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T.A. Kaeding

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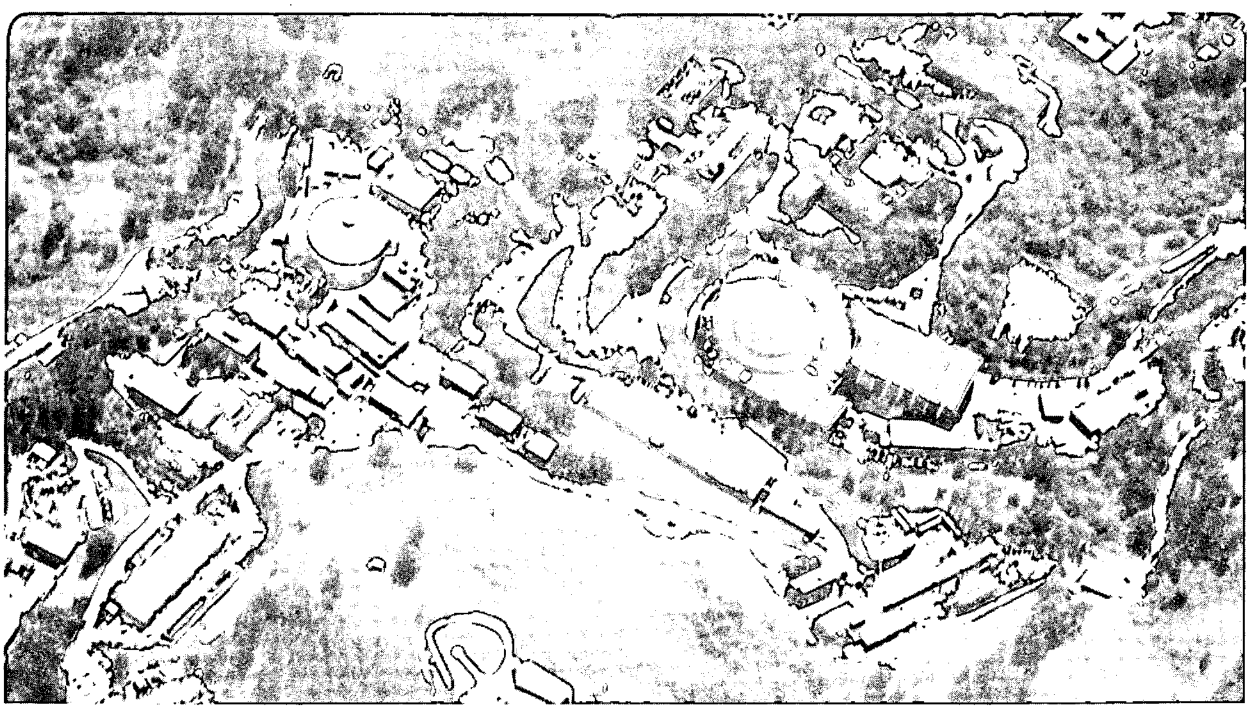

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D-Meson Mixing in Broken $SU(3)$*

Thomas A. Kaeding†
Theoretical Physics Group
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A fit of amplitudes to the experimental branching ratios to two mesons is used to construct a new estimate of neutral D mixing which includes $SU(3)$ breaking. The result is dominated by the experimental uncertainties. This suggests that the charm sector may not be as sensitive to new physics as previously thought and that long-distance calculations may not be useful.

Key words: D mesons, charm, mixing, $SU(3)$, $SU(3)$ breaking

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†Electronic address: `kaedin@theorm.lbl.gov`.

Introduction

The prospects of probing new physics in D^0-$\overline{D}^0$ mixing has been a topic recent discussion [1] [2]. In this letter we will present a new estimate of Δm_D due to mixing via two-body hadronic intermediate states. Because the couplings between the D mesons and the possible intermediate states are not all known, we will rely on the results of a fit to the branching ratios [3]. This fit utilizes a complete parameterization of the decays in the framework of broken flavor $SU(3)$ and allows us to extract the (complex) couplings needed to estimate Δm_D. Intermediate states from the fit are limited to pseudoscalar-pseudoscalar (PP), pseudoscalar-vector (PV), and vector-vector (VV). We will find that the results are overwhelmed by their uncertainties. These uncertainties owe their origin to the large (sometimes ~30%) uncertainties on the experimentally measured branching fractions. Other estimates, based on short-distance processes or on the heavy-quark effective theory, give Δm_D to be two orders of magnitude smaller that our central values and uncertainties. In the absence of cancellations among the hadronic modes, our estimate allows the standard-model contributions to D^0-$\overline{D}^0$ mixing to be close to the current experimental limit $\sim 10^{-13}$ GeV [4].

1 Formalism

Weak interactions to second order in the coupling G_F give an off-diagonal part to the Hamiltonian which represents $D^0 \leftrightarrow \overline{D}^0$ transitions. Such a term is responsible for the mixing. Its real part enters the mass matrix and generates a mass difference between the two eigenstates. In the absence of CP violation, we can write this as

$$\mathcal{H}_{\text{mass}} = \left(D^0 \; \overline{D}^0\right) \begin{pmatrix} M & \frac{1}{2}\Delta m \\ \frac{1}{2}\Delta m & M \end{pmatrix} \left(\; D^0 \overline{D}^0 \; \right). \tag{1}$$

The mass eigenstates are

$$\begin{aligned} D_1 &= \tfrac{1}{\sqrt{2}}(D^0 + \overline{D}^0), \\ D_2 &= \tfrac{1}{\sqrt{2}}(D^0 - \overline{D}^0), \end{aligned} \tag{2}$$

with masses

$$\begin{aligned} m_1 &= M + \tfrac{1}{2}\Delta m, \\ m_2 &= M - \tfrac{1}{2}\Delta m. \end{aligned} \tag{3}$$

The mass difference Δm is a convenient parameter for the size of the mixing. It is the quantity whose measurement and calculation are important in the question of probing new physics in D^0-$\overline{D}^0$ mixing.

Before continuing, we must draw the distinction between short- and long-distance estimates to Δm. Short-distance contributions come from the calculation of the "box" diagrams in the quark model. Long-distance contributions come from the dispersion due to intermediate hadronic states. In this case, the approach is only valid for internal momenta that are below the scale at which QCD becomes nonperturbative. Typically this scale is taken to be

$$\mu \sim 1 \text{ GeV}. \tag{4}$$

The dependence on μ in our estimate will be explicit.

2 Modelling the Couplings

In order to construct our estimate, we need to know the couplings $D^0 M_1 M_2$ between the neutral D mesons and the lowest-lying nonets of pseudoscalars and vectors. To this end, we have parameterized these couplings in the $SU(3)$ framework with flavor-symmetry breaking by an octet. The details can be found in [3]. Here we will only sketch the calculation.

The particles are organized into $SU(3)$ multiplets. The D mesons (D^0, D^+, D_s) form an antitriplet $\overline{\mathbf{3}}$. The pseudoscalars ($\pi^\pm$, π^0, $K^\pm$, K^0, $\overline{K}^0$, η, η') fit into an octet and a singlet. The physical η and η' are mixtures of the octet and singlet pieces, with mixing angle given by experiment [5]. The vectors ($\rho^\pm$, ρ^0, $K^{*\pm}$, K^{*0}, $\overline{K}^{*0}$, ω, ϕ) are likewise arranged, with a vector mixing angle that is found in [4].

The charm-changing Hamiltonian is proportional to a product of quark currents

$$H_{\Delta C=1} \sim \overline{u}\gamma^\mu(1-\gamma_5)q\,\overline{q}'\gamma_\mu(1-\gamma_5)c, \tag{5}$$

where

$$\begin{aligned} q &= \cos\theta_C d - \sin\theta_C s, \\ q' &= \sin\theta_C d + \cos\theta_C s. \end{aligned} \tag{6}$$

Since the quark-annihilation operators in H transform as antitriplets, and the $\overline{q}$ operators as triplets, we can expand H in terms of $SU(3)$ representations. The

Hamiltonian is found to transform as **15** and $\overline{\mathbf{6}}$. The Clebsch-Gordan factors in this expansion and in the following were calculated by computer [6] [7].

The amplitude for each decay of the type $D \to$ PP, PV, VV can now be written as the sum of reduced matrix elements with appropriate Clebsch factors. For each of PP, PV, and VV there are 48 (complex) reduced matrix elements. The number of parameters is too large to be fit by the available 45 measured modes and 13 modes with experimental limits [4] [8]. Therefore, we must make some assumptions to limit their number. first, we assume that corresponding reduced matrix elements of PP, PV, and VV are proportional in magnitude. This proportionality is represented by two new parameters $A_{\mathrm{PV/PP}}$ and $A_{\mathrm{VV/PP}}$. Second, we assume that the phase of each reduced matrix element is given by the phase of the representation into which the daughter mesons are contracted. These are the $(\eta_1\eta_1)_1$, $(\eta_1\mathrm{P})_8$, $(\mathrm{PP})_1$, $(\mathrm{PP})_8$, $(\mathrm{PP})_{27}$, $(\eta_1\omega_1)_1$, $(\eta_1\mathrm{V})_8$, $(\omega_1\mathrm{P})_8$, $(\mathrm{PV})_1$, $(\mathrm{PV})_8$, $(\mathrm{PV})_{8'}$, $(\mathrm{PV})_{10}$, $(\mathrm{PV})_{\overline{10}}$, $(\mathrm{PV})_{27}$, $(\omega_1\omega_1)_1$, $(\omega_1\mathrm{V})_8$, $(\mathrm{VV})_1$, $(\mathrm{VV})_8$, $(\mathrm{VV})_{27}$. The phases then become new parameters. With these assumptions, the number of linear combinations of reduced matrix elements that contribute to any decay is reduced to 40.

There are now far fewer parameters than we began with. The data constrain all of them, except for three combinations of reduced matrix elements and the phases of $(\eta_1\eta_1)_1$, $(\omega_1\omega_1)_1$, and $(\eta_1\omega_1)_1$. There are too many free parameters in the singlet-singlet cases, and so we do not attempt to make any estimates of their values. However, one of the remaining combinations of matrix elements that involve the D^0 can be constrained by reasonable estimates of one additional decay mode. In order to see the effect of our lack of knowledge in this case, we make two different estimates, called schemes A and B. In the former we use

$$B(D^0 \to \eta K^0) = 3\tan^4\theta_C\ B(D^0 \to \eta\overline{K}^0), \tag{7}$$

and in the latter we take

$$B(D^0 \to \phi K^0) = 3\tan^4\theta_C\ B(D^0 \to \phi\overline{K}^0). \tag{8}$$

The coefficient 3 is motivated by the size of the recently measured mode $D^0 \to K^+\pi^-$ [9]. Interestingly, a coefficient of 1 does not result in a consistent fit.

3 Estimate of Δm

The long-distance contributions to Δm arise from dispersive effects involving intermediate states. The two-body hadronic intermediate states were considered by [10] are were estimated by

$$\begin{aligned}\Delta m^{K^\pm,\pi^\pm} &= \tfrac{1}{2\pi}\ln\tfrac{m_D^2}{\mu^2}\ \Big[\Gamma(D^0\to K^+K^-)+\Gamma(D^0\to\pi^+\pi^-) \\ &\qquad -2\sqrt{\Gamma(D^0\to K^+\pi^-)\Gamma(D^0\to K^-\pi^+)}\Big]\,. \end{aligned} \tag{9}$$

Here μ is the cutoff discussed in Section 2. Notice the implicit assumption that the couplings are relatively real. If we insert the most recent values for these rates [4] [9], we obtain

$$\Delta m^{K^\pm,\pi^\pm} = (\text{-}0.75 \text{ to } 0.29)\times 10^{-15}\text{GeV}. \tag{10}$$

The range of values is due to the uncertainty on the branching fraction to $K^+\pi^-$ [9]. The purpose of this exercise is to show that large $SU(3)$ breaking may give large long-distance contributions to neutral D mixing. Should the $SU(3)$ breaking be due only to the K-π mass difference, the rates would be related by

$$\begin{aligned}\Gamma(D^0\to K^+K^-)\Phi(KK) &= \Gamma(D^0\to\pi^+\pi^-)\Phi(\pi\pi) \\ &= \tan^2\theta_C\Gamma(D^0\to K^-\pi^+)\Phi(K\pi) \\ &= \cot^2\theta_C\Gamma(D^0\to K^+\pi^-)\Phi(K\pi),\end{aligned} \tag{11}$$

where Φ represents the phase-space corrections. These corrections are on the order of a percent, and so in this case

$$\Delta m \simeq 10^{-17}\text{GeV}, \tag{12}$$

a value consistent with other estimates, as discussed below.

We will make two improvements on this approach. First, we will include all PP, PV, and VV intermediate states, with the exception of the singlet-singlet states. Second, we will allow the couplings to take complex values. Equation 9 is replaced by

$$\Delta m_D^{\text{L-D}} = \frac{1}{2\pi}\ln\frac{m_D^2}{\mu^2}\times N\sum_I \mathcal{A}(D^0\to I)\mathcal{A}^*(D^0\to\overline{I}), \tag{13}$$

where N is a normalization factor given by

$$\Gamma(D^0\to K^-\pi^+) = N|\mathcal{A}(D^0\to K^-\pi^+)|^2. \tag{14}$$

The coupling are extracted from the fit of the previous section.

Our estimates for Δm are presented in the Table. The estimates due to PV intermediate states are consistent with zero and have uncertainties on the order of 50 $\times 10^{-15}$ GeV. The estimates due to PP and VV intermediate states vary according to our choice of estimate for the unconstrained modes (Equation 7 or 8) and are at most one standard deviation from zero. The source of the large uncertainties is the uncertainty with which we know the individual branching fractions used to constrain our parameterization. The contributions from individual modes enter with differing phases, but the uncertainties are cumulative. The result is that our attempt to estimate Δm is nearly overwhelmed by uncertainties. It is also worth noting that these uncertainties are all greater than the prior estimate which used only the modes $\pi^+\pi^-$, $K^+\pi^-$, $K^-\pi^+$, and K^+K^- (see Equation 10).

4 Discussion

We have found that a long-distance calculation of Δm_D is dominated by the experimental uncertainties. The treatment of D-meson mixing by Donoghue *et al.* [10] has underestimated these uncertainties. On the other hand, there also exist estimates based on the underlying quark processes. The short-distance calculation based on the "box" diagrams has been done in [11] [12] [13] [10] [14]. For example, [10] finds that

$$\Delta m^{\text{box}} = -\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi \sin^2\theta_{\text{w}}} \cos^2\theta_C \sin^2\theta_C \frac{(m_s^2 - m_d^2)^2}{M_W^2 m_c^2} m_D F_D^2 (B_D - 2B'_D). \quad (15)$$

Here B and B' are hadronic factors defined in [10] and F_D is the pseudoscalar decay constant:

$$\left\langle 0|A_\mu|D^0(p)\right\rangle = iF_D p_\mu. \quad (16)$$

If we assume that the hadronic factors $B \simeq B' \simeq 1$, and take $F_D \simeq 300$ MeV [4] and $m_s \simeq 250$ MeV, then the contribution of the box diagrams to Δm is

$$\Delta m^{\text{box}} \simeq 4.8 \times 10^{-17} \text{GeV}. \quad (17)$$

In addition, an estimate of the mass difference in the heavy-quark effective field theory (HQEFT) was performed in [15] [16]. These arrive at estimates for the contributions of 4-, 6-, and 8-quark operators:

$$\begin{aligned}\Delta m_D^{\text{4-quark}} &\simeq 1 \quad \times 10^{-17}\ \text{GeV},\\ \Delta m_D^{\text{6-quark}} &\simeq 2 \quad \times 10^{-17}\ \text{GeV},\\ \Delta m_D^{\text{8-quark}} &\simeq 0.5 \ \times 10^{-17}\ \text{GeV}.\end{aligned} \tag{18}$$

These estimates replace both the long-distance calculation and the box calculation. In order to reconcile the HQEFT estimates with our approach, there must be cancellation of the individual long-distance contributions to about one percent. In addition, it may be that the HQEFT estimate has underestimated the $SU(3)$ breaking involved in the D system. The breaking of $SU(3)$ is known from the splittings of hadronic masses to be at the level of 20-30%. However, in the charm system, we know from the branching fractions that $SU(3)$ is broken at the 100% level. In the HQEFT estimate, only the quark-mass differences were used to break the flavor symmetry. We conjecture that this is the reason that the HQEFT estimate is on the same order of magnitude as the box and much smaller than the long-distance estimates.

The cancellation between contributions from different hadronic intermediate states needed to reconcile the HQEFT approach with that of the long-distance estimates would have to be among, rather than within, the individual $SU(3)$ representations. We can see this by considering the contribution due to the complete octet of pseudoscalar mesons. Although the singlet-singlet pieces of the PP decay modes are completely unconstrained and the singlet-octet only partially constrained, the octet-octet parts of the amplitudes are completely determined by data. Therefore we are able to extract the octet-octet parts from the fit without relying on the assumption of Equation 7 or 8. It is then possible to construct an estimate of Δm that includes the entire pseudoscalar octet, without the mixing in of singlet pieces. We find that this contribution is

$$\Delta m_D^{\text{P octet}} = (9.6 \pm 2.2) \times 10^{-15}\ \text{GeV}. \tag{19}$$

This estimate differs significantly from zero, and is therefore an indication that cancellations among hadronic modes must be between the various $SU(3)$ representations.

It is not possible for us to determine whether cancellation occurs within the complete set of PP intermediate states, including both $SU(3)$ octet and singlet pieces. The expected size of the PP singlet-octet contribution can be seen in the difference between Equation 19 and the entries in the first column of Table 1. These singlet-octet contributions vary by our estimation schemes for the unconstrained modes, and are on the order of half of $\Delta m_D^{\mathrm{P\,octet}}$. We have no expectations on the size of the singlet-singlet contribution, but can remark that it would be entirely due to $SU(3)$ breaking. It is possible that PP modes will cancel among themselves, once the singlet-singlet and singlet-octet pieces are fully included. But should that not occur, then the only hope of cancellation would be between the PP, PV, and VV modes taken together.

Due to the large uncertainties (sometimes $\sim 30\%$) on the experimentally measured branching fractions, the uncertainties on our estimates of the long-distance contribution are very large. There are two things that could be learned from this. First, the large uncertainties indicate that we are yet unable to make a useful calculation of Δm_D using intermediate hadronic states. It is not reasonable for the experimental situation to improve to the point where the uncertainties on Δm_D reach the precision of the box or HQEFT estimates in the near future. Continued endeavors using one-particle (resonant) or three-particle intermediate states are discouraged. Second, in the absence of cancellations among the hadronic intermediate states, the large range of possible values allows the standard-physics contributions to D-meson mixing to be as large as

$$|\Delta m_D| = 10^{-13}\mathrm{GeV}. \tag{20}$$

This is near the experimental limit [4], and therefore this process would be less likely to be useful as a probe of new physics than previously thought [1] [2]. We are left with the hope that a direct measurement of the D-meson mass difference can be obtained and that these questions can be resolved.

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Table : Estimates of Δm_D. PP, PV, and VV refer to pseudoscalar-pseudoscalar, pseudoscalar-vector, and vector-vector intermediate states. Schemes A and B for estimating some doubly suppressed neutral modes are explained in Section 2. The first line neglects those modes. The last line includes the full pseudoscalar octet. In it, the octet-octet parts of $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ are included and the singlet-octet parts are excluded. All values are in 10^{-15} GeV.

	PP	VV	PV
no estimates	7.3 ± 3.9	19.1 ± 11.3	-60.3 ± 63.3
scheme A	10.1 ± 4.4	25.5 ± 11.9	-56.5 ± 63.9
scheme B	4.6 ± 4.5	14.7 ± 12.1	-65.5 ± 63.9
$K^{\pm}$ and $\pi^{\pm}$	3.7 ± 1.3		
full octet	9.6 ± 2.2		