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Neutrino oscillations and uncertainty in the solar model

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The Mikheyev-Smirnov-Wolfenstein (MSW) resonant neutrino oscillation mechanism is investigated for the Sun using a detailed numerical solar model and a modified version of the Parke-Walker technique for following the neutrino phases through the oscillation resonance. We present overall solar-neutrino spectra and the associated expected neutrino count rates for the ^{37}Cl, ^{71}Ga, and Kamiokande detectors for ranges of masses and vacuum mixing angles for two neutrino species. We also investigate the effects of uncertainties in the solar model. In particular, we examine the effect of opacity changes on the expected solar-neutrino spectrum and resulting parameter space for the MSW mechanism. We find that plausible uncertainties in the standard solar model, and in particular the opacity, translate into significant expansion in the constraints on neutrino masses and vacuum mixing angles from neutrino experiments. It is shown, however, that forthcoming results from the Kamiokande solar-neutrino experiment could put stringent constraints on even the expanded MSW parameter space.

I. INTRODUCTION

The results of the ^{37}Cl solar-neutrino experiment of Davis, Harmes, and Hoffman[1] have perplexed astrophysicists for some time because the observed neutrino count rate [2.1±0.3 SNU (solar-neutrino unit) where 1 SNU$=10^{-36}$ captures/atom of chlorine/sec] is inconsistent with the rate predicted from the standard solar model ($=5.8$ SNU, Ref. 2). There has been considerable speculation as to the cause of this discrepancy but most "classical" solutions examine ways of reducing the central temperature by about 4% (cf. Ref. 3).

Recently, however, Mikheyev and Smirnov[4] have shown that neutrino oscillations could have a resonant character in the Sun, where the presumably small vacuum mixing of the electron neutrino ν_e with some other flavor of neutrino, ν_x, would be enhanced to the maximal value for certain density regimes in the Sun determined by the masses of ν_e and ν_x and the vacuum mixing angle. Additionally, an adiabatic condition may be obtained in which, for a range of neutrino energies and vacuum mixing angles, all ν_e may be transformed to ν_x (cf. the discussions in Refs. 4–7).

Since the degree of adiabaticity and hence the completeness of the $\nu_e \rightarrow \nu_x$ transformations depends on the neutrino energies, vacuum mixing angles, and masses there is a large region of this parameter space which yields a ^{37}Cl ν_e count rate consistent with the result of Davis, Harmer, and Hoffman.[1] Because the ^{37}Cl detector is most sensitive to the high-energy solar neutrinos ($E_\nu > 814$ keV) from the ^{8}B and ^{7}Be decays, transformation of these high-energy neutrinos is most effective in lowering the overall neutrino count rate. If the parameters of neutrino masses and vacuum mixing angles are chosen so that these highest-energy neutrinos go through the resonance adiabatically then the lowest-energy solar neutrinos will not go through the resonance adiabatically and, thus, will not be completely transformed. Alternatively, if the neutrino masses and vacuum mixing angles are chosen so that the low-energy solar neutrinos go through the resonance adiabatically, then the highest-energy neutrinos are only partially transformed. Either of these scenarios can yield the observed ^{37}Cl count rate.[5–7]

Since the proposed ^{71}Ga experiment has a low threshold energy ($E_\nu = 236$ keV) it is sensitive to the solar *pp* neutrinos, with the result that the standard solar model predicts a count rate of ~ 120 SNU in this work (cf. Refs. 2 and 3). The ^{71}Ga experiment is currently being carried out and there are several proposals to do similar experiments.[8] The two scenarios outlined above for giving the observed ^{37}Cl count rate would yield very different ^{71}Ga results. If the high-energy ^{8}B neutrinos are adiabatically transformed, then we expect the low-energy *pp* neutrinos to be only partially transformed and the ^{71}Ga count rate should be near the standard solar model value, whereas partial transformation of the high-energy neutrinos may imply adiabatic transformation of the low-energy *pp* neutrinos and hence a much lower ^{71}Ga count rate. Detailed calculations and predicted ^{71}Ga count rates for the parameter space of neutrino masses and mixing angles which are consistent with the observed ^{37}Cl count rate are given in Refs. 5–9.

Determinations of the range of neutrino mixing parameters which make the standard solar model consistent with the observations have typically neglected the quite significant uncertainty (± 2.2 SNU for ^{37}Cl) in that model. As we will show, solar models within this stated uncertainty require a considerably different range of mixing parameters. In a somewhat more extreme case, a few percent reduction in the solar interior temperature can bring the predicted ^{37}Cl count rate into agreement with the observed rate, so that the ^{37}Cl result alone is not convincing proof that neutrino oscillations are occurring. Many schemes have been devised to bring about such a lowering of the core temperature;[2,3] while none of these schemes have been convincingly demonstrated, and some have been proven wrong, there are still concerns over the precision of the standard model which must be considered. Because the *pp* neutrinos are relatively insensitive to these "classical" solutions, a low ^{71}Ga count rate would be taken as fair evidence that neutrino oscillations are occurring in the Sun. A ^{71}Ga count rate close to the value predicted for the standard solar model will be relatively inconclusive for neutrino oscillations. Similar information can be obtained by doing other kinds of solar-neutrino experiments which have different thresholds: for example, there is an effort to use the Kamiokande experiment as a solar-neutrino detector with a threshold of about 10 MeV (Ref. 10). While there are possible solutions which do not involve neutrino mixing, we will examine a more conservative range of solar models that are within the range of uncertainty given by Bahcall *et al.*[3]

II. SOLAR MODEL CALCULATIONS

We developed a stellar evolution code capable of reproducing the standard solar model of Bahcall, Cleveland, Davis, and Rowley[2] (5.8 ± 2.2 SNU, 1982). In order to avoid the ambiguity caused by using opacity tables generated at different epochs and with different compositions, we obtained a copy of the table used by Ulrich.[11] The "potential lowering" and Coulomb pressure effects in our equation of state were then modified to agree with that used in the standard model.

The model contained 500 mass zones and was evolved to an age of 4.55 Gyr in 75 time steps, each convergent to an accuracy of 10^{-5}. We obtained a model giving 5.6 SNU for the ^{37}Cl detection. The remaining discrepancy is attributed to residual difference in the equation of state.

We have used a modified version of the Parke-Walker[9] technique to calculate the solar-neutrino spectra corresponding to various neutrino mass differences and vacuum mixing angles. We were then able to modify our solar model, allow for self-consistent changes in solar opacity, and perform a detailed numerical integration of neutrino mixing throughout the Sun.

The main sources of uncertainty in the standard model are related to the reaction rates and the opacity (with contributions from both composition and physics approximations). While the reaction rate uncertainty is based on data, the opacity uncertainty is based on comparison of computer code calculations using many approximations to the microphysics. While the accuracy of the opacity tables provided by Los Alamos[12] are clearly adequate for most astrophysical applications this accuracy has been called into question at the 20–50 % level in a number of astrophysical situations.[11,13] However, the observations which conflict with the stellar models are mainly sensitive to temperatures lower than that of the solar core. Concern for the accuracy of the opacities has led to new attempts to calculate the opacity.[14,15] These efforts generally include improved atomic physics, and a reexamination of other issues such as occupation numbers in a dense gas, line widths, mixing in a fully coupled plasma, etc. In the absence of experimental verification of the physics approximations used in these codes under the condition of local thermodynamic equilibrium it is difficult to assign a real estimate of the accuracy of the opacity in use.

Comparison of new code calculations at Livermore indicate differences as much as 50% are possible in cooler regions, but in the interior where the neutrinos are formed, the accuracy is probably better than 15% (unless there is some gross composition difference). This is not enough in itself to solve the solar-neutrino problem but does result in a considerable change in the parameters required for neutrino oscillations to be the solution.

III. CALCULATION OF NEUTRINO OSCILLATIONS

We have used the numerical model to the Sun described above to compute neutrino spectra and emissivities as a function of radius. We can then numerically compute the phase of a neutrino as it propagates from its production site in the Sun through the actual density run in the solar model and eventually to Earth. To numerically calculate the neutrino phase we use a modified version of the Parke and Walker technique.[9] In this method if $|\nu_e\rangle$ and $|\nu_x\rangle$ are the neutrino flavor eigenstates (where $|\nu_x\rangle$ could equally well be $|\nu_\mu\rangle$ or $|\nu_\tau\rangle$) then the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ are given by

$$|\nu_1\rangle = \cos\theta_N|\nu_e\rangle - \sin\theta_N|\nu_x\rangle \ , \tag{1a}$$

$$|\nu_2\rangle = \sin\theta_N|\nu_e\rangle + \cos\theta_N|\nu_x\rangle \ , \tag{1b}$$

where θ_N is the instantaneous matter mixing angle at electron density N. Mikheyev and Smirnov[4] have shown that this matter mixing angle is related to the vacuum mixing angle θ_0 by

$$\sin 2\theta_N = \frac{\Delta_0}{\Delta_N}\sin 2\theta_0 \ , \tag{2a}$$

where

$$\Delta_N = [(A - A_\Delta \cos 2\theta_0)^2 + \Delta_0^2 \sin^2 2\theta_0]^{1/2} \ , \tag{2b}$$

and where $\Delta_0 = m_2^2 - m_1^2$ is the vacuum neutrino mass difference and Δ_N is the matter neutrino mass difference at electron density N, and where A is the effective neutrino mass contribution from the charged-current electron-neutrino exchange graph for a neutrino of energy E_ν at density ρ (in g cm^{-3}) with electron fraction Y_e:

$$A \simeq 1.5184\times 10^{-7}\ \text{eV}^2(\rho Y_e)(E_\nu/\text{MeV})\ . \tag{2c}$$

We note that the resonance condition from Eq. (2b), $A=\Delta_0\cos 2\theta_0$, determines the density at resonance for a given neutrino energy.

The oscillation length at resonance L_{res} is given by

$$L_{\text{res}}=\frac{4\pi E_\nu}{\Delta_0\sin 2\theta_0} \approx (247.6\ \text{cm})\frac{E_\nu/\text{MeV}}{(\Delta_0/\text{eV}^2)\sin 2\theta_0}\ . \tag{3a}$$

This oscillation length is to be compared with the characteristic solar density gradient length scalar δr, at resonance

$$\delta r=\tan 2\theta_0\left[\frac{d\ln\rho}{dr}\right]^{-1}\ . \tag{3b}$$

If $L_{\text{res}} \ll \delta r$ the neutrino goes through the resonance adiabatically. Parke and Walker[9] have shown that the probability of detecting a solar neutrino at Earth is

$$\bar{P}_{\nu_e}=\tfrac{1}{2}+(\tfrac{1}{2}-P_x)(\cos 2\theta_0)(\cos 2\theta_N)\ , \tag{4a}$$

where θ_N refers to the matter mixing angle at the neutrino production site in the Sun and P_x is the Landau-Zener probability of a neutrino remaining unchanged through the resonance:

$$P_x=\exp\left[-\frac{\pi^2}{2}\frac{\delta r}{L_{\text{res}}}\right]\ . \tag{4b}$$

For neutrinos produced on the far side of the Sun which go through the resonance twice we employ

$$\bar{P}_{\nu_e}=\tfrac{1}{2}+[\tfrac{1}{2}-(P_{x1})][1-2(P_{x2})]\cos(2\theta_0)\cos(2\theta_N)\ , \tag{4c}$$

where P_{x1} is the Landau-Zener probability for the second resonance and P_{x2} corresponds to the first.

For each neutrino produced at radius r_p in the Sun in energy bin E_i we compute the appropriate Δ_N. With Δ_N for each energy bin at point r_p we compute ρ_{res}, L_{res}, δr, and θ_N for

$$\rho_p \gg (\rho_{\text{res}})_i \quad \text{means } \theta_N=\pi/2\ ,$$

$$\rho_p \ge (\rho_{\text{res}})_i \quad \text{means } \pi/4 \le \theta_N \le \pi/2\ ,$$

$$\rho_p \le (\rho_{\text{res}}) \quad \text{means } \theta_0 \le \theta_N \le \pi/4\ ,$$

$$\rho_p \ll \rho_{\text{res}} \quad \text{means } \theta_N \approx \theta_0\ .$$

We can then use Eqs. (4a)–(4c) to calculate the figures shown below.

IV. RESULTS

The free parameters in the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism are the mass difference between the electron neutrino and some other species, and the mixing angle. A standard solar model of 5.8 SNU's can be made consistent with the observations of Davis if the parameters fall in a triangular region shown in Fig. 1(a).

As described above, the MSW theory applied to the standard solar model results in a solution to the solar-neutrino problem for a triangular region of parameter space. The limits of this region are typically set by the formal uncertainty in the measurement of Davis, Harmer, and Hoffman. We argue that the uncertainties in the required parameter space are instead due to the uncertainties in the solar model.

While the uncertainties are presented as 3 σ limits, problems such as opacity are not amenable to assigning error in a formal sense. As discussed above, accuracy of

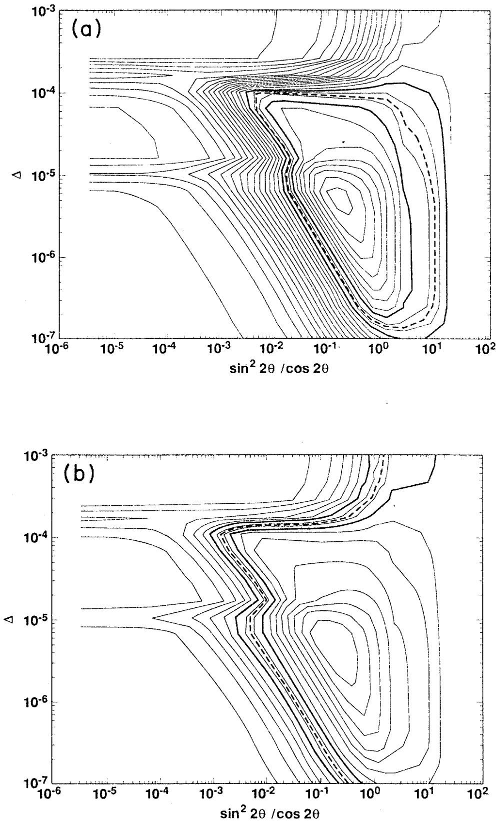

FIG. 1. (a) ^{37}Cl experiment iso-SNU contour plot for the standard solar model (5.6 SNU) as a function of neutrino mass difference $\Delta(=m_1^2-m_2^2)$ in eV^2 and $\sin^2 2\theta/\cos 2\theta$ where θ is the vacuum mixing angle. The dashed contour is 2.1 SNU (the bold solid contours are at 2.4 SNU and 1.8 SNU, respectively). The envelope defined by the solid curves gives the result of Davis, Harmer, and Hoffman. The other contours are 0.3 SNU apart. (b) Same as (a) but now the solar opacity has been reduced by 15% in the solar model.

the opacity calculations are assessed by comparing the results of calculations at Livermore and Los Alamos for a representative mixture of elements at specified temperatures and densities. The Livermore results are generally not more than 10% lower than the Los Alamos opacities, and typically 5% lower in the core. In the absence of any experimental verification of the calculated opacities this is the only means of assessing the uncertainty due to opacity.

As discussed above, an opacity calculation depends on several distinct physical processes, each of which is approximated in some fashion. These codes do utilize fundamentally different approximations, but, unfortunately, are not *entirely* independent. Comparisons have been performed, and whenever differences were found, the approximations were examined to determine the cause. Modifications then result in a pair of codes that give the best opacities possible given the physical models used, but also give similar values. In the face of more precise observations such as those of solar and stellar pulsations, concern on the accuracy of the opacity calculations has risen, and new efforts have begun to calculate opacities.

In the absence of experimental verification, we believe that actual uncertainty due to the opacity physics is greater than previous estimates. Igelsias, Rogers, and Wilson[15] have only published opacities at lower temperatures than we are concerned with for determining the neutrino spectrum from the Sun, but they find differences of $>50\%$. The difference between their calculation and one from Los Alamos would be less in the core of the Sun, but even a 15% change in the opacity produces a considerable effect on the spectrum of neutrinos produced, and the parameter space required to solve the solar problem.

To illustrate this point we have calculated a model in which the opacity at temperatures greater than 3×10^6 K were reduced by 15% (less than the contribution of the heavy elements to the opacity). This results in a drop of the solar-neutrino rate to 3.6 SNU, or a drop of 2.2 SNU, comparable to the limit set by Bahcall, Cleveland, Davis, and Rowley[2]. As can be seen in Fig. 1(b), the high mixing angle solutions present in the standard solution no longer exist. Additionally, the mass difference between neutrino species has been extended to higher and lower values. Similar we show iso-SNU plots for a ^{71}Ga experiment for the standard solar model [Fig. 2(a)], and for the reduced opacity [Fig. 2(b)]. Neutrino energy spectra for the standard solar model are shown for various neutrino mixing parameters in Fig. 3.

The changes in opacities can go both ways, but most of the modifications in the new opacity efforts are expected to raise the opacities. Increasing the opacities by an amount consistent with a 2.2-SNU raise results in a parameter space for MSW resonance oscillations that is smaller than the present limits, a triangular region inside of the one defined for the standard model.

In addition to these results for the ^{37}Cl and ^{71}Ga experiments, the detection of a solar-neutrino signal from the Kamiokande detector could be very important in constraining ν-oscillation parameters. Since the effective threshold for the Kamiokande detector would be about 10 MeV this experiment samples the high-energy end of the ^{8}B neutrino spectrum. Used in combination with the ^{37}Cl detector result, a Kamiokande limit or detection could tell whether or not the high-energy neutrinos have been adiabatically transformed (Fig. 4). In fact the two experiments taken together can significantly narrow the parameter space of vacuum mixing angle and mass difference, so that a prediction can be made for future ^{71}Ga experiment count rates.

If the effective threshold for the Kamiokande experiment is taken as 10 MeV and a mean count rate of $0.45\mp.15$ of the total standard solar model ^{8}B neutrino flux[10] is observed then the parameter region for neutrino oscillations to explain the reduction has a very small overlap with the parameter region required to explain the ^{37}Cl detector result. We show this graphically in an iso-SNU contour plot for the standard solar model (Fig. 3). If the Kamiokande results were really as low as 0.45 of the standard ^{8}B flux then the region of overlap with the ^{37}Cl experiment solution is centered on $\Delta m^2\approx10^{-5}$ eV^2, and $\sin^2(2\theta)/\cos(2\theta)=2\times10^{-2}$. This gives a predicted ^{71}Ga detector count rate of about 88 SNU. Even with the

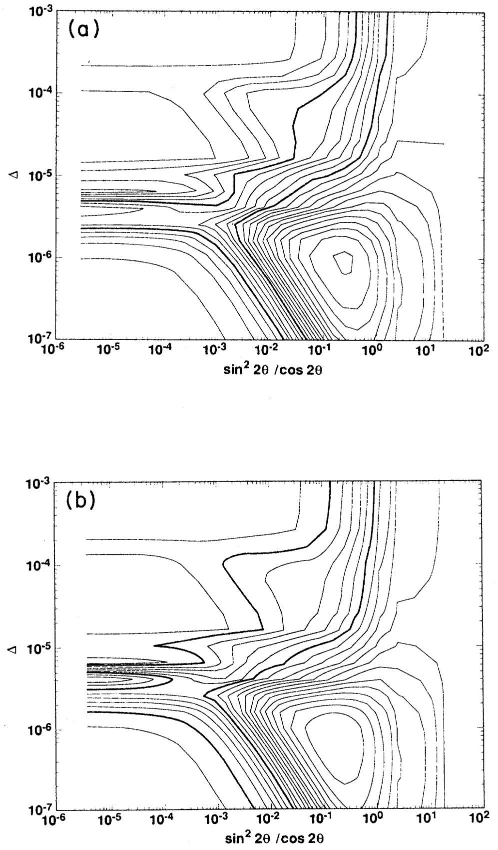

FIG. 2. (a) ^{71}Ga experiment iso-SNU contour plot for the standard solar model. Axes are as in Fig. 1(a). Contours are 5 SNU apart. Solid lines denote 75, 100, and 125 SNU's. (b) Same as (a) but now the solar model has its opacity reduced by 15%.

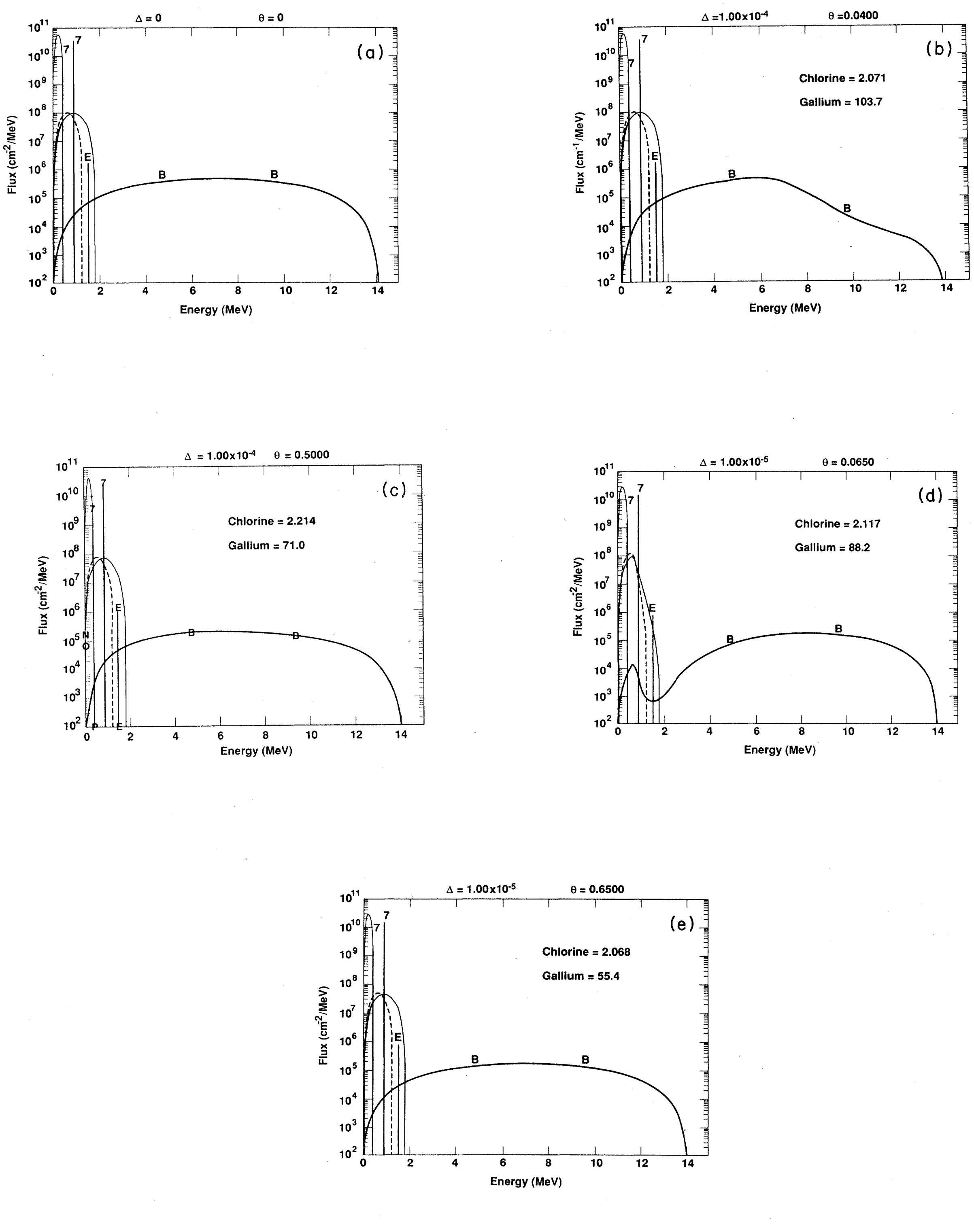

FIG. 3.. Neutrino energy spectra for the standard solar model where "B" stands for ^{8}B; "E" for the PEP neutrinos; "7" for ^{7}Be (two vertical lines); ^{13}N decay gives the dashed curve; ^{15}O decay gives the heavy solid curve; the light curve corresponds to the *pp* neutrinos. Each figure is labeled by the appropriate SNU count rate for a ^{37}C and ^{71}Ga experiment and each figure is labeled at the top of the difference in neutrino masses $\Delta(=m_1^2-m_2^2)$ in eV^2 and by the vacuum mixing angle θ in radians. The figures are (a) no neutrino oscillations; (b) a typical adiabatic solution to ^{37}Cl for high-energy neutrinos which leaves the ^{71}Ga result high; (c) partial transformation of the high-energy neutrinos; (d) and (e) partial transformation of high-energy neutrinos and adiabatic transformation of low-energy neutrinos which give the ^{37}Cl result but would predict low ^{71}Ga.

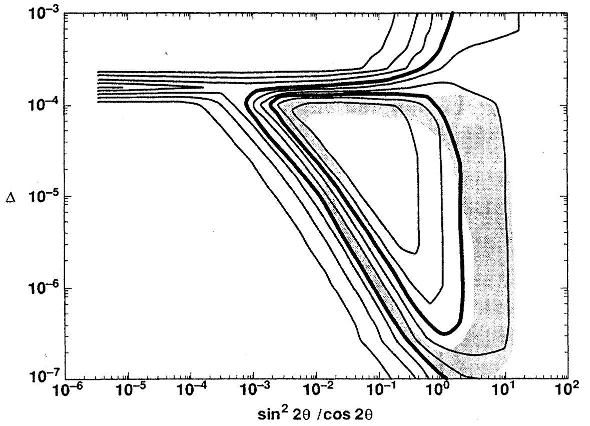

FIG. 4. An iso-SNU contour plot for the standard solar model in which the shaded area represents the result of Davis, Harmer, and Hoffman for the ^{37}Cl experiment as in Fig. 1(a). contours are in units of 0.1 of the total standard-model ^{8}B neutrino flux for this threshold. The bold solid lines are for a hypothetical Kamiokande solar neutrino result with a threshold of 10 MeV and count rates between 0.3 and 0.6 of the ^{8}B neutrino flux.

uncertainty for the Kamiokande result assumed as above, the overlap region is small. Improvement in this result could easily raise doubts about any neutrino oscillation solutions to the solar-neutrino problem.

V. CONCLUSIONS

Stellar evolution theory has been remarkably successful in reproducing the luminosity functions observed in clusters, the changes in surface composition observed in various types of stars, and in linking various types of stars together in an evolutionary sequence. From this we believe that the opacities used are adequate for most astrophysical applications. The magnitude of the solar-neutrino problem is however tremendously sensitive to the central temperature of the Sun, and is affected by opacity changes which would not be noticed elsewhere in astronomy.

We believe the range of neutrino properties that might resolve the solar-neutrino problem is limited by the uncertainty in the solar models, and not the uncertainty of the detection of Davis, Harmer, and Hoffman. This must be considered when developing experiments to test the MSW parameter space.

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[1] R. Davis, D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. **21**, 1205 (1968).

[2] J. N. Bahcall, B. T. Cleveland, R. Davis, and J. K. Rowley, Astrophys. J. **292**, L79 (1985); also J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).

[3] J. N. Bahcall, W. F. Huebner, S. H. Lubow, P. D. Parker, and R. K. Ulrich, Rev. Mod. Phys. **54**, 767 (1982).

[4] S. P. Mikheyev and A. Yu. Smirnov, Nuovo Cimento **9C**, 17 (1986). See also L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[5] H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1985).

[6] W. C. Haxton, Phys. Rev. Lett. **57**, 1271 (1986).

[7] S. J. Parke, Phys. Rev. Lett. **57**, 1275 (1986).

[8] R. Davis (private communication).

[9] S. J. Parke and T. P. Walker, Phys. Rev. Lett. **57**, 2322 (1986).

[10] Y. Totsuka (private communication).

[11] R. Ulrich, Bull. Am. Astron. Soc. **18**, 989 (1986).

[12] A. Cox, in *Stars and Stellar System,* edited by L. H. Allen and D. B. McLaughlin (University of Chicago Press, Chicago, 1965), Vol. 8, p. 195; W. F. Hubner, A. L. Merts, N. U. Mager, and M. F. Arso, Los Alamos Scientific Report No. LA-6760-M, 1977 (unpublished).

[13] G. S. Stringfellow, F. J. Swenson, and J. Faulkner, Bull. Am. Astron. Soc. **19**, 1020 (1987).

[14] D. G. Hummer and D. Mihalas, Astrophys. J. **331**, 794 (1988).

[15] C. A. Iglesias, F. J. Rogers, and B. G. Wilson, Astrophys. J. **322**, L45 (1987).

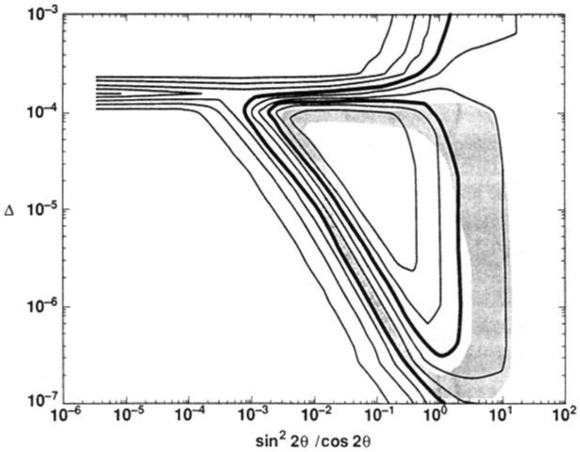

FIG. 4. An iso-SNU contour plot for the standard solar model in which the shaded area represents the result of Davis, Harmer, and Hoffman for the ^{37}Cl experiment as in Fig. 1(a). contours are in units of 0.1 of the total standard-model ^{8}B neutrino flux for this threshold. The bold solid lines are for a hypothetical Kamiokande solar neutrino result with a threshold of 10 MeV and count rates between 0.3 and 0.6 of the ^{8}B neutrino flux.