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NONPARAMETRIC ESTIMATION OF
DYNAMIC HEDONIC PRICE MODELS AND
THE CONSTRUCTION OF RESIDENTIAL
HOUSING PRICE INDICES

By

Richard Meese
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Nonparametric Estimation of Dynamic Hedonic Price Models and the Construction of Residential Housing Price Indices

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I. INTRODUCTION

Accurate measurement of residential real estate price trends is an issue of great concern to the primary and secondary mortgage markets, producers of housing services, and consumers of owner occupied housing. Unfortunately there remain important shortcomings with the techniques commonly used to represent real estate price trends. The most widely reported measure is the National Association of Realtors quarterly publication of median sales prices of existing single family homes from fifty four metropolitan areas. The U.S. Bureau of the Census also publishes an index of new single-family housing prices.[1] A well known limitation of the median sales price index is that it does not standardize for the characteristics of housing sold in any given period. The Census index is also flawed because it reflects only new homes in the "speculative builders" category. This sample selection rule excludes approximately one third of total new home sales (Nourse (1982)), and ignores sales of existing homes. Strategies for dealing with the shortcomings of these real estate price indices are suggested by Palmquist (1979, 1980), Case and Shiller (1987, 1989), Shiller (1991), Case et al. (1991), and Case and Quigley (1991). Despite the recent interest in this area, a consensus has yet to emerge.

There are several technical difficulties in developing improved measures of real estate price indices. First, the changing weight problem affects price index construction

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[1]This is a "quality" adjusted price index based on a standard 1977 quality house. Hedonic estimates for the house price index are obtained by regressing actual price data on a vector of housing characteristics in each year.

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for all categories of goods and services. Second, residential real estate price indices, and price indices for most other consumer and producer durables, must also contend with the change in the quality of goods (repackaging of attributes) over time. Strategies used to address the quality shift problem include; 1) estimation of hedonic price indices (Griliches (1961, 1971)) that allow for time variation in attribute coefficients, and 2) paired sales approaches (Bailey, Muth and Nourse (1963)) that control for quality change by restricting the sample to repeat sales observations where houses do not experience quality shift.

A third difficulty is that there remains considerable debate on the construction of residential real estate price indices once quality change is accounted for. Last, theoretical issues concerning the appropriate functional form for the hedonic price equation, and the relationship between this functional specification and the price index formula employed must also be addressed.

The purpose of this paper is to compare several common parametric specifications for hedonic prices with a nonparametric technique called locally weighted regression. The estimated hedonic functions are then used to derive Fisher Ideal real housing price indices following recent developments in the economic theory of index numbers (Samuelson and Swamy (1974) and Diewert (1976, 1978, and 1981)). The data set for the analysis is a sample of nineteen years of residential housing prices and characteristics for sixteen municipalities in two urban counties; Alameda and San Francisco in Northern California. Analysis is conducted by municipality because land is regulated and

taxed at that level. We anticipate that different land-use policies and property tax rates will have differential effects on housing price trends.

The paper is organized as follows. Section II critiques quality adjustment strategies in the construction of house price indices. Section III reports on three parametric models that are used to estimated hedonic price indices, and Section IV introduces a nonparametric approach to the same problem. In Section V we then examine theoretical issues relevant to the construction of house price indices, and argue for the use of nonparametric regression techniques when estimating dynamic hedonics. Section VI concludes.

II. QUALITY ADJUSTMENT STRATEGIES

Hedonic price indices were first introduced by Griliches (1961) as a way of using market information to obtain price adjustments for quality change. The theory of hedonic price indices treats quality change in a given good as a repackaging of the bundle of its underlying attributes. Application of the technique to residential real estate is direct and a very extensive literature exists in which hedonic price equations are estimated using housing data. For the most part this research applies regression techniques to explain observed housing prices as a function of property attributes, locational attributes, and a time trend if appropriate. There are two methods of applying hedonic price estimates in the extant literature on quality adjusted real estate price indices. The first method pools data from adjacent time periods (or chains adjacent periods), assumes that the estimated attribute prices are time invariant, and includes time dummies (intercept shifters) for each period. A price index is then constructed

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directly from the coefficients on the time dependent intercepts; see Ferri (1977), Palmquist (1980), Bryan and Colwell (1982), and Mark and Goldberg (1984). The second method estimates implicit prices for attributes in a separate hedonic regression for each time period (Greenlees (1982), Mark and Goldberg (1984)). The estimated hedonic price equations are then used to construct standard Laspeyres[2] or Paasche[3] price indices.

Criticisms of the hedonic quality adjustment strategy focus primarily on the problems associated with the estimation of hedonic price equations in general; see Palmquist (1982), Sirmans (1982), and Case and Shiller (1987, 1989), among others. For a hedonically adjusted price index to globally reflect the "true" price index; 1) the correct set of property attributes must be included in the regression,[4] 2) the appropriate functional form must be selected for the estimating equation, and 3) the attributes must

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[2]The Laspeyres price index uses the initial period quantities as weights. It is defined as

$$P_{LA} = \Sigma p_i^t x_i^0 / \Sigma p_i^0 x_i^0$$

where p_i is the estimated implicit price for the ith attribute and the x_i is the initial period quantity of attribute i.

[3] The Paasche price index uses current period (T) quantities as weights. It is defined as

$$P_{PA} = \Sigma p_i^t x_i^T / \Sigma p_i^0 x_i^T$$

where p_i is the estimated implicit price for the ith attribute and x_i is the current period quantity of ith attribute.

[4] As noted by Epple (1987), the omitted variable bias is more severe in the context of hedonic regression models, because the equilibrium conditions in the markets for implicit attributes imply functional dependence between the characteristics of suppliers, demanders and products. It is therefore less likely that omitted or "unmeasured" attributes are uncorrelated with the included attributes of the model.

enter the "true" utility and production functions through the "bundle" rather than directly. Functional form selection techniques are criticized because the theoretical guidelines for selection criteria are weak, and implementation of Box-Cox or flexible functional form approaches requires considerable effort. The flexible functional forms may also limit the number of characteristics estimated because these specifications tend to proliferate parameters. Finally, the hedonic strategy is criticized because it requires large and costly data sets that include actual sales prices and property characteristics.

The paired sales strategy is a direct response to the criticisms of hedonic price estimation. The paired sales strategy for properties with unchanged attribute bundles does not require estimation of a hedonic price equation and, by self standardizing characteristics, it does not require a correctly selected and measured set of property attributes (Palmquist (1980), Mark and Goldberg (1984), Case (1986), Case and Shiller (1987, 1989), and Shiller (1991)). In the paired sales formulation the change in the selling price of the property is estimated as a function of the timing of, or the time interval between, the two sales. In its more general specification, attribute change can be accounted for in the paired sales approach (Palmquist (1980), Case and Quigley (1991), and Shiller (1991)). For the paired sales strategy to globally reflect the "true" price index; 1) coefficients of the "true" underlying hedonic price equation must remain stable between sales (or else the attribute function will not cancel out), 2) account must be made of the one quality attribute which cannot remain constant between sales, the age of the property, because this attribute is collinear with the year of sale variable and would bias estimates on the time parameters (Palmquist (1980)), 3) there is no sample

selection bias using only properties that sold multiply in a given metropolitan area (Case and Quigley (1991)), 4) there are no important differences in mean or median housing characteristics across municipalities, if aggregate indices are to be constructed, and as before, (5) the attributes of residential real estate enter the actual utility and production functions through the "bundle" rather than directly.

The undeniable beauty of the paired sales technique is that real estate price indices can be estimated with smaller data sets that are less costly to assemble. The problem with this suggestion is that the maintained hypotheses that attribute prices remain stable between sales, or that there is no sample selection bias cannot be tested. Violation of either assumption, of course, invalidates the technique. Thus, proper application of the paired sales technique should imply prior testing of the maintained hypotheses using hedonic price estimates. Another advantage of the hedonic price equation strategy is that it allows for the construction of both price and quantity indices, whereas the pure quality-constant paired sales approach allows only for price index construction. Both strategies require weighting schemes for the measurement of quality shift, as we discuss below.

III. HEDONIC ESTIMATES OF IMPLICIT HOUSING PRICES

For each municipality, suppose the price of a home in period t, $p(i,t)$, varies with its quantitative characteristics $x(i,t)$ and its qualitative binary characteristics $d(i,t)$ according to a hedonic equation:

$$p(i,t) = m(t) + \beta'G[x(i,t),d(i,t)] + u(i), \qquad (1)$$

where m(t) captures the changing mean in housing prices over time, ß denotes a (kx1) vector of parameters, G is a function of the attributes, and u(i) is an additive error term. The index i runs from one to the total sum of home sales in a given quarter t, where t runs from 1970 quarter 1 through 1988 quarter 4. The x(.) variables in our sample include number of bathrooms, number of bedrooms, finished square footage, total number of rooms, an index for quality, and dwelling age. Binary attributes d(.) include dummy variables for pools, fireplaces, assumability of mortgage, mortgage type (FHA or VA), and zoning type. Data sources are given in the Appendix.

The nonstationary mean in housing prices is attributed to the drift m(t) which is modeled as

$$m(t) = \alpha(t)dum(t) + e(t), \qquad (2)$$

where dum(t) is a dummy variable equal to one for each quarterly observation period t and zero otherwise, α(t) is the regression parameter (intercept) measuring the mean level of prices in quarter t once the average attributes ß'G[x(-,t),d(-,t)] have been accounted for, and e(t) is the time series error component assumed to be white noise. Combining (1) and (2) yields

$$p(i,t) = \beta'G[\ x(i,t),d(i,t)\ ] + b(t)dum(t) + (e(t)+u(i)) \qquad (3)$$

Since few of the data points in our sample are repeat sales (roughly 15% of total observations), it is unreasonable to consider a first difference specification or stochastic

trend to control for the nonstationary mean in housing prices[5]. Case and Shiller (1987, 1989) discard 93% of the observations from their Alameda county data set (which is also a subset of the data set used here) in order to control for the nonstationary mean in housing prices while avoiding an explicit control for quality change over time.[6] Our procedure allows for a time dependent intercept in housing prices, while simultaneously controlling for quality change in the stock of housing within each municipality and across municipalities.

As is well known, the hedonic price equation specified in Equation (3) is a reduced form equation reflecting both supply and demand effects. The functional form for the attributes function $G(.)$ cannot in general be specified on theoretical grounds. However, the choice of functional form does restrict the class of admissible functions for the underlying supply and demand equations. Flexible functional forms can be used to provide second order approximations for arbitrary twice differentiable functions, and to avoid the imposition of theoretically unwarranted restrictions (Halvorsen and Pollakowski (1981), Diewert (1976)). Possible parametric specifications for the hedonic include the generalized Box-Cox (Box and Cox (1964)), the Translog (Christensen Jorgenson, Lau (1973)), or a simple log-linear function. A semi-nonparametric specification with more rigorous second-order approximation properties is the Fourier

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[5]Case and Quigley (1991) provide a method alternative to the one employed here for calculating housing price indexes that controls for changing attributes over time.

[6] In order to assess any peculiarities of repeat sales in our sample we include an additional dummy variable equal to zero for homes that sold more that once during the sample, and one otherwise. This brings the total number of attributes to twelve; six attribute variables and six dummy variables.

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flexible functional form of Gallant (1982). Finally, nonparametric techniques which "curve fit" every data point, or set of data points, are also suitable for this application, as we discuss below.

Two parametric specifications are considered here; the translog and the log linear; both are restricted versions of the quadratic Box-Cox. The translog can be represented as

$$P(i,t) = b_0 + b'X(i,t) + 1/2X(i,t)'AX(i,t) + c(t)dum(t) + v(i,t) \tag{4}$$

where $P(i,t)$ is $\ln(p(i,t))$, the elements of b and A are parameters with symmetry requiring $\{a_{ij}\} = \{a_{ji}\}$, $X(i,t)$ is a stacked vector of the $\ln(x(i,t))$ and $d(i,t)$ attributes, the intercept dummies are as before, and the composite error term is $v(i,t)$. The log linear specification can be obtained by setting all $A = 0$. These parametric forms are common in the literature, see for example Halvorsen and Pollakowski (1981) or Palmquist (1980). In order to conduct inference, we need some auxiliary statistical assumptions on equation (4); time invariant parameters, constant variance of the composite error term, and lack of serial correlation in the time series component of the composite error. Additionally, all estimation is undertaken in real terms, because preliminary data analysis indicates that parameter constancy is implausible in nominal terms.[7]

We estimate two versions of the log-linear specification, neither of which is nested in the translog. Our most profligate specification of (3) is a regression of $P(i,t)$ on $X(i,t)$

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[7] For example, the implicit price of bathrooms should not be modeled as constant over the 19 year period. Instead, we deflate housing prices using the San Francisco-Oakland CPI for all items less shelter. The relative prices of the underlying attributes from the real price hedonic are much more likely to be stable over long subperiods of our sample.

that allows for different coefficients on the attributes every year.[8] Below we refer to this specification as model 1.

Next we consider model 2, a constrained version of model 1 with constant attribute parameters for three subperiods; 1970Q1 to 1977Q4, 1978Q1 to 1982Q4, and 1983Q3 to 1988Q4, where Q denotes quarter. The dates correspond to institutional breaks in the California real estate market; see Jaffee (1984). The Wellenkamp v. Bank of America decision occurs in 1978; it sets a legal precedent for voiding due-on-sale clauses. But after a 1982 Supreme Court decision and the Garn - St. Germain Act, due-on-sale clauses were again generally enforced, Jaffee (1984, p.9). The 1982 Garn - St. Germain Act also legalizes adjustable rate mortgages. Last, this sample subdivision conveniently coincides with the years (but not exact quarters) of the Federal Reserve experiment with monetary targeting.

A conventional F-statistic can be used to test the hypothesis that the three regime specification, model 2, can be used in place of the yearly model 1. In almost all cases we can reject the three regime model as a constrained version of model 1. Model selection criteria, defined below, do not usually corroborate the results of the classical F procedure.

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[8] Given variation in the attribute data, it is not possible to have separate coefficients for all dummy variables in every year; for example there are years when no homes with swimming pools were sold. Dummies for residential and multiple use zoning also fall in this category. Additionally, there are two variables which appear in only 10 subperiods. These are FHA/VA financing and assumable mortgages.

Our translog specification (4) is called model 3.[9] Because the log-linear specifications are not nested in the translog, we use the Schwarz (1978) model selection criterion to distinguish between models.[10] In contrast to classical hypothesis testing, order selection criteria can be interpreted as choosing the model specification (from the set of models considered) that best approximates the data. Model selection criteria do not rely on a fixed significance level as sample size increases, as is often the case when empirical researchers rely on classical methods. Last, the Schwarz criterion - like classical hypothesis testing - will asymptotically select the correct model if it is in the set of models being considered.

As shown in Table 1, the results of our specification search on the house price hedonic (3) indicate that no single parameterization is best for all municipalities in our sample. The F statistic generally indicates that model (1) should be employed instead of model (2), yet the Schwarz criterion never selects model (1). The Schwarz criterion selects the more parsimonious models (2) and (3) on roughly an equal basis. The selection of a single specification for all municipalities would greatly simplify the calculation of price indices, because the same program could be used to calculate the

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[9] This model includes linear (in logarithm) attribute variables, and where possible, quadratic and interaction terms for the attributes. Clearly, redundant terms (like squares of a binary variable) are discarded to avoid perfect collinearity.

[10] The Schwarz (1978) criteria chooses a model for which the sum of the logarithm of the (maximum likelihood estimate) of residual variance plus a penalty function for estimated parameters is a minimum. The penalty function is number of parameters times logarithm of sample size divided by sample size. The number of parameters in models (1-3) are respectively, 223, 106, and 152. See Geweke and Meese (1981) for details on the performance of order selection criteria in regression models.

implicit attribute prices. For these reasons we choose to employ a nonparametric estimator of the hedonic (3). It does not impose a specific functional from on (3) a priori, other than G(X(i,t)) be a smooth function. In addition, the same flexible algorithm can be applied to the estimation of implicit attribute prices and the construction of price indices for each municipality.

IV. LOCALLY WEIGHTED REGRESSION

The nonparametric technique used to estimate the hedonic equation (3) is locally weighted regression (LWR). The LWR procedure is described in Cleveland and Devlin (1988), and Cleveland, Devlin and Grosse (1988); they discuss both the asymptotic and the small sample properties of the estimator. LWR is a technique for estimating a regression surface in a moving average manner and a wide range of smooth functions can be approximated using the methodology. For example consider a version of the regression model (3);

$$P(i,t) - P(i,-) = G(X(i,t)) + v(i,t), \qquad i=1,...,I(t), \quad t=,...,T. \qquad (5)$$

where $P(i,-)$ is the quarterly mean of the logarithm of housing prices, and all other terms are defined earlier. In our application the object of interest is an estimate of G at the median values of the explanatory variables. In what follows we denote the vector of median attributes for quarter t by $X(m,t)$.

LWR uses a fraction n, $0 < n < 1$, of the total number of observations $\Gamma = T^* \Sigma (I(t))$ closest to $X(m,t)$, where proximity to $X(m,t)$ is assessed using the Euclidian distance between all points in the sample and $X(m,t)$. The distance metric is defined by

$$D[X(m,t),X(i,t)] \; = \; [\Sigma (X(m,t)-X(i,t))^2)]^{1/2} \qquad (6)$$

where the summation runs over the k elements of X. The regression surface at X(m,t) is

estimated by a weighted least squares (WLS) regression of p on X for the $n\Gamma$

observations nearest X(m,t).[11] The weights are given by

$$W \; = \; V[D(X(m,t),X(i,t))/D(x(m,t),X(n,\Gamma))], \qquad (7)$$

where $D(X(m,t),X(n\Gamma))$ is the distance from the median X to its $n\Gamma^{th}$ nearest neighbor.

In addition, Cleveland and Devlin (1988) suggest the "tricube" function be chosen for

V[.];

$$
\begin{aligned}
V[s] &= (1-s^3)^3, \quad \text{if} \;\; s < 1 \\
&= 0, \quad \text{otherwise}
\end{aligned}
\qquad (8)
$$

Other smooth V functions result in consistent estimation of G(X(m,t)), but Cleveland

and Devlin prefer the tricube on the basis of their experimental work. The tricube

function has the advantage that it has smooth contact at both 0 and 1. Typically, it is

functions with discontinuities at the 0 and 1 endpoints that perform poorly in finite

samples; see Cleveland Devlin (1988) section 10.4 for further discussion.

In conventional applications of LWR a weighted least squares regression of the

dependent variable on the set of independent variables is performed at every point in the

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[11]At a minimum, the X variables need to be standardized to have unit variance, because the distance measure is sensitive to the units of measure of X. In addition, we must transform the dependent variable to induce stationarity in the mean of this series. To do this we subtract the quarterly mean of the logarithm of housing price P(i,-) from each observation in the sample; P(i,t)-P(i,-) as in equation (5). This corresponds to the dummy variable procedure used earlier in our parametric estimation of the house price hedonic (3).

sample. Clearly, LWR is a highly machine intensive technique. In our application we are only interested in the curvature of the dynamic hedonic G at the median characteristics X(m,t), t=1,...76. However, the cost savings from running only 76 regressions is mitigated by our need to use a large fraction n of all observations when estimating G(.) each quarter. Because two of our binary attributes X are either zero or one for about 98% of the total observations Γ, we must use a value of n equal to one to avoid singularity of the X matrix.[12]

In practice (finite samples) n can be chosen by the user to balance the tradeoff between bias and sampling error. As n gets small the number of implicit parameters increases. As the number of implicit parameters increases, bias in the estimator of G(.) decreases, while sampling variability in the estimator of G(.) increases. Large sample theory requires that the ratio of implicit parameters to observations goes to zero as both numerator and denominator of the ratio get arbitrarily large.

As n approaches one, the number of parameters implicit in LWR is the same as a parametric OLS regression model where the coefficients are allowed to vary each quarter. The LWR estimator for n=1 will be different from an OLS estimator, because the LWR procedure weights those homes most like the median X(m,t) the most heavily when determining the implicit prices of the attributes in quarter t. Our choice of n is driven by data considerations. In order to experiment with a window size n in the range

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[12] For example, suppose the "window size" n were 0.96 for the Piedmont hedonic. If the distance metric for observation X(m,t) happens to select only those homes that are zoned residential (over 99% of the homes are zoned residential in Piedmont), then values of X(i,t) for this dummy variable would all be zero and the regression could not be run for quarter t.

of .3 to 1.0 (a range considered reasonable by Cleveland and Devlin) it is necessary to drop all of the dummy variables from the analysis; see below.

Next we present some descriptive statistics for the estimated implicit prices of (5) when X consists of 11 attributes.[13] Given space constraints we have chosen to report implicit prices for only one municipality, and a weighted average using all 16 municipalities. The implicit prices are in real terms; they are relative to the San Francisco, Oakland CPI net of shelter costs, 1970 = 1.0 The implicit prices are calculated as the derivative of the hedonic (transformed from logarithmic form) with respect to the attribute of interest.

In Table 2 we present both the mean and standard deviation of the implicit price for the 11 attributes in the LWR fit of equation (5) for Piedmont in Alameda County. We had no priors on the signs of both the age variable and the multiple sales dummy (they are positive and negative respectively for Piedmont.) In all other cases, the estimated prices accord with our intuition. Bathrooms, square footage, total rooms, house quality, mortgage assumability, swimming pools and fireplaces are positively priced. The FHA/VA dummy is negatively priced, as it is an indicator of less expensive homes (FHA and VA federal mortgages have ceilings well below the median price of the homes in our sample.) Last, the coefficient of variation for each implicit price is roughly one tenth, indicating moderate variation in real attribute prices over time.

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[13] We dropped number of bedrooms from the analysis, because of the collinearity between the variables for number of bathrooms, total rooms, number of bedrooms, and total square footage. While any subset of three of these attributes produces a fit with more stable coefficients than the model with all four, dropping the bedrooms variable produces a model with estimated signs that accord with our aforementioned priors.

In Table 3 we report the same statistics as in Table 2, for a sample weighted average across all 16 municipalities. For the entire sample, average implicit prices are much smaller than for the exclusive Piedmont area, and two of the binary attributes - the mortgage assumability and residential zoning dummies - have negative prices. As noted earlier, both these attributes experience little variation in our sample, which may help explain the lack of precision in their estimation. While informative, Table 3 runs counter to the spirit of our disaggregate analysis. The heterogeneity apparent from Table 2 and 3 only reinforces our approach of considering the 16 municipalities separately.

Additional evidence on the LWR fit can be gleaned from the Table of regression diagnostics. In Table 3 we report Garcia-Bera tests for normality of LWR regression errors, a Lagrange multiplier test for serial correlation in lags 1 through 4 of the regression error, an Engle test for ARCH in the first four lagged squared regression disturbances, and a White test for heteroskedasticity in the error terms using only the squares of the contemporaneous values of the first five attribute variables.[14] All diagnostics are performed on the set of 76 time series errors associated with the median attributes in each quarter, X(m,t).

The normality test of zero skewness and no excess kurtosis (column one) indicates the error distribution is symmetric and bell shaped (with the exception of Hayward where a single observation gives rise to an asymmetric error distribution), so a window size of n equal to one does not seem to induce bias in the estimate of G(.). The

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[14] See Godfrey (1988) for a comprehensive discussion of error diagnostic tests.

evidence of serial correlation and ARCH (columns two and three respectively) suggests some additional dynamics could be exploited in the estimation of roughly two-thirds of the municipalities. Contemporaneous heteroskedasticity (column four) is also evident in the majority of municipalities. Since the $X(i,t)$ are exogenous and the error distribution is symmetric, we still have consistent estimates of the implicit attribute prices and corresponding price indices. However, the rejections in columns two through four suggest more efficient estimates of the implicit prices and price indices may be available. In limited experiments with Piedmont (the municipality with the smallest number of observations), reducing window size n (by dropping dummy variables from the analysis) mitigates some of the heteroskedasticity. It is also possible that a different weighting function V[-] may improve the fit.

V. THE CONSTRUCTION OF PRICE INDICES BY MUNICIPALITY

In the last twenty years there has been a resurgence of interest in the construction of index numbers. This revitalization largely reflects the discovery by Diewert (1976) that index number formulae can be derived directly from production or utility functions. The insight is important because it implies that one can specify a utility or production function with desirable theoretical characteristics, or expenditure or cost functions that are dual to these functions, and then derive the appropriate index number formulae. Despite this innovation many researchers still rely on ad hoc formulae for price index construction, or ignore the index number problem entirely.

Diewert (1976) calls a price index "exact" when it can be derived from the underlying utility or production function. He also strongly argues for limiting the

admissible class of utility or production functions to those that can provide a second order approximation to any arbitrary utility or production function.[15] These functions are termed flexible and the price indices that are derived from them are termed superlative.

The Fisher Ideal index has been shown by Diewert (1976) to be both exact and superlative if the flexible approximating function is restricted to be linearly homogenous of degree one, a standard regularity condition for neoclassical cost or expenditure functions.[16] Diewert thus recommends the use of the Fisher Ideal price index for approximating functions of order 2 such as the translog because (1) their functional form is simple, (2) the index is a function of the "sufficient statistics" from revealed preference theory, and (3) the pairing of the Fisher Ideal price and quantity indices can be consistent with both linear cost or expenditure functions (infinite substitutability between goods to be aggregated) and Leontief functions (zero substitutability between goods to be aggregated); see Diewert (1976, pp. 138-139).

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[15] Rosen (1974) shows that as long as there is increasing marginal cost of attributes for producer/sellers and a constraint on unbundling the attribute package, the hedonic function is likely be nonlinear. Thus, the use of approximation functions that are at least second order flexible would be appropriate for hedonic price equations (Halvorsen and Pollakowski (1981)).

[16] Rosen argues that if the demander/buyers can be assumed to be identical and only the sellers differ, the estimated hedonic price equation is the compensated demand function. Similarly if producer/sellers are identical and face identical cost conditions, the estimated hedonic price equation is the compensated supply function; see Rosen (1974, p 50-51). In these two special cases the linear homogeneity condition would hold for the hedonic price equation. Even in the case where linear homogeneity cannot necessarily be assumed, use of a Fisher Ideal price index is a good strategy to employ in conjunction with a second order approximating function for the hedonic.

The Fisher Ideal price index[17] is defined as the geometric average of the Laspeyres and Paasches price indices

$$P_{ID} = ((\ \Sigma p_i^t x_i^0 / \Sigma p_i^0 x_i^0\ )(\ \Sigma p_i^t x_i^T / \Sigma p_i^0 x_i^T))^{1/2} \tag{9}$$

Additionally, the Fisher Ideal satisfies Fisher's (1922) factor reversal and time reversal tests. The former test requires that the product of the price index times the quantity index should yield the expenditure ratio between the two periods, and the latter test requires that price level comparisons be invariant to the base year chosen.

In order to construct a Fisher Ideal price index using the parametric hedonics estimated above, we must evaluate the regression function at a particular value of the X(i,t). We choose the median attributes in quarter t as the appropriate weights in the price index, since they are less sensitive than the sample mean to extreme characteristics.[18] When the hedonic model consists entirely of attribute parameters with no time variation, such as the translog specification (4), the Fisher Ideal price index collapses to the ratio of the parameter estimates on the quarterly time dummies. Likewise, the three regime model (2) leads to Fisher Ideal price index that collapses to ratios of the quarterly time dummy variables within each of the three regimes. Only

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[17] Strictly, prices in the Fisher Ideal price index are rental rates per unit. In our application, we interpret the implicit prices per unit of attribute as capitalized rents.

[18] In quarters associated with recession, the number of house sale observations upon which the median is based can be quite small for some municipalities. This small sample problem applies with equal force to the use of mean attribute values as price index weights. Below, we further examine the effect of a small number of observations per quarter on the construction of price indices.

model (1), which allows attribute parameter variation at an annual (but not quarterly) frequency, is suited to the construction of Fisher Ideal indices.

A comparison between price indices constructed from the three parametric models of Section III reveals similarity in overall trend, but considerable difference in the autocorrelation structure of each price index series. In other words a reasonable estimate of longer term price change is obtainable from any model with a flexible trend parameterization (quarterly dummies in our case), but subsequent data analysis relating price indices to supply and demand fundamentals say, will depend more heavily on the form of the estimated hedonic via the induced autocorrelation in the price index series. Since we are also interested in a model of house price determination at the municipality level (see our companion paper; Meese and Wallace (1991)), we feel it is important to calculate Fisher Ideal price indices using hedonics that are as flexibly estimated as possible.

The quarterly implicit price estimates generated by LWR lead to well defined Fisher Ideal price index for the entire sample period of 76 quarters, while not imposing constancy of functional form across the 16 different municipalities. In addition, the flexibility of LWR when estimating hedonics is likely to result in a generated price index series with more reliable short run dynamics than parametric approaches such as model (1-3). Last, the LWR approach has the further advantage that the same software can be used to construct price indices for each municipality.

As shown in Figure 1, the Fisher ideal price indices generated by LWR regression have a number of distinguishing features. First, there is a decided upward trend in the

cost of housing relative to all other goods. From 1970 through 1987, the real cost of housing increased by a factor of roughly 250%. For some of the smaller municipalities in trendy homogeneous neighborhoods, the relative price increase is much higher. For example house prices in Piedmont increased sixfold and those in Albany fourfold over the 19 year period.

Second, the trend in real housing prices appears stochastic. The trend is relatively flat from 1970 through 1973 or 1974, as house prices just kept pace with the general price level. An upward trend in real housing prices appears in the mid 1970's after the first oil shock. It is interrupted by the great recession of 1981-82. The level of real housing prices resumes its upward trend around 1983. An application of standard unit root tests confirms our ocular analysis of stochastic trend; all series appear to have a stationary mean after first differencing.[19]

The periods of change in the trend of real housing prices can be associated with both macroeconomic phenomena (as noted above) and with institutional change in the California real estate market. Both proposition 13, which limits property taxes (1978), and the advent of adjustable rate mortgages (ARMs) in 1982 have affected Bay Area housing prices. A decrease in personal property tax rates reduces the capitalization rate, so that higher house prices can be associated with the same stream of rental services,

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[19] See Stock and Watson (1988) for a discussion of stochastic trends, and related references. The results of unit root tests applied to our Fisher Ideal price indices are reported in a companion paper; see Meese and Wallace (1991). We also take up the issue of the fundamental determinants of both the trend and seasonality in Bay Area real housing prices over the 1970 to 1988 period.

and the teaser rates associated with ARMs made it easier for individuals to qualify for homes in the high interest rate environment of the 1980's.

A third feature of our LWR price index plots is the seasonality in real housing prices. Housing prices are highest on average in the second quarter, followed by the third, fourth and first quarters respectively. However, the autocorrelation of the first difference of the house price indices indicates only weak seasonality, as the autocorrelations at lag 4 span a range of roughly 1.5 to 3.0 times their standard deviation.

Another candidate explanation for the pronounced variation in house prices around trend is a small sample problem associated with the construction of house price indices at a quarterly frequency. During recessions the number of observations per quarter can get as low as 12 for the smaller municipalities. The median characteristics of such a small sample of homes can be atypical of sales in nearby time periods. This small sample problem is manifest in the nonseasonal variation in house prices around trend, especially during the 1980-1982 period.[20]

The last test carried out in our evaluation of Fisher Ideal Price indices using LWR involved the computation of confidence bounds for the computed price indices. Since the price index is a highly nonlinear function of the implicit price estimates, confidence intervals are best obtained by resampling techniques. The regression diagnostics for Piedmont indicate that the iid disturbance specification is reasonable.

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[20] One obvious solution to this problem is to take median attributes from a moving average of observations centered at the quarter of interest. A drawback is that data points at the beginning and end of the sample are treated differently than those in the middle.

Since bootstrapping LWR is incredibly machine intensive it is also convenient to use the municipality with the smallest sample size (also Piedmont). A 95% bootstrap confidence interval for the Fisher ideal index is calculated every quarter, and is graphed in Figure 2. The bootstrap simulations indicate considerable precision in the estimation of the Fisher ideal index.[21]

VI. CONCLUDING REMARKS

In this paper we advocate the use of nonparametric regression techniques to construct housing price indices. We address the theoretical issues relevant to the construction of a Fisher ideal price index, and we tackle the practical problems of estimating dynamic hedonic models for housing prices from an unbalanced panel of individual home sales data from 16 different municipalities in Northern California over the period 1970-1988. The analysis includes an examination of the variation in the implicit price of house attributes over time, diagnostic checks of the adequacy of the fitted hedonics, and simulated confidence intervals for the Fisher ideal price index. In a companion paper, we use the estimated price indices to study the relation between housing prices and their fundamental determinants.

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[21] The bootstrap experiments are conducted by holding the regressors fixed in each quarter, and then drawing with replacement from the set of 2,258 errors for the Piedmont hedonic at time t. 100 bootstrap samples are so generated and the confidence interval obtained is the 5th and 95th ordered price index generated for each quarter.

TABLE 1

HEDONIC MODELS FOR HOUSING ATTRIBUTES

CITY	N	K=223 ESS(1)	K=106 ESS(2)	K=144 ESS(3)	S(1)	S(2)	S(3)	F
ALAMEDA	5,380	162.45	169.70	166.43	-3.144	-3.287*	-3.246	1.97
ALBANY	2,669	78.89	83.97	80.58	-2.862	-3.152*	-3.075	1.21
BERKELEY	5,688	353.30	367.58	351.72	-2.439	-2.578*	-2.564	1.89
CASTRO VALLEY	5,148	162.87	169.35	159.81	-3.083	-3.238*	-3.233	1.67
DUBLIN	2,927	30.68	34.93	29.52	-3.950	-4.139	-4.204*	3.20
FREMONT	20,252	504.11	522.18	467.10	-3.584	-3.606	-3.699*	6.14
HAYWARD	7,872	199.45	206.78	185.05	-3.421	-3.519	-3.732*	2.40
LIVERMORE	7,968	147.11	157.70	131.80	-3.741	-3.803	-3.939*	4.77
NEWARK	4,905	72.02	75.86	68.57	-3.835	-3.985	-4.241*	2.13
OAKLAND	23,947	895.24	1910.95	1864.35	-2.443	-2.484	-2.493*	1.68
PIEDMONT	2,258	76.10	85.02	75.07	-2.628	-2.917*	-2.911	2.04
PLEASANTON	8,164	145.63	156.59	149.55	-3.780	-3.837	-3.841*	5.11
SAN FRANCISCO	36,197	2541.67	2414.25	2229.12	-2.591	-2.677	-2.745*	NA*

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TABLE 1
HEDONIC MODELS FOR HOUSING ATTRIBUTES

CITY	N	K=223 ESS(1)	K=106 ESS(2)	K=144 ESS(3)	S(1)	S(2)	S(3)	F
SAN LEANDRO	6,791	183.72	190.88	179.09	-3.320	-3.434	-3.448*	2.19
SAN LORENZO	3,380	76.075	83.67	77.27	-3.258	-3.444*	-3.132	2.69
UNION CITY	4,398	79.99	86.18	77.62	-3.582	-3.730	-3.762*	2.76

Table 1 notes: S(i), Schwarz criterion for model i=1,2,3. An asterisk denotes the model for which the Schwarz criterion is at a minimum. The F statistic has 117 numerator degrees of freedom in all cases. Model 1 has 223 fitted parameters, Model 2 has 106 parameters, and Model 3 has 144 parameters. The 5% (1%) critical values for an F(100,1000) = 1.26 (1.3). The F statistic is not available for San Francisco because Model 1 is not nested in Model 2 due to deleted dummy variables in Model 1.

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TABLE 2

AVERAGES AND STANDARD DEVIATIONS FOR LWR ESTIMATED ATTRIBUTE PRICES IN PIEDMONT, (1970Q1 - 1988Q4)[*]

Attribute	Average	Std. Dev.
Number of bathrooms	9398.59	856.24
Sq. ft. of floor space	29.08	2.66
Number of total rooms	400.90	30.69
Index of house condition	10530.55	964.44
Age of dwelling (years)	117.94	9.54
FHA/VA dummy variable	-15861.28	1401.57
Multiple sales dummy variable	-6934.16	616.87
Mortgage assumability dummy	2492.72	220.64
Residential zoning dummy	1069.01	96.75
Swimming pool dummy	7106.48	648.64
Fireplace dummy	1277.58	106.18

[*] Statistics based on 75 quarters for Piedmont, as 1988Q4 has one observation, 1970 dollars.

TABLE 3

WEIGHTED AVERAGES AND STANDARD DEVIATIONS FOR LWR ESTIMATED ATTRIBUTE PRICES IN SAN FRANCISCO/BAY AREA, (1970Q1 - 1988Q4)

Attribute	Average	Std. Dev.
Number of bathrooms	1974.13	147.78
Sq. ft. of floor space	11.38	.85
Number of total rooms	195.99	42.05
Index of house condition	2347.96	175.23
Age of dwelling (years)	29.39	4.23
FHA/VA dummy variable	-2899.99	130.52
Multiple sales dummy variable	506.18	30.05
Mortgage assumability dummy	-346.57	75.80
Residential zoning dummy	-416.33	32.89
Swimming pool dummy	1717.48	173.27
Fireplace dummy	469.27	49.27

*Statistics based on LWR estimates for 76 (in some cases 75) quarters for 16 municipalities, 1970 dollars.

TABLE 4

CHI-SQUARE REGRESSION DIAGNOSTICS FOR THE LWR FIT OF EQUATION (3)

Municipality	Normality Df = 2	Serial Cor. Df = 4	ARCH Df = 4	White Df = 5
		Diagnostic test**		
Alameda	2.24	35.26*	21.32*	40.19*
Albany	2.80	8.57	3.11	17.80*
Berkeley	2.79	12.23*	2.35	21.31*
Castro Valley	2.42	25.33*	16.82*	7.29
Dublin	3.90	8.68	7.79	30.65*
Hayward	11.51*	1.03	4.48	28.09*
Fremont	2.38	30.28*	20.01*	26.90*
Livermore	3.63	2.39	2.38	19.56*
Newark	3.00	10.88*	4.38	9.73
Oakland	5.73	11.07*	1.46	26.16*
Piedmont	3.65	7.56	16.20*	9.77
Pleasanton	4.34	9.20	9.96*	23.56*
San Francisco	2.69	9.08	.94	13.96*
San Leandro	3.47	14.41*	14.49*	44.55*
San Lorenzo	4.14	2.35	3.56	15.57*
Union City	2.78	14.71*	7.49	10.09

** 5% and (1%) critical values for the appropriate Chi-square variables:

Df =	2	4	5
	5.99 (6.63)	9.49 (13.28)	11.07 (15.09)

* Denotes significance at the 5% level.

FIGURE 1

CITY OF ALAMEDA
Real Price Index
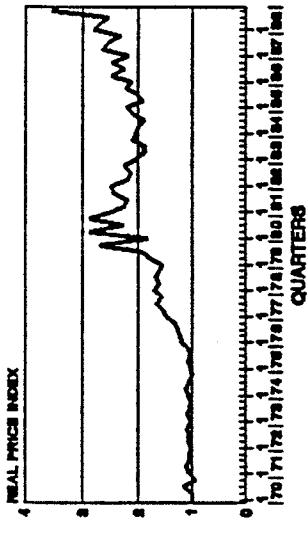
CITY OF ALBANY
Real Price Index
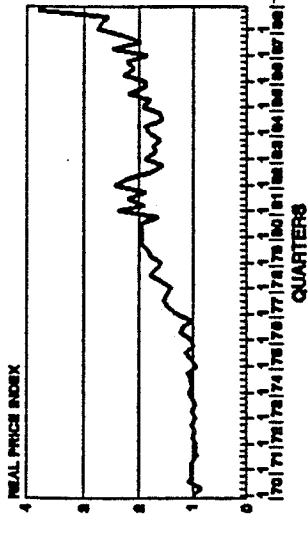
CITY OF BERKELEY
Real Price Index
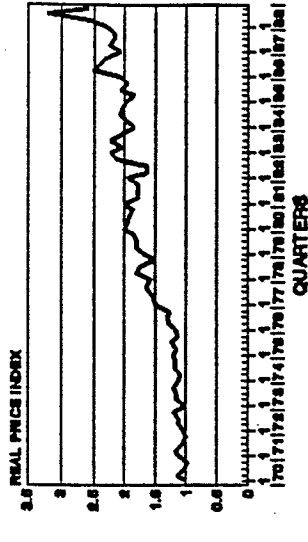
CITY OF CASTRO VALLEY
Real Price Index
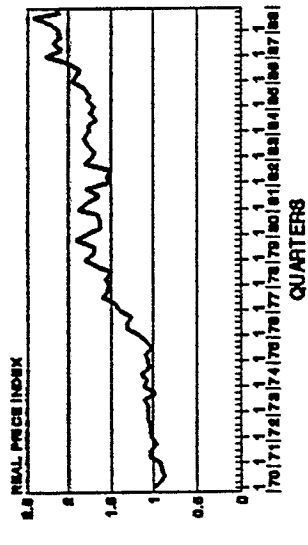
CITY OF DUBLIN
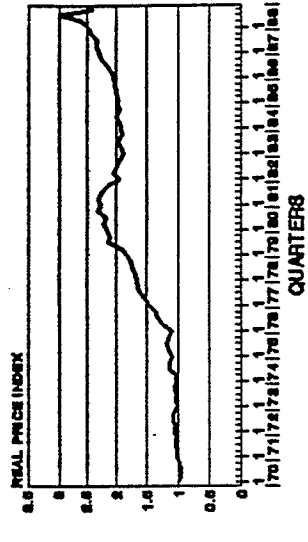Real Price Index
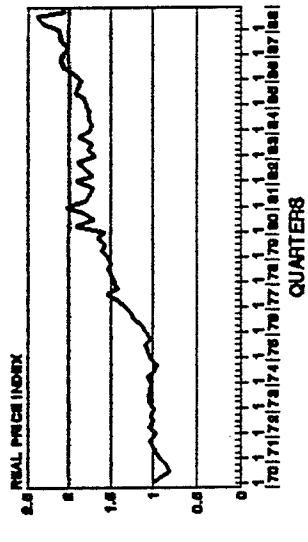
CITY OF FREMONT
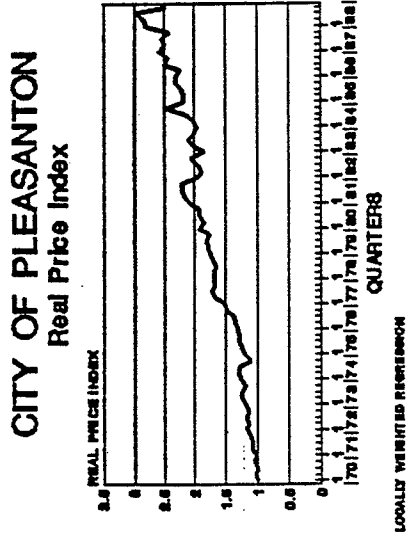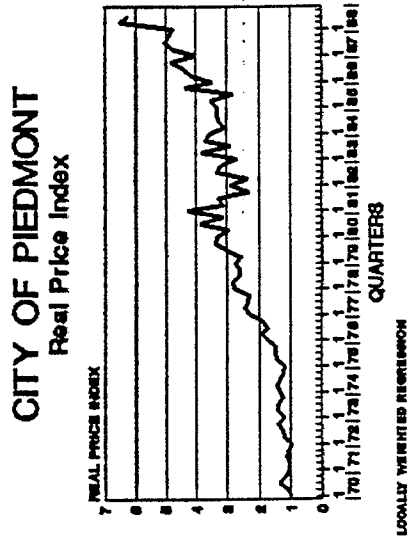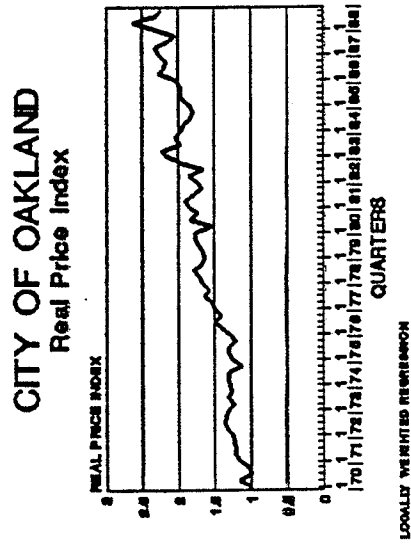Real Price Index

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FIGURE 1

CITY OF HAYWARD
Real Price Index

CITY OF LIVERMORE
Real Price Index

CITY OF NEWARK
Real Price Index

CITY OF OAKLAND
Real Price Index

CITY OF PIEDMONT
Real Price Index

CITY OF PLEASANTON
Real Price Index

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FIGURE 1

CITY OF SAN FRANCISCO
Real Price Index

CITY OF SAN LEANDRO
Real Price Index

CITY OF SAN LORENZO
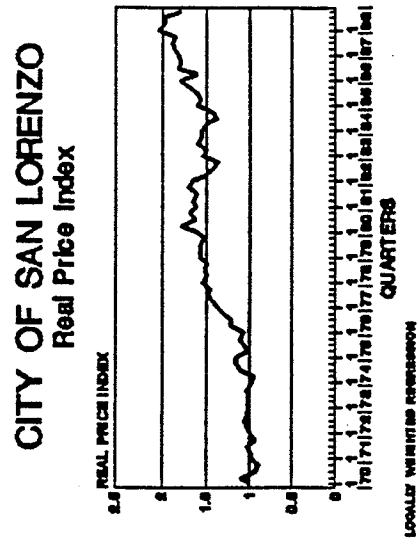Real Price Index

CITY OF UNION CITY
Real Price Index
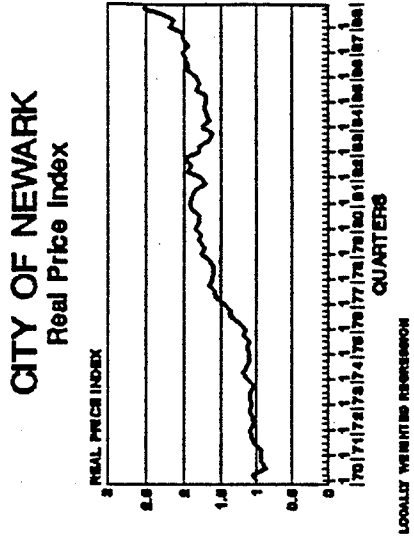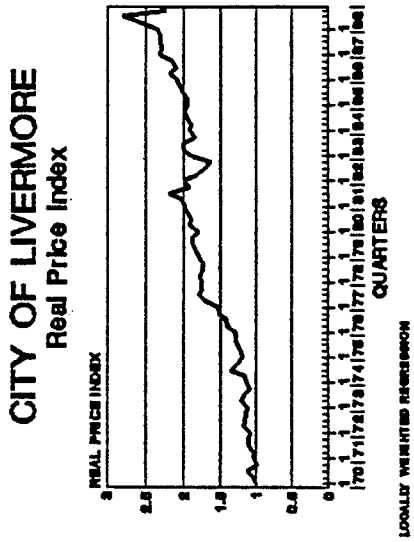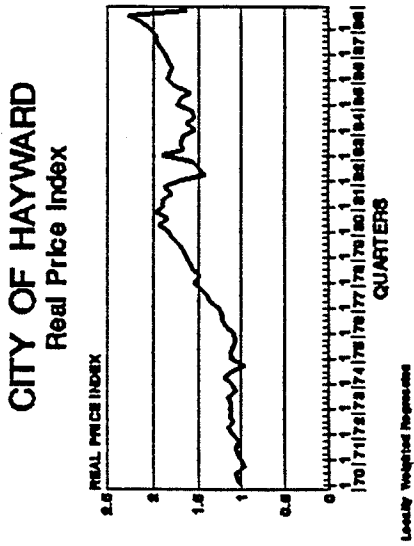
31
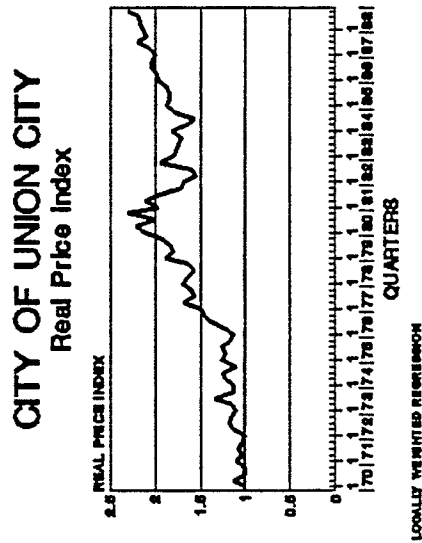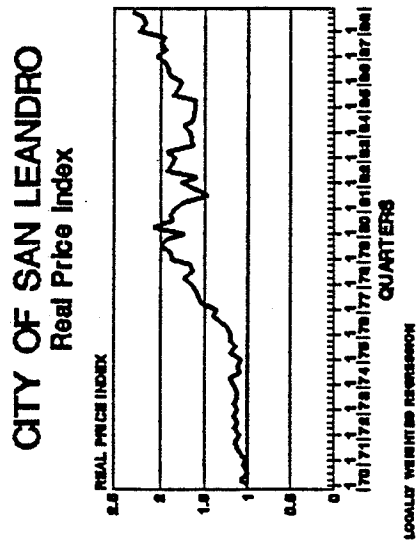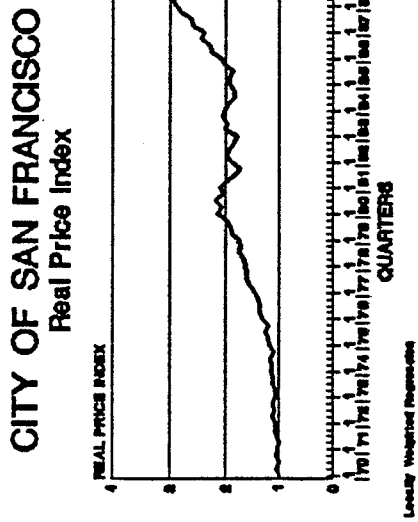
FIGURE 2

CITY OF PIEDMONT
CONFIDENCE INTERVALS FOR PRICE INDICES

Price Index

EXPANSION 1970-74

QUARTERS
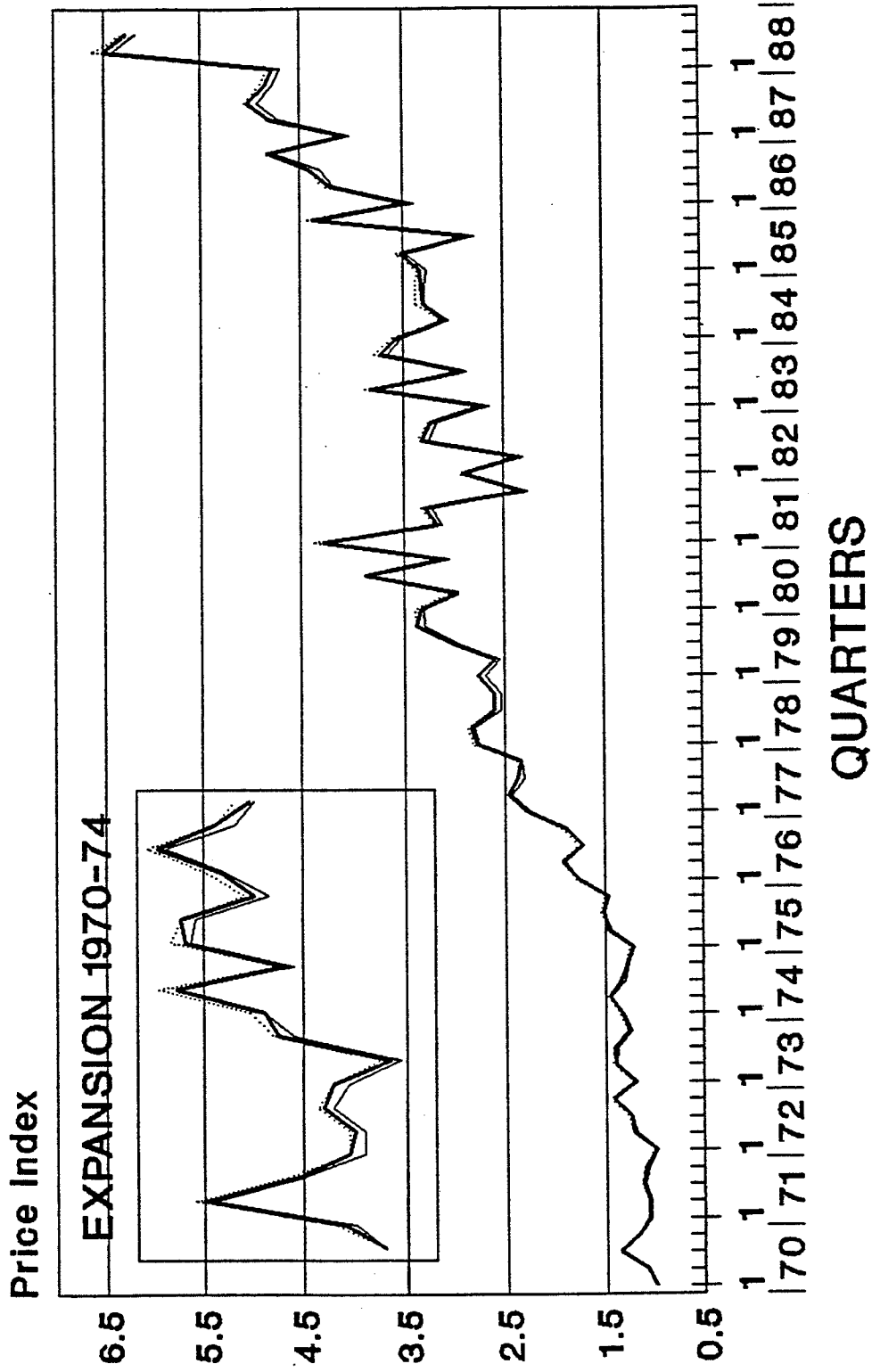
LOCALLY WEIGHTED REGRESSION

Data Appendix

The hedonic price index (3) is estimated using residential housing sale prices and characteristics for Alameda County (136,782 observations) and San Francisco County (44,686 observations) from 1970 through 1988. We obtained the data from the California Market Data Cooperative, which compiles data for the California Association of Realtors. Our data set reflects a very complete panel of cross sections of actual home sales prices and characteristics, and is thought to comprise approximately ninety percent of all recorded arms-length sales over the period. We identified all multiple sales over the sample period and obtained geographic data such as census tract, zoning and property tax juridictions for each parcel. We also included a distance to work measure, however, in some communities , such as Alameda (a small island in the San Francisco Bay) and Albany, there was insufficient variance in this variable.

Based on the distribution of home sales over the 19 year period, we decided that a quarterly model was the appropriate frequeny at which to estimate our hedonic model. We used Chow-Lin (1971) procedures to extrapolate the San Francisco/Oakland CPI less shelter index for missing quarters from 1970Q1 through 1976Q1. The related series used for the extrapolation was the U.S. CPI less shelter series for the 76 quarters.

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