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A Unified Model for Small-Strain Shear Modulus of Variably Saturated Soil

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Abstract: The small-strain shear modulus G_0 is a property fundamental to the deformation response of soils to dynamic loading. It depends on soil mineralogy, particle characteristics, void ratio, effective stress, and the degree of saturation. Some theories have been established to predict G_0 of soils under either saturated or unsaturated conditions, but not both. Accordingly, a new unified model was developed for G_0 of different soils under variably saturated conditions. The model is based upon two conceptual mechanisms: material hardening/softening, which is considered using the degree of saturation as a variable, and inter-particle contact forces carried by the soil skeleton, which is considered using the effective stress as a variable. By utilizing suction stress theory, it is demonstrated that G_0 can be correlated to the parameters of the soil-water retention curve. Results from the literature for 22 soils along with new results for 7 soils were used to validate the model under variably saturated conditions and a range of total stresses.

Keywords: Small-strain shear modulus, unsaturated soils, bender element, soil-water retention, suction stress.

Introduction

The shear modulus of soils at strain levels less than 0.001% is referred to as the initial, maximum, or small-strain shear modulus, and is typically denoted as G_0 or $G_{\max}$. It is a fundamental parameter of soils in geotechnical problems such as earthquake ground response analysis, static and dynamic soil-structure interactions, and liquefaction potential evaluation (Clayton 2011; Likitlersuang et al. 2013; Yang and Gu 2013). The small-strain shear modulus, G_0 represents the stiffness of the soil skeleton under the conditions where slippage at particle contacts is negligible and the soil fabric is fixed. In this case, the soil exhibits an elastic response where the deformation is governed by contact behavior and is reversible without hysteresis (Cho and Santamarina 2001; Santamarina et al. 2001).

A large number of investigations have been conducted on dry or saturated soils examining the effects of void ratio, over-consolidation ratio (OCR), stress-strain history, and strain rate on G_0 (Hardin and Richart 1963; Viggiani and Atkinson 1995; Rampello et al. 1997; Sorensen et al. 2010). Several studies have also been conducted to characterize the G_0 of unsaturated soils considering the effect of particle size, compaction energy, degree of saturation, matric suction, and hysteresis (Cho and Santamarina 2001; Shibuya and Mitachi 2005; Santagata et al. 2005; Sawangsuriya et al. 2009; Ng et al. 2009; Sorensen et al. 2010; Khosravi and McCartney 2012; Yang and Gu 2013; Heitor et al. 2013; Oh et al. 2014). However, there is not a universal model that can generalize the small-strain shear modulus for soils with changes in effective stress under both saturated and unsaturated conditions.

The small-strain shear modulus can be measured by using various types of tests, both in the field and in the laboratory (Kurtulus and Stokoe 2008; Rampello et al. 1997; Viggiani and

Atkinson 1995). The bender element method, among all techniques, is the most versatile and simple laboratory test to determine G_0 for all soils (Blewett et al. 2000; Lee and Santamarina 2005; Leong et al. 2005; Clayton 2011). Based upon current testing procedures, however, the measurement of G_0 for unsaturated soils is time-consuming and cumbersome to have the soil samples reach equilibrium under the applied net normal stress and matric suction. Hence, the experimental investigations of G_0 for unsaturated soils in the literature cannot cover the complete soil spectrum. In this case, empirical equations available for estimating G_0 are limited in that they cannot be extended to other soil types without additional experimental testing. Lu and Kaya (2013) developed a simple and rapid approach, namely the Drying Cake (DC) method, to dry soil specimens and characterize their mechanical and hydraulic properties over a wider range of degree of saturation than any other method. In this study, the DC test was modified to incorporate shear-wave measurements using the bender element so that the evolution of G_0 with changes in degree of saturation following the primary drainage path can be assessed for different soil types. The experimental results from these tests along with experimental results from the literature were also used to verify a newly proposed unified model of small-strain shear modulus for all types of soils under saturated and unsaturated conditions.

Suction-Stress-Based Unified Conceptual Model of G_0

Micromechanical analyses suggest that the macro-scale small-strain stiffness is stress dependent and can be fitted well by the general power equation of the following form (Hardin and Richart 1963):

$$G_0 = A \cdot F(e) (\sigma')^{\gamma_1} \quad (1)$$

where A and γ_1 are fitting parameters, $F(e)$ is an empirical function of void ratio e , and σ' is the effective stress. This equation applies for dry and fully saturated soils. To extend this principle to unsaturated soils, several investigators used *Bishop's* (1959) effective stress concept, and introduced the effect of net normal stress and matric suction (Mancuso et al. 2002; Mendoza et al. 2005; Ng et al. 2009; Sawangsuriya et al. 2009; Khosravi and McCartney 2012; Heitor et al. 2013; Oh et al. 2014) to study the dependence of G_0 on saturation and matric suction for limited types of soil. For instance, Sawangsuriya et al. (2009) proposed a G_0 model for unsaturated soils in the following form:

$$G_0 = A \cdot F(e) \left[(\sigma - u_a) + S^\kappa (u_a - u_w) \right]^{\gamma_2} \quad (2)$$

where S is the degree of saturation, u_a and u_w are the pore air and water pressures, respectively, and κ and γ_2 are empirical parameters. To avoid the difficulty of determination of *Bishop's* effective stress parameter χ , the authors adopted the experimental relationship $\chi = S^\kappa$ that was originally proposed by Vanapalli and Fredlund (2000) in order to evaluate the effective stress in unsaturated soils (i.e., the term in brackets). However, Eq. 2 only works well for a limited range of net confining stress values. Another example is the G_0 model for unsaturated soils proposed by Oh et al. (2014), which considers the non-linear relationship between G_0 and matric suction:

$$G_0^{\text{unsat}} = G_0^{\text{sat}} \cdot \left[1 + \zeta (u_a - u_w) S^\xi \right] \quad (3)$$

where G_0^{sat} and G_0^{unsat} represent the small-strain shear modulus under saturated and unsaturated conditions, respectively, and ζ and ξ are fitting parameters. This model applies only to non-plastic sandy soils under a certain range of net confining stress. Khosravi and McCartney (2012) proposed a model for G_0 to consider the effects of hydraulic hysteresis involving the concept of material hardening proposed by Wheeler et al. (2003). However, this model requires several

additional parameters beyond those in the models mentioned above, so it was not used in this study for comparisons.

Despite the fact that matric suction is inter-correlated with the net normal stress in describing the state of stress in unsaturated soils, matric suction is not a stress state variable as it does not directly generate the inter-particle stress acting on particle contacts (Lu 2008). External confining and matric suction are intrinsically coupled when both are applied to unsaturated soils, which means that it is necessary to up-scale these two variables acting on the solid particles and air-water interfaces into stress state variables acting on the faces of the air-water-solid representative elementary volume (Lu 2008). As the models in Eqs. 2 and 3 do not consider this up-scaling, they cannot uncouple the effect of matric suction (or degree of saturation) from that of the external confining stress for unsaturated soils. Lu and Likos (2006) and Lu et al. (2010) proposed the concept of suction stress, which up-scales and combines different types of inter-particle forces dependent on matric suction, and is intrinsically related to the soil-water retention curve (SWRC) of a given type of soil. Through the suction stress framework, the effective stress calculated in following equations can truly represent the contribution of contact behavior to the small-strain shear modulus for unsaturated soils (Lu et al. 2010):

$$\sigma' = (\sigma - u_a) - \sigma^s \quad (4)$$

$$\sigma^s = -S_e (u_a - u_w) = -(S_e / \alpha) \left[S_e^{n/(1-n)} - 1 \right]^{1/n} \quad (5)$$

where σ^s is the suction stress, S_e is the effective degree of saturation, and α and n are empirical fitting parameters in the van Genuchten (1980) SWRC model:

$$S_e = \frac{S - S_r}{1 - S_r} = \left\{ 1 + \left[\alpha (u_a - u_w)^n \right] \right\}^{1/n-1} \quad (6)$$

where S_r is the residual degree of saturation. Parameter α is related to the inverse of the air-entry suction of a soil [kPa^{-1}] and parameter n is a dimensionless pore-size distribution parameter of a soil.

Although the degree of saturation plays a role in definition of effective stress (Lu et al. 2010), Khalili et al. (2004) found that changes in degree of saturation may have independent effects on material hardening/softening and effective stress. To investigate the role of degree of saturation on material hardening/softening, Lu and Kaya (2014) performed compression tests on specimens brought to equilibrium at different initial degrees of saturation using the DC method to develop an empirical relationship for the elastic moduli of unsaturated soils at finite-strain levels ($\sim 1\%$). Sandy, silty and clayey soils at different degrees of saturation were compressed under vertical stresses up to 100 kPa to determine the finite-strain Young's and shear moduli values. The empirical relationship indicates that the finite-strain shear modulus is proportional to a power of the effective degree of saturation, as follows:

$$G'(S_e) \propto (1/S_e)^\beta \quad (7)$$

where β is an experimental fitting parameter, and the inverse of S_e is shown in parentheses to reflect that increases in the effective degree of saturation result in a material softening mechanism. Several other studies have also observed material hardening/softening during decreases and increases in the effective degree of saturation under constant external stress (Ng et al. 2009; Khosravi and McCartney 2012).

Although their approach may have successfully quantified the general trend between S and G_0 , the finite-strain shear moduli values measured by Lu and Kaya (2014) are much smaller in magnitude than the value of G_0 because of significant degradation in stiffness with the greater

strain level in the compression tests (e.g., $<0.0001\%$ to 1%). In this case, the contact or skeleton forces may have redistributed during the loading to high strain levels caused by the change of soil texture and fabric. This suggests that the enhancement in G_0 stemming from transmission of contact forces through the soil skeleton without redistribution of stresses needs to be considered separately from the material hardening/softening effects better represent the G_0 for a constant soil fabric regime at small strains.

Based on the above discussion of different types of existing models for G_0 and the role of effective stress in unsaturated soils, we propose a new unified model for G_0 extended from the finite-strain power law in Eq. 7 and covering all soils under both saturated and unsaturated conditions. For the small-strain shear modulus, the role of effective stress acting on the soil skeleton is considered as a separate mechanism from the role of material softening/hardening due to changes in the degree of saturation. A schematic principle of the conceptual model in a representative elementary volume (REV) of the soil is shown in Figure 1. Specifically, illustrations of the stiffness development due to material softening/hardening of soil matrix and the contact forces formed by hydration or capillary water at the two different soil water regimes, hydration water and capillary water regime are shown in Figures 1(a) and 1(b), respectively. Suction stress incorporates the effects of soil-water interaction in both regimes, and has been used to calculate the effective stress for unsaturated soils. The effective stress is assumed to primarily affect the shear modulus of unsaturated soils through the soil skeleton or network of particle contacts. A conceptual model for the proposed mechanisms contributing to the shear modulus development in an REV is shown in Figure 1(c). In this figure, $G'(S_e)$ and $G''(\sigma')$ represent the components of the shear modulus arising from the role of the effective degree of

saturation and from the role of the effective stress, respectively. In such an REV, the two components can be linked together in a multiplicative form.

As the effective degree of saturation varies under constant external confining stress, the soil matrix changes in stiffness. This component can be quantified using a form of Eq. 7 (Lu and Kaya 2014). The externally applied net stress and the suction stress σ^s control the stresses at the particle contacts. For unsaturated soils, the effective stress can be determined using Eq. 4 (Lu et al. 2010). Thus, the second component can be quantified as a similar proportionality of power law of effective stress, consistent with the Hardin's equation, as follows:

$$G''(\sigma') \propto (\sigma'/P_{\text{atm}} + 1)^{\gamma_0} \quad (8)$$

where effective stress σ' is normalized by the value of the standard absolute atmospheric pressure P_{atm} ($\cong 101.3$ kPa), and γ_0 is an empirical fitting parameter. Note that although the standard absolute atmospheric pressure is used as a constant normalization pressure, laboratory tests such as those in this study commonly use the gage pressure as the pore air pressure (e.g., for a specimen with $S_e = 1$, pore air pressure $u_a = 0$). Based on the conceptualized principles illustrated in Figure 1, the small-strain shear modulus can be determined by a soil matrix hardening/softening component related to the effects of the effective degree of saturation and a contact force component solely due to the effective stress. The proposed model can be written as follows:

$$G_0 = G'(S_e) \cdot G''(\sigma') = A_0 \left(\frac{1}{S_e} \right)^\beta \cdot \left(\frac{\sigma'}{P_{\text{atm}}} + 1 \right)^{\gamma_0} \quad (9)$$

where A_0 is an experimental fitting parameter having the same units as the small-strain shear modulus (typically MPa). When soil is saturated ($S_e = 1$), no external confining stress is applied

($\sigma = 0$), and the suction stress is zero ($\sigma' = \sigma^s = 0$), A_0 can be defined as the small-strain shear modulus of saturated soil without confinement G_0^{sat} . The value of G_0^{sat} incorporates the effect of void ratio, particle interlocking, initial conditions, and stress history. By replacing A_0 with G_0^{sat} , Eq. 9 can be rewritten as follows:

$$G_0 = G_0^{\text{sat}} \left(\frac{1}{S_e} \right)^\beta \cdot \left(\frac{\sigma'}{P_{\text{atm}}} + 1 \right)^{\gamma_0} \quad (10)$$

when soil is saturated ($S_e = 1$), the proposed equation reduces to a form similar to Hardin's equation (Eq. 1):

$$G_0 = G_0^{\text{sat}} \left(\frac{\sigma'}{P_{\text{atm}}} + 1 \right)^{\gamma_0} \quad (11)$$

where the effective stress reduces to Terzaghi's effective stress in saturated soil, $\sigma' = \sigma - u_w$. Therefore, the proposed equation unifies the small-strain shear modulus extended from saturated soil to unsaturated soil.

Experimental Program

To examine the evolution of G_0 for different soils with respect to degree of saturation, tests on 7 soils were conducted by incorporating shear-wave velocity measurement following the DC method of Lu and Kaya (2013). A schematic experimental setup of the shear-wave velocity measuring system is shown in Figure 2. First, soil cake specimens were statically compacted using a GeoTac loading frame, and then were saturated by submersion in water within a desiccator and applying vacuum for sufficient time (usually 1~2 days). All soil cake specimens were prepared with a diameter of 76.2 mm and a thickness of ~19 mm. Next, soil cake specimens were dried in a chamber with limited opening for lowering the evaporation rate and suction

gradient between soil sample and surrounding air to ensure the soil has homogeneous distribution in degree of saturation throughout the cake during the evaporation drying process. The system was maintained at a constant room temperature and a relative humidity of ~10%, which is equivalent to a boundary total suction of approximately 317 MPa applied on the DC system, according to the Kelvin's equation (e.g., Lu and Likos 2004). To monitor the weight change, the soil cake with the supporting plate was placed on a digital balance for monitoring of changes in the degree of saturation.

The shear-wave measuring system includes a Tektronix MDO3012 oscilloscope with an AFG function generator module and a Krohn-Hite 3940 digital filter/amplifier. A 50 Hz step function was sent to the transmitter and the received signal was passed through a 100 Hz high-pass channel and a 50 kHz low-pass channel, with 20 dB of gain amplification for each channel. The signals are digitized at a sampling rate of 2.5 GHz. 10k data points were recorded for each signal to determine the first arrival of the shear-wave. The stacking of 256 signals permits reduction of the non-coherent noise. A pair of bender elements was installed at the center of the top and bottom surfaces of the soil cake. The transmitter sent an S-wave through the soil cake and the vibration was detected by the receiver. The shear wave velocity V_s was obtained from the first arrival of the S-wave. The G_0 values can be calculated using $G_0 = \rho \cdot (V_s)^2$, where ρ is the mass density of soil.

Validation of the Proposed Model

A classification of all types of soils studied in this paper according to their sources is shown in Table 1, which includes 29 soils in total: 7 soils tested in this study using bender elements with suction applied using the DC method (soils No. 1-7, reference a); 5 soils from

Sawangsuriya et al. (2009) (soil No. 8-12, reference b) with suction control using axis translation; 6 soils from Khosravi and McCartney (2012) (soil No. 13-18, reference c) with suction control using axis translation; 2 soils from Ng et al. (2009) (soil No. 19-20, reference d) with suction control with axis translation; 5 soils from Mendoza et al. (2006) (soil No. 21-25, reference e) with suction control using air drying; and 4 soils from Yoon et al. (2008) (soil No. 26-29, reference f) with suction control using axis translation. The soil classifications, index properties (porosity, void ratio, and plasticity index), loading condition and applied net confining stress, van Genuchten (1980) SWRC model parameters, and the fitting parameters for G_0 in the proposed unified model, are all summarized in Table 1. The SWRCs of soils 1-25 were experimentally obtained or adopted from the literature. The different data sets from Khosravi and McCartney (2012) and Ng et al. (2009) are all for the same soil, but for wetting and drying SWRC paths under different net normal stresses.

The least squares fitting technique was used for determination of the fitting parameters for Eq. 10 for each soil. The soils studied cover a wide range of soils, from sandy to silty to clayey soils, and considered different ranges of externally applied stresses. Data from references a and e are under no net normal stress and only experience changes in matric suction; data from references b, c and d are under constant net normal stress and experience changes in matric suction; and data from reference f are under fully saturated conditions under different net normal stresses. The general behavior of proposed unified model fits well with the experimental results. The value of G_0^{sat} varies from 15 to 125 MPa, depending on soil type and initial void ratio. The parameter β varies from 0.001 to 3, while the parameter γ_0 varies from 0.01 to 0.78. All soils have the coefficient of correlation R^2 value between 0.88 and 0.99, indicating that the proposed Eq. 10 can reasonably represent the small-strain shear modulus.

The SWRCs of the selected soils types (e.g., SC, ML, CL, CH, MH) examined in this study are shown in Figure 3. All data points from the original papers shown in Table 1 are re-plotted in Figures 3(a) and 3(c), along with the fitted van Genuchten (1980) SWRC curves. The suction stress characteristic curves (SSCC) defined by Eq. 5 using the fitted SWRC parameters from Figures 3(a) and 3(c) are shown in Figures 3(b) and 3(d). As shown, the suction stress changes significantly, spanning the full saturation range with values ranging over 2~4 orders of magnitude. This implies dramatic variation in effective stress under unsaturated conditions, as well as potentially strong influences of material hardening/softening.

Comparisons between the measured small-strain shear modulus and numerically fitted values by using proposed unified model for selected soils according to different external stresses and saturation conditions are shown in Figures 4 through 6. The relationships between G_0 and the effective degree of saturation are shown in Figures 4(a) and 4(c), while the relationships between G_0 and the effective stress are shown in Figures 4(b) and 4(d). All of these specimens were tested under no external confining stress using the DC method. For soils tested under constant confining stress with varying matric suction, the relationships between G_0 and the effective degree of saturation are shown in Figures 5(a) and 5(c), while the relationships between G_0 and the effective stress are shown in Figures 5(b) and 5(d). For sandy soils at fully saturated conditions and various constant external confining stresses, the relationship between G_0 and the effective stress for measured data and model fittings is shown in Figure 6. The data and curves shown in Figures 4 through 6 indicate that the proposed unified model fits well with experimental data. The G_0 evolution with respect to the effective degree of saturation has different patterns (concave upward or downward, or S-shape), depending on the soil type and

confining stresses. In general, it is clear that G_0 increases with the increasing effective stress and with the reducing degree of saturation.

Correlation with Soil-Water Retention Parameters

The intrinsic correlation between soil water retention and suction stress implies the effective stress dependency of small-strain shear modulus is inter-related to soil water retention. In the SWRC model of van Genuchten (1980), the α parameter (the inverse of the air-entry suction) reflects the largest pore size and also the density of soil samples. On the other hand, the effect of initial porosity or void ratio has been embedded into parameter G_0^{sat} . Hence, the correlation between fitting parameters of the proposed unified model and α was not observed and only correlations with the soil type or pore size distribution parameter n were considered in this paper. The relationship between G_0^{sat} and n is shown in Figure 7(a). In this case, lower values of n , which correspond to fine-grained cohesive soils have higher G_0 values under saturated conditions. The dependency of G_0^{sat} on initial void ratio is shown in Figure 7(b) with soils labeled by the “Soil No.” in Table 1. The results indicate that denser soils with lower initial e generally have higher G_0^{sat} values, although the trend may not be clear due to overconsolidation effects and the fact that the initial void ratio before application of the confining stress is plotted in this figure. Specifically, for a given soil type, a constant initial e with increasing external confinement leads to increasing G_0^{sat} values, while constant external confinement with increasing initial e results in a decrease in G_0^{sat} . However, both trends are not strong enough when considering all of the soils together to develop a quantitative empirical relationship. It should be noted that such correlations between G_0^{sat} and n or e for an individual soil may show stronger trends, and quantitative

relationships can be considered on a project-specific basis. In general, because the parameter G_0^{sat} depends on initial soil relative density or void ratio, and solid particle properties, it needs to be experimentally determined as well for a given set of initial conditions for a soil.

The relationships between the other fitting parameters of the proposed model and the n parameter of the SWRC are shown in Figure 8. Parameter β was observed to follow a power-law correlation with n in Figure 8(a). Lower β values were observed for high n values (sandy soils), while the β value increases as the n value increases and the pore size distribution decreases (i.e., a greater clay fraction). An empirical fitting equation was identified as $\beta = 9.6 \times n^{-6.0}$, which can be used for estimation of G_0 of unsaturated soil using the van Genuchten (1980) SWRC model for different soil types. This inter-correlation between material hardening/softening parameter β and soil pore size gradation parameter n implies that the material softening/hardening mechanism stem from the cementation effect of hydration of fine content of the soil.

More clayey soil with lower n value has higher parameter β numbers, reflecting higher sensitivity of G_0 to the effective degree of saturation for fine soils, and higher material softening/hardening effect during wetting/drying. The effective stress term is not only determined by suction stress in unsaturated conditions, but also depends on the external net normal stress. For this reason, the power γ_0 of the effective stress term does not show clear correlation with the n parameter as shown in Figure 8(b), and should be experimentally determined to estimate the G_0 of soils for saturated and unsaturated cases. The average value of γ_0 for the soils investigated in this study is 0.5, which corresponds with the theoretical value expected from the Hertzian contact theory, and the range of parameters is consistent with those observed by Stokoe et al. (1999) for different soils.

Comparison with Other G_0 Models

Based on the correlation between proposed unified model and soil water retention, given the van Genuchten (1980) SWRC model parameters the unified model can be written into following equation form for prediction of G_0 of saturated or unsaturated soils:

$$G_0 = G_0^{\text{sat}} \left(\frac{1}{S_e} \right)^{9.6 \cdot n^{-6.0}} \cdot \left(\frac{\sigma'}{P_{\text{atm}}} + 1 \right)^{\gamma_0} \quad (12)$$

The G_0 of all 29 soils were predicted using Eq. 12 and were compared with fitted curves for the models in Eqs. 2 and 3. Comparisons of the predicted values of G_0 from the proposed unified model prediction with those from the other two fitted models are shown in Figures 9 and 10. Generally, the newly proposed model has similar performance with the models described by Eqs. 2 and 3. For most of the soils, the proposed model has a R^2 value that is slightly higher than that for the models in Eqs. 2 and 3. However, the proposed unified model correlates with the fitting parameters to the SWRC of the soils, while the fitting parameters of the models in Eqs. 2 and 3 have no solid physical meaning. A statistical comparison shows that for the total of 25 unsaturated soils under investigation, the proposed unified model has better or equivalent prediction for 15 soils, has middle predictability for 3 soils, and has relatively worse prediction behavior in the other 7 soils. Most of the R^2 values for the unified model prediction are in the range of 0.9 to 1.0, with only few values of approximately 0.88.

For Eq. 1, the model is established based on Terzaghi's effective stress theory for saturated soils. The effect of matric suction or suction stress when soil is desaturated is not taken in account. Hence, this model cannot predict well the material softening/hardening effect for unsaturated soils. For Eqs. 2 and 3, the matric suction is multiplied by a power function of the

effective degree of saturation as an independent variable. In this case, matric suction is intrinsically inter-related to the effective degree of saturation. However, the changes of matric suction and degree of saturation have converse effects: matric suction increases as the effective degree of saturation decreasing and vice versa. This nature makes the fitting parameters (i.e., the exponent κ of S of Eq. 2), compete with the exponent γ_2 of the effective stress term. Thus, the fitting results for the parameters κ and γ_2 for the model in Eq. 2 do not have a solid physical meaning or correlate with constitutive soil properties (e.g., the SWRC). The same case is also observed for the fitting parameters ζ and ξ for the model in Eq. 3. Note that while Eq. 12 can be used to predict G_0 for all types of soil under both mechanical (external stress) and environmental (degree of saturation) loading conditions, it cannot predict G_0 under a completely dry condition, (i.e., S or $S_e = 0$), due to the appearance of S_e in the denominator. However, should such situation arise as in the commonly encountered case of dry sand, the denominator S_e in the second term in Eq. 12 can be substituted by $S_e + S_0$ with S_0 set to be a dimensionless non-zero constant.

Conclusions

This study addresses the roles of material hardening/softening and effective stress using suction stress theory on the small-strain shear modulus across a wide range of soil types, degrees of saturation, and externally applied stresses. A conceptual model was established that consists of two individual mechanisms related to the impacts of effective stress and material hardening/softening on the soil matrix and contact behavior. Following the conceptual model, a quantitative model was proposed by extending the power law approach for estimating small-strain shear modulus under different effective stress levels to both fully saturated and unsaturated soils. An advantage of the proposed unified model is that its parameters have physical meaning,

and can be correlated with the soil water retention curve to predict the small-strain shear modulus for unsaturated soils. Specifically, the material softening/hardening effect quantified using the effective degree of saturation was found to correlate well with the pore-size distribution parameter of the van Genuchten SWRC model. The evolution of small-strain shear modulus with the effective degree of saturation was observed to be more sensitive for silty and clayey soils than sandy soils. Further, the model was found to apply for cases of different loading conditions, which include both no external stress and various external net confining stress conditions.

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490

Soil No.	Soil name and reference *	USCS	Porosity ϕ	Void ratio e	van Genuchten parameters θ_r α n [kPa ⁻¹]	PI	Testing conditions	Net confining stress $\sigma_0 - u_a$ [kPa]	Fitting parameters of proposed model G_0^{sat} β γ_0 R^2
1	Bonny silt ^a	ML	0.47	0.90	0.02 0.06 1.52	4	Unsaturated	0	33.5 1.00 0.06 0.93
2	Hopi silt ^a	SC	0.42	0.72	0.01 0.03 1.58	13	Unsaturated	0	32.1 1.13 0.47 0.99
3	BALT silt ^a	ML	0.40	0.67	0.02 0.01 1.65	6	Unsaturated	0	119.3 0.44 0.19 0.88
4	Iowa silt ^a	ML	0.49	0.97	0.01 0.05 1.60	11	Unsaturated	0	22.2 0.75 0.65 0.99
5	Denver claystone ^a	CL	0.58	1.37	0.06 0.02 1.38	21	Unsaturated	0	48.1 0.98 0.21 0.99
6	Denver bentonite ^a	CH	0.69	2.21	0.10 0.02 1.40	73	Unsaturated	0	13.4 1.84 0.17 0.98
7	Missouri clay ^a	-	0.40	0.67	0.08 0.02 1.44	-	Unsaturated	0	73.9 1.25 0.28 0.95
8	SC-Std-Opt ^b	SC	0.27	0.37	0.00 0.007 1.33	14	Unsaturated	35	111.6 3.09 0.10 0.98
9	ML-Std-Opt ^b	ML	0.39	0.51	0.00 0.03 1.32	11	Unsaturated	35	39.3 1.58 0.10 0.94
10	CL1-Std-Opt ^b	CL	0.38	0.60	0.01 0.004 1.30	24	Unsaturated	35	64.3 3.59 0.09 0.99
11	CL2-Std-Opt ^b	CL	0.33	0.48	0.01 0.005 1.27	9	Unsaturated	35	121.6 4.05 0.08 0.92
12	CH-Std-Opt ^b	CH	0.54	1.15	0.01 0.002 1.36	52	Unsaturated	35	19.2 4.39 0.47 0.94
13	d-p100 ^c	ML	0.35	0.54	0.02 0.03 1.71	4	Unsaturated	100	38.4 0.12 0.29 0.89
14	d-p150 ^c	ML	0.35	0.54	0.02 0.03 1.33	4	Unsaturated	150	39.6 0.33 0.35 0.89
15	d-p200 ^c	ML	0.35	0.54	0.02 0.04 1.44	4	Unsaturated	200	41.5 0.14 0.41 0.97
16	w-p100 ^c	ML	0.35	0.54	0.02 0.08 1.72	4	Unsaturated	100	31.2 0.16 0.58 0.95
17	w-p150 ^c	ML	0.35	0.54	0.02 0.1 1.81	4	Unsaturated	150	46.5 0.22 0.25 0.94
18	w-p200 ^c	ML	0.35	0.54	0.02 0.1 1.82	4	Unsaturated	200	40.9 0.03 0.47 0.94
19	dw-p110 ^d	ML	0.39	0.64	0.00 0.001 1.96	4	Unsaturated	110	38.1 0.32 0.97 0.99
20	dw-p300 ^d	ML	0.39	0.65	0.00 0.006 2.22	4	Unsaturated	300	52.6 0.31 0.82 0.99
21	S1 ^e	MH	0.50	0.99	0.02 0.0006 1.90	35	Unsaturated	0	61.7 0.14 0.51 0.98
22	S2 ^e	MH	0.56	1.29	0.00 0.0006 1.87	35	Unsaturated	0	53.5 0.57 0.02 0.92
23	S3 ^e	MH	0.56	1.29	0.00 0.0007 1.72	35	Unsaturated	0	38.1 0.15 0.41 0.96
24	S4 ^e	MH	0.54	1.18	0.01 0.0006 1.79	35	Unsaturated	0	48.9 0.21 0.40 0.97
25	S5 ^e	MH	0.56	1.29	0.00 0.0004 1.86	35	Unsaturated	0	6.7 0.08 0.89 0.91
26	TS-d ^f	SP	0.41	0.70	- - -	-	Saturated	50,100,200,400	17.1 - 0.41 0.99
27	TS-l ^f	SP	0.44	0.80	- - -	-	Saturated	50,100,200,400	13.1 - 0.44 0.99
28	SS-d ^f	SP	0.41	0.70	- - -	-	Saturated	50,100,200,400	16.3 - 0.42 0.99
29	SS-l ^f	SP	0.43	0.75	- - -	-	Saturated	50,100,200,400	14.6 - 0.42 0.99

* Note: a: current study; b: Sawangsuriya et al. (2009); c: Khosravi and McCartney (2012); d: Ng et al. (2009); e: Mendoza et al. (2006); g: Yoon et al. (2008).

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Figure 1. Illustrations of the impacts of inter-particle contact forces and pore water: (a) Schematic of soil in the hydration regime; (b) Schematic of unsaturated soil in the capillary regime; (c) The conceptual model consisting of two counterparts of small-strain shear modulus arising from the contributions of effective stress and degree of saturation.

Figure 2

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