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Linear magneto-conductivity as a probe of time-reversal symmetry breaking

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Several optical experiments have shown that in magnetic materials the principal axes of response tensors can rotate as an odd function of an applied magnetic field. Here we offer a microscopic explanation of this effect, and propose a closely related DC transport phenomenon — an off-diagonal *symmetric* conductivity, linear and *odd* in a magnetic field, which we refer to as linear magneto-conductivity (LMC). Although LMC has the same functional dependence on magnetic field as the Hall effect, its origin is fundamentally different: LMC requires time-reversal symmetry to be broken even before a magnetic field is applied, and is therefore a sensitive probe of magnetism. We demonstrate LMC in three different ways: *via* a tight-binding toy model, a density functional theory calculation on MnPSe₃, and a semiclassical treatment. The third approach identifies two distinct mechanisms yielding LMC: momentum-dependent band magnetization and Berry curvature. Finally, we propose an experimental geometry suitable for detecting LMC, and demonstrate its applicability using Landauer–Büttiker simulations. Our results emphasize the importance of measuring the full conductivity tensor in magnetic materials, and introduce LMC as a new transport probe of symmetry.

I. INTRODUCTION

One of the central tasks of condensed matter physicists is to uncover and manipulate novel phases of matter, and the exotic phenomena that they host. The key to achieving this goal is the correct identification of the symmetries associated with each phase. Time reversal symmetry (TRS) is an example of a symmetry that is broken in many materials of both fundamental and practical interest, but it can be challenging to experimentally detect.

Part of the difficulty is that the breaking of TRS is *necessary* but not *sufficient* to observe some of the most commonly measured signatures of TRS-breaking, such as the linear anomalous Hall effect (AHE) [1] and optical circular dichroism (CD). In other words, while observation of AHE or CD guarantees that TRS is broken, the lack of such observations does not prove the presence of TRS. This motivates the establishment of complementary experimental probes of time reversal symmetry breaking.

The AHE, the CD, and other quantities introduced below revolve around the conductivity tensor

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \quad (1)$$

from which we can define the symmetric and antisymmetric parts as:

$$\sigma_{ij}^s = \frac{1}{2}(\sigma_{ij} + \sigma_{ji}), \quad (2)$$

$$\sigma_{ij}^a = \frac{1}{2}(\sigma_{ij} - \sigma_{ji}), \quad (3)$$

where the frequency dependence is implicitly assumed. In that sense, the well-known Hall conductivity is the DC component of σ_{ij}^a .

Besides the AHE and CD, another TRS probe is based on the so-called optical linear magneto-birefringence (LMB) effect [2–5], whereby the optical birefringence (difference in reflectivity between the two orthogonal linear light polarizations) has a component that is proportional to an applied magnetic field, H . This effect stems from a component of the optical conductivity tensor that is symmetric under exchange of indices, but an odd function of an applied magnetic field: $\sigma_{ij}^s(\omega) \sim H$. It can be shown that this term is prohibited in the presence of time reversal symmetry, and that it can be present even in point groups that do not allow circular dichroism, or the anomalous Hall effect, making it a powerful complementary probe of TRS.

Even more information about the order parameter can be gleaned when we consider the tensorial nature of the LMB effect: the magnetic field can change the magnitude of the optical birefringence, or rotate its principal optical axes - we refer to these effects as diagonal, and off-diagonal respectively [4]. While TRS needs to be broken to yield both the diagonal and off-diagonal LMB, distinct spatial symmetries allow the two effects. This richness of information contained in the optical responses has been used several times in the literature to uniquely pinpoint to the magnetic order parameter in a complex magnet [2–5]. However, the mechanisms that yields these effects, in particular the off-diagonal LMB, are poorly understood.

Each symmetry-sensitive effect that is detected at op-

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tical frequencies has its zero-frequency DC counterpart - after all, the symmetry of the conductivity tensor is independent of the frequency at which it is probed. For example, AHE and CD both stem from the antisymmetric part of the conductivity tensor ($\sigma_{ij}^a(\omega) = -\sigma_{ji}^a(\omega)$). Therefore, both the diagonal and the off-diagonal linear magneto-birefringence have to have DC counterparts, which we term diagonal and the off-diagonal linear magneto-conductivity (LMC). Indeed H -linear magnetoresistance was recently reported in several experiments [6–8], and it was analyzed theoretically how it might arise in Weyl and Dirac semimetals [9–15] and altermagnets [16].

Even if a measurement of linear magneto-conductivity is not the goal of the experiment, this signal might influence the measurement of other transport coefficients, in particular the Hall effect. The Hall effect is typically measured in a “Hall bar” configuration, in which a current parallel to the x direction (for example) generates a transverse voltage proportional to a magnetic field applied normal to the bar. This measurement assumes implicitly that only the Hall effect contributes to the H -odd component of σ , i.e. that the symmetric off-diagonal component $\sigma_{xy}^s = (\sigma_{xy}(H) + \sigma_{yx}(H))/2$ is H -even. In *non-magnetic* materials the validity of this assumption is guaranteed by the Onsager’s reciprocity relations, which encode microscopic time reversibility. However, similarly to the presence of LMB, LMC can also contribute to the measured signal in magnetic materials. It is therefore essential for an experimentalist working on magnetic materials to be aware of LMC, both to benefit from the insights that can be obtained through measurements of these effects, and to avoid misinterpreting Hall effect measurements.

In this paper we analyze both LMB and LMC through a series of theoretical approaches aimed to develop an understanding of these signals: what are the conditions that allow them, how do they arise from the electronic structure of a magnetic material, and when are they expected to be large? The main focus of the paper is the off-diagonal component of the LMC tensor, since it is least discussed in the literature to date, and most likely to be missed in experiments due to the similarities of the signatures to the Hall effect.

We start by a symmetry analysis of both LMC and LMB. We then offer a microscopic understanding of the off-diagonal LMB in a system of localized orbitals in a magnetic field, and use the obtained insights to construct a tight-binding (TB) toy model which yields a nonzero DC LMC. Within this model the effect originates from a field-induced rotation of the Fermi surface (FS), resembling the rotation of principal optical axes that characterizes LMB. To gain insight into the physical origin of this effect, we derive the field-linear σ_{xy}^s from semiclassical equations of motion and identify two mechanisms: Berry curvature and momentum-dependent band magnetization. Although TRS needs to be broken, neither of these mechanisms relies on the existence of a net magne-

tization, suggesting that LMC may be present in a wide range of magnetic materials. We propose an experimental geometry suitable for measuring the full conductivity tensor, and demonstrate it by a mesoscopic transport calculation. Finally, we use density-functional theory (DFT) calculations on an example material, MnPSe₃, to show that a large LMC is present beyond simple toy models, and needs to be seriously considered in experimental investigations.

II. SYMMETRY ANALYSIS

To clarify the distinct symmetry constraints imposed by time reversal symmetry on the conductivity tensor, we start from the Onsager’s relation, which states that in TRS-invariant systems:

$$\sigma_{ij}(H) = \sigma_{ji}(-H). \quad (4)$$

To analyze the field-linear response, we expand the conductivity tensor as $\sigma_{ij}(H) = \sigma_{ij}(H=0) + \gamma_{ijk}H_k$, where we introduced the tensor γ_{ijk} . Onsager’s relation now imposes the following two constraints on the transport coefficients:

$$\sigma_{ij}(H=0) = \sigma_{ji}(H=0), \gamma_{ijk} = -\gamma_{jik}. \quad (5)$$

The violation of these two constraints indicates TRS breaking and leads to two complementary probes of TRS breaking: anomalous Hall effect and linear magneto-conductivity.

To see this even more explicitly, we separately analyze the antisymmetric and symmetric components of conductivity tensor: $\sigma_{ij}^a = \sigma_{ij}^a(H=0) + \gamma_{ijk}^a H_k$ and $\sigma_{ij}^s = \sigma_{ij}^s(H=0) + \gamma_{ijk}^s H_k$, respectively. The first condition in Eq. (5) is always satisfied for σ_{ij}^s : the symmetric, field independent conductivity contains no information about time reversal symmetry breaking. The second condition is always satisfied for $\gamma_{ijk}^a H_k$, as expected: the field linear Hall effect is always allowed. Conversely, observing non-zero $\sigma_{ij}^a(H=0)$ or γ_{ijk}^s proves that time reversal symmetry is broken. Specifically, limiting the analysis to a 2D case, detecting $\sigma_{xy}(H=0) = -\sigma_{yx}(H=0)$ indicates breaking of TRS (AHE), as does observing $\gamma_{xyz} = \gamma_{yxz}$ (LMC).

As is clear from Eq. 5, the tensors responsible for AHE and LMC are different, and therefore experience distinct constraints imposed by spatial symmetries. To emphasize this point further, in Table I we list the symmetries that allow (✓) or prohibit (×) the two effects. In particular, AHE is prohibited by mirror symmetries m_x and m_y , which allow for LMC. In contrast, LMC requires the z -axis to have at most the C_{2z} rotational symmetry, while AHE is not constrained by rotational symmetry. AHE and LMC are therefore truly complementary probes of time reversal symmetry breaking.

Symmetry	AHE	LMC
$\mathcal{T}$	×	×
$\mathcal{P}$	✓	✓
$\mathcal{PT}$	×	×
m_x	×	✓
m_y	×	✓
m_z	✓	✓
m'_x	✓	×
m'_y	✓	×
m'_z	×	×
$\mathcal{C}_{2z}$	✓	✓
$\mathcal{C}_{3z}$	✓	×
$\mathcal{C}_{4z}$	✓	×
$\mathcal{C}'_{2z}$	✓	×
$\mathcal{C}'_{3z}$	✓	×
$\mathcal{C}'_{4z}$	✓	✓

TABLE I. Symmetry requirements for the anomalous Hall effect (AHE) and linear magneto-conductivity (LMC). ✓ and × denote that the effect is and is not allowed by the corresponding symmetry, respectively. We emphasize the symmetries that prohibit AHE but allow LMC by shading.

III. OPTICAL LINEAR MAGNETO-BIREFRINGENCE

In the previous section we outlined the symmetry requirements to observe symmetric conductivity that is a linear function of magnetic field. In the rest of the paper we analyze the different microscopic mechanisms that may yield observable signals reporting on TRS-breaking. Here, we start by studying the optical effect in the local orbital picture.

We develop a minimal model of EuCd_2P_2 , a material in which off-diagonal LMB has been observed [4], and use it to calculate the optical conductivity tensor by employing the Kubo formula [17]. For simplicity we consider an isolated Eu^{2+} ion in an octahedral crystal field of D_{3d} symmetry, and model a one-electron $f \rightarrow d$ transition (Fig. 1a). The initial state configuration ($^8S_{7/2}$, $L = 0$, $S = 7/2$) consists of seven degenerate spin-polarized f states, which are not split by crystal field or spin-orbit coupling, since $L = 0$. The final state $5d$ manifold is split by a crystal field (Δ_{CF}), spin-orbit coupling (λ) and

a Zeeman term induced by a magnetic field $\mathbf{H}$:

$$\hat{H}_L(\Delta_{CF}, \lambda, \mathbf{H}) = \Delta_{CF} \hat{H}_{CF} + \lambda \hat{\mathbf{L}} \cdot \hat{\mathbf{S}} + \mu_B \mathbf{H} \cdot (2 + \hat{\mathbf{S}}) \cdot \hat{\mathbf{L}}$$

where $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$ are orbital angular momentum (OAM) and spin operators, respectively, and μ_B is the Bohr magneton (see Appendix B for details). The resulting energy spectrum, assuming $\Delta_{CF} \gg \lambda \gg H$, is shown in Fig. 1a. The conductivity tensor can now be evaluated for transitions between the degenerate initial state manifold and each of the final states, calculated assuming an applied $\mathbf{H}$ within the yz plane. This field captures the effect of an intrinsic magnetization M_y that acquires a component along the z direction (M_z) due to $\mathbf{H}$. The optical conductivity is calculated using the standard Kubo formula:

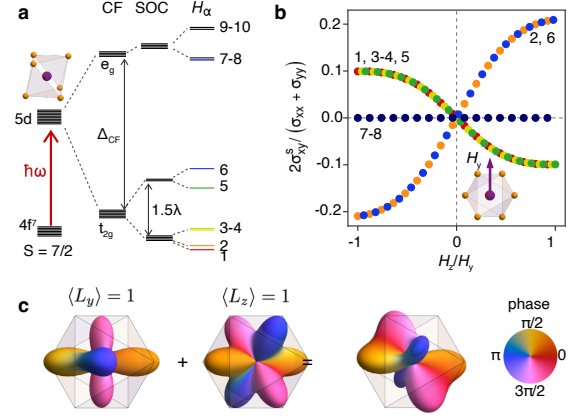


FIG. 1. (a) The model considers a one-electron $f \rightarrow d$ transition, with a degenerate initial state manifold and a final state manifold split by the three terms in Eq. (6): Δ_{CF} separates the d manifold into states of e_g and t_{2g} symmetry. (b) The ratio of off-diagonal symmetric conductivity to the average diagonal conductivity as a function of H_z/H_y , calculated for the different final states in Fig. 1a. (c) The orbital wavefunctions corresponding to $\langle L_\alpha \rangle = 1$, for $\alpha = y, z, (z + y)/\sqrt{2}$.

$$\sigma_{\alpha\beta}(\omega) = -\frac{ie^2}{V\hbar} \times \sum_{n,m} (f_m - f_n) (E_m - E_n) \times \frac{\langle n | \hat{\alpha} | m \rangle \langle m | \hat{\beta} | n \rangle}{\hbar(\omega + i\eta) - (E_m - E_n)}, \quad (7)$$

where $\hat{\alpha}$ and $\hat{\beta}$ are position operators ($\alpha, \beta = x, y, z$), ω is energy, n and m states, f_n and f_m their respective occupations, E_n and E_m their energies, and η is a small damping. Initial states are taken to be seven degenerate spin-polarized f -orbitals, while the final states are d -orbitals split by a trigonal crystal field and spin-orbit coupling.

We plot Eq.(7) in Fig. 1b, revealing a non-zero $\sigma_{xy}^s \sim H_z/H_y$ that is finite only for t_{2g} final states, indicating the importance of OAM, which is quenched in the e_g manifold. The LMB is induced by a subtle interference effect between wave functions with non-zero OAM in the y and z directions (L_y^e and L_z^e , Fig. 1c). Both the amplitude and the phase of the wave functions have a non-trivial angular dependence; the former is caused by the CF which encodes spatial symmetries, and the latter by the field-induced OAM. Their interference results in a rotation of charge density, leading to the principal axis rotation. A rotation is therefore expected for optical transitions in which either the initial or the final state supports a non-zero OAM, despite the lowering of symmetry by the CF, as is the case for t_{2g} orbitals.

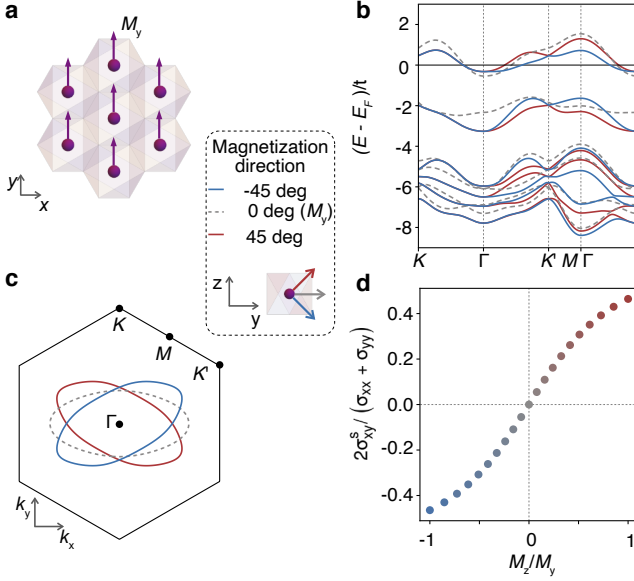


FIG. 2. Rotation of Fermi surfaces and change of conductivity in a tight binding model as a function of the orientation of the triangular lattice of magnetic ions with moment $\mathbf{M} = (0, M_y, M_z)$ in the $y-z$ plane. The panels (b-d) study quantities when $\mathbf{M}$ points along -45° , 0° , or $+45^\circ$ from the y axis, represented by red, grey, and blue arrows. (b) The band dispersion along high symmetry path in the Brillouin zone (BZ) for the three orientations of $\mathbf{M}$. t is the overall hopping energy scale and E_F is the Fermi energy. (c) FS at the Fermi energy $E = E_F$ for the three orientations of $\mathbf{M}$. As $\mathbf{M}$ is oriented from -45° to $+45^\circ$, the FS rotates accordingly in a clockwise way. (d) The conductivity angle $2\sigma_{xy}^s/(\sigma_{xx} + \sigma_{yy})$ as a function of M_z/M_y . For (c) and (d) we assumed the filling at $E = E_F$.

IV. TIGHT BINDING MODEL

The insight thus gained as to the mechanism for off-diagonal LMB at optical frequencies suggests that an analogous effect in the DC limit might be achieved in a system with t_{2g} orbitals at the Fermi level. We therefore construct such a tight-binding model, based on hopping between local d orbitals on a triangular lattice that experience a strong trigonally-distorted octahedral crystal field splitting (Fig. 2a, D_{3d} symmetry). The TB Hamiltonian has the form

$$\hat{H}_{TB} = \hat{H}_L + \sum_{\mathbf{r}\mathbf{r}', \ell\ell', \sigma\sigma'} t_{\mathbf{r}, \mathbf{r}', \ell\ell', \sigma\sigma'} d_{\mathbf{r}, \ell, \sigma}^\dagger d_{\mathbf{r}', \ell', \sigma'}, \quad (8)$$

where ℓ (σ) label the d orbitals (spins). The first term, $\hat{H}_L = \sum_{\mathbf{r}} d_{\mathbf{r}}^\dagger H(\Delta_{\text{CF}}, \lambda, \mathbf{M}) d_{\mathbf{r}}$, is equivalent to the local term in Eq. (6), with magnetization $\mathbf{M}$ in place of an external field, while the second term describes near-neighbor hopping between such orbitals; the hopping matrix elements $t_{\mathbf{r}\mathbf{r}', \ell\ell', \sigma\sigma'}$ are obtained from the Slater-Koster parameterization [18] (see details in Appendix B). In our toy model we apply a magnetic field along $\hat{z}$,

H_z , to a ferromagnetic material, with intrinsic magnetization along $\hat{y}$, M_y . The applied H_z tilts the magnetization towards $\hat{z}$, inducing a component of magnetization $M_z \sim H_z$, and therefore changing the symmetry of the magnetic structure. LMC then captures the difference of the symmetric part of the conductivity tensor for positive and negative M_z , while keeping M_y fixed: $\sigma_{xy}^s(M_y, M_z) - \sigma_{xy}^s(M_y, -M_z) \neq 0$. In order to evaluate this quantity, we first explore the corresponding electronic structure. In Fig. 2b we show the band structure of the t_{2g} -derived bands calculated for three orientations of magnetization $\mathbf{M}$: along y axis (equivalent to zero applied field) and $\pm 45^\circ$ away from the y axis in the yz plane (corresponding to opposite signs of applied field). The bands are clearly modified by the magnetization. More insights can be gleaned from the effect of $\mathbf{M}$ on the FS (Fig. 2c): the FS is elliptical for all three orientations of $\mathbf{M}$, and it rotates in the $k_x k_y$ plane as the magnetization $\mathbf{M}$ rotates in the yz plane.

The rotation of the FS induced by the magnetization has profound consequences on transport. The conductivity tensor depends on the geometry of an elliptical FS as:

$$\sigma = \sigma_0 \mathbb{1} + \sigma_0 \mathcal{E}^2 \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \quad (9)$$

where $\mathbb{1}$ is the unit matrix, and θ and $\mathcal{E}$ denote the orientation of the major axis of the ellipse and its eccentricity, respectively. The isotropic part $\sigma_0 \mathbb{1}$ denotes the conductivity in the absence of $\mathbf{M}$, as enforced by the threefold rotation and mirror symmetries of the lattice and the model. Importantly, the principal axes of the conductivity tensor correspond to the major and minor axes of the ellipse, and a nonzero field-induced FS rotation θ induces a nonzero off-diagonal conductivity. Specifically, Eq. (9) reveals that $\sigma_{xy} = \sigma_{yx}$, which implies that a nonzero σ_{yx}^s is expected. Also, since σ_{xy} is directly proportional to $\sin 2\theta$, we expect the opposite sign of σ_{xy} for the two FSs shown in Fig. 2c, calculated for the magnetization along $\pm 45^\circ$.

We confirm the above picture by calculating σ_{xy}^s as a function of the orientation of $\mathbf{M}$ using the Kubo formula. In Fig. 2d we plot the ratio $2\sigma_{xy}^s/(\sigma_{xx} + \sigma_{yy})$, which is independent of the relaxation time τ , as a function of M_z/M_y . Importantly, σ_{xy}^s is an odd function of M_z , and exhibits the same linear dependence at small M_z as the (anti-symmetric) Hall conductivity. Therefore, extracting the H -linear component of the transverse voltage measured in a single geometry, as is usually done, is not sufficient to isolate the Hall signal in a magnetic material. Our results thus call for independent measurements of σ_{xy} and σ_{yx} to separate σ_{xy}^a and σ_{xy}^s .

V. SEMICLASSICAL DERIVATION

Our TB model shows that a magnetization can induce LMC, but it does not prove that magnetization is *nec-*

essary for this effect to take place; this is reminiscent of the AHE, which can be induced by a magnetization, but it does not require it [19]. To explore this analogy, we have derived the conductivity σ_{ij}^s of a two-dimensional (2D) system to linear order in $\mathbf{B} = B_z \hat{z}$ within the semiclassical approximation [20–23],

$$\sigma_{xy}^s = B_z \frac{\tau e^2}{\hbar} \int \frac{d^2 k}{(2\pi)^2} \times \left[\frac{1}{2} (v_x^0 \partial_y m_{\mathbf{k}} + v_y^0 \partial_x m_{\mathbf{k}}) - m_{\mathbf{k}} v_{xy}^0 + e v_x^0 v_y^0 \Omega_{\mathbf{k}} \right] f'_0, \quad (10)$$

where τ is the relaxation time and e the elementary charge. Note that in semiclassics the electrons are affected by the field $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. The full derivation is given in [17], including corrections due to a field-induced shift of the chemical potential [24]. Inside the integral, f'_0 is the derivative of the Fermi function $f_0(\varepsilon_{0,\mathbf{k}})$ with respect to the single-particle energy $\varepsilon_{0,\mathbf{k}}$,

$v_i^0, i = x, y$ are the components of the Fermi velocity and we defined their derivative with respect to momentum as $v_{ij}^0 = \partial_i v_j^0 = \partial_j v_i^0$, $\Omega_{\mathbf{k}}$ is the Berry curvature, and $\mathbf{m}_{\mathbf{k}} = m_{\mathbf{k}} \hat{z}$ is the magnetization of the conduction electrons, defined by the shift of the single-particle energy in an external field, $\tilde{\varepsilon}_{\mathbf{k}} = \varepsilon_{0,\mathbf{k}} - \mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}$ [25]. We find two distinct contributions at linear order in B_z coming from magnetization and from Berry curvature. The first three terms in the integral are the magnetization contribution; it is further divided into a “velocity” part (first two terms) and a “band curvature” part (third term). The fourth term describes the “Berry curvature” contribution.

The magnetization contribution arises from momentum-dependent changes of $\mathbf{m}_{\mathbf{k}}$ and band velocities. Note that if the magnetization is constant or linear in momentum across the momentum space, the velocity part of the magnetization contribution to LMC vanishes. In contrast, as illustrated in Fig. 1, orbital mixing – promoted by crystal field and spin-orbit coupling – leads to non-zero expected value of OAM. Therefore, we speculate that systems with mixed and varying orbital character throughout the Fermi level can exhibit a net contribution of such velocity component in Eq. (10). On the other hand, the band curvature part of the magnetization contribution to LMC vanishes if the band disperses linearly in momentum space. Finally, the Berry curvature contribution is proportional to $\Omega_{\mathbf{k}}$ multiplied by the components of the Fermi velocity, and arises from Berry phase correction to the phase space volume [26]. This term can in principle give a non-zero contribution even if $\Omega_{\mathbf{k}}$ is constant across the momentum space.

We note that although AHE can also arise from $\Omega_{\mathbf{k}}$, its functional form differs substantially from that of LMC:

$$\sigma_{xy}^{\text{AHE}} = -\frac{e^2}{\hbar} \int \frac{d^2 k}{(2\pi)^2} \Omega_{\mathbf{k}} f_0. \quad (11)$$

First, σ_{ij}^{AHE} does not depend on the Fermi velocity or the

relaxation time. Second, the integral in Eq. (11) includes all occupied states, whereas the integrand in Eq. (10) is nonzero only near the Fermi surface. These differences make it clear that the AHE and LMC are two distinct probes of TRS breaking, as previously deduced from symmetry considerations in Section II.

VI. DENSITY FUNCTIONAL THEORY

Finally, to go beyond symmetry analysis and toy models, we demonstrate that LMC can be captured by *ab initio* calculations. We choose a monolayer of MnPSe_3 as our model system (for details of the calculation see [17], and Refs. [27–38]). This compound is a member of the family of magnetic 2D transition metal phosphorous trichalcogenides that exhibit a wealth of interesting magnetic, optical and topological properties [39–42]. MnPSe_3 adopts the trigonal $R\bar{3}$ space group where each layer of magnetic Mn atoms form a honeycomb lattice surrounded by PSe_3 layers above and below it (Fig. 3a). Although its ground state magnetic order is antiferromagnetic, for illustrative purposes here we consider the metastable ferromagnetically ordered case previously studied in Refs. [41, 42]. MnPSe_3 is an appropriate system to validate our theory within the DFT method both because the magnetic symmetry allows for LMC, and because its low-energy electronic structure contains t_{2g} orbitals of Mn.

We performed DFT calculations for two orientations of Mn magnetic moments within the yz plane (Fig. 3a, right panel), and used the Kubo formula to evaluate $2\sigma_{xy}^s / (\sigma_{xx} + \sigma_{yy})$ as a function of the Fermi level. As can be seen in Fig. 3b, a non-zero σ_{xy}^s is predicted across a wide range of Fermi energies. Its sign depends on the orientation of the magnetic moment, thus validating our understanding of the LMC and its presence in MnPSe_3 . An important finding is the magnitude of the effect. For a Fermi energy close to the valence band maximum (VBM) σ_{xy}^s reaches values as large as 25% of the diagonal conductivity. This dramatic enhancement occurs because the small FSs close to the VBM can be significantly perturbed by the magnetization; this suggests that the LMC is particularly relevant to the study of magnetic semiconductors and semimetals. Even with the Fermi energy far from the band edge, σ_{xy}^s reaches 1%–3% of the diagonal conductivity over a wide range of Fermi energies, which is substantial comparable to observed Hall conductivities. Indeed, anomalous Hall effect [43] is considered “giant” when it corresponds to a few percent of longitudinal conductivity. The observation that such large effects can be found in *ab initio* calculations motivates both large-scale computational searches and experiments on magnetic materials in which LMC is expected on symmetry grounds.

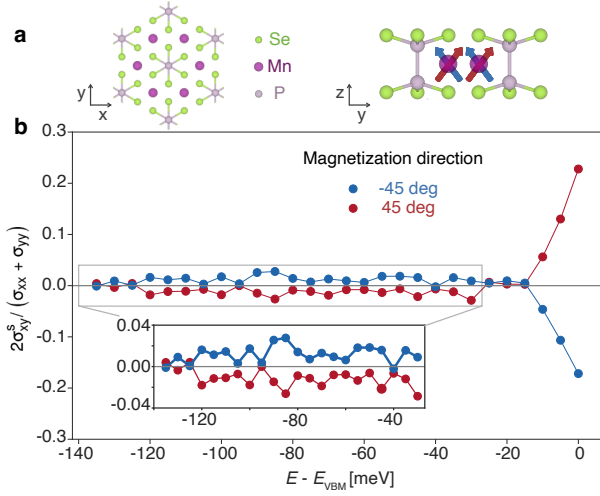


FIG. 3. (a) Top view and side view of the MnPSe_3 crystal structure. The red and blue arrows indicate the two magnetization directions used in the calculations. (b) Conductivity angle as a function of energy for magnetization orientation along $\pm 45^\circ$ with respect to the yz plane.

VII. EXPERIMENTAL DETECTION

Having demonstrated that LMC is to be expected in a range of magnetic materials, we propose an experimental strategy to detect it, and distinguish it from the Hall effect. Long Hall bars are typically used for transport measurement to achieve a homogeneous current density between the longitudinal contacts. Exchanging current and voltage contacts in a single device, which would theoretically probe σ_{xy} and σ_{yx} , would sacrifice the uniform current condition. Another strategy, fabricating devices with the bars along two orthogonal crystal directions would introduce complications such as sample-to-sample variation. These considerations lead us to propose an L-shaped device, illustrated in Fig. 4(a), to independently measure σ_{xy} and σ_{yx} in the same geometry and in the same crystal, allowing for robust extraction of σ^a and σ^s . Such devices can be structured from single crystals using focused ion beam milling [44], or created in thin flakes using lithographic techniques.

To demonstrate this approach, we performed Landauer-Büttiker calculations in the L-shaped geometry (for more details see [17], and Refs. [45–50]), assuming bands given by the TB model above (Eq. (8), Fig. 2). The resulting $2\sigma^s_{xy} / (\sigma_{xx} + \sigma_{yy})$ is plotted as a function of the orientation of $\mathbf{M}$ in Fig. 4b. The qualitative agreement of the Landauer-Büttiker calculations with the Kubo formula (see Fig. 2d) is excellent. For comparison, we also plot $2\sigma^a_{xy} / (\sigma_{xx} + \sigma_{yy})$, from which we see that σ^s_{xy} is larger than the Hall conductivity for our choice of parameters, demonstrating that this regime can indeed be reached. We note that the ratio $\sigma^s_{xy} / \sigma^a_{xy}$ depends on the scattering time, and becomes larger with increasing purity.

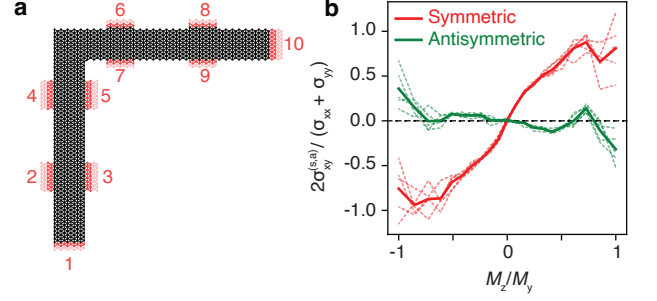


FIG. 4. (a) L-shape device geometry. The scattering region (leads) is denoted by the black (red) lattice. Leads 1 and 10 are the source and drain electrodes, while leads 2-8 are voltage probes. (b) Conductivity angle for the symmetric (red) and antisymmetric (green) conductivity as a function of M_z/M_y . The calculations are done based on the tight binding model (Eq. (8), Fig. 2), with a Fermi level of -1.95 . Solid line is the average value performed over 5 disorder realizations (shown in dashed, paler color).

VIII. CONCLUSION

To conclude, the impact of our work is threefold. First, we have identified a microscopic mechanism responsible for the rotation of principal optical axes in a magnetic field. Furthermore, we demonstrated a fundamentally new transport effect, referred to as linear magnetoconductivity, using three complementary approaches: a tight-binding toy model, DFT calculations and a semiclassical calculation. The latter identifies two distinct mechanisms yielding LMC: momentum-dependent band magnetization, and Berry curvature. Finally, we have shown the importance of measuring the full conductivity tensor in magnetic materials, both because the Hall effect can otherwise be misidentified, and because such measurements serve as a powerful probe of symmetry. We hope that our results inspire new ways of probing the effect of time-reversal symmetry breaking on conduction electrons in future measurements.

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**SUPPLEMENTARY MATERIAL FOR “LINEAR MAGNETO-CONDUCTIVITY AS A DC PROBE OF
TIME-REVERSAL SYMMETRY BREAKING”**

Appendix A: Semiclassical calculation

In this section, we derive the most generic semiclassical expression for linear magnetoconductivity, assuming that the Fermi energy lies within a single non-degenerate band. We adopt the approach discussed in, e.g., Refs. [21, 22]. We start with the semiclassical equations of motion having the form [23]

$$\begin{aligned}\hbar \dot{\mathbf{r}} &= \nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} - \hbar \dot{\mathbf{k}} \times \boldsymbol{\Omega}_{\mathbf{k}}, \\ \hbar \dot{\mathbf{k}} &= -e\mathbf{E} - e\dot{\mathbf{r}} \times \mathbf{B},\end{aligned}\tag{S1}$$

where $\boldsymbol{\Omega}_{\mathbf{k}} = i\langle \nabla_{\mathbf{k}} u_{\mathbf{k}} | \times \nabla_{\mathbf{k}} u_{\mathbf{k}} \rangle$ is the Berry curvature. The quasiparticle dispersion relation is modified in the presence of an external magnetic field $\mathbf{B}$:

$$\tilde{\varepsilon}_{\mathbf{k}} = \varepsilon^0 - \mathbf{m}_{\mathbf{k}} \cdot \frac{\mathbf{B}}{B},\tag{S2}$$

where ε^0 is the bare band energy at $\mathbf{B} = 0$, $H_{\mathbf{k}} | u_{\mathbf{k}} \rangle = \varepsilon^0 | u_{\mathbf{k}} \rangle$, and the orbital magnetic moment equals

$$\mathbf{m}_{\mathbf{k}} = -i \frac{e}{2\hbar} \langle \nabla_{\mathbf{k}} u_{\mathbf{k}} | \times (H_{\mathbf{k}} - \varepsilon^0) | \nabla_{\mathbf{k}} u_{\mathbf{k}} \rangle.\tag{S3}$$

The solution of the equations of motion reads as

$$\begin{aligned}\dot{\mathbf{r}} &= \frac{1}{\hbar D_{\mathbf{k}}} \left\{ \nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} + e\mathbf{E} \times \boldsymbol{\Omega}_{\mathbf{k}} + \frac{e}{\hbar} \mathbf{B} (\boldsymbol{\Omega}_{\mathbf{k}} \cdot \nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}}) \right\}, \\ \dot{\mathbf{k}} &= \frac{1}{\hbar D_{\mathbf{k}}} \left\{ -e\mathbf{E} - \frac{e}{\hbar} \nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} \times \mathbf{B} - \frac{e^2}{\hbar} (\mathbf{E} \cdot \mathbf{B}) \boldsymbol{\Omega}_{\mathbf{k}} \right\},\end{aligned}\tag{S4}$$

where the factor $D_{\mathbf{k}} = 1 + (e/\hbar)(\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{k}})$ accounts for the density of states modification [26].

The current density in a uniform system equals

$$\mathbf{j} = -e \int \frac{d^d k}{(2\pi)^d} D_{\mathbf{k}} \dot{\mathbf{r}} f = -\frac{e}{\hbar} \int \frac{d^d k}{(2\pi)^d} \left[\nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} + e\mathbf{E} \times \boldsymbol{\Omega}_{\mathbf{k}} + \frac{e}{\hbar} \mathbf{B} (\nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} \cdot \boldsymbol{\Omega}_{\mathbf{k}}) \right] f,\tag{S5}$$

where d is the dimensionality of the system. The semiclassical distribution function f satisfies the conventional kinetic equation. We assume the relaxation time approximation for simplicity, resulting in

$$\frac{\partial f}{\partial t} + \dot{\mathbf{k}} \cdot \nabla_{\mathbf{k}} f = -\frac{f - f_0}{\tau},\tag{S6}$$

where $\dot{\mathbf{k}}$ is given by Eq. (S4), τ is the relaxation time, and $f_0 = f_0(\tilde{\varepsilon}_{\mathbf{k}})$ is the equilibrium Fermi-Dirac distribution, which at zero temperature merely equals $f_0 = \Theta(\mu - \tilde{\varepsilon}_{\mathbf{k}})$. We emphasize that f_0 depends on the modified quasiparticle energy $\tilde{\varepsilon}_{\mathbf{k}}$ rather than on the bare band energy ε^0 . Assuming a monochromatic electric field,

$$\mathbf{E}(t) = \mathbf{E}_{\omega} e^{-i\omega t} + \mathbf{E}_{\omega}^* e^{i\omega t},\tag{S7}$$

the distribution function equals

$$f(t) = f_0 + f_1 e^{-i\omega t} + f_1^* e^{i\omega t} + \dots,\tag{S8}$$

where $f_1 \propto E = |\mathbf{E}_{\omega}|$. Physical electric field $\mathbf{E}(t)$ is real, implying that $\mathbf{E}_{-\omega} = \mathbf{E}_{\omega}^*$. The kinetic equation for f_1 then becomes

$$D_{\mathbf{k}} \left(i\omega - \frac{1}{\tau} \right) f_1 = -\frac{e}{\hbar} \left(\mathbf{E}_{\omega} + \frac{e}{\hbar} (\mathbf{E}_{\omega} \cdot \mathbf{B}) \boldsymbol{\Omega}_{\mathbf{k}} \right) \cdot \nabla_{\mathbf{k}} f_0 - \frac{e}{\hbar^2} \nabla_{\mathbf{k}} f_1 \cdot \nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} \times \mathbf{B}.\tag{S9}$$

At small magnetic fields, the solution of this equation up to linear order in $\mathbf{B}$ equals

$$f_1(\mathbf{k}) \approx \frac{e\tau}{\hbar D_{\mathbf{k}}(1 - i\omega\tau)} \left\{ \left(\mathbf{E}_{\omega} + \frac{e}{\hbar} (\mathbf{E}_{\omega} \cdot \mathbf{B}) \boldsymbol{\Omega}_{\mathbf{k}} \right) \cdot \nabla_{\mathbf{k}} f_0(\tilde{\varepsilon}_{\mathbf{k}}) + \frac{e\tau}{\hbar^2(1 - i\omega\tau)} (\nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} \times \mathbf{B}) \cdot \nabla_{\mathbf{k}} (\mathbf{E}_{\omega} \cdot \nabla_{\mathbf{k}} f_0(\tilde{\varepsilon}_{\mathbf{k}})) \right\},\tag{S10}$$

and we explicitly restored the argument of f_0 . Expanding further terms with $\tilde{\varepsilon}_{\mathbf{k}} = \varepsilon^0 - \mathbf{m}_{\mathbf{k}} \cdot \mathbf{B}$ up to linear order in $\mathbf{m}_{\mathbf{k}} \cdot \mathbf{B}$ and introducing bare band velocity $\mathbf{v} = \nabla_{\mathbf{k}} \varepsilon^0 / \hbar$, we find

$$f_1(\mathbf{k}) \approx \frac{e\tau}{\hbar(1-i\omega\tau)} \left\{ \hbar (\mathbf{E}_{\omega} \cdot \mathbf{v}) f_0'(\varepsilon^0) - e(\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}}) (\mathbf{E}_{\omega} \cdot \mathbf{v}) f_0'(\varepsilon^0) - (\mathbf{E}_{\omega} \cdot \nabla_{\mathbf{k}} (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}})) f_0'(\varepsilon^0) \right. \\ \left. + e(\mathbf{E}_{\omega} \cdot \mathbf{B}) (\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{v}) f_0'(\varepsilon^0) - \hbar (\mathbf{E}_{\omega} \cdot \mathbf{v}) (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) f_0''(\varepsilon^0) + \frac{e\tau}{1-i\omega\tau} f_0'(\varepsilon^0) (\mathbf{v} \times \mathbf{B} \cdot \nabla_{\mathbf{k}} (\mathbf{E}_{\omega} \cdot \mathbf{v})) \right\}. \quad (\text{S11})$$

Importantly, f_0' and f_0'' now have argument ε^0 , and we defined $f_0'(\varepsilon) = \partial f_0(\varepsilon) / \partial \varepsilon$ and $f_0''(\varepsilon) = \partial^2 f_0(\varepsilon) / \partial \varepsilon^2$. From Eq. (S5), we find that the linear in $\mathbf{B}$ part of the current equals $\mathbf{j}_1 e^{-i\omega t} + \mathbf{j}_1^* e^{i\omega t}$, with

$$\mathbf{j}_1 = -\frac{e}{\hbar} \int \frac{d^d k}{(2\pi)^d} \left\{ \left[\nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} + \frac{e}{\hbar} (\nabla_{\mathbf{k}} \tilde{\varepsilon}_{\mathbf{k}} \cdot \mathbf{\Omega}_{\mathbf{k}}) \mathbf{B} \right] f_1(\mathbf{k}) + e \mathbf{E}_{\omega} \times \mathbf{\Omega}_{\mathbf{k}} f_0(\tilde{\varepsilon}_{\mathbf{k}}) \right\}. \quad (\text{S12})$$

Expanding again up to the linear order in $\mathbf{B}$, we find

$$\mathbf{j}_1 = \mathbf{j}_1^{(0)} + \mathbf{j}_1^{(1)}, \quad (\text{S13})$$

where $\mathbf{B}$ -independent part is given by

$$\mathbf{j}_1^{(0)} = -\frac{e^2}{\hbar} \int \frac{d^d k}{(2\pi)^d} \left\{ f_0(\varepsilon_{\mathbf{k}}^0) \mathbf{E}_{\omega} \times \mathbf{\Omega}_{\mathbf{k}} + \frac{\hbar \tau f_0'(\varepsilon_{\mathbf{k}}^0)}{1-i\omega\tau} (\mathbf{E}_{\omega} \cdot \mathbf{v}) \mathbf{v} \right\}, \quad (\text{S14})$$

and linear in $\mathbf{B}$ contribution has form

$$\mathbf{j}_1^{(1)} = \frac{e^2}{\hbar} \int \frac{d^d k}{(2\pi)^d} (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) (\mathbf{E}_{\omega} \times \mathbf{\Omega}_{\mathbf{k}}) f_0'(\varepsilon_{\mathbf{k}}^0) + \\ + \frac{e^2 \tau}{\hbar(1-i\omega\tau)} \int \frac{d^d k}{(2\pi)^d} f_0'(\varepsilon_{\mathbf{k}}^0) \left\{ (\mathbf{E}_{\omega} \cdot \mathbf{v}) \nabla_{\mathbf{k}} (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) + \mathbf{v} (\mathbf{E}_{\omega} \cdot \nabla_{\mathbf{k}} (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}})) + \right. \\ \left. + e \left[\mathbf{v} (\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}}) (\mathbf{E}_{\omega} \cdot \mathbf{v}) - \mathbf{v} (\mathbf{E}_{\omega} \cdot \mathbf{B}) (\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{v}) - \mathbf{B} (\mathbf{E}_{\omega} \cdot \mathbf{v}) (\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{v}) \right] \right\} + \\ + \frac{e^2 \tau}{1-i\omega\tau} \int \frac{d^d k}{(2\pi)^d} \mathbf{v}_{\mathbf{k}} \left\{ f_0''(\varepsilon_{\mathbf{k}}^0) (\mathbf{E}_{\omega} \cdot \mathbf{v}) (\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) - \frac{e\tau}{\hbar(1-i\omega\tau)} f_0'(\varepsilon_{\mathbf{k}}^0) (\mathbf{v} \times \mathbf{B} \cdot \nabla_{\mathbf{k}} (\mathbf{E}_{\omega} \cdot \mathbf{v})) \right\}. \quad (\text{S15})$$

The first term in this expression corresponds to the anomalous Hall effect correction due to finite magnetic field. This term is fully antisymmetric and thus does not contribute to σ_{ij}^s . The last term is entirely due to Lorentz force and is not related to the geometry of the band structure. It may have both symmetric and anti-symmetric components that we specify below. Finally, in $d = 2$ which is the main focus of this paper, both $\mathbf{\Omega}_{\mathbf{k}}$ and $\mathbf{m}_{\mathbf{k}}$ are pointing in the z -direction, hence, the two terms containing $\mathbf{\Omega}_{\mathbf{k}} \cdot \mathbf{v}$ are equal to 0.

So far we only considered the grand canonical ensemble, i.e., assumed that the chemical potential is fixed and does not depend on $\mathbf{B}$. If we instead focus on the canonical ensemble and assume the fixed density of particles n , we find that the chemical potential is renormalized as $\mu \rightarrow \mu - \delta\mu$. The correction $\delta\mu$ due to finite magnetic field is found from the condition

$$n = \int \frac{d^d k}{(2\pi)^d} D_{\mathbf{k}} f_0(\varepsilon_{\mathbf{k}}^0 + \delta\mu - \mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) = \int \frac{d^d k}{(2\pi)^d} f_0(\varepsilon_{\mathbf{k}}^0) \quad (\text{S16})$$

and equals to the leading order in $\mathbf{B}$

$$\delta\mu = \delta\mu_{\mathbf{m}} + \delta\mu_{\mathbf{\Omega}} = \frac{1}{N_0} \int \frac{d^d k}{(2\pi)^d} \left\{ -(\mathbf{B} \cdot \mathbf{m}_{\mathbf{k}}) f_0'(\varepsilon_{\mathbf{k}}^0) + \frac{e}{\hbar} (\mathbf{B} \cdot \mathbf{\Omega}_{\mathbf{k}}) f_0(\varepsilon_{\mathbf{k}}^0) \right\}, \quad N_0 = - \int \frac{d^d k}{(2\pi)^d} f_0'(\varepsilon_{\mathbf{k}}^0). \quad (\text{S17})$$

At zero temperature,

$$N_0 = \int \frac{d^d k}{(2\pi)^d} \delta(\varepsilon_{\mathbf{k}}^0 - \mu), \quad (\text{S18})$$

which is the zero-field density of states per unit volume at the Fermi level.

The corresponding correction to the current comes from the zero-order contribution $\mathbf{j}_1^{(0)}$, Eq. (S14), and equals

$$\delta \mathbf{j}_{\Gamma}^{(\mu)-} = \frac{e^2}{\hbar} \delta \mu \int \frac{d^2 k}{(2\pi)^d} \left\{ f'_0(\varepsilon_{\mathbf{k}}^0) \mathbf{E}_\omega \times \mathbf{\Omega}_{\mathbf{k}} + \frac{\hbar \tau f''(\varepsilon_{\mathbf{k}}^0)}{1 - i\omega\tau} (\mathbf{E}_\omega \cdot \mathbf{v}_{\mathbf{k}}^0) \mathbf{v}_{\mathbf{k}}^0 \right\}. \quad (\text{S19})$$

Again, the first term is the correction to the anomalous Hall effect, which is antisymmetric. The second term is the correction to the Drude conductivity: it is symmetric and contributes to σ_{ij}^s . The information about the band structure geometry is hidden in the correction $\delta \mu$.

Focusing on the case $d = 2$, we have $\mathbf{\Omega}_{\mathbf{k}} = \Omega_{\mathbf{k}} \hat{z}$ and $\mathbf{m}_{\mathbf{k}} = m_{\mathbf{k}} \hat{z}$. Assuming further that $\mathbf{B} = B_z \hat{z}$ and extracting the symmetric part of the conductivity tensor, $\sigma_{ij}^s = (\sigma_{ij} + \sigma_{ji})/2$, we find

$$\begin{aligned} \sigma_{ij}^s = \sigma_{ij}^{D_+} &= \frac{e^2 \tau B_z}{\hbar(1 - i\omega\tau)} \int \frac{d^2 k}{(2\pi)^2} \left\{ f'_0(v_i^0 \partial_j m + v_j^0 \partial_i m + e v_i^0 v_j^0 \Omega) + \hbar f''_0 m v_i^0 v_j^0 \right\} \\ &- \frac{e^2 \tau}{1 - i\omega\tau} \delta \mu \int \frac{d^2 k}{(2\pi)^2} f''_0 v_i^0 v_j^0 \end{aligned} \quad (\text{S20})$$

where $\varepsilon^{\alpha\beta}$ is the antisymmetric Levi-Civita tensor (summation over repeated indices is implied) and σ_{ij}^D is the zero-field Drude conductivity:

$$\sigma_{ij}^D = \frac{e^2 \tau}{1 - i\omega\tau} \int \frac{d^2 k}{(2\pi)^2} f'_0 v_i^0 v_j^0 \quad (\text{S21})$$

Here we defined $\partial_i \equiv \partial/\partial k_i$ and suppressed index $\mathbf{k}$ and argument ε^0 in f'_0, f''_0 for brevity.

The second-order term in $\tau/(1 - i\omega\tau)$ originating from the Lorentz force can be rewritten as the full derivative, hence, it vanishes after the integration over the whole Brillouin zone:

$$\hbar \int \frac{d^2 k}{(2\pi)^2} f'_0 \varepsilon^{\alpha\beta} \partial_\alpha \partial_\beta (v_i^0 v_j^0) = \int \frac{d^2 k}{(2\pi)^2} \varepsilon^{\alpha\beta} (\partial_\alpha f_0) \partial_\beta (v_i^0 v_j^0) = \int \frac{d^2 k}{(2\pi)^2} \varepsilon^{\alpha\beta} \partial_\alpha [f_0 \partial_\beta (v_i^0 v_j^0)] = 0, \quad (\text{S22})$$

where we used the identity $\varepsilon^{\alpha\beta} \partial_\alpha \partial_\beta (v_i^0 v_j^0) = 0$ and assumed no singularities in the velocity components.

Next, the components involving orbital magnetic moment $m_{\mathbf{k}}$ can be regrouped using the identity

$$\hbar f''_0 m v_i^0 v_j^0 = \frac{1}{2} [(\partial_i f_0) m v_j^0 + (\partial_j f_0) m v_i^0] = \frac{1}{2} [\partial_i (f_0 m v_j^0) + \partial_j (f_0 m v_i^0)] - \frac{f'_0}{2} (v_j^0 \partial_i m + v_i^0 \partial_j m) - f'_0 m v_{ij}^0 \quad (\text{S23})$$

where we defined $v_{ij}^0 = \partial_i v_j^0 = \partial_j v_i^0$. The first two terms in this expression are full derivatives which vanish upon integration over momenta. Collecting all the non-vanishing contributions in Eq. (S20), we obtain

$$\sigma_{ij}^s = \sigma_{ij}^{D_+} = \frac{e^2 \tau B_z}{\hbar(1 - i\omega\tau)} \int \frac{d^2 k}{(2\pi)^2} f'_0 \left\{ \frac{1}{2} (v_j^0 \partial_i m_{\mathbf{k}} + v_i^0 \partial_j m_{\mathbf{k}}) - m_{\mathbf{k}} v_{ij}^0 + e v_i^0 v_j^0 \Omega_{\mathbf{k}} \right\} + \frac{e^2 \tau \delta \mu}{\hbar(1 - i\omega\tau)} \int \frac{d^2 k}{(2\pi)^2} f''_0 v_i^0 v_j^0 \quad (\text{S24})$$

where we also integrated by parts in the last term.

Equation (S24) is the most general expression for the symmetric part of the conductivity tensor in two-dimensional systems with the magnetic field along the z -axis. If mirror symmetry m_y is additionally present, the term with $\delta \mu$ and σ^D do not contribute to σ_{xy}^s , and, in the limit $\omega \rightarrow 0$, we reproduce Eq. (10) from the main text.

Appendix B: Symmetry-allowed tight-binding model

Denote the d orbital creation operators as $d^\dagger = (\hat{d}_{xz}^\dagger, \hat{d}_{yz}^\dagger, \hat{d}_{xy}^\dagger, \hat{d}_{x^2-y^2}^\dagger, \hat{d}_{z^2}^\dagger)$ (where we have suppressed the spin component $\hat{d}_\ell^\dagger = (\hat{d}_{\ell,\uparrow}^\dagger, \hat{d}_{\ell,\downarrow}^\dagger)$) which correspond to the following orbital basis

$$|d_{xz}\rangle = \frac{1}{\sqrt{2}}(|2, -1\rangle - |2, 1\rangle), \quad (\text{S25a})$$

$$|d_{yz}\rangle = \frac{i}{\sqrt{2}}(|2, -1\rangle + |2, 1\rangle), \quad (\text{S25b})$$

$$|d_{xy}\rangle = \frac{i}{\sqrt{2}}(|2, -2\rangle - |2, 2\rangle), \quad (\text{S25c})$$

$$|d_{x^2-y^2}\rangle = \frac{1}{\sqrt{2}}(|2, -2\rangle + |2, 2\rangle), \quad (\text{S25d})$$

$$|d_{z^2}\rangle = |2, 0\rangle, \quad (\text{S25e})$$

the expressions on the right is written in the $|L, L_z\rangle$ basis. The onsite trigonal crystal field Hamiltonian has the form

$$\hat{H}_{CF} = d^\dagger \begin{pmatrix} \frac{2}{3} & 0 & 0 & \frac{\sqrt{2}}{3} & 0 \\ 0 & \frac{2}{3} & -\frac{\sqrt{2}}{3} & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{3} & \frac{1}{3} & 0 & 0 \\ \frac{\sqrt{2}}{3} & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} d, \quad (\text{S26})$$

whereas the onsite spin-orbit coupling term has the form

$$\hat{H}_{SO} = d^\dagger \hat{\mathbf{L}} \cdot \hat{\mathbf{S}} d, \quad (\text{S27})$$

where

$$\hat{L}_x = \begin{pmatrix} 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & -i & -i\sqrt{3} \\ -i & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & i\sqrt{3} & 0 & 0 & 0 \end{pmatrix}, \quad \hat{L}_y = \begin{pmatrix} 0 & 0 & 0 & -i & i\sqrt{3} \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ -i\sqrt{3} & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \hat{L}_z = \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i & 0 \\ 0 & 0 & -2i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (\text{S28})$$

and $\hat{\mathbf{S}}$ are written in terms of the standard Pauli matrix operators. We have set $\hbar = 1$.

The triangular lattice is spanned by the lattice vectors $\hat{\mathbf{e}}_1 = \frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$ and $\hat{\mathbf{e}}_2 = -\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{y}$, where we have set the lattice constant to unity. We provide the matrix for hopping along the bond $\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2$ (written in the basis of d orbitals given above):

$$[t]_{\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2} = \begin{pmatrix} V_{dd\delta} & 0 & 0 & 0 & 0 \\ 0 & V_{dd\pi} & 0 & 0 & 0 \\ 0 & 0 & V_{dd\pi} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4}(V_{dd\delta} + 3V_{dd\sigma}) & \frac{\sqrt{3}}{4}(-V_{dd\delta} + V_{dd\sigma}) \\ 0 & 0 & 0 & \frac{\sqrt{3}}{4}(-V_{dd\delta} + V_{dd\sigma}) & \frac{1}{4}(3V_{dd\delta} + V_{dd\sigma}) \end{pmatrix}, \quad (\text{S29})$$

and the other nearest neighbor hoppings can be obtained through translation or threefold rotation (by applying the matrix $\hat{R}_z^{\text{orb}}\left(\frac{2\pi}{3}\right) \equiv e^{-i\frac{2\pi}{3}\hat{L}_z}$). Here $V_{dd\sigma}$, $V_{dd\pi}$ and $V_{dd\delta}$ are the standard Slater-Koster parameters [18]. We used the following values in Fig. 2 of the main text:

$$\lambda_{SO} = 1.2, \quad V_{dd\sigma} = -1.0, \quad V_{dd\delta} = 0.4, \quad V_{dd\delta} = 0, \quad \Delta_{CF} = 20.0, \quad (\text{S30})$$

and a magnetization of norm $|\mathbf{M}| = 1.4$.

Appendix C: Landauer-Büttiker implementation

The Landauer-Büttiker formalism is employed to calculate σ_{xy}^s from the tight-binding model Eq. (8) in the main text. To that end, we use the Kwant package [45] to implement our model in the L-shaped device shown in Fig.4(a). The Hamiltonian of the semi-infinite leads is also Eq. (8). The parameters are kept the same as those used for calculating σ_{xy}^s in Fig. 2(d). The width of the device channel and of the leads is taken to be 100 lattice sites, while the length of each arm is 700 sites, resulting in a total device length of ~ 1400 sites (see next section for the reasoning behind these values).

Components of the conductivity tensor are not directly obtained in a Landauer-Büttiker calculation, but rather they are extracted from inverting the resistivity tensor

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} = \frac{1}{\rho_{xx}\rho_{yy} - \rho_{xy}\rho_{yx}} \begin{pmatrix} \rho_{yy} & -\rho_{xy} \\ -\rho_{yx} & \rho_{xx} \end{pmatrix} \quad (\text{S31})$$

whose components can be computed in the following way. The main outputs from Landauer-Büttiker are the transmission probabilities T_{ij} between each pair of leads i, j coupled to a scattering region. Then, imposing the current conditions at each lead (source, drain or floating), the voltages can be obtained with the Büttiker formula

$I_i = \frac{e}{h} \sum_j T_{ji} V_i - T_{ij} V_j$ [46, 47]. With the voltage at each lead and a current $I = I_{\text{source}} = -I_{\text{drain}}$, a resistance measurement is given by $R_{ij} = (V_i - V_j)/I$ and hence the resistivity becomes $\rho_{ij} = \frac{w}{d_{ij}} R_{ij}$, with w and d_{ij} being the cross section or width of the device and distance between leads i and j , respectively. To minimize the variances produced by the random disorder (see next section), we perform an average over 5 disorder configurations and calculate all components of the resistivity tensor in the L-shaped device. In this way, and taking lead 1 (10) as the source (drain), the resistivities read $\rho_{xx} = \rho_{68}$, $\rho_{yy} = \rho_{24}$, $\rho_{xy} = \rho_{32}$ and $\rho_{yx} = \rho_{89}$.

1. Diffusive transport in Landauer–Büttiker calculations

Most of mesoscopic transport experiments occur in the diffusive transport regime (as opposed to ballistic or localized transport). In this way, to have diffusive transport in a Landauer–Büttiker calculation, we include random Anderson disorder potential $\sum_{\mathbf{r}} d_{\mathbf{r},\ell,\sigma}^\dagger d_{\mathbf{r},\ell,\sigma}$ at each site $\mathbf{r}$ uniformly distributed in the interval $[-U/2, U/2]$. Because of the quasi-one-dimensional nature of our finite device, if U is too large, electrons will localize, whereas if U is too small, most of the transport could be ballistic inside our device. Therefore, the choice of U impacts on the choice of the device length and width. To elucidate what is a good value of U so that diffusive transport occurs *throughout* the entire device, we perform the following procedure [48, 49]. We make a two-terminal device with the same width as our device of interest (Fig. 4(a)) and calculate the two-terminal conductance, G_{2T} , for different channel lengths L and fixed U . Then, we use the relation $G_{2T} = \sigma w$ to obtain the longitudinal conductivity, and plot σ as a function

of L . For an extensive enough range of L values, σ will first increase to then remain constant and finally decrease. These three behaviors correspond to ballistic, diffusive and localized transport regimes. Increasing the width will help maximize the range of lengths for which transport is diffusive, and resemble more a mesoscopic two-dimensional system. However, that comes at a higher computational time and memory demand. We found a width of $w = 100a$ (a the lattice constant) to be big enough for our device, and computationally affordable. Finally, by looking at the scaling of σ with L , we choose a value of U that allows for a long enough device to probe the physics we are interested in but still within computational reach. To illustrate this, we plot in Fig. S1 the longitudinal conductivity for two different strengths of disorder and for devices oriented along the x and y direction. The Fermi level is taken to be the same as the one used for Fig. 4(b) in the main text, i.e. -1.95 , and $M_z/M_y = 0.32$ is used. From this plot, it is clear that $\sigma_{xx} \neq \sigma_{yy}$, which is expected due to the broken c_3 symmetry. Most importantly, one can see that for the strongest disorder case, the onset of localization appears rather quickly, around $L \sim 500a$. This is not a good value of U , since we cannot build the L-shape device we want within $500a$ if the width of the device and leads is $100a$. In contrast, for $U = 0.3V_{dd\sigma}$, the transport is diffusive for both directions at $L \sim 266a$ (see vertical dashed line) and remains constant for long enough lengths. Hence, we choose the inter-lead distance to be at least $L = 266a$. This, together with the width $w = 100a$, results in a total length of the L-shaped device of $\sim 1400a$, which still falls within diffusive transport (vertical dotted line in Fig. S1).

Finally, one also can obtain meaningful quantities such as the mean free path. In the diffusive regime, $G_{2T} = \sigma w/L$, while from quantum transport it is known that $G_{2T} = Nl_e/L$ [47, 50], where N is the number of propagating modes and l_e the mean free path. Consequently, $\sigma w = Nl_e$. Given the leads are described by the same Hamiltonian as the scattering region, N can be extracted from their band structure, whereas σ is readily obtained from Fig. S1. This gives us a mean free path of $l_e^x = 31a$ and $l_e^y = 17a$ for the x and y directions, respectively.

Appendix D: Density functional theory implementation

DFT calculations were carried out using the Vienna *ab initio* simulation package (VASP) [27–29]. The Perdew-Burke-Ernzerhof (PBE) generalized gradient approximation (GGA) was employed for the exchange-correlation potential [30] and the wave functions and pseudopotentials were generated within the projector-augmented wave (PAW) method [31]. In our calculations, the monolayer of MnPSe₃ was modeled using a periodic slab geometry in a supercell setup. To avoid spurious interactions with periodic images, a vacuum region of 28 Å in the out-of-plane direction was used. We employed a plane-wave basis with an energy cutoff of 520 eV and used a dense k-point grid sampling of $13 \times 13 \times 1$ in the BZ according to the Monkhorst-Pack scheme [32]. Both the lattice constant and positions of all atoms were relaxed until the force was less than 2 meV/Å while keeping the volume fixed. The optimized in-plane lattice constant was found to be $a = b = 6.363$ Å, consistent with experimental value ($a = b = 6.394$ Å) [33]. For the electronic self-consistent loop, the total energy convergence criterion was set to 10^{-7} eV by using GGA (without U). This choice yielded consistent results when compared to calculations with different values of U , as reported in [41].

We considered the collinear ferromagnetic (FM) order of the magnetic moment on Mn with spin-orbit coupling (SOC) with its orientation described as $\hat{\mathbf{m}} = (0, \sin \Theta, \cos \Theta)$, where Θ varied between 0° , $+45^\circ$, and -45° .

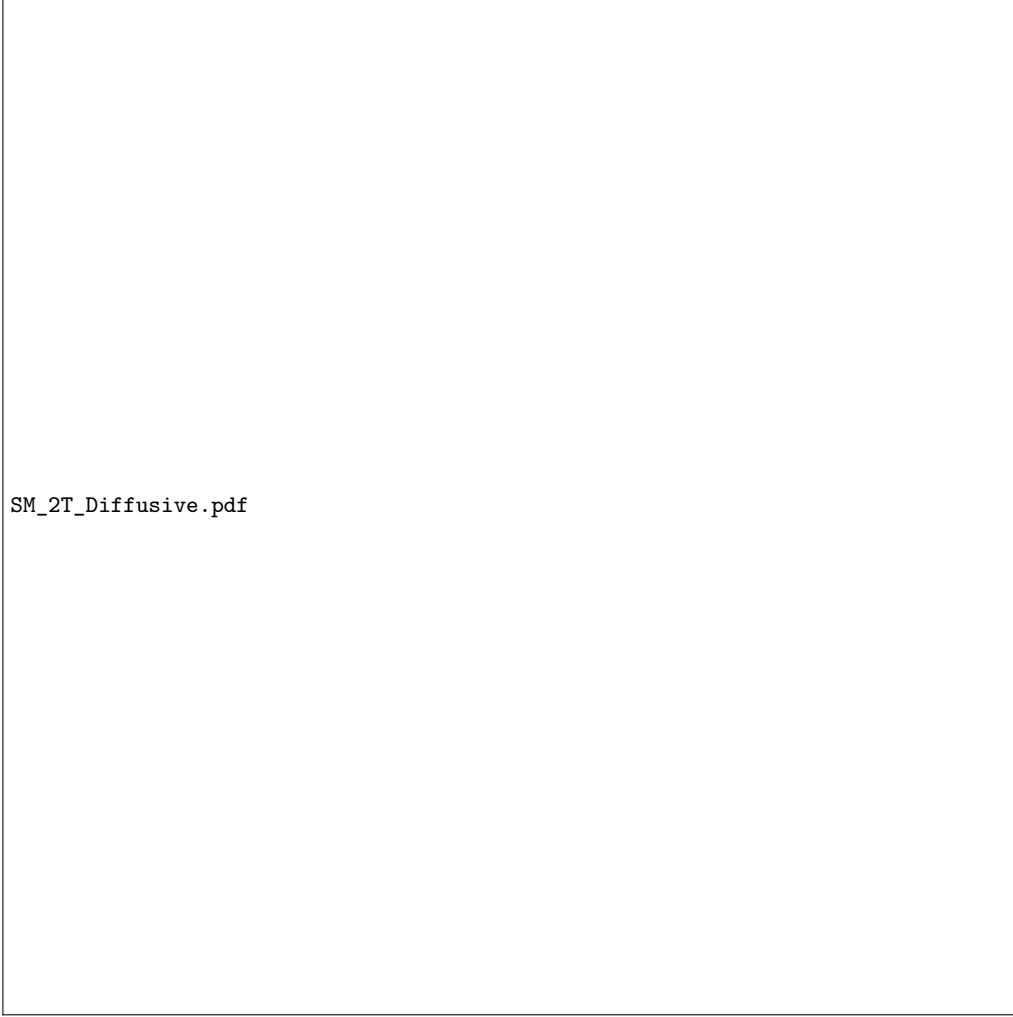


FIG. S1. Longitudinal conductivity extracted from the two-terminal conductance for two strengths of Anderson disorder and for a device oriented along x (black) and along y (red). 10 disorder averages have been carried out. The diffusive regime exists for lengths in which the conductivity remains constant. The vertical dashed (dotted) line denotes the chosen inter-lead distance (total length) of the L-shaped device. Importantly, lengths within these two vertical lines fall within diffusive transport.

The symmetric and anti-symmetric components of the off-diagonal conductivity tensor were analyzed through explicit calculation of the Fermi velocity and Berry curvature associated with the given Fermi level (chemical potential). We used the Wannier90 code [34–36] to obtain the Wannier tight-binding Hamiltonian based on maximally localized Wannier functions (MLWFs). To perform the Wannierization procedure, we carried out DFT calculations using a dense k -point grid of $18 \times 18 \times 1$. The choice of basis set for the Wannierization included $p_{x,y,z}$ orbitals for the six Se atoms and $d_{xz,yz}$ orbitals for two Mn atoms present in the supercell [42]. This selection allowed us to construct a Wannier tight-binding Hamiltonian of rank 44. Once we obtained the Wannier tight-binding Hamiltonian, we explicitly calculated the Fermi velocity and the Berry curvature [37, 38] from this Hamiltonian.

1. Supporting DFT data for MnPSe₃

In this section, we present DFT calculations to complement the calculation of σ_{xy}^s showed in the main text for MnPSe₃.

We argued that in order to observe linear magneto-conductivity, the states at the Fermi level should possess a finite orbital angular momentum even in the presence of a strong crystal field, and demonstrated that with a tight-binding made from t_{2g} orbitals. Therefore, we plot in Fig. S2 the orbital-resolved density of states with magnetization orientation at $+45^\circ$ with respect to the z axis in the yz plane. As expected from the nonzero σ_{xy}^s numerically computed

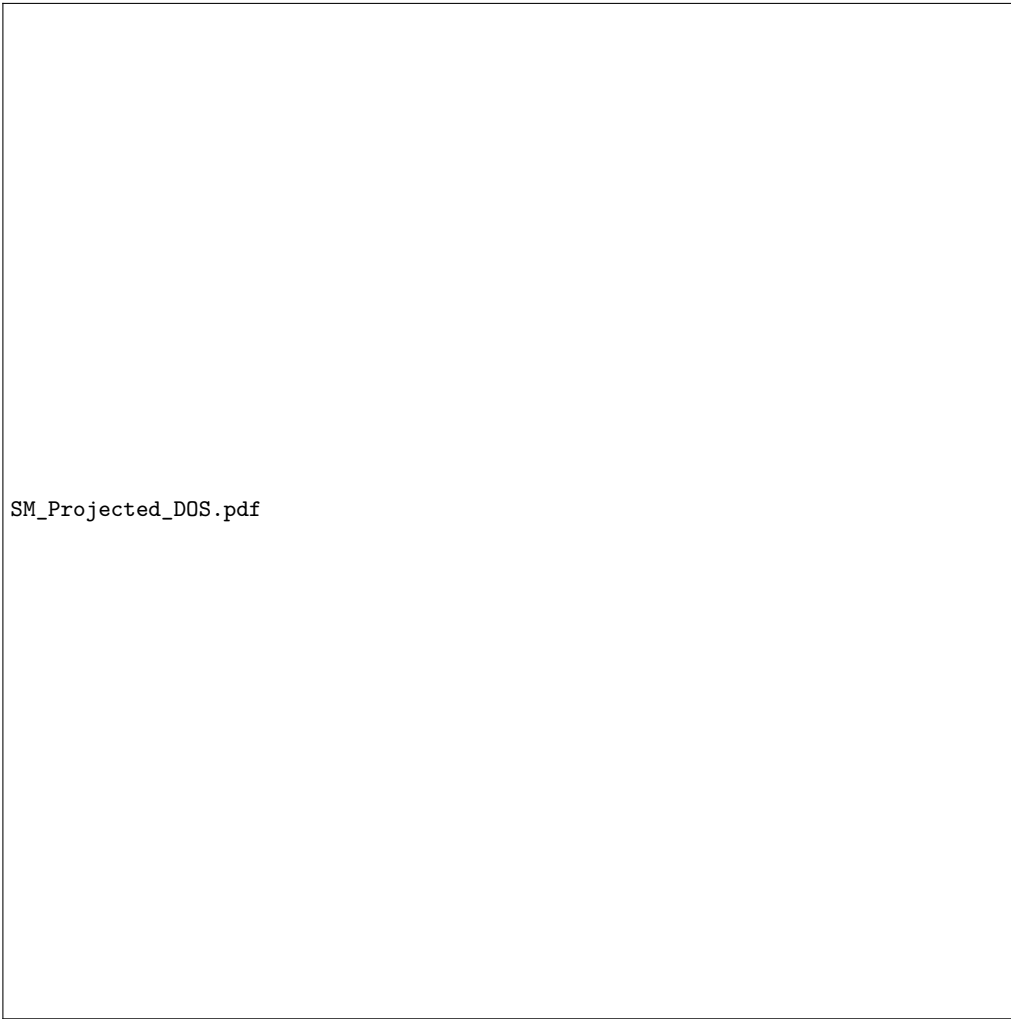


FIG. S2. Projected density of states of Mn and Se atoms near the Fermi level (set to 0 eV), for MnPSe₃ with magnetization orientation along +45° with respect to the z axis in the yz plane.

in Fig. 3 in the main text, MnPSe₃ shows considerable weight of t_{2g} orbitals (d_{xy} , d_{xz} , d_{yz}) near the Fermi level, thus validating our understanding of the importance of having a finite orbital angular momentum.

Another property that we argued is tied to the appearance of σ_{xy}^s is the rotation of the Fermi surface with respect to the principal axis. Fig. S3 displays the Fermi surface at an energy -110 meV with respect to the valence band maximum for magnetizations $\pm 45^\circ$. The Fermi surfaces have pockets around the Brillouin zone corners (K points) and round the Γ point. In the K points, the Fermi surface rotation between the two magnetization configurations is not very clear, but each Fermi surfaces clearly shows the breaking of both c_3 and m_y symmetries. On the other hand, the Fermi surfaces at the Γ point reveal opposite rotation in the xy plane. This result, together with Fig. S2 and Fig. 3 in the main text, demonstrate the correlation between orbital angular momentum, Fermi surface rotation and nonzero σ_{xy}^s in realistic materials.

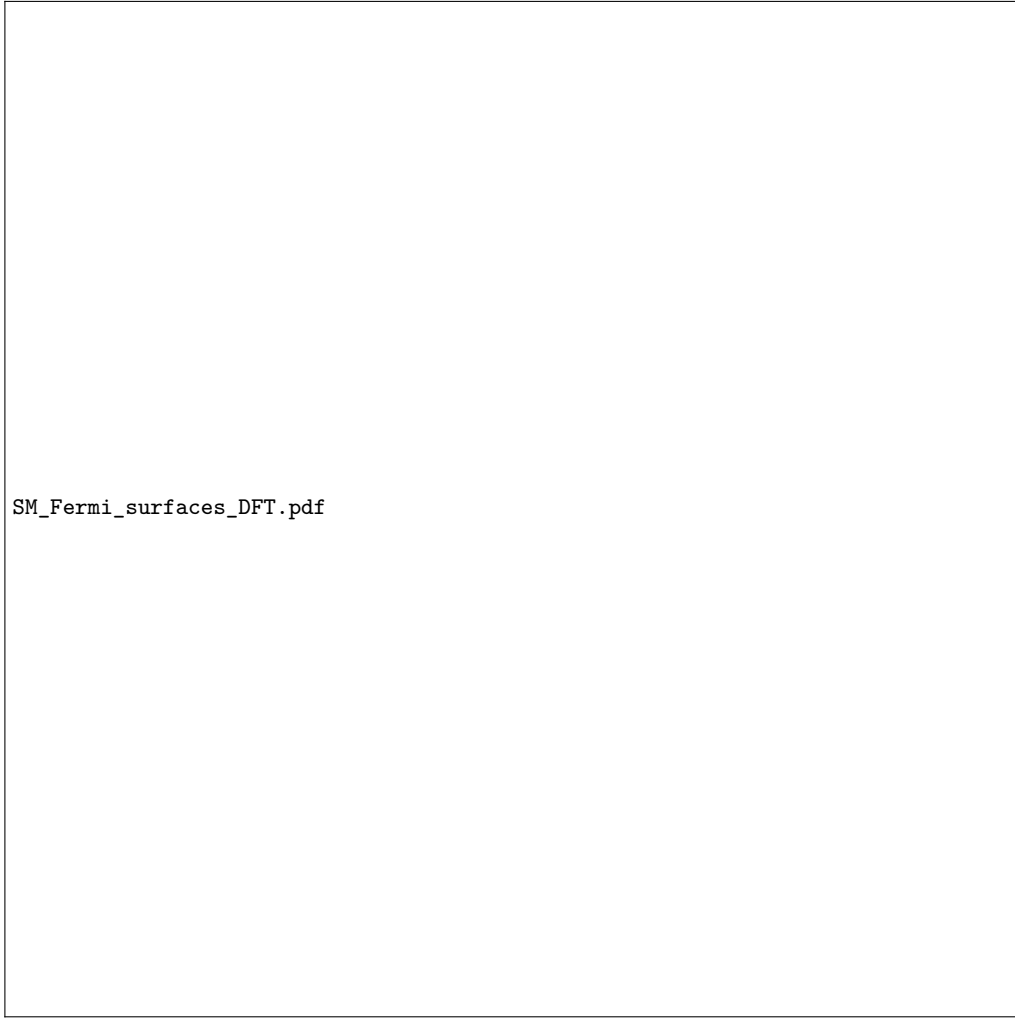


FIG. S3. Fermi surface at energy -110 meV with respect to the valence band maximum for MnPSe_3 with magnetization orientation along $+45^\circ$ (red) and -45° (blue). Left (right) panel show the Fermi surface in the whole Brillouin zone (around the Γ point).