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VECTOR-AXIAL VECTOR INTERFERENCE TERMS IN INELASTIC NEUTRINO REACTIONS EXTRAPOLATED FROM SINGLE-PION PHOTOPRODUCTION*

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ABSTRACT

By use of the generalized scaling sum rules the vector-axial vector interference terms in the scaling limit of the inelastic neutrino reactions are calculated from the absorptive parts of the single-pion photoproduction amplitudes at zero momentum transfer. The photoproduction amplitudes are evaluated by use of the results of multipole analysis. With the optimum values for the two parameters of the generalized scaling as determined in the previous works, we have obtained $F_3^{\nu N}(\omega)$ which leads to a good agreement with experiment in the ratio of $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$. However, the zeroth moment in $x\ (=\frac{1}{\omega})$, $\int_0^1 dx(F_3^{\bar{\nu}p} + F_3^{\nu p})$ turns out to be smaller than a half of the value predicted in the fractionally charged quark model.

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1. INTRODUCTION

The extrapolation of the inelastic structure functions in the deep inelastic scaling limit to the small Q^2 region has been attempted for the electroproduction process [1], [2], and subsequently the partially conserved axial vector current hypothesis (PCAC) has been combined with it to relate the inelastic structure functions with the πN total cross sections [3], [4], [5]. For instance, $F_2(\omega = \infty)$ has been predicted independently of the parameters involved and even of specific forms of correspondence, and a good overall fit to the large ω region has been achieved [3]. Supported by this successful result, the $F_2(\omega)$ functions have been calculated from the threshold to $\omega = \infty$ in terms of the total cross sections of pion-nucleon scattering [5]. The moment $\int_0^1 dx(F_2^{\bar{\nu}p} + F_2^{\nu p})$ has turned out to be in a good agreement with the recent analysis of CERN experiment.

Encouraged with these successful attempts incorporating the PCAC, we will proceed here to explore into the vector-axial vector interference term $F_3(\omega)$. We will first show that this structure function is related through the generalized scaling to the absorptive part of the single-pion photoproduction amplitude in the forward direction $(t = 0)$. We will estimate the latter up to 500 MeV in the incident photon energy by use of the results of multipole analysis, and then extrapolate it to the scaling limit through the generalized scaling sum rules of right moment. The fits thus obtained will be compared with the predictions of the quark model as well as the experimental results at CERN and NAL. Since the scaling function $F_3(\omega)$ is most sensitively dependent on the underlying structure of currents, it is powerful in discriminating among various models of hadrons to have some information of $F_3(\omega)$.

2. THE RELATION BETWEEN νW_3 AND THE SINGLE-PION PHOTOPRODUCTION AMPLITUDES

We will first relate $\nu W_3(\nu, Q^2)$ of the vector-axial vector interference term to the absorptive part of the photoproduction amplitude of a single pion at zero momentum transfer. Since the kinematical factor of the photoproduction amplitude vanishes in the forward direction for the initial and final nucleons with spin averaged over, we go off the forward direction to relate them. The vector-axial vector interference amplitudes are defined as

$$\frac{1}{2\pi}\left(\frac{E\,E'}{m^2}\right)^{1/2}\int d^4x\, e^{iqx}\,\langle\, \vec{p}'\,|\, A_\mu^{(i)}(x)\, V_\nu^{(j)}(0)\,|\,\vec{p}\,\rangle$$

$$= \frac{-i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, p^\lambda k^\kappa A - \frac{i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, q^\lambda k^\kappa B\,, \tag{2.1}$$

where $E = (\vec{p}^2 + m^2)^{1/2}$, $E' = (\vec{p}'^2 + m^2)^{1/2}$, A and B are invariant functions of $\nu = (pk)/m$, $t = (p - p')^2 = (q - k)^2$, q^2, and k^2 $(k \equiv p' + q - p)$. The nucleon mass is meant by m, and the nucleon spins of $|\,\vec{p}\,\rangle$ and $\langle\,\vec{p}'|$ are averaged over. The conservation of the vector current $V_\nu^{(j)}$ is taken into account in (2.1). The superscripts attached to A_μ and V_ν refer to the isospins running over 1, 2, and 3. The isospins are suppressed in A and B. In the forward limit $k = q$ the right-hand side reduces to

$$-\frac{i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, p^\lambda q^\kappa A(\nu, Q^2)\,. \tag{2.2}$$

The absorptive part of the single-pion photoproduction amplitude is written as

$$\frac{1}{2}\left(\frac{E\,E'}{m^2}\right)^{1/2}\int d^4x\, e^{iqx}\,\langle\, \vec{p}'\,|\, j^{(i)}(x)\, V_\nu^{(j)}(0)\,|\,\vec{p}\,\rangle$$

$$= -\frac{1}{2m^2}\,\epsilon_{\nu\lambda\kappa\mu}\, p^\lambda k^\kappa q^\mu C\,, \tag{2.3}$$

where $j^{(i)}(x)$ is the pion source function and C is an invariant function of ν and t $(q^2 = m_\pi^2$ and $k^2 = 0)$. Let us choose $q^2 = k^2$ in (2.1) and (2.3), ignoring the pion mass m_π^2 in (2.3) as we usually do in using the PCAC. The PCAC relation leads us to

$$A(\nu,t;\ q^2{=}k^2{=}0) = \left(\frac{f_\pi}{\pi}\right) C(\nu,t;\ q^2{=}k^2{=}0)\,, \tag{2.4}$$

where f_π is the pion decay constant $(\sqrt{2}\, f_\pi = 0.97\, m_\pi)$.

We define likewise the photon emission by the incident pion and the corresponding vector-axial vector interference term,

$$\frac{1}{2\pi}\left(\frac{E\,E'}{m^2}\right)^{1/2}\int d^4x\, e^{iqx}\,\langle\, \vec{p}'\,|\, V_\mu^{(i)}(x)\, A_\nu^{(j)}(0)\,|\,\vec{p}\,\rangle$$

$$= -\frac{i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, p^\lambda q^\kappa \bar{A} - \frac{i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, k^\lambda q^\kappa \bar{B}\,, \tag{2.5}$$

and

$$\frac{1}{2}\left(\frac{E\,E'}{m^2}\right)^{1/2}\int d^4x\, e^{iqx}\,\langle\, \vec{p}'\,|\, V_\mu^{(i)}(x)\, j^{(j)}(0)\,|\,\vec{p}\,\rangle$$

$$= -\frac{1}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, p^\lambda k^\kappa q^\mu\, \bar{C}\,. \tag{2.6}$$

The PCAC relates them as

$$\bar{A}(\nu,t;\ q^2{=}k^2{=}0) = \left(\frac{f_\pi}{\pi}\right) \bar{C}(\nu,t;\ q^2{=}k^2{=}0)\,. \tag{2.7}$$

Comparing (2.4) and (2.7) with the usual definition of the W_3 structure function in the neutrino reactions

$$\frac{1}{2\pi}\left(\frac{E}{m}\right)\int d^4x\, e^{iqx} \langle \vec{p} \mid \left[V_\mu^{(i)}(x)A_\nu^{(j)}(0) + A_\mu^{(i)}(x)V_\nu^{(j)}(0)\right] \mid \vec{p} \rangle$$

$$= -\frac{i}{2m^2}\,\epsilon_{\mu\nu\lambda\kappa}\, p^\lambda q^\kappa W_3(\nu, Q^2) \tag{2.8}$$

where $Q^2 = -q^2 = -k^2$, we find

$$W_3(\nu, 0) = \left(\frac{f_\pi}{\pi}\right) [C(\nu,0,0) + \bar{C}(\nu,0,0)] , \tag{2.9}$$

where the second and third arguments in C and $\bar{C}$ refer to t and Q^2, respectively.

According to the parton model and light-cone singularities in the free quark model the function $\nu W_3(\nu, Q^2)$ scales in the deep inelastic limit of $Q^2 \to \infty$ with $\omega = 2m\nu/Q^2$ fixed. We will assume in the following that this scaling rule should hold valid,

$$\lim_{\substack{Q^2 \to \infty \\ 2m\nu/Q^2 \text{fixed}}} \nu W_3(\nu, Q^2) = F_3(\omega) . \tag{2.10}$$

3. GENERALIZED SCALING SUM RULES

It has been shown in many different approaches that $F_3(\omega)$ may be expanded in the Regge asymptotic region at $\omega \to \infty$ as ordinary hadronic amplitudes are. The generalized scaling was originally stated as follows: $\nu W_3(\nu, Q^2)$ may be extrapolated on the average with the variable $\omega' = (2m\nu + M^2)/(Q^2 + a^2)$ from the deep inelastic region to the finite Q^2 region. It is interpreted here that the

Regge asymptotic terms in ω of $F_3(\omega)$ are given by those in ν with Q^2 fixed of $\nu W_3(\nu, Q^2)$ with replacement of ω by $(2m\nu + M^2)/(Q^2 + a^2)$. At $\omega \to \infty$, $F_3(\omega)$ is expanded as

$$F_3(\omega) \underset{\omega \to \infty}{\sim} \sum_i \beta_i\, \omega^{\alpha_i(0)} , \tag{3.1}$$

while $W_3(\nu, Q^2)$ may be expanded with Q^2 fixed as

$$W_3(\nu, Q^2) \sim \sum_i \gamma_i (Q^2)\, \nu^{\alpha_i(0)-1} . \tag{3.2}$$

The generalized scaling rule relates β_i to $\gamma_i(Q^2)$ through

$$\gamma_i(Q^2) = (Q^2 + a^2)^{-\alpha_i(0)} \beta_i . \tag{3.3}$$

The parameter a^2 in the right-hand side is, in general, dependent on the trajectory i. The generalized scaling sum rules are derived with (3.3) for the $\nu W_3(\nu, Q^2)$ functions of even and odd crossing symmetry separately, when one takes appropriate differences between $\nu W_3(\nu, Q^2)$ and $F_3(\omega)$ so as to cancel their higher asymptotic powers in ν and ω.

Let us construct $\nu W_3^{(\pm)}(\nu, Q^2)$ of even and odd symmetry in ν. We will be interested throughout this paper only in the strangeness conserving currents since the strangeness changing part is small enough. Define $W_3^{\bar{\nu}p}$ and $W_3^{\nu p}$ through

$$\frac{1}{2\pi}\left(\frac{E}{m}\right)\int d^4x\; e^{iqx} \langle \vec{p} \mid \left[V_\mu^{(1)+i(2)}(x)\, A_V^{(1)-i(2)}(0) + A_\mu^{(1)+i(2)}(x)\, V_\nu^{(1)-i(2)}(0) \right] \mid \vec{p} \rangle$$

$$= -\frac{i}{2m^2}\; \epsilon_{\mu\nu\lambda\kappa}\, p^\lambda q^\kappa\, W_3^{\bar{\nu}p,\nu p}(\nu, Q^2)\,, \qquad (3.4)$$

where (1) $\pm$ i(2) refer to the strangeness conserving weak currents of $\Delta Q = \pm 1$ associated with the $\bar{\nu}p$ and νp reactions. The linear combinations

$$W_3^{(\pm)}(\nu, Q^2) = W_3^{\bar{\nu}p}(\nu, Q^2) \pm W_3^{\nu p}(\nu, Q^2) \qquad (3.5)$$

satisfy the crossing relations

$$W_3^{(\pm)}(-\nu, Q^2) = \mp W_3^{(\pm)}(\nu, Q^2)\,. \qquad (3.6)$$

The generalized scaling rule as expressed in (3.3) then leads us to a set of sum rules

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} d\omega\, F_3^{(-)}(\omega)/\omega - \int_{\nu_0(Q^2)}^{\nu_{max}} d\nu\, W_3^{(-)}(\nu,Q^2) \right] = 0 \qquad (3.7)$$

and

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} d\omega\, F_3^{(+)}(\omega) + \frac{2m}{Q^2+a^2}\int_{\nu_0(Q^2)}^{\nu_{max}} d\nu\, \nu W_3^{(+)}(\nu,Q^2) \right] = 0, \qquad (3.8)$$

where

$$F_3^{(\pm)}(\omega) = \lim_{\substack{Q^2\to\infty \\ 2m\nu/Q^2 \text{ fixed}}} \nu\, W_3^{(\pm)}(\nu, Q^2)\,, \qquad (3.9)$$

the upper limits of the integrals ω_{max} and ν_{max} are kept related through

$$\omega_{max} = (2m\nu_{max} + M^2)/(Q^2 + a^2)\,, \qquad (3.10)$$

and the lower limit $\nu_0(Q^2)$ should be chosen to include the nucleon pole contribution in the integral over ν. The parameters a^2 and M^2 are to be determined from fits to experimental data in the Regge asymptotic region. With a^2 chosen appropriately, the leading powers are canceled in (3.7) and (3.8) because of (3.3). The value of M^2 is then to be chosen in such a way that the nonleading powers and the Khuri satellite terms of the leading powers left out may be maximally canceled between the first and second integrals in (3.7) and (3.8). Since we have no experimental information whatsoever of $F_3(\omega)$ at $\omega \to \infty$, we will later make some theoretical speculation on the values for a^2 and M^2. The investigation of νW_2 from the deep inelastic region to the shallow inelastic region has suggested that a^2 should be somewhere around 0.4 GeV^2 [2] while M^2 should be of the order of 1 GeV^2 [1], [2]. The fit to the asymptotic behavior of $\nu W_2^{\gamma N}$ with the smoothed πN total cross sections also supports the value of a^2 between 0.2 and 0.4 GeV^2 [3]. The fit to $F_2^{\nu N}(\omega)$ in the entire range of ω based on the generalized scaling sum rules has given the optimum value of a^2 as [5]

$$a^2 = 0.3 \text{ GeV}^2. \qquad (3.11)$$

In writing the sum rules (3.7) and (3.8) we have implicitly assumed that there should be no fixed pole of right signature in the complex J plane, or if any its residue should be independent of Q^2. The latter is the case for $W_2^{(-)}(\nu, Q^2)$, as is known in the Adler-Dashen-Gell-Mann-Fubini sum rule. It is expected to be true generally for the fixed poles associated with equal-time commutators of local operators. If the parameters a^2 and M^2 are common to all the higher Regge powers, the sum rules of higher moment would be written since the substitution law (3.10) would cancel nonleading powers simultaneously. If there is no wrong signature fixed pole whose residue is dependent on Q^2, the sum rules of wrong moment would hold,

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} d\omega\, F_3^{(-)}(\omega) - \frac{2m}{Q^2+a^2} \int_{\nu_0(Q^2)}^{\nu_{max}} d\nu\, \nu W_3^{(-)}(\nu, Q^2) \right] = 0\,, \tag{3.12}$$

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} d\omega\, F_3^{(+)}(\omega)/\omega - \int_{\nu_0(Q^2)}^{\nu_{max}} d\nu\, W_3^{(+)}(\nu, Q^2) \right] = 0\,. \tag{3.13}$$

These are less likely to hold valid than (3.7) and (3.8) are, since there is no theoretical nor experimental evidence for the wrong signature fixed poles being Q^2 independent.

4. DETERMINATION OF $F_3^{\bar{\nu}p,\nu p}$

We will use the sum rules (3.7) and (3.8) of right moment to determine $F_3(\omega)$. We start with postulating the functional form for $F_3(\omega)$. The A_2 and ω trajectories dominate in $F_3^{(\pm)}(\omega)$ in the large ω region. Their intercepts are $\sim \frac{1}{2}$ at $t = 0$, so we require

$$F_3^{(\pm)}(\omega) \underset{\omega\to\infty}{\sim} \beta_{A_2,\omega}\, \omega^{1/2}\,. \tag{4.1}$$

Near the threshold $\omega = 1$, $F_3^{(\pm)}(\omega)$ is expected to approach zero. It may be argued in the same way as for $F_2^{(\pm)}(\omega)$ [6], [7] that $F_3^{(\pm)}(\omega)$ should vanish like $(\omega - 1)^3$ as $\omega \to 1$ if both of the vector and axial vector form factors fall off like $(Q^2)^{-2}$ as $Q^2 \to \infty$. We therefore parametrize $F_3^{(\pm)}(\omega)$ as

$$F_3^{(\pm)}(\omega) = A^{(\pm)}\omega^{1/2}\left(1 - \frac{1}{\omega}\right)^3 \tag{4.2}$$

for $\omega \gtrsim 1$. $A^{(\pm)}$ are going to be determined through the sum rules of the lowest right moment.

To obtain the absorptive part of the single-pion photoproduction amplitude, we use the results of multipole analysis in the resonance region. The accurate and model independent analysis has been carried out by Pfeil and Schwela [8] in the region of the Δ resonance (1236 MeV). The analysis has been extended by Moorhouse and collaborators [9], [10], [11] using dispersion relations and by Walker [12] in a simple dynamical model up to just above the third resonance (1688 MeV). We use the multipole analysis by these people. The integrals in the right-hand side of (3.7) and (3.8) are dominated by the Δ resonance as well as by the nucleon pole contribution. The relative importance of the second and third resonances are far smaller in the present case than in the pion-nucleon scattering.

We have first integrated the right-hand side of (3.7) and (3.8) over ν up to 500 MeV using the multipoles given in [8] - [12]. Assuming that the sum rules are saturated enough at ν_{max} = 500 MeV, we have

$$\int_1^{\omega_{max}} d\omega\, F_3^{(-)}(\omega)/\omega \simeq \int_{\nu_0}^{500\ \mathrm{MeV}} d\nu\, W_3^{(-)}(\nu,0)\ ; \qquad (4.3)$$

$$\int_1^{\omega_{max}} d\omega\, F_3^{(+)}(\omega) \simeq \frac{2m}{a^2} \int_{\nu_0}^{500\ \mathrm{MeV}} d\nu\, \nu\, W_3^{(-)}(\nu,0)\ , \qquad (4.4)$$

where $\omega_{max} = (2m \times 500\ \mathrm{GeV} + M^2)/a^2$. $W_3^{(\pm)}(\nu,0)$ are related through (2.9) to the photoproduction amplitudes in the notations of Chew, Goldberger, Low, and Nambu [13] as

$$W_3^{(\pm)}(\nu,0) = \left(\frac{4f_\pi}{e}\right)\Bigg[- \frac{eg_{\pi N}}{2m}(\mu_p - \mu_n)\delta(\nu) + \frac{4}{[(E_2+m)(E_1+m)]^{\frac{1}{2}}}\Bigg\{\sum_{\ell=0}^{\infty}(\ell+1)(\ell+2)(\ell M_{\ell+}^{(\pm)} + E_{\ell+}^{(\pm)}) + \sum_{\ell=2}^{\infty}(\ell-1)\ell\Big((\ell+1)M_{\ell-}^{(\pm)} + E_{\ell-}^{(\pm)}\Big)\Bigg\} - \left(\frac{W+m}{W-m}\right)\left(\frac{E_2+m}{E_2-m}\right)^{\frac{1}{2}}\sum_{\ell=1}^{\infty}\ell(\ell+1)\Big((\ell+1)M_{\ell+}^{(\pm)} + \ell M_{\ell-}^{(\pm)}\Big)\Bigg]\ , \qquad (4.5)$$

$$\begin{Bmatrix} E_{\ell\pm}^{(+)} \\ M_{\ell\pm}^{(+)} \end{Bmatrix} = \frac{2}{3}\begin{Bmatrix} E_{\ell\pm}^{(3/2)} \\ M_{\ell\pm}^{(3/2)} \end{Bmatrix} + \frac{1}{3}\begin{Bmatrix} E_{\ell\pm}^{(\frac{1}{2})} \\ M_{\ell\pm}^{(\frac{1}{2})} \end{Bmatrix}\ , \qquad (4.6)$$

$$\begin{Bmatrix} E_{\ell\pm}^{(-)} \\ M_{\ell\pm}^{(-)} \end{Bmatrix} = -\frac{1}{3}\begin{Bmatrix} E_{\ell\pm}^{(3/2)} \\ M_{\ell\pm}^{(3/2)} \end{Bmatrix} + \frac{1}{3}\begin{Bmatrix} E_{\ell\pm}^{(\frac{1}{2})} \\ M_{\ell\pm}^{(3/2)} \end{Bmatrix}\ , \qquad (4.7)$$

where μ_p and μ_n are the total magnetic moment of the proton and the neutron, respectively ($\mu_p - \mu_n = 4.71$), $g_{\pi N}$ is the pseudoscalar coupling constant ($g_{\pi N}^2/4\pi = 14.5$), W is the center-of-mass energy, $E_{\ell\pm}$ and $M_{\ell\pm}$ are the electric and magnetic transitions to the πN system in the state of orbital angular momentum ℓ and total angular momentum $J = \ell \pm \frac{1}{2}$, and the superscripts (3/2) and (1/2) stand for the total isospins. Note the difference in the normalization of (4.6) and (4.7) from (3.5). The multipoles of Pfeil and Schwela [8] have been substituted in the Δ resonance region by supplementing them with the results by Moorhouse and collaborators [9], [10] above 450 MeV. Carrying out the numerical integrals we have found

$$\text{r.h.s. of } (4.3) = -3.19\ , \qquad (4.8)$$

$$\text{r.h.s. of } (4.4) = -5.19\ \mathrm{GeV} \times (2m/a^2)\ . \qquad (4.9)$$

The overall sign of these are not determined from the single-pion photoproduction experiment alone although the relative sign is fixed between (4.8) and (4.9). However we can determine the overall sign common to (4.8) and (4.9) with recourse to a bit of theory, the PCAC hypothesis. The Goldberger-Treiman relation

$$\left.\begin{aligned} f_\pi &= \frac{2m\, g_A}{g_{\pi N}\, K(0)} , \\ K(0) &> 0 \quad \text{and} \quad g_A = +1.25 \end{aligned}\right\} \qquad (4.10)$$

determines the sign of the product $f_\pi\, g_{\pi N}$. This fixes the sign of the nucleon pole term in (4.5). The relative sign of $\mathrm{Im}\, M_{1+}^{(3/2)}$ due to the Δ resonance to that of $eg_{\pi N}$ was found in the original paper of CGLN [13] to be positive, and the subsequent multipole analyses [8] - [12] have been done with the convention of $eg_{\pi N} > 0$. Therefore the sign of the continuum contribution relative to that of the nucleon pole is known. In fact, we should choose f_π and $g_{\pi N}$ to be both positive if we substitute the multipoles in the prevailing convention. With all of these observations the right-hand sides of (4.8) and (4.9) turn out to be negative. We will later see that this overall negative sign is crucially important in reproducing $\sigma^{\bar{\nu}N}/\sigma^{\nu N} < 1$ instead of the opposite. In (4.8) there are two large contributions, one from the nucleon pole and the other from $\mathrm{Im}\, M_{1+}^{(3/2)}$ due to Δ(1236 MeV). However, the nucleon term and the Δ term largely cancel each other leaving about 30% of the nucleon term. On the other hand (4.9) comes almost entirely from Δ since the extra factor ν in the integrand kills the nucleon term. We therefore expect that (4.9) for $F_3^{(+)}$ is more stable against details of the multipoles than (4.8).

We now substitute the function $F_3(\omega)$ as parametrized in (4.2) into the left-hand sides of (4.3) and (4.4) to determine the coefficient $A^{(\pm)}$. The parameter a^2 and M^2 were determined previously in the fits to $F_2(\omega)$. They are most generally dependent

on the quantum numbers of the currents as well as the Regge poles exchanged between the nucleon and the current. For instance, the values for a^2 and M^2 determined in [2] apply to the A_2 trajector which couples with the two vector currents, and those in [3] are for the ρ and f trajectories which couple with the two axial vector currents. But, it has so far been turned out that a^2 and M^2 take the common values within possible errors for all the Regge poles of $\alpha_i(0) \simeq 1/2$ whether the external lines are the vector currents or the axial vector currents. We therefore assume here as we did in [5] that the values for a^2 and M^2 be dependent only on the intercepts of Regge trajectories. We take over the optimum values as determined in the previous works

$$a^2 = 0.3\ \mathrm{GeV}^2 \quad \text{and} \quad M^2 = 1.0\ \mathrm{GeV}^2, \qquad (4.11)$$

and substitute them into the sum rules.

For $\nu_{max} = 500$ MeV we find

$$A^{(-)} = -3.1 \quad \text{and} \quad A^{(+)} = -7.9\,. \qquad (4.12)$$

As we remarked before, $A^{(+)}$ is very stable against errors in the multipoles while $|A^{(-)}|$ may decrease as the Δ contribution increases. But the numbers given in (4.12) are fairly reliable once ν_{max} is fixed at 500 MeV, just above the Δ resonance tail. A large source of ambiguity probably exists in the choice of ν_{max} and the multipoles at higher energies. If we extend the integrals on the right-hand sides of (3.12) and (3.13) to 1200 MeV by picking up only the resonance contributions in the multipole analysis by Walker and the recent result by Moorhouse and Oberlack [11], $A^{(+)}$ may decrease as much as by a factor of $2 \sim 3$ while $A^{(-)}$ may decrease less significantly. This should probably not be trusted since above the second resonance region the single-pion photoproduction involves rather high background contributions. We would have to take into account the nonresonant multipoles there. In addition, the multipoles of the resonant amplitudes contain large errors above the second resonances, and the values are not considered as final yet. We would rather quote the results in (4.12) derived from the firmly established lower energy multipole data.

By taking linear combinations of $F_3^{(\pm)}(\omega)$, we obtain $F_3(\omega)$ for the $\bar{\nu}p$ and νp reactions as

$$F_3^{\bar{\nu}p}(\omega) = -5.5\,\omega^{\frac{1}{2}}\left(1 - \frac{1}{\omega}\right)^3 , \qquad (4.13)$$

$$F_3^{\nu p}(\omega) = -2.4\,\omega^{\frac{1}{2}}\left(1 - \frac{1}{\omega}\right)^3 , \qquad (4.14)$$

where of course $F_3^{\nu n}(\omega) = F_3^{\bar{\nu}p}(\omega)$ and $F_3^{\bar{\nu}n}(\omega) = F_3^{\nu p}(\omega)$. Experiment measures directly through the ratio of the total cross sections $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ the integral

$$\int_0^1 dx\, x(F_3^{\bar{\nu}p}(x) + F_3^{\nu p}(x)) , \qquad (4.15)$$

where $x = 1/\omega$. According to the analysis of the CERN data [14],

$$\sigma^{\bar{\nu}N}/\sigma^{\nu N} = 0.38 \pm 0.02 , \qquad (4.16)$$

leading to

$$B \equiv -\left[\int_0^1 dx\, x\, F_3^{\nu N}(x)\right] \bigg/ \left[\int_0^1 dx\, F_2^{\nu N}(x)\right]$$
$$= 0.90 \pm 0.04 \qquad (4.17)$$

through the formula $\sigma^{\bar{\nu}N}/\sigma^{\nu N} = (2 - B)/(2 + B)$. The superscripts νN and $\bar{\nu}N$ stand for the average over the proton and the neutron,

$$F_{2,3}^{\nu N,\bar{\nu}N} \equiv \frac{1}{2}\,(F_{2,3}^{\nu p,\bar{\nu}p} + F_{2,3}^{\nu n,\bar{\nu}n}) . \qquad (4.18)$$

From our results in (4.12) we find

$$\int_0^1 dx\, x\, F_3^{\nu N}(x) = -0.40 . \qquad (4.19)$$

If we devide it with the experimental observed value for $\int_0^1 dx\, F_2^{\nu N}(x)$ [14],

$$\int_0^1 dx\, F_2^{\nu N}(x) = 0.49 \pm 0.07 , \qquad (4.20)$$

we obtain $B = 0.82$. If we choose $F_2^{\nu N}(x)$ determined from the πN total cross sections through the same extrapolation technique [5] as done in this paper

$$\int_0^1 dx\, F_2^{\nu N}(x) = 0.51 , \qquad (4.21)$$

we find

$$B = 0.79 . \qquad (4.22)$$

In either way the agreement with the experimental value $B = 0.90 \pm 0.04$ is very satisfactory. The number quoted in (4.22) may be affected by the values for a^2 and M^2 as well as by errors in the multipoles and the cutoff ν_{max}. The value for a^2 may be off by 10% from 0.3 GeV^2 without causing serious discrepancy elsewhere. The value for M^2 may have even a larger ambiguity. The good agreement with experiment in $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ may be regarded as justifying a posteriori our choice of the values for a^2 and M^2.

5. COMPARISON WITH THE PREDICTIONS IN THE QUARK MODEL

Theorists have speculated that the fractionally charged quarks may be the fundamental constituents of hadrons and therefore of the hadronic weak currents. A few sum rules have been derived for the inelastic structure functions in the quark model. They are [15]

$$\int_0^1 dx \left(F_3^{\bar{\nu}p}(x) + F_3^{\nu p}(x)\right) = -6 , \tag{5.1}$$

and

$$\int_0^1 dx\, x \left(F_3^{\bar{\nu}p}(x) - F_3^{\nu p}(x)\right) = -6 \int_0^1 dx\left(F_2^{\gamma p}(x) - F_2^{\gamma n}(x)\right)$$

$$\simeq -0.30 \pm 0.12 . \tag{5.2}$$

The factor 6 in the right-hand sides of (5.1) and (5.2) is characteristic of the fractionally charged quarks. We evaluate the integrals in the left-hand sides with our function as obtained in (4.13) and (4.14). They turn out to be

$$\int_0^1 dx\left(F_3^{\bar{\nu}p}(x) + F_3^{\nu p}(x)\right) = -2.4 , \tag{5.3}$$

$$\int_0^1 dx\, x\left(F_3^{\bar{\nu}p}(x) - F_3^{\nu p}(x)\right) = -0.32 . \tag{5.4}$$

Notice that the signs are in agreement with those in (5.1) and (5.2). This is highly nontrivial. As we emphasized before, $F_3^{\bar{\nu}p} - F_3^{\nu p}$ may be subject to some errors since it comes from the small difference between the two large contributions. But, the agreement of (5.4) with the prediction of the fractionally charged quark model is still

significant. On the other hand the sum $F_3^{\bar{\nu}p} + F_3^{\nu p}$ is more stable against errors in the multipoles. It is highly unlikely that it changes by more than 10% once ν_{max} is fixed at 500 MeV. We would draw the conclusion in the present analysis based on the generalized scaling sum rules that the zeroth-moment integral $\int_0^1 dx\, (F_3^{\bar{\nu}p} + F_3^{\nu p})$ is much smaller, approximately by a factor of two, than the value predicted in the fractionally charged quark model. But it may be worthwhile to mention that the integrals of the zeroth moment and of the first moment are tightly correlated to each other once we parametrize $F_3^{(\pm)}(\omega)$ with a single parameter $A^{(\pm)}$ as in (4.2). It is much desired to determine $F_3(\omega)$ through the sum rules by parametrizing it with more parameters. More accurate multipoles are needed for this purpose in the energy region above the second resonance.

6. SUMMARY AND CONCLUSIONS

We have related through the generalized scaling rule the absorptive part at $t = 0$ of the single-pion photoproduction amplitude to the scaling function $F_3(\omega)$ in the highly inelastic neutrino reactions. The results are given in (4.13) and (4.14). Combining it with the other scaling function $F_2(\omega)$ which was calculated previously in the same technique of extrapolation from the πN total cross sections, we can find the complete x (or ω) dependence of $d\sigma^{\nu,\bar{\nu}}/dx\, dy$, where $y = \nu/E$. The double differential cross sections are given as

$$d\sigma^{\nu p, \bar{\nu}p}/dx\, dy = \frac{G^2 mE}{\pi}\left[\left(1 - y + \frac{y^2}{2}\right)F_2^{\nu p,\bar{\nu}p}(x) \mp \left(y - \frac{y^2}{2}\right)x\, F_3^{\nu p,\bar{\nu}p}(x)\right] , \tag{6.1}$$

where E is the incident neutrino energy, G is the Fermi constant, and the Callan-Gross relation $2x\, F_1(x) = F_2(x)$ has been used. The minus sign should be chosen for the νp reaction and the plus sign should be chosen for the $\bar{\nu} p$ reaction in front of the second term inside of the square bracket in (6.1). The same formulae hold with the proton p replaced with the neutron n . The functions $F_2^{\nu,\bar{\nu}}$ were calculated in [5] as

$$F_2^{\bar{\nu}p}(x) = F_2^{\nu n}(x) = 0.80(1 - x^2)^3 + 1.79\, x^{\frac{1}{2}}(1 - x)^3 + 6.7\, x^{3/2}(1 - x)^3, \tag{6.2}$$

$$F_2^{\nu p}(x) = F_2^{\bar{\nu}n}(x) = 0.80(1 - x^2)^3 + 1.08\, x^{\frac{1}{2}}(1 - x)^3 - 6.7\, x^{3/2}(1 - x)^3. \tag{6.3}$$

Combining these with the present results for $F_3(x)$

$$F_3^{\bar{\nu}p}(x) = -5.5\, x^{-\frac{1}{2}}(1 - x)^3 , \tag{6.4}$$

$$F_3^{\nu p}(x) = -2.4\, x^{-\frac{1}{2}}(1 - x)^3 , \tag{6.5}$$

we can evaluate the double differential cross sections in the entire region of x and y. The cross sections $d\sigma^{\nu N}/dx\, dy$ and $d\sigma^{\bar{\nu}N}/dx\, dy$ are plotted in Fig. 1 for the nucleus target of $N = Z$ at several different values of y . The $\bar{\nu}N$ cross sections show a strong y dependence, while the νN cross sections remain practically unchanged as y varies except very near $x = 0$.

It is important to make sure whether or not our results violate the positivity conditions based on unitarity, not on a specific quark model. The relevant one to the present results is [17], [18]

$$|F_3(x)| \leqq 2\, F_2(x) , \tag{6.6}$$

which hold for νp and $\bar{\nu} p$. The positivity condition is well satisfied for the $\bar{\nu} p$ (and therefore νn) reaction. It is almost satisfied for the νp ($\bar{\nu} n$) reaction except above $x \simeq 0.9$ (or equivalently at $1 \leqq \omega \lesssim 1.1$), where a tiny violation shows up

$$\left[2\, F_2^{\bar{\nu}p}(x) - |\, F_3^{\bar{\nu}p}(x)\, |\right] \Big/ \left[2\, F_2^{\bar{\nu}p}(x) + |\, F_3^{\bar{\nu}p}(x)\, |\right] \lesssim 0.1 . \tag{6.7}$$

The violation is small and certainly within the errors in the multipole analysis and the dependence on the cutoff ν_{max}. Moreover F_2 and F_3 themselves are less than 3×10^{-3} in the region of $1 \leqq \omega \lesssim 1.1$. We have not imposed a positivity condition in any form throughout our investigation. We do not consider that this vanishing tiny violation of the positivity condition is of any serious flaw in the present approach. We therefore conclude that the extrapolation procedure combined with the PCAC hypothesis has successfully given the scaling functions $F_3^{\nu,\bar{\nu}}(\omega)$ as well as $F_2(\omega)$ in the highly inelastic neutrino reactions.

7. ACKNOWLEDGMENT

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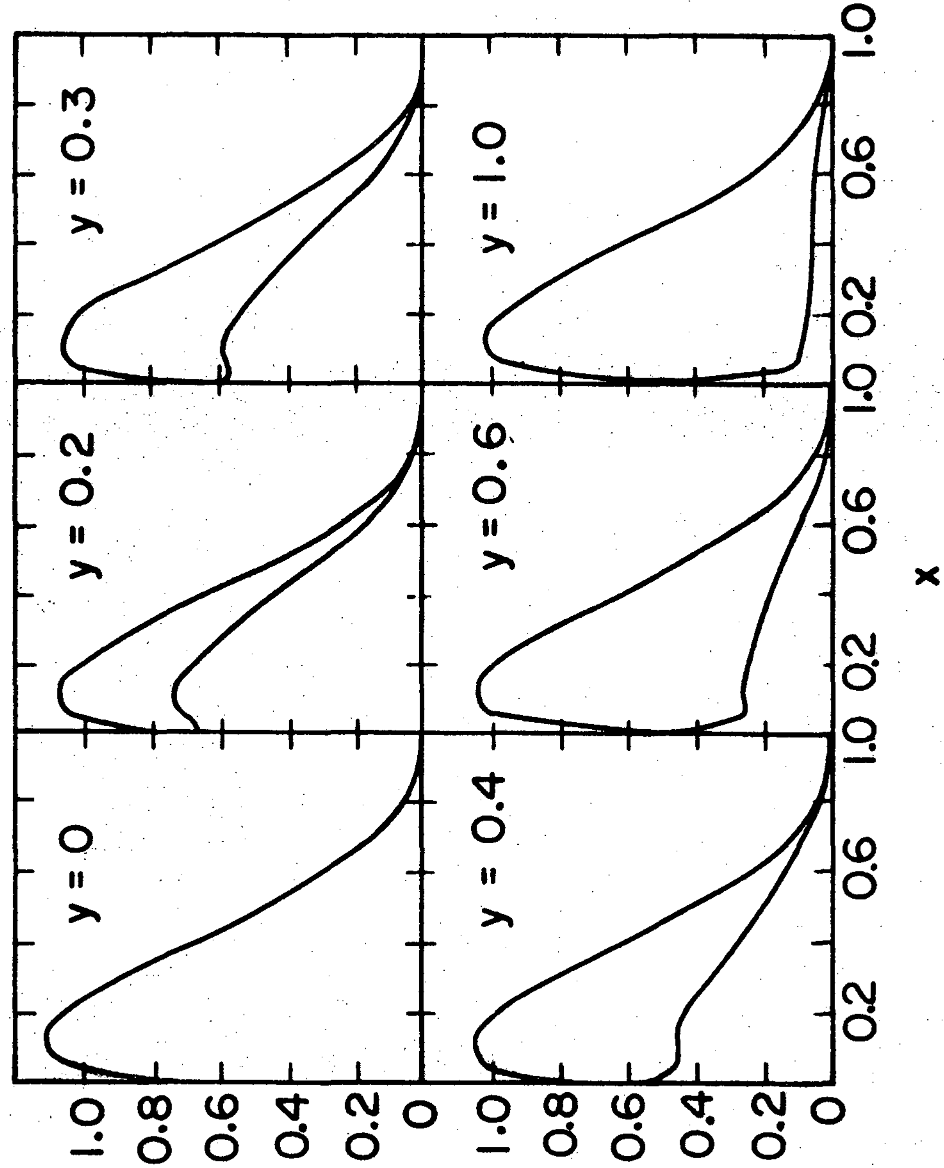
FIGURE CAPTION

Fig. 1: The double differential cross sections $d\sigma^{\nu N, \bar{\nu} N}/dx\,dy$ in the unit of GmE/π ($= 1.56\ E(\text{in GeV}) \times 10^{-38}\ cm^2$). The upper curve stands for the νN reaction and the lower one for the $\bar{\nu} N$ reaction. They coincide at $y = 0$.



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