

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

Meaning making in a college mathematics lecture format : the intersection of mathematics, language, and cultural meaning systems

Permalink

<https://escholarship.org/uc/item/44w8d9fh>

Author

Kailikole, Kathryn Leimomi

Publication Date

2009

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA, SAN DIEGO

Meaning Making in a College Mathematics Lecture Format:
The Intersection of Mathematics, Language, and Cultural Meaning Systems

A Dissertation submitted in partial satisfaction of the requirement for the degree

Doctor of Education

in

Teaching and Learning

by

Kathryn Leimomi Kailikole

Committee in charge:

Professor Barbara Sawrey, Chair
Professor Jim Lin
Professor Hugh Mehan
Professor Ana Celia Zentella

2009

Copyright

Kathryn Leimomi Kailikole, 2009

All rights reserved.

The Dissertation of Kathryn Leimomi Kailikole is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2009

TABLE OF CONTENTS

Signature Page	iii
Table of Contents	iv
List of Figures	ix
List of Tables	xi
Vita	xiii
Abstract of the Dissertation.....	xiv
Chapter 1: Developing and Assigning Meaning While Learning and Re-Learning Mathematics	1
1.1 Background for the Study	2
1.1.1 The Developmental Mathematics Conundrum	2
1.1.2 Learning in a College Lecture Format	4
1.1.3 The Complications of Precision in Comprehending Mathematics and Developing Proper Mathematical Meaning.....	7
1.2 The Framework Underlying the Study.....	12
1.2.1 Elements in the Framework for Examining Student Meaning-Making and Learning in Mathematics	12
1.2.2 Language and Discourse in the Mathematics Classroom	14
1.2.3 Cultural Meaning Systems and Discourse in the Mathematics Classroom	15
1.3 Overview of Methodology	18

1.3.1 Research Questions	19
1.4 Conclusion	20
Chapter 2: Linguistic, Sociological and Cultural Perspectives	
On Teaching-Learning Mathematics.....	22
2.1 What Mathematics Do We Teach and Learn?	22
2.2 Teaching-Learning Mathematics at Individual, Interactional And Institutional Levels of Analysis	24
2.3 The Role of Language and Communication in the Teaching-Learning of Mathematics	26
2.3.1 How We Perceive and Talk about Mathematics	26
2.3.2 Types of Language and Communication in Teaching-Learning of Mathematics	27
2.4 Theoretical Foundations for the Role of Language and Communication in Teaching-Learning Mathematics.....	36
2.4.1 Constructivism and Functional Linguistics	31
2.4.2 Social Constructivism, Socioculturalism and Interactionism	32
2.5 The Role of Communication and Pedagogy in the College Lecture Format	36
2.6 Conclusion	38
Chapter 3: Methodology.....	42
3.1 Research Design and Methodological Approach	42
3.2 Setting and Sampling.....	43

3.2.1 Setting	43
3.2.2 Sampling Procedures.....	45
3.2.3 Sampling Implications.....	46
3.3 Measures	47
3.3.1 Mathematics Register Terminology Survey.....	47
3.3.2 Student-Annotated Notes.....	49
3.3.3 Student and Instructor Interviews	49
3.3.4 Videotape Lecture Observations	50
3.4 Data Collection Procedures	51
3.5 Data Reduction	55
3.5.1 Coding the Survey, Annotated Notes, and Student Interviews	55
3.5.2 Decisions about Using Data	61
3.6 Pilot Testing	61
3.7 Data Analysis Procedure.....	62
3.8 Limitations of the Study.....	63
Chapter 4: Meaning Making and the Mathematics Register.....	65
4.1 Introduction.....	65
4.2 Knowing Meaning and Situational Uses.....	66
4.2.1 Multiple Meanings	66
4.2.2 Skewed Meaning and Partial Knowledge.....	74
4.2.3 Knowing Meaning: Simplicity and Repetition	86
4.3 Shared Meanings in Symbols and Visual Representation.....	88

4.4 Assigning Meaning to the Mathematics Register: Objectivism and Cultural Meaning Systems	99
4.5 Conclusion	104
Chapter 5: Developing versus Replicating Meaning-Making.....	108
5.1 Introduction.....	108
5.2 Lecture A and Collected Annotated Notes: Diana, Sally and Melinda.....	109
5.2.1 Perceptions about the Lecture.....	109
5.2.2 Student Interaction with the Mathematics.....	113
5.2.3 The Mathematics Register: Developing versus Replicating Meaning-Making	119
5.3 Lecture B and Collected Annotated Notes: Alicia, Linda and Frank.....	124
5.3.1 Perceptions about the Lecture.....	124
5.3.2 Student Interaction with the Mathematics.....	129
5.3.3 The Mathematics Register: Developing versus Replicating Meaning-Making	133
5.4 Conclusion.....	136
Chapter 6: Discussion and Implications.....	138
6.1 Faulty Mathematical Meaning-Making	138
6.2 Conditions Contributing to Faulty Mathematical Meaning-Making	140
6.2.1 Objectivism in the Teaching-Learning Environment...	140

6.2.2 Facility with Language.....	143
6.2.3 Algorithmic Thinking	144
6.2.4 The College Lecture Format: Pace, Capacity, and Assessment.....	147
6.3 Compensating for Limited Teacher-Student Interaction.....	149
6.4 Implications for Teaching and Learning.....	151
6.5 Future Research	152
Appendix A: Selected Annotated Notes	154
Appendix B: Post-Course Surveys for Student Note-Takers.....	179
References.....	185

LIST OF FIGURES

Figure 1.1: The Angle of Depression.....	9
Figure 4.1: Student’s Understanding of Reciprocal and the Inverse Tangent Function	73
Figure 4.2: Annotated Notes from Lecture A, examples of communicating the notion of identity	81
Figure 4.3: Annotated Notes from Lecture B, examples of communicating the notion of identity	82
Figure 4.4: Example of foil from the Mathematics Register Survey	83
Figure 4.5: Visual Representations of Tangent.....	89
Figure 4.6: Visual Representations of One-to-One.....	92
Figure 4.7: Visual Representations of Negative.....	95
Figure 5.1: Melinda’s Notes: Missing an example from lecture, bottom of page 2 and top of page 3.....	110
Figure 5.2: Diana’s Notes: same material in Melinda’s notes in Figure 5.1	111
Figure 5.3: Diana’s Annotations on Her Notes	114
Figure 5.4: A Page from Diana’s Notes: Distance-Rate-Time Word Problem	115
Figure 5.5: A Page from Melinda’s Notes: Distance-Rate-Time Word Problem	116
Figure 5.6: A Page from Sally’s Notes: Distance-Rate-Time Word	

Problem	117
Figure 5.7: Melinda’s Annotated Notes, referred to in Table 5.4	121
Figure 5.8: Frank’s Notes: annotation about the instructor.....	125
Figure 5.9: Frank’s Notes: annotation that he “got lost”.....	126
Figure 5.10: Linda’s Notes: annotation about the “undo” button.....	128
Figure 5.11: Alicia’s Notes: annotation about the “undo” button.....	129
Figure 5.12: Alicia’s Notes: an example of her using previous knowledge to annotate.....	130
Figure 5.13: Frank’s Notes: annotation about a helpful example.....	131
Figure 5.14: Notes Matching the Interview Excerpt in Table 5.7.....	133
Figure 5.15: Notes Matching the Interview Excerpt in Table 5.8.....	135

LIST OF TABLES

Table 3.1: Summary of Collected Data.....	43
Table 3.2: Words and Phrases Used in the Mathematics Register Terminology Survey	49
Table 3.3: Codebook	57
Table 4.1: Oxford Dictionary On-line: Definitions for <i>root</i>	67
Table 4.2: Student Everyday and Mathematical Meanings for the Word: <i>root</i>	68
Table 4.3: Percentage of Student Survey Responses for the Word: <i>root</i> ...	69
Table 4.4: Oxford Dictionary On-line: Definitions for <i>dependent</i>	75
Table 4.5: Percentage of Student Survey Responses for the Word: <i>identity</i>	77
Table 4.6: Percentage of Student Survey Responses for the Words: <i>distribute</i> and <i>foil</i>	84
Table 4.7 Oxford Dictionary On-line: Definitions for <i>product</i> and <i>even</i>	95
Table 4.8: Survey Responses for <i>tangent</i>	90
Table 4.9 Survey Responses for <i>one to one</i>	93
Table 4.10: Survey Responses for the Mathematical Meaning for <i>negative</i>	95
Table 4.11: Interview Excerpts: Angle of Depression	98
Table 4.12: Interview Excerpts: Half-life versus Half a Life	104
Table 5.1: Interview with Melinda about the pace of the lecture.....	112
Table 5.2: Interview with Sally about the pace of the lecture.....	113

Table 5.3: Interview with Sally about word problems.....	118
Table 5.4: Interview with Melinda about question marks of her notes.....	121
Table 5.5: Interviews with Frank about the instructor.....	126
Table 5.6: Interview with Linda about the instructor.....	126
Table 5.7: Interview with Linda about difficulty learning logarithms	132
Table 5.8: Interview with Alicia about the angle of depression.....	134

VITA

- 2009 Ed.D. University of California at San Diego, Education
- 1990 M.A. University of California at Santa Cruz, Education
- 1985 B.A. University of California at Berkeley, Applied Mathematics
- 2008 – Director, Louis Stokes Institute for Opportunity in STEM Education, Council for Opportunity in Education, Washington DC
- 2006 – 2008 Executive Director, STEM Program Development, Council for Opportunity in Education, Washington DC
- 1998 – 2006 Teaching Visitor, Mathematics, University of California, San Diego
- 2000 – 2006 Assistant Director, Office of Academic Support and Instructional Services, University of California, San Diego
- 1996 – 2006 Coordinator, Math and Science Tutorial Program, University of California, San Diego
- 1992 – 1995 Adjunct Instructor, Mathematics, Santa Monica College, CA
- 1993 – 1995 Instructor, Mathematics, University of California, Riverside
- 1991 – 1995 Coordinator of Math Programs, Learning Center, University of California, Riverside
- 1989 – 1991 Coordinator, Academic Excellence Program, Division of Natural Sciences, University of California, Santa Cruz
- 1988, 1989 Mathematics Instructor and Coordinator of Math Programs, Summer Bridge Program, University of California, Santa Cruz
- 1987 – 1991 Teaching Assistant, Department of Mathematics, University of California, Santa Cruz
- 1986, 1987 Mathematics Instructor, Minority Scholars Program, University of California, Berkeley
- 1985 Pre-school Teacher, Katmandu Pre-School, Richmond, CA

ABSTRACT OF THE DISSERTATION

Meaning-Making in a College Mathematics Lecture Format:
The Intersection of Mathematics, Language, and Cultural Meaning Systems

by

Kathryn Leimomi Kailikole

Doctor of Education in Teaching and Learning

University of California, San Diego, 2008

Professor Barbara Sawrey, Chair

Students and teachers use language to communicate mathematical knowledge and understanding. This communication is compounded by the underlying requirement for students to acquire language with specialized meaning and to have facility with this

meaning as a member of a mathematics discourse community. The lecture format is a long-standing means of communicating knowledge in the university mathematics classroom.

Instructors and students approach the mathematics classroom with cultural meaning systems that contain diverse and divergent assumptions, histories, personalities, and social and cultural norms. The instructor assembles a lecture from a perspective that includes a deep understanding of mathematics that may not match the perspective and mathematical knowledge of the student audience. Given the multiple and complex concepts in mathematics, assigning a precise meaning can be difficult for students, producing a high likelihood that connotation, implication, and relevance can be confused. Even with precise mathematical definitions, students' interpretation of the meaning of a definition being taught is very much dependent on the precision with which the student assigns connotation, implication, and relevance to each symbol, picture, metaphor and word within the framework of the definition.

This study examines the question of how students assign meaning to terms in the mathematics register and how these assigned meanings determine the ways in which students understand mathematical concepts. The students in this study are drawn from a college-level remedial mathematics course. The depth of students' pre-existing knowledge of mathematics determined the complexity and breadth of their current meaning-making. The students' use of language and meaning in an "everyday" context influenced the ways meaning was assigned to terms in the mathematics register and the lack of precision in understanding and assigning mathematical meanings. The subtle variations in meanings and the seemingly reasonable connections students made to

arrive at these meanings suggests instructors need to take cultural meanings that students bring to the classroom into account when teaching the mathematics course. Instructional planning should include ways for instructors to decipher previous student meaning-making about and understanding of mathematical concepts in order to correct students' misunderstandings and to help students develop more precise meanings assigned to terminology and concepts.

Chapter 1: Developing and Assigning Meaning While Learning and Re-Learning Mathematics

As a mathematics instructor, I am concerned about the level and depth of mathematical understanding my college students have and how well they critically problem solve by applying their mathematical understanding, particularly in a remedial or developmental class. Although I could assume that the students are all at the same knowledge level since they are enrolled in the same developmental mathematics class, I know that the students' mathematical skill levels are as diverse as their individual learning experiences in mathematics. Moreover, I know that the diversity of skill levels and experiences leads to students assigning a wide range of meanings to symbols, pictures, and language used in mathematics.

This diversity of skill levels and experiences can be associated with a number of factors – a factor under consideration in this study is belief systems tied to mathematics instruction and learning. That is, belief systems tied to mathematics and how mathematics is taught in general and with specific populations; belief systems about mathematics and education tied to individual instructors and students; and belief systems tied to the role of language in thought processes. Indeed, the demographics of many developmental mathematics classes may be connected to belief systems imposed during mathematics instruction from Kindergarten to twelfth grade and beyond.

The developmental mathematics class under investigation in this study has historically been mostly women (over 60%), minority (approximately 50%), low-

income (approximately 60%) and first-generation (approximately 60%) students.¹ While this demography cannot be ignored, it is not the focus of this study. Instead, the focus is on student meaning-making – those developed, assigned, and maintained meanings they hold – as a result of belief systems at work during the learning and re-learning of mathematics.

In this chapter, I present the background for the study, the elements and framework underlying the study, an overview of the methodology, and conclude with the organization of the related literature and results of the study.

1.1 Background for the Study

1.1.1 The Developmental Mathematics Conundrum

A developmental mathematics lecture is comprised of mathematical knowledge that an entering college freshman should have already mastered – for example Algebra, Trigonometry, Precalculus. The difficulty in teaching, in my case, or taking, in the students' case, a developmental mathematics class is that most students have been or should have been exposed to the concepts that I will cover. This previous exposure presents a conundrum for instructor and students. The students enter the class with established notions or assigned meanings – some right, some wrong, some vague, some confused – about mathematics and the words, symbols and pictures used to explain it. College administrators, instructors and students assume that the students only need a

¹ Percentages calculated using registrar enrollment and demographic records for 2004-2005 at the university in this study.

review of critical concepts and they can then move on to college-level mathematics. This assumption is not illogical. However, the conundrum is that a review of the critical concepts does not get to the core of why the students are in the developmental class in the first place. And, why high numbers of students are failing or withdrawing from developmental mathematics classes nationally (Conley and Bodone, 2002). Organizing a course to review concepts implies that the students' knowledge base is solid, and not filled with misconceptions and misunderstandings; only a minimal amount of effort by both instructor and students is needed for students to advance to the next class.

The catch is that the instructor may have little or no experience with the mathematical history – the depth of the lack of preparation and inadequate skills to develop cogent mathematical meanings – a student brings to the developmental mathematics lecture (American Diploma Project, 2004). Thus, the instructor doesn't see the fallacy a "minimal amount of effort" represents. Most instructors have had vastly different experiences with mathematics than their students – and this is usually positive. They understand that students have some exposure to mathematics knowledge whether that exposure is positive or negative does not necessarily matter. Furthermore, if the students have enrolled in the class, then instructors assume at some level that the students have met the pre-requisite standards for the class. What is unfortunate about this situation is that although student pre-requisite knowledge can be measured by previous coursework or placement test (American Diploma Project, 2004), we don't measure how the students understand and process this pre-requisite knowledge in relation to their skill level. That is, instructors don't necessarily know how students are

developing and assigning meaning relative to their knowledge base and subsequently processing and understanding the mathematics. So both instructor and students may overlook how the established notions or assigned meanings that students bring to a developmental math course cause fundamental problems in learning and re-learning mathematics.

To clarify, these established notions or assigned meanings are shaped by students' learning experiences and attitudes about mathematics – for example, “Math knowledge is procedural and algorithmic,” “There is only one way to do mathematics,” “I’ve seen this all before,” or “My math teachers were confusing.” Students potentially can develop improved or new mathematical meanings. However this achievement hinges on instructional interaction that leads to more profound meaning-making for the students. This instructional interaction includes the assumptions the instructor and students make; the expectations the instructor and students have; the resulting teaching-learning interactions that occur between the instructor and students; and the subsequent mathematical meanings and understandings the students gain. Unfortunately, the learning environment that these processes play out, primarily in the lecture format – is in itself problematic.

1.1.2 Learning in a College Lecture Format

One of the more restrictive mathematics classroom environments is the college lecture format. With the large size of classes at the college, the lecture format has become the taken-for-granted option. Lectures provide a means to get sizeable amounts of material communicated to a large audience and help to meet requirements of the

syllabus under the financial constraints of a public college/university system and the time constraints of the quarter/semester system. As a traditional method of instruction, the lecture method is seen as successful (Brown & Race, 2002). It need not be replaced but only needs to be improved (Pascarella & Terenzini, 2005). However, the mathematics lecture format is mostly unidirectional with the lecturer providing the information and language for students; the students presumably digesting the information with little or no other interaction (Seymour & Hewitt, 1997; Brown & Race, 2002; Veel, 2000). This unidirectional communication seems to be particularly problematic for women and ethnic minorities (Corson, 1993; Seymour & Hewitt, 1997; Tinto, 2000). Moreover, the lecture format has been linked to reasons why women and ethnic minorities leave the sciences (Seymour & Hewitt, 1997) and college in general (Tinto, 1987; Tinto, 2000).

In addition to the restrictions of the mathematics lecture format, many college mathematics faculty, as well as faculty in other disciplines, have received minimal teacher training as graduate students (Pruitt-Logan, et al, 2002). The issue of teacher training for future college faculty has become a serious enough concern that the Council of Graduate Schools has partnered with organizations like the National Science Foundation to institute Preparing Future Faculty programs (Pruitt-Logan, et al). One of the goals of these programs is improving teaching and instruction in higher education by exposing future faculty to new educational technologies and other changes such as increased diversity of students in the classroom. Example programs are project-based learning in physics and integrated curriculum for calculus courses (Pruitt-Logan, et al). Unfortunately, faculty participation and support for these programs is weaker in math

and science than in the humanities and social sciences (Pruitt-Logan, et al, 2002). Moreover, although curricular changes may have occurred, there is no indication that changes to the lecture format in large freshman and sophomore gateway math and science courses have been attempted.

Thus, with faculty having minimal teacher training and using a limited instructional format, the college mathematics lecture favors students who can learn with little or no interaction, no mutual discourse, nor developmental guidance. For students to be most successful in lecture courses, they must have not only the prerequisite knowledge and preparation but also the prerequisite facility with the specialized mathematical discourse. That is, a more successful student will be able to perform the technical requirements of solving a problem while also understanding the discourse surrounding the problem. Unfortunately, this assumed knowledge and specialized discourse are not readily accessible for many students, particularly women, students from underrepresented ethnic backgrounds, low-income and first-generation students, and English language learners. With the unidirectional nature of the lecture and the presumption of equal access to knowledge and specialized language, the college lecture format is fertile ground for miscommunications of mathematical knowledge, principally the subtle nature of the precision needed to understand mathematics. These potential miscommunications contribute particularly to the disaffection of women and ethnic minorities with mathematics and generate frustration, for the lecturer and the students, over the lack of successful learning.

Next, I want to make clear, for those who have not extensively studied mathematics, the nature of the precision needed to understand mathematics and for assigning meaning in mathematics.

1.1.3 The Complications of Precision in Comprehending Mathematics and Developing Proper Mathematical Meanings

“Precision in definition” is a fundamental but unwritten rule in doing mathematics. This rule is often overlooked in college-preparatory and developmental mathematics classes. For those who don’t study mathematics this rule may require explanation. This explanation at times is dense in that it builds from one statement to the next. Each statement requires reflection before moving onto the next one, and I have tried not to make a miraculous leap in logic though the explanation.

For a mathematical definition to be precise, the definition must have meaning within a mathematical framework. The symbols, pictures and descriptive language in the definition take on specialized meaning in relation to each other, to previous mathematical knowledge and to the theoretical boundaries of the conditions of the current mathematical context. Consequently, the symbols, pictures and descriptive language come together in a package to provide a succinct exposition of a concept. For this exposition to be mathematically relevant, the reader/user must have a clear understanding of not only the relationship between the symbols, pictures, and descriptive language but also the importance of the arrangement and status of these items. However, each symbol, picture, metaphor or word in the definition may have more than one mathematical implication within more than one arrangement of

mathematical connotations for more than one mathematical concept, hence the need for precision. Given the complexity of relationship and arrangement in mathematics, a reader/user may likely confuse or disarrange the meanings and relevance of symbols, pictures, metaphors and words. Moreover, the ability of the reader/user to assign a precise meaning is predicated on his or her previous mathematical knowledge and their understanding of the theoretical boundaries that produce the conditions of the mathematical context. So even if we have an institutional and precise mathematical definition, the interpretation of the meaning of the definition is very much dependent upon whether the reader/user assigns reliable past knowledge and understands the boundaries and conditions to further develop an institutionally accurate understanding of the implication of the relationship and arrangement of connotations of each symbol, picture, metaphor and/or word.

The meaning a reader/user assigns will depend on his or her previous mathematical knowledge and level of understanding of this previous knowledge. The manner in which the reader/user interacts with his or her world will also influence the way in which meaning is assigned (Lakoff and Johnson, 2003). For example, a student has no need to use the mathematical expression, *angle of depression*, to interact in his or her everyday world. The challenge in mathematics instruction is shifting the students' attention away from the meaning of the expression in the everyday world to the world of mathematics. Will students have the mathematically intended meaning for the mathematical expression? Moreover, will the student understand the overarching conditions, relationships, and arrangement of connotations in which the mathematical expression is being applied?

The previous example seems minimal in that the focus is on the single expression, *angle of depression*. However, this single expression embodies a physical depiction that in itself is a combination of connotations and meanings— an angle with an initial side or ray on a horizontal axis and the terminating side sweeping downwards from the horizontal axis forming an acute angle (Figure 1.1). A student working with this expression must have an understanding of the meanings of horizontal axis, initial and terminal sides, rays and acute angle in the context of the mathematical world, not the everyday world.

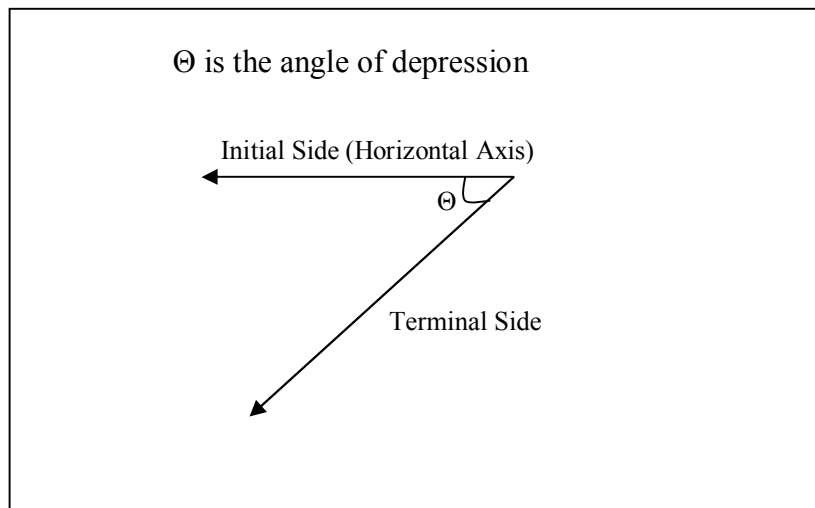


Figure 1.1: The Angle of Depression

To complicate matters, now add “real world” conditions such as, “A pilot flying a plane uses a landmark on the ground north of the plane’s current location. The angle of depression from the plane to the landmark is 30° . The problem then continues...” The student must now transform the expression, *angle of depression*, into the embodied physical depiction and apply it to the given “real world” conditions. Does the direction “north” matter? Will the speed of the plane matter if it is given in the exposition of the

problem? Are the conditions of “the pilot flying a plane” the same as other conditions? For example, “A student begins to walk at 2 miles/hour south over a bridge at the same time a boat in the river 20 feet below travels east at 5 miles/hour; what does the angle of depression from the student to the boat look like when she begins to walk over the bridge? Does the angle of depression change 2 minutes later?” These seemingly minimal applications of a single mathematical expression, like the *angle of depression*, become more complicated when the student tries to assign meaning within the specialized arrangement of the embodied physical depiction in relation to both the “real world” conditions and the boundaries of the specialized mathematics world.

In order for this complex meaning-making exercise to end successfully, a number of elements must fall into place. The student must have facility with the mathematical terminology and with the boundaries presented in a specialized view of the world – the relationship of a person, a bridge, and a boat in this example. More importantly, previous or current teaching and learning interactions should have laid the foundation for a student to work toward a solution in which they can piece together the specialized meanings, relationships and arrangements of the expression, *angle of depression*, and associated conditions.

Ironically, a student may muddle through a problem to a successful end and still not have a clear meaning assigned to each word and symbol in a definition or an understanding of the order in which he or she finds them. Instead, the student may have attached a skewed meaning that is close enough to the correct meaning. And, the exposition of the problem may be such that the skewed meaning is sufficient for the student to arrive at an adequate solution. The exposition of the problem is therefore

critical in teaching-learning situations. Say the exposition of the problem leads to an algorithmic or formulaic solution. A student may have a skewed meaning attached to definitions and concepts embodied in the solution but this student does not need the correct meaning if s/he has memorized an algorithmic way to solve the problem. Worse yet, the student may have stumbled upon a skewed meaning that led to an adequate solution once and continues to use it successfully because the expositions of subsequent problems do not challenge the student's skewed meaning-making.

Is this skewed meaning-making part of the conundrum in developmental math then? Students may have skewed meanings about mathematical concepts that they have repeatedly covered in high school mathematics courses. Then they enroll in a college developmental mathematics course. The unidirectional nature of the lecture may allow the student to maintain the skewed meanings *in class*. However, the expositions of the problems *on exams* may challenge the students' skewed meaning-making and they perform poorly on the exam. Subsequently, the instructor and students are frustrated over the outcome because the lectures reviewing the concepts should have been sufficient for the students to perform well on the exam. Overall, the question that perplexes me is this: do the complications of "precision in definition" combined with the college lecture format spur the developmental mathematics conundrum? This study looks at the developmental mathematics conundrum through the lens of student meaning-making as they play out over students' learning experiences concerning mathematics in a college lecture format.

1.2 The Framework Underlying the Study

1.2.1 Elements in the Framework for Examining Student Meaning-Making and Learning in Mathematics

Student meaning-making and learning in a college-level remedial mathematics course can be shaped by a number of elements – the mathematics itself, the quality and quantity of instruction, students’ abilities and motivation to name a few. For this study, I focus on students’ perspectives, conceptions, and beliefs about mathematics under the umbrella of three elements which shape student-meaning making in the college-level remedial mathematics course. The first element is the nature of mathematics as an institutional, objective reality – the social and cultural implications and assumptions about how one learns mathematics, how mathematics is taught, and how mathematical thought coincides with a universal world view (Lakoff and Nunez, 2000). This discursive conception of mathematics determines how we all – mathematician, teacher, student, interested or uninterested bystander – develop and assign meaning to mathematical notions.

The second element is the impact of the college lecture format on student meaning-making. That is, the act of teaching mathematics in a lecture format, the interaction between instructor and students – the supposed transfer of mathematical meaning from instructor to student – has bearing on the nature and quality of the meanings students maintain, develop or assign. The lecture format can be limited in substantive interactions between instructor and students because developmental mathematics courses in large public colleges often have one instructor for seventy-five

students, three hours of lecture, one hour discussion section – this is the case for the institution in this study. Lecture notes become one of few resources a student has to make sense of the information delivered in lectures. Students use the lecture and notes to develop a collection of meanings, new and old, that are then appended to what constitutes the student's mathematics knowledge base.

The third element is the individual student's experience – their history with mathematics and more importantly how they are experiencing the mathematics learning event, the developmental math course, under consideration for this study. Student's past and concurrent experiences in mathematics learning environments can shape their perspectives, conceptions, and beliefs about mathematics which in turn can guide their meaning-making about mathematical notions. These three elements – the objectivist nature of mathematics, the college lecture format, and the students' experiences – make up the framework for examining students' perspectives, conceptions, and beliefs which can affect their meaning-making and learning in mathematics.

The simplicity of the three-element framework belies the dynamic and complex schemas at work within the framework. That is, this framework by no means is static; there are constantly changing connections, exchanges and resources, what Sewell (1992) calls schema, between the three elements. The way we view the institution of mathematics and its resources determines how we teach mathematical concepts and make connections between mathematics and everyday life. The way we explain mathematics to students determines how they view the nature of the institution of mathematics, its place in society and its relationship to them, and so on. Potentially, there are many schemas to examine within this framework. However the focus of this

study is on how students develop and assign mathematical meaning in particular for the language, symbols and pictures used to define mathematical concepts. To do so, I explore language, discourse and cultural meaning systems related to mathematics and the college lecture format.

1.2.2 Language and Discourse in the Mathematics Classroom

In mathematics, the assumption is that people communicate in numbers, symbols, graphs, operations and equations (Pimm, 1987). However, all of these aspects of mathematics must be verbally described and verbally interpreted for student understanding (Veel, 2000). Thus, language is the critical means of communication in the mathematics classroom. Students, teachers, and researchers use language in order to facilitate, explain, or justify mathematical reasoning and understanding. Language in the mathematics classroom ranges from everyday discourse to the mathematics register, the technical language of mathematics, from improvisations on terminology to traditional mathematical metaphors. Everyday discourse can be used in the mathematics classroom to ease introduction of new concepts. The mathematics register includes a specialized vocabulary, mathematical metaphors, analogies, idioms, and the grammar and syntax of mathematical sentences, symbols and equations (Schleppegrell, 2004; Pimm, 1987). Improvisations are non-mathematical words and phrases developed and used as mathematical terminology within the context of a classroom or introduced in a textbook. Traditional mathematical metaphors are established, universally accepted words and phrases the meanings of which have been transferred to the context of mathematics (Schleppegrell, 2004; Pimm, 1987).

1.2.3 Cultural Meaning Systems and Discourse in the Mathematics Classroom

Cultural meaning systems “consist of learned systems of meaning, communicated by means of natural language and other symbol systems, having representational, directive and affective functions, and capable of creating cultural entities and particular senses of reality” (D’Andrade, 1984, p. 116). Because cultural meaning systems are learned systems, the cultural meaning systems may then seem inherent to the cultural entities and realities that they define. That is, as people who employ them may perceive them. Yet, these same people may develop and change these seemingly inherent cultural meaning systems as language, symbols and realities change over time. So, cultural meaning systems can seem moderately oxymoronic – an acquired innateness – and leads back, in some ways, to the developmental mathematics conundrum.

The mathematics community employs a cultural meaning system specific to mathematical entities and senses of reality. The metaphorical nature of mathematics, the assigning of specific meanings, which are not commonly used meanings, to words and combinations of words (e.g. *angle of depression*) (Lakoff and Nunez, 2000), can be viewed as one such cultural meaning system with an air of an “attained innateness” about it. The meaning system seems innate in that it embodies physical, scientific realities that can be interpreted as universal or natural truths (Lakoff and Nunez, 2000) that effect humanity in general. Thus, we all should be able to employ this system of meaning because it implicitly describes our world. Conversely, this cultural meaning system or metaphorical nature of mathematics seems attained in that it represents the

specific world views of the creators of the meaning system. Moreover, it has been developed and changed over time as mathematical knowledge has advanced and demands to describe emergent scientific knowledge grow. For someone outside the mathematics community, this system of meaning may be difficult to employ – it may be antithetical to their world view; the creators of the meaning system and outsiders may have different cultural and lived experiences.

Whatever the reasons for the difficulty to employ this cultural meaning system, the outsider may not view the metaphorical nature of mathematics as a learned system of meaning developed by a specific community (mathematicians and scientists) to create specific cultural entities and particular senses of mathematical and scientific reality. Rather, the outsider may assume the cultural meaning system is an inherent, objective set of truths – confirmed by terminology such as identity, axiom, property, and definition (Lakoff and Nunez, 2000; Lakoff and Johnson, 2003) – that governs everyone. Likewise, community members may view this system in the same way. So the outsider, as well as the community members, may see difficulty in employing this system as an ingrained deficit in his/her-self or the student rather than an issue of unsound assumptions about learning, divergent world views, and communication and instruction relevant to employing an unfamiliar cultural meaning system.

Thus, in the mathematics teaching and learning environment, instructors and students must negotiate a mine field of assumed shared knowledge of the cultural meaning system – knowledge about mathematical concepts, definition and use of words and expressions, similar experiences and histories with mathematics, and beliefs about who uses mathematics and what mathematics is. Moreover, broader unspoken

assumptions influence the nature of interactions in the classroom; for example, how males and females should behave in a mathematics classroom; the type of mathematical knowledge an ethnic minority brings to the classroom; and, what counts as valid knowledge in the classroom. Within these interactions, “speakers and writers simultaneously present content, negotiate role relationships, and structure texts through particular grammatical choices which make a text the kind of text it is” (Schleppegrell, 2004. p.18). These unspoken assumptions indicate that differences may exist in the ways in which we perceive communication and understanding to occur in the mathematics classroom. Nevertheless, the underlying base for these unspoken assumptions – the notion we all share the same mathematical knowledge and membership in a specialized mathematical discourse community – in turn may influence language and talk in the mathematics classroom (Cazden, 2001; Pirie, 1998), and ultimately, may direct and determine communication and interactions in mathematics learning environments.

This specialized mathematical discourse community does not mean that the mathematics register exists to exclude people from mathematics. In fact, the mathematics register exists to allow for efficient communication among a community. Exclusion from mathematics more likely is fostered by how we view mathematics as an idealized institution; how this view manifests itself in teaching and learning interactions; and how individual beliefs play out in the teaching and learning process.

1.3 Overview of Methodology

In order to study the different ways that students and teachers communicate in the mathematics classroom, researchers have predominantly used ethnographic methods. The bulk of the data comes from classroom observations and interviews, with subsequent analysis and discussion of the transcripts. The approaches to these observations are constructivist, sociocultural, and sometimes a little of both. The research discussed in this paper includes and reflects theories, methods, and notions of teaching and learning that are not strictly sociocultural in perspective. Looking for the linguistic implications of learning mathematics led to research in functional linguistics (Schleppegrell, 2004) or a linguistic-pragmatic perspective (Pimm, 1987). Other research focuses on communication from the individual's perspective, on how one person makes sense and interprets another's words and sentences and then develops one's own cognitive mental structures in the tradition of Piaget (Sierpinska, 1998). Social interactions with language give rise to individual cognitive conflicts and the individual resolves this conflict, reorganizing cognitive concepts with respect to the language and the social interaction (Cobb, Yackel & Wood, 1993). Certain researchers hold to a sociocultural perspective on learning in the tradition of Vygotsky, the social and cultural factors that influence the individual's development. For example, "our use of language determines our learning, and our learning determines our use of language" (Wink & Putney, 2002.p.59). Yet other researchers look at communication in the mathematics classroom as discourse in the tradition of Bruner or interactionism.

Mathematics is a discourse where the language and the interactions follow shared conventions, context and pretext (Sierpinska, 1998).

1.3.1 Research Questions

The guiding research question for this study is: How are students making meaning during the social construction of mathematical knowledge in a college-level Precalculus lecture? Given the broad nature of this guiding question, I chose to address it by starting with the taken-for-granted view that “mathematics is objective” (Lakoff and Nunez, 2000). Often accompanying the objective nature of mathematics is a mechanistic pedagogy in which knowledgeable teachers transfer knowledge to less-knowledgeable students (Kohlberg & Mayer, 1992).

The premise that mathematics is objective and that student learning flows effortlessly from teachers provides the opportunity to examine a different notion: student development of mathematical concepts can also be shaped by social and cultural factors. To narrow the scope of the guiding question, I focused on three themes which would connect the objectivism of mathematics with social and cultural factors affecting learning: (1) the mathematics register and student meaning-making, (2) assumed mathematical knowledge and assigned meaning to the mathematics register, and (3) an examination of student processes for enacting meaning-making within the college-level Precalculus lecture structure.

More specifically, I ask: **(1)** What meanings do students employ in the mathematics classroom when there are multiple mathematical and everyday meanings, metaphors and symbols that can represent terminology in the mathematics register?

(2) How are students assigning meaning to specific words and terminology in the mathematics register?

(3) How do students perceive the meaning-making process within the Precalculus lecture structure? How do students insert themselves within the meaning-making process? And, where do students place their meaning-making process within the larger context of the specialized Precalculus discourse community?

1.4 Conclusion

The aim of my research is to describe the nature of meaning-making and construction of knowledge in a college-level remedial mathematics course. I examine the question of how students are assigning meaning and interacting with the mathematics. My overarching interest is to highlight the impact of the lecture format in a mathematics course that is considered a “gatekeeper” course for student participation in mathematics and science majors. This gatekeeper course often has a larger percentage of women and ethnic minority students enrolled.

Chapters two and three provide a more detailed accounting of the literature related to mathematics learning, meaning-making and language and of the methodology used in this study, respectively. In chapters four and five, I examine the social, cultural, and linguistic context of lecture instruction for college-level lower-division mathematics learners. I analyze and describe the nature of student meaning-making – the relationships between: language and meaning, lecturer practices and students’ practices, learning mathematics and students’ construction of social and academic

identities through their interpretation of cultural meaning systems. I use ethnographic methods to elicit, analyze and describe college students' perspectives and experiences in learning mathematics. In chapter six, I discuss the results of the study which help to illuminate ways in which daily lecture practices in college-level lower-division mathematics are linked to larger social discourses and lead to recommendations for the specific class in this study.

Chapter 2: Linguistic, Sociological and Cultural Perspectives on Teaching-Learning Mathematics

2.1 What Mathematics Do We Teach and Learn?

Mathematics has been perceived either as a “man-made” institution (Rismann, 1998) or a construct that occurs naturally that man has stumbled upon (Lakoff & Nunez, 2000). Adherence to one point of view or the other can influence the discourse around the practice, learning and teaching of mathematics, and lead to an overarching tension in mathematics: *What is mathematics? Who is it for? Who has access to it?* A prevalent assumption holds that mathematics is an unconditional, legitimate, and sacred truth (Lakoff & Nunez, 2000). For some, mathematics is a human, biological construct. “Mathematics is a natural part of being human. It arises from our bodies, our brains, and our everyday experiences in the world.” (Lakoff & Nunez, 2000, p. 377) For others, mathematics is a social construct envisioned and communicated in different ways by different cultures. These differences are accentuated by the different languages used by these cultures to express meaning (Lakoff & Nunez, 2000, Cole & Scribner, 1974). Mathematics is contained within the individual, yet it is also enacted through interactions and upheld through assumed social and cultural norms.

How and why then do social and cultural norms and doing mathematics converge across cultures? According to Sewell (1992), a multiplicity of structures exists, each with a “set of mutually sustaining schemas and resources that empower and constrain social action and tend to be reproduced by that action.” (Sewell, p. 19) These structures can intersect and schemas and resources can be transposed. Language,

cultural meaning systems, and mathematics intersect when an alternative view of mathematics as a social construct is applied. The mathematics that I pose for this construct is the mathematics learned in classrooms throughout the United States and the majority of the industrial world. This mathematics is “an unconditional, legitimate, and sacred truth,” “a natural part of being human,” and an ordered and logical way of thinking (Lakoff and Nunez, 2000). Such assumptions mediate the conundrum in the developmental mathematics lecture.

If mathematics is treated as natural, ordered, and logical, then it is important to study the way in which instructors teach and students subsequently make meaning. Instructors can assume that the act of teaching is one of transferring knowledge, often in the form of algorithms about the natural order and logic of mathematics to students. If a “knowledge transfer” view of teaching-learning (Kohlberg & Mayer, 1992) is adopted, then students’ meaning-making becomes secondary to teaching algorithms which is the accepted as normal practice particularly with a lecture instructional format that does not allow for an investigation of the quality of student meaning-making. When mathematics is conceived to be a natural order, and teaching-learning is treated as a mechanical task, rigid roles and behaviors for instructors and students in the learning process accrue. These are static roles where the instructor as expert transfers knowledge by telling to students who are expected to absorb the transferred knowledge correctly. This view of mathematics instruction and learning not only promotes unidirectional interactions but also creates a forum where individual (both instructors and students) belief systems and personal intention can be misunderstood. When mathematics is treated as a self contained logical system, and taught in a manner that

minimizes teacher-student interaction, this practice ironically minimizes the precision and importance of language in understanding mathematics. Belief systems, language and mathematics are enacted as situated practices in which individual experiences, socialization, social constructs, and cultural norms converge in a cognitive act (West and Zimmerman, 1987). It is this convergence of language, mathematics, and belief systems that I study in this thesis.

2.2 Teaching-Learning Mathematics at Individual, Interactional and Institutional Levels of Analysis

At this point, I will position belief systems, language and doing mathematics in a social framework and discuss the importance of the individual, interactional and institution levels of analysis (Rismann, 1998) in this process. Mathematics can be heuristically treated as a cultural meaning system. That is, mathematics is contained within the individual (the lecturer and student), yet it is also enacted through interactions (classroom communications) and upheld through assumed and institutionalized social and cultural norms (mathematics as natural, objective and neutral). Cultural meaning systems allow for the processes taking place in the mind as well as the social and cultural motivations and implications on processes taking place between people (D'Andrade; cf Mehan 19xx – the “between the ears and beneath the skin paper”). In addition, cultural meaning systems, fluid by definition, change and adjust with the variability of individual preference (personality) and discursive practices. So treating mathematics as a cultural meaning system provides me the

flexibility to discuss the structure of belief systems and mathematics mediated by communication and language at the individual, interactional, and institutional levels.

The cultural meaning system that incorporates the notion that mathematics is an unconditional, legitimate and sacred truth promotes a mathematics that is politically, socially, and culturally neutral and accessible to all students. Therefore, any academic success and the onus of failure occur at an individual level. Comparatively, mathematics as a natural part of being human promotes mathematics as being accessible only to those with biologically innate mental facilities. Students with these higher mental facilities will succeed equally. Or students who can develop these higher mental facilities will succeed. But if students do not succeed, then they are blamed for their own failure.

While mathematics seems to be a matter of individual learning, the cultural meaning systems related to “doing belief systems, doing language, and doing mathematics” occur in the interactions between teachers and students. That is, the cultural meaning system of mathematics is actualized and mediated in social interactions through the language and discourse practices of mathematics coupled with the objectivist and neutral vision of mathematics.

New dimensions in the discussion of cultural meaning systems are added when we focus on the college mathematics lecture, particularly at freshman or first year level. First year students enrolled in a transitional course, such as Precalculus and Calculus, are at critical stages of personal development (Chickering and Reisser, 1993). Thus, while engaged in a learning process fraught with social and cultural undertones, students are also developing identity, managing emotions, learning to become

interdependent (Chickering and Reisser). Furthermore, if these students cannot make connections between the learning process in the classroom and their personal development, they become disengaged with and disinterested in the university setting (Tinto, 1987, 2000; Dey & Hurtado, 2005; Pascarella & Terenzini, 2005).

2.3 The Role of Language and Communication in the Teaching-Learning of Mathematics

2.3.1 How We Perceive and Talk about Mathematics

To discuss communication, language, and mathematics learning properly, I must first elaborate on an overarching tension in mathematics education in general and in this field of mathematics education in particular. The way one views and defines mathematics determines the manner in which one teaches and talks about mathematics and studies mathematics learning. Moreover, the way one views and defines mathematics determines the underlying and often unspoken assumptions about mathematics as an academic discourse community. The romanticized version treats mathematics as a neutral fact independent of humans' constructions. Mathematical tenets and canons are obvious and easily attainable. Consequently, there are preferred ways to envision, talk about and learn mathematics. This version leads people to believe that mathematics is language- and culture-free. Because of its stability and universality, this version of mathematics encourages the notion that the use of language in mathematics should be more or less uniform.

However, if we treat mathematics as a human construct, we find it exists in some form in all cultures. Mathematics is stable, precise, generalizable, symbolizable, calculable, consistent, universally available, and effective for precisely conceptualizing a large number of aspects of the world as we experience it (Lakoff & Nunez, 2000). The human construct view of mathematics recognizes that mathematical knowledge is variable and can be learned differently. This vision of mathematics leads people to believe there may be more than one right way to talk about and learn mathematics. But no matter how mathematics is envisioned language plays a critical role.

2.3.2 Types of Language and Communication in Teaching-Learning

Mathematics

Over the past forty years, research and commentary on communication, language, and mathematics learning has ranged from reading comprehension to writing in mathematics to discourse in the mathematics classroom. In studying mathematics learning, critical means for communicating knowledge and understanding emerge: everyday language, improvisations on terminology, mathematical symbols, visual representations, unspoken but shared assumptions, and the mathematics register (Schleppegrell, 2004; Pirie, 1998; Pimm, 1987). Everyday language and improvisations often have some social or cultural significance that are perceived to aid the student in comprehension of mathematical concepts.

For example, the metaphorical phrase *family of curves* can be used instead of a *set of curves*. “Family” is not part of the mathematical register but the metaphor aids in students’ understanding of what characteristics a set of curves should have.

Improvisations can also lead to misuse of the mathematics register. For example, two students using the phrase “ x squared” to mean “2 times x ” may both understand the phrase in the same way and may calculate the correct answer, but their use of the phrase is in conflict with the mathematics register (Pirie, 1998).

Communication in a mathematics classroom can be conducted through mathematical symbolic and graphical means as well. Some mathematical symbols are considered a part of the mathematics register (Pimm, 1987). Whether a part of the register or not, mathematical symbols and graphs are verbally described and verbally interpreted for student understanding. These verbal descriptions and interpretations can be made using the different types of language discussed so far. Beyond the type of language used, the description and interpretation of symbols and graphs depends greatly on the social and cultural pretext and context of the students and teachers.

Unspoken assumptions about shared knowledge and membership in a specialized mathematical discourse community also influence language and talk in the classroom (Cazden, 2001; Pirie, 1998). Unspoken assumptions include how students should behave in a classroom, the type of knowledge a student brings to the classroom, what counts as valid knowledge in the classroom, and the way a student is interpreting and understanding a concept. Such unspoken assumptions can direct and determine communication and interactions in mathematics learning environments.

The mathematic register, itself, is a text and context which allows “speakers and writers simultaneously to present content, negotiate role relationships, and structure texts through particular grammatical choices which make a text the kind of text it is” (Schleppegrell, 2004. p.18). In sum, regardless of the type of communication –

everyday language, symbols, graphs or the mathematics register – discussion and talk in the mathematics classroom is fundamentally social and cultural.

Research on communication, language, and mathematics education has existed for many years. For a historical perspective, I have relied on Aiken's (1972) review of research. Aiken reports that vocabulary and syntax significantly affected the readability of mathematics materials. Students had difficulty understanding of mathematics concepts because of their inability to understand the technical vocabulary and syntax of the mathematics texts. Aiken further finds that student ability to verbalize mathematics and conduct verbal interactions in technical language influenced the learning of mathematics. Problems in learning mathematics often resembled problems in learning a second language. The issues Aiken discusses are similar to knowledge of and facility with the mathematics register however his discussion in regards to language reflects a behaviorist perspective: Students react to and interact with an established mathematics vocabulary and syntax. And his recommendations focus on how to get the students to better comprehend the technical language.

2.4 Theoretical Foundations for the Role of Language and Communication in Teaching-Learning Mathematics

More recent research falls into the following categories: constructivism, social constructivism, socioculturalism, interactionism and functional linguistics. These theories certainly have differences but they also have commonalities. In the following discussion, I group some theories together because the commonalities outweigh the

differences. Firstly, I have grouped research that is constructivist with those from functional linguistics because these groups are looking specifically at the vocabulary, metaphors, and other ingredients of the mathematics register. The constructivists are more interested in the content of the mathematics whereas the functional linguists are more interested in the lexical implications of the research. There is not much research regarding the mathematics register from functional linguistics. In fact, the following research comes primarily from a recent volume of the journal *Language and Education* specifically devoted to the mathematics register. The second grouping is the social constructivists, socioculturalists and interactionists because these groups are primarily looking at discourse and language interactions in the classroom. Social constructivists are more interested in process, socioculturalists are interested equally in process and outcomes, and interactionists are more interested in the outcomes.

2.4.1 Constructivism and Functional Linguistics

Some researchers approach and design their observations from a constructivist viewpoint; the individual and the manner in which the individual makes meaning and develops cognitive structures is the focus of the observations and analysis. In the classroom, cognitive conflict and reorganization of knowledge is critical. Constructivist approaches usually have the students are talking and the teacher is listening to determine whether the students are making conceptual progress. In terms of research, the student or the teacher is talking and the researcher is listening for cognitive and procedural development.

In his review of commonly shared “frames”, Davis (1984) discusses two linguistic issues in learning mathematics: mathematical metaphors and ambiguity. The metaphors *borrow* and *carry* are part of the mathematics register used in addition and subtraction operations. The confusion students have with borrow and carry are confounded with the structure or form of the addition or subtraction problem. When carry and borrow are repeated in the problem students have more difficulty reproducing the carrying or borrowing action. The question that arises is whether students truly understand the notion of carry and borrow if they can’t repeat the process in the same problem. The second topic involves ambiguity in defining terms, using precise language and making a metaphoric extension of the equals sign (Pimm, 1987). Word problems are usually the focus in this research. Students use ambiguous descriptors like “vinegar” instead of the correct terminology “milliliters of vinegar.” The metaphoric extension of the equals sign happens when students use “vinegar” to represent unit measurement such as milliliters. Leung (2005) found however that having student “pinpoint” the meaning of the word could reduce this ambiguity. For example, in the vinegar case, the students should clearly define and differentiate between what vinegar means and milliliters of vinegar.

In contrast, Barwell (2005) found that ambiguity in the classroom opened up opportunities to explore the meaning of the term “dimension”. The teacher in the study engages the students in a productive conversation about dimension by pointing out different perspectives on dimension can introduce ambiguity about the difference between two and three dimensional objects.

Greer's (1992) review of research on multiplication and division includes ways in which individual students make meaning of mathematical concepts through interpreting and struggling with the mathematical register. The discussion revolves around extensive quantities, describing aspects of the world by counting or measuring, and intensive quantities, derived by the application of mathematical operations. The vocabulary used to describe extensive and intensive quantities are often common words used in everyday conversation. However, when the vocabulary is used in this mathematical context the meaning of the word is not the same as the everyday usage. The vocabulary words in the mathematics register have a different context and are good example of the confusion between the mathematics register and common English language use. The vocabulary words "price", an intensive quantity, and "cost", an extensive quantity, are distinctly defined and have separate meanings in mathematics. However, in common English usage and in different contexts, the distinction between the words is not clear. Particularly in a barter culture, price and cost have overlapping meanings.

2.4.2 Social Constructivism, Socioculturalism, and Interactionism

Kanes (1998) focuses on "the intersection between language as a meaning-making action and language as an aspect of meaning-making that corresponds to the lived purposes, intentions, and contexts of people" (Kanes, 1998.p.113). Whereas Kanes' review of this research domain takes on a sociocultural perspective, his focus is on the individual student and the way in which the pretext and the context of the language determine the student's meaning-making actions in learning mathematics. In sum, Kanes has a social constructivist approach. Another of Kanes' colleagues also

reflects this social constructivist approach. Cestari (1998) looked at classroom interactions or “speech event” between teacher and student and teacher and class. Her analysis of the interaction demonstrates a situation in which there are different meanings for the same event. The teacher asks for quantities in terms of monetary value and the student is giving quantities in terms of the number of pieces or slips of money. The interaction shows the student in cognitive conflict when the teacher uses new language and then shows the social procedure of the teacher questioning the student to help him reorganize the concepts. Cestari (1998) emphasizes the importance of the teacher having context for the student’s language choice in description of the money to lead him through the cognitive process. More importantly, the interactions showed the teacher tending to make up metaphors when questioning more confused students and they also showed how difficult it was for the teacher to vary communication patterns from the traditional question-answer exchange. With the teacher controlling many of the interactions, the teacher-student question-answer exchange often becomes stagnant and students are not engaged in learning mathematics (Cestari, 1998; Yackel & Cobb, 1993). However, when students are allowed to generate solutions instead of following procedure, student explanations were conceptual rather than calculational and they used language that carried the significance of actions on taken-as-shared mathematical objects (Yackel & Cobb, 1993).

Some researchers approach and design their observations from a sociocultural point of view; the dialogue and context of the classroom or the dialogue and context surrounding a concept is the focus of the observations and analysis. In the classroom, socioculturalism takes on different meanings. In one case it means that the teacher is

talking and the student is listening to make meaning of concepts within the context of classroom and the teacher's talk. In another case it means that teacher-student or student-student dialogues are occurring with both talking and listening to enhance the meaning-making and learning process.

The work by Ball and Bass (2000) is also sociocultural. Ball and Bass (2000) and other researchers look at the way in which individual teachers make meaning by using language and tie together content, pedagogy, and communication. Their commentaries not only include the social and cultural implications in the classroom of a teacher who is able to bring together content knowledge with a well thought out pedagogical plan but also reflect the importance of knowledge of and facility with the mathematics register. Learning and teaching mathematics requires an understanding of the language used to relay the concepts of mathematics. Having an understanding of mathematics subject matter necessitates having a facility with the mathematics register. However, to teach mathematics subject matter effectively, teachers should also have pedagogical content knowledge. Having pedagogical content knowledge necessitates that a teacher has a deeper facility with the mathematics register.

The mathematics register is critical in a teacher's role in helping students make meaning, in constructing a way of thinking mathematically and way of understanding mathematics. Pedagogical content knowledge involves teachers unpacking subject matter knowledge so that they can better see this knowledge from a learner's perspective and determine how they can best present and communicate this knowledge to their students. The mathematics register is part of this unpacking process. A teacher must have facility with the specialized language to have a deeper understanding of the

mathematical concepts and also be able to translate this specialized language into a discourse that allows students to learn. Thompson and Thompson (1994) highlight the problem of a teacher having subject matter knowledge but not having sufficient language to teach a concept to the students in a manner that they would understand. Thompson and Thompson show with this example how critical it is for a teacher to be able to unpack the mathematics register into everyday language so that students can understand it and then make connections between everyday language, the mathematical content and the mathematics register.

A teacher's ability to move between linguistic registers is highlighted in Cahnmann and Remillard's (2002) work on mathematics teaching in urban settings. Their work is a case study of two teachers, one bilingual and one monolingual. The teachers work in culturally, linguistically and socioeconomically diverse classroom settings. The authors found that although the bilingual teacher provided more culturally relevant learning activities she lacked the facility with the mathematics register to provide a better base for student mathematical comprehension. In fact, she was confused herself when students started to interpret the assignment differently than she had intended. On the other hand, the monolingual teacher had better facility with the mathematics register, but she lacked the ability to communicate this knowledge in a discourse that students would understand.

Ball and Bass (2000) further the discussion of the mathematics register and pedagogical content knowledge by discussing the need for developing pedagogical strategies for dealing with metaphors and analogies in mathematics like "take-away" and "borrow" that cause problems in students' understanding of subtraction (Davis,

1984). Ball and Bass suggest that teachers must contemplate, struggle with and cognitively reorganize these metaphors just as students have.

Seeger (1998) and Bartolini Bussi (1998) look at the way in which learning mathematics is the “introduction into, and the sharing of, a culture” (Seeger.p.85). Students participating in classroom environments that encouraged discussion and validated student language moved back and forth more often from personal senses to meaning (Bartolini Bussi). Unfortunately, these types of classrooms are the exception and not the rule (Seeger). Krummheuer’s (1998) interactionist approach to argumentation in the mathematics classroom has similar outcomes. Students explicate their own thinking and thus achieve a shared interpretation of a situation (Krummheuer). Students accepting responsibility for contributing to the discussion generate ways of constructing mutual understanding, leading to increased intersubjectivity.

2.5 The Role of Communication and Pedagogy in the College Lecture Format

With the National Science Foundation actively pursuing the preparation of future faculty (Pruitt-Logan, et al, 2002), lecturing and note-taking have become topics of interest in general and in specific (Brown & Race, 2002). In general, lecturers and students can have different ideas of what a lecture is supposed to be (Brown & Race) and the lack of preparation and motivation on the lecturer and students part become problematic for all students (Leamnsion, 1999). Unorganized lectures most often confuse students (Leamnsion). A lecturer’s poor presentation skills such as low

projection of voice, sloppy writing on the chalkboard, and poor basic classroom management skills, lead to students disaffection with the class and subject matter (Tinto, 2000; Braxton, Bray & Berger, 2000; Seymour & Hewitt, 1997) and the severity of grading leads to stressful academic competition (Astin, 1968). In addition to these classroom issues, content knowledge and pedagogic content knowledge are critical for the lecturer. To be successful, the lecturer must not only provide the mathematical knowledge but also must be able to unpack the content knowledge (Ball & Bass, 2000). In unpacking the content knowledge, a lecturer must be able to assess and repack the knowledge so as to best address the variety of interpretations of the material the audience can make (Ball & Bass).

The previous list of problems that arise for the general population can be exacerbated for women and students from underrepresented ethnicities (Xie & Shauman, 2003; Seymour & Hewitt, 1997). Problems with mathematical performance can be connected influenced by “stereotype threat” (Spencer, Steele, & Quinn, 1999) and negative attitudes and attributions toward mathematics by women (Pedro, Wolleat, Fennema, & Becker, 1981). The mathematics lecture format interpreted as a literacy activity offers another perspective on barriers that women and ethnic minorities face. By literacy activity, I mean any activity that requires the ability to read, write, listen and speak. In terms of the barriers facing women and minorities, Corson (1993) found that “often culturally different students will approach literacy activities in majority culture classrooms in ways that are inconsistent with school norms but consistent with their own cultural norms and values. The evidence confirms that teachers can and do make incorrect assessments of their students’ ability because of this problem, which affects all

modes of language and which is clearly more likely to occur when the distance between the teacher's culture and the child's is greater." (p. 52) Therefore, preparing future faculty to be sensitive not only to the pedagogic needs and outcomes of the general student population but also to the specific pedagogic needs and outcomes for women and ethnic minorities is critical to increasing student success and continuation in mathematics (Pruitt-Logan, et al, 2002).

2.6 Conclusion

Student meaning-making and learning in a college-level remedial mathematics course can be influenced by a number of factors – the mathematics itself, the quality and quantity of instruction, students' abilities and motivation to name a few. For this study, I focus on three elements of the college-level remedial mathematics lecture that influence student meaning-making and learning: (1) the nature of mathematics as an institution, (2) the outcomes of the act of teaching mathematics in the college lecture format, and (3) students' perspectives and experiences that guide their meaning-making and specifically how they make sense of mathematical notions that are unfamiliar or familiar yet still foreign. These features of mathematics determines how we all – mathematician, teacher, student, interested or uninterested bystander – develop and assign meaning to mathematical notions.

The second element is the outcomes of the act of teaching mathematics in the college lecture format. That is, the act of teaching mathematics, the interaction between instructor and students – the supposed transfer of mathematical meaning from instructor

to student – has bearing on the nature and quality of the meanings students maintain, develop or assign. Although the lecture format can be limited in what could be called substantive interactions between instructor and students, particularly with a one instructor to seventy-five student ratio, the three hours of lecture – the only classroom time besides a one hour discussion section – has significant impact on student learning and meaning-making outcomes. The lecture notes are one of few resources a student has to make sense of the transferred information. They use the lecture and notes to develop a collection of meanings, new and old, that are then appended to what constitutes the student's mathematics knowledge base.

The third element is the individual student's perspectives and experiences that guide their meaning-making about the world at large and specifically how they make sense of mathematical notions that are unfamiliar or familiar yet still foreign. Do the conclusions of their meaning-making lead the students to a deeper and more comprehensive understanding of the mathematics they are learning? These three elements – more generally the institution (mathematics), the interactions (act of teaching), and the individual (the student) – make up the framework for examining student meaning-making and learning in the college lecture format.

The simplicity of the three-element framework belies the dynamic and complex schemas at work within the framework. That is, this framework by no means is static; there are constantly changing connections, exchanges and resources between the three elements. The way we view the institution of mathematics and its resources determines how we teach mathematical concepts and make connections between mathematics and society. The way we explain mathematics to students determines how they view the

nature of the institution of mathematics, its place in society and its relationship to them, and so on. The focus of this study is on how students develop and assign mathematical meaning. In particular, I am interested in how students use the language, symbols and pictures to define mathematical concepts. Then, the language, discourse and cultural meaning systems are examined within the college lecture format.

I examine how communication and the language and discourse of mathematics are viewed and used between teachers and the resulting impact on student meaning-making. Teaching, meaning-making and learning processes are compounded by the underlying requirement for students to acquire language with specialized meaning and to have facility with this meaning as a member of a mathematics discourse community. The use of specialized mathematical vocabulary, grammar, metaphors, and symbols complicates the way meaning is communicated. Moreover, a student's ability to understand these mathematical communications becomes more problematic when social and cultural norms and assumptions shape the exchange. Because meaning-making (Lakoff and Johnson, 2000; D'Andrade, 1984; Cole and Scribner, 1974), language and discourse (Schleppegrell, 2004; Cazden, 2001) can be influenced by social and cultural norms, I also examine how these norms or cultural meaning systems (D'Andrade) are viewed and used by individuals and between individuals. For example, a mathematics instructor who has facility with the specialized mathematics discourse approaches the planning of a lecture from particular perspectives and cultural meaning systems that may not match the perspectives and cultural meaning systems of the student audience. These cultural meaning systems, both for the instructor and students, develop and exist under the auspices of diverse and often divergent assumptions, histories, personalities,

and social and cultural norms. Subsequently, the transfer of mathematical knowledge and the resulting student meaning-making can become confused and tangled with incongruous notions.

Finally, I examine student meaning-making at the individual or student level of analysis and also examined how the interactional and institutional levels mediate or impede students meaning-making. Therefore, at each level of analysis the units of analysis were students in a college level remedial mathematics course. I also specifically focused on constructs related to language, communication and cultural meaning systems to evaluate how students interpreted the use of mathematics discourse and register in reconciling learning processes within the institution of mathematics and the mathematics classroom. That is, how they assigned and interpreted mathematical meaning.

Chapter 3: Methodology

3.1 Research Design and Methodological Approach

The study is informed by a constitutive ethnographic approach (Mehan, 1979) including in-depth student interviews over a three month period of instruction, analysis of student annotated notes and a pre- and post-mathematics register terminology survey to understand better the role of language, communication, and cultural meaning systems in the social construction of mathematical knowledge in a college-level Precalculus lecture. The constitutive ethnographic design allowed me to develop and amend student profiles on their meaning-making habits as data collection progressed thus informing subsequent interviews and analysis of annotated notes and the construction of the post-terminology survey. A summary of collected data can be found in Table 3.1.

I collected exit interviews with the instructors, two graduate students, a male and a female, who had advanced to candidacy, to record their perceptions of the class in general and their teaching experiences – both were novice instructors. The instructors also completed the mathematics register survey which I compared their responses to their students' survey responses. I used the instructors' responses as the benchmark for “institutional” knowledge. I also videotaped the lectures during three different time periods in the quarter – at the beginning of the quarter, in the middle, and at the end.

I collected pre-course mathematics register terminology surveys from all the students enrolled in the target lectures during the first week of the quarter. That is, if they attended discussion section; attendance in discussion section is mandatory for the

course. The post-course survey was administered during the last week of the quarter and all students still registered in the course were asked to complete the survey during discussion. There were fourteen students who volunteered to be note-takers. Their surveys were marked with their assigned note-taker identification number. The note-takers took annotated notes for the entire quarter. In-depth interviews with the note-takers were conducted three times during the quarter. Their most recent annotated notes were collected at each interview.

Table 3.1: Summary of Collected Data

Interviews	Documents	Video or Audio Recordings
2 exit interviews – one with each instructor	2 mathematics register terminology surveys – one for each instructor	24 lectures were videotaped; 12 for each instructor
26 in-depth interviews with 14 student note-takers	108 pre-course mathematics register terminology surveys	26 in-depth interviews with student note-takers
	94 post-course mathematics register terminology surveys	
	11 sets of annotated notes; 5 complete sets of annotated notes	
	14 student note-taker demographic survey	

To maintain focus on the research questions at hand I chose to examine solely the students' experiences, perceptions and meaning making. Data collected through the

videotape lecture observations, instructor exit interviews and surveys were used only in support of or in contrast to student data.

Data collected through the mathematics register terminology survey was used to evaluate students' applications and interpretations of word meanings. Specifically, the survey data helped to examine questions about student meaning-making, assigned meaning, assumed mathematical knowledge and the mathematics register. Data collected through the student interviews and annotated notes provided a window to the means by which students account for, explain, and describe their processes and understandings of the means of communication and language used in the classroom interaction and the objective nature of the mathematics presented by the lecturer. Lastly, the overall collected data allowed for the examination of interactions taking place in the Pre-calculus lecture. The data supplied examples in which students performed meaning-making acts in reaction to the lecture being delivered by the instructor. In examining these meaning-making acts, I found that the students' social and cultural sense of the everyday was linked to their sense of mathematical realities and that their comfort level with the mathematical register was linked to their knowledge of concepts embedded in it.

3.2 Setting and Sampling

3.2.1 Setting

The setting for the research was a subset of lectures for a college level remedial mathematics course. Two of four lectures were used in this study. These two lectures

were chosen because they were similar in enrollment size, and male and female student distribution. The two lectures were also larger than the other lectures, 75 students compared to 60 students respectively, and the location of both lectures facilitated the periodic video recordings of the lectures. There were three male and one female advanced graduate student instructors assigned to the four lectures. Of the two lectures chosen for the study, one of the lectures had the female instructor and the other lecture had a male instructor. Since the overall enrollment for these lectures historically had been approximately 70% female, a purposive sampling decision was made to select the course with the female instructor (Tashakkori and Teddlie, 1998). All of the students enrolled in the two lectures were asked to volunteer to complete the mathematics register terminology survey during their discussions at the beginning and at the end of the 10-week academic quarter. The discussions met once a week on Thursday and had class sizes between 25 and 35 students.

3.2.2 Sampling Procedures

Multistage Cluster Sampling was used to determine the different samples for the study (Tashakkori and Teddlie, 1998). The lecture samples were selected and then a sample of students attending the two lectures was selected using purposive sampling methods (Tashakkori and Teddlie). Students, interested in volunteering to be note-takers, filled out a selection criteria questionnaire demographic survey for student note-takers. The survey was used to determine a sample that was representative of the historic enrollment of the course, predominantly female, low income, first generation, and underrepresented. The questionnaire was also designed to try to determine the

students' consistency in taking and using notes in a mathematics classroom. The initial size for this sample had been set at 12 students however only 14 students met the deadline for indicating an interest in volunteering. Since all the students met the criteria for participating in the sample, the sample size was increased to 14 rather than the proposed 12.

3.2.3 Sampling Implications

The two lectures and the sample of students attending the lecture met the historic enrollment percentages of female and male students in these courses however data was unavailable for the breakdown of ethnicity, income level, and first generation status. The selected lectures for this study were afternoon classes and may have led to the larger enrollments and could mean that students enrolled in the course because they were avoiding morning classes. However, there is no documented study or report on students' self selection of class time and their ability to understand the mathematics being communicated to them. The hope is that in avoiding the morning classes, the students were alert and aware during the afternoon lectures.

The selected sample of volunteer note-takers although small (14 students) approximately, 85% female, reflects the historic class population, 70% female. The note-takers sample is approximately 60% Hispanic. Because the students self-selected to be in the sample, they may be predisposed to being overly helpful in the collection of data. In addition, the students were offered extra help in exchange for participating in the study. Therefore, they may be participating in the note-taking for the additional help and may not provide consistent or helpful feedback in the data collection.

The female instructor was concerned that she was given the lecture position because she was a woman and the study was taking place. She indicated that the general climate for women performing mathematics graduate work was not supportive and she feared that others would see her as getting the position because of her status as a woman and not her abilities as a mathematician. I reassured her that this was not the case. The Graduate Advisor, who also serves on the dissertation committee for this study, selected her from the qualified pool of graduate students. The study may have made the Graduate Advisor more sensitive to the needs of the students (approximately 70% women) in the remedial course. This female instructor is one of three female graduate students to be assigned the lecturer position in a span of nine years. I am uncertain but doubtful that her concern had changed the way she addressed the students or taught the course.

3.3 Measures

3.3.1 Mathematics Register Terminology Survey

The mathematics register terminology survey was constructed by reviewing the text book, Fundamentals of Precalculus (Mark Dugopolski, 1st edition, 2003) for the course. Twenty words and phrases (Table 3.2) were chosen by the range of everyday and mathematical meanings that could be ascribed to them. Because the chosen words are part of the mathematical register used in prerequisite courses, the majority of the words should have been familiar to students before entering the course in this study. The twenty words were split into a pre-course and post-course survey. The pre-course

survey had twelve words all of which should have been prerequisite knowledge. The post-course survey had twelve words four of which were repeated from the pre-course survey. The four words were selected after preliminary analysis of the pre-course survey responses. One of the four, *Inequality*, students had given consistent everyday and mathematical meanings ascribed to it. Another, *Transformation*, students had few mathematical meanings reported and a high rate of incorrect mathematical meanings. The third word, *Variable*, students had a high rate of mathematical meanings reported that indicated they had some mathematical understanding of the term but not a complete understanding. The fourth word, *Identity*, students had unique and diverse everyday meanings ascribed to it and also had a high rate of misreading the word. That is, the respondents provided definitions for the word “Identify.” The pre- and post-course surveys were constructed in a table with three columns. The first column contained the word; the second column provided space to write the everyday meaning of the word; the third column provided space to write the mathematical meaning of the word. Students could leave the space blank or write “not sure” for those words they did not know. The mnemonic *FOIL* was on the pre-course survey. *FOIL* stands for First-Inside-Outside-Last and is used to teach students how to use the distributive property in the multiplication of two binomials. After conducting a preliminary analysis of the pre-course surveys for the student note-takers, I included data collection on gender, ethnicity, and enrollment in a basic writing or English as a Second Language course on the post-course survey for all students.

Table 3.2: Words and Phrases Used in the Mathematics Register Terminology Survey

Pre-course Survey		Post-course Survey	
Root	Rational	Composition	Imaginary
Foil	Reciprocal	Even	Interval
Radical	Dependent	Prime	Negative
Distribute	Tangent	One to One	Product
Pre- and Post-Course Surveys			
Identity	Inequality	Transformation	Variable

3.3.2 Student-Annotated Notes

The student annotated notes were collected and spanned class lectures from week 2 through week 10 of the academic quarter. The student notes captured the way in which students transcribed the written lecturer notes from the chalk board and the non-written verbal communication of the lecturer. The student annotations to the notes captured instances in which the students were reacting to the flow of learning events occurring during the lecture. The annotations were “+” or “?” marks that captured reactions that in the students’ estimation were either positive – for example, a helpful and clear explanation, or negative – student did not understand the concept. The annotations symbolically captured when students felt that mathematical meaning was successfully or unsuccessfully communicated to them.

3.3.3 Student and Instructor Interviews

Field notes and interviewer’s notes on the copies of the annotated notes were collected during the interviews. The interviews were also either video- or audio-taped. The in-depth student interviews revolved around the students’ annotated notes and

produced empirical data representing the students' explanation of their annotations, recollections and descriptions of what was happening in the lecture, corresponding to their notes, and accountings of their understanding of the learning event and of the mathematical concept involved. Other data points were students' explanations, recollections, descriptions, and accountings for the lectures in general and in specific situations where students would comment on notes even though they did not make an annotation.

The instructor interviews captured empirical data representing the instructors' recollections, descriptions, and accountings of the lecture learning environment, their perceptions of their teaching experience, and their perceptions of the students and the students' learning processes.

3.3.4 Videotape Lecture Observations

The selected mathematics lecturers for this study were videotaped during lecture at three time periods. The videotaped time periods occurred during weeks 3/4, weeks 6/7, and weeks 9/10 during the 10-week academic quarter. Four lectures were recorded during each time period. The time periods occurred before major exams (midterms 1 and 2 and the final exam). The video recorder was focused on the lecturer during the entire class time, although instructor-student interactions were audio recorded, provided the student consented to have his or her voice taped. The video recordings captured the lecturers' notes as they were written on the board and the manner in which the lecturer verbally and non-verbally communicated the written notes to the class. In addition,

verbal communication that was not written on the board was captured along with non-verbal communication that corresponded to it.

3.4 Data Collection Procedures

All data collection events took place during the fall academic quarter, September through December, of 2006 for a Precalculus course at a California University. (Documents used to collect data can be found in Appendix A.) There were nine data collection events for this study and are listed by category:

- Student Note-Taking:
 - 14 demographic surveys for student note-takers
 - 11 sets of annotated notes recorded by student note-takers
 - 26 interviews with student note-takers
 - 5 exit interview questionnaires for student note-takers
- Mathematics Register Terminology Survey
 - 108 pre-course student surveys
 - 94 post-course student surveys
 - 2 instructor surveys
- Instructor and Lecture
 - 24 videotaped sessions of selected lectures
 - 2 interviews one with each instructor

The pre-course survey and the demographic survey for student note-takers occurred during the first week of instruction for the fall quarter. The Mathematics Register

Terminology Survey for the instructors contained all twenty words and the instructors only filled out the survey once. For scheduling reasons, the instructors took the surveys home, filled them out, and returned them by the end of the fourth week of the quarter. Video taping of selected lectures, the annotation of notes recorded by student note-takers, and interviews with student note-takers took place throughout the fall quarter. The interview with instructors, final interview questionnaire for student note-takers, and the post-course survey took place in the last week of instruction for the fall quarter.

The pre-course survey was administered during four discussion sections led by teaching assistants in the first week of instruction. The four discussion sections corresponded to the Mathematics lectures chosen for this study and had class sizes from 25 to 35 students. The discussion sections were chosen for the survey distribution because the 15 minutes required for the completion of the survey did not interfere with the instruction planned for the first week's discussion section. I explained to the students how to fill out the survey and indicated that the survey was voluntary. The students were allowed a maximum of 15 minutes at the beginning of each section to fill out the survey. One hundred and eight pre-course surveys were collected. I compiled and reviewed the pre-course survey responses to determine changes to the post-course survey. Preliminary analysis of the pre-course survey responses led to the selection of four words to be repeated on the post-course survey responses; the addition of demographic data requests for ethnicity and basic writing course enrollment; and larger font size of the words that student's would define. The post-course survey was administered using the same procedures in the same four discussion sections as the pre-course survey. The sections were used again because the 15 minute time period did not

interfere with the planned instruction for the last week of classes. There was some attrition in both lectures and resulted in the collection of 94 post-course surveys.

The demographic survey for student note-takers was distributed with student permission forms for videotaping the lecture during the two Mathematics lectures in the first week of instruction. I explained the study and requested that only students interested in volunteering to be a note-taker fill out the form. The demographic survey forms were collected and reviewed. The number of volunteers for one lecture was eight and for the other lecture, six. Since I had set the note-taker limit to six students for each lecture, I decided to use all the volunteers for the annotation of lecture notes. I met with the student note-takers after the pre-course surveys in their discussion sections. We met outside the discussion section room for approximately ten minutes. I distributed notebooks with duplicating pages to each volunteer, explained the annotation procedures during their note-taking, asked that they only use the notebook for lecture notes, gave them a schedule for the collection of notes and interview time periods, and collected the necessary permissions. The student note-takers started annotating their notes on the Friday of the first week of instruction and continued to annotate their notes through the last week of instruction. Five of the note-takers dropped out of the study at various points during the quarter and five note-takers were inconsistent about meeting for interviews and delivering their annotated notes during the scheduled collection time periods.

Three time periods were scheduled for the collection of the annotated notes and to conduct student interviews which involved the review of the annotations made in the notes. These time periods were scheduled for the fourth, seventh, and tenth weeks of

instruction. These time periods were chosen because they occurred directly before a midterm or final exam and directly after the videotaped lectures. Before each of these time periods, the instructors for the selected Mathematics lectures were videotaped for four consecutive lectures.

The student interviews varied from individual to group interviews during the first scheduled time period. I met with eleven of the fourteen note-takers. The group interviews at times would facilitate discussion of the annotations of the notes and at other times restricted some note-takers to acquiesce to the dominant speaker in the group. As note-takers made comments about language, communication and/or math learning that were notable, I would add an interview question related to the comment to ask in future interviews. For the second and third scheduled time periods, I was able to interview students individually because the sample size became smaller and I had allotted more time to collect the interview data. During the interviews, I would address math questions as we discussed the annotations rather than leave them to the end of the interview. In order to get to the cause of the communication and language issues in understanding the mathematics, I often times had to discuss the mathematical concept and misunderstandings that were occurring. Therefore, to be consistent during the interview, I maintained a pattern of simultaneously discussing the annotations and correcting mathematical misunderstandings. All the interviews were either videotaped or audio recorded. During the last data collection time period, the student interviews included a set of questions about their general experiences in the lecture, their ability to understand the instructor, and their perceptions of the instructor's ability to communicate in general and specifically the course content. At the end of the

interview, the students filled out an exit questionnaire regarding their study habits and experience with the course.

During the tenth week, the two instructors for the selected Mathematics lectures were interviewed. The interview consisted of grand tour questions regarding the instruction of the course, focused questions on their ability to communicate the mathematical concepts to the students, and focused questions on their perceptions of the students during the lectures and during office hours. The interviews were audio recorded.

3.5 Data Reduction

Since the data collected for this study was qualitative, the data reduction and analysis often occurred simultaneously. The reduction of the data occurred through preliminary coding of the survey data, annotated notes and student interviews. I had a priori notions about codes however decided to review the empirical data before coding. A constant comparison analysis of the empirical data resulted in emergent codes which were applied to all three data sources (See Codebook, Table 3.3). However, not all codes produced results for the three data sources. A member check of emergent codes was used in the initial coding (Tashakkori and Teddlie, 1998).

3.5.1 Coding the Survey, Annotated Notes, and Student Interviews

A constant comparison analysis was used to reduce the empirical data collected in the survey. Initially a priori codes existed however a review of the survey data was

performed to establish an emergent coding system. Although themes of a priori codes existed in the emergent codes, the new codes better reflected the themes found in the data. A review was conducted for each word or phrase in the survey. I had started to enter the data into an electronic datasheet but found that in some cases what was drawn or the way an entry was written was more meaningful. So I made copies of the surveys and continually reviewed them with colored pencils to code. The emergent codes for each word or phrase were compared against the other words and phrases to combine or eliminate codes. The resulting emergent codes were compared against the survey data to find contradictory cases. A review of the contradictory cases with the emergent codes produced a set of codes representing the survey data. In a second phase of reduction, the codes were then applied to the interview data and annotated notes.

During this second phase of coding additional codes emerged to identify themes unique to the annotated notes and interviews. A constant comparison analysis was used to reduce empirical data in the student annotated notes and interviews. The codes and corresponding empirical data were tested against a comprehensive review of the annotated notes and student interviews. The review included a triangulation of data in which the annotated notes codes were compared and measured against the interview codes and then compared and measured against the survey codes.

Table 3.3: Codebook

Code Categories	Codes	Examples
Literacy	NEM = No Everyday Meaning	<ul style="list-style-type: none"> Interval and Tangent high response for not sure Radical had a high response but not as high as Interval and Tangent
	DEF = not a common meaning or meaning not exactly right	<ul style="list-style-type: none"> Radical = awesome, excellent Dependent as independent Identity related to Identify Distribute = handing out EVENLY or EQUALLY
Limited Math	NMM = No Math meaning	<ul style="list-style-type: none"> Transformation, Dependent, Tangent, Composition, Identity, and Rational Transformation and Identity post-survey
	SA = some attempt but incorrect	<ul style="list-style-type: none"> All words on survey Dependent
	PC = answer has some connection to the concept	<ul style="list-style-type: none"> All words on survey Variable, Rational, Reciprocal, Distribute, and Interval
	SYM = Used symbol	<ul style="list-style-type: none"> Variable = x Radical = $\sqrt{\quad}$ Inequality = either \neq or/and \leq or/and \geq Imaginary = i
	VR = Used visual representation	<ul style="list-style-type: none"> Tangent = picture of circle with a line touching at one point Negative = picture of number line Transformation = picture of houses with opposite shading Distribute $\rightarrow a(b+c) = ab + ac$

Code Categories	Codes	Examples
Social and Cultural	JAR = Jargon	<ul style="list-style-type: none"> • Radical and Prime • Transformation = make over
	DEF = not a common meaning TT= twist on how meaning is interpreted	<ul style="list-style-type: none"> • Root, angle of depression • Identity = identify • Half life = half a life • Dependent is independent • Reciprocal and inverse particularly notation • Imaginary = non-existent
Organizational Practices	SC = Socially Charged meanings	<ul style="list-style-type: none"> • Inequality, even, negative
	PRO = Procedural knowledge	Performing tasks and procedures ok Example is procedural
	OK = Organizing Knowledge	How to translate and solve word problems Keywords in notes but no clear definition
	OB = treated concept as an object	<ul style="list-style-type: none"> • Radical = $\sqrt{\quad}$, root = base, tangent = opposite over adjacent
	SYM = Used symbol	<ul style="list-style-type: none"> • Variable = x • Radical = $\sqrt{\quad}$ • Inequality = either \neq or/and \leq or/and \geq • Imaginary = i • Even \rightarrow 2, 4, 6 ... and Prime \rightarrow 2, 3, 5... • Interval \rightarrow notation
	VR = Used Visual representation	<ul style="list-style-type: none"> • Tangent = picture of circle with a line touching at one point • Negative = picture of number line • Transformation = picture of houses with opposite shading • Distribute $\rightarrow a(b+c) = ab + ac$

Code Categories	Codes	Examples
Organizational Practices	CON = Context	<ul style="list-style-type: none"> • Tangent either picture or opposite over adjacent • Identity algebraic equality versus trigonometric equality but no context for truth
Objectivism (Lakoff & Johnson, 2003, reference pg.186)	OB = Treated concept as an object	<ul style="list-style-type: none"> • Variable = x, Radical = $\sqrt{\quad}$ • Inequality = either \neq or/and \leq or/and \geq • Imaginary = i • Even $\rightarrow 2, 4, 6 \dots$ and Prime $\rightarrow 2, 3, 5 \dots$ • Interval \rightarrow notation
	PO= Properties of Objects	<ul style="list-style-type: none"> • Tangent = picture of circle with a line touching at one point • Negative = picture of number line • Transformation = picture of houses with opposite shading • Distribute $\rightarrow a(b+c) = ab + ac$
	WR = using word to define word	<ul style="list-style-type: none"> • Root is the root of the number, variable is a variable in equation, prime is prime numbers
	CAT = Categories	<ul style="list-style-type: none"> • Tangent either picture or opposite over adjacent • Identity algebraic equality versus trigonometric equality but no context for truth
	TRU = Things objectively true or false	Example is procedural Example is or is not well described or defined in procedure or organization
	FIX = Words have fixed meanings	<ul style="list-style-type: none"> • Interval, Tangent, and Radical had meaning in mathematics but not everyday • “same meaning” as math meaning • “never use” in everyday meaning

Code Categories	Codes	Examples
Linking Everyday and Math Meanings	CON = Context	<ul style="list-style-type: none"> • Tangent either picture or opposite over adjacent • Identity algebraic equality versus trigonometric equality but no context for truth
	DEF = not a common meaning or meaning not exactly right	<ul style="list-style-type: none"> • Radical = awesome, excellent • Dependent as independent • Identity related to Identify • Distribute = handing out EVENLY or EQUALLY
	TT= twist on how meaning is interpreted	<ul style="list-style-type: none"> • Identity = identify • Half life = half a life • Dependent is independent • Reciprocal and inverse particularly notation • Imaginary = non-existent
	NEM = No Everyday Meaning	<ul style="list-style-type: none"> • Interval and Tangent high response for not sure • Radical had a high response but not as high as Interval and Tangent
	NMM = No Math meaning	<ul style="list-style-type: none"> • Transformation, Dependent, Tangent, Composition, Identity, and Rational • Transformation and Identity post-survey
	MAT = Math is math, no everyday	<ul style="list-style-type: none"> • Interval, Tangent, and Radical had meaning in mathematics but not everyday • “same meaning” as math meaning • “never use” in everyday meaning

3.5.2 Decisions about Using Data

The interviews posed a challenge because the interviews occurred in conjunction with “tutoring sessions.” Recall, to keep the flow of the interview process, I would clarify misconceptions and answer questions during the interview. Additionally, the first interviews were small group interviews.

Because there was more data collected than needed to address the research questions for this study, decisions to exclude data occurred. The videotape lecture and interview data has been excluded for more than one reason. The original intent for the videotape lecture was to have students view the lecture during the in-depth interviews. However, the pilot-testing showed that this process became too time consuming and impinged upon the expected time commitment for the student participating in the study. In addition, problems with video equipment resulted in two-thirds of the lecture and interview videos having no sound. I felt that the student data provide sufficient points for triangulating patterns and results. Although the lecture data was being excluded, the instructor interviews and surveys were used however minimally to support or contradict student results.

3.6 Pilot Testing

The data collection procedures for the student note-taker annotations of lecture notes and the corresponding student interviews were pilot tested during a summer Mathematics course at the California University. From the pilot test I was able to clarify and refine the interview procedures, the methods for annotating notes, the selection of

the students who volunteered for the research study, and the feasibility of completing the proposed research plan.

The pilot test revealed that the interviews and videotapes of the lecture did not produce significant differences in the data collected when the interviews and videotaping was done continuously through the course. The pilot test also revealed that the student interviews were time consuming thus limiting the number of students in the research project would be necessary. As a result, the number of students to be selected as student note-takers was reduced from a minimum of fifteen in each Mathematics lecture to six student note-takers in each. In addition, viewing the videos during the interview did not enhance the data collected during the interviews and was time consuming. Therefore, the videotaped lectures were not used during the interviews. The number of times I conducted interviews and collected annotated notes was reduced from continuously throughout the fall quarter to three time periods in the fall quarter, each time period being before an exam. The videotaping of the lecture was also reduced to three time periods.

The student note-taker participant questionnaire for the fall data collection was revised so that more information about how the student viewed note taking and using notes was collected. Also, the procedure to provide a detailed explanation about the requirements for volunteering to be a note-taker was revised.

3.7 Data Analysis Procedures

Similar to the data reduction, data analysis was a combination of constant comparison analysis and triangulation of the data and codes. The codes were measured against a comprehensive review of the interviews to organize codes into code categories (See Table 3.3: Codebook). I repeatedly reviewed the data and used recursive coding rules to develop emergent code categories. The code categories were triangulated with the student annotated notes and surveys and a member check of code categories provided additional perspective (Tashakkori and Teddlie, 1998). The categories were also compared to theories related to metaphorical meaning (Lakoff and Johnson, 2003), mathematics and metaphor (Lakoff and Nunez, 2000), the mathematics register (Pimm, 1987; Schleppegrell, 2004), and cultural meaning systems (D'Andrade, 1987) guiding this study. Negative case analysis of code categories was used to measure the categories against interview data, annotated notes, and survey data (Tashakkori and Teddlie). The code categories provided a basis for the reporting of the results in Chapter 4 and 5. The code categories fit to the theories used in this study will be discussed in more detail in the closing chapter (Chapter 6).

3.8 Limitations of Study

The student note-taker sample was small compared to the size of the class however the smaller sample size allowed for longer and more in depth student interviews than a larger sample would have allowed. The note-taker sample was primarily female however given the over representation of women in the selected mathematics lectures, the note-taker sample was representative of the lecture. In

addition, the college remedial mathematics course was selected for its historic high percentage of women, particularly underrepresented women, enrolled in the course. Therefore findings from this study may be generalizable only for similar populations of women entering college remedial mathematics courses. No participant check was conducted on the overall results.

Chapter 4: Meaning Making and the Mathematics Register

4.1 Introduction

In this chapter, I report on student meaning-making and use of the mathematics register in a college level Precalculus class. I also attempt to provide answers for my first two focused research questions:

(1) What meanings do students employ in using the mathematics register when there are multiple mathematical and everyday meanings, metaphors and symbols that can represent terminology in the mathematics register?

(2) How are students assigning meaning to specific words and terminology in the mathematics register?

Students assigned a number of different meanings to words and symbols used on the mathematics register survey. Students demonstrated knowledge about meanings in the mathematics register and everyday life however the demonstrated knowledge was varied, could be considered incorrect in particular situations, and at times was skewed from normally accepted definitions. In many instances, the meanings students assigned to a word for everyday use and for use in mathematics were often closely linked. Either the link was to ascribe mathematics meaning to everyday meaning or to ascribe everyday meaning to mathematics meaning. Excerpts from student note-taker interviews provide examples of how meanings were developing during the interview itself. Also, examples from student annotated notes show how meanings were developed or assigned during the lecture. In the examples from the notes, the student

meaning-making understandably was most often connected to the lecturers' comments or notes (written on the chalkboard) about a topic or concept. The examples of student meaning-making – whether from the survey, interviews, or annotated notes – seemed for the most part to be connected to students' existing knowledge or personal experiences.

4.2 Knowing Meaning and Situational Uses

4.2.1 Multiple Meanings

The word: root.

A simple word such as *root* has 17 definitions (Table 4.1), 8 noun definitions and 9 verb definitions listed in the on-line version of the Oxford Dictionary². These numbers do not include phrases, for example “put down roots,” also listed in Table 4.1. These definitions include a mathematics definition (noun definition #8): “a number or quantity that when multiplied by itself one or more times gives a specified number or quantity.” The corresponding equation for this definition is:

$$\text{For all positive real numbers } a, \quad a \cdot a \cdot a \cdot a \cdot a \cdots a = a^n = b$$

(*a* multiplied *n* times)

Interestingly, the mathematics definition is specific to the concept of an exponential expression yielding a number and is a mathematical meaning often assigned to the word, *root*. However, there are other possible descriptions for *root* in

² root. (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/root_1?view=ok

mathematics. For example, the *root* of a function is a solution when the function equals zero. A mathematical example for this description is:

The number “2” is a root of the function $f(x) = x^2 + 3x - 10$.

This statement means that if I substitute “2” for the variable “x” in the given function $x^2 + 3x - 10$, the resulting calculation is “0.”

Table 4.1: Oxford Dictionary On-line: Definitions for Root

***Root*¹**

• **noun** **1** a part of a plant normally below ground, which acts as a support and collects water and nourishment. **2** the embedded part of a bodily organ or structure such as a hair. **3** (also **root vegetable**) a turnip, carrot, or other vegetable which grows as the root of a plant. **4** the basic cause, source, or origin: *money is the root of all evil*. **5** (**roots**) family, ethnic, or cultural origins. **6** (also **root note**) Music the fundamental note of a chord. **7** Linguistics a form from which words have been made by the addition of prefixes or suffixes or by other modification. **8** Mathematics a number or quantity that when multiplied by itself one or more times gives a specified number or quantity.

• **verb** **1** cause (a plant or cutting) to establish roots. **2** (usu. **be rooted**) establish deeply and firmly. **3** (**be rooted in**) have as a source or origin. **4** (**be rooted**) stand immobile through fear or amazement. **5** (**root out/up**) find and get rid of. **6** Austral./NZ & Irish vulgar slang have sexual intercourse with.

— PHRASES **at root** fundamentally. **put down roots** begin to have a settled life in a place. **root and branch** (of a process or operation) thorough or radical. **take root** become fixed or established.

— DERIVATIVES **rootless** adjective.

— ORIGIN Old English, related to WORT.

***root*²**

• **verb** **1** (of an animal) turn up the ground with its snout in search of food. **2** rummage. **3** (**root for**) informal support enthusiastically.

— ORIGIN Old English.

I was able to reduce the student-assigned meanings to approximately nine mathematical meanings for the word *root* and approximately ten everyday meanings (Table 4.2). Of course, students did not always write the meaning exactly in the same words but the essence of the meaning was the same. Only 11% of the student responses were “not sure” on the mathematical meaning of *root* and 3% of the student responses were “not sure” on the everyday meaning. Students could respond with a description for the everyday and mathematical meanings for *root* however the responses and rate of responses indicate that the students did not have a precise or clear definition for the word.

Table 4.2: Student Meanings for the Word Root

The meanings from the pre and post course surveys were reduced and summarized to reflect the essence of the descriptions employed by the student. The list is in order by percentage of responses.

Everyday Meaning:

1 tree/plant roots **2** the root of a tree help provide nutrition to the tree in order to grow **3** nationality, ancestry, family roots **4** base of a tree **5** anchor for a tree or plant **6** base of something **7** at the core of something **8** the prefix of a word **9** to cheer **10** a cavity – root is decayed

Mathematical Meaning: (not all are necessarily correct)

1 root of a number, square root of a number, $\sqrt{\quad}$ **2** the base of a number **3** the greatest common factor **4** the zero **5** irrational **6** the base number of a sequence **7** the number multiplied by itself “x” times to give you some other number **8** half the original number **9** original/solid starting point

Most students in both the pre and post surveys gave descriptions that more closely resembled the definition associated with exponential expressions. However, the

descriptions (pre and post) were vague. One student response, that *root* is “one of these things you can’t explain but still know,” although not exactly echoed by other students, reflected the essence of how students were assigning some mathematical meanings. That is, in some cases students used the word in the survey to define the word. For example, students reported that *root* is “a root of a number” or “a square root of a number” for approximately 66% of the mathematical meanings assigned (Table 4.3). Only 14% of the responses – *root* is a “base” or “power” – were near but not exact to the Oxford Dictionary definition #8. Some students used the symbol, $\sqrt{\quad}$, to assign meaning without adequately describing or defining the symbol. In this case, I believe the students thought that the symbol, $\sqrt{\quad}$, was assumed shared knowledge. They assumed I would understand what they meant.

Table 4.3: Percentage of Student Survey Responses for the Word: *Root*

Mathematical Description of the Word: <i>Root</i>	Percentage of Student Responses (Pre and Post Surveys)
“A root of a number” or similar	32%
“ $\sqrt{\quad}$ ” or similar	34%
“A base” or “power” or similar	14%
Solution to an equation	1%
Not sure	11%
Incorrect Meanings	8%

(Percentages are rounded.)

The Oxford mathematical definition however is not the only mathematics meaning that can be assigned to *root* in a Precalculus course. Other meanings often depend on the Precalculus instructor and textbook however the conditions under which

the word is used are critical to the meaning. Returning to the meaning a “*root* of a function,” only 1% gave a near response “solution to an equation.” However, none indicated that the equation must equal “0,” a subtle but important piece of information. For example, “2” is a *solution* of the equation $(x + 2)(x + 3) = 24$. That is, when “2” replaces the variable “ x ” to the left of the “ $=$ ” then the resulting calculation will equal the right side of the “ $=$,” 24. In contrast, *root* is used in the following way: “2” is a *root* of the function $f(x) = (x - 2)(x + 3)$, as opposed to “2” is a *solution* of the equation $(x - 2)(x + 3) = 0$. In this situation, “2” is a *root* because when it is plugged into $f(x) = (x - 2)(x + 3)$ the resulting answer is zero. The number “2” is also a solution to the equation for the same reason – the equation establishes that plugging “2” into the expression equals zero and not some other number. To make the situation more confusing the above equation example could be written using the metaphor *satisfies* instead of the word *solution*. For example, 2 *satisfies* the equation $(x - 2)(x + 3) = 0$. *Satisfies* is used to represent that “2” is a solution to the equation. Students would have to understand what it means “to *satisfy* an equation.” One of the instructors gave, “where a function = 0” as the meaning for *root*. This instructor’s meaning presupposes students sharing his understanding of the metaphor “function.”

Clearly, there is more than one mathematical meaning assigned to *root*. So, how meaning is assigned depends on the situation in which *root* is being used. For the student, it may have meant what situation they were able to recall when they filled out the survey. An instructional concern arises when one considers the previous discussion about “*root* of a function,” “*solution* to an equation” and “*satisfies* an equation.”

Solution and *satisfies* mean something different from *root* but the nature of the situational meaning of “*root* of a function” seeming to be so similar to “*solution* to an equation” allows students to make skewed connections resulting in skewed meaning and misunderstanding. A discussion of skewed meanings follows in section 4.2.2.

Similar to results Pimm (1987) and Orr (1987) found, a student’s ability to give a mathematical meaning is not the same as knowing a mathematical meaning that fits with an assumed institutional definition. Clearly, from the meanings given, most students have some understanding, both everyday and mathematical, of the word *root*. The diversity and variations in responses could reflect the transformation of meaning during the Interpersonal and Intrapersonal phases of communication (Wink and Putney, 2002). Students learn meaning through communication from an instructor (Interpersonal) but then the student internalizes the meaning to fit with their experiences, cultural meaning systems (D’Andrade, 1984) and previous knowledge (Wink and Putney).

For example, students who assigned irrational (Mathematical Meaning #5 in Table 4.2) as the meaning for *root* may have made this connection because of previous mathematical knowledge. The Set of Irrational Numbers is one of the first concepts covered in the Precalculus class and is also prerequisite knowledge for the course. The pre-course survey was given after the first three class lectures. The Irrational Number Set contains numbers such as $\sqrt{2}$. The symbol “ $\sqrt{}$ ” is a square root symbol so *root* becomes connected to “irrational”.

The word: inverse.

Although the word *inverse* was not on the pre- or post-course survey, student note-takers indicated that the multiple mathematical meanings for *inverse* were problematic. In addition to additive and multiplicative *inverse*, there are *inverse* functions with an algebraic interpretation and a graphical interpretation, *inversely* proportional, and students often connect *inverse* to a reciprocal (*reciprocal* was on the mathematics register survey). In each case written above, *inverse* has a situational mathematical meaning. The additive *inverse* is $-a$ such that $a + (-a) = (-a) + a = 0$ for a belonging to the set of real numbers. The multiplicative *inverse* is $1/a$ such that $a(1/a) = (1/a)a = 1$, for a belonging to the set of real numbers and $a \neq 0$. The expression $1/a$ also is the *reciprocal* of a . Hence, the possible reason for students connecting *reciprocal* to *inverse*. *Inversely* proportional may also lead a student to make connection between *inverse* and *reciprocal*. For example, if I say k is inversely proportional to n , the corresponding mathematical statement would be: $k \propto (1/n)$. *Inverse* function is a more involved explanation. Rather than go into this explanation, the important point for this discussion is that the *inverse* function is not a *reciprocal*.

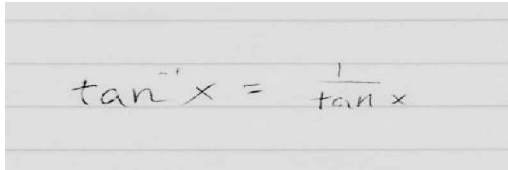
A student's understanding of one situational use for *inverse* did not mean understanding of other situational uses. For example, Jackie, a student note-taker, had some facility with the notion of *inverse* as reciprocal however had difficulty with the notion of *inverse* functions. In Figure 4.1, we see Jackie's response for *reciprocal* on the mathematics register survey. Although she misuses the "=" sign in her response students commonly make this mistake. Rather than write $\frac{1}{2}$ is the reciprocal of 2, she

use the “=” sign to imply that connection. Her everyday response is also connected to the mathematical meaning in that she wrote “flip over” – which is what she did to 2 to get $\frac{1}{2}$. In Figure 4.1, we also see Jackie’s interpretation of the *inverse* tangent function during an interview. She understood that the reciprocal or multiplicative *inverse* of a number or expression was one over that number or expression, i.e. she flipped it over. Unfortunately, she applied that same meaning to the *inverse* tangent function – in Figure 4.1 she “flips the function over.” Notation, also a part of the mathematics register, may have contributed to the misunderstanding. For example, the *inverse* function of $f(x)$ is written as $f^{-1}(x)$; the *inverse* tangent function is $f^{-1}(x) = \tan^{-1}(x)$. The “- 1” in this situation signifies an *inverse* where as “- 1” in the situation of an exponent indicates a reciprocal, for example $a^{-1} = 1/a$ or $(3/4)^{-1} = 4/3$. She displayed her misunderstanding of the *inverse* tangent after the interview when I was answering her questions about the material. I asked if I could make a copy of her work.

Jackie’s Survey Response for Reciprocal:

	Everyday Meaning	and	Math Meaning
Reciprocal	flip over		$\frac{1}{2} = \frac{2}{1}$

Notes: Student Incorrect Interpretation of Inverse Tangent



The figure shows a student's handwritten work. At the top, there is a table with two columns: 'Everyday Meaning' and 'Math Meaning'. Under 'Everyday Meaning', the student has written 'flip over'. Under 'Math Meaning', the student has written the equation $\frac{1}{2} = \frac{2}{1}$. Below the table, there is a note: 'Notes: Student Incorrect Interpretation of Inverse Tangent'. Underneath the note, there is a photograph of a piece of lined paper with the handwritten equation $\tan^{-1} x = \frac{1}{\tan x}$.

Figure 4.1: Student’s Understanding of Reciprocal and Inverse Tangent Function

The situational uses of *inverse* occur throughout a Precalculus course as well as throughout prerequisite courses such as Algebra I and II. So students may have entered the Precalculus course, under consideration for this study, with one or more established mathematical meanings for *inverse*. With multiple mathematical meanings and situational uses, students may then have difficulty determining which meaning is attached to a specific situational use. The multiple mathematical meanings that can be assigned to *inverse* coupled with the close connections to notions of the *reciprocal* could certainly lead students to develop a skewed or incorrect meaning for *inverse* functions.

For words that have many multiple mathematics meanings like *root* and *inverse*, communicating and knowing the situational use for the mathematical meaning was critical for student understanding. Unfortunately, because of students' existing knowledge, the multiple meanings and situational uses, specific mathematical meanings for *root* and *inverse* often became skewed.

4.2.2 Skewed Meanings and Partial Knowledge

The word: dependent.

Dependent has only four definitions (Table 4.4) in the Oxford dictionary³, not as many as *root*. However, *dependent* was problematic not for students reporting multiple mathematical or everyday meanings but for a specific incorrect meaning being assigned. Students had two major everyday responses for *dependent*, 80% gave the meaning

³ dependent. (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/dependent?view=ok

“relying on someone or something” and 20% gave a meaning for the antonym *independent*. Rather than say the students were incorrect for reporting a meaning for *independent*, I prefer to look at this as a skewed meaning or skewed meaning-making. I am choosing to look at it this way because *dependent* and *independent* are related concepts and physically the words differ by the prefix “in.” This skewed meaning-making in everyday use had consequences for the reporting of mathematical meanings.

Table 4.4: Oxford Dictionary On-line: Definitions for Dependent

Dependent

• **adjective 1 (dependent on)** contingent on or determined by. **2** relying on someone or something for financial or other support. **3 (dependent on)** unable to do without. **4** Grammar subordinate to another clause, phrase, or word.

• **noun** variant spelling of DEPENDANT.

— DERIVATIVES **dependence** noun **dependently** adverb.

The same 20% of students reporting an everyday meaning for *independent* rather than *dependent* were reporting a mathematical meaning for *independent* as well. For example, a student gave the everyday and math meaning for *dependent* that implied independence. The student reported an everyday meaning: “something that stands on its own.” This same student reported the mathematical meaning: “a variable or number that stands alone.” Students reporting the more accurate everyday meaning for *dependent* such as “rely on another or something,” were able to report a more accurate mathematical meaning such as “a variable determined by another.” This skewed

meaning-making is problematic because *independent* is also a part of the mathematics register. And, the mathematical meanings for *dependent* and *independent* are very similar to the everyday use definitions. Although the percentage, 20%, of students with skewed meaning-making was small, this incidence showed that when students had a skewed everyday meaning, they subsequently assigned a skewed mathematical meaning. Conversely, when students had a more accurate everyday meaning, they subsequently assigned a more accurate mathematical meaning. The reported meaning for *dependent* also demonstrated that when the everyday and mathematical meanings are more directly linked to one another, knowing the everyday meaning may help a student process or develop the mathematical meaning.

The word: identity.

The everyday meanings assigned to *identity* were not always exactly precise or accurate. *Identity* was used on both the pre-course and post-course surveys (Table 4.5). Approximately 45% and 43% of the responses for the mathematical meaning on the pre- and the post-course surveys, respectively, were blank, “not sure” or don’t know.” So from the beginning to the end of the course almost half of the students could not give a mathematical meaning for *identity*. Nonetheless the concept of *identity* was covered both at the beginning and toward the end of the course, and is prerequisite knowledge for the course. That is, *identity* is a basic concept covered in Algebra, Intermediate Algebra and High School Precalculus courses.

In some cases, the mathematical meaning reported by students was skewed because students assigned a skewed everyday meaning to *identity*. Approximately 35% of the everyday meanings on the pre-course survey described “identify,” similar to your

identity being an “identification of who you are as a person” or “identification card” (See Table 4.5). Given the age group of the students, college freshmen, the sociocultural notion of identification being linked to identity may have been high on their radar screen. In turn, a percentage of these students assigned “identifying numbers or values of numbers,” “identifying equations,” and “identifying the meaning of something” to the mathematical meaning of *identity*. The notion of *identity* being “identify” in mathematics is not correct. Fortunately, only about 5% of the post-course responses assigned “identify” to the mathematics meaning for *identity*. So, for some student notions of *identity* did change over the 10-week course.

Table 4.5: Percentage of Student Survey Responses for the Word: *Identity*

Everyday Meaning	Pre-Course Percentage	Post-Course Percentage	Math Meaning	Pre-Course Percentage	Post-Course Percentage
Identify or Identification	35%	4%	Identify	14%	5%
Self or Fact of Being Who or What a Person Thing is	46%	45%	A rule or property	6%	11%
			What number is, etc.	8%	9%
			Additive or Mult. Identity	20%	18%
Personal Characteristics Or Association with culture, race, ethnicity	16%	51%	A (true) statement	0%	2%
			Trig Equation	1%	8%
			Algebraic Equation	6%	4%
Blank or Not sure	3%	0%	Blank or Not sure	45%	43%

Like many words in the mathematics register, *identity* has variations in its meaning depending on the situation. The *identity* property for addition, the *identity* property for multiplication, algebraic *identity*, and trigonometric *identity* can be in a Precalculus syllabus. The notion of truth is fundamental in each of the cases listed above and often truth is implied in the mathematical statement, property, theorem, or definition. For example, the additive *identity* is 0 such that $a + 0 = 0 + a = a$, for a belonging to the set of real numbers. The implication of the statement is that the *identity* property for addition is always true for the set of real numbers. That is, $a + 0 = 0 + a = a$ is always true for real numbers.

The Oxford Dictionary listed the mathematical definition for *identity* as: “An equation expressing the equality of two expressions for all values of the variables, e.g. $(x + 1)^2 = x^2 + 2x + 1$.”⁴ Interestingly, both instructors gave a slightly different meaning than the Oxford dictionary however their meanings comparatively were almost exactly the same. “A statement that is always true;” and “a statement that is always true no matter what value the variables take.” The instructors’ mathematical meaning is correct whether discussing algebraic or trigonometric identities or properties. The notion of truth in an identity is not used by the Oxford dictionary however is implied in the phrase: “for all variables.” In contrast none of the students from the pre-course survey connected truth or “for all” to the mathematical meaning of *identity*. And, in the post-course survey, only two students (one for each instructor) out of the 94 reported responses connected truth of an equation or “for all values” to *identity*. (A third student

⁴ identity (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/identity?view=ok

used truth as: “a number’s true self.”) *Identity* was covered at least twice but briefly in the syllabus the *Identity* Property for Addition and Multiplication and Trigonometric Identities. Only basic trigonometric identities were covered. Students wrote “trig equation” or gave an example of a basic trigonometric *identity*, “ $\sin^2 x + \cos^2 x = 1$,” more often in the post-course survey (approximately 8% on the post-course survey compared to 1% in the pre-course survey). Trigonometric Identities were covered toward the end of the academic quarter. In the pre-course survey, approximately 6% of the students wrote “algebraic equation” or gave an example such as “ $(x + 1)^2 = x^2 + 2x + 1$.”

The *Identity* Property for Addition and Multiplication was covered in the first lecture for the course and prior to the pre-course survey. So it was not surprising to record 20% of the responses in the pre-course survey were connected to the *Identity* Property. The *Identity* Property or a related meaning was 18% of the responses on the post-course survey for *identity*. Ten percentage points more than Trigonometric *Identity* which was covered at the end of the class when the post-course survey was administered. Students’ preference for assigning meaning connected to the *Identity* Property may be occurring because the concept is usually first introduced in Algebra I or Pre-Algebra in high school. The *Identity* Property is then consistently reviewed in every mathematics course that follows through to Precalculus. Trigonometric Identities are usually introduced in a Trigonometry or Precalculus course.

In some case, the students only wrote 0 (the additive *identity*) or 1 (the multiplicative *identity*). For some students, the meanings they assigned seemed connected to what had been covered in the lecture however many only reported incomplete knowledge, for example only writing a 0 or 1 without any other explanation.

In the cases where a student wrote a math equation, algebraic or trigonometric, they did not indicate conditions (i.e. when it is true) for the equation.

I looked at the annotated notes to see if the instructors had used true or truth when lecturing on *identity*. (See scanned sections of notes in Figure 4.2 for lecture A and Figure 4.3 for lecture B.) Students started annotating their notes after the first lecture so I could not find a reference to algebraic *identity* or the *Identity* property. However the annotated notes covered the rest of the course. In reviewing the annotated notes, six student note-takers had turned in lecture notes covering trigonometric identities. Two students were in lecture A and four students were in lecture B. The two sets of student notes from lecture A did not have “true” or “truth” written in the notes. In the notes there is the statement “For any angle or real number, $\sin^2 \alpha + \cos^2 \alpha = 1$.” Similar to the Oxford Dictionary description, truth is implied in the statement because the instructor for lecture A indicated “for angle or real number.” For lecture B, two of four sets of notes had “True for any Alpha” or “True for any α !” The other two sets had “for any angle” or “for any α !” Given the different ways in which the statements were written and that “true” was missing from the other two sets of notes, I suspect that the instructor said “true” or “true for any alpha” but perhaps did not write it down. (Videotaped recording of the lecture could not have helped here. This lecture was not videotaped.)

This situation may demonstrate instructors’ assumed knowledge about who has facility with the specialized mathematics discourse community (Pirie, 1998; Cazden, 2001). That is, the instructors assumed that students understood the discourse context of “for any angle” to imply truth for the equation. (On a larger scale the Oxford dictionary

mathematics definition presupposes the assumption of truth.) This assumed knowledge may also explain why only two students on the post-survey and none on the pre-survey had matching mathematical meanings to the instructors. Because truth in *identity* was assumed, students may have felt that writing “for all” was redundant. Moreover, the assumption of shared knowledge meant that no one checked to see if the students really understood the implication of truth. Thus, leaving the student survey responses of written algebraic or trigonometric identities such as $\sin^2 \alpha + \cos^2 \alpha = 1$ only partially correct because students did not write a phrase “for all real values of α ” or something similar.

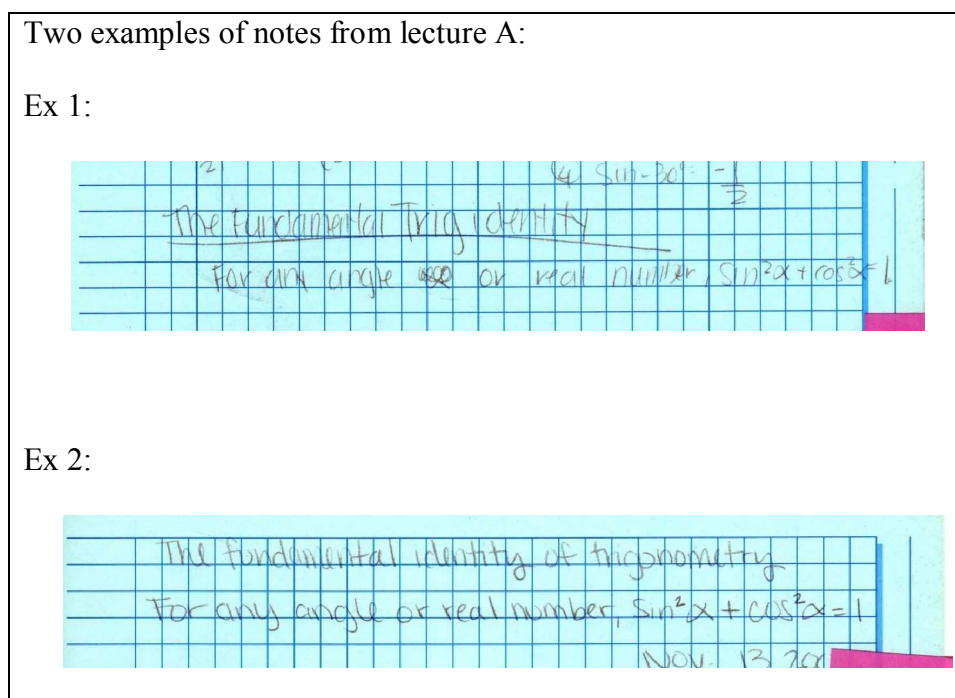
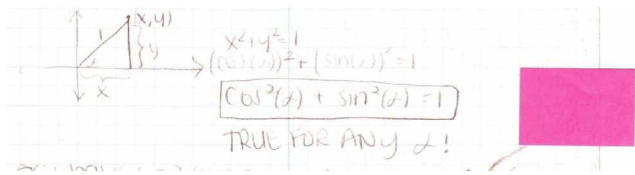


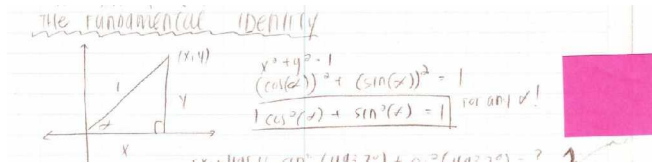
Figure 4.2: Annotated Notes from Lecture A, examples of communicating the notion of identity

Four of the five examples of notes from lecture B:

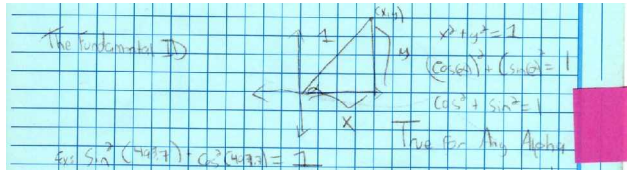
Ex 1:



Ex 2:



Ex 3:



Ex 4:

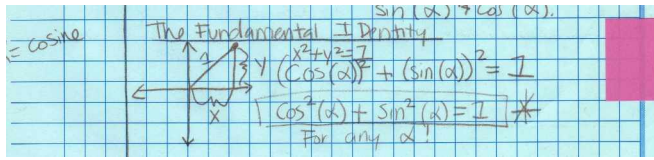


Figure 4.3: Annotated Notes from Lecture B, examples of communicating the notion of identity

The word: distribute and the mnemonic: foil.

Many students, approximately 56% and 58% respectively, gave clear and correct everyday and mathematical meanings for *distribute*. However *distribute* was another term that resulted in student everyday responses that were slightly skewed. In this case,

the skewing of the everyday meaning may have helped students in their mathematical meaning-making. For the everyday meaning for *distribute*, students overall gave the similar meaning that is listed in the Oxford dictionary: to divide, allocate, spread out or pass out. Approximately 30% finished the thought with the skewed meaning “evenly or equally.” To distribute does not necessitate even or equal distribution. This notion of even or equal distribution may have helped to connect to the idea of mathematical distribution: in multiplying expressions, terms in one expression are allocated in equal or same proportions to all terms in the other expression through multiplication. The opposite, the student’s mathematics meaning influencing the student’s everyday meaning, may also have been occurring. Many students also gave an example, $a(b + c) = ab + ac$. This equation is often used when covering the Distributive Property.

Students were more certain about the mnemonic meaning of *foil*. No student left the mathematical meaning blank or wrote “not sure.” They either wrote out the mnemonic – First, Outside, Inside, Last. Or, they wrote the mathematical equation: $(a + b)(a + b) = a^2 + ab + ba + b^2$ (see other examples in Figure 4.4). Or, they wrote the Distributive Property.

Foil	than qualification or terms of comparison Thin aluminum sheet used to wrap food.	First Outer Inner Last $(a + b)(c + d)$
Foil	to run a plan	Front Outer Inner Last $(a + b)(b + a)$ $ab + b^2 + b^2 + ab$ F O I L

Figure 4.4: Examples of foil from the Mathematics Register Survey

Table 4.6: Percentage of Student Survey Responses for the Words: *distribute* and *foil*

Everyday Meaning	<i>distribute</i>	<i>foil</i>
To divide, allocate, spread out or pass out	56%	
To divide, allocate, spread out or pass out evenly or equally	30%	
Give to people, friends	5%	
Household product, aluminum sheets used for cooking, etc.		72%
To ruin someone's plans		9%
Aluminum		18%
Fencing tool, weapon		1%
Blank or Not sure	9%	0%
Math Meaning		
	<i>distribute</i>	<i>foil</i>
Multiple a variable or number to every term in an expression or $a(b + c) = ab + ac$	58%	2%
A rule or property	6%	3%
First Outside Inside Last	5%	56%
$(a+b)(a+b) = a^2 + ab + ba + b^2$ or similar	1%	32%
The Distributive Property	8%	7%
Other Incorrect meaning	8%	0%
Blank or Not sure	14%	0%

Before I continue with the discussion of *foil*, I want to state that I do not believe *foil* is a part of the mathematics register. The mnemonic supposedly helps students remember how to apply the distributive property to the multiplication of two binomial (two terms added or subtracted) expressions. However, the mnemonic only works for this particular application of the distributive property. Unfortunately, students then use the mnemonic to all applications of the distributive property – that is, in other multiplications or products of two or more equations. In these cases, the mnemonic is an incorrect application of the distributive property. So, why then did I include it in the mathematics register?

Over fifteen years ago, I learned about *foil* from students in my Precalculus lecture at another college. I was appalled and thought that the mnemonic was an abomination of the distributive property. The students however thought that I was lacking critically important mathematical knowledge and clearly my ability to teach the topic hinged on my knowing the mnemonic. In my negative reaction to *foil*, I did not win over any students to the notion that the distributive property was more than *foil*. I got the impression they thought I was trying to make the mathematics more difficult for them. After that experience, I incorporated *foil* into my lectures as a way to move students eventually away from using the mnemonic and to applying the distributive property. In this way, I was able to win over some students to the notion that the distributive property was more than *foil*.

I showed the Mathematics Register Survey to faculty in the Mathematics Department at the university. None who saw the survey knew what *foil* was. When I explained, they were appalled for much the same reasons I was appalled all those years ago. They insisted I take it out of the survey. I chose not to take it out because I was interested in seeing the students' and graduate student instructors' responses. As I suspected, to a one the students and the graduate student instructors all knew what *foil* represented. In fact, one instructor and two student note-takers used the mnemonic as a verb – “you foil the expressions” or “I foiled it to get that answer.” This acceptance of the mnemonic is problematic.

In some cases on the survey, students seemed to be interpreting the mnemonic as a formal procedure or law. Students related the mnemonic to “a process” or called it “the distributive property.” It is not the distributive property. Some students assigned

a loftier context to the mnemonic. That is, *foil* was a “process to multiply all polynomial or algebraic expressions.” If a student attempted to use the mnemonic, assuming they applied it appropriately, on a multiplication of two trinomials (three terms added or subtracted), the resulting multiplication would have been impossible to complete. Many students assigned a grander purpose (a process to multiply all polynomials) and thus skewed meaning (the distributive property) to the mnemonic. This skewed meaning-making potential leads students to having only a partial understanding of the distributive property.

4.2.3 Knowing Meaning: Simplicity and Repetition

The words: product and even.

Two words on the mathematics register survey in which students provided correct everyday and mathematical meanings were *product*⁵ and *even*⁶. Both words have multiple everyday meanings (Table 4.7) and students reported many of these meanings for the everyday uses of the words. For the mathematical meanings however the students overwhelmingly gave one meaning for each word. For *product*, approximately 81% of the responses were essentially “the result of multiplication.” For *even*, approximately 78% of the responses indicated “numbers that are multiples of 2.” This percentage response included the listing of even numbers, “2, 4, 6, 8” as well.

⁵ *product*. (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/product?view=ok

⁶ *even*. (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/even?view=ok

Only 4% of the responses for *product* and 2% of the responses for *even* were “not sure” or “don’t know.”

Table 4.7: Oxford Dictionary On-line: Definitions for Product and Even

<p><i>Product</i></p> <ul style="list-style-type: none"> • noun 1 an article or substance manufactured for sale. 2 a result of an action or process. 3 a substance produced during a natural, chemical, or manufacturing process. 4 Mathematics a quantity obtained by multiplying quantities together. <p>— ORIGIN Latin <i>productum</i> ‘something produced’.</p> <p><i>Even</i></p> <ul style="list-style-type: none"> • adjective 1 flat and smooth; level. 2 equal in number, amount, or value. 3 having little variation in quality; regular. 4 equally balanced: <i>the match was even</i>. 5 (of a person’s temper or disposition) placid; calm. 6 (of a number) divisible by two without a remainder. • verb make or become even. • adverb used for emphasis: <i>he knows even less than I do</i>. <p>— PHRASES even as at the very same time as. even if despite the possibility that. even now (or then) 1 now (or then) as well as before. 2 in spite of what has (or had) happened. 3 at this (or that) very moment. even so nevertheless. even though despite the fact that.</p> <p>— DERIVATIVES evenly adverb evenness noun.</p> <p>— ORIGIN Old English.</p>
--

The question that arose with *product* and *even* was: why were students able to give correct responses for these words when they had difficulty with many of the others on the survey? I chose *product* and *even* because like the other words on the survey they are prerequisite knowledge for the Precalculus course. Moreover, they are words

and concepts that students should learn starting as early as elementary school in basic mathematics classes (Nation Council for Teachers of Mathematics, 2000). So one would hope by their freshman year in college most students would know the meanings for *product* and *even*. Nevertheless, the level of knowing the meanings may also come from the repetition of learning the concepts in more than one mathematics class over many years and the simplicity of the meanings assigned to them.

4.3 Shared Meanings in Symbols and Visual Representations

For the words *tangent*, *negative* and *inequality* and the phrase *one-to-one*, students communicated mathematical meaning by using visual cues: symbols, numbers, drawings and words in drawings. There were no drawings used to communicate everyday meanings for any of the words on the pre and post course surveys. Moreover, symbols and numbers were rarely used for everyday meanings as well. In expressing mathematical meaning by using visual cues rather than words, the students may not have had sufficient words to describe the mathematical meaning (Lakoff and Nunez, 2000). However, the symbols, numbers and drawings presented by students had substance in terms of relaying mathematical understanding. In order for their use of symbols, numbers, drawings and words in drawings to convey correct meaning, there had to have been some assumption on the students' part that the mathematical meaning was implicit in the visual cues and that the mathematical meaning was shared knowledge. That is, students must have assumed I would understand what they were drawing on the survey.

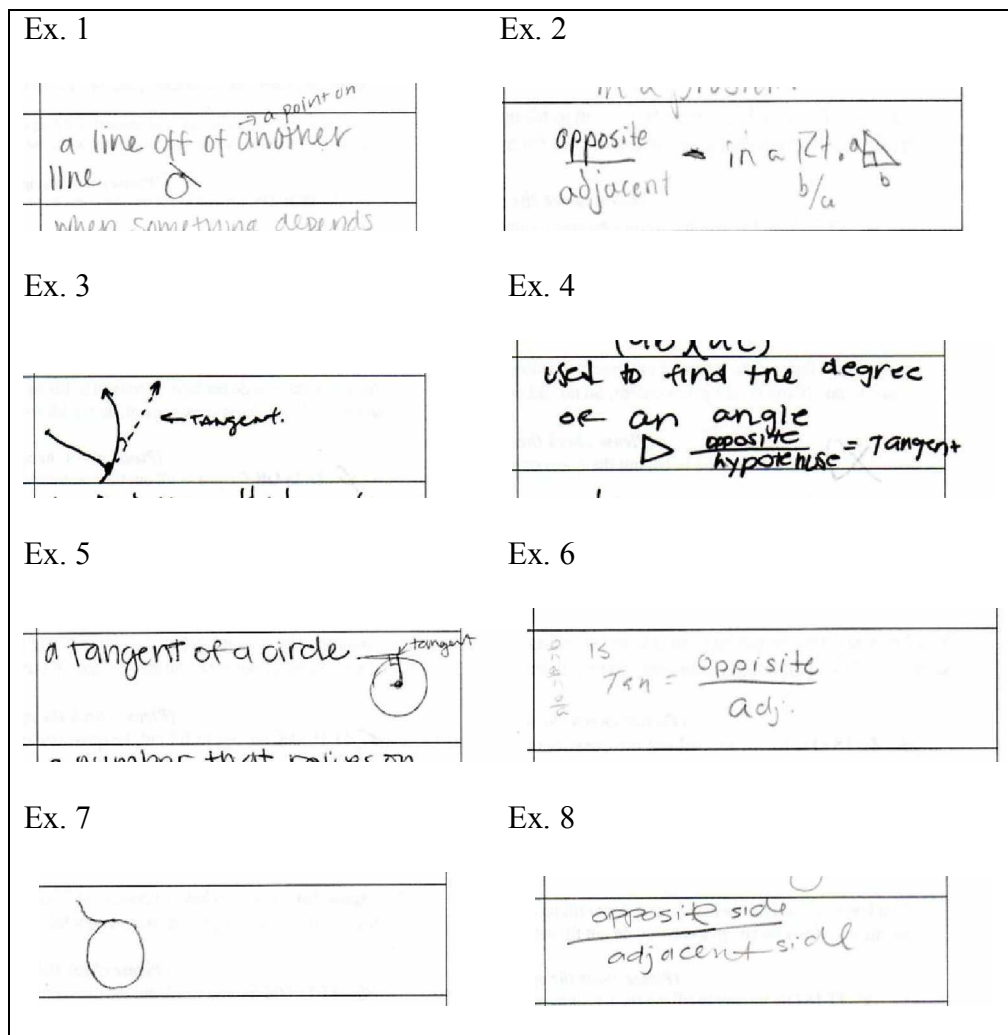


Figure 4.5: Visual Representations of Tangent

The word: tangent.

The drawings that students used had a specific connection to a specific word. For example, with the word *tangent*, students provided two different visual cues; each represented a different situational use of *tangent*. One visual cue was a circle and a line (Examples 1 and 5 in Figure 4.5). Students did not draw a random circle and line; the students drew the circle and line to touch in one place so that the line is adjacent to the edge of the circle. One student wrote: "A line touching the circle at one point that is,

tangent to the circle.” Other drawings depicted what looked like two curves touching at one point and another depicted a line touching a graph of a function at one point (Examples 3 and 7 in Figure 4.5). In all of these types of drawings, one of the shapes was always curved and most often a circle. Only 7% of the students used these types of drawings while 24% of the students wrote a description of “a line touching a circle in one place” (Table 4.8)

Table 4.8: Survey Responses for *tangent*

Everyday meanings	Percent response	Mathematical Meaning	Percent Response
To go off topic when talking	21%	Drawing of line tangent to circle, line tangent to curve, etc.	7%
Only use in math not everyday	6%	A line touching a circle in one place	24%
Trigonometry, angle	35%	Trigonometry, angle, tangent, tan	29%
Blank or “not sure”	38%	Drawing of the right triangle tangent ratio	8%
		Blank or “not sure”	32%

The other visual cue for tangent was words in a drawing that referenced the derivation of the trigonometric ratio from a geometric figure (Examples 2, 4, 6, and 8 in Figure 4.5). Interestingly, the tangent of an angle or the tangent function is derived from using the Unit Circle and the line tangent to the Unit Circle at the coordinate point (1,0). No student made that connection however this particular derivation is not in the text book used in the course. I did not see this derivation in the annotated notes either. The derivations of trigonometric ratios using the Unit Circle lead to the notion of right triangle trigonometric ratios. The right triangle is commonly used to introduce

trigonometric ratios. This method is described in the textbook and was in the annotated notes for both lecturers. Using the right triangle, the tangent of an angle is equal to the ratio of the side opposite from that angle over the side adjacent to that angle. Students could have drawn the rectangular graph of the tangent function but none did. The students seemed to hone in on the right triangle ratio. This preference may have occurred because as noted before the right triangle is usually used to introduce trigonometric functions. From the review of annotated notes, the instructors spent more time on the right triangle ratios than on the rectangular graph of the tangent function. I placed this example in this section because although students used words for the meaning, they configured more than one word into a ratio and not a sentence. Many students, 38% and 32% respectively, did not know or give an everyday or mathematical meaning. Another 35% and 29% respectively of the students gave “tan”, “tangent” or “trig” and “angle” as both the everyday and mathematical meaning (Table 4.7).

The phrase: one-to-one.

The phrase *one-to-one* was on the post-course survey. In the Precalculus course under consideration for this study, *one-to-one* functions were introduced in the sixth week of the quarter. Understanding the properties of *one-to-one* functions is essential to understanding the properties of inverse functions. Inverse functions are a major concept covered in the last four weeks of the quarter. So *one-to-one* functions are also critical however, student understanding of the properties of *one-to-one* functions is usually limited to the properties observable on the graphs of *one-to-one* functions. The observable property is that the graph of a *one-to-one* function either always increases (moves upward from left to right) or always decreases (moves downward from left to

right) on its domain. This observable property can be verified by the Horizontal Line Test.

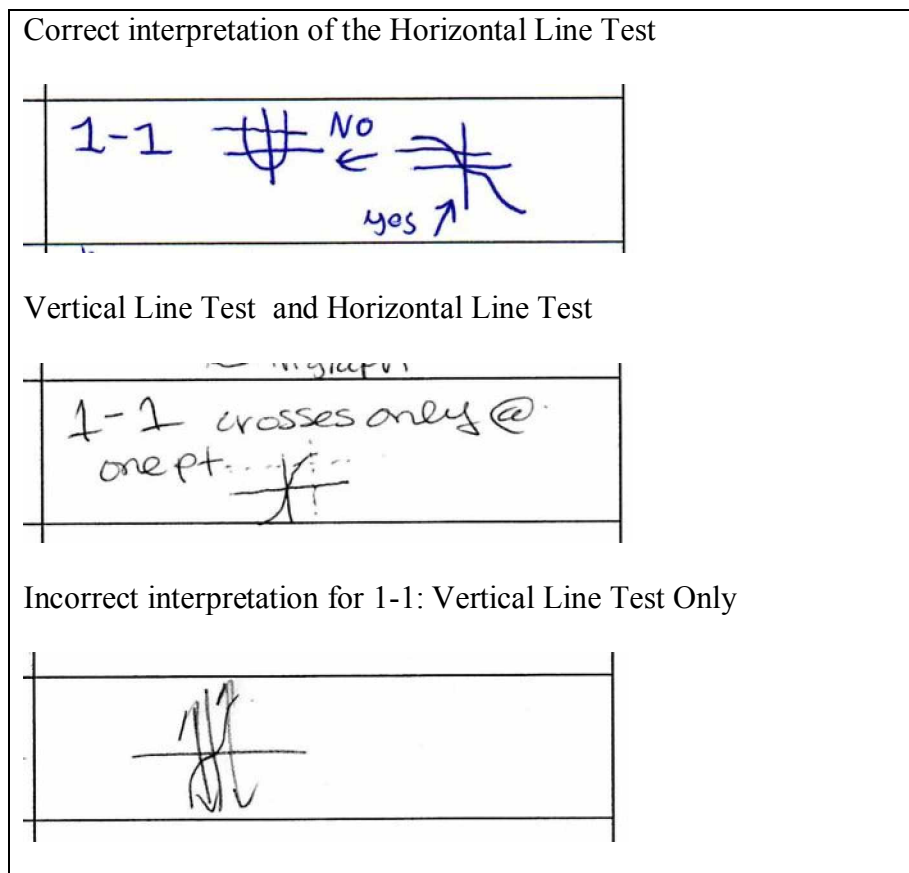


Figure 4.6: Visual Representations of One-to-One

Students did give visual representations for *one-to-one* functions. These visual representations, 22% of the responses (Table 4.9), were for the most part a demonstration of the Horizontal Line Test (Figure 4.6). In some cases, students, 21% of responses (Table 4.9) wrote “horizontal line test” rather than draw a picture. Some students gave a demonstration of the Vertical Line Test (Figure 4.6) – a few with the Horizontal Line Test and a few with only the Vertical Line Test. The Vertical Line Test

is used to confirm that a drawing or graph of a relation is a function of x . The Horizontal Line Test is used to confirm that the function of x is also a *one-to-one* function of x . So it is not necessarily incorrect to give the Horizontal Line Test or the Horizontal Line Test and the Vertical Line Test – however it may be demonstrating that the student has partial or limited knowledge of the meaning of *one-to-one*. To give only the Vertical Line Test is incorrect. For some students, the “test” – correct or incorrect – became the meaning of *one-to-one* even though the test is only a procedure to confirm the properties of *one-to-one* functions if a graph of the function is available (as opposed to an algebraic, exponential, or trigonometric equation).

Table 4.9: Survey Responses for *one to one*

Everyday meanings	Percent response	Mathematical Meaning	Percent Response
Talking one person to another, private conversation	31%	Drawing a graph of a function of x with a horizontal line, and sometimes a vertical line as well	22%
Relationship, couple	4%	The Horizontal Line Test	21%
Only use in math not everyday, horizontal line test	17%	Function is invertible	28%
Blank or “not sure”	48%	Incorrect meaning or drawing	9%
		Blank or “not sure”	20%

The problem with the drawings was that there was no way of knowing the level of understanding the student had about the drawing. We don’t know if they were producing the drawing from a deeper understanding of the properties of *one-to-one* functions (only increasing or decreasing on its domain). Or if they were reproducing a

drawing or procedure that they connect to *one-to-one*, because the Horizontal Line Test is a common strategy that instructors employ as a learning aid. The response given most often, 28% (Table 4.9), was that *one-to-one* means that a function is invertible – that is, the function has an inverse. In fact, to establish the parameters of an inverse function, one first must determine whether the function is *one-to-one* or not. If the function is *one-to-one* on its domain then the function is invertible. So “invertibility” is a logical meaning for students to assign to *one-to-one*.

The word: negative.

Students provided a number of different visual cues to communicate the same meaning they were assigning to *negative* (Figure 4.7). The symbol “ < 0 ” (meaning less than zero), the numbers “-1, -2, -3 ...”, the drawing of a real number line shaded to the left of zero, and the words “numbers to the left of zero” are all ways of indicating negative numbers – although “-1, -2, -3 ...” only represents negative integers. Table 4.10 shows that six different ways to represent *negative* numbers were reported. Moreover, the percentage response rates were fairly even in distribution. Like *product* and *even*, *negative* should have produced a number of correct responses (approximately 2% of the responses for the mathematical meaning of *negative* were “not sure”) however the concept of a *negative* number is not the easiest concept to understand (Lakoff and Nunez, 2000). The number of different ways (symbols, numbers, drawings, and words) in which students represented *negative* numbers could be a confirmation of the difficulty in understanding or communicating the concept of *negative* as opposed to another concept like *product* in which students tended to give one mathematical meaning – the result of multiplication. But with *negative*, students

often used *negative* to describe meaning – that is, “a negative number” or “negative numbers.” In some cases, a student would write a random negative number like “-7”, “-2”, and “-5.”

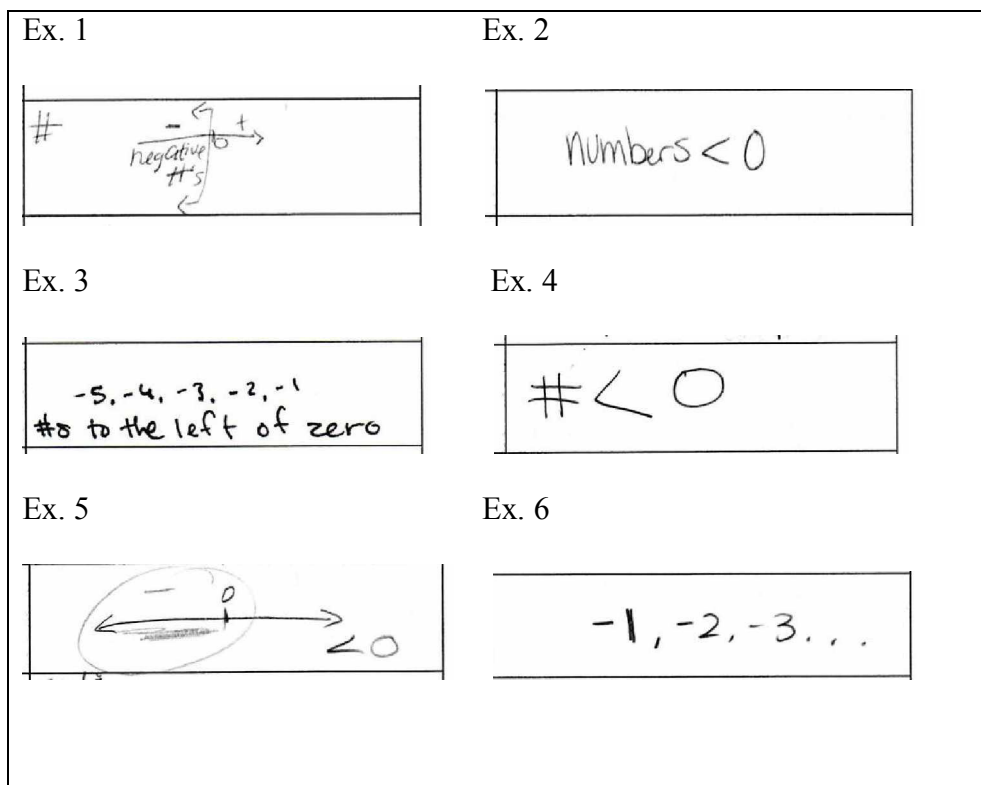


Figure 4.7: Visual Representations of Negative

Table 4.10: Survey Responses for the Mathematical Meaning for *negative*

Mathematical Meaning	Percent Response
A negative number, negative numbers	18%
Numbers less than zero, or < 0	21%
(list of integers) -1, -2, -3,... or -1,-2,-3,-4,-5	18%
(only one number) -7 or -2 or -4, etc	11%
Drawing of a real number line shaded to the left of zero	14%
Numbers to the left of zero	16%
Blank or “not sure”	2%

The word: transformation.

Visual cues can convey a wealth of meaning however, a visual cue needs other reinforcements to successfully situate student mathematical understanding. The word *transformation* was on the pre- and post-course surveys. *Transformation* was repeated on the post-course survey because 69% of the responses for mathematical meaning on the pre-course survey were “not sure”, “don’t know” or left blank. On the post-course survey, this response dropped to 30%. In addition to the drop in “not sure” there was an increase in students (55%) relating *transformation* to the change in a graph through shifting, reflecting or stretching or wrote some form of the equation, $y = a(x - h)^2 + k$. In fact, no student gave this type of response on the pre-course survey. I looked at the student-annotated notes to see how the instructors were explaining *transformation*. The instructors discussed transformations of the graphs of function in six lectures over the 10-week course, 20% of the lectures. In addition, the instructors provided additional supports to the visual cues of the graph by using and writing out the transformation in words and gave procedures to perform the transformation. It was clear in the collected annotated notes that both instructors consistently connected transformations to changes in a graph. The notion was understood well enough by the students that only one student used a graph or drawing to display a correct mathematical meaning for transformation on the post-course survey. (Other reported graphs were incorrect.)

The phrase: angle of depression.

In other situations reinforcement to visual cues is necessary to develop student comprehension when the student does not understand the word or phrase connected to the visual cue. Two note-takers had particular problems with the notion of an *angle of*

depression. During separate interviews both note-takers brought up and asked about an example that included an *angle of depression* (Table 4.11). I looked at the students' annotated notes, the drawing in the notes seemed clear and the *angle of depression* was implicit in the drawing. Both note-takers said they understood the mathematical procedures in solving the given example. The issue for both was the meaning for the *angle of depression* however each had different issues.

For Melinda, she had no mathematical meaning for the assumed knowledge of depression as being a decline or drop, for example, an angle dropping down from a horizontal axis (*an angle of depression*). For her the meaning was depression: "I think of depression like being depressed, you know being sad." I discussed this example in the section regarding cultural meaning systems instead and will focus on the other note-taker.

Alicia's confusion over *angle of depression* was a better fit for the notion of meaning and visual cue. Alicia wanted to have a better understanding for why the picture, the visual cue, of the angle was drawn. She wanted meaning for conditions in relation to how the concept of angle of depression fit with the techniques being used and how to draw a correct triangle. She wanted to connect to visual cues to mathematical procedure. She was intent on having the necessary supports for understanding the visual cue.

Visual representations, graphs, symbols, numbers, etc., can be used to communicate mathematical meaning and situate student understanding. For this communication to be most effective the relative nature of the connotations and conditions for the visual representations must be clearly and repeatedly explained.

Moreover, shared knowledge about the meaning and conditions of the visual cues should not be assumed.

Table 4.11: Interview Excerpts: Angle of Depression

Melinda's Interview

Melinda: I kinda get how he did it. I know why he used the tangent and the steps he took to solve it.

I: Angle of depression, what does that mean?

Melinda: I don't know . . . I think of depression as being depressed . . . you know being sad.

I: Did he (instructor) describe it?

Melinda: No

Alicia's Interview

Alicia: And then page 15 that was when we hit the word problems and that was just confusing because she came up . . . she explained angle of depression and just the concept of angle of depression and using it in word problems was confusing.

I: Had you heard this before?

Alicia: No, I don't remember doing this kind of word problem and then the technique . . . how do we come to using these techniques with angle of depression. Just I mean . . . even though I know that . . . ok I can take these steps but I have a question mark because I am like . . . how do I get to those steps? How do I come to the conclusion that I have to use those steps in the word problem? . . . And then we go on to doing an actual word problem . . . and I guess the concept of getting like sine of 30 degrees is clear to me but just how we know how to draw the angle of depression triangle is what confused me. I wasn't sure . . . like . . . she did give an example. Like . . . if you are looking a certain eye level then you would go up or down. If it was an angle of depression or the other angle (laughter) which I don't remember at this time but just knowing which way to draw the triangle when it is angle of depression which is what confused me.

4.4 Assigning Meaning to the Mathematics Register: Objectivism and Cultural Meaning Systems

In this section, cultural meaning systems and the conflicts between it and objectivism in the mathematical meaning-making process are examined. To provide context for this discussion, I recall the “The Objectivists Theory of Communication:”

Meanings are objects.
Linguistic expressions are objects.
Linguistic expressions have meanings (in them).
In communication, a speaker sends a fixed meaning to a hearer
via the linguistic expression associated with that meaning.⁷

So from an objectivist point of view math is math. Like the communication describe above, mathematics exists outside of ourselves and the interactions we have with the world. Thus mathematics is an assumed and a shared knowledge. An objectivist would say there is no cultural perspective to mathematical meaning. The objectivists’ view of teaching of mathematics is much like the objectivists’ theory of communication. The mathematics lecture structure can be seen as an objectivist structure. The lecturer sends a fixed mathematical meaning to the students via the mathematics register associated with that meaning. If a student does not learn the mathematics, then it is due to subjective errors, the lecturer misuses the mathematics register or the lecturer is misunderstood (Lakoff and Johnson, 2003) because the assumed and shared knowledge never changes.

⁷ p. 206. Lakoff and Johnson . 2003.

When students assigned symbolic meanings to the mathematical register, they displayed objectivist qualities – whether this depended on the Precalculus lecture structure is not particularly clear. In contrast, when students assigned skewed meanings to the mathematics register, the students seemed to be reacting to two things. The skewed meanings were the results of multiple meanings and situational uses, thus mathematics is not as simplistic as having one way of doing or understanding a concept. The skewed meanings could also have been a result of students reacting to terms in the mathematics register from a cultural perspective, for example connecting *identity* to identify or identification.

The objectivists' mathematical understanding of *identity* is that “it is a statement that is always true.” Students in the Precalculus course did not share this mathematical meaning. Approximately 46% (Table 4.6) of the students gave everyday meanings for *identity* that matched “the fact of being who or what a person or thing is.”⁸ Although this everyday meaning was correct, these students connected this meaning to their mathematical meanings. Statements such as “what the real value of the variable is,” “what the number is,” and “a number’s true self” emphasized the character or essence of something mathematical (variable, number) rather than a statement or equation always being true. So in the case of *identity*, we see that a correct everyday meaning can have consequences on mathematical meaning-making. One could say that the additive identity, 0, keeps the essence of a number when zero is added to it. (i.e., $a + 0 = a$). Approximately, 20% of the responses assigned additive or multiplicative identity to

⁸ identity. (n.d.). *AskOxford.com Oxford University Press*. Retrieved May 24, 2007, from AskOxford.com website: http://www.askoxford.com/concise_oed/identity?view=ok

the mathematical meaning (Table 4.6). The responses were either equations or the numbers “0” or “1.” The cultural meaning of *identity* as character or essence was strongly held by the students to the point that some ascribed the meaning “how you are identified to the world.” This notion of character, essence and identify dominated student assignment of mathematical meaning for the word *identity*.

A student may be assuming that since they have such a strong feeling or understanding of an everyday meaning that meaning then also applies to the mathematical meaning and, why not? If the notion of *identity* as something true never impinged on how the student interacted with their world, including their previous mathematics classes then why not assign the notion of character and essence to the mathematical meaning.

The word: imaginary.

Imaginary is another example similar to *identity*. The common responses to the everyday meaning of *imaginary* were “nonexistent” or “make believe.” About 30% of the mathematical meanings were also “nonexistent” or “make believe” however 50% gave the symbol (object) “ i ” as the mathematical meaning. “ i ” is the symbol that represents the square root of -1. Although “ i ” was the common response, this response did not mean that students did not believe that *imaginary* numbers were make believe or didn’t really exist. Rather, “ i ” was the objectivist response to *imaginary*.

Most Precalculus students don’t have a mathematical context for how understanding the existence of “ i ” impacts the way they interact with their world. So believing that “ i ” is “make believe” has no mathematical or everyday consequences particularly if the student can correctly use the mathematical object “ i ” in basic

mathematics procedures. So objectivism and cultural meaning systems coexist in the student mathematical meaning-making but in this case neither progress student conceptual understanding.

The word: inequality.

Many students had definite responses to the everyday meaning for *inequality*. *Inequality* was “unfair” and “unequal.” In fact, students produced longer everyday descriptions about *inequality* than any other word on the pre-and post-course survey. Interestingly, for the students who assigned “unfair” and/or “unequal” to the everyday, the predominant mathematical meaning these students assigned was unequal or “ \neq .” For a few, the mathematical meaning was less than or “ $<$.” These students however never gave greater than or “ $>$ ” as a single response. If the term greater than or “ $>$ ” showed up it was with the other two options (unequal or less than). Similar to *identity*, the everyday meaning for *inequality* carried a definite social or political meaning for some students. Students who provided everyday meanings about unfairness, discrimination, etc. tended to assign unequal to the mathematical meaning. In contrast, an opposite direction on assigning meaning could be happening in the case of *inequality*. Students may have such a confirmed understanding that *inequality* in mathematics means unequal they then ascribed the same meaning for everyday.

The phrase: half-life.

Lastly, the phrases *angle of depression* and *half-life* provided glimpses into the conflict of cultural meaning systems with the objectivist view that mathematical meanings are assumed and shared. For both phrases, the student note-takers understood the procedural aspects of solving the specific types of word problems. The *angle of*

depression was used in right triangle word problems and *half-life* was used in growth and decay word problems. The *angle of depression* was problematic for Alicia and Melinda because they did not quite comprehend the connotation of depression as part of the angle. Melinda had some understanding about right triangle word problems. She did not understand why depression would be used to describe an angle. In Melinda's case as well as Alicia's, the way in which Melinda and Alicia had been interacting with their world did not necessitate knowing about the *angle of depression*.

Similarly, Alicia's confusion over the phrase *half-life* was connected to connotation and procedure (Table 4.12). Alicia's conflict was that she related *half-life* to the notion of *half a life*. During the part of the interview when we discussed *half-life*, Alicia's facial expressions and physical reaction indicated that she seemed to find the idea of *half a life* morally repugnant. The idea that someone or something could have only *half a life* seemed to be causing the barrier for her to solving *half-life* problems. Once we discussed the concept of *half-life* that it is the amount of time it takes for a substance to lose $\frac{1}{2}$ its mass rather than something having *half a life*, Alicia was able to complete the *half-life* problems without any assistance. That she needed no further explanation to do a *half-life* problem while I watch indicated that she had the mathematical skills to complete exponential word problems but the notion of *half a life* was keeping her from doing the specific exponential problems, *half-life*.

Table 4.12: Interview Excerpt about Half-Life versus Half a Life

I:	Ok...what was happening there?
Alicia:	This is when we get to half-life and ... oh it was more of the bottom part (of the page)
I:	Okso the strategies are okay
Alicia:	Yeah ...
I:	It's the radio active decay?
Alicia:	Yeah ... so it was more just when we got into half life and the "A" naught and all that stuff
I:	Ok so what would you say is most confusing about this?
Alicia:	Um I think the half-life... the half-life is what's confusing me the most...I mean . . . just . . . because . . . how can you have <i>half of a life</i> . . . (long pause) ...and I mean just connecting that concept to math basically and how we use it with math...

4.5 Conclusion

This section examined the results of the study in terms of answering the first two research questions.

(Research Question One) What meanings do students employ in using the mathematics register when there are multiple mathematical and everyday meanings, metaphors and symbols that can represent terminology in the mathematics register?

Students had limited capabilities in providing a meaning for words both in the everyday and mathematics. The rate of “not sure” was higher in the mathematics use.

However I was surprised to see students writing not sure for everyday meanings for words such as *inequality*, *variable* and *composition*. Students also used the word itself to define it. The meaning written by one student about root: “one of these things you can’t explain but still know,” exemplifies the notion that the word itself embodies the meaning and needs little explanation. Students seemed to add to the multiple meanings for many of the words in the register because they would give definitions that would be just slightly skewed from a commonly used definition.

Students employed a variety of meanings in using the mathematics register. The multiple meanings that exist in everyday and mathematical use were certainly confusing. However, students seemed to compensate for multiple meanings by assigning similar meaning to everyday and mathematics use. Even in cases where the mathematics meaning was used both in everyday and math contexts. For example, about 35% of the students gave a trigonometric meaning to the everyday use of *tangent* (Table 4.8). The implication was that *tangent* only has a mathematical meaning. In some cases, students would write “same meaning” or “same meaning as math.” In a couple of cases for *tangent*, *one-to-one*, and *interval*, students wrote “never use” under everyday meaning.

(Research Question Two) How are students assigning meaning to specific words and terminology in the mathematics register?

Depending on how students assigned meaning, the students demonstrated ability to interact in the specialized mathematics discourse community or remained on the periphery of the community. For example, the student can better navigate the specialized mathematics discourse community by demonstrating ability to understand

and to employ the phrase “for all” or “for all values” to imply truth. Consequently, connotation and condition seemed to be major factors in assigning or not assigning mathematical meaning. Some assigned meanings were clearly given as a result of taking the Precalculus course. For example, students were better able to assign meaning to *transformation* at the end of the course after the concept had been covered in the lecture. However, students’ inability to give a mathematical meaning to *identity* did not change over the course of the quarter.

When a definite everyday meaning was being assigned by many students the meaning tended to manifest in assigned mathematical meanings. *Inequality* became unequal rather than a range of possible solutions to an expression given numerical boundaries. In other cases definite meaning about symbols emerged. The symbol $\sqrt{\quad}$ tended to embody meaning for the words *root* and *radical*, and the symbol x embodied the word *variable*. Definite meaning was never clearly assigned to *angle of depression* by Melinda. Alicia did however develop a more definite meaning for *angle of depression* and *half-life*. Melinda and Alicia may have continued to assign meaning from their existing cultural meaning systems. However, Alicia struggles with assigning a new meaning by asking questions and trying to understand the uses and connotations of particular notions like *angle of depression* and *half-life*.

Lastly, I return to a problem with visual cues (symbols, graphs, numbers, etc.) that students submitted as meaning. The level of understanding the students connected to the visual cue was difficult to determine. Students could have been producing the visual cue from a deeper understanding of the concept. In comparison, the students could have been reproducing the visual cues by imitating the meaning-making process

displayed in the lecture structure. In this case assumed shared meanings and conditions are used but not necessarily understood. This sentiment could be applied to how students' assigned meanings to the mathematics register overall.

Chapter 5: Developing versus Replicating Meaning-Making

5.1 Introduction

In this chapter, I attempted to answer the third focused research question, actually set of questions:

(Research Question Three) How do students perceive the meaning-making process within the Precalculus lecture structure? How do students insert themselves within the meaning-making process? And, where do students place their meaning-making process within the larger context the specialized Precalculus discourse community?

Case studies describing student meaning-making in each lecture was employed to examine the set of focused research questions. Overall, the lectures were similar to other college lectures of medium size, approximately 75 students. The lectures were traditional in college pedagogy (Brown & Race, 2002; Leamson 1999). The instructor displayed a meaning making process at the front of the class. Students took notes to record the meaning making process. The annotated notes from volunteer student note-takers were collected as an artifact representing the meaning making process occurring in the Precalculus lecture. Discussions about these notes with note-takers are also included in the case study. The analyses of the case studies focus on:

1. (Student perception of the meaning-making process) the annotated notes;
2. (How the student inserted themselves in the meaning-making process) Student interaction with their annotated notes and instructors; and

3. (Where they place their meaning-making process within the larger context of the specialized Precalculus discourse community) Student facility with the mathematics register and whether they produced or imitated meaning making.

Two case studies follow: one case study from Lecture A with the instructor of record, John and one case study from Lecture B with instructor of record, Mary. (Pseudonyms have been used for the instructors and student note-takers.)

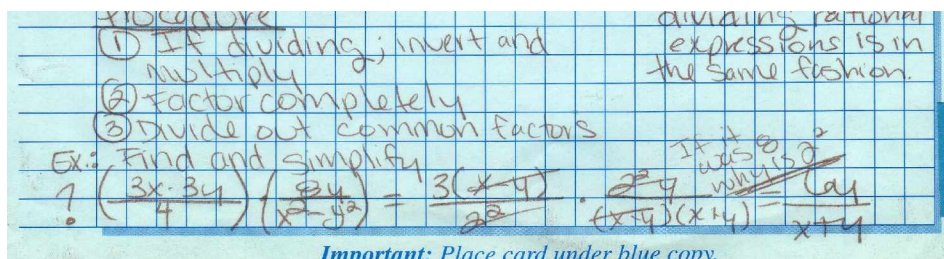
5.2 Lecture A and Collected Annotated Notes: Diana, Sally and Melinda

5.2.1 Perceptions about the Lecture

Diana, Sally and Melinda were three student note-takers in John's lecture. After reviewing the notes from all the note-takers in John's lecture, it became apparent that John wrote a lot of information on the chalkboard. In the case of Diana, her notes seem less crowded and she did not write everything that John wrote on the board. This was apparent when comparing her notes to other note-takers. During one interview, Diana also said she didn't write everything the instructor wrote on the board. Sally seemed to write almost everything down and her note pages, although crowded, are neat and organized. Melinda seemed to write everything down as best she could. Her notes feel crowded and slightly disorganized.

Within the first pages of the annotated-notes, one could see how Diana, Melinda and Sally perceived the lecture in the notes. Melinda was having trouble keeping up with the pace of the lecture immediately. Melinda (Figure 5.1) was missing an example

that Diana (Figure 5.2) and Sally had on their notes. Melinda did not understand the example preceding the missing example. She placed a question mark on the notes and we spoke about it during an interview. She “slowed down because she didn’t understand the example” and then missed the next example that John gave.



The missing example should have been here.

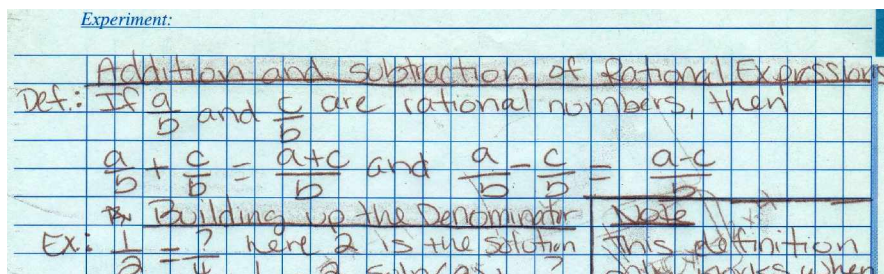


Figure 5.1: Melinda’s Notes: Missing an example from lecture bottom of page 2 and top of page 3

Melinda’s notes were almost overwhelming with the amount of writing on each page of her notes. The notes in certain sections were crowded and hard to read. You could almost feel the sense of panic and pressing weight of the class that she often displayed during interviews. In the last interview, Melinda said that John wrote too

many examples and definitions, but didn't explain them. She felt that she couldn't listen to him because she was always writing. She would have like to have "seen more explaining and less writing."

The dark blue rectangular marker indicates the example that Melinda missed.

• invert & multiply
 • factor completely
 • divide out common factor.

+ Ex: find & simplify (Multiply)

$$\left(\frac{3x-3y}{4}\right) \cdot \left(\frac{8y}{x^2-y^2}\right) = \frac{2(x-y)}{2^2} \cdot \frac{2y}{(x-y)(x+y)}$$

$$= \frac{2y}{x+y}$$

+ Ex: $\left(\frac{2+x}{2}\right) \div \left(\frac{x^2-4}{4}\right)$ (Divide)

$$= \left(\frac{2+x}{2}\right) \cdot \left(\frac{4}{x^2-4}\right) \leftarrow \text{invert}$$

$$= \frac{2+x}{2} \cdot \frac{2^2}{(x+2)(x-2)} \leftarrow \text{factor out completely \& cancel out common factor}$$

$$= \frac{2}{x-2}$$

Addition and subtraction of Rational Expressions.
 + - If $\frac{a}{b}$ & $\frac{c}{d}$ are rational #'s then $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$
 • only work when there's a common denominator.
 • must find one if one does not exist (find LCD)

Figure 5.2: Diana's Notes on same material in Melinda's notes in Figure 5.1

In contrast, Diana's notes are clear easy to follow and concise. They feel light in comparison to Melinda's notes. Diana however seemed more comfortable with the mathematics than Melinda and seemed worry-free about the class in our meetings. Sally's notes were similar to Melinda's but she was more similar to Diana in that she was also worry-free about the class. Certainly Diana and Sally cared about the class; they just seemed to display more control over the material than did Melinda. Melinda thought that John covered too much information and gave too many examples (Table 5.1). In reviewing Melinda's notes, I found them crowded and they seemed to be

nothing more than a copy notes from the board. Sally, on the other hand, thought the class was just like high school not too fast and not too slow. Her description of the lecture was, “We copied notes, then we would do this examples.” Sally described a pretty typical math lecture at the college where study took place. The only difference that Sally noted was that John could not walk around and check with students (Table 5.2). Sally also liked math where as Melinda did not. Interestingly, Diana seemed indifferent; she was taking the class to fill a general education requirement.

Table 5.1: Interview with Melinda about the pace of the lecture

I: When you are sitting in lecture do you feel like you are understanding everything that is being said?

Melinda: No

I: So what do you feel you are doing in lecture?

Melinda: Taking notes... writing what he’s writing on the board

I: Do you go back and look at your notes?

Melinda: When I do my homework but otherwise no

I: How does John compare to other math teachers you’ve had?

Melinda: John writes too much examples and he just writes the definitions write out of the book but didn’t talk about it

Table 5.2: Interview with Sally about the pace of the lecture

Sally: ...mmm ... I guess they were always just... I don't know how to explain ... everything just made sense. I usually had a good relationship with my math teachers. So they would always give me their undivided attention. I don't know ... it was always like they always helped me. They were always really helpful with me cause they knew I liked math.

I: So if you had to compare your professor with your high school teachers what would be the similarities and differences?

Sally: Well ...he (John) was good like he has ...uh... its basically almost the same thing. We copied notes, then we would do examples. The only thing that's different... but of course its different from high school... not college... its just that they (high school teachers) go around and make sure you know everything. John didn't have time to do that. So they're not that much different. It s just that in high school they're more helpful because they have the time to go around and ask questions.

5.2.2 Student Interaction with the Mathematics

Diana inserted herself into the notes immediately. She annotated her notes in the example in Figure 5.2 near the dark blue tab. The annotation was “factor out completely and cancel out common factors.” Although Diana’s annotations highlighted procedures, the procedures were not evident on Sally’s notes. An enlargement of Diana’s annotations is below (Figure 5.3). Diana had said “I like to put math in my own words.” Her own words or annotations were found throughout her notes (Appendix A). She also was selective about the information that she would write down. This reason was why her notes seemed less crowded than Melinda’s and Sally’s. Melinda’s and Sally’s notes contained more writing than Diana’s but it was easy to

follow Diana's notes. In addition, the content of mathematics that the instructor had written on the board did not seem to be compromised by Diana's simultaneous editing. Diana annotated important reminders as well. On the page of notes in Figure 5.4 she laid out the work for the distance-rate-time word problem in such a way that her notes were easy to follow. Melinda's and Sally's notes for this same example were less organized (Figures 5.5 and 5.6 respectively).

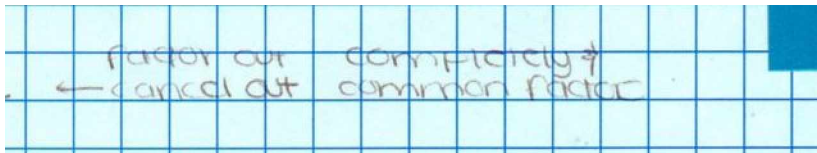


Figure 5.3: Diana's Annotations on her Notes

Diana annotated the reminder: "ALWAYS DEFINE VARIABLES." Comparing Diana's notes with Sally's and Melinda's, I found that Diana continued the problem beyond Sally and Melinda. After the first variable value was found, Diana continued find the values of all the other variables although it was not necessary. Neither Melinda nor Sally did this which makes me believe that neither did the instructor. Diana may have done this because it was a practice that she had learned from another instructor. I found this specific example of differences between Diana's, Sally's and Melinda's notes after the data collection was completed. So I did not have the opportunity to ask Diana why she found the values for all the variables. Melinda's notes were hard to follow particularly with the problems spanning two pages. Melinda's notes may have been clearer if she would have started the example on a new

page in the notebook. Sally's notes were easier to follow but her work jumps around the page. Word problems were Sally's least favorite.

25

Name: #12 Date: 10/11/04

Experiment:

+ Distance/Rate problems.
- recall: $d=rt$

+ Ex. on route to LA (90 miles away) a car leaves San Diego driving with heavy traffic which makes it slow. Assuming you can drive twice as fast when there is no traffic it took the entire trip took $4\frac{1}{2}$ hours. How fast does the car drive during rush hours.

90 =

let t_1 = time of first trip
let t_2 = time of second trip

t_2 = time of second trip
 t_2 = time of second trip * ALWAYS DEFINE VARIABLES *

$d_1 = d_2$

$r_1 t_1 = r_2 t_2$
 $4.5 r_1 = 2 r_2 t_2$
 $2 r_2 t_2 = 4.5 r_1$
 $r_2 t_2 = 2.25 r_1$

$r_1 t_1 = 2 r_2 t_2$
 $r_1 t_1 = 2 (2.25 r_1)$
 $r_1 t_1 = 4.5 r_1$
 $t_1 = 4.5$

$t_2 = 4.5 - t_1$
 $t_2 = 4.5 - 4.5$
 $t_2 = 0$

$r_1 = \frac{90}{4.5}$
 $r_1 = 20$
 $r_2 = 40$

+

Important: Place card under blue copy.

Figure 5.4: A page from Diana's Notes: Distance-Rate-Time Word Problem

Melinda's notes span the bottom of one page to the top of another.

Distance/Rate Problems $d = rt$
 $r = 17$
 Recall: $d = rt$
 Ex/ On a commute to LA (90 miles away) a car leaves San Diego during rush hour and returns when there is no traffic. Assuming you can drive twice as fast when there is no traffic and that the entire trip took 4.5 hours. How fast can the car drive during rush hour?
Important: Place card under blue copy.

Name: 13 Date: Oct. 11, 2006
 Experiment:

Let $t_1 =$ time of the 1st trip
 $r_1 =$ rate of the 1st trip
 $t_2 =$ time of the 2nd trip
 $r_2 =$ rate of the 2nd trip

$d = d_1 = d_2$ $r_1 t_1 = r_2 t_2 \rightarrow r_1 t_1 = 2r_1 t_2$
 $t_1 t_2 = 4.5 \rightarrow t_1 = 4.5/t_2$
 $2t_1 = t_2$ $t_2 = 4.5/t_1$
 $2(4.5/t_1) = 4.5/t_1$
 $9/t_1 = 4.5/t_1$
 $9 = 4.5$
 $3 = 1.5$
 $t_1 = 3$
 $t_2 = 1.5$
 $r_1 = \frac{90}{3} = 30$
 $r_2 = \frac{90}{1.5} = 60$

Chemical Solution Type Problems $t_1 = 3$

Ex/ A chemist has a solution containing a 10% concentration of acid and one containing 50% concentration of acid. How many milliliters of each solution should be mixed to obtain 47.5 ml of a solution having 20% concentration of acid?

Figure 5.5: A page from Melinda's Notes: Distance-Rate-Time Word Problem

Distance / Rate Problems

Recall: $d=rt$

On a commute to LA (90 miles away) a car leaves San Diego during rush hour and returns when there is no traffic. Assuming that you can drive twice as fast when there is no traffic and that the entire trip took 4.5 hours. How fast does car drive during rush hour

Let $t_1 =$ time of the 1st trip
 $r_1 =$ rate of the 1st trip
 $t_2 =$ time of the 2nd trip
 $r_2 =$ rate of the 2nd trip

$t_1 = 3$
 $t_2 = 1.5$

$d_1 = d_2$
 $r_1 t_1 = r_2 t_2$
 $t_1 + t_2 = 4.5$
 $2r_1 = r_2$
 $r_1 t_1 = 2r_1 t_2$

$t_2 = 4.5 - t_1$

$r_1 t_1 = 2r_1 (4.5 - t_1)$
 $t_1 = 2(4.5 - t_1)$
 $t_1 = 9 - 2t_1$
 $3t_1 = 9$
 $t_1 = 3$

$v = \frac{90}{3} = 30$

Chemical Solution Type Problem

ex: a chemist has a solution containing 12% concentration of acid and one containing a 50% of acid. How many milliliters of each solution should be mixed to obtain 475 milliliters of a solution having a 20% concentration of acid

Let $x =$ the amount of 12% solution need
 $y =$ the amount of 50% solution need

$x + y = 475$
 $0.12x + 0.5y = 20(x + y)$

$12x + 50y = 20(x + y)$
 $x = 7$

Important: Place card under blue cov.

Figure 5.6: A page from Sally's Notes: Distance-Rate-Time Word Problem

In Sally's case, she very rarely inserted herself in the notes. In fact, she did not make many annotations to review for this project. And her few annotations, she dismissed in the interview as "oh, I figured that out right after I wrote it down." She felt

everything was clear and the class was a review of material she already knew. Sally, more than the other two, seemed to have an objectivist view of mathematics. So inserting herself into the notes would not fit her view of how mathematics was done. In fact, she didn't "really pay attention to what it (in this case an Interest Function) means. [She] just did the equation or graph. [She] always understood everything just by problems." Sally said this while we were talking about words relating to concepts.

Table 5.3: Interview with Sally

Sally: The word problems... no those are the ones actually... yeah... the only ones that I do have trouble with are word problems...I don't like them.

I: What is it about them that you don't like?

Sally: I could never put them together.

I: Which ones were easier?

Sally: The age ones were easier than the distance equals rate times time

Sally: I have seen all the material before... except these kinds of word problems... I had through calculus in high school.

I: So when you are studying for the exam, how do you know what math vocabulary, what words are important to know?

Sally: Well I usually don't study vocabulary. I study only math like equations and the concepts.

I: How do you know which terms relate to which concept?

Sally: Um?

I: Like Interest Function, how do you know what that means?

Sally: I usually just go by the equation ...I don't really pay attention to what it means. I just do the equation...graph... I always understand everything just by problems.

In the interview excerpt (Table 5.3) with Sally, Sally voices an opinion about doing mathematics that she continually displayed during my data collection. She did not seem interested in what things meant – words, definitions, theorems. She seemed more interested in knowing basic procedures for solving equations; drawing graphs; finding inverse functions, to name a few. The word problems may have been troublesome for her because they do not always have a set procedure for solving them. My conversations with Sally always seemed to revolve back to algorithmic ways of doing mathematics whenever I tried to steer the interview to her meaning-making.

In Melinda's case, she inserted herself into her notes through the annotations for the project. The “?” became a way to ask for help. The number of “?” marks she placed in the notes seemed to also reflect her level of frustration with the instructor and the course. In working with Melinda on her questions after the interviews, I found that Melinda should have been enrolled in a College Algebra course (unfortunately not offered at the university in this study). Her skill level and comprehension of the material limited her to performing basic algorithmic processes. Although I spent a limited time with Melinda, I do not believe Melinda's skill level allowed her to develop substantive mathematical meanings to help her succeed in the Precalculus course.

5.2.3 The Mathematics Register: Developing versus Replicating Meaning-Making

In this section, I discuss developing versus imitating meaning-making. By developing meaning-making, I mean that the student note-takers displayed behaviors which indicated they were actively trying to understand the meanings of the

mathematical concepts represented in the mathematics register. In contrast, by imitating meaning-making, I mean that the student note-takers displayed behaviors which indicated they approached the mathematics using algorithms and rote memory of procedures.

Melinda had incredible difficulty with the mathematics register. In the end, she felt she had not mastered the vocabulary. One poignant example was her difficulty in understanding amplitude, phase shift, and period (Table 5.4 and Figure 5.7). In reviewing the notes with her, it was one word after the other that she did not know. She was frustrated because she felt John had assumed she would know the terminology and thus he did not explain it thoroughly. The circled words continued on for three pages. I asked at the end of the third page, “What happened here?” because her notes stop in the middle of a problem. She said she was writing so much to keep up that she no longer knew what he was saying. Given Melinda’s quiet nature, I can only presume she did not ask John for help because she was overwhelmed. During our meetings, she also indicated she felt that asking John for help would be a waste of time; she wouldn’t understand him.

Table 5.4: Interview with Melinda about question marks on her notes

I: What is it that made you put the question mark?

Melinda: I don't know what the theorem is saying. I don't know why it is like the absolute value of a?

I: Do you know what amplitude is?

Melinda: No

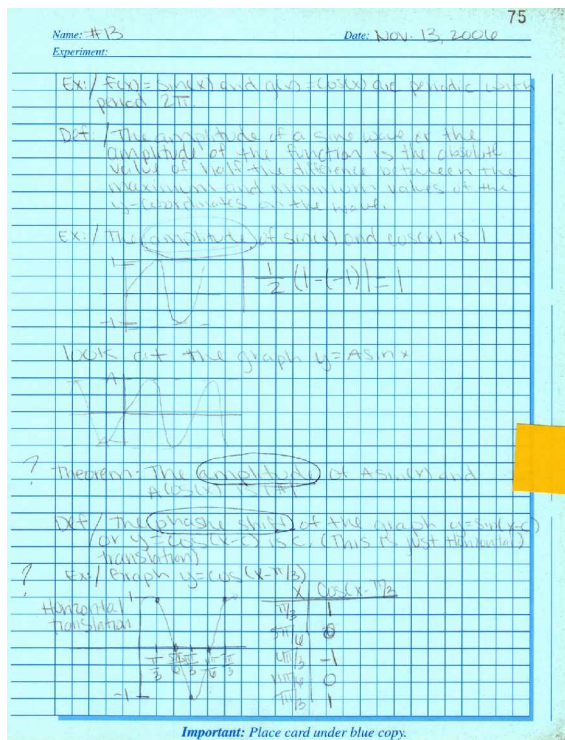
I: Ok, have you seen the word before?

Melinda: No. He just said it and didn't explain what it was.

I: Here...(on notes) ... is says phase shift. Do you know what that means?

Melinda: No

One page of the notes being referred to in interview:

**Figure 5.7:** Melinda's Annotated Notes referred to in Table 5.4

In comparing their post-course Mathematics Register Surveys (Appendix B), Diana had given everyday meanings for all 12 words and was unsure about 4 math meanings. She gave good descriptive mathematics meanings otherwise. Sally left 4 everyday meanings blank and those she gave were usually one word. She left 3 math meanings blank. Those she gave were correct but in some cases her meanings were not as descriptive as Diana's. Melinda seemed to misunderstand the survey. She left 5 everyday meanings blank but otherwise gave math meanings for the rest. In the math meaning column she gave symbolic examples, for example i for *imaginary*, -7 for *negative*. She did not do this in the pre-course survey although she wrote "not sure" for 11 out of the 24 entries. Melinda's responses on the pre- and post-course surveys reflected the low level of mathematics ability she displayed on her notes and in the interviews.

Diana seemed to have the best facility with the mathematics register. Because she liked to put things in her own words, this may have facilitated her capabilities with the mathematics register. In contrast, Sally saw no need to understand vocabulary at all. Doing math was working only with equations and problems. Diana and Sally had enough of a background in mathematics to be able to manage the amount of information presented in lecture. Most likely neither needed additional explanations for the course material in the same way Melinda needed. Diana did edit the notes from the chalkboard and would put things in her own words on her notes – adjusting and establishing her understanding of the material presented in the lecture. Of the three, Diana seemed to be

the only one actively developing mathematical meanings within the mathematics register.

Melinda's skill level, as discussed, limited her ability to develop mathematical meanings within the mathematics register. Unfortunately, this limitation seemed to lessen Melinda's confidence in her abilities as the course progressed and may have contributed to her filling out the post-course survey incorrectly when she had filled out the pre-course survey correctly. Sally, on the other hand, had the confidence and skill level to develop mathematical meanings but displayed no interest in developing meaning beyond solidifying her abilities to perform procedures for solving problems. Sally's conclusions about how she learned mathematics influenced me to review objectivism in particular with mathematics learning. Sally tended toward the notions that mathematics is algorithmic – a collection of objects, equations and operations connected by procedure – an objectivist's point of view. So long as she knew the procedures and her meaning-making was not challenged (as it was with word problems), Sally would be able to do the mathematics.

In the end, Diana seemed to produce mathematical meaning-making rather than imitating it. Sally seemed to embody the imitation of mathematical meaning-making. She felt she did not have to think about what mathematics means rather just do the problems. Melinda unfortunately never seemed to reach a comfort level in this class even to imitate meaning-making let alone develop it.

5.3 Lecture B and Collected Annotated Notes: Alicia, Linda, and Frank

5.3.1 Perceptions about the Lecture

Alicia, Linda and Frank were three note-takers in Mary's lecture. Unlike the note-takers in the other lecture, the notes did not immediately reveal students' perceptions of the lecture. However this aspect of their notes did develop over time. For Frank, Alicia and Linda, their notes were not as busy as Melinda's or Sally's. Overall, the note-takers from Mary's lecture turned in notes that were well spaced out and were not as overwhelming as the notes from John's class. Mary did not seem to write as much on the board as John. The note-takers in Mary's lecture also wrote down things that Mary said during lecture but did not write on the chalkboard. Overall, the note-takers in Mary's lecture felt that she was easy to follow and for the most part explained the mathematics clearly. This sentiment was reiterated during the interview with the possible exception of Frank.

Frank's perceptions of the lecture were the first of the three note-takers to become evident. Frank felt that Mary was okay as a teacher and would indicate when she did something that was helpful in his annotations on the notes. However, Frank had some antagonism toward and impatience with Mary. This sentiment came through in the notes at a variety of places and during the interviews. He made a note about her giving the wrong problem and answer (Figure 5.8). Since I didn't see these comments on other students' notes, I asked them about it. These students said she had made a mistake but they didn't bother to write down what she had done. The point did not seem to bother the other student note-takers as much as Frank.

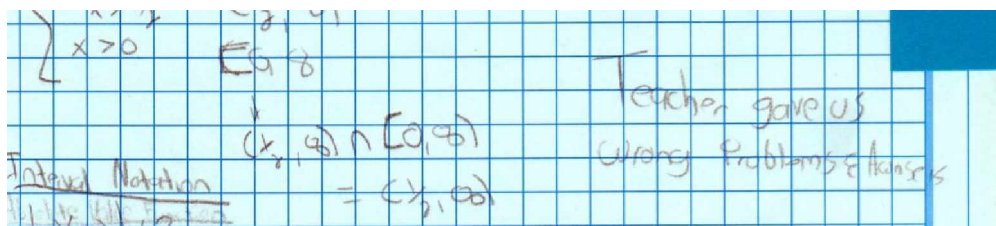


Figure 5.8: Frank's Notes: annotation about the instructor

At another point in the notes (Figure 5.9), Frank pointedly wrote “got lost.” His annotation expressed his frustration. He knew he could have just written a “?” mark. But he seemed to want to be able to express what he thought was an exercise in futility. He made a point of discussing it with me in the interview. What made matters worse for Frank in this case was that she referred to all the lines going everywhere as the “monster.” Frank did not appreciate Mary’s attempt to lighten the situation and would have appreciated a clearer explanation.

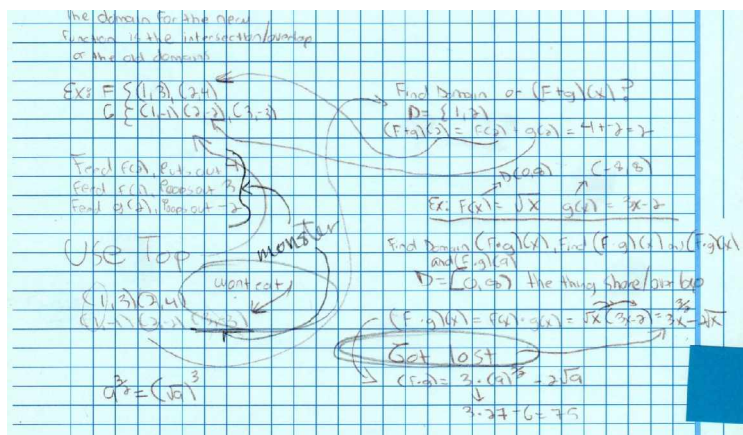


Figure 5.9: Frank's Notes: annotation that he “got lost”

Table 5.5: Interviews with Frank about the instructor

Excerpt 1

I: How does Mary compare with other math teachers that you have had?

Frank: She's ok...she's the same...she could explain better.

I: What do you mean?

Frank: Like... sometimes Mary will make a problem longer that it needs to be... like its really simple

Excerpt 2

I: Is there anything in particular that the instructor was saying that was confusing or was it the concept?

Frank: Well like some of the people in the class... they... uh ...I guess they're not that good at trig. So I guess the professor expects us to know that you know pi over 4 is 45 degrees and to know our 30 60 90 triangles and just to ...kinda like already have an idea of what this would be... you know ... so when she explained it. You know sometimes when we ask her to show us the full ... the full steps to do it and if not ... she'll just go from step a to c and skip step b because she's assuming we already know b. Lately like... I don't know... a lot of people they ask questions when they were doing the review. But when she was going over all the trig if somebody would ask a question... you know ...like let's say... show me how you got that? She ...she would like answer and then she would also tell us its something you would already have to know in order to do this.

I: Ok and your feeling was that you didn't know it

Frank: Yeah ...you know ... like I guess once you go into that class you're supposed to know all that trig stuff and how to graph. You know... what's the period and what's the phase shift.... the amplitude... stuff like that. You're supposed to already know it.

Frank was the only male note-taker in the project. With only one male, I could not say his antagonism had anything to do with gender. He never made an issue of her

gender in the interviews. The overall impression that I got was that he perceived that Mary was not treating the class as a college course. She made-up words to describe concepts and she overly explained problems when she didn't need to (Excerpt 1, Table 5.5). Yet in another situation, Mary did not explain well enough for Frank. He felt that she was assuming students should already know the material. In this case, he seemed more concerned for other students in the class (Excerpt 2, Table 5.5). In fact, he would often refer to his friends, two females, in the class and intimated that they shared his impatience with Mary's style. None of his friends were note-takers however so I could not confirm this sentiment.

Unlike Frank, Alicia and Linda gave a less biased observation of the lecture. Mary seemed to interact more with the students in her lecture than John. And, it seemed that she didn't write as much information on the chalkboard. In the end, Mary's and John's lecture styles was a matter of preference by the note-takers. Both Alicia and Linda liked Mary's style. Linda liked the silly words she would use and the clear explanations (Table 5.6). In an example, from Linda's and Alicia's notes, Mary had called the inverse the "undo button" (Figure 5.10 and 5.11 respectively). Frank also had it on his notes, written exactly like Linda's and Alicia's. Interestingly, the "undo button" occurred in Alicia's notes before it showed up in Frank's and Linda's notes. That is, Alicia had written "undo button" a few lecture examples before the lecture example where all the note-takers wrote "undo button." This may have happened because Alicia had the tendency of editing her notes as she took them, much like Diana in John's class. Alicia often wrote down cue words that Mary would say but not always

write on the chalkboard. So most likely, Mary was saying “undo button” before she wrote it down on the chalkboard.

Table 5.6: Interview with Linda

I: Does Mary ask questions in class?

Linda: Yeah...she ... asks if we are doing ok...

I: How do you think she can tell if the class understands?

Linda: ... she looks at our faces... I think if we look confused she explains it again ...

I: Do you think she gauged the class well

Linda: yes

I: how is Mary compared to your high school teachers?

Linda: ... she uses kinda silly things to refer the math problem. So it kinda keeps us awake and engaged ... its like...oh... funny and yet its math. So I would say that she is like my math teacher (high school teacher)

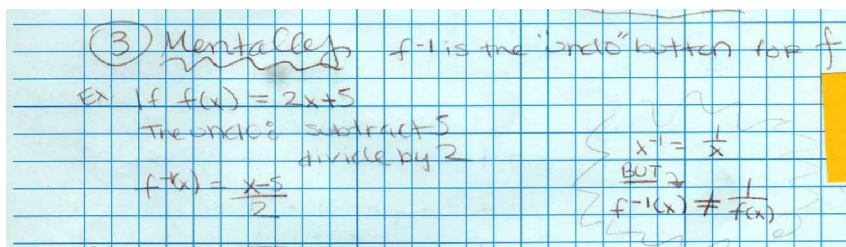


Figure 5.10: Linda’s Notes: annotation about the “undo” button

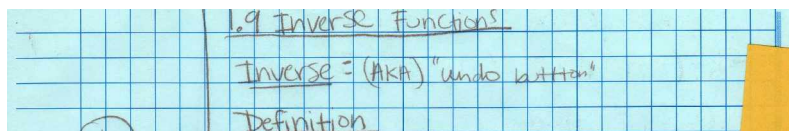


Figure 5.11: Alicia’s Notes: annotation about the “undo” button

5.3.2 Student Interaction with the Mathematics

Alicia, Linda and Frank all inserted them selves into their notes, just in different ways. Alicia used a note taking method she learned in AVID⁹. You can see in all her notes the left-hand margin was used to write concept questions and other comments. She also used the margin to write “+” and “?” for the project annotations. Her notes were easy to follow. What I found I had to do most often with Alicia was to ask if the annotations were hers or Mary’s. She would often write down things Mary said but would also write her own comments about keywords, concepts and procedures.

The sample page of her notes in Figure 5.12 demonstrates an example where Alicia’s previous knowledge and facility with the economics register came into conflict with Mary’s interpretation of a concept. The problem for Alicia occurred when Mary misused a term in the economics register. Mary was giving an example of a Profit-Cost-Revenue problem. Mary did not use the term “revenue” but instead used “earnings.” The problem for Alicia was that she had taken economics and “revenue” and “earnings” are related but different concepts in economics. Alicia was always trying to make connections between meaning, concepts and procedures. So Mary’s misuse of “earnings” really bothered Alicia.

⁹ AVID is Advancement Via Individual Determination, a high school program to prepare “at risk” students for college study.

1.8 Operations with Functions

Every suit a store sells costs them \$100, plus \$4000 in rent & staff per week. They sell each suit for \$350. What is the profit per week given that they sell x suits?

Costs = $C(x) = 4000 + 100(x)$

Revenue Earnings = $E(x) = 350x$

Profit = $P(x) = E(x) - C(x) = 350x - (4000 + 100x)$

How many suits do they need to sell in a week to break even?

$250x = 4000$

$0 = 250x - 4000$

$4000 = 250x$

$16 = x$

We can add, subtract, multiply, or divide.

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ → as long as $g(x) \neq 0$

The domain for the new fn is the intersection (overlap) of the old domains.

EX: $f = \{(1,3), (2,4)\}$, $g = \{(1,1), (2,-2), (3,-3)\}$

Find domain of $(f+g)(x)$

$D = \{1, 2\}$

↑
c has 3
b/c don't have both 3

Important: Place card under blue copy.

Figure 5.12: Alicia's Notes: an example of her using previous knowledge to annotate

Other than the antagonism that Frank inserted into his notes, Frank would also mark when he found something particularly helpful for his learning (Figure 5.13). He seemed to find Mary's visual cues particularly helpful. He annotated the visual cues more so than the "silly words" that Mary used. The "helped" written in the notes was an annotation I made during the interview – Frank's annotation was the star.

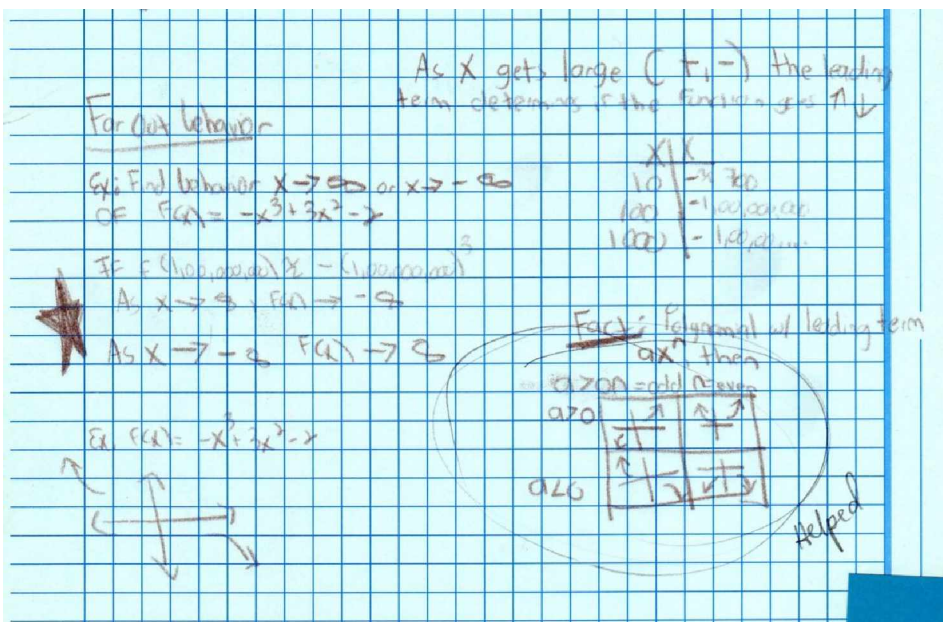


Figure 5.13: Frank's Notes: annotation about a helpful example

Linda inserted herself the least of the three. Linda was more like Sally in terms of her approach to mathematics however Linda seemed more engaged in the learning process. Her “?” annotations usually indicated a question about concept and how it related to procedure. In the interview excerpt (Table 5.7) and matching notes (Figure 5.14), Linda was struggling with understanding Logarithms. Although she only made one “?” annotation, she essentially did not understand two pages of examples. In general Linda did not write a lot of “?” but when she did it usually encompassed more than just a single problem but a whole concept.

Table 5.7: Interview with Linda about difficulty learning logarithms

Linda and Interviewer discussing a “?” annotation on her notes:

Linda: I’ve never seen logs but just in chemistry

I: Ok

Linda: Just in chemistry

I: Did you use these rules in chemistry

Linda: No we just used it to find the Ph

I: Ok, did you understand what the instructor was saying when she was talking about that (rules on the bottom of page 94)?

Linda: I kinda didn’t because I guess I really don’t know what a log is...I don’t really understand it that well. So like after reviewing it I understand it but it just doesn’t stick to me.

I: So the whole concept of what a logarithm is... is not sticking with you?

Linda: No

I: So on number (page) 95?

Linda: Yeah... I’m just writing things down but I’m not understanding it. This is where we start using the rules.

Like Melinda in John’s lecture, Linda copied the notes from the chalkboard even though she did not understand what Mary was doing. Fortunately for Linda, Mary did not seem to write as much as John, and Linda’s skill level was more developed than Melinda’s. Like Sally in John’s lecture, Linda focused on solidifying her knowledge of procedures for solving problems. However, unlike Sally, during our interviews and conversations, Linda would sometimes ask questions to try to make connections between procedure and concepts.

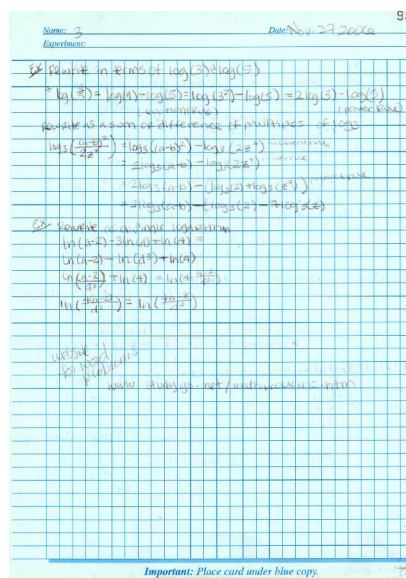
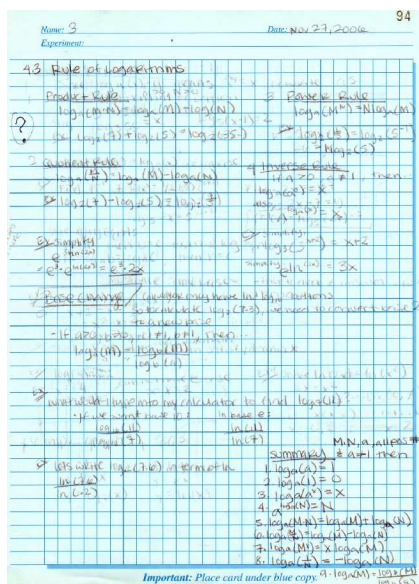


Figure 5.14: Notes Matching the Interview Excerpt in Table 5.7

5.3.3 The Mathematics Register: Developing versus Replicating Meaning-Making

In comparing the post-course Mathematics Register Surveys, Alicia, Linda, and Frank gave interesting responses in the survey (Appendix B). All three tended to give symbols or other visual representations for the mathematics meanings particularly Frank. Alicia gave everyday meanings to all the words whereas Frank was not sure about three words and Linda was not sure about two words. Linda and Frank gave mathematics meanings (not all were correct) to all the words whereas Alicia was not sure about two words. For example, Alicia wrote not sure for the math meaning for *composition* while Linda and Frank gave incorrect meanings. During interviews and in the annotated notes, Alicia always seemed clearer about what she knew, what she didn't, and what she almost understood.

Alicia had more questions about the mathematics register but her questions always seemed to be because she was trying to make connections from connotation to procedure to concept (Table 5.8). Alicia was the source for the examples about *angle of depression* and *half-life* in Chapter 4. In Figure 5.15, you can see in her notes the techniques that she is concerned about in terms of understanding how the context and meaning of *angle of depression* fit together to make a cohesive concept.

Table 5.8: Interview with Alicia about the angle of depression

Alicia: And then page 15 that was when we hit the word problems and that was just confusing because she came up . . . she explained angle of depression and just the concept of angle of depression and using it in word problems was confusing.

I: Had you heard this before?

Alicia: No, I don't remember doing this kind of word problem. And then the technique . . . how do we come to using these techniques with angle of depression. Just I mean . . . even though I know that . . . ok I can take these steps but I have a question mark because I am like . . . how do I get to those steps? How do I come to the conclusion that I have to use those steps in the word problem? . . . And then we go on to doing an actual word problem . . . and I guess the concept of getting like sine of 30 degrees is clear to me but just how we know how to draw the angle of depression triangle is what confused me. I wasn't sure . . . like . . . she did give an example. Like . . . if you are looking a certain eye level then you would go up or down. If it was an angle of depression or the other angle (laughter) which I don't remember at this time but just knowing which way to draw the triangle when it is angle of depression which is what confused me.

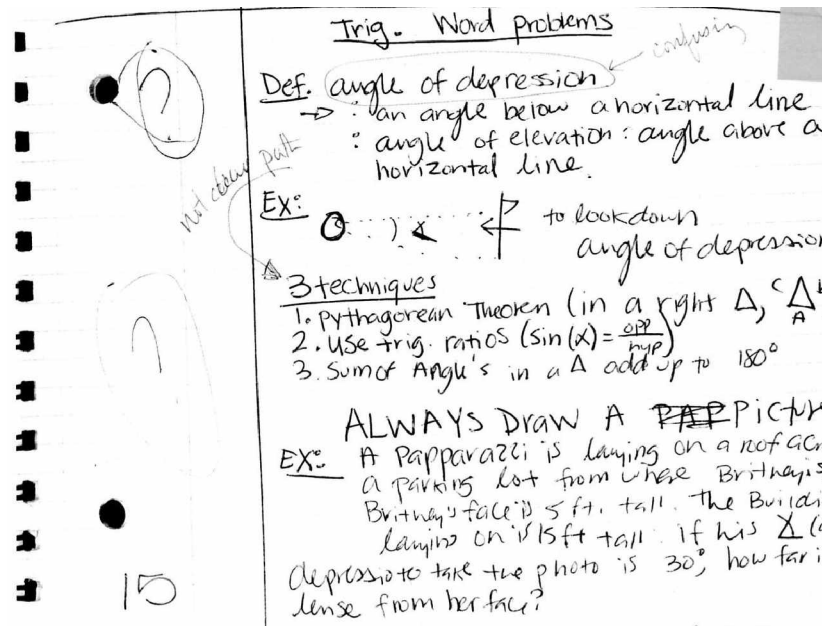


Figure 5.15: Notes Matching the Interview Excerpt in Table 5.8

Even though Alicia had more “?” annotations on her annotated notes than Frank and Linda, Alicia seemed to have the most facility with the mathematics register and was more intent on understanding and producing meaning-making than Frank and Linda. Frank’s facility with the register was definitely more visual in nature. I could not determine whether Frank was consistently creating the visual representations from deeper knowledge of items in the mathematics register or if he was reproducing them from rote memory. During the interviews, Frank sometimes demonstrated a deeper understanding of visual representations such as his visual representation for *one-to-one* on the post-survey (Appendix B) and at other times his understanding was incomplete such as his visual representation for *transformation*. Linda seemed to view the register more as just another part of math to learn. Of the three, she seemed to imitate meaning-making. That is, during the interviews she would recall mathematical knowledge in

terms of steps in a procedure. Although her annotations indicated problems with understanding a concept overall, when helping her with the mathematics she preferred to rely on procedure.

5.4 Conclusion

This chapter examined the results of the study by answering my third research question.

(Research Question Three) How do students perceive the meaning-making process within the Precalculus lecture structure? How do students insert themselves within the meaning-making process? And, where do students place their meaning-making process within the larger context the specialized Precalculus discourse community?

The note-takers in both lectures were sincere in their efforts to learn the mathematics and in their participation in the project. Of the six, Alicia and Diana seemed to be the two note-takers who were more actively participating in their learning process. They both inserted themselves into their notes. Their annotations demonstrated that they were engaged and thinking about the mathematics and the communication from their instructors. Frank also seemed to be more reflective during the notetaking process in the class than the others. Nevertheless, he did not take his process to the level that Alicia and Diana did. The other note-takers behaved more like participants watching a show that they would later talk about over coffee. That is, they took notes put annotations for the project but then dealt with the meaning-making

occurring in their notes when we met. They did say they used their notes to do homework and review. They didn't however reflect much upon the meanings and concept found in their notes. In Melinda's defense, I believe she was overwhelmed by a class that she was not prepared to take.

Chapter 6: Discussion and Implications

6.1 Faulty Mathematical Meaning-Making

To address my first two research questions, I focused, in Chapter 4, on the meanings students employ in a college developmental math class, Precalculus, and the meanings they assign to terms in the mathematics register. The student subjects, both survey-takers and note-takers, seemed genuine in providing the meanings they make use of in mathematics. In analyzing the data, I found that most if not all the students at some point in the study employed faulty mathematical meaning-making or assigned meanings that were not quite appropriate or completely off the mark.

For many of the students, mathematical meanings are “one of these things you can’t explain but still know.” The quote was written by a student in the space provided for the mathematical meaning of the word *root* on the mathematics register survey (See Section 4.2.1). As an educator and especially after conducting this study, this brief statement means different things to me. The statement may mean that the student has minimal understanding of the concept and thus cannot explain or describe it. The statement may mean that the student has an understanding of the concept but never developed the language or mathematical discourse to explain or describe it. Or perhaps the statement reveals that in the course of this student’s history of learning mathematics the concept may have been taught with cursory and shallow treatment. That is, the student may not have been given the opportunity to learn the concept in depth and detail to develop the understanding and language to explain or describe it. Subsequently, as a

teacher this statement steers me to one action – ask what the student means by this statement. What and how do they know and don't know?

The statement floated in the back of my mind as a read through the student surveys, annotated notes, and interviews. Student prerequisite knowledge required for the Precalculus course was evident in the data (surveys, notes, and interviews) however the depth and detail of understanding of this prerequisite knowledge was clearly a complication to student mathematical meaning-making. Students demonstrated a limited depth of understanding in different and subtle manifestations. Overwhelmingly, many students did not have sufficient language to describe their understanding of mathematical meanings. Many meanings were connected to objects which implied the students believed these objects encompassed a mathematical meaning for a word that was apparent and shared – for example, *root* became $\sqrt{\quad}$, *imaginary* became i , *inequality* became $>$ or $<$ or \neq . Many students provided distorted descriptions of mathematical meanings which could lead to misapplications of mathematical concepts – for some students, *identity* meant to identify. In other cases, students applied incorrect situational uses of a concept which could lead to fundamental mistakes in mathematical operations – for example, an inverse function became the reciprocal of the function. Yet in other cases, students connected meaning to procedures or algorithms which may be sufficient only to demonstrate superficial understanding of a concept – some students identified the mnemonic FOIL as the distributive property. Lastly, precision and rigor in the students' mathematical meaning-making was rarely evident. This faulty mathematical meaning-making for prerequisite knowledge may have landed many of the students in the Precalculus course. However from the responses on the pre-

and post-course surveys, the students' faulty meaning-making seemed minimally ameliorated as a result from taking the Precalculus course.

6.2 Conditions Contributing to Faulty Mathematical Meaning-Making

No matter how faulty their mathematical meaning-making may have or may have not been many of the students in this Precalculus course passed the class. Herein lays the teaching-learning quandary. Although students applied faulty mathematical meaning-making, they were able to demonstrate sufficient mathematical abilities on the course assessments to pass the class. So, what are the conditions that allow for students to continue using faulty mathematical meaning-making while also progressing, supposedly, from one mathematics course to the next?

6.2.1 Objectivism in the Teaching-Learning Environment

Mathematics is objective (Lakoff and Nunez, 2000). This objectivism is a necessary part of mathematical thinking. Objects, such as $+$, $=$, i , $\sqrt{\quad}$, even $!$, are assigned mathematical meanings under particular situational uses. Without objects as a part of the mathematics register, mathematical treatises – proofs, problem solving and such – would be cumbersome and tedious. For the mathematics register surveys, although they were asked to provide a written description, the students often responded by providing an object as the description or meaning for many of the words on the surveys. For the most part, the objects they provided were a good match to the words in the mathematics register. So, objectivism in mathematics is not particularly the problem for student

meaning-making. The conditions, on the other hand, that are created when objectivism exists in the teaching-learning environment are the problem for student meaning-making. In the surveys, students did not have to use language or other means to describe meaning when they provided an object. The assumption was that the mathematical meaning of the object is uniformly and unconditionally apparent and shared. This condition comes about because we use objects so often and repeatedly in mathematics that they become mundane particularly for the mathematician. Nonetheless, the underlying mathematical connotations, implications and situational uses of these objects are more complicated than the mundane. These connotations, implications and situational uses frequently are translated and transferred as mundane knowledge in the mathematics classroom. That is, the connotations, implications and situational uses of the object are thought to be as commonplace as the object and therefore may not require much reflection or deeper inspection. The teaching-learning problem becomes how deeply or superficially do students understand the mathematics related to an object. Moreover, what does an instructor do to make improvements to the students' existing knowledge?

Unfortunately, I could not ask every student who completed a mathematics register survey what they meant by or how they understood the object they wrote down – a limitation of this study. Although this situation narrowed my analysis of the data collected on the mathematics register survey, the survey revealed the multitude of ways students applied meaning even when using objects. Subsequently, the survey data generated further research questions.

For example, for the word *negative*, the student who responded with the single number “-7” demonstrated more limited meaning-making than the student who responded with (negative integers) “...-3, -2, -1”. Both of these students who responded with the single number “-7” and “...-3, -2, -1”, respectively, demonstrated more limited meaning-making than the student who responded with “<0”. The “<0” implies a broadening of the set of numbers beyond negative integers. However, the students who responded with “<0” demonstrated analogous meaning-making to the student who responded with “numbers left of zero” or to the student who responded with a drawing of a number line shaded to the left of zero. “Numbers left of zero”, “<0”, and the drawing of the shaded number line are three different ways of demonstrating the same thing but is the student meaning-making the same or different? Is one description preferable knowledge and meaning-making than the others? Do the students have the same level of understanding of the concept or does one have a deeper, more complex understanding of the concept?

All the responses above for *negative*, “-7”, “...-3, -2, -1”, “<0”, “numbers left of zero”, and the drawing of the shaded number line, are considered objects under the objectivist theory of communication (section 4.4, p 99). Regardless of the level of meaning-making demonstrated in each response, these responses are problematic in that each is a brief representation that does not fully capture the concept. Additionally, the brief representations may demonstrate adequate knowledge (in some cases) but they do not fully reveal the quality of the mathematical meaning-making that led to them. Objects and brief statements limit the use of language that might otherwise expose the student meaning-making process.

6.2.2 Facility with Language

Limited math abilities were assumed in this study given the sample population was drawn from a remedial math class. I expected students to have limited facility with the mathematics register because of their limited facility with the subject matter meaning (Pimm, 1987; Schleppegrell, 2004). However, I found that many students had difficulty defining and identifying meanings for the everyday use for words on the mathematics register survey. Students' limited vocabulary was surprising given that the college in this study is considered a Tier 1 Research University. However Precalculus is considered a remedial math class at this university. So, students in Precalculus may have also had low verbal and writing skills as well. I collected data on student enrollment in basic writing and English as a Second Language. Separating data by writing categories did not produce any significant difference in the types of responses. The writing level data was self reported by the student and most likely not consistent. For example, Sally and Melinda were in the basic writing class but did not self report it. I found out they were in the class later during the exit interview.

Thirty-four percent of all responses to the pre- and post-course surveys were missing everyday and math meanings for the same word. This result indicated that students may not have been able to provide a math meaning because they were not able to provide an everyday meaning. This result is consistent with Pimm's (1987) and Schleppegrell's (2004) findings that students' facility with vocabulary in general impacts their ability to work within a specialized register. Students' limited vocabulary, in turn, limits their ability to participate verbally in the mathematics discourse of the

Precalculus class. In spite of this, students' limited vocabulary may not limit their ability to process mathematical procedure. As with an object or objectivism, a student or the instructor may use minimal language to communicate algorithmic processes.

6.2.3 Algorithmic Thinking

Algorithms are not a part of the mathematics register however they are a part of the mathematics discourse community. Algorithms define logical step-by-step procedures for solving mathematical problems; in certain cases, they simplify recursive processes that would otherwise be unwieldy. Similar to objectivism, algorithmic thinking is not the particular problem for student mathematical meaning-making. The problem is the way in which algorithmic thinking is employed to teach and learn mathematical concepts. Many times, the algorithm or procedure is substituted for the mathematical concept. This substitution allows a student to have a superficial understanding of the mathematics while at the same time the capability to perform basic and unchallenging mathematical problems.

The mnemonic *foil* is an example of algorithmic thinking that diminishes the mathematical concept – the distributive property – that underlies it. Of all the words on the mathematics register survey this word is the one word that every student assigned a mathematical meaning. Ironically, the mathematics faculty (at the university) who viewed the survey did not know what mathematical meaning *foil* had and do not consider *foil* to be part of the mathematics register. Neither do I; *foil* is a learning device for a very narrow interpretation of the distributive property. The mathematics faculty, once they found out what *foil* was, immediately deemed it invalid and

determinedly requested that I remove it from the survey. I left it on because I wanted to know what the students and instructors thought about *foil*.

The teaching-learning dilemma is that *foil* is universally used by mathematics teachers and authors of mathematics texts. It has become a so commonplace device that students and teachers treat *foil* as part of the mathematics register. That is, it has valid mathematical meaning attached to it. In fact, not only did all the students assign mathematical meaning to the mnemonic *foil* but the two graduate student instructors did as well. The graduate student instructors' response to *foil* was vastly different than the mathematics faculty. The graduate student instructors' treated *foil* as a commonplace term and did not take exception to it being on the survey. During the exit interviews with the graduate student instructors, they each indicated that they had learned about *foil* in pre-college mathematics courses. They saw nothing exceptional about the term however they both used the mnemonic in the lectures where they discuss multiplication of polynomial expressions and the distributive property.

This situation poses a challenge to the idea that the mathematics register is fixed and established. As these graduate students progress in their studies to become mathematicians, will their view of *foil* change? Will they adopt the view the mathematics faculty hold or will they (and other graduate students with similar learning experiences) irrevocably change the mathematics register? Regardless of whether *foil* is a part of the register or not, now or in the future, *foil* as it is used perpetuates a limited understanding of a mathematical concept and increases the potential for students to employ faulty mathematical meaning-making.

One of the student-note takers provided an example of algorithmic thinking limiting a deeper understanding of mathematical concepts. The student note-taker, Sally thought of mathematics algorithmically – doing mathematics was procedural for her. She preferred to follow prescribed steps to arrive at a solution; to be given an equation rather than construct an equation from a word problem, and to graph easily identifiable functions rather than build a function from a given graph. Sally equated conceptual meaning to standardized and mundane procedures that required little use of the vocabulary in the mathematics register. More significantly, employing standardized and mundane procedures allowed Sally to perform mathematical operations adequately without having a deeper and detailed understanding of the relevant mathematical concepts. Sally's penchant for procedures may be the reason she was in the Precalculus class. She had taken math courses through Calculus (see interview excerpt, p. 118) in high school but tested into the Precalculus class. The placement test for the course, although multiple choice, contained several word problems; function building from graphs; and problems with no prescribed steps.

While working with Sally, I clearly saw that she was proficient in solving mathematical problems that did not challenge her mathematical meaning-making or the degree to which she understood concepts in Precalculus. The teaching-learning problem in this Precalculus course was that the majority of the problems assigned for homework and many of the examples in Sally's notes did not challenge her mathematical meaning-making. Many of the problems and examples required rote procedures whether the problems involved equations, functions or graphs. In fact, the homework and lecture examples, most likely reinforced her belief that mathematics is procedural. Her

performance on the two midterms was above average. In reviewing the midterms with her, I saw that the midterms contained many problems with which Sally would be proficient. When she had significant difficulty, these problems were usually word problems and problems with no prescribed steps. However, there were not many of these problems on the midterms. So, although Sally's mathematical meaning-making was not as advanced and thorough as other note-takers, her skill in performing procedure coupled with the manner in which student knowledge was assessed aided Sally's progression through the Precalculus course.

6.2.4 The College Lecture Format: Pace, Capacity, and Assessment

The syllabus for the Precalculus course did not allow for much reflection or deeper inspection of concepts – a limitation of developmental math courses. Without going into the specifics of the syllabus, a college level Precalculus course covers the same content of a nine month high school course in 10 weeks. (The university is on a quarter system.) The pace of the course therefore is relentlessly quick and students cannot afford to fall behind. Because of the amount of material in the course, the graduate student instructors could not devote sufficient time to delve into detail on any one topic. Moreover, the Precalculus course covers so much material that it is difficult to write a test that assesses student knowledge of all the material covered. In addition, the instructors are novice teachers who may not have had the experience in writing problems that assess both content knowledge and the quality of that knowledge.

In reviewing the annotated notes, I found that both graduate student instructors covered at least one to two sections of a chapter each lecture. During the lectures the

instructors provided explanations and examples for the concepts however it seemed that most of the note-takers were just copying down the notes from the board without thinking about what they were writing. Unfortunately, the size of each lecture along with the pace of the class did not allow for much interaction between instructor and students.

The lecture structure of the classroom tends toward objectivism in the manner in which communication is performed (Lakoff and Johnson, 2003). The annotated notes and student interviews revealed that the lectures consisted mostly of the instructors giving information and the students writing notes. The student note-taker, Melinda, who had limited proficiency with Precalculus concepts, expressed frustration with the lecture. During the interviews, Melinda indicated a sense of failure similar to students in studies examining college lectures and college leaving behaviors ((Tinto, 2000; Braxton, Bray & Berger, 2000; Seymour & Hewitt, 1997). In addition, similar to other studies, Frank and Melinda also expressed frustration and disappointment with instructors' lecture styles and teaching abilities (Brown & Race, 2002). Frank, in particular, seemed to view Mary's lecture style as an obstacle to his learning (Pascarella & Terenzini, 2005).

Precalculus is a gateway to advance mathematics and science courses – the content of this course is fundamental and required knowledge for success in future math and science coursework. Unfortunately, the lecture format supports unproductive teaching-learning models: too much material is covered so that it is most likely covered with cursory and shallow treatment; there is no space or time in the course for student discussion or interaction with the instructor; assignments, examples and assessments

that do not challenge the quality of student mathematical meaning-making; and novice instructors teaching a course and students that really require more experienced teachers. As a result, a Precalculus course that could be an entry point into a math-science pipeline may instead become a barrier or detractor for students to pursue math-based fields.

6.3 Compensating for Limited Teacher-Student Interactions

Because the Precalculus course did not allow for much student-teacher interaction in the lecture, students seemed to rely on existing knowledge or social/cultural knowledge to make connections between the everyday world, the mathematics register, and mathematical concepts. For example, some students' mathematical definitions for terms such as *identity*, *inequality*, *angle of depression*, and *half-life*, seemed to be influenced by social and cultural interpretations of the words in everyday use. This compensation to define meaning seemed to be happening when students were developing their understanding of a term in the register but did not have enough interaction with a more knowledgeable other (like the instructor) to reach intertextuality with the mathematics register (Wink & Putney, 2002). Melinda and Alicia were confused about the phrase, *angle of depression*; however each remediated their confusion differently. Alicia relied on prior knowledge where Melinda used an everyday understanding of depression. In working with both Alicia and Melinda, we discussed their confusion and each student attempted to pinpoint the source of their ambiguity over the phrase. Leung (2005) found that having students "pinpoint" the

meaning of a word could reduce ambiguity. This may have been the case with Melinda who was focusing on the depression in the angle of depression. However, Alicia was struggling with a new phrase, a new mathematical concept and a new set of visual cues. She was trying to find a coherent strategy to connect all of it. Alicia was working through a Zone of Proximal Development to solidify her understanding of the new information (Wink & Putney). In addition, Alicia's practice of contributing to her own learning process helped her to create intersubjectivity with the subject matter (Krummheuer, 1998).

In another result, students gave responses that indicated that they believed that *imaginary* numbers don't exist, that they are not real-life or corporeal. Their responses for mathematical meaning matched with their responses for the everyday meaning of *imaginary*. This action was consistent with other words where students attributed their everyday meaning to their mathematical meaning. Similarly, Pimm (1987) found that students thought that odd numbers were even because odd numbered items could be shared evenly amongst a group of students. These misinterpretations of words in the mathematics register demonstrated that without instructional support students relied on other means to make connections and balance their understanding. In some cases, this balance was achieved by students employing meanings from their world and worldview rather than from a mathematics world and worldview (Lakoff and Johnson, 2003).

Students' ability to interact in the specialized mathematics discourse community was connected to their ability to understand the multiple ways in which the specific term was being used. For example, students often gave examples of both *rational numbers* and *rational expressions* for the term *rational*. In some cases it was clear they

understood both were fractions but different from one another. This understanding may have been developed during the Precalculus lectures because it was a term being discussed when the pre-course survey was administered. Similar to findings in Cestari's (1998) work, in the data analysis, situational uses of terms in the mathematics register continually surfaced as a factor in students' meaning-making. In this study, I found students employed either social or cultural situational uses for words to assign mathematical meaning. However, I also found that some of the assigned meanings demonstrated limited understanding of the situational uses in the mathematics and symbol representations (Lakoff and Nunez, 2000).

6.4 Implications for Teaching-Learning Mathematics

The setting for this study was a course that I had taught for years. After completing this study, my recommendations for the Precalculus course are focused on facilitating teacher training and student learning.

For teacher training, I recommend training that facilitates teacher's use of discussion in the lecture. The discussion should focus on (1) more and clearer explanations regarding word usage and meanings in mathematics; and (2) enlisting student understandings of vocabulary to clarify ambiguity over word, symbols, and metaphors in the mathematics register.

For student learning, I recommend developing strategies (1) to increase student vocabulary, both everyday meanings and mathematical meanings and (2) to improve

student skills in distinguishing between word meanings in mathematics and mathematical definitions.

6.5 Future Research

Although this study focused on meaning-making in a college Precalculus lecture, much of the study was conducted outside of the lecture environment. I focused on the student and used student in-depth interviews, analysis of student annotated notes, and student surveys. Although some analysis of the lectures had been planned, technical problems with video recording and my inability to observe the lectures consistently did not allow for this analysis to occur. The surveys and interviews focused on student meaning-making and student perceptions of meaning-making in a college level Precalculus course and limited the scope of this study to the student. The setting of the study also resulted in a sample that was overwhelmingly female. Given the nature of the sample, the results of this study may only represent a female perspective of the meaning-making and perceptions of the meaning-making occurring in the Precalculus class. However, the data collected included a small set of males. The results for the males did not differ drastically from the females. Therefore, I could not tease out any difference between women's and men's mathematical meaning-making and perceptions of meaning-making in this Precalculus lecture. Clearly more research similar in nature could only strengthen and expand the current work.

Other avenues of research are:

1) Everyday use of word meanings in some cases helped students make connections to more accurate mathematical meanings comprehension. However in other cases the everyday use led students to assign faulty or incorrect meanings to terms in the mathematics register. The following questions seem pertinent:

- What are the optimal ways of helping students switch from everyday meaning to a mathematics meaning and use of the mathematics register?
- What are the long-term effects of using everyday meanings to mediate student development of and facility with the mathematics register?
- What are the long-term effects of using everyday meanings to mediate the way students think about and understand mathematics?

2) Given how critical it is for an instructor to be able to not only have pedagogical content knowledge but also to move between everyday meaning, mathematical meaning and the mathematics register, how can mathematics teacher education at the college level be developed to enhance student engagement and learning in a lecture structure?

3) There has not been much discussion in the literature that I have found about ethnic or gender differences in how students work with the mathematics register. Some work has been done on African American students (Orr, 1987) and women (Fonzi & Smith, 1998); however too little is available to permit any definitive conclusion. And this study is also limited in providing definitive results.

- How does a student's history of meaning-making in mathematics affect his/her success and continuation in mathematics?
- Are there gender or ethnicity differences in the type of meaning-making in a mathematics classroom?

Appendix A

Selected Scanned Student-Annotated Notes

Diana

05

Name: #12 Date: 10/02/08

Experiment:

(Continuation) A.4
 Multiplication and Division of Rational #'s.

+ - If $\frac{a}{b}$ & $\frac{c}{d}$ are rational #'s then,

- Multiplication is defined as $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- and if $c \neq 0$ then $\frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{b} = \frac{ad}{bc}$

↓
Multiply Reciprocal

- Multiplying & dividing rational expressions is in the same fashion.

Procedure for dividing rational expressions.

+ - if you're dividing...

- invert & multiply
- factor completely
- divide out common factors.

+ • Ex find & simplify (Multiply)

$$\left(\frac{3x-2y}{4}\right) \cdot \left(\frac{8y}{x^2-y^2}\right) = \frac{3(x-y)}{4} \cdot \frac{2y}{(x-y)(x+y)}$$

$$= \frac{6y}{4(x+y)}$$

+ • Ex $\left(\frac{2+x}{2}\right) \div \left(\frac{x^2-4}{4}\right)$ (Divide)

$$= \left(\frac{2+x}{2}\right) \cdot \left(\frac{4}{x^2-4}\right) \leftarrow \text{invert}$$

$$= \frac{2+x}{2} \cdot \frac{2^2}{(x+2)(x-2)} \leftarrow \text{factor out completely \& cancel out common factor}$$

$$= \frac{2}{x-2}$$

Addition and Subtraction of Rational Expressions.

+ - If $\frac{a}{b}$ & $\frac{c}{d}$ are rational #'s, then $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$

- $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

- Only work when there's a common denominator.
 ↳ if not find one is one does not exist. (find up denominator)

+ • Ex $\frac{2}{x-1} - \frac{3}{x^2+x-14}$ com; $\frac{2}{x-1} = \frac{2}{(x+2)(x-1)} = 2(x+2)$

Important: Place card under blue copy.

Name: #12

Date: 10/03/04

Experiment:

Miscellaneous Absolute Value problems.

$$+ - |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

+ ② $x = -4, y = 2, z = -1$

Find $\frac{|x+z|+|y|}{|x+y+z|}$

$$= \frac{|-4+(-1)|+|2|}{|-4+2+(-1)|}$$

$$= \frac{|-5|+|2|}{|-3|}$$

$$= \frac{5+2}{3} = \frac{7}{3}$$

+ ③ $|x^2+17| = x^2+17$

+ ④ $|2-\pi|+2$

$$= -(2-\pi)+2$$

$$= -2+\pi+2$$

$$= \pi$$

+ ⑤ $x < 6$, what is $|x-5|+(6-x)$

$$= -(x-5)+(6-x)$$

$$= -x+5+6-x$$

$$= 11-2x$$

Solve the following Equation.

⑥ $7+2x-4+4x=21$

$$(4x+3)=21$$

$$-3-3$$

$$\frac{4x}{4} = \frac{18}{4} \quad |x=2.25|$$

+ ⑦ $\frac{2}{x}+4 = \frac{1}{x} - \frac{6}{2x}$

~~$$\frac{2}{x}+4 = \frac{2}{2x} - \frac{6}{2x}$$~~

~~$$\frac{2}{x}+4 = \frac{-4}{2x}$$~~

~~$$\frac{2}{x}+4 = \frac{-2}{x}$$~~

~~$$\frac{2}{x}+4 = \frac{-2}{x}$$~~

~~$$\frac{2}{x}+4 = \frac{-2}{x}$$~~

$$\left(\frac{2}{x}\right)\frac{2}{2} + \left(\frac{2}{2}\right)4 = \frac{1}{x}\left(\frac{2}{2}\right) - \frac{6}{2x}$$

$$\frac{4}{x} + \frac{4x}{2x} = \frac{2}{2x} - \frac{6}{2x}$$

$$\left(\frac{2x}{2x}\right)\frac{4+8x}{2x} = \frac{2-6}{2x} \quad \left(\frac{2x}{2x}\right) \leftarrow \text{Numerators are equal}$$

$$4+8x = 2-6$$

$$8x = -8$$

$$x = -1$$

Important: Place card under blue copy.

Name: #12

Date: 10/24/06

Experiment:

inequality	set	interval notation	type	graph
$x > a$	$\{x x > a\}$	(a, ∞)	open	
$x < a$	$\{x x < a\}$	$(-\infty, a)$	open	
$x \geq a$	$\{x x \geq a\}$	$[a, \infty)$	closed	
$x \leq a$	$\{x x \leq a\}$	$(-\infty, a]$	closed	

Ex. $(-3, \infty)$ $\rightarrow -3 < x < \infty \rightarrow -3 < x$
 $[-2, 1]$ $\rightarrow -2 \leq x \leq 1 \rightarrow (-\infty, 1]$

Properties of Inequality

+ If A & B are algebraic expressions & C is a nonzero real #, then the inequality $A < B$ is equivalent to

- 1) $A < B$ if $C > 0$
- 2) $AC < BC$ (if C is positive)
 $AC > BC$ (if C is negative.)
- 3) $\frac{A}{C} < \frac{B}{C}$ (if C is positive)
 $\frac{A}{C} > \frac{B}{C}$ (if C is negative.)

Solving linear inequalities.

- Defn: $ax + b <, >, \leq, \geq 0$

+ Ex. write in interval notation given

$$\begin{array}{l} -2x + 4 < 0 \\ -4 \quad -4 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{isolate } x$$

$$\begin{array}{l} -2x < -4 \\ -2 \quad -2 \end{array}$$

$$\boxed{x > 2} \quad \leftarrow \text{solution set}$$

$$2 < x < \infty \quad \leftarrow \begin{array}{c} | \\ 0 \quad 2 \end{array}$$

$$(2, \infty)$$

+ Ex. $(\frac{1}{3}x + 2) \leq (\frac{1}{5}x + 10)$ — multiply by least common den.

$$= 3x + 27 \leq x + 54 \quad \leftarrow \text{isolate } x$$

$$= \begin{array}{l} 3x + 27 \leq x + 54 \\ -x \quad -x \\ \hline 2x + 27 \leq 54 \\ -27 \quad -27 \\ \hline 2x \leq 27 \\ \hline x \leq \frac{27}{2} \end{array} \quad \leftarrow \begin{array}{c} | \\ 0 \quad \frac{27}{2} \end{array}$$

$$(-\infty, \frac{27}{2}]$$

Important: Place card under blue copy.

Name: #12

Date: 10/11/04

Experiment:

+ Distance (Rate) problems.

- recall: $d = rt$

+ *EX. on route to LA (90 miles away) a car leaves San Diego during rush hour & returns when there is no traffic. Assuming you can drive twice as fast when there is no traffic & that the entire trip took $4\frac{1}{2}$ hours, how fast does the car drive during rush hour.

$$90 =$$

let $t_1 =$ time of first triplet $t_2 =$ time of second trip $t_1 =$ time of -rush hour $t_2 =$ time of -no traffic

* ALWAYS DEFINE VARIABLES *

$$d_1 = d_2$$

$$r_1 t_1 = r_2 t_2$$

$$t_1 t_2 = 4.5$$

$$2r_1 t_2$$

$$t_2 = 4.5 - t_1$$

$$r_1 t_1 = 2r_1 t_2$$

$$\frac{r_1 t_1}{r_1} = \frac{2r_1 (4.5 - t_1)}{r_1}$$

$$t_1 = 2(4.5 - t_1)$$

$$t_1 = 9 - 2t_1$$

$$3t_1 = 9$$

$$t_1 = 3$$

$$t_2 = 1\frac{1}{2}$$

$$r_1 = \frac{d}{t_1}$$

$$r_1 = \frac{90}{3}$$

$$r_1 = 30$$

$$r_2 = 60$$

$$r_2 = 60$$

$$r_2 = 60$$

$$r_2 = 60$$

Important: Place card under blue copy.

Name: #12

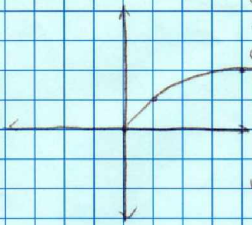
Date: 10/11/09

Experiment:

(continuation)

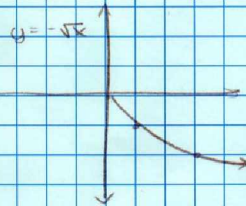
+ Ex #3 $y = \sqrt{x}$

x	y
0	0
1	1
4	2



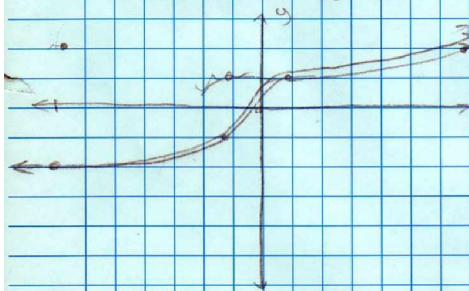
NOTE: if we took equation from ex. #2 ($x = y^2$) & took square root of both sides we get $y = \sqrt{x}$

is a function



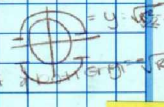
+ Ex #4 Graph $y = \sqrt[3]{x}$

x	y
0	0
1	1
-1	-1
8	2
-8	-2



+ Getting $x^2 + y^2 = r^2$ is a circle centered @ (0,0) w/ radius r

if we solve for y $y^2 = r^2 - x^2 \Rightarrow y = \pm \sqrt{r^2 - x^2}$
 solving for y splits the circle into two halves
 separately, they are functions.



Important: Place card under blue copy.

Name: #12

Date: 10/11/04

Experiment:

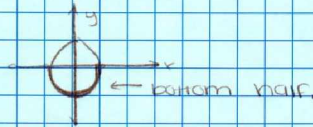
(continuation) semi-circles
 $y = -\sqrt{1-x^2}$ = bottom 1/2
 $y = +\sqrt{1-x^2}$ = top 1/2

Each separately are functions

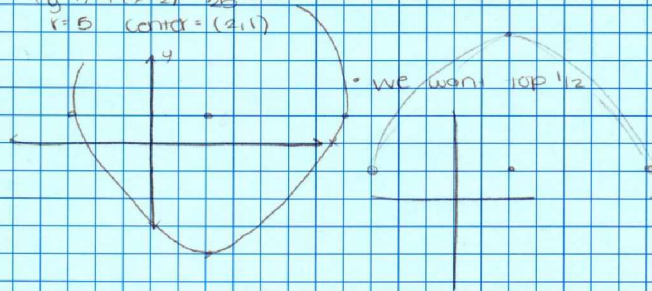
- Ex #5

Graph $y = -\sqrt{1-x^2}$ • re-write in standard form of circle
 $y^2 = 1-x^2$
 $x^2 + y^2 = 1$

• circle of $r=1$, centered @ $(0,0)$ • remember which 1/2 of circle you were prescribed w/ first

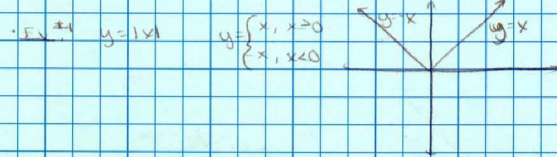


Graph $y = \sqrt{25-(x-2)^2} + 1$
 $(y-1) = \sqrt{25-(x-2)^2}$
 $(y-1)^2 = 25-(x-2)^2$ • circle into standard form of circle
 $(y-1)^2 + (x-2)^2 = 25$
 $r=5$ center = $(2,1)$



Piece wise functions:

- dir. formulas are used for dir. regions of the domain



Important: Place card under blue copy.

Sally

Section A.4 CONT'D
Multiplication and Division of Rational Numbers

Def - If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
 and if $c \neq 0$ then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Note Multiplying and dividing rational expressions is in the same fashion

Procedure

- ① If dividing invert and multiply
- ② Factor completely
- ③ Divide out common factors

ex - Find and simplify

$$\left(\frac{3x-3y}{4}\right) \cdot \left(\frac{8y}{x^2-y^2}\right) = \frac{3(x-y)}{2^2} \cdot \frac{2^3 y}{(x-y)(x+y)} = 2 \cdot \frac{(3(x-y))}{(x-y)(x+y)}$$

$$\boxed{\frac{6y}{x+y}}$$

③ $\left(\frac{2+x}{2}\right) \div \left(\frac{x^2-4}{4}\right) = \left(\frac{2+x}{2}\right) \cdot \left(\frac{4}{x^2-4}\right) = \frac{(2+x) \cdot 2}{(x+2)(x-2)} = \frac{2}{x-2}$

Addition and Subtraction of Rational Expressions

Def - If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+c}{bd}$
 and $\frac{a}{b} - \frac{c}{d} = \frac{ad-c}{bd}$

Note This definition only works when there is a common denominator, so if there is not a common denominator we must find one

Building up the denominator

ex - $\frac{1}{2} = \frac{?}{4}$, here 2 is the solution $\frac{1}{2} = \frac{2}{4}$

solution $\frac{1}{2} = \frac{?}{2 \cdot 2}$ factor everything to find ?

Note We can also build up the denominator in rational expressions

Important: Place card under blue copy.

Miscellaneous Absolute Value Problems

recall: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Suppose $x=-4, y=2, z=-1$

$$\textcircled{2} \text{ Find } \frac{|x+z| + |y|}{|x+y+z|} = \frac{|-4+(-1)| + |2|}{|-4+2+(-1)|} = \frac{|-5| + |2|}{|-3|} +$$

$$= \frac{5+2}{3} = \frac{7}{3}$$

Rewrite the Expression without using absolute values

$$\textcircled{3} |x^2+17| = x^2+17$$

$$\textcircled{4} |2-x|+2$$

$$-(2-x)+2 = -2+x+2 = x +$$

$$\textcircled{5} \text{ given that } x < 5, \text{ what is } |x-5| + |6-x|? +$$

$$-(x-5) + 6-x$$

$$-x+5+6-x = 11-2x$$

Solve the following equations

$$\textcircled{6} 7+2x-4+4x=21$$

$$3+6x=21$$

$$-3 \quad -3$$

$$6x=18$$

$$\underline{\underline{6 \quad 6}} \quad x=3$$

$$\textcircled{7} \frac{2}{x} + 4 = \frac{1}{x} - \frac{6}{2x}$$

$$\frac{-2}{x} = \frac{-2}{x}$$

$$4 = \frac{1}{x} - \frac{6}{2x} - 2$$

$$\frac{4^2-1}{2x} = \frac{6}{2x(x)}$$

$$4 = \frac{-2x}{(2x)(x)} = \frac{6x}{2x(x)}$$

$$4 = \frac{-8x}{3x}$$

$$4+8x = -4$$

$$\frac{8x}{8} = \frac{-8}{8} \quad x = -1$$

$$\textcircled{7} \frac{2}{x} + 4 = \frac{1}{x} - \frac{6}{2x}$$

$$\frac{(2)}{2x} + \frac{4(2x)}{(2x)(2x)} = \frac{1}{2x} - \frac{6}{2x}$$

$$\frac{4+8x}{2x} = \frac{2-6}{2x} \quad \frac{4+8x}{2x} = \frac{2-6}{2x}$$

CANCEL

$$4+8x = -4$$

$$\frac{8x}{8} = \frac{-8}{8} \quad x = -1$$

Important: Place card under blue copy.

Name: _____

Date: _____

Experiment: _____

Solving Linear Inequalities

Defn A linear ~~equation~~ inequality is of the form $ax + b \leq 0$.

ex - Write the solution set in interval notation and graph.

$$-2x + 4 < 0$$

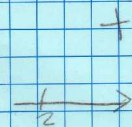
$$-4 -4$$

$$-2x < -4$$

$$-2 = -2$$

$$x > 2$$

$$2 < x < \infty, (2, \infty)$$



ex:

$$\left(\frac{1}{3}x + 3\right) \leq \left(\frac{1}{9}x + 6\right) \Rightarrow \frac{1}{3}x + 3 \leq \frac{1}{9}x + 6$$

$$-x + 27 \leq -x + 54$$

$$2x + 27 \leq 54$$

$$2x \leq 27$$

$$x \leq \frac{27}{2}$$

$$-\infty < x \leq \frac{27}{2} \Rightarrow (-\infty, \frac{27}{2}]$$

Compound Inequality

Defn - A compound inequality is a sentence containing two simple inequalities connected by "and" or "or".

ex $x \geq -1$ and $x \leq 2 \rightarrow -1 \leq x \leq 2$
in interval notation $[-1, 2]$

Note This interval does not contain the infinity symbol and is therefore called a bounded interval.

Summary of Bounded Inequalities

Inequality	Set	Interval Notation	type	graph
$a < x < b$	$\{x \mid a < x < b\}$	(a, b)	open	
$a \leq x \leq b$	$\{x \mid a \leq x \leq b\}$	$[a, b]$	closed	
$a \leq x < b$	$\{x \mid a \leq x < b\}$	$[a, b)$	half-open	
$a < x \leq b$	$\{x \mid a < x \leq b\}$	$(a, b]$	" "	

Important: Place card under blue copy.

Distance / Rate Problems

Recall: $d = rt$

On a commute to LA (90 miles away) a car leaves San Diego during rush hour and returns when there is no traffic. Assuming that you can drive twice as fast when there's no traffic and that the entire trip took 4.5 hours. How fast does car drive during rush hour

Let $t_1 = t$ (time of the 1st trip)

$r_1 =$ rate of the 1st trip

$t_2 =$ time of the 2nd trip

$r_2 =$ rate of the 2nd trip

$t_1 = 3$

$t_2 = 1.5$

$d_1 = r_1 t_1$

$r_1 = \frac{d_1}{t_1}$

$$r = \frac{90}{3} = 30$$

$d_1 = d_2$

$r_1 t_1 = r_2 t_2$

$t_1 + t_2 = 4.5$

$2r = r_2$

~~$r_1 = 30$~~

$r_1 t_1 = 2r_2 t_2$

$t_2 = 4.5 - t_1$

$$r_1 t_1 = 2r_2 (4.5 - t_1)$$

$$t_1 = 2(4.5 - t_1)$$

$$t_1 = 9 - 2t_1$$

$$3t_1 = 9$$

$$\frac{3t_1}{3} = \frac{9}{3}$$

$$t_1 = 3$$

Chemical Solution Type Problem

ex: a chemist has a solution containing 20% concentration of acid and one containing a 50% of acid. How many milliliters of each solution should be mixed to obtain 475 milliliters of a solution having a 30% concentration of acid

Let $x =$ the amount of 20% solution used

$y =$ the amount of 50% solution needed

$$x + y = 475$$

$$.2x + .5y = .3(475)$$

$.2x + .5y =$ acid amount of acid

$$x = 7$$

Important: Place card under blue copy.

Ex #3 graph $y = \sqrt{x}$

x	y
1	1
4	2

(1,0) (2,0) (4,2)

This is a function

Notice ~~we~~ if we took the equation from ex #2 $x = y^2$ and the square root of both sides $\pm \sqrt{x} = y$

Ex #4 graph $y = \sqrt[3]{x}$

x	y
0	0
1	1
-1	-1
8	2
-8	-2

Semi-circles

recall that $x^2 + y^2 = r^2$ is the equation of a circle centered at (0,0) of radius r. If we solve for y, $x^2 + y^2 = r^2 \rightarrow y^2 = r^2 - x^2 \rightarrow y = \pm \sqrt{r^2 - x^2}$

So solving for y split the circle into a top half (semicircle) and a bottom half.

$y = \sqrt{r^2 - x^2}$
 $y = -\sqrt{r^2 - x^2}$

$x^2 + y^2 = r^2$ is not a function, but $y = \sqrt{r^2 - x^2}$ and $y = -\sqrt{r^2 - x^2}$ are functions.

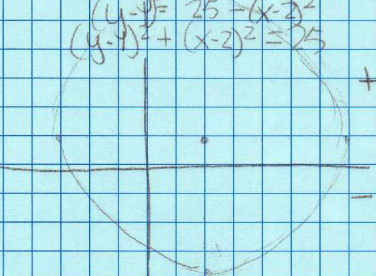
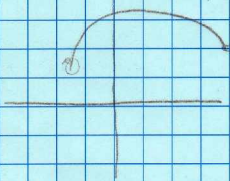
Ex #5 graph $\Delta y = \sqrt{1-x^2}$ First if we rewrite this in standard form for a circle, $y^2 = 1 - x^2$ $x^2 + y^2 = 1$

Circle of radius one and center (0,0). We need bottom half of the circle.

Important: Place card under blue copy.

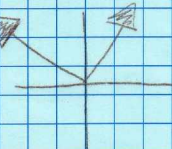
(14) Example
 $y = \sqrt{25 - (x-2)^2} + 1$
 $(y-1)^2 = 25 - (x-2)^2$
 $(y-1)^2 + (x-2)^2 = 25$

We want the top half

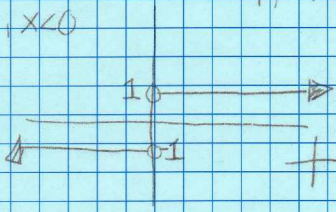



Piecewise Functions
 For some functions, different formulas are used for different regions of the domain, such functions are called piecewise functions.

Ex #6 $y = |x|$ $y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



Example $f(x) = \frac{|x|}{x}$ $f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$ $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

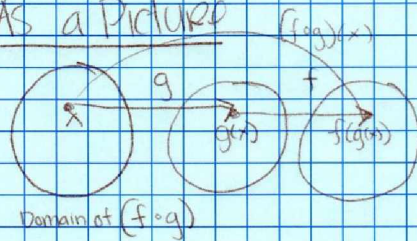


Important: Place card under blue copy.

Name:

Date:

Experiment:

As a Picture

EX

$$\text{Let } f(x) = x^2, g(x) = \sqrt{x}$$

$$h(x) = 3x + 1$$

Find the following and state the domain

$$\textcircled{1} (f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2$$

$$\text{Domain } [0, \infty)$$

$$\textcircled{2} (g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$$

$$\text{Domain } (-\infty, \infty)$$

$$\textcircled{3} (f \circ g \circ h)(x) = f(g(h(x))) = f(g(3x+1)) = f(\sqrt{3x+1}) = (\sqrt{3x+1})^2$$

$$\text{Domain}$$

$$[-1/3, \infty)$$

$$x \geq -1/3$$

Writing Functions as CompositionsWrite the following as the compositions of two functions f and g

$$\textcircled{1} h(x) = \frac{1}{\sqrt{x+3}} \quad \text{let } g(x) = \sqrt{x+3}, \text{ let } f(x) \text{ be } \frac{1}{x}, \text{ then } (f \circ g)(x) = \frac{1}{\sqrt{x+3}}$$

$$\textcircled{2} h(x) = (x+1)^2$$

$$\text{let } g(x) = x+1$$

$$\text{let } f(x) = x^2$$

Important: Place card under blue copy.

Melinda

Experiment:

Continued Notes

Basic principle of rational numbers: $\frac{ac}{bc} = \frac{a}{b}$

IF a, b, c are real numbers with $c \neq 0$
 In other words, you can cancel common factors from the numerator and denominator as long as its not zero

This gives us a way to reduce rational expressions
Reducing to lowest terms

$$\text{Ex.: } \frac{2x+4}{x^2+5x+6} = \frac{2(x+2)}{(x+3)(x+2)} = \frac{2}{(x+3)}$$

$$\text{Ex.: } \frac{b-a}{a^2-b^2} = \frac{b-a}{(a-b)(a+b)} = \frac{-(a-b)}{(a-b)(a+b)} = \frac{-1}{(a+b)}$$

$$\text{Ex.: } \frac{x^6y^2}{xy} = \frac{x^{6-1}y^{2-1}}{x^0y^0} = x^5y^1 \text{ (old way)}$$

$$= \frac{x^5y(x^0y^0)}{x^0y^0} = x^5y$$

Note

$$\text{NO } \frac{x+3}{3} \neq x$$

$$\text{YES } \frac{3(x+3)}{3} = x+3$$

Example

$$? \frac{(x-1)}{(x-1)} = 1$$

? NO, this is not true when $x=1$

The left hand side is undefined when $x=1$, the RHS IS Defined.

10/2/10

Section A.4 cont'D

Multiplication and Division of rational numbers

IF $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers
 then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and if $c \neq 0$ then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Note

Multiplying and dividing rational expressions is in the same fashion.

Procedure

- ① IF dividing; invert and multiply
- ② Factor completely
- ③ Divide out common factors

Ex.: Find and simplify

$$? \left(\frac{3x-3y}{4} \right) \left(\frac{2y}{x^2-y^2} \right) = \frac{3(x-y)}{2} \cdot \frac{2y}{(x-y)(x+y)} = \frac{3y}{x+y}$$

If it was $\frac{2y}{x^2-y^2}$

$$\frac{2y}{(x-y)(x+y)} = \frac{2y}{x+y}$$

Important: Place card under blue copy.

Name: 13

Date: 10/2/06

U3

Experiment:

Addition and subtraction of Rational Expressions

Def.: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then

$\frac{a}{b} + \frac{c}{d} = \frac{a+d}{b}$ and $\frac{a}{b} - \frac{c}{d} = \frac{a-c}{b}$

Building up the Denominator

Ex.: $\frac{1}{2} = \frac{?}{4}$, here 2 is the solution. This definition only works when there is a common denominator so if there is NOT a common denominator we must find one.

Note

we can also build up the denominator in rational expressions. Factor everything to find? There is NOT a common denominator we must find one.

Ex.: $\frac{2}{x-7} = \frac{?}{x^2-5x+4}$

Procedure

- (1) Factor each denominator completely.
(2) Write a product using each factor that appears in a denominator.
(3) For each factor use the highest power of that factor that appears.

Ex.: Find the LCD of $\frac{2}{x-2}$ and $\frac{3}{x^2-28}$

lcd = 2 * 7 * (x-2)^2. The LCD is 2 * 7 * (x-2)^2 * (x+2). The LCD contains the factors 2, 7, (x-2)^2, (x+2).

Ex: of finding LCD in sum/diff of rational expressions

Procedure

- (1) Find the LCD
(2) Build up each denominator to the LCD
(3) Add or subtract the fractions
(4) Simplify

Ex: $\frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3}$

Important: Place card under blue copy.

Solve the following

$$\textcircled{6} 7+2x-4+4x=2$$

$$6x+3=2 \quad x=3$$

$$\begin{array}{r} -3 \quad -3 \\ 6x = 18 \\ \hline x = 3 \end{array}$$

Simplify, Assume
 $x+y$ could be Negative

$$\textcircled{7} \frac{2}{x} + 4 = \frac{1}{x} - \frac{6}{2x}$$

$$\textcircled{8} \frac{2}{x} - \frac{4}{x} + \frac{8}{2x} = \frac{1}{x} - \frac{6}{2x}$$

$$\frac{4}{2x} + \frac{8x}{2x} = \frac{2}{2x} - \frac{6}{2x}$$

$$4+8x = -4$$

$$8x = -8 \quad x = -1$$

$$\textcircled{9} \sqrt{x^2} = |x|$$

$$\textcircled{10} \sqrt[3]{x^{12}y^9} = x^4y^3$$

$$\textcircled{10} \sqrt{x^4} = x^2$$

$$\textcircled{11} \sqrt{x^4y^6} = x^2y^3$$

Rationalize the denominator & Simplify

$$\textcircled{12} \frac{1}{2+\sqrt{5}} \cdot \frac{(2-\sqrt{5})}{(2-\sqrt{5})} = \frac{2-\sqrt{5}}{4-5} = \frac{2-\sqrt{5}}{-1} = \sqrt{5}-2$$

$$\textcircled{13} \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$$

$$\textcircled{14} \frac{\log_2(\sqrt[3]{4(y+1)})^2}{\sqrt[3]{4(y+1)} (\sqrt[3]{4(y+1)})^2} = \frac{\log_2(4(y+1)^{2/3})}{2 \sqrt[3]{4(y+1)}}$$

Section 1.2

Linear and Absolute Value Inequalities

Def.: The symbol ∞ represents infinity, is NOT a number, but a concept meaning there is no bound.

Interval Notation

Suppose we have an inequality, $x > 4$
The solution set to this inequality is $\{x | x > 4\}$

We can write $x > 4$ as $4 < x < \infty$
 $x \geq 4$ as $x \leq 4 < \infty$
 $\downarrow \quad \downarrow$
 $[4, \infty) \quad (4, \infty)$

Important: Place card under blue copy.

Solving linear inequalities

Def.: A linear inequality is of the form $ax + b < 0$

Ex.: Write the solution set in interval notation and graph it.

$$-2x + 4 < 0$$

$$\begin{matrix} -4 & + \\ -2x & < -4/2 \\ x & > 2 \end{matrix}$$

Ex.: $(\frac{1}{3}x + 3) \leq (\frac{1}{9}x + 6)$

~~$3x + 27 \leq x + 54$~~

$$3x + 27 \leq x + 54$$

$$2x + 27 \leq 54$$

$$2x \leq 27/2$$

$$x \leq 27/4$$

$$a < x \leq 27/4$$

$$(-\infty, \frac{27}{4}]$$

Compound inequalities

Def.: A compound inequality is a sentence containing two simple inequalities connected by "and" or "or"

Ex.: $x \geq -1$ and $x \leq 2 \rightarrow -1 \leq x \leq 2$ ^{interval notation} $[-1, 2]$

Note

This interval does NOT contain the ∞ symbol and is therefore called a bounded interval

Summary of Bounded Inequalities

inequality set	Interval Notation	Type	Graph
$a < x < b$ $\{x a < x < b\}$	(a, b)	open	(\quad)
$a \leq x \leq b$ $\{x a \leq x \leq b\}$	$[a, b]$	closed	$[\quad]$
$a \leq x < b$ $\{x a \leq x < b\}$	$[a, b)$	half open	$[\quad)$
$a < x \leq b$ $\{x a < x \leq b\}$	$(a, b]$	half open or half closed	$(\quad]$

Important: Place card under blue copy.

Solving linear inequalities

Def: A linear inequality is of the form $ax + b \leq 0$

Ex.: Write the solution set in interval notation and graph it.

$$-2x + 4 < 0$$

$$\begin{array}{r} -2x < -4/2 \\ \underline{-2} \\ x > 2 \end{array}$$

Ex.: $\left(\frac{1}{3}x + 3\right) \leq \left(\frac{1}{2}x + 6\right)$

~~1/3x + 3~~

$$3x + 27 \leq x + 54$$

$$2x + 27 \leq 54$$

$$2x \leq 27/2$$

$$x \leq 27/2$$

$$-\infty < x \leq 27/2$$

$$(-\infty, 27/2]$$

Compound inequalities

Def.: A compound inequality is a sentence containing two simple inequalities connected by "and" or "or"

Ex.: $x \geq -1$ and $x \leq 2 \rightarrow -1 \leq x \leq 2$ Interval notation $[-1, 2]$

Note

This interval does NOT contain the ∞ symbol and is therefore called a bounded interval

Summary of Bounded Inequalities

Inequality set	Interval Notation	Type	Graph
$a < x < b$ $\{x a < x < b\}$	(a, b)	open	
$a \leq x \leq b$ $\{x a \leq x \leq b\}$	$[a, b]$	closed	
$a \leq x < b$ $\{x a \leq x < b\}$	$[a, b)$	half open	
$a < x \leq b$ $\{x a < x \leq b\}$	$(a, b]$	half open or half closed	

Important: Place card under blue copy.

Name: 13

Date: Oct. 11, 2006

Experiment:

Let t_1 = time of the 1st trip
 r_1, r_2 = rate of the 1st trip

t_2 = time of the 2nd trip
 r_2 = rate of the 2nd trip

$$d = d_2 \quad r_1 t_1 = r_2 t_2 \rightarrow r_1 t_1 = 2 r_1 t_2$$

$$t_1 + t_2 = 45 \rightarrow t_1 + 45 t_2 = 45$$

$$2 t_1 = r_2$$

$$r_2 = 45 t_1$$

$$t_1 = 2(45 t_2)$$

$$t_1 = 3$$

$$t_2 = 15$$

Let $r_1 = r_2$

$$r = d_1$$

$$t_1 = 3$$

$$r = \frac{d_1}{t_1} = \frac{10}{3} = 30$$

$$t_1 = 2(45 t_2)$$

$$t_1 = 9 - 2 t_2$$

$$3 t_1 = 9 - 2 t_2$$

$$t_1 = 3$$

Chemical Solution Type Problems

Ex/ A chemist has a solution containing a 10% concentration of acid and one containing 50% concentration of acid.

How many milliliters of each solution should be mixed to obtain 47.5 milliliters of a solution having 20% concentration of acid?

Let x = the amount of 10% solution needed
 y = the amount of 50% solution needed

$$\begin{cases} x + y = 47.5 \\ .12x + .5y = .2(x + y) \end{cases} \quad \left(\frac{.12x + .5y}{x + y} \right) = .2$$

$$y = 47.5 - x$$

$$\begin{aligned} .12x + .5y &= .2x + .2y \\ -2x - .2y &= -2x - .2y \end{aligned}$$

$$-.8x + .3y = 0 \rightarrow -.8x + .3(47.5 - x) = 0$$

$$x = 37.5$$

$$y = 10$$

Important: Place card under blue copy.

Name: 13

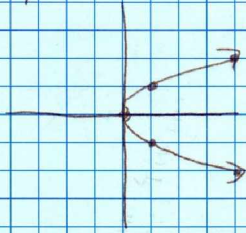
Date: Oct. 16, 06

27

Experiment:

Ex. 2/ $x=y^2$

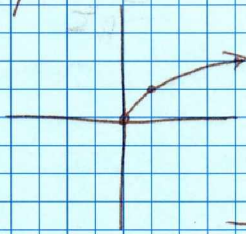
x	y
0	0
1	1
1	-1
4	2
4	-2



This is also a parabola (opens to right)
 This is NOT the graph of a function.

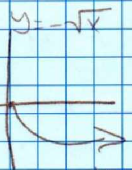
Ex. 3/ Graph $y=\sqrt{x}$

x	y
0	0
1	1
4	2



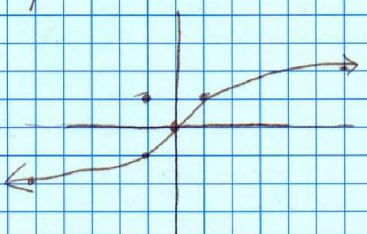
This IS a function

Notice if we took the equation from Ex. #2 $x=y^2$ and take the square root from both sides.



Ex. 4/ Graph $y=\sqrt[3]{x}$

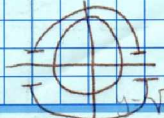
x	y
0	0
1	1
-1	-1
8	2
-8	-2



Semi-Circles

Recall that $x^2+y^2=r^2$ is the equation of a circle centered at (0,0) of radius r. If we solve the equation for y,

$x^2+y^2=r^2 \rightarrow y^2=r^2-x^2 \rightarrow y=\sqrt{r^2-x^2}$
 so solving for y splits the circle into a top half.



IS NOT a function, but

Important: Place card under blue copy.

$y=\sqrt{r^2-x^2}$ & $y=-\sqrt{r^2-x^2}$

Experiment:

Ex #5 / Graph $y = -\sqrt{1-x^2}$, first if we rewrite in standard form for circle,

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

Circle of radius 1, center (0,0)

We need the bottom half of this circle

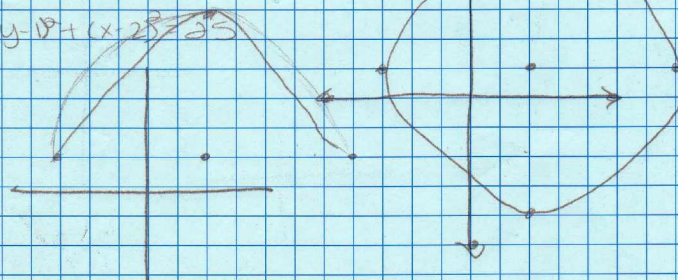
b) $y = \sqrt{25 - (x-2)^2} + 1$

$$y - 1 = \sqrt{25 - (x-2)^2}$$

$$y - 1 = 25 - (x-2)^2$$

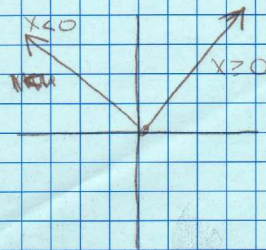
$$(y-1)^2 + (x-2)^2 = 25$$

circle with center (2,1) radius 5

Piece wise functions

For some functions, different formulas are used for different regions of the domain, such functions are called piecewise functions.

Ex. 6/ $y = |x|$ $y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

**Important:** Place card under blue copy.

$$f(x) = \sqrt{x+3}, \quad g(x) = \frac{1}{x}, \quad h(x) = 2x+1$$

$$(1) (f+g)(x) = f(x) + g(x) = \sqrt{x+3} + \frac{1}{x} \quad \text{Domain of } f: [-3, \infty)$$

$$g: (-\infty, 0) \cup (0, \infty)$$

$$(2) (f \cdot g)(x) = \sqrt{x+3} \cdot \left(\frac{1}{x}\right) = \frac{\sqrt{x+3}}{x} \quad h: (-\infty, \infty)$$

$$(3) (f/h)(x) = \frac{\sqrt{x+3}}{2x+1} \quad x \geq -3$$

$$x \neq -\frac{1}{2}$$

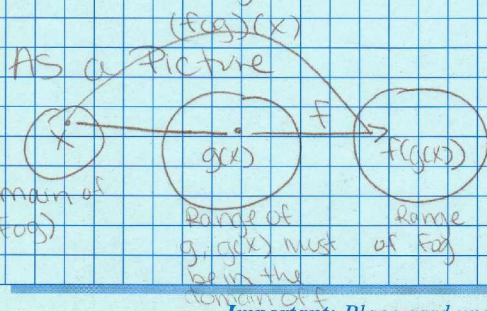
$$\left(-3, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$$

Composition of Functions

If f and g are two functions, the composition of f and g written $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$, provided that $g(x)$ is in the domain of $f(x)$.

The composition of g and f , $g \circ f$ is

$(g \circ f)(x) = g(f(x))$, provided that $f(x)$ is in the domain of $g(x)$.



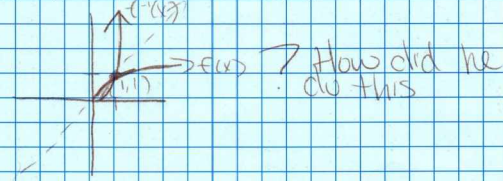
Important: Place card under blue copy.

Name: 13

Date: Oct. 25, 2000

40

Experiment:

 $f(x) = 7x$ and its inverse

Finding Inverses Mentally

Ex/ Find the inverse of

a) $f(x) = 6x - 3$

f multiply by 6
then subtract 3f⁻¹ - add 3
divide by 6

$$f^{-1}(x) = \frac{x+3}{6}$$

b) $f(x) = x^3 + 4$

f - cubes the #
add adds 4f⁻¹ subtract 4
then take's cube root

$$f^{-1}(x) = \sqrt[3]{x-4}$$

Important: Place card under blue copy.

75


Name: #13 Date: Nov. 13, 2006

Experiment:

Ex: $f(x) = \sin(x)$ and $g(x) = \cos(x)$ are periodic with period 2π .

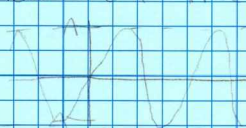
Def: The amplitude of a sine wave or the amplitude of the function is the absolute value of half the difference between the maximum and minimum values of the y -coordinates on the wave.

Ex: The amplitude of $\sin(x)$ and $\cos(x)$ is 1



$$\frac{1}{2} (1 - (-1)) = 1$$

LOOK at the graph $y = A \sin x$

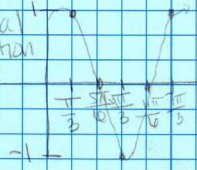


? Theorem: The amplitude of $A \sin(x)$ and $A \cos(x)$ is $|A|$.

Def: The phase shift of the graph $y = \sin(x-c)$ or $y = \cos(x-c)$ is c . (This is just horizontal translation)

? Ex: Graph $y = \cos(x - \pi/3)$

Horizontal translation



x	$\cos(x - \pi/3)$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
π	0
$4\pi/3$	1

Important: Place card under blue copy.

Name: #13

Date: Nov 20, 02

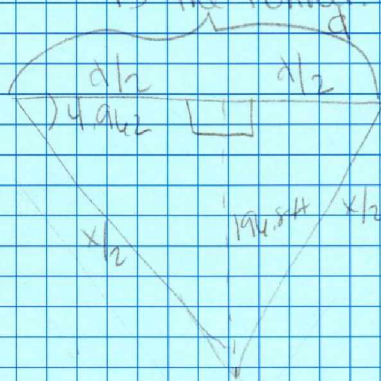
Experiment:

$$\text{Solve } \sin^2 x - 1 + \cos x$$

$$\begin{aligned} \sin^2 x - 1 + \cos x &= 0 \\ 1 - \cos^2 x - 1 + \cos x &= 0 \\ 0 &= \cos^2 x - \cos x \\ 0 &= \cos x (\cos x + 1) \\ \cos x &= 0 \text{ or } \cos x = -1 \\ \sum x, x &= \frac{\pi}{2} + \pi k \text{ or } \pi \\ \sum x &= \pi + 2\pi k \end{aligned}$$

3.6 Trig Word Problems

#36 / length of a tunnel. A tunnel under a mountain is 1916.8 ft. below the surface at its lowest point as shown in the drawing. If the angle of depression of the tunnel is 4.962° from how high apart on the surface are the entries to the tunnel? How long is the tunnel?



$$\tan 4.962^\circ = \frac{1916.8}{d/2}$$

$$\frac{(1916.8)(2)}{d}$$

$$d = \frac{2(1916.8)}{\tan 4.962^\circ}$$

Let x = length of tunnel

$$\boxed{x = \frac{2(1916.8)}{\sin 4.962^\circ}} \quad \sin 4.962^\circ = \frac{1916.8}{x/2} = \frac{2(1916.8)}{x}$$

Important: Place card under blue copy.

Appendix B

Post-Course Surveys for Student Note-Takers

Alicia

Gender: Male Female Ethnicity: Mexican-American (SDCC 1) or SDCC 4
 Please fill-in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.



WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition	composing Art, music, etc	Not sure
Even	Balanced or referring to a #	An even #
Interval	Cycle	not sure
Imaginary	NOT Real	referring to imaginary #'s
Prime	referring to early years "prime years"	#'s such as 1, 3, 5, 7...
Product	A result of.	The answer after multiplying two #'s or variable
Negative	Something bad, awful etc	a negative # any # below zero
One to One	A conversation	Being one to one is being invertible
Identity	A person's background or meaning of oneself	ex: $\sin^2(x) + \cos^2(x) = 1$
Variable	Variety	A letter used in place of an unknown #.
Transformation	A change	Graphing an expanded equation
Inequality	Injustice or unfair	ex: $x < 0$

NOTE-TAKER NUMBER OR NAME:

(Use only if you are a note-taker for the study.)

Linda

Gender: Male Female Ethnicity: Mexican-American/Chicana ^(B) SDCC 1 or SDCC 4
 Please fill-in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.


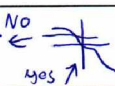
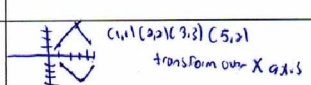
WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition	Composition of words/ music	use numbers
Even	Today the sun is out <u>even</u> though cold. could also be even #	even # compared to odd #
Interval	section	(a, b) interval notation
Imaginary	Something that's fictional not true	$0 + bi$ $i = \text{imag.}$
Prime	(Not sure)	Prime #'s 1, 3, 5...
Product	Something/ item	Result of one thing to another
Negative	Don't think so <u>negative</u> (down)	-3, -2, -1 negative #'s  in graph
One to One	Relation ship.	f^{-1} crosses only @ one pt. 
Identity	Who you are what you identify yourself as	Identity of #'s
Variable	(Not sure)	a, b, c, \dots
Transformation	changing of something girl transformed in to woman	$-2(x+3)^2 - 4$ way graph will be @ end of all transformations
Inequality	Unequal	unequal

NOTE-TAKER NUMBER OR NAME:

(Use only if you are a note-taker for the study.)

Frank

Gender: Male Female Ethnicity: Chicano #7 SDCC 1 or SDCC 4
 Please fill in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.

WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition	N/A	$(2x + 5y)(3x + 7y)$ One compositions
Even	even candy is good	2, 4, 6, 8, 10
Interval	interval notation is \rightarrow	$(-\infty, \infty)$ if your graphing 
Imaginary	Something that can't be seen	$i, \pi, \pm i$
Prime	Prime time	# that can be multiplied by 1 and it self. 1, 3, 5, 7
Product	The product of 2 e $6 = 12$	multiplying numbers
Negative	Owing someone, debt	-1, -2, -3, -4, -5
One to One	N/A	1-1 no 
Identity	N/A	there are theorems that use identity
Variable	X, Y, Z	X, Y, Z, A, b, c
Transformation	to transform is to change	
Inequality	not equal	$\neq \leq <$

NOTE-TAKER NUMBER OR NAME: #7

(Use only if you are a note-taker for the study.)

Diana

Gender: Male (Female) Ethnicity: Mexican American, SDCC 1 or SDCC 4
 Please fill-in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.


WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition	the contents of an object, how it is set up	I don't know
Even	fair, just, equal	a number that is divisible by two without a remainder
Interval	a period of time	a set of points that repeat themselves.
Imaginary	not real	a square root of a negative #.
Prime	to condition or give a mind set to someone.	prime # ...? not sure.
Product	the resulting object, action, or #.	the resulting # after a multiplication problem.
Negative	bad, not good	a number less than zero.
One to One	a private, two-person meeting	a function which passes the horizontal line test & is invertible.
Identity	the group/ideology one associates with	Not sure.
Variable	allowable degree of variety.	an unknown entity in a math problem.
Transformation	to shift shapes, feelings, or position. to change.	Not sure.
Inequality	Not equal, not even.	A problem showing the relationship b/w # two entities which are not equal.

NOTE-TAKER NUMBER OR NAME: #12

(Use only if you are a note-taker for the study.)

Melinda

Gender: Male Female Ethnicity: Hispanic # 13 SDCC 1 or SDCC 4
 Please fill in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.

WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition	+ Dont know	Dont know
Even	All numbers that are even.	2, 4, 6, 8
Interval	+ I dont know	DK
Imaginary	A number that is not Real	i
Prime	Prime #'s are odd #'s	1, 3, 5, 7
Product	Product is given by 2 #'s that are being multiplied	$5 \times 7 = \boxed{35}$ - Product
Negative	All numbers that are to the left of 0 on number line.	-7
One to One	one-to-one is when a graph is not hit twice by the vertical line test	
Identity	+ Dont know	DK
Variable	x, y are variables	$2x + y = 0$ x & y are variables
Transformation	+ Dont know	DK
Inequality	+ Dont know	DK

NOTE-TAKER NUMBER OR NAME:

(Use only if you are a note-taker for the study.)

Sally

Gender: Male Female Ethnicity: Mexican SDCC 1 or SDCC 4
 Please fill-in the requested information in the table below. You may be brief in writing your definition. If you are not sure about a response, write "not sure" however you also may write your best guess.

WORD	Please define the meaning(s) of the word in everyday use:	Please define the meaning(s) of the word as used in mathematics
Composition		
Even	Fair	multiple of 2
Interval		numbers in between
Imaginary	make-believe; done	non-real numbers
Prime	Best	number that can be divided by one and itself
Product	item	multiplying
Negative	wrong	number less than zero
One to One		crosses horizontal line that crosses the graph once
Identity	who + what you are	
Variable		x, y, z
Transformation	to change	transforming sd path from the origin to graph
Inequality	not-fair	

NOTE-TAKER NUMBER OR NAME:

9

(Use only if you are a note-taker for the study.)

References

- Aiken, L. (1972). Language factors in learning mathematics. *Review of Educational Research*, 42 (3), 359-385.
- Alanti, C. (1995). Primary school teachers' explanation of boys' disruptiveness in the classroom: a gender specific aspect of the hidden curriculum. In S. Mills (ed.), *Language and Gender: Interdisciplinary Perspectives*. (pp. 149 – 159). New York, NY: Longman Group Limited.
- American Diploma Project. (2004). *Ready or Not: Creating a High School Diploma That Counts*. The American Diploma Project. Achieve, Inc.
- American Mathematical Association of Two Year Colleges. (1995). *Crossroads in mathematics: Standards for introductory college mathematics before calculus*. D. Cohen, ed. Memphis, TN: American Mathematical Association of Two Year Colleges.
- American Mathematical Association of Two Year Colleges. (2002). *The Crossroads Revisited Project: The American Mathematical Association of Two-Year Colleges Executive Summary of Research on the Effects of Crossroads*. <http://www.amatyc.org/Crossroads/execsummaryARG.html>.
- Astin, A. (1968). *The College Environment*. Washington, DC: The American Council on Education.
- Ball, D. & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: knowing and using mathematics. In J. Boaler (ed.), *Multiple Perspectives on the Teaching and Learning of Mathematics* (pp. 83- 104). Westport, CT: Ablex.
- Bartolini Bussi, M. (1998). Verbal interaction in the mathematics classroom: a vygotskian analysis. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 155-166). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Barwell, R. (2005). Ambiguity in the mathematics classroom. *Language and Education*, 19 (2), 118-126
- Blake, M. & Van Sickle, S. (2001). Helping linguistically diverse students share what they know. *Journal of Adolescent & Adult Literacy*, 44 (5), 468-475.
- Bell, J. (2002). Framing and text interpretation across languages and cultures: a case study. In J. Flowerdew (ed.), *Academic Discourse*. (pp. 287 – 301). New York, NY: Person Education.

- Bourdieu, P. (1998). *Masculine Domination*. Stanford, CA: Stanford University Press.
- Braxton, J. M., Bray, N. J. & Berger, J. B. (2000). Faculty teaching skills and their influence on the college student departure process. *Journal of College Student Development*, 41 (2), 215 – 227.
- Brown, S. & Race, P. (2002). *Lecturing: A Practical Guide*. Sterling, VA: Stylus Publishing Inc.
- Butler, J. (1999). *Gender Trouble: Feminism and the Subversion of Identity*. New York, NY: Routledge.
- Cahnmann, M. & Remillard, J. (2002). What counts and how: mathematics teaching in culturally, linguistically, and socioeconomically diverse urban settings. *The Urban Review*, 34 (3), 179-204.
- Cazden, C. (2001). *Classroom Discourse: The Language of Teaching and Learning*. Portsmouth, NH: Heinemann.
- Cestari, M. (1998). Teacher-student communication in traditional and constructivist approaches to teaching. In H. Steinbring, M. Bartolini Bussi, & A. Sierpinska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 155-166). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Chickering, A. & Reisser, L. (1993). *Education and Identity*. San Francisco, CA: Jossey-Bass Publishers.
- Cobb, P. Yackel, E. & Wood, T. (1993). Chapter 3: theoretical orientation. *Journal for Research in Mathematics Education, Monograph*, (6), 21-32.
- Cole, M. & Scribner, S. (1974). *Culture and Thought: A Psychological Introduction*. New York, NY: John Wiley & Sons, Inc.
- Conley, D. and F. Bodone. (2002). *University expectations for student success: Implications for system alignment and state standard and assessment policies*. Center for Education Policy Research, University of Oregon. ERIC, ED464922.
- Corson, D. (1993). *Language, Minority Education and Gender: Linking Social Justice and Power*. Bristol, PA: Multilingual Matters LTD.
- D'Andrade, RG. (1984). Cultural meaning systems. In R. Shweder & R. LeVine (ed.), *Culture Theory: Essays on Mind, Self, and Emotion*. (pp. 88 – 119). New York, NY: Cambridge University Press.

- Davis, R. (1984). A few commonly-shared frames. In *Learning Mathematics: The Cognitive Science Approach to Mathematics Education*, Chapter 8. Westport, CT: Ablex.
- Dey, E. & Hurtado, S. (2005). College students in changing contexts. In Altbach, P, Berdahl, R. & Gumport, P. (ed.), *American Higher Education in the Twenty-First Century: Social, Political, and Economic Challenges*. (pp. 315 – 339). Baltimore, MD: The John Hopkins University Press.
- Fonzi, J. & Smith, C. (1998). Communication in a secondary mathematics classroom: some images. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 155-166). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Gee, J.P. (2005). *An Introduction to Discourse Analysis: Theory and Method*. New York, NY: Routledge, Taylor & Francis Group.
- Gershman, J. (2005). “Harvard president posts transcript of divisive speech.” *The New York Sun*. New York, NY. February 18, 2005.
- Greer, B. (1992). Multiplication and division as models of situations. In D. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 276-295). New York: Macmillan Publishing Company.
- Johnson, C. (2005). “Do genes play a role . . .” *Boston Globe*. Boston, MA. January 25, 2005.
- Kanes, C. (1998). Examining the Linguistic Mediation of Pedagogic Interactions in Mathematics. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 120-139). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Kiewra, K., Benton, S., Kim, S., Risch, N. & Christensen, M. (1995). Effects of note-taking format and study technique and recall and relational performance. *Contemporary Educational Psychology*, 20, 172 – 187.
- Kobayashi, K. (2005). What limits the encoding effect of note-taking? A meta-analytic examination. *Contemporary Educational Psychology*, 30, 242 – 262.
- Krummheuer, G. (1998). Formats of argumentation in the mathematics classroom. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 155-166). Reston, VA: The National Council of Teachers of Mathematics, Inc.

- Lakoff, G. & Nunez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York, NY: Basic Books.
- Lakoff, RT. (2004). *Language and women's place: text and commentaries*. Edited by M. Bucholtz. New York, NY: Oxford University Press.
- Leammon, R. (1999). *Thinking About Teaching and Learning: Developing Habits of Learning with First Year College and University Students*. Sterling, VA: Stylus.
- Leung, C. (2005). Mathematical vocabulary: fixers of knowledge or points of exploration? *Language and Education*, 19 (2), 127-135
- Lynch, J., Leder, G. & Forgasz, H. (2001). Mathematics: a dilemma for feminists. In MacNabb, E., Cherry, M., Popham, S. & R. Prys (ed.), *Transforming the Disciplines A Women's Studies Primer*. (pp. 185 – 192). New York, NY: The Haworth Press, Inc.
- MacLachlan, G. & Reid, I. (1994). *Framing and Interpretation*. Portland, OR: Internationalized Specialized Book Services Inc.
- Mercer, M. (1995). Language and the guided construction of knowledge. In G.M. Blue and R. Mitchell (ed.), *Language and Education* (pp. 28- 40). Bristol, PA: Multilingual Matters, Ltd.
- Orr, E. (1987). *Twice as Less*. New York, NY: Basic Books.
- Pascarella, E. & Terenzini, P. (2005). *How College Affects Students: Volume 2, A Third Decade of Research*. San Francisco, CA: John Wiley & Sons, Inc.
- Pedro, J. Wolleat, P. Fennema, E. & Becker, A. (1981) Election of high school mathematics by females and males: attributions and attitudes. *American Educational Research Journal*, 18, (2), 207-218.
- Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. New York, NY: Routledge and Kegan Paul.
- Piolat, A., Olive, T. & Kellog, R. (2005). Cognitive effort during note taking. *Applied Cognitive Psychology*, 19, 291 – 312.
- Pirie, S. (1998). Crossing the gulf between thought and symbol: language as (slippery) stepping-stone. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 7-29). Reston, VA: The National Council of Teachers of Mathematics, Inc.

- Pollitt, K. (2005). "Summers of our discontent." *The Nation*. Washington, DC. February 21, 2005.
- Pruitt-Logan, A., Gaff, J., & Jentoff, J. (2002) *Preparing Future Faculty in the Sciences and Mathematics: A Guide for Change*. Council of Graduate Schools. Washington, DC: Association of American Colleges and Universities.
- Rickards, J., Fajen, B., Sullivan, J. & Gillespie, G. (1997). Signaling, notetaking, and field independence-dependence in text comprehension and recall. *Journal of Educational Psychology*, 89, (3), 508 – 517.
- Rismann, B. J. (1998). *Gender Vertigo*. New Haven, CT: Yale University Press.
- Schleppegrell, M. (2004). *The Language of Schooling: A Functional Linguistic Perspective*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Secada, W. (1991). Degree of bilingualism and arithmetic problem solving in hispanic first graders. *The Elementary School Journal*, 92 (2), 213-231.
- Secada, W. (1996). Urban students acquiring English and learning mathematics in the context of reform. *Urban Education*, 30 (4), 422-448.
- Secada, W. (1988). [Review of *Twice as Less: Black English and the Performance of Black Students in Mathematics and Science* and *Speaking Mathematically: Communication in Mathematics Classrooms*]. *Journal for Research in Mathematics Education*, 19 (4), 362-366.
- Seeger, F. (1998). Discourse and beyond: on the ethnography of classroom discourse. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 155-166). Reston, VA: The National Council of Teachers of Mathematics, Inc.
- Sewell Jr., W. H. (1992). A theory of structure: duality, agency, and transformation. *The American Journal of Sociology*, 98, (1), 1 – 29.
- Seymour, E. & Hewitt, N. (1997). *Talking About Leaving: Why Undergraduates Leave the Sciences*. Boulder, CO: Westview Press.
- Sierpiska, A. (1998). Three epistemologies, three views of classroom communication: constructivism, sociocultural approaches, interactionism. In H. Steinbring, M. Bartolini Bussi, & A. Sierpiska (ed.), *Language and Communication in the Mathematics Classroom* (pp. 30-62). Reston, VA: The National Council of Teachers of Mathematics, Inc.

- Spencer, S. Steele, C. & Quinn, D. (1999) Stereotype threat and women's math performance. *Journal of Experimental Social Psychology*, 35, 4-28.
- Stigler, J. & Baranes, R. (1988 - 1989). Culture and mathematics learning. *Review of Research in Education*, 15, 253-306.
- Tannen, D. (1993). The relativity of linguistic strategies: rethinking power and solidarity in gender and dominance. In D. Tannen (ed.), *Gender and Conversational Interaction* (pp. 165 – 188). New York, NY: Oxford University Press.
- Tannen, D. (1994). *Gender and Discourse*. New York, NY: Oxford University Press.
- Tinto, V. (1987). *Leaving College*. Chicago, IL:
- Tinto, V. (2000). Linking learning and leaving: exploring the role of the college classroom in student departure. In J. Braxton (ed.), *Reworking the Student Departure Puzzle*. (pp. 81 – 94). Nashville, TN: Vanderbilt University Press.
- Titsworth, B. & Kiewra, K. (2004). Spoken organizational lecture cues and student notetaking as facilitators of student learning. *Contemporary Educational Psychology*, 29, 447 – 461.
- Valdes, G. (2001). *Learning and Not Learning English: Latino Students in American Schools*. New York, NY: Teachers College Press.
- Veel, R. (2000). Language, knowledge and authority in school mathematics. In F. Christie (ed.), *Pedagogy and the Shaping of Consciousness: Linguistic and Social Processes*. (pp. 185 – 216). New York, NY: Continuum.
- West, C. & Zimmerman, D. (1987). Doing gender. *Gender and Society*, 1, (2), 125 – 151. New York, NY: Sage Publications, Inc.
- Wink, J. & Putney, L. (2002). *A Vision of Vygotsky*. Boston, MA: Allyn and Bacon, A Pearson Education Company.
- Xie, Y. & Shauman, K. (2003). *Women in Science: Career Processes and Outcomes*. Cambridge, MA: Harvard University Press.
- Yackel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, Monograph*, 27 (6), 458-477.