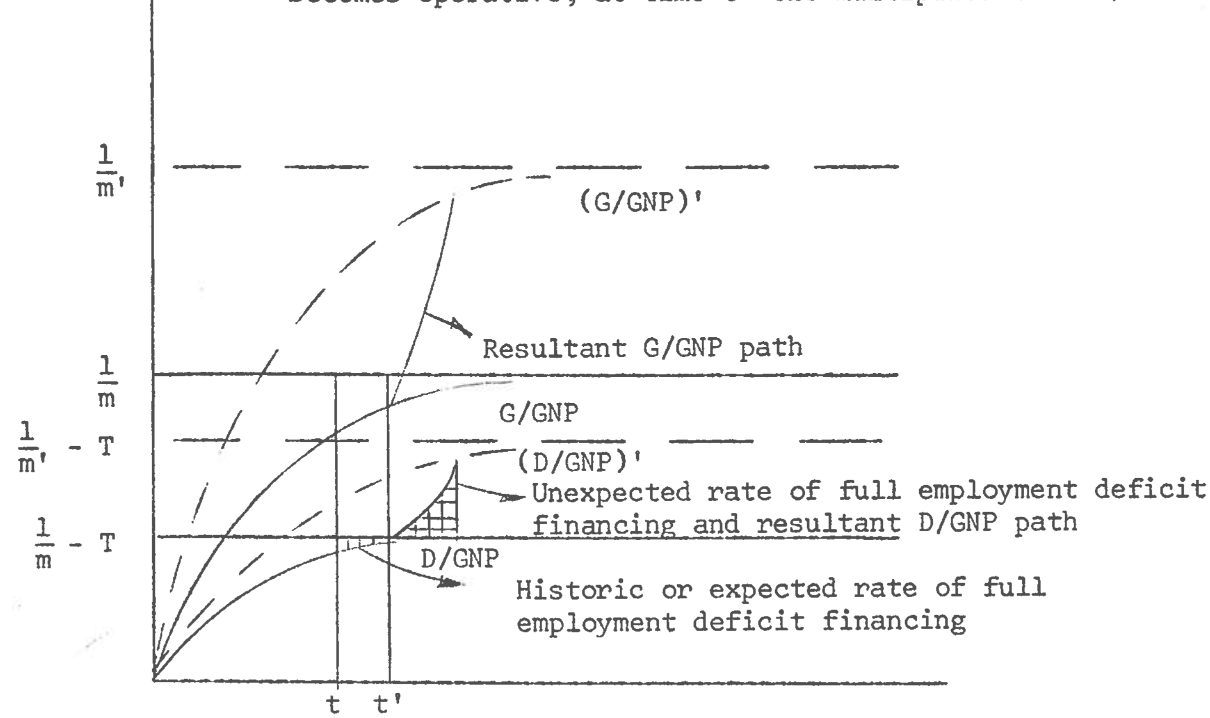


Figure 3

Note, of course, that there is a lower limit to the multiplier of one. When m = 1, government expenditure must fill the entire gap between the level of unemployment and full employment, and, full employment maintenance can then no longer be the guide to government spending. At that point, as we show in Section IV, the continued operation of the balanced growth constraint requires that government must in effect compete with the private sector for resources. Then we are in a "Baumol" world in which the effective tax rate on the private sector must rise, and government spending policy must be consciously geared to preserve balanced growth. We can speculate here that the long run problems of the economy will be less those of full employment maintenance and more those of the increasing competition between the public and private sector for the economy's resources.[20]

However, before we explore the dynamics of the Baumol world, let us first explore some of the possible factors that lie behind the balanced growth constraint.

SECTION III

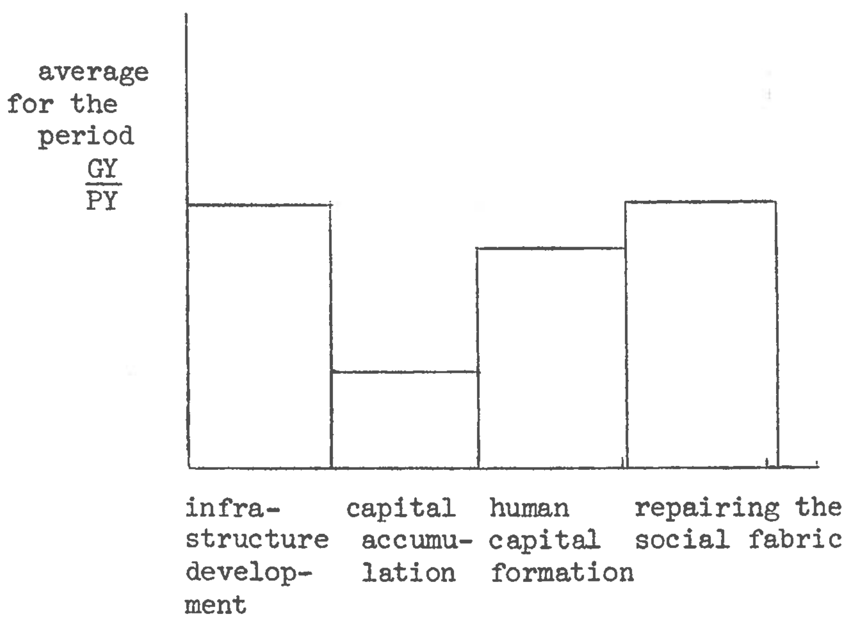
In this section, we want to outline some of the determinants of the balanced growth constraint. To do this, we present a set of skeletal hypotheses concerning the role of the public sector in different stages of economic development. Let us consider a brief "four stage theory" of public sector growth.

In the initial stages of development, we assume that the public sector must grow faster than the private sector to provide the necessary economic infrastructure for successful private sector growth. Thus public sector spending will be typically concentrated in transportation and communications infrastructure investment. In the second stage, which might be identified with Rostow's "take off point," the private sector begins to expand rapidly under the impact of the newly developed infrastructure. Typically, there will be an influx of labor from the rural areas, a high level of savings, and productivity growth due to the consequent rapid increase in the size of the capital stock. In this stage, the public sector will recede in importance as capital accumulation provides the impetus for productivity growth.

The third stage begins when the bulk of productivity growth can no longer come from the accumulation of physical capital based on a high rate of savings, but rather from investments in human capital. The limitations to productivity growth based in a strict accumulation of physical capital come from three sources: (1) a declining rate of in-migration from the rural areas (or from abroad); (2) a declining rate of savings as consumption demands increase; and (3) an increasing demand on the technical proficiency of the population as the industrial and administrative technology becomes more elaborate. In this period rapid expansion in both the health and education of the population is required for economic growth. Thus the rate of public sector growth must once again increase in order to provide the human capital for overall productivity growth. In effect, economic development based on the classic appropriation and, often depletion of labor (in the form of child labor, deteriorating health, factory related deaths, etc.) must be replaced by economic development based on the upgrading of the population. <u>The public sector then becomes the vehicle for this upgrading process, i.e. for the growth in human capital</u>.

Finally, the fourth stage begins when society starts to "repair" those parts of its social and natural environment that have been depleted or damaged by the industrial process. Thus society must undertake vast pollution control expenditures to correct ecological imbalances, and it must embark on an urban reconstruction program to rectify dysfunctional land use patterns and upgrade the rejects of, or those who are unprepared for, the industrial process (e.g. youth, minorities, the structurally unemployed). <u>Because such expenditures are related to private productivity, primarily by correcting negative externalities, the public sector must serve as the vehicle for these expenditures and investments</u>.

We then hypothesize that at present the balanced growth constraint operates through an intensification of stage three and the beginnings of stage four. Because both stages represent deeply structural elements in the overall process of economic development, we can expect the consequences of these stages to be deep and lasting.[21] (Thus for example if, as some argue, the U.S. economy is an "innovation economy", then continuing increases in income will require a continuing flow of new products and processes. If, as is likely, the new products and processes embody more complex technological, administrative, and marketing processes, human capital per capita will have to rise along with per capita income. Similarly, since "social repair" expenditures correct market imbalances, and since the latter will most likely rise with the level of economic activity, social repair expenditures will rise with income. [Note in the latter case even though social repair expenditures are enforced through the political system and appear in the short run as strictly "consumption items," they represent in the long run factor inputs since they preserve the "readiness" and potential productivity of human and natural resources.] Social repair expenditures are functionally like "maintenance" costs for a piece of capital equipment.) We summarize this typology in Figure 4 on a bar graph.



(the relative sizes are purely suggestive)

FIGURE 4

SECTION IV

In this section we want to analyze the consequences of the balanced growth constraint when the multiplier is one. It is clear that when the multiplier is one, government spending must fill the entire gap between the employed and the total labor force. But it can also be shown that it must in fact draw labor out of the private sector, and that the effective tax rate on the private sector must rise without limit.

Let us write the balanced growth constraint as an equality. (This simplification does not in any way affect the substance of our conclusions.)

25) $GY/PY = V$, where V is again the "social infrastructure" ratio. Then the production function for the private and public sector will be

26) $PY(t) = ae^{k't}PL(t)$ and $GY(t) = ze^{dt}GL(t)$, where we date $t = 0$ at the point where the multiplier is one, and government spending policy can no longer be based on full employment maintenance alone.

Then, substituting these production functions into (25) and rearranging some terms, we have

27) $GL(t) = V(a/z)e^{(k' - d)t}PL(t)$

Now turning to equation (5) the total number of available unemployed laborers will be given as

28) $L(0)e^{nt}(1 - se^{(n' - n)t})$ where we add the factor s,

$s = e^{(n' - n)t'}$ where t' is the number of periods from our "original" zero date to our present zero date (i.e. when m reaches 1). This adjustment allows us to combine equations (5) and (27).

Now it can also be shown that a given r corresponds to a given proportion of labor that is hired directly for government employment, as opposed to the labor that is hired through government procurement contracts. Call that proportion h. ($r = \dfrac{1}{1 + (\frac{h}{1-h}) \frac{1}{f}}$).

Then let L*(t) be the amount of labor that must be transferred from the private to the public sector. Then the number of laborers that will be working in the public sector will be

$$29) \quad hL(0)e^{nt}(1 - se^{(n' - n)t}) + L^*(t)$$

Let us write this for simplicity of calculationas $hA + L^*$, $L^* > 0$. The number of laborers working in private sector production will be

$$30) \quad PL(0)e^{n't} + L(0)e^{nt}(1 - se^{(n' - n)t})(1 - h) - L^*(t)$$

which we can abbreviate as $B + A(1 - h) - L^*$.

Note of course that we have assumed a multiplier of one, so that government spending does not indirectly create any private income. Then, letting VA/Z = Q and (k' - d) = y and placing all our abbreviations into equation (27) we have

$$31) \quad hA + L^* = Qe^{yt}(B + A(1 - h) - L^*),$$ then with some manipulation,[22] we have

$$32) \quad L^*(1 + Qe^{yt}) = Qe^{yt}(B + A(1 - h)) - hA$$

$$33) \quad L^* = (Qe^{yt}B + Qe^{yt}A - hA(Qe^{yt} + 1)) / (1 + Qe^{yt})$$

Now as time rises this expression will approach

$$34) \quad Qe^{yt}(B + A - ha)/Qe^{yt} = B + A - hA$$

The last expression, the right hand side of (34), is the amount of labor that must be in the limit transferred from the private to the public sector. (It is clearly positive.) Thus the total amount of labor working in the public sector at the limit will be

35) $hA + L^* = B + A - hA + hA = B + A$

Returning to equations (29) and (30) we see that this represents

36) $L(0)e^{nt}(1 - se^{(n' - n)t}) + PL(0)e^{n't}$, or in effect the entire labor force. In other words, even in the presence of unemployment, the balanced growth constraint will force an increasing proportion of the economy's resources into the private sector.

Now when the multiplier was greater than one, the G/GNP ratio could not be identified with the effective tax rate on the economy, since government spending created income in the private sector. That is, a given level of G did not represent a subtraction from the private sector. But with m = 1 the balanced growth constraint forces the government sector to compete with the private sector. In this case the rise in government spending represents a subtraction of resources from the private sector, so that it must result in a rise in the effective tax rate. Since the public sector must in the limit appropriate all resources, the tax rate rises toward one. In effect, despite the presence of unemployed resources, the balanced growth constraint forces the tax rate on the private sector to rise. In Figure 5, we demonstrate the transition from the stage of deficit financing to the stage of taxation.

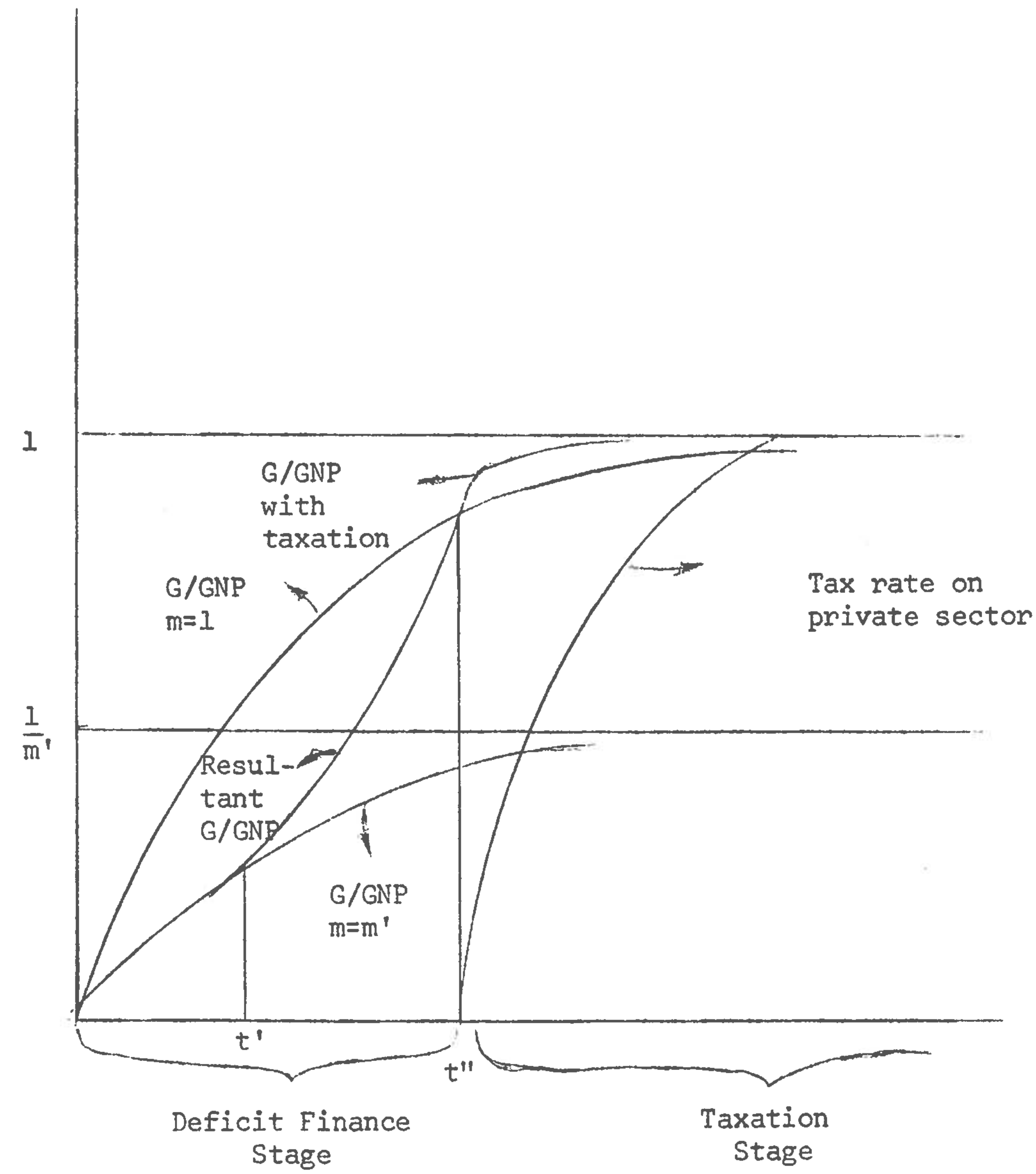

1
G/GNP
with
taxation
G/GNP
m=1
Tax rate on
private sector
1
m'
Resul-
tant
G/GNP
G/GNP
m=m'
t'
t"
Deficit Finance
Stage
Taxation
Stage

Figure 5

At t' the multiplier begins to fall with a consequent rise of the G/GNP ratio toward the path given by m = 1. When m = 1 the tax rate on the private sector begins to rise so that G/GNP actually rises above its previous m = 1 path. Then as t increases, both the G/GNP path and the tax rate converge toward one.

Summary

To summarize, we have seen that an employment problem alone, no matter how severe, cannot in the long run account for a rising ratio of government spending to GNP. A full employment maintenance policy is of itself fully stabilizing However, such a policy leads to imbalanced growth between public and private sector output. We then argued that where balanced growth was a constraint on economic development, that is, a certain amount of social infrastructure was required to support a given unit of private sector output, then a full employment maintenance policy would preserve balanced growth through a drop in the multiplier. But this drop in the multiplier would in turn raise the steady state ratio of government spending to GNP. There would thus be a tendency for G/GNP to rise to its limit of one.

We then argued that the falling multiplier would lead to unexpected rates of deficit financing as government decision-makers based their budgetary estimates on the basis of past values of the multiplier. The resultant insufficient government spending and higher than expected unemployment would thus force a higher rate of deficit financing than expected.

We then suggested that the balanced growth constraint was operating through the human capital investment requirement and the more recent upsurge in "social repair" expenditures. These expenditures seem to be rooted in a new "stage" of economic development and thus their impact would appear to be of a lasting nature.

Finally, we showed that, when the multiplier reached one, the continued operation of the balanced growth constraint would force the tax rate on the private sector to rise without limit, despite the presence of unemployed labor.

One critical policy question emerges from our analysis. There are essentially four ways to overcome the balanced growth constraint: 1) the productivity differential between public and private sector production can be overcome through a government "rationalization" program, 2) the "non-substitutability" between private and public goods, implied by the balanced growth constraint, might be relaxed through a process of "reprivatization", i.e. turning certain social-infrastructure production (e.g. schooling) over to the market. If the productivity differential remains, then this would simply result in a "services inflation". If, as some argue, the private sector were more efficient, then this policy might close the productivity gap, 3) government services might be "contracted out", in which case the funding will come from public sources, but again the possible greater efficiency of the private sector might close the productivity gap, 4) the efficiency of a given unit of social infrastructure, e.g. the manner in which it is rationed or used, might be continuously increased. In such a case the required level of government production to support some level of private production would fall. In other words, the productivity gap in production could be overcome through greater efficiency in use. It might prove interesting to speculate on the different political and economic consequences of these policies.

APPENDIX I

Section I: The amount of employment created with G(t) of government spending

The amount of direct government employment created will be $rG(t)/fae^{k't}$ where $rG(t)$ is government spending for government wages and $fae^{k't}$ is the economy wide wage rate. Now $rG(t)$ spending by the government will create through the multiplier effect an additional $(m - 1)rG(t)$ of private sector income, and so $(m - 1)rG(t)/ae^{k't}$ additional employment in the private sector (where $ae^{k't}$ is labor productivity in the private sector). But since the government also spends $(1 - r)G(t)$ directly in the private sector, it will create via the multiplier effect an additional $m(1 - r)G(t)/ae^{k't}$ jobs in the private sector. Therefore the total jobs created by government spending will be

1) $rG(t)/fae^{k't} + (m - 1)rG(t)/ae^{k't} + m(1 - r)G(t)/ae^{k't} =$

2) $rG(t)/fae^{k't} + (G(t)/ae^{k't})(mr - r + m - mr) =$

3) $rG(t)/fae^{k't} + (G(t)/ae^{k't})(m - r)$

Section II: The calculation of the value lim ('Y'(t)/G(t))

1) $\dfrac{G(t)}{ae^{k't}}(r/f + m - r) = L(0)e^{nt}(1 - e^{(n' - n)t})$

2) $G(t) = L(0)ae^{(n + k')t}(1 - e^{(n' - n)t})/r(1 - f)/f + m$

3) $'Y'(t) = e^{k't}L(0)ae^{n't} = L(0)ae^{(k' + n')t}$

4) $$G(t)/'Y'(t) = \frac{L(0)ae^{(n + k')t}(1 - e^{(n' - n)t})}{L(0)ae^{(n' + k')t}}/B =$$

$$e^{(n - n')t}(1 - e^{(n' - n)t})/B =$$

$$(e^{(n - n')t} - 1)/B = e^{(k' - k)t} - 1/B$$

5) $$\lim_{t \to \infty} 'Y'(t)/G(t) = \lim_{t \to \infty} B/e^{(k' - k)t} - 1 = 0$$

Section III: The calculation of the value (GY(t)/PY(t))

1) $$GY(t)/PY(t) = GY(t)/'Y'(t) + (m - r)G(t) = 1/\frac{'Y'(t)}{GY(t)} + (m - r)\frac{G(t)}{GY(t)} =$$

2) $$\frac{1}{\frac{fa}{rze^{(d - k')t}}\left[\frac{'Y'(t)}{G(t)} + (m - r)\right]}$$

since $GY(t) = rG(t)ze^{(d - k')t}1/fa$

3) $$\frac{1}{\frac{fa}{rze^{(d - k')t}}\left[\frac{B}{e^{(k' - k)t} - 1} + (m - r)\right]} \cong$$

4) $$\frac{1}{\frac{fa}{rz}(e^{(k - d)t} + (m - r)e^{(k' - d)t})}$$

APPENDIX II

The estimation of r

We can estimate r in two ways, directly with national income statistics, or indirectly with the assumptions of our model. Thus our definition of r should roughly correspond to the ratio of government wages and salaries (military included) to the total of government purchases of goods and services. In 1968 it was $95.7/$200 or about .5. (Statistical Abstract 1971, pp. 307, 309)

We can also calculate r in the following way: If we let GW and PW be the wage rates in the public and private sectors respectively, and we let GL and 'PL' be the number of workers on government payrolls and the number of private sector workers on government contracts respectively, and if we assume again that the wage share in the private sector is .75, then we have

$$1) \quad r = (GW)(GL)/(GW)(GL) + \frac{(PW)('PL')}{.75}$$

But since we assume that GW = PW, this reduces to

$$2) \quad r = 1/1 + \frac{'PL'}{GL\ (.75)}$$

Now we can obtain an estimate of 'PL'/GL for 1968 (Supplement to the Manpower Report of the President, 1969, p. 107). It is 1/2. So we have

$$3) \quad r = 1/1 + \frac{1}{2(.75)} = 3/5 = .6$$

Clearly our estimates differ, and it would be tempting to regard the divergence as a test of our assumption that GW = PW (that is, PW/GW = 3/2, if the second way of deriving r is to result in a value for r of .5). But since 'PL'/GL from the manpower reports is no doubt only a rough estimate, and since data from a single year may reflect particular distortions of that year (i.e. the greater than average proportion of military employment in overall government employment, which could pull down the average government wage) we have chosen to simply split the difference between the two figures and settle on r = .55.

APPENDIX III

Deriving the rate of divergence between the warranted and actual growth rates

We have that

1) $G/GNP = G/mG + 'Y' = 1/m + 'Y'/G$

But we know from equation (13) that

2) $'Y'/G = B/e^{(k' - k)t} - 1$

But we also know that in 1968 G/GNP = .21, so we can write

3) $.21 = 1/m + 'Y'/G$

and assuming that m = 2.5 we have

4) $'Y'/G = 1/.21 - m = 4.8 - 2.5 = 2.3$

Thus substituting (4) into (2) we have

5) $2.3 = B/e^{(k' - k)t} - 1$

Now we know from equation (8) that

6) $B = r(1 - f)/f + m = .55(.25)/.75 + 2.5 = 2.68$

So that we have

7) $2.3 = 2.68/e^{(k' - k)t} - 1$

8) $2.3(e^{(k' - k)t} - 1) = 2.68$

9) $e^{(k' - k)t} = 4.98/2.3 = 2.1$

Now to derive (k' - k) we must make some assumption about the value t, i.e. how long government spending has been a function of full employment maintenance. If we assume that the commitment to full employment begins with the passage of the Full Employment Act in 1948 then t = 20.

Or we might assume that full employment spending begins effectively with wartime mobilization so that $t = 28$. Clearly the choice of t is somewhat arbitrary, and our result must be taken with a grain of salt.

Now from an exponential table we know that

10) $e^{(.7)} = 2.01$

So we can say roughly that $(k' - k)t = .7$ so that if $t = 20$

11) $k' - k = .7/20 = .035$ and if $t = 28$

12) $k' - k = .7/28 = .025$

which suggests an average degree of divergence of 3%. This estimate is probably on the high side, which no doubt reflects the fact that not all government spending is undertaken for the sake of employment maintenance.

APPENDIX IV

<u>The percentage drop in the multiplier (assuming k = d)</u>

Let us turn to equation (24) where we assume that k = d. Then if the balanced growth constraint is to be maintained, the percentage fall in the denominator, dD/D, of our expression must equal the negative of the percentage increase in the numerator, dN/N. Now since

1) $D = \frac{fa}{rz}(m - r)$ then $dD/D = \frac{fa}{rz}\, dm/dt \;/ \frac{fa}{rz}(m - r) =$

$dm/dt \;/\; (m - r)$

and since $N = e^{(k' - d)t}$

2) $dN/N = (k' - d)e^{(k' - d)t} \;/\; e^{(k' - d)t} = (k' - d)$

so we have

3) $dm/dt \;/\; (m - r) = -(k' - d)$ or $dm/dt = -(k' - d)(m - r)$

then we know

4) $dm/dt \;/\; m = -(k' - d)(m - r)/m$

The last expression is the percentage change in the multiplier required to satisfy the balanced growth constraint.

If we assume again that m = 2.5 and r = .55 we have that m - r/m = .78. We cannot of course empirically discover k' - d. But let us assume that it ranges between two and three percent. Thus we have that dm/dt / m will range between 1.4% and 2.3%. Then, taking the average of our two estimates we can say that the multiplier will be falling at a rate of 1.8% to maintain the balanced growth constraint.

Note of course that, as m falls, the percentage change in m required to maintain the balanced growth constraint will rise since

$$5)\quad \frac{d(m - r)/m}{dm} = \frac{r}{m^2} > 0$$

FOOTNOTES

1. "States' Spending Excedes Revenue," New York Times, July 17, 1972.

2. Report of the Tax Foundation, as reported in "Tax Bite Deeper; No Relief Sighted," New York Times, January 31, 1972.

3. "Study Finds U.S. May Face a Need for a Big Tax Rise," New York Times, May 25, 1971.

4. William Baumol, "The Macro-Economics of Unbalanced Growth; Anatomy of Urban Crisis," American Economic Review, June 1967.

5. It is of course difficult to measure the precise degree of employment stagnation in the private sector. Measuring the ratio of government and government contracted employment to total private sector employment is not entirely convincing, since government induced employment may represent a withdrawal of resources from the private sector, i.e. resources that would have been otherwise used by the private sector. However, the record of less than full employment throughout much of the fifties and early sixties suggests that all government employment cannot represent a subtraction of resources available to the private sector. A more convincing measure is perhaps the comparative rates of growth of the different components of total employment. In Most Notorious Victory, (New York, 1966, p. 210) B. Seligman estimates that from 1953-1963 the production for profit component of the private sector actually contributed to unemployment rather than employment in that period. He suggests that this sector offered 200,000 fewer full time jobs by the end of the '53-'63 decade. The total employment impact of government spending is often not appreciated. In 1968 close to 30% of the non-farm employed population was employed either by the government or by businesses that were directly on government contracts (e.g. military, construction etc). This number excludes any multiplier effect. See The Manpower Report of the President, 1969, Supplement, p. 107.

6. Explaining such a divergence remains one of the unmet challenges of Keynesian long run theory. The Neo-classical models ignore the challenge by simply assuming that investment will always equal savings at the full employment level.

7. The rationale for this assumption is based less on market mechanisms, e.g. the mobility of labor between the public and private sector, and more on the role of the growing unions of government employees. The Tax Foundation Inc. of New York reports that government payroll costs increased some 88% over the past decade with the average annual salary of the government worker increasing 64% in that decade. This rapid growth in income is probably due to the rapid rate of unionization of government employees in the past decade. Current membership in the largest union, the American Federation of State, County and Municipal Employees, is about 550,000 and growing at about 10,000 a week. In 1960 there were only 36 public employee strikes in the nation, while in 1970 the number stood at 412. (See "Government Pay up 88% in Decade," New York Times, August 2, 1972) In a study in New York it was found that pay gains for city clerical workers for the year 1970-71 equalled that of their private sector counterparts. Pay gains for skilled maintenance workers in the government sector were actually greater than those of their counterparts in the private sector for the same time period. (See "Pay Parity Found in Study Here," New York Times, August 13, 1972)

8. See Appendix I for this derivation.

9. Note that with $m \geq 1$ the tax rate must be less than G/GNP. In this case the rise in G does not represent a subtraction from private income. Rather, through deficit finance, the government mobilizes the savings that could not find investment outlets, so that at full employment $G + I = S$.

10. For the derivation of this estimate see Appendix II.

11. In 1968 payments to capital (profits, interest and rent) come to about 21% of national income net of government (civilian and military) wages and salaries. If we include proprietary income it rises to 31%. We choose 25% as a compromise figure, favoring the lower estimate since a certain fraction of proprietary income will include payments that functionally resemble wage and salary income (e.g. small businessmen). Thus the share of labor will be about 75%. See Statistical Abstract, 1971, p. 309.

12. Council of Economic Advisors, "The Workings of the Multiplier," in The Battle Against Unemployment (A. Okun, ed), New York, 1972.

13. The latter figure includes military employment but excludes farm employment. See Manpower Report of the President, 1970, pp. 216, 265.

14. On the basis of our estimates we can calculate the likely degree of employment stagnation, i.e. the degree of divergence between the warranted and actual growth rate. It comes to about 3%. The estimate is no doubt on the high side reflecting the fact that not all government spending can be attributed to employment maintenance alone. See Appendix III.

15. There are several grounds for this assumption. 1) Government production is not subject to market discipline and is thus not exposed to the competitive pressures that make for rapidly rising productivity. 2) Government output is difficult to measure, thus making strategies for increasing productivity difficult to formulate. 3) Much government production is service-oriented and thus cannot draw on capital intensive techniques to increase labor productivity. 4) There are many strictly political factors (e.g. patronage) that affect both the size of government work forces and the structure of government work, often functioning as barriers to productivity increases.

16. In other words, looking at equation (21) we see that government expenditure grows more rapidly than government output. Since a full employment spending policy implies a convergence to a constant ratio of government spending to private output (inclusive of multiplier effects) the ratio of government output to private output must fall.

17. This is not the case when the multiplier reaches its lower limit of one. We discuss this in Section IV.

18. Our analysis does not, of course, logically preclude the possibility that r will rise to satisfy the balanced growth constraint. State and local spending may in fact be more attuned to balanced growth rather than full employment policy. Thus it is possible that both r will rise and m will fall simultaneously. In the short run the two can offset each other so that actual G/GNP will remain unchanged. (See equations 7, 8, and 12 for the impact of r on G/GNP.) But in the long run, as r rises toward its limit of some fraction less than one, and m falls toward its limit of one, the G/GNP ratio must rise.

19. Since the multiplier falls toward one, the steady state rate of deficit financing will be given as 1 - (tax rate) or for a tax rate of 30%, a 70% rate of deficit financing. So long as interest on the debt is taxable income, then debt obligations incurred to maintain full employment can of course never completely confiscate private income, even though the rate of deficit financing continuously rises. Of course the debt incurred need not be sold to the public, i.e. the Treasury can sell its bonds directly to the Federal Reserve, in which case it incurs no interest charges and effectively "prints money". The money-printing will not be inflationary since the economy is at less than full employment.

Note that we assumed the tax rate was constant, since raising the tax rate to finance government spending will lower the multiplier.

Estimating the steady state rate of deficit financing for m = 2.5 and a tax rate of about 30% we find: D/GNP = .40 - .30 = .10.

20. This competition is reflected not only in the growing size of government budgets, but also in the nation's capital markets. It was recently estimated that, in 1972, combined Treasury and federally assisted borrowing will come to 50% of total credit demands

in the economy. One reason for the rapid expansion in federal credit programs lies in the fact that they are not included in the federal budget and are thus not subject to the budget review process (such as FHA and VA guaranteed mortgages). See, "Has Federal Borrowing Hit Limit?", The Wall Street Journal, August 18, 1972.

21. Some speculative guesses as to the appearance of these stages in U.S. economic history suggests the following: Stage one: 1812 to the end of the Civil War; Stage two: 1865 to the end of World War One; Stage three: 1918 to the present; Stage four: the present. Of course, the dating of stages must be taken with a grain of salt. "Stage" is primarily a concept of proportions. All aspects of public sector growth outlined here "existed" in all the stages suggested here. What counts is the relative significance of the different aspects in these different periods.

22. Note that we can treat L* units of labor in the public and private sector as equivalent, since we continue to assume that wage rates are equalized in the two sectors.

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