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<https://escholarship.org/uc/item/45t2z352>

Journal

Physical Review C, 113(2)

ISSN

2469-9985

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Publication Date

2026

DOI

10.1103/hrd3-246d

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Probing the j -Dependence of Angular Distributions and $N = 20$ Shell Rigidity via the $^{36}\text{S}(p, d)^{35}\text{S}$ Reaction

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(Dated: December 11, 2025)

An investigation of the $N = 20$ ^{36}S nucleus has been performed through a detailed study of the $^{36}\text{S}(p, d)^{35}\text{S}$ neutron-removal reaction, employing a 66 MeV proton beam at iThemba LABS and an innovative target design. Using the high-resolution $K=600$ magnetic spectrometer, 98 states in ^{35}S were identified up to 16 MeV excitation energy, including 47 previously unobserved states. Angular distributions and spectroscopic factors, including isobaric analog state contributions, were extracted for 81 levels. A pronounced j -dependence in the angular distributions of $\ell = 2$ states provides refined insights into the spin-orbit splitting. Finite-range adiabatic distorted wave approximation calculations qualitatively reproduce the observed j -dependence. Comparisons of the measured $1d_{5/2}$ spectroscopic strength distribution with large-scale shell-model and ab-initio calculations show good agreement overall, and the robustness of the $N = 20$ shell closure in ^{36}S is confirmed when comparing the relatively low fp orbital occupancies in ^{40}Ca and ^{36}S across the Fermi surface. This study underscores the utility of neutron-removal reactions in probing nuclear structure and the Fermi surface of sd nuclei and beyond. The findings advance our understanding of shell evolution and offer constraining data for theoretical models.

I. INTRODUCTION

Nuclear structure studies are fundamentally tied to understanding the intricate interplay between the nucleus and the nuclear forces, as well as the resulting manifestation of shell effects across the nuclear chart. Precision measurements of nuclear properties, such as level energies, spins, parities, and spectroscopic factors serve as powerful probes into the underlying structure of the nucleus and provide critical benchmarks for refining nuclear theories. Significant advancements in experimental techniques and theoretical models have highlighted that shell closures are not immutable. A notable example is the $N = 20$ shell gap, which presents a compelling case

of shell evolution, where the ‘traditional’ magic number weakens due to a complex interplay of monopole interactions, correlations, and the influence of intruder orbitals (see, e.g. Refs. [1, 2]). The availability of detailed nuclear data plays a pivotal role in advancing and validating our understanding of these phenomena.

Over the last decades, much experimental effort has been undertaken to investigate the evolution of the $N = 20$ shell closure between ^{40}Ca and the border of the Island of Inversion [3–11]. Such studies typically enhance our understanding of the level structure, shedding light on how the one- and two-particle-hole (ph) states evolve, as e.g. in the cases of ^{35}P [4, 10, 11] and ^{34}Si [6, 8]. Likewise, investigations into the occupancies of orbitals located below and above $N = 20$ (forming the so-called Fermi surface) provide deeper insights into the effectiveness of the shell closure, as for instance in ^{34}Si [9], ^{32}Mg [12], ^{30}Na [13], and ^{30}Ne [14]. Note that the missing link in this list is ^{36}S . Besides the presence of a shell gap that can be large or moderate, a magic nucleus is best char-

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acterized by the weak occupancy of valence orbitals, as was discussed already in Ref. [15] for the cases of ^{40}Ca and ^{48}Ca .

In order to characterize another $N = 20$ isotone beyond ^{35}P , en route from ^{40}Ca to the Island of Inversion, the $^{36}\text{S}(p, d)$ neutron-removal reaction has been used in the present work to investigate neutron single-particle states in ^{36}S . In this paper, we report on a detailed measurement of nuclear level energies, angular distributions and their spectroscopic factors, which made it possible to investigate the rigidity of the $N = 20$ shell closure by probing the Fermi surface of ^{36}S and comparing it with similar data from the $N = 20$ doubly magic nucleus ^{40}Ca [16].

The $^{36}\text{S}(p, d)^{35}\text{S}$ neutron-removal reaction, performed at the iThemba Laboratory for Accelerator Based Sciences (iThemba LABS) in South Africa, revealed a strong j -dependence in the angular distribution of differential cross sections for $\ell = 2$ states. This phenomenon was first observed in 1964 with the (d, p) [17], (d, t) [18], and (p, d) [19] transfer reactions. More experimental evidence for the j -dependence was reported in the following years, see e.g. Refs. [20–27]. Theoretical efforts to explain the j dependence have also been made (see for instance Refs. [28–31]). In this work we revisit the study of this phenomenon by using different parameterizations of the optical model potential with a specific focus on the spin-orbit potential.

The access to high excitation energies up to 16 MeV allowed the determination of orbital spectroscopic factor strengths including the identification of the isobaric analog states (IAS). As for the latter, a comparison thereof with results obtained from the $(-1p)$ reaction using appropriate Clebsch-Gordan coefficients is made. Such a detailed analysis of hole strengths makes a comparison of the fragmentation of the deeply bound $d_{5/2}$ orbital with state-of-the-art models possible.

Sections II and III of this paper provide a detailed account of the experimental setup and analytical procedure, respectively. Section IV discusses the results on A) the observed j -dependence of the angular distributions, B) IAS, C) the determination of energy centroids, iv) a comparison of the strength distributions of $5/2^+$ neutron-removal states with model calculations, and v) the shell evolution in $N = 20$ isotones. Concluding remarks are made in Section V.

II. EXPERIMENTAL SETUP

The $^{36}\text{S}(p, d)^{35}\text{S}$ reaction was studied at the Separated Sector Cyclotron (SSC) facility of iThemba LABS. A 66 MeV dispersion-matched proton beam, with beam intensities ranging from 4 to 8 nA, impinged on a periodically moving target of highly enriched ^{36}S . This innovative target solution was required to minimize the potential impact of sublimation during beam bombardment, without introducing heavy contaminants typically used

in sublimation-resistant sulphur compounds (e.g. AgS or SbS). In direct reaction studies, these contaminants will result in the population of numerous unwanted excited states obscuring many of the states in ^{35}S . To this end, ^{36}S powder, enriched to 99.24%, was encapsulated between two thin ($0.5\ \mu\text{m}$) Mylar foils, preventing the potential loss of material to vacuum due to sublimation. A secure seal was ensured by fusing the Mylar foils together with a temperature-controlled cylinder [32]. Target survival was ensured by moving the target to a new position every sixty seconds, using two independent motors connected to the target frame via off-axis rods and counterbalanced by a spring, resulting in a periodic swiveling motion of the target in the plane perpendicular to the beam. The thickness of these ^{36}S targets, $1 \pm 0.05\ \text{mg}/\text{cm}^2$, was determined with the alpha energy-loss technique. In addition, a $1\ \text{mg}/\text{cm}^2\ ^{\text{nat}}\text{S}$ target, manufactured with the same technique as the ^{36}S target, was utilized to identify possible contaminant states.

The outgoing deuteron and triton reaction products were momentum analyzed in the high-resolution K=600 Q2D magnetic spectrometer and detected at its medium-dispersion focal plane [33]. The focal-plane detectors consisted of two multiwire drift chambers (MWDCs), and a plastic scintillator detector downstream from the MWDCs. The scintillator was utilized to construct the data acquisition trigger, and particle identification (PID) was achieved through the standard method of combining information on energy deposition in the scintillator with a relative measurement of the time of flight through the spectrometer.

Due to the momentum acceptance associated with the medium-dispersion focal plane ($p_{\text{max}}/p_{\text{min}} = 1.097$), the excitation-energy range of 0–10.8 MeV was covered using two magnetic fields with ample allowance (a few MeV) for spectra overlap, ensuring proper normalization of results from different fieldsets. Measurements at various laboratory scattering angles (θ_{lab}) of $4^\circ, 7^\circ, 10^\circ, 15^\circ, 21^\circ$ and 28° enabled the extraction of angular distributions of the cross sections for the states of interest. Angle calibration of the $\pm 2^\circ$ circular entrance aperture of the spectrometer using a multi-hole collimator made it possible to subdivide the K=600 angular acceptance into four angular regions. This provides up to twenty-two datapoints for angular distributions, taking into account the acceptance overlap between the spectrometer angles $4^\circ, 7^\circ$, and 10° . Further exploration of the excitation-energy region up to 16 MeV was performed with a third magnetic fieldset at $\theta_{\text{lab}} = 7^\circ$ and 15° to search for strength at higher excitation energies. An experimental energy resolution of 35 keV FWHM was achieved. Excitation-energy spectra representing the full acceptance for all angles are shown in Fig. 1.

For each angular selection, the optimized focal-plane position spectrum was converted to ^{35}S excitation energy following a rigidity calibration procedure [34]. The well-established low-lying ^{35}S states at $E_x = 0, 1.572, 1.991, 2.717, 3.421$, and $3.801\ \text{MeV}$ were used in the calibration

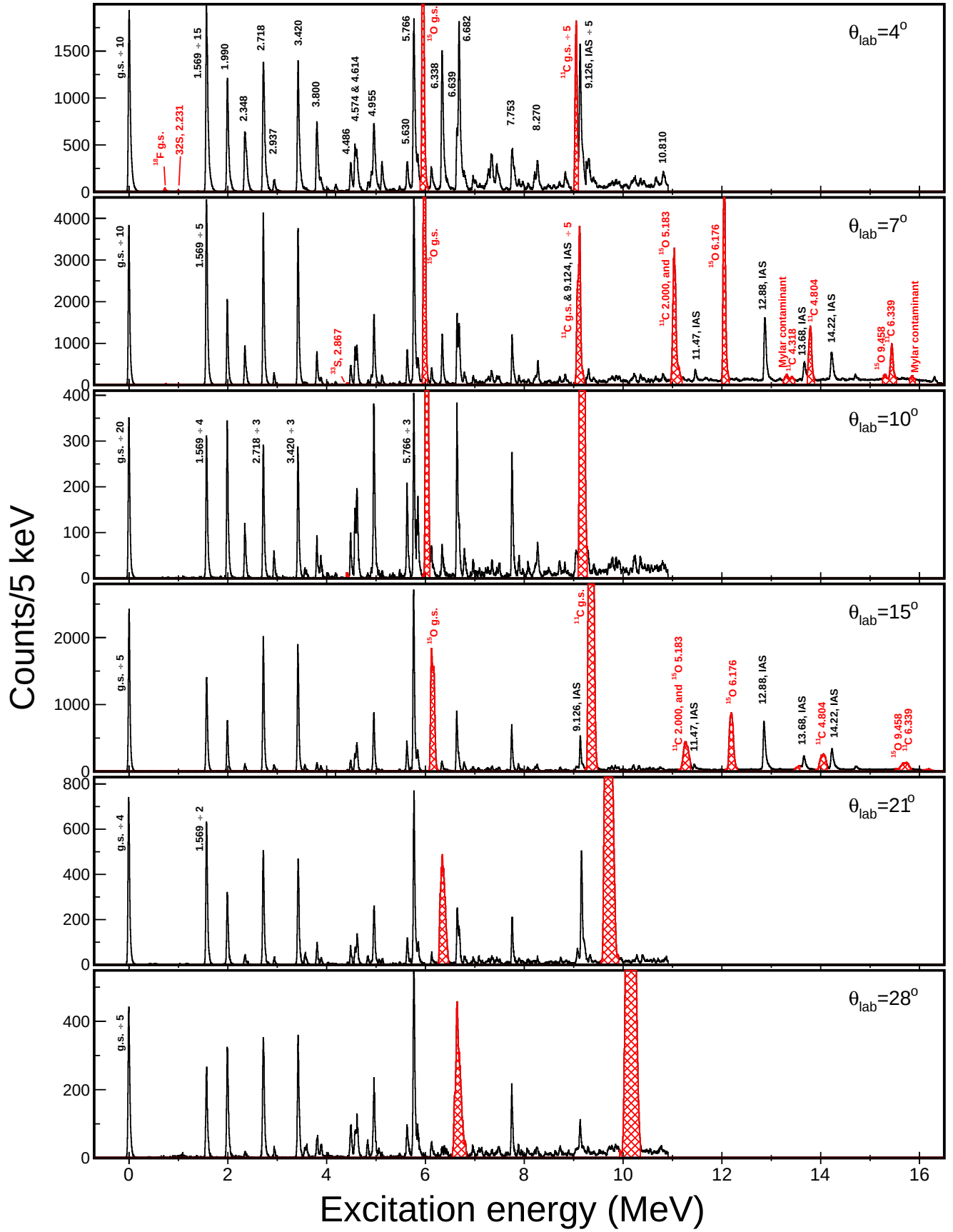


FIG. 1: Excitation-energy spectra for the $^{36}\text{S}(p, d)^{35}\text{S}$ reaction at different laboratory scattering angles. Excitation energies associated with prominent peaks are indicated, and contaminant peaks are shown as red hatched areas. The higher excitation-energy region accessible at 7° and 15° reveals the $3/2^+$ and $5/2^+$ IAS. The division symbols and accompanying numbers indicate the factors by which the peak intensities were reduced for clarity in the plot.

procedure. Additionally, the presence of prominent contaminant peaks, such as the ground states of ^{15}O and ^{11}C as well as the 2.0 MeV excited state in ^{11}C , attributed to the Mylar windows of the ^{36}S target, provided calibration points at higher excitation energies. Data acquired with a Mylar target confirmed the peak assignments and showed no significant population of states from carbon or oxygen isotopes other than ^{11}C and ^{15}O .

The $^{\text{nat}}\text{S}(p, d)$ reaction was studied to identify possible contaminant states due to the presence of $^{32,33,34}\text{S}$, with an isotopic distribution of 0.07%, 0.05% and 0.64%, respectively, in the enriched ^{36}S target. These differ significantly from the natural isotopic distribution of 94.99% for ^{32}S , 0.75% for ^{33}S and 4.25% for ^{34}S . The excitation-energy spectrum for the $^{\text{nat}}\text{S}(p, d)$ reaction was compared to that of the ^{36}S target (see Ref. [34]). Only the weak free-standing 2.231 MeV state in ^{32}S , as well as the 2.867 MeV $5/2^+$ state in ^{33}S , could be clearly identified as sulphur contaminant peaks in the final ^{35}S excitation-energy spectrum. Both appear in regions devoid of known ^{35}S candidates, with the ^{32}S candidate found around 1 MeV between the ground state and first-excited state, and the 2.867 MeV ^{33}S state barely visible at 4.41 MeV. The presence of the ground state and 0.84 MeV excited state from ^{33}S underneath the 1.572 MeV $1/2^+$ and 2.348 MeV $3/2^-$ ^{35}S states, respectively, is adequately addressed through peak-fitting procedures. Finally, the ground state of ^{18}F , which originates from a 530 ppm ^{19}F contaminant in the target material, is very weakly observed.

III. ANALYSIS

A. Experimental data analysis

Ninety-eight (98) states in ^{35}S were identified up to an excitation energy of 16 MeV. At high excitation energies, the non-contaminant strongly populated states observed at 9.126, 11.47, 12.88, 13.68, and 14.22 MeV have been identified as IAS of ^{35}P . All identified states are listed in Table II. Peak positions for known states agree excellently with the known ^{35}S states listed in Ref. [35]. Forty-seven (47) of the states ascribed to ^{35}S in this study are new. These states, most of which are above 8.4 MeV, are considered to belong to ^{35}S because of their non-observation in measurements with the Mylar or $^{\text{nat}}\text{S}$ targets, and the fact that the variation in their excitation-energy peak position for the full angle range $\theta_{\text{lab}} = 2^\circ$ to 30° is on a par with results from well-established ^{35}S states.

The yield for individual states was extracted by a peak-fitting procedure applied to the excitation-energy spectra. For strongly populated states, the $K=600$ angular acceptance was subdivided into four angular regions to yield four excitation-energy spectra per $K=600$ angle placement. All states were fitted with a convolution of a Gaussian and Landau function (~ 35 keV FWHM), which takes into account the presence of the peak tail

on the high excitation-energy side caused by straggling. The width parameters used in the fitting routine were obtained by scrutinizing the quality of fit for the isolated ground state and 1.991 MeV $7/2^-$ state of ^{35}S . Cutoff parameters were employed in the peak convolution procedure 85 keV below and 300 keV above the peak centroid. The experimental cross-section distributions of prominent states are shown in Fig. 2.

Due to the swiveling motion of the target, a different area was exposed to the beam every sixty seconds. Since the range of motion covered an area slightly larger than the actual size of the ^{36}S target, the beam sometimes hit the edge of the ^{36}S target with a reduced thickness. This resulted in clearly identified one-minute-long periods with reduced beam-intensity-weighted count rates. Normalization factors taking this effective target-thickness variation into account contributed $\sim 10\%$ towards the systematic uncertainty of the cross sections. The other major contribution to the systematic error is the nominal target-thickness uncertainty (5%). Added in quadrature, these give a total systematic uncertainty of 11.2%.

B. Adiabatic distorted wave approximation analysis

The experimental angular distributions were compared with results from adiabatic distorted wave approximation (ADWA) calculations [36] performed using the zero-range code DWUCK4 [37] in order to help with the identification of the orbital angular momentum transferred in the reaction. To facilitate comparison with previous $N = 20$ datasets, the same methodology as Ref. [16] was followed in order to minimize the impact of the well-known uncertainties in spectroscopic factors extracted from ADWA analyses introduced by different choices of distorting optical potentials and, above all, bound-state wave functions.

The entrance channel proton optical potential was calculated using the global parameter set of Ref. [38]. The parameter values appropriate for 66 MeV $p + ^{36}\text{S}$ were calculated according to Eq. (2) of Ref. [16]. For the exit channel deuterons, an adiabatic potential [39], which includes the effect of deuteron breakup in an approximate way was constructed from the global neutron and proton optical potential parameters of Ref. [40] according to the prescription given by Ref. [41]. The parameters were calculated following Eq. (3) of Ref. [16] and were re-calculated at the appropriate deuteron energy for each final state. Matoba *et al.* [16] reduced the strength of the spin-orbit component of the adiabatic potential by a factor of 2/3 to improve the description of their analysing-power data. However, since this had little effect on the cross-section angular distributions and no analyzing-power data are available for $^{36}\text{S}(p, d)^{35}\text{S}$, here the SO potential was kept at the original value and not reduced. The wave function of the picked-up neutron was

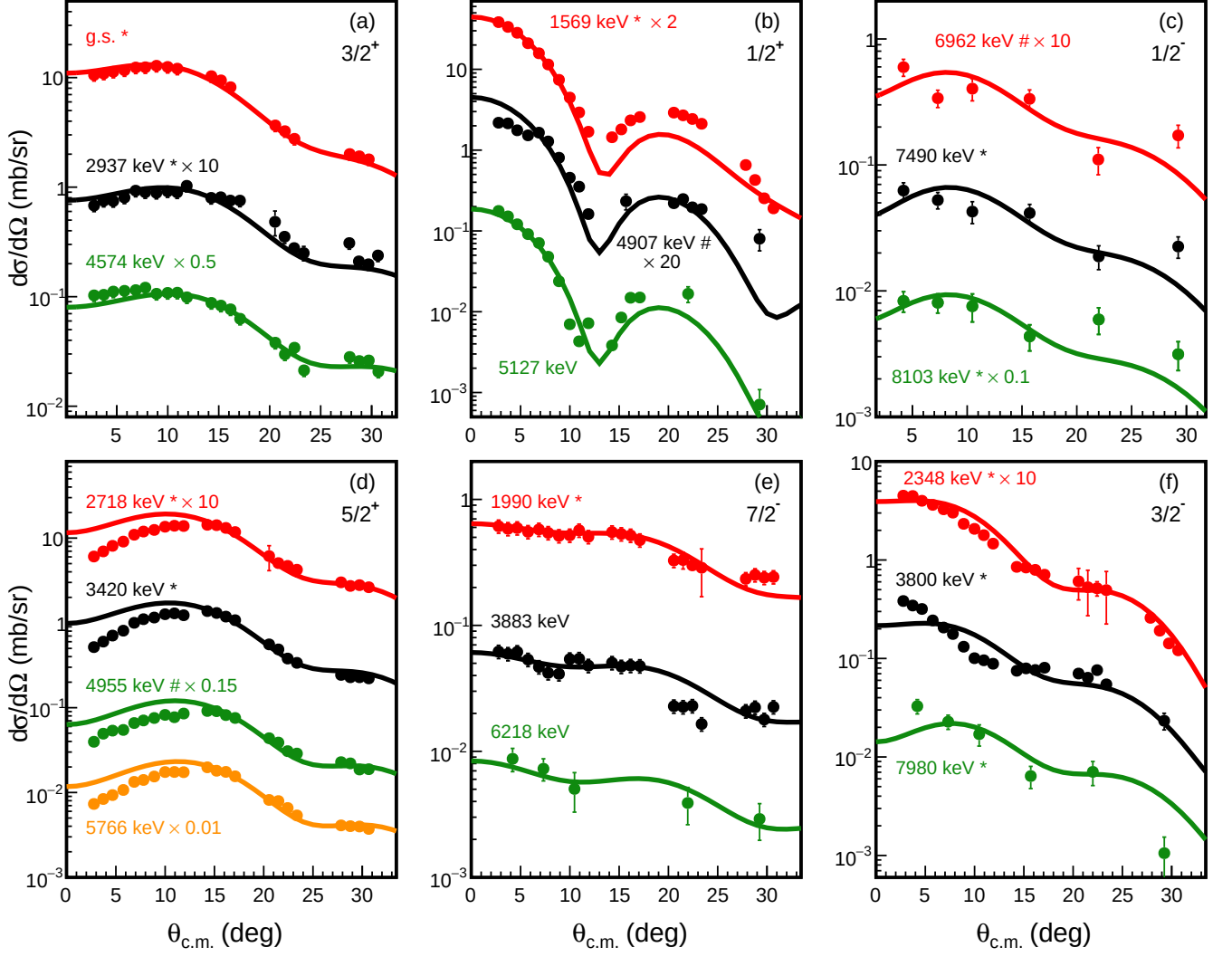


FIG. 2: Representative angular distributions with fitted ZR-ADWA calculations for $\ell \leq 3$, some with arbitrary normalization factors for display purposes. States labeled with an asterisk (*) represent those with established j^π assignments from ENSDF [35], while the three states labeled with a hash (#) represent new assignments in contrast to what is listed in the literature. Error bars associated with the experimental cross-section results represent a combination of systematic and statistical errors added in quadrature.

calculated using the separation energy method whereby the neutron is bound to the core in a Woods-Saxon well with the potential depth adjusted to reproduce the experimental binding energy. Since the same neutron orbitals as in the work of Ref. [16] are relevant in this study, the same parameters were used including for the IAS. The radius and diffuseness parameters were $r_0 = 1.27$ fm and $a_0 = 0.70$ fm, respectively. A spin-orbit term of Thomas form with strength factor $\lambda = 25$ was also included. These parameters were adjusted to give the expected summed occupation number for the neutron $3/2^+$, $1/2^+$ and $7/2^-$ states in ^{40}Ca (total neutron number ≈ 6). Finite-range effects were included via the local energy approximation with a finite-range parameter of 0.629. Nonlocality effects for the neutron, proton, and deuteron potentials were corrected for using nonlocality

parameters $\beta_n = 0.85$, $\beta_p = 0.85$ and $\beta_d = 0.54$, respectively.

To distinguish between the zero-range and the finite-range adiabatic distorted Wave approximation calculations, described in section IV, the zero-range calculations following the methodology of Ref. [16] will be referred to as “ZR-ADWA”, and the finite-range calculations as “FR-ADWA”. It should be recalled that the ZR-ADWA calculations equally include the adiabatic treatment of deuteron breakup via the substitution of the adiabatic potential $\bar{V}(R)$ of Ref. [36] (in the approximate form given in Ref. [41]) for the usual deuteron optical potential in the exit channel.

The ℓ assignments and the determination of neutron-removal spectroscopic factor values C^2S^- were made by the best match of the cross sections between the ex-

perimental angular distributions and ZR-ADWA calculations, which are shown in Fig. 2. The spectroscopic factors for 81 of the 98 states, together with j^π assignments from the ENSDF [35] and this study, are listed in Table II. The angular distributions of known $1/2^+$, $3/2^+$, $1/2^-$, $3/2^-$, and $7/2^-$ states are adequately reproduced by the ZR-ADWA calculations, on a par with what was found previously, e.g. in Ref. [16]. The Woods-Saxon parameters a_0 and r_0 for the neutron bound state have been adjusted [3] in this work to yield expected summed occupation numbers for the neutron $1f_{7/2}$, $2s_{1/2}$, and $1d_{3/2}$ states (total neutron number ≈ 6) in the ^{40}Ca nucleus, resulting in occupancies of 3.75 and 1.85 for the $1d_{3/2}$ and $2s_{1/2}$ orbitals, respectively, as shown in Table III.

There is a clear signal of a strong j -dependence for the known $\ell = 2$ states in the shape of the angular distributions at forward angles. This is clear from a comparison of panels (a) and (d) of Fig. 2, and allowed for unambiguous j^π assignments for all $\ell = 2$ states. This phenomenon will be discussed in more detail in section IV A.

Although there also appears to be a difference in forward-angle behavior for the $\ell = 1$ states, the absence of strongly populated $1/2^-$ states prevents any definite conclusions about j -dependence for these states. For all $\ell = 1$ states we therefore restrict definitive assignments only to those states with known j values reported in the literature.

For the majority of states, j^π assignments are in agreement with those listed in the ENSDF [35], although there are a few cases where a different j^π value is suggested when confronted with strong evidence, e.g. the states at 4.907, 4.955 and 6.962 MeV, as shown in Fig. 2.

IV. RESULTS AND DISCUSSION

A. Investigating j -dependent angular distributions

As mentioned above, there is a clear sign of a strong j -dependence in the shape of the angular distributions at forward angles observed for the known $\ell = 2$ states. This is illustrated in Fig. 2 by the cross-section distributions of the $3/2^+$ ground state and previously assigned excited state at 2.938 MeV [42] in ^{35}S , as compared to the known $5/2^+$ states at 2.718 and 3.421 MeV. A more detailed comparison, highlighting this discrepancy for all strong $\ell = 2$ states, is shown in Fig. 3.

This observed j -dependence allows for the unambiguous assignment of j^π values to all observed $\ell = 2$ states, including those previously unknown. Although this phenomenon has received relatively little attention in recent years, it was first observed over half a century ago by Lee and Schiffer [17] in the context of the (d, p) transfer reaction at low incident energies (8–12 MeV). Since then, numerous examples of j -dependent cross section angular distributions have been reported across a range of transfer reactions, including (p, d) [19], (p, α) [25], $(d, ^3\text{He})$ [43], and $(^3\text{He}, \alpha)$ [21]. Specifically, for the (p, d) reac-

tion, a j -dependence has been observed in neutron pickup processes involving $\ell = 1, 2$, and 3 from light to medium-mass nuclei (e.g. ^{32}S , ^{36}Ar , ^{40}Ca , ^{56}Fe , ^{58}Ni) at beam energies ranging from 17.5 MeV to 52 MeV [19–22, 44]. A consistent conclusion from these studies is that the angular distribution for the $\ell - 1/2$ state peaks at smaller angles than that of the $\ell + 1/2$ state, exhibits a more rapid decline between the first maximum and the first minimum, and shows a more pronounced oscillatory structure at backward angles.

The influence of the spin-orbit component in the distorting potential on j -dependence was investigated in early studies [28]. While calculations for the $(d, ^3\text{He})$ reaction strongly suggested that the j -dependent effect arises mainly from the deuteron spin-orbit potential [23, 43], calculations investigating the impact of the spin-orbit potential as well as non-locality in the case of (p, d) and (d, p) reactions were not very conclusive [29, 43]. Although some j -dependence was observed in distorted wave calculations that include the spin-orbit component for $\ell = 1$ states at backward angles [29, 45] for the (d, p) reaction off fp-shell nuclei, the effect was not reproduced for $\ell = 3$ states [29].

Due to the limited theoretical understanding of this behavior at the time, empirical rules were developed to assign spin values. However, these were not considered sufficiently definitive [46]. In contrast, j -dependence observed in analyzing power measurements, enabled by the advent of polarized particle beams, was regarded as having a solid theoretical basis, independent of the j -dependence seen in cross section measurements, and thus considered more reliable [24]. As a result, reliance on cross-section angular distributions for spin assignments gradually declined.

In a renewed attempt to understand the origin of the j -dependence in the cross section results, and its different manifestation in the present work compared to that of Kozub *et al.* [22], several tests have been performed. The main observations are discussed below.

As is well known, the shape of the angular distributions is influenced by the properties of the final state, and by the reaction mechanisms, e.g., multi-step *versus* one-step [47–49]. In the major part of the present work, and in particular in Fig. 2 and Table II, the reaction model used to analyze the (p, d) reaction data on ^{40}Ca and ^{36}S is a zero-range adiabatic model, ZR-ADWA. It was chosen to compare the present results on ^{36}S to those of ^{40}Ca performed in Ref. [16].

In the present subsection, the FR-ADWA [50] is considered. It has the advantage of treating the finite-range of the transition operator and compares well with the exact approach [51]. Another important advantage of the FR-ADWA, as for the ZR-ADWA, is that these take as input only nucleon-nucleus potentials, which can be derived from the same global optical potentials.

For the investigation of the j -dependence of the transfer angular distributions, the FR-ADWA code NLAT [54] and optical potentials derived from the KDUQ global

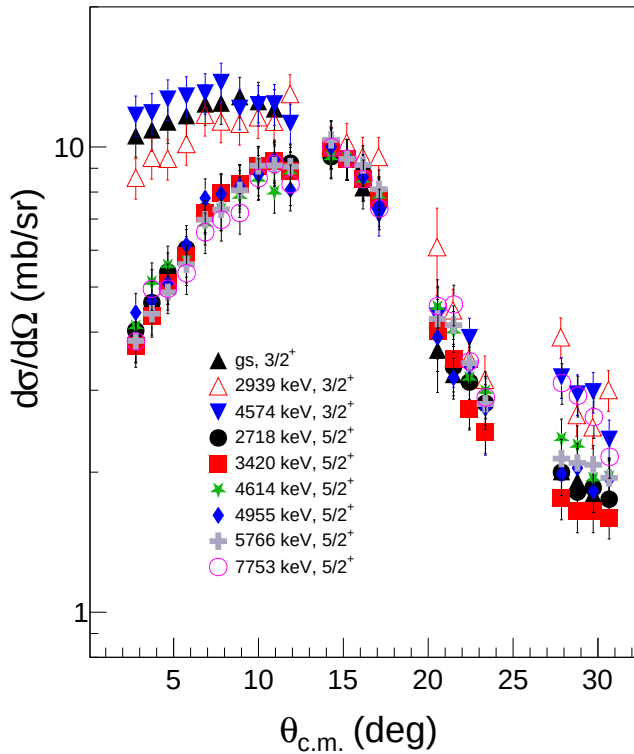


FIG. 3: Angular distributions of strong $3/2^+$ and $5/2^+$ states resulting from the $^{36}\text{S}(p, d)^{35}\text{S}$ reaction at $E_p = 66$ MeV. For ease of comparison, all the cross sections were normalized to the ground-state distribution at $\sim 15.2^\circ$.

nucleon potentials, which include quantified uncertainties [53] are used. The theoretical 1σ and 2σ bands for the cross sections are obtained by propagating the 416 sets of the KDUQ democratic posterior distributions [53] in the FR-ADWA model and considering a single-particle bound state wave function generated using a Woods-Saxon potential of range $r_0 = 1.25$ fm and diffuseness $a = 0.65$ fm. The resulting 1σ and 2σ theoretical bands for the transfer distributions for $^{36}\text{S}(p, d)$ and $^{40}\text{Ca}(p, d)$, after normalization to the transfer data, are shown in blue in Fig. 4. Note that the $^{40}\text{Ca}(p, d)$ data [16] have no uncertainties reported. A 10% uncertainty on all data points has been assumed in the analysis presented here. For both isotopes, the theoretical prediction reproduces well the shape of the transfer distribution populating the $3/2^+$ ground state (left panel) at forward angles. It is seen that the theory predictions overestimate the data at larger angles. This might indicate the importance of more complex reaction mechanisms, i.e., multi-step transfer, not included in the present model and/or unaccounted-for uncertainties in the optical potentials.

Interestingly, the theoretical predictions for $^{36}\text{S}(p, d)$ and $^{40}\text{Ca}(p, d)$ populating the $5/2^+$ first-excited state (right panel) reproduce partly the decrease of the cross sections at forward angles. This indicates that the FR-ADWA together with the KDUQ potentials predict quali-

tatively the j -dependence of the cross sections. Nevertheless, there are still discrepancies: the theory predictions overestimate (resp. underestimate) the forward angles in ^{36}S (resp. ^{40}Ca). These results suggest that 1) the FR-ADWA calculations using KDUQ capture partly the j -dependence of the transfer cross sections; 2) even when accounting for the uncertainties of the optical potentials, the theoretical and experimental transfer distributions for $^{36}\text{S}(p, d)^{35}\text{S}(5/2^+)$ are inconsistent within 1σ at forward angles; 3) the inclusion of the finite-range effects in the reaction model cannot fully resolve this discrepancy; and 4) this discrepancy is nucleus-dependent, i.e., the theoretical predictions overestimate the data for ^{36}S and underestimate them for ^{40}Ca .

As mentioned above, one main ingredient of FR-ADWA calculations are optical potentials. These potentials are used to generate the scattering wave functions; necessary to calculate the transfer T -matrix and cross sections. When one populates a $3/2^+$ or a $5/2^+$ state, contributions from scattering waves with different spins interfere. Since the spin-dependence of the optical potentials influences these scattering wave functions, it can also impact the shape of the angular distribution [28]. To test the effect of the optical potential spin-orbit terms on the transfer cross sections, the spin-orbit real and imaginary depths of all entrance and exit optical potentials have been multiplied by a constant factor. The distributions obtained when dividing the spin-orbit terms by two (green bands) or multiplying by two (red bands) are shown in Fig. 4.

By comparing the results populating the $3/2^+$ and $5/2^+$ states, one can see that the spin-orbit terms significantly impact the shape of the $5/2^+$ transfer distribution. In particular, they show that increasing the spin-orbit depths allows a better reproduction of the dip at forward angles for the $^{36}\text{S}(p, d)^{35}\text{S}(5/2^+)$, and of the larger angles for $^{36}\text{S}(p, d)^{35}\text{S}(3/2^+)$. On the contrary, decreasing the spin-orbit amplitude by a factor of two (green bands) almost removes the j -dependence and clearly worsens the agreement for the $(5/2^+)$ state.

For clarification, we are not arguing that such a simplistic adjustment of spin-orbit strengths should be adopted, but rather that these results suggest possible inaccuracies in the amplitude of the KDUQ spin-orbit terms. This is not surprising as the KDUQ parametrization was fitted on existing scattering data on stable nuclei, and the spin-orbit terms were mostly constrained with the few available polarization observables. Thus, the better agreement for $^{40}\text{Ca}(p, d)$ than for $^{36}\text{S}(p, d)$ can be understood by the fact that the KDUQ corpus of data includes scattering and polarization observables on ^{40}Ca but not on ^{36}S . This study hence highlights possible inaccuracies in current phenomenological optical models when used for nuclei that were not included in the calibration of their parameters and emphasizes the need for the development of more accurate optical model potentials.

Cross-checks were performed using both the corrected

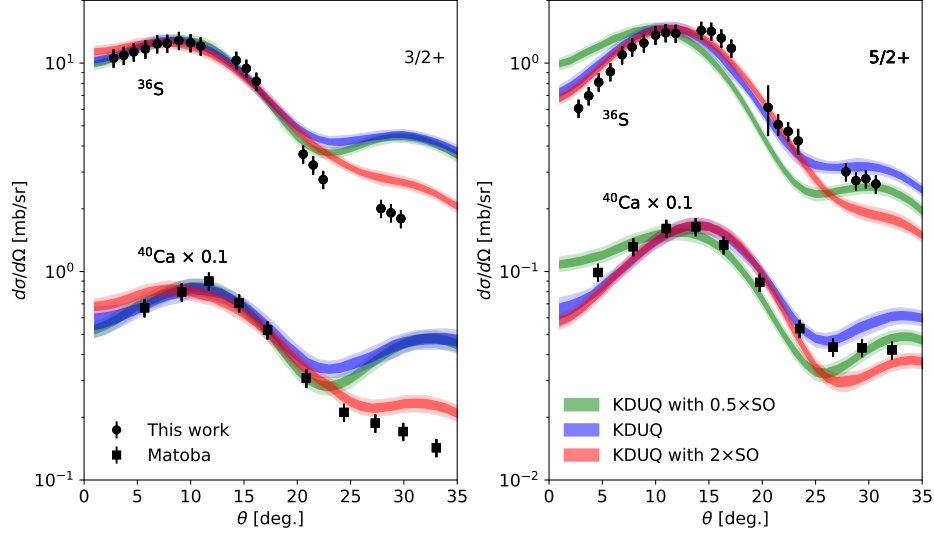


FIG. 4: Experimental cross sections and normalized FR-ADWA calculations for (left) the ground state of both the $^{36}\text{S}(p,d)$ and $^{40}\text{Ca}(p,d)$ reactions, and (right) for the 2.717 MeV $5/2^+$ state in ^{35}S and the 5.128 MeV $5/2^+$ state in ^{39}Ca . Data for the $^{40}\text{Ca}(p,d)$ reaction from Matoba *et al.* [16] were sourced from EXFOR [52] (subentry E1556005 for the 5.128 MeV state) and 10% uncertainties on all data points were assumed. The 1σ and 2σ (light and dark shaded) theoretical bands are obtained from the 416 samples of the KDUQ optical potentials [53]. The green, blue and red bands correspond to modified amplitudes of the spin-orbit potentials with respect to the KDUQ parametrization by factors of 1/2, 1 and 2, respectively.

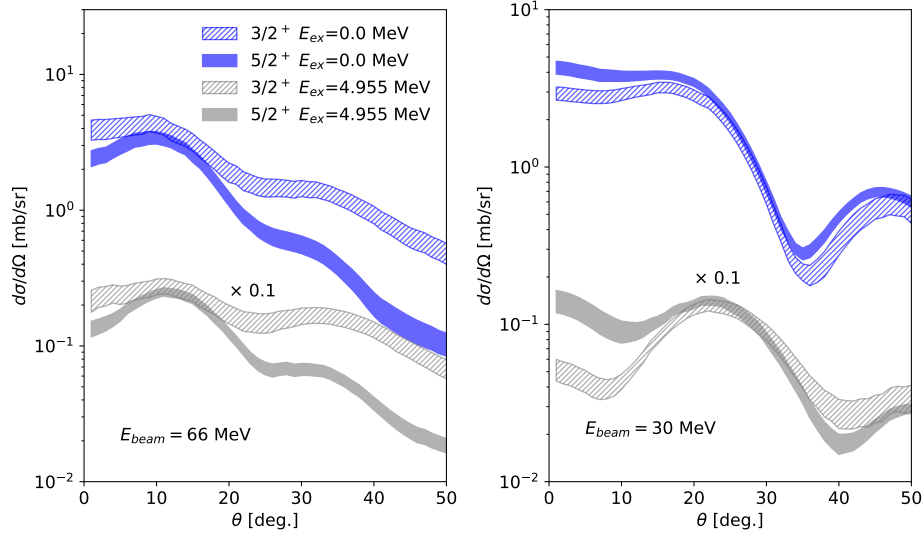


FIG. 5: Theoretical FR-ADWA transfer angular cross sections for $^{36}\text{S}(p,d)$ at 66 MeV (left) and 30 MeV (right). The 1σ theoretical bands are obtained from the 416 samples of the KDUQ optical potentials [53]. The different bands correspond to $3/2^+$ and $5/2^+$ states located at various excitation energies.

potentials following the procedure of Matoba *et al.* and those based on the Koning-Delaroche parameterization [55]. The resulting FR-ADWA and ZR-ADWA calculations show only minor variations at the most forward angles, mainly for the $3/2^+$ ground state, while overall differences in angular distributions and extracted spec-

troscopic factors are negligible, confirming the robustness of the analysis.

The consistency of the conclusions concerning the j -dependence with the work of Kozub *et al.* [22], in which they observe for $^{32}\text{S}(p,d)$ at 33.6 MeV a stronger increase in the measured cross section at forward angles for the

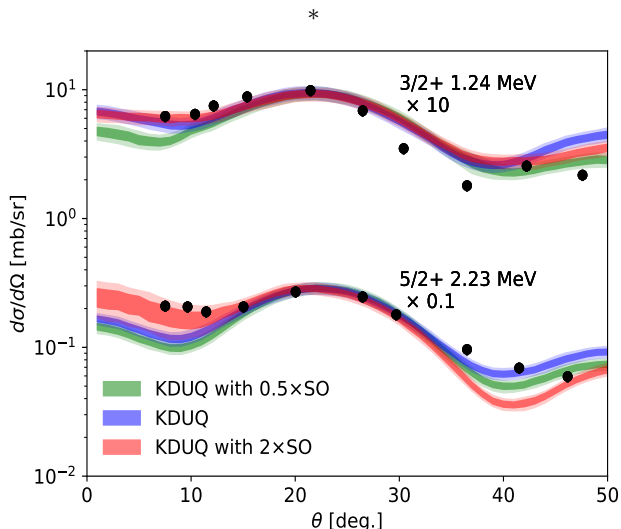


FIG. 6: FR-ADWA calculations for the $3/2^+$ state at 1.24 MeV and the $5/2^+$ state at 2.23 MeV in ^{31}S populated with the $^{32}\text{S}(p,d)$ reaction at 33.6 MeV beam energy.

$3/2^+$ states than $5/2^+$ states is now discussed. There are two main differences between the reaction analyzed in the present work and that in Ref. [22]: the beam energy, i.e., 66 MeV versus 33.6 MeV, and the Q -value of the reaction. To understand if both analyses can be reconciled, the calculations for $^{36}\text{S}(p,d)^{35}\text{S}(3/2^+)$ and $^{36}\text{S}(p,d)^{35}\text{S}(5/2^+)$ at 30 MeV using the KDUQ optical potentials and considering two different excitation energies were repeated (see Fig. 5). This figure shows two important features. First, the Q -value, which depends on the excitation energy of the final state of ^{35}S , has a greater influence on the shape of the distribution at forward angles for low beam energy, while its influence is more modest at higher beam energy. Second, the j -dependence of the transfer angular distributions manifests itself differently at the two beam energies considered here. This observation clarifies the j -dependence observed in the present work and in the study by Kozub *et al.* [22], although the sensitivity to j is less pronounced in their data. Together with the theoretical predictions for the $^{32}\text{S}(p,d)$ reaction shown in Fig. 6, these results contribute to an understanding of the observed j -dependence.

These additional tests also suggest that to make an accurate j assignment from transfer cross sections: 1) theoretical predictions are crucial, since the shape of the cross section depends on many features, i.e. beam energy, Q -value, and spin-parity of the final state; and 2) measuring several states with similar ℓj is necessary to infer systematics of the differential cross sections.

It is important to mention that other effects, not considered here, might influence the shape of the transfer angular distribution and be sensitive to the spin of the final state. Among these effects are multi-step paths, in which ^{36}S could be inelastically excited to its 2^+ state

before the neutron is transferred. This two-step mechanism will be sensitive to single-particle configurations of $^{36}\text{S}(2^+)$. Although likely instructive, such an analysis goes beyond the scope of this work.

Finally, upon closer inspection of existing datasets, more examples of this small-angle j -dependence may become apparent. Consider the recent study of the $^{37}\text{Ca}(p,d)$ transfer reaction from an odd- N nucleus, where the angular distributions for a $1d_{3/2}$ neutron removal leading to the ground state of ^{36}Ca differ at small angles from the $1d_{5/2}$ neutron removal leading to, e.g., the 6.54 and 7.84-MeV excited states (see Ref. [56] and its supplemental material).

B. IAS states

In an effort to have an accurate reconstruction of the effective single-particle energies of the different orbitals, the excitation-energy spectrum was explored to much higher values (~ 16 MeV) than is usually the case, allowing for all possible contributions from the different orbitals to be identified. In this and following sections the C^2S^- values were deduced from the ZR-ADWA calculations detailed in section III B. The $^{36}\text{S}(p,d)^{35}\text{S}$ reaction can produce states with two different isospin values $T = 3/2$ and $T = 5/2$, in contrast to the $^{40}\text{Ca}(p,d)^{39}\text{Ca}$ reaction where only $T = 1/2$ is possible. A decomposition into $T_>$ and $T_<$ is not possible from the data. However, states below 9 MeV in ^{35}S likely have $T = 3/2$, while those observed at 9.126, 11.47, 12.88, 13.68, and 14.22 MeV likely have $T = 5/2$, given that basic considerations of binding and Coulomb energy differences and proton-neutron mass differences suggest that the IAS of the ground state in ^{35}P should be at 9.017 MeV. Indeed, these five IAS are identified in ^{35}S through similar energy differences relative to the $1/2^+$ state at 9.126 MeV (within less than 100 keV) compared to the $T = 5/2$ states in the ^{35}P nucleus. These include the ground state ($1/2^+$) as well as the excited states at 2.387 ($3/2^+$), 3.860 ($5/2^+$), 4.664 ($5/2^+$), and 5.198 ($5/2^+$) of ^{35}P . These states in ^{35}P are typically populated through the one-proton removal reaction such as $^{36}\text{S}(-1p)^{35}\text{P}$ [4, 11] or $^{36}\text{S}(d,^3\text{He})^{35}\text{P}$ [57]. The spectroscopic values of the IAS states have been obtained by normalizing the calculations to the experimental differential cross sections. Since the same neutron orbitals as in the work of Ref. [16] are relevant in this study, the same parameters provided by Ref. [16] were also used for the IAS.

The values of C^2S^- for the $T = 5/2$, $3/2^+$ and $5/2^+$ states from the $^{36}\text{S}(p,d)^{35}\text{S}$ reaction have been obtained from cross-section data at two angles. However, as their spin assignment is known, spectroscopic factors can be derived and compared to those of the $T = 5/2$ states from the $^{36}\text{S}(d,^3\text{He})^{35}\text{P}$ reaction [35, 57]. It is known [58] that, for the case of neutron removal, the square of the Clebsch-Gordon coefficient $C^2(-1n)$ must be $1/(2T + 2)$, and in the case of proton removal, $C^2(-1p)$ amounts to $(2T +$

TABLE I: Comparison between the C^2S^- values for the $T = 5/2$ states with $j^\pi = 3/2^+$ (first row) and $5/2^+$ (following rows), obtained from the $^{36}\text{S}(d, ^3\text{He})^{35}\text{P}$ proton-removal reaction [57] (first three columns) and from the present $^{36}\text{S}(p, d)^{35}\text{S}$ neutron-removal reaction for IAS (last two columns).

^{35}P			^{35}S	
E_x (MeV)	$C^2S^-(-1p)$	$C^2S^-(-1p) / (2T+1)$	E_x (MeV)	$C^2S^-(-1n)$
2.387	0.31	0.052	11.47	0.052
3.860	2.91	0.486	12.88	0.489
4.664	1.06	0.177	13.68	0.156
5.198	1.38	0.230	14.22	0.247

$1)/(2T+2)$. In these relations, the ^{36}S nucleus has $T = 2$ and the $^{36}\text{S}(p, d)^{35}\text{S}$ reaction can therefore populate both $T = 3/2$ and $T = 5/2$ states. It is noted that the spectroscopic factor S , which describes the initial and final overlap of populated states in the reaction, is the same for both reactions. It follows, when comparing the present data for the $T = 5/2$ (IAS) states in ^{35}S to the $T = 5/2$ states in ^{35}P , that the expected ratio is $[1/(2T+2)] \div [(2T+1)/(2T+2)] = 1/(2T+1) = 0.167$. As shown in Table I, the $C^2S^-(-1p)$ values found in the $^{36}\text{S}(d, ^3\text{He})^{35}\text{P}$ reaction for the $3/2^+$ state amount to 0.31 at 2.387 MeV, while for the $5/2^+$ states values are 2.91, 1.06 and 1.38 at excitation energies, E_x , of 3.860, 4.664 and 5.198 MeV, respectively. When multiplied by the ratio of Clebsch-Gordon coefficients to account for the neutron- versus proton-removal reactions, a very good correspondence is found between the C^2S^- values of the $T = 5/2$ IAS, obtained from the neutron-removal reaction, and those of the $T = 5/2$ states derived from the proton-removal reaction [57], bearing in mind that these results have been obtained from different experimental probes and optical model potentials.

C. Determination of energy centroids

The sum of C^2S^- values for all states of a particular j^π , which relates to the occupancy of that orbit with a maximum value of $(2j+1)$ in the case of a fully occupied orbital, are listed in Table III and compared with results for the doubly magic nucleus ^{40}Ca . Systematic errors associated with the effective target thickness are indicated for the case of ^{36}S . The C^2S^- weighted mean energy for each orbit, obtained from

$$\bar{E}^-(j^\pi) = \frac{\sum E_x(j^\pi) C^2S^-(j^\pi)}{\sum C^2S^-(j^\pi)}, \quad (1)$$

and the centroid binding energy of the hole states obtained from

$$\langle \epsilon_{j^\pi}^- \rangle = -S_n(^{36}\text{S}) - \bar{E}^-(j^\pi), \quad (2)$$

are also listed. The summation is made for states having C^2S^- values as low as 1×10^{-3} up to $E_x = 9.5$ MeV in ^{39}Ca , and extended to 16 MeV in ^{35}S in order to include the contribution of the IAS.

For the determination of the binding energies $\langle \epsilon_{1/2^+} \rangle$ and $\langle \epsilon_{3/2^+} \rangle$ of the neutron $2s_{1/2}$ and $1d_{3/2}$ orbitals in ^{36}S and ^{40}Ca , one needs information from the neutron-removal and neutron-adding reactions, as these orbitals are located close to the valence fp orbitals. As for the more deeply bound $1d_{5/2}$ orbital, $\langle \epsilon_{5/2^+} \rangle$ can be reasonably well determined from the neutron-removal reaction only. Spectroscopic information related to the neutron-removal and -adding parts of ^{40}Ca was obtained using $^{40}\text{Ca}(p, d)^{39}\text{Ca}$ [16] and $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ [59] studies, respectively. As for ^{36}S , the neutron-adding spectroscopic information is known from the $^{36}\text{S}(d, p)^{37}\text{S}$ [42] study. The $^{36}\text{S}(p, d)^{35}\text{S}$ neutron-removal has never been measured prior to this work.

The centroid binding energy derived from the $^{36}\text{S}(d, p)^{37}\text{S}$ neutron-adding reaction is

$$\langle \epsilon_{j^\pi}^+ \rangle = -S_n(^{37}\text{S}) + \bar{E}^+(j^\pi), \quad (3)$$

with

$$\bar{E}^+(j^\pi) = \frac{\sum E_x(j^\pi)(2j+1)C^2S^+(j^\pi)}{\sum (2j+1)C^2S^+(j^\pi)}. \quad (4)$$

Combining the particle- and hole-strength information from Eqs. 2 and 3, the energy centroid can be expressed as

$$\langle \epsilon_{j^\pi} \rangle = \frac{\langle \epsilon_{j^\pi}^- \rangle \sum C^2S^-(j^\pi) + \langle \epsilon_{j^\pi}^+ \rangle \sum (2j+1)C^2S^+(j^\pi)}{\sum C^2S(j^\pi)}, \quad (5)$$

with

$$\sum C^2S(j^\pi) = \sum C^2S^-(j^\pi) + \sum (2j+1)C^2S^+(j^\pi). \quad (6)$$

Uncertainties related to the energy centroids originate from the choice of states used in the calculation of the binding energy of each orbit, specifically, whether tentatively assigned states are included in the calculation or not. The contribution of uncertainties associated with excitation energies of individual states to the binding energies is generally small (< 10 keV). Moreover, error bars related to the determination of the C^2S values due to the choice of optical model potential or to the uncertainty in the target thickness are correlated for all states and thus do not affect the binding-energy centroids. Consequently, the reported value of the energy centroids of the neutron-removal single-particle strength and binding energies reported in Table III were calculated as the average of the two calculations that either include or exclude tentatively assigned states, with uncertainties calculated to cover the range within one sigma. In the case of ^{39}Ca [16], there is only one reported $1d_{3/2}$ state and three $2p_{3/2}$ states, all definitively assigned with no tentative others.

Consequently, there are no uncertainties quoted for their energy centroids in Table III.

The results, obtained for each ℓ orbital, are now discussed in more detail.

1. $\ell = 0$

Four $\ell = 0$, $1/2^+$ states were found in ^{39}Ca [16] with $\sum C^2S^- = 1.79(17)$. The major strength is found at $E_x = 2.463$ MeV ($C^2S^- = 1.64(15)$) and the energy centroid at $\bar{E}^-(1/2^+) = 2.612(37)$ MeV. By summing this neutron-removal strength with the neutron-adding value of $C^2S^+ = 0.08$ [59], one obtains $\sum C^2S(1/2^+) = 1.87(17)$. Concerning ^{35}S , most of the $1/2^+$ strength is localized in the first state at $E_x = 1.569$ MeV ($C^2S^- = 1.28(14)$). A significant contribution of $C^2S^- = 0.348(30)$ is, however, also observed in the IAS at 9.126 MeV. In total, the summed value of $\sum C^2S^-(1/2^+) = 1.85(20)$ is very similar to that in ^{39}Ca , and the deduced energy centroid is $\bar{E}^-(1/2^+) = 3.650(13)$ MeV.

2. $\ell = 1, 3$

From the present experiment, it is determined that the intruder orbitals $0f_{7/2}$ and $1p_{3/2}$ are partially filled, as evidenced by their C^2S^- values of 0.29(3) and 0.12(1), respectively. This corresponds to neutrons that are scattered from the generally occupied sd orbitals to the valence fp orbits, removable by using the (p, d) reaction. It is remarkable that, despite using different experimental techniques and sets of optical model potentials, the derived number of scattered neutrons across the $N = 20$ gap from the sd to the pf shells, $\sum C^2S^-(fp)$, is very similar to the number of neutrons that can be added to the sd orbits $\sum C^2S^+(sd)$ determined from the (d, p) reaction. Indeed, from Table III for ^{36}S , $\sum C^2S^-(fp) = 0.43(3)$ compared to $\sum C^2S^+(sd) = 0.33$.

3. $\ell = 2$

Only the $3/2^+$ ground state of ^{39}Ca was populated in the neutron-removal reaction [16] with a strength of $\sum C^2S^-(3/2^+) = 3.74(20)$. The neutron-adding (d, p) reaction [59] to the four states at 2.009, 4.418, 5.095 and 5.411 MeV gives an additional contribution of $(2j+1)\sum C^2S^+(3/2^+) = 0.42$, leading in total to $\sum C^2S(3/2^+) = 4.16(20)$, compatible with the expected full $1d_{3/2}$ occupancy by 4 neutrons. Including this neutron addition shifts the binding-energy centroid, determined from the neutron-removal reaction, by about 1 MeV, leading to $\langle\epsilon_{3/2^+}\rangle = -14.577(20)$ MeV.

Overall, the $1d_{3/2}$ hole strength is more fragmented in ^{36}S compared to ^{40}Ca . However, the largest contributor is also the ground state with a C^2S^- value of 3.50(39), the other fragments each having $C^2S^- \leq 0.080$. The

summed strength $\sum C^2S^-$ amounts to 3.75(41), to which the neutron-adding (d, p) contribution is 0.33, giving rise to $\sum C^2S = 4.08(41)$, very similar to the value of 4.16(20) in ^{39}Ca . The neutron-adding part is extracted using the three states at 1.397, 3.355 and 3.666 MeV from Ref. [42]. Taking the full strength, the binding-energy centroid is $\langle\epsilon_{3/2^+}\rangle = -9.560(41)$ MeV, which is 702 keV less bound than the value of $\langle\epsilon_{3/2^+}^- \rangle$.

The strength of the $1d_{5/2}$ orbital is significantly spread in energy for ^{39}Ca . In ^{35}S , it is even more fragmented and distributed between $E_x = 2.718$ and the $5/2^+$ IAS at $E_x = 14.22$ MeV. The value of $\sum C^2S^-(5/2^+)$ is 3.87(43), which includes all tentatively assigned states. The $5/2^+$ neutron-adding strength is not considered here as there are no identified $5/2^+$ states in ^{37}S populated by means of the neutron-adding (d, p) reaction, and those identified in ^{41}Ca likely originate from the $2d_{5/2}$ orbital rather than from the deeply bound $1d_{5/2}$ orbital presently being studied. This hypothesis is further substantiated by the fact that the $5/2^+$ states populated in ^{41}Ca are located above the $9/2^+$ states from the $1g_{9/2}$ orbital and lie as much as 17-20 MeV away from the $\langle\epsilon_{5/2^+}\rangle$ centroid. It follows that the centroid binding energy can be derived directly in ^{36}S and ^{40}Ca from the neutron-removal part only, leading to $\langle\epsilon_{5/2^+}\rangle = -16.840(10)$ MeV in ^{36}S . The resultant energy differences between spin-orbit partners are listed in Table III.

Although it has no effect on the centroid value $\langle\epsilon_{5/2^+}\rangle$, it is interesting to compare the absolute value of the $\sum C^2S^-(5/2^+)$ strength between ^{39}Ca [16] and ^{35}S . As for ^{39}Ca , Ref. [16] reported a summed strength of 4.72. However, inconsistencies on the order of 10% in the cross section results of the prominent 5.128 MeV $5/2^+$ state in ^{39}Ca reported in Figs. 1 and 5 of Ref. [16] prompted a new measurement of the $^{40}\text{Ca}(p, d)$ reaction at iThemba LABS, to investigate the yield ratio of the prominent $5/2^+$ peaks in ^{39}Ca relative to the $3/2^+$ ground state. This measurement was performed at only two laboratory scattering angles (8° , 16°) at an incident beam energy of 66 MeV. We have confirmed a systematic 10% yield ratio reduction for the most prominent $5/2^+$ peaks in ^{39}Ca . By taking this factor into account as well as states with firm $5/2^+$ assignments only, the $\sum C^2S^-(5/2^+)$ value is found to be about the same in ^{39}Ca (3.78) as in ^{35}S (3.61(40)).

4. Quenching of the $1d_{5/2}$ strength

Based on our normalization of the summed spectroscopic factor strength for the sdf states to 6, we should in principle obtain a summed value for the $5/2^+$ states of $(2J+1) \simeq 6$. Yet, similar to the work of Matoba *et al.* [16], we observe a deficit of about 35-40% (depending on the number of adopted $5/2^+$ states) for the deeply bound $1d_{5/2}$ orbital, that is to be added to the commonly used quenching factor of 0.6 [60]. A very similar additional quenching factor was also observed for the deeply-bound

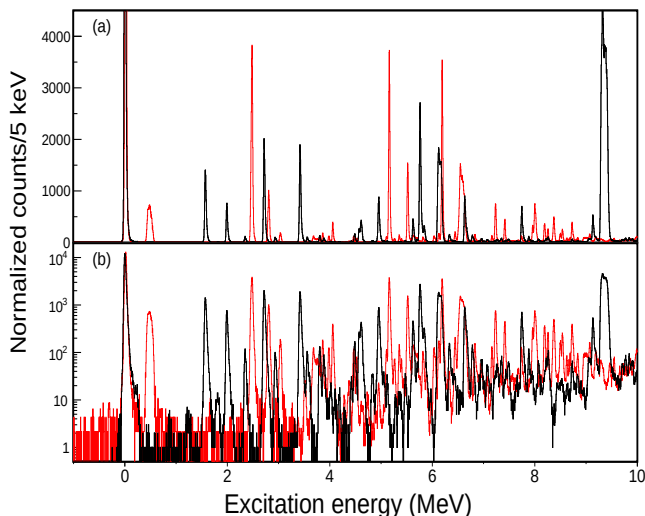


FIG. 7: Excitation energy spectra for the $^{36}\text{S}(p, d)$ (in black) and $^{40}\text{Ca}(p, d)$ (red) reactions measured at 66 MeV incident energy at the K=600 magnetic spectrometer at iThemba LABS. The spectra, normalized on the ground state, represent data acquired at 15° for ^{36}S and 16° for ^{40}Ca , with a circular aperture that covers up to 2° on either side of the indicated spectrometer central angle. Plots (a) and (b) illustrate the same data, plotted on linear and logarithmic scales, respectively, to highlight the good background conditions of the measurements as well as the growth of a continuous component starting at ~ 4.5 MeV.

proton $1d_{5/2}$ orbital, populated through the proton removal reactions $^{48}\text{Ca}(d, ^3\text{He})$ [61] and $^{48}\text{Ca}(e, e'p)$ [60], although this deficit of strength may be due to the limitation in the excitation energy range studied (around 9 MeV). This significant amount of missing strength, also observed for the proton $1d_{5/2}$ orbital in ^{48}Ca [62], is discussed as well in Table 3 of Ref. [3].

Another case of interest is the ratio of the C^2S^- values between the 6 MeV $3/2^-$ state and $1/2^-$ ground state in ^{15}O , both populated by the $^{16}\text{O}(p, d)^{15}\text{O}$ reaction. This ratio amounts to $\simeq 1$ when the transfer reaction is performed at a “low” energy of 45 MeV [63]. It is at variance with the value of about 2 expected when assuming that the respective occupancies of the $1p_{3/2}$ and $1p_{1/2}$ orbitals scale with $(2J + 1)$. A similar conclusion on the added quenching factor of a deeply bound orbital can be found from the work of Kawase *et al.* [64]. There, the quenching factors for the deeply bound $1p_{3/2}$ and $1p_{1/2}$ ground state, obtained in the $^{14}\text{O}(p, 2p)^{13}\text{N}$ reaction at the quasi-free condition, are about 0.55 and 0.75, respectively. The extra quenching of $\sim 30\%$ for the deeply bound $1p_{3/2}$ orbital is similar to the value we find based on the ratio of the summed $5/2^+$ over the $3/2^+$ strength.

This can be understood by the limited excitation energies that can be accessed by the experiments, causing more apparent quenching of the deeply-bound orbital.

To illustrate this, we consider the spectral function for the neutron in ^{40}Ca below the Fermi energy, calculated within the Dispersive Optical Model (DOM) in Fig. 2 of Ref. [65]. That work constrains the self-energy from which the spectral function can be obtained, with scattering data, bound-state energies, charge density and particle number. Fig. 2 of that work shows a large spreading of the hole strength for the deeply bound $1d_{5/2}$ orbital, in contrast to that of the $1d_{3/2}$ orbital that is mostly contained in a narrow single peak. Consequently, when integrating the theoretical hole spectral strength up to an excitation energy of about 9 MeV, as done in the experimental work of Matoba *et al.* [16] or in Fig. 7 of our work, an approximately 15% weaker fraction of the $1d_{5/2}$ is obtained, as compared to that of the $1d_{3/2}$ orbital. This added quenching value of 15% for the most deeply bound orbital is smaller than the one of about 35% observed experimentally. This difference can come from the fact that the DOM calculations are not sufficiently constrained by data that exhibit fragmentation of deeply bound states in order to trust in a quantitative value for this additional quenching. Moreover, this can also be due to the fact that the experimental spectrum of the neutron-removal reaction ^{40}Ca (Fig. 7) already displays a continuum of states, rather than only well defined peaks, above 5 MeV excitation energy. Note that interestingly, the extra-quenching of the $d_{5/2}$ states does not impact significantly the value of the $1d_{5/2} - 1d_{3/2}$ SO splitting deduced from Ref. [65]. Indeed for ^{40}Ca we find that the result changes by only about 100 keV, regardless of whether a bounding excitation-energy condition is imposed on the spectral strength or the spectrum is truncated at 10 MeV.

In Fig. 7 we also notice that both ^{40}Ca and ^{36}S display a similar continuous strength which starts at the same energy and with the same slope. This suggests that the extra-quenching of the $d_{5/2}$ orbital in ^{36}S may be explained similarly as for ^{40}Ca , with a similar consequence on its determination of the $1d_{5/2} - 1d_{3/2}$ SO splitting.

D. Comparison to model calculations

Regardless of the missing $\sum C^2S^-(5/2^+)$ strength, it is nonetheless very instructive to compare the experimental strength distribution of $5/2^+$ neutron-removal states with theoretical model predictions. This is addressed below using the Large-Scale Shell Model (LSSM) and *ab initio* approaches.

1. Large-Scale Shell-Model calculations

Large-Scale Shell-Model (LSSM) calculations are performed using an ^{28}O core and a valence space consisting of the sd shell for protons and $sd - pf$ orbitals for neutrons, allowing core excitations across the $N = 20$ shell gap. The effective interaction is the SDPF-MIX20

interaction, which is a version of the SDPF-MIX interaction [8, 66], with better constraints toward neutron-rich ^{28}O and the Island of Inversion at $N = 20$ [2, 67, 68]. The spectroscopic sum rule states are computed for the neutron $1d_{5/2}$ and $1d_{3/2}$ removal from ^{36}S and then the fragmentation into physical states in ^{35}S is generated. The calculations use the Lanczos Structure Function method [69] over 300 final states and include up to 4 particle-hole excitations across $N = 20$ for ^{36}S and ^{35}S . The results are presented in Fig. 8. The detailed fragmentation of the spectroscopic strength compares well with the present experimentally observed strength but with an overall excess in the theoretical sum rule for the $5/2^+$ states. Using a bare one-particle removal operator, the total theoretical C^2S^- value amounts to 5.97, to be compared with the present experimental extraction of 3.61(40) for $5/2^+$ states with well-defined assigned spin values. In the lower panel of Fig. 8, we observe a progressive weakening of the experimental spectroscopic strength as the excitation energy (or binding energy) increases for $T = 3/2$ states. This is in line with several observations outlined in the previous section that revealed more quenching for deeply-bound states. Conversely, the experimental strength located around 13-14 MeV, which corresponds to the $T = 5/2$ states, is comparable to theory.

2. Ab-initio calculations

Ab-initio calculations employ the self-consistent Green's function (SCGF) approach, which tackles the A -body problem via a correlation expansion truncated to low orders to produce systematically improvable solutions [70]. In the present work, the algebraic diagrammatic construction scheme at second order (ADC(2)) [71, 72] is adopted starting from two- and three-nucleon forces derived within the framework of chiral effective field theory. Specifically, four different models of the nuclear Hamiltonian are employed: (i) the NNLO_{sat} Hamiltonian [73], characterized by a simultaneous fit of all coupling constants on binding energies and charge radii of selected carbon and oxygen isotopes, in addition to two-, three- and four-nucleon observables; (ii-iii) the Δ NNLO_{GO} Hamiltonian [74], which explicitly includes Δ -isobar degrees of freedom and is fitted on few-body observables as well as nuclear matter, available with two different momentum cutoffs, $\Lambda = 394$ and 450 MeV; and (iv) the 1.8/2.0 (EM 7.5) Hamiltonian [75], composed of a low-resolution two-body interaction combined with three-nucleon forces fitted to the ground-state energies of ^3H and ^{16}O as well as the charge radius of ^{16}O . More details on the computational scheme and the associated evaluation of uncertainties can be found in Ref. [76].

While a SCGF calculation usually targets the A -body ground state of an even-even nucleus, the spectral representation of the one-body propagator at the same time gives access to energies and spectroscopic probabilities

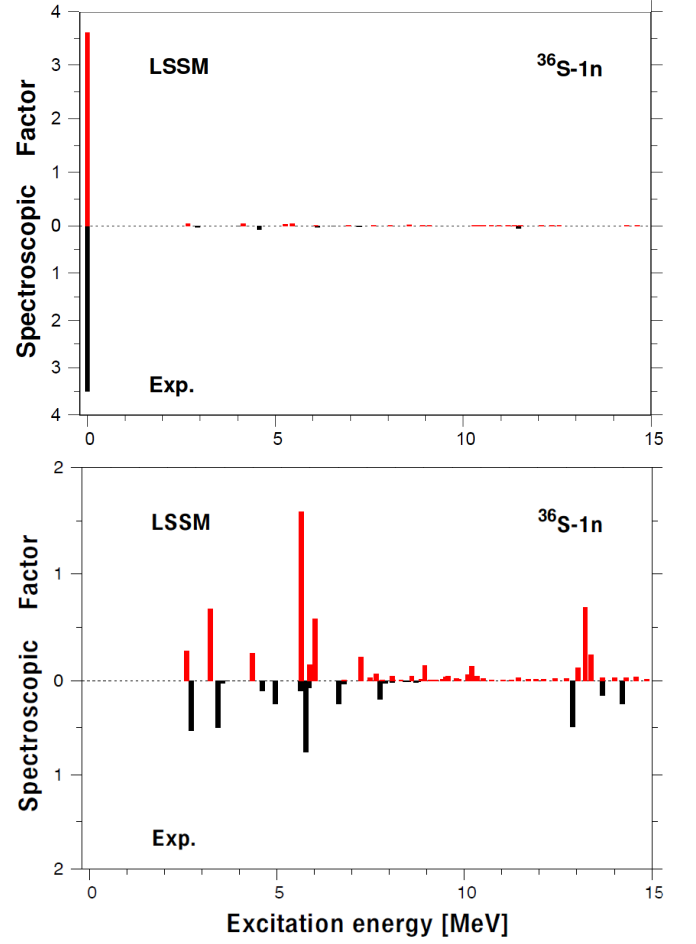


FIG. 8: Comparison of experimental and LSSM $d_{3/2}$ (top) and $d_{5/2}$ (bottom) spectroscopic factors.

associated with the $A \pm 1$ -body eigenstates [70]. These quantities are then typically combined to build the spectral function S . Its diagonal part $S^{-,hh}$, for a given one-nucleon removal state h , is the quantity that can be most meaningfully compared with the experimental strength distribution in the corresponding channel [77].

Figure 9 shows the diagonal spectral function $S^{-,hh}(E_x)$ for $h = 1d_{5/2}$ computed within SCGF-ADC(2) using the four interactions (i)-(iv). For comparison, the strength distribution from the $^{36}\text{S}(p,d)^{35}\text{S}$ neutron-removal reaction is displayed along the negative y-axis. The x-axis scans the excitation energies, E_x , of $j^\pi = 5/2^+$ final states relative to the $j^\pi = 3/2^+$ ground state in ^{35}S . For all interactions, the theoretical spectral function shows substantial fragmentation, with the majority of the strength being concentrated around $E_x \approx 4 - 6$ MeV, i.e. between two main experimental peaks. All calculations, however, underestimate the energy of the highest fragment if compared with the measured $E_x = 5.99$ MeV. The associated spectroscopic factors, around $C^2S^- \approx 1.5 - 2$, are globally higher than

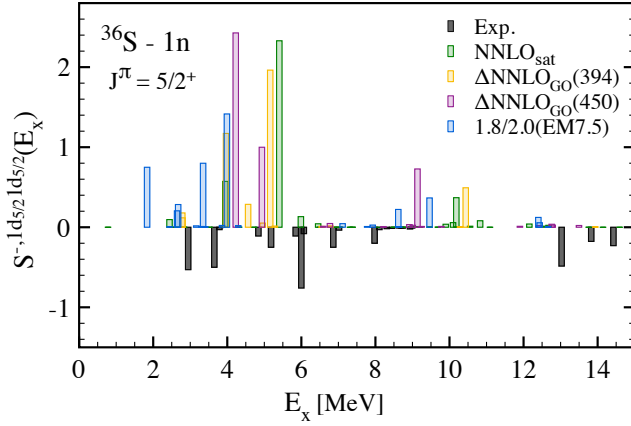


FIG. 9: Diagonal part of the $1d_{5/2}$ spectral function for one-neutron removal from the ^{36}S ground state to $j^\pi = 5/2^+$ final states in ^{35}S . SCGF-ADC(2) calculations using four different ab-initio interactions are shown on the positive y-axis; the strength distribution derived in the experimental analysis is displayed along the negative y-axis. On the x-axis, E_x represents excitation energies with respect to the $j^\pi = 3/2^+$ ground state of ^{35}S .

those resulting from the experimental analysis, indicating that the ADC(2) calculations still lack some important correlations. Unsurprisingly, the degree of fragmentation is inversely proportional to the resolution scale of the employed interaction, with the $\Delta\text{NNLO}_{\text{GO}}(394)$ and 1.8/2.0 (EM 7.5) generating more states than the other two Hamiltonians. In all cases, similarly to LSSM calculations, in SCGF no missing strength is observed for $j^\pi = 5/2^+$ compared to $j^\pi = 3/2^+$ states, with both total occupancies being close to their maximum values. While all calculations show some strength around $E_x \approx 10$ MeV, none of them can successfully reproduce the IAS with $E_x \gtrsim 13$ MeV. This last feature, as well as the overall degree of fragmentation, is expected to be significantly improved if one moves to a higher many-body truncation level, as suggested by the SCGF-ADC(3) results discussed in Ref. [77].

E. $N = 20$ shell closure

The existence of the $N = 20$ magic number is well known to be characterized by a large energy gap between the normally occupied $2s_{1/2}$ and $1d_{3/2}$ orbitals below $N = 20$, and the valence $2p_{1/2}$, $2p_{3/2}$ and $1f_{7/2}$ orbitals above $N = 20$. When present, such large gaps lead to well-known properties such as a sudden increase of the 2^+ state energy, a decrease of transition probability $B(E2)$, a drop in one or two-neutron separation energies above $N = 20$, to name a few. A detailed experimental review can be found in Section VI of Ref. [1].

When experimentally possible, a more stringent probe of the effectiveness of a shell closure is the determina-

tion of the Fermi surface of a nucleus, that is the fraction of occupancy of orbitals located below and above the Fermi energy. An extreme case of magicity would be the full occupancy of all (sd) orbitals and no occupancy of the (fp) orbitals, which would yield a Fermi surface with the shape of a step function. However, such an idealized case does not exist as nuclei are correlated systems and exhibit a more or less diffuse Fermi function, which characterizes the degree of magicity. Such studies require neutron-removal or neutron-adding reactions, such as (p, d) or (d, p) transfer reactions, with high accuracy, as for instance described in Ref. [15] for probing the $N = 20$ and $N = 28$ shell gaps in ^{40}Ca and ^{48}Ca , respectively.

As can be derived from Table III, the fraction of occupancy in the upper fp shell over the sum of the full sd and fp shells is weak for both nuclei, at 2.7% for ^{40}Ca and 4.6% for ^{36}S . This new result points to a significant and similar $N = 20$ shell closure and magicity for both ^{40}Ca and ^{36}S . Owing to the weak correlation observed for ^{36}S , the amplitude of its $N = 20$ gap can be estimated to be 5.58 MeV from the $S_n(^{36}\text{S}) - S_n(^{37}\text{S})$ difference.

Other experiments were carried out to probe the neutron Fermi surface below ^{36}S using neutron-knockout reactions at intermediate energy on a Be target. Although there are some uncertainties related to the competition between the knockout and two-step reactions in populating negative-parity states [9, 11, 78], the fraction of intruder states has been found to be 16(7)% in ^{34}Si [9] and as much as 30(5)% in ^{32}Mg [12], which is located in the Island of Inversion.

V. CONCLUSION

The $^{36}\text{S}(p, d)^{35}\text{S}$ reaction was studied at iThemba LABS to probe the nuclear structure of ^{36}S . Using the K=600 high-resolution magnetic spectrometer and an innovative target-encapsulation technique to prevent contamination, 98 states in ^{35}S , including Isobaric Analog States (IAS), were identified up to an excitation energy of 16 MeV; 47 of these had not been observed previously. Angular distributions and spectroscopic factors were extracted for 81 of these levels. The C^2S^- values for the $T = 5/2$ IAS located above 9 MeV have been obtained using the same optical potential treatment as other $T = 3/2$ states. They agree with spectroscopic strengths deduced from the proton-removal reaction from the same nucleus, after applying the expected coefficient of $1/(2T+1)$, with $T=2$ being the isospin value of ^{36}S . This result also shows the need to include these IAS components for the accurate determination of single-particle centroids of states.

A j -dependence in the angular distributions of $\ell = 2$ states has been observed, demonstrating that the neutron-removal (p, d) reaction provides additional spectroscopic sensitivity, as this j -dependence enables unambiguous assignments of j^π values for $\ell = 2$ levels. Finite-range adiabatic distorted-wave approximation cal-

culations using the KDUQ global optical model qualitatively reproduce the observed j -dependence at intermediate beam energies and small detection angles. Further experimental and theoretical studies, especially to constrain the amplitude of the spin-orbit term in the optical potential, are highly desirable to explore the underlying mechanism.

The summed spectroscopic strength for the $1d_{5/2}$ orbitals shows significant fragmentation and comparison with large-scale shell-model and ab-initio self-consistent Green's function results shows good overall agreement, with the latter calculations displaying substantial fragmentation but failing to reproduce the exact position of the main experimental peaks (including the IAS). The agreement with theoretical strengths holds despite the experimentally observed quenching at high excitation energies, apart from the IAS contribution. The cause of this quenching has not yet been identified, but the same effect has been reported in other studies involving the neutron or proton $d_{5/2}$ orbital. Further work is required to understand its origin.

Data from the (p, d) reaction allow the magicity of nuclei to be probed by determining orbital occupancies across the Fermi surface. With more than 95% occupancy of the valence sd shell and less than 5% contribution from fp orbitals, the robustness of the $N = 20$ shell closure in ^{36}S is confirmed. This approach, which provides one of the best probes of the rigidity of a shell closure, can be applied for an improved study of shell evolutions throughout the chart of the nuclides. There exist already successful attempts at the $N = 8$ and $N = 20$ shell gaps.

Finally, the advantage of the (p, d) reaction over the (d, p) reaction, which is typically limited to lower beam energies due to deuteron breakup, is that it can be employed at medium beam energies where the j -dependent phenomenon is observed that allows for j^π identification of states. This reaction may therefore be instrumental in the systematic experimental study of the role of the spin-orbit and tensor forces in inducing changes to the spin-orbit splitting across the chart of nuclides, including nuclei accessible with radioactive ion beams. The mechanism through which the tensor force impacts the spin-orbit splitting [79] should result, in most cases, in a decrease of the spin-orbit splitting with increase in mass that exceeds the impact of the sole action of the spin-orbit force.

ACKNOWLEDGMENTS

We extend our gratitude to W.H. Dickhoff and A.O. Macchiavelli for valuable discussions and feedback. Additionally, our appreciation goes to C. Vandevoorde for the support during the experimentation phase. M.W. is grateful for insightful discussions about the IAS with the late J. Schiffer. O.S. thanks A. Brown for important discussions related to the expected fraction of

isospin values neutron-removal reactions. The authors thank the Accelerator and Technical Services Department of iThemba LABS for providing stable running conditions during the experiment. This work is based on research supported in part by the National Research Foundation of South Africa (Grant Numbers: 118846, 85509, REP_SARC180529336567) and through contributions by iThemba LABS. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Contract No. DE-AC02-05CH11231 (LBNL) and the U.S. Nuclear Data Program. SCGF calculations were performed using HPC resources from GENCI-TGCC (Contracts No. A0130513012 and A0170513012). The FR-ADWA calculations relied on iCER and the High Performance Computing Center at Michigan State University for computational resources.

REFERENCES

- [1] T. Otsuka, A. Gade, O. Sorlin, T. Suzuki, and Y. Utsuno, Evolution of shell structure in exotic nuclei, *Rev. Mod. Phys.* **92**, 015002 (2020).
- [2] F. Nowacki, A. Obertelli, and A. Poves, The neutron-rich edge of the nuclear landscape: experiment and theory., *Prog. Part. Nucl. Phys.* **120**, 103866 (2021).
- [3] O. Sorlin and M.-G. Porquet, Nuclear magic numbers: New features far from stability, *Prog. Part. Nucl. Phys.* **61**, 602 (2008).
- [4] M. Wiedeking, E. Rodriguez-Vieitez, P. Fallon, M. P. Carpenter, R. M. Clark, D. Cline, M. Cromaz, M. Descovich, R. V. F. Janssens, I.-Y. Lee, M.-A. Deleplanque, A. O. Macchiavelli, F. S. Stephens, R. Teng, X. Wang, D. Ward, C. Y. Wu, S. Zhu, T. Otsuka, Y. Utsuno, and A. Volya, Intruder excitations in ^{35}P , *Phys. Rev. C* **78**, 037302 (2008).
- [5] P. C. Bender, C. R. Hoffman, M. Wiedeking, J. M. Allmond, L. A. Bernstein, J. T. Harke, D. L. Bleuel, R. M. Clark, P. Fallon, B. L. Goldblum, T. A. Hinnners, H. B. Jeppesen, S. Lee, I.-Y. Lee, S. R. Leshner, A. O. Macchiavelli, M. A. McMahan, D. Morris, M. Perry, L. Phair, N. D. Scielzo, S. L. Tabor, V. Tripathi, and A. Volya, Approaching the “island of inversion”: ^{34}P , *Phys. Rev. C* **80**, 014302 (2009).
- [6] F. Rotaru, F. Negoita, S. Grévy, J. Mrazek, S. Lukyanov, F. Nowacki, A. Poves, O. Sorlin, C. Borcea, R. Borcea, A. Buta, L. Cáceres, S. Calinescu, R. Chevrier, Z. Dombrádi, J. M. Daugas, D. Lehbertz, Y. Penionzhkevich, C. Petrone, D. Sohler, M. Stanoiu, and J. C. Thomas, Unveiling the intruder deformed 0_2^+ state in ^{34}Si , *Phys. Rev. Lett.* **109**, 092503 (2012).
- [7] R. Han, X. Li, W. Jiang, Z. Li, H. Hua, S. Zhang, C. Yuan, D. Jiang, Y. Ye, J. Li, Z. Li, F. Xu, Q. Chen, J. Meng, J. Wang, C. Xu, Y. Sun, C. Wang, H. Wu, C. Niu, C. Li, C. He, W. Jiang, P. Li, H. Zang, J. Feng, S. Chen, Q. Liu, X. Chen, H. Xu, Z. Hu, Y. Yang, P. Ma, J. Ma, S. Jin, Z. Bai, M. Huang, Y. Zhou, W. Ma, Y. Li, X. Zhou, Y. Zhang, G. Xiao, and W. Zhan, Northern boundary of the “island of inversion” and triaxiality in ^{34}Si , *Physics Letters B* **772**, 529 (2017).

TABLE II: Observed states in ^{35}S at excitation energies E_x with spin and parity j^π as well as neutron-removal spectroscopic factors C^2S^- determined in this work. Excitation energies as well as spin and parity assignments from Ref. [35] are also shown. For states with $j^\pi = (1/2^-, 3/2^-)$, the C^2S^- values have been calculated on the basis of a $3/2^-$ assignment. Since absolute spectroscopic factor values strongly depend on the choice of binding potential parameters, the present values were obtained with the r_0 and a_0 parameters of Ref. [16], chosen so that the summed C^2S^- value for the $3/2^+$, $1/2^+$, and $7/2^-$ states equals 6. A systematic uncertainty of 11.2% is associated with all spectroscopic factors, due to target thickness uncertainties as discussed in the text.

E_x (MeV) (ENSDF [35])	E_x (MeV) (this work)	j^π (ENSDF [35])	j^π (this work)	C^2S^- (this work)
0	0.000 (5)	$3/2^+$	$3/2^+$	3.50
1.572378 (19)	1.569 (5)	$1/2^+$	$1/2^+$	1.28
1.99128 (5)	1.990 (5)	$7/2^-$	$7/2^-$	0.17
2.347789 (18)	2.348 (5)	$3/2^-$	$3/2^-$	0.033
2.71700 (3)	2.718 (5)	$5/2^+$	$5/2^+$	0.53
2.93864 (5)	2.937 (5)	$3/2^+$	$3/2^+$	0.034
3.4211 (3)	3.420 (5)	$5/2^+$	$5/2^+$	0.50
3.55809 (3)	3.558 (5)	$(3/2^-, 5/2^+)$	$(5/2^+)$	0.03
3.5948 (4)	3.592 (5)	$(1/2:7/2)^+$	$(5/2^+)$	0.004
3.801958 (23)	3.800 (5)	$3/2^-$	$3/2^-$	0.02
3.8853 (4)	3.883 (5)	$(3/2^-, 5/2)$	$7/2^-$	0.018
4.0259 (4)	4.023 (5)	$(1/2:5/2)^+$	$(3/2^+)$	0.0056
4.1064 (4)	4.107 (7)	$(1/2, 3/2, 5/2)^+$	$(3/2^+)$	0.0023
4.189280 (21)	4.182 (5)	$1/2^-$	$1/2^-$	0.001
4.3026 (7)	4.300 (7)	$(1/2:5/2, 7/2^-)$	$(1/2^-, 3/2^-)$	0.0003
4.47764 (7)	4.486 (7)	$(1/2, 3/2, 5/2)^+$	$7/2^-$	0.045
4.576 (8)	4.574 (5)	$(1/2:5/2)^+$	$3/2^+$	0.08
4.617 (10)	4.614 (5)		$5/2^+$	0.11
4.839 (8)	4.838 (5)	$(1/2:9/2)^+$	$(1/2^+)$	0.0035
4.903 (8)	4.907 (5)	$1/2^-$	$1/2^+$	0.011
4.96309 (3)	4.955 (5)	$3/2^-$	$5/2^+$	0.25
5.127 (10)	5.121 (5)	$(1/2:9/2)^+$	$1/2^+$	0.0092
-	5.286 (5)		$(1/2^-, 3/2^-)$	0.00065
5.475 (10)	5.476 (5)		$3/2^+$	0.009
-	5.630 (5)		$5/2^+$	0.11
5.771 (10)	5.766 (5)	$(1/2:9/2)^+$	$5/2^+$	0.76
5.8413 (10)	5.844 (5)	$(1/2:7/2^-)$	$(5/2^+)$	0.08
6.129 (10)	6.121 (5)	$(1/2, 3/2, 5/2)^+$	$(3/2^+)$	0.035
-	6.218 (5)		$(7/2^-)$	0.003
6.334 (8)	6.338 (5)		$1/2^+$	0.044
6.446 (8)	6.439 (5)		$(1/2^+)$	0.0026
6.5451 (13)	6.550 (5)		$3/2^+$	0.0035
6.6352 (13)	6.639 (5)		$5/2^+$	0.25
6.684 (9)	6.682 (5)	$(1/2:9/2)^+$	$1/2^+$	0.056
-	6.790 (5)		$(5/2^+)$	0.037
6.98610(3)	6.962 (5)	$1/2^+$	$(1/2^-, 3/2^-)$	0.005
7.01895 (4)	7.019 (5)		$(1/2^-, 3/2^-)$	0.0015
7.10079 (4)	7.102 (5)	$3/2$	$(1/2^-, 3/2^-)$	0.002
7.14333 (11)	7.151 (5)		$(7/2^-)$	0.009
7.2181 (4)	7.215 (5)		$3/2^+$	0.012
7.2532 (6)	7.249 (5)		$(3/2^+)$	0.012
7.27599 (5)	7.279 (5)	$1/2^+$	$1/2^+$	0.0068
7.33106 (5)	7.326 (5)	$1/2^+$	$1/2^+$	0.0073
7.34397 (20)	7.349 (5)		$(1/2^+)$	0.0095
7.44221 (5)	7.441 (5)	$1/2^+$	$1/2^+$	0.0094
7.48122 (7)	7.490 (5)	$1/2^-$	$1/2^-$	0.006
7.7498 (4)	7.753 (5)		$5/2^+$	0.20
7.8808 (12)	7.885 (5)		$(5/2^+)$	0.03
7.97427 (11)	7.980 (7)	$3/2^-$	$3/2^-$	0.002

TABLE II continued^a

E_x (MeV) (ENSDF [35])	E_x (MeV) (this work)	j^π (ENSDF [35])	j^π (this work)	C^2S^- (this work)
8.078 (3)	8.073 (7)		(5/2 ⁺)	0.018
8.093 (5)	8.103 (5)	1/2 ⁻	1/2 ⁻	0.0017
8.2243 (4)	8.221 (5)	1/2 ⁻	1/2 ⁻	0.004
8.2734 (4)	8.270 (5)	1/2	1/2 ⁻	0.009
8.4189 (5)	8.410 (7)	(1/2:9/2) ⁺	(5/2 ⁺)	0.01
-	8.465 (7)		(5/2 ⁺)	0.01
-	8.509 (5)		5/2 ⁺	0.012
-	8.557 (7)		(7/2 ⁻)	0.003
-	8.602 (5)		(7/2 ⁻)	0.01
-	8.649 (7)		(7/2 ⁻)	0.0065
-	8.707 (5)		(5/2 ⁺)	0.022
-	8.761 (7)		(5/2 ⁺)	0.011
-	8.822 (5)		(1/2 ⁻ , 3/2 ⁻)	0.006
-	8.876 (5)		(1/2 ⁻ , 3/2 ⁻)	0.0025
-	8.947 (7)		(1/2 ⁻ , 3/2 ⁻)	0.001
-	9.032 (15)		(1/2 ⁻ , 3/2 ⁻)	0.007
-	9.065 (15)		(1/2 ⁻ , 3/2 ⁻)	0.007
-	9.126*(7)		1/2 ⁺	0.348
9.155 (10)	9.180 (5)	(1/2:9/2) ⁺	1/2 ⁺	0.06
-	9.219 (15)		(1/2 ⁻ , 3/2 ⁻)	0.003
-	9.294 (8)		(1/2 ⁻ , 3/2 ⁻)	0.01
-	9.339 (8)		(1/2 ⁻ , 3/2 ⁻)	0.002
-	9.395 (5)		(1/2 ⁻ , 3/2 ⁻)	0.0045
-	9.446 (7)		(1/2 ⁻ , 3/2 ⁻)	0.0018
-	9.510 (10)		(1/2 ⁻ , 3/2 ⁻)	0.0035
-	9.565 (20)		(1/2 ⁻ , 3/2 ⁻)	0.0033
-	9.615 (10)		(1/2 ⁻ , 3/2 ⁻)	0.0028
-	9.666 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	9.716 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	9.774 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	9.849 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	9.911 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	9.989 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.049 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.166 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.225 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.277 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.350 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.421 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.493 (10)		(7/2 ⁻)	0.02
-	10.563 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.663 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.746 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	10.810 (10)		(1/2 ⁻ , 3/2 ⁻)	-
-	11.47*(2)		3/2 ⁺	0.052
-	12.88*(10)		5/2 ⁺	0.489
-	13.68*(10)		5/2 ⁺	0.156
-	14.22*(10)		5/2 ⁺	0.247
-	14.70(10)	-	-	-

^a States marked with an asterisk (*) are identified as isobaric analog states (IAS).

TABLE III: A comparison of $C^2S^{-(+)}$ values, the energy centroids of the single-particle strength (weighted mean energies) $\bar{E}^{-(+)}$, and the binding energy values $\langle\epsilon_{j\pi}^{-(+)}\rangle$ for different orbitals, as obtained from the neutron-removal (p, d) and adding reactions on ^{40}Ca and ^{36}S . Values for energy centroids of the neutron-removal single-particle strength and associated binding energies were calculated as the average of the two calculations that either include or exclude tentatively assigned states, with uncertainties calculated to cover the range within one sigma. The fp valence orbits are mostly populated in the (d, p) rather than the (p, d) reaction, which leads to relatively small C^2S^- values and large uncertainties for the $p_{3/2}$, $f_{7/2}$, and $p_{1/2}$ orbits. Finally, the summed spectroscopic factors and binding energies from the combined adding and removal reactions are listed.

	^{40}Ca			^{36}S		
Orbit	neutron-removal Ref. [16]			neutron-removal (this work)		
	$\sum C^2S^-$ ^a	$\bar{E}^-$ (MeV)	$\langle\epsilon_{j\pi}^- \rangle$ (MeV)	$\sum C^2S^-$	$\bar{E}^-$ (MeV)	$\langle\epsilon_{j\pi}^- \rangle$ (MeV)
$s_{1/2}$	1.79(17)	2.612(37)	-18.247(37)	1.85(20)	3.650(13)	-13.539(13)
$d_{3/2}$	3.74(20)	0.000	-15.635	3.75(41)	0.373(41)	-10.262(41)
$d_{5/2}$	4.25 ^b	6.533(77)	-22.168(77)	3.87(43) ^c	6.951(10)	-16.840(10)
$p_{3/2}$	0.02	4.537	-11.098	0.12(1)	4.632(1.551)	-5.257(1.551)
$f_{7/2}$	0.26	3.718(530)	-11.917(530)	0.29(3)	3.191(781)	-6.698(781)
$p_{1/2}$	-	-	-	0.022(2)	8.221(377)	-1.668(377)
$\sum s_{1/2}, d_{3/2}$	5.53			5.60(46)		
$\sum fp$	0.28			0.43(3)		
Orbit	neutron-adding Ref. [59]			neutron-adding Ref. [42]		
	$(2j+1)\sum C^2S^+$	$\bar{E}^+$ (MeV)	$\langle\epsilon_{j\pi}^+ \rangle$ (MeV)	$(2j+1)\sum C^2S^+$	$\bar{E}^+$ (MeV)	$\langle\epsilon_{j\pi}^+ \rangle$ (MeV)
$s_{1/2}$	0.08	3.38	-4.98	-	-	-
$d_{3/2}$	0.42	3.21	-5.16	0.33	2.65	-1.65
Orbit	neutron-removal and adding ^d			neutron-removal and adding ^e		
	$\sum C^2S$	$\langle\epsilon_{j\pi} \rangle$ (MeV)		$\sum C^2S$	$\langle\epsilon_{j\pi} \rangle$ (MeV)	
$s_{1/2}$	1.87(17)	-17.68(37)		1.85(20)	-13.539(13)	
$d_{3/2}$	4.16(20)	-14.577(20)		4.08(41)	-9.560(41)	
$d_{5/2}$	4.25	-22.168(77)		3.87(43)	-16.840(10)	
$\ell = 2$	$\Delta_{1d}^{\text{SO}^-}$ (MeV)	Δ_{1d}^{SO} (MeV)		$\Delta_{1d}^{\text{SO}^-}$ (MeV)	Δ_{1d}^{SO} (MeV)	
	6.533(77)	7.590(77)		6.578(44)	7.280(45)	

^a In Ref. [16] an error is quoted only for two states, but the general statement is made that the accuracy of the normalization due to the ambiguity of the procedure for fitting the theoretical to experimental angular distributions is estimated to be better than 10%. Errors are displayed here only for the 2 orbits that were explicitly assigned errors in the work of Ref. [16].

^b Ref. [16] reports a value for $\sum C^2S^-$ of 4.72 for all $d_{5/2}$ states. However, a 10% yield ratio reduction as discussed in subsection IV C is taken into account here. We have found that $\sum C^2S^- = 3.78$ for states with a definitive $5/2^+$ assignment, including this 10% reduction.

^c 3.61(40) for only states with firm $5/2^+$ assignment.

^d The uncertainties reflect the values from the neutron-removal work of Ref. [16].

^e The uncertainties reflect the values from the present neutron-removal work.

[8] R. Lică, F. Rotaru, M. J. G. Borge, S. Grévy, F. Negoită, A. Poves, O. Sorlin, A. N. Andreyev, R. Borcea, C. Costache, H. De Witte, L. M. Fraile, P. T. Greenlees, M. Huyse, A. Ionescu, S. Kisiov, J. Konki, I. Lazarus, M. Madurga, N. Mărginean, R. Mărginean, C. Mihai, R. E. Mihai, A. Negret, F. Nowacki, R. D. Page, J. Pakarinen, V. Pucknell, P. Rahkila, E. Rapisarda, A. Șerban, C. O. Sotty, L. Stan, M. Stănoiu, O. Tengblad, A. Turturică, P. Van Duppen, N. Warr, P. Dessagne, T. Stora, C. Borcea, S. Călinescu, J. M. Daugas, D. Filipescu, I. Kuti, S. Franchoo, I. Gheorghe, P. Morfouace, P. Morel, J. Mrazek, D. Pietreanu, D. Sohler, I. Stefan, R. Șuvăilă, S. Toma, and C. A. Ur (IDS Collaboration), Normal and intruder configurations in ^{34}Si populated in the β^- decay of ^{34}Mg and ^{34}Al , *Phys. Rev. C* **100**, 034306 (2019)

[9] S. Jongile, A. Lemasson, O. Sorlin, M. Wiedeking, P. Papka, D. Bazin, C. Borcea, R. Borcea, A. Gade, H. Iwasaki, E. Khan, A. Lepailleur, A. Mutschler, F. Nowacki, F. Recchia, T. Roger, F. Rotaru, M. Stănoiu, S. R. Stroberg, J. A. Tostevin, M. Vandebrouck, D. Weishaar, and K. Wimmer, Structure of ^{33}Si and the magicity of the $N = 20$ gap at $Z = 14$, *Phys. Rev. C* **102**, 024321 (2020).

- [10] M. Salathe, H. L. Crawford, A. O. Macchiavelli, B. P. Kay, C. R. Hoffman, A. D. Ayangeakaa, C. M. Campbell, R. M. Clark, M. Cromaz, P. Fallon, M. D. Jones, S. A. Kuvin, J. Sethi, M. Wiedeking, J. R. Winkelbauer, and A. H. Wuosmaa, Search for the $1/2^+$ intruder state in ^{35}P , *Phys. Rev. C* **102**, 064317 (2020).
- [11] A. Mutschler, O. Sorlin, A. Lemasson, D. Bazin, C. Borcea, R. Borcea, A. Gade, H. Iwasaki, E. Khan, A. Lepailleur, F. Recchia, T. Roger, F. Rotaru, M. Stanoiu, S. R. Stroberg, J. A. Tostevin, M. Vandebrouck, D. Weisshaar, and K. Wimmer, Spectroscopy of ^{35}P using the one-proton knockout reaction, *Phys. Rev. C* **93**, 034333 (2016).
- [12] J. R. Terry, B. A. Brown, C. M. Campbell, J. M. Cook, A. D. Davies, D.-C. Dinca, A. Gade, T. Glasmacher, P. G. Hansen, B. M. Sherrill, H. Zwahlen, D. Bazin, K. Yoneda, J. A. Tostevin, T. Otsuka, Y. Utsuno, and B. Pritychenko, Single-neutron knockout from intermediate energy beams of $^{30,32}\text{Mg}$: Mapping the transition into the “island of inversion”, *Phys. Rev. C* **77**, 014316 (2008).
- [13] M. Petri, P. Fallon, A. Macchiavelli, S. Heil, E. Rodriguez-Vieitez, D. Bazin, C. Campbell, R. Clark, M. Cromaz, A. Gade, T. Glasmacher, I. Lee, S. Malbrunot-Ettenauer, S. Paschalis, A. Ratkiewicz, J. Terry, D. Weisshaar, and M. Wiedeking, Competing particle-hole excitations in ^{30}Na : Constraining state-of-the-art effective interactions, *Physics Letters B* **748**, 173 (2015).
- [14] P. Fallon, E. Rodriguez-Vieitez, A. O. Macchiavelli, A. Gade, J. A. Tostevin, P. Adrich, D. Bazin, M. Bowen, C. M. Campbell, R. M. Clark, J. M. Cook, M. Cromaz, D. C. Dinca, T. Glasmacher, I. Y. Lee, S. McDaniel, W. F. Mueller, S. G. Prussin, A. Ratkiewicz, K. Siwek, J. R. Terry, D. Weisshaar, M. Wiedeking, K. Yoneda, B. A. Brown, T. Otsuka, and Y. Utsuno, Two-proton knockout from ^{32}Mg : Intruder amplitudes in ^{30}Ne and implications for the binding of $^{29,31}\text{F}$, *Phys. Rev. C* **81**, 041302 (2010).
- [15] Y. Uozumi, O. Iwamoto, S. Widodo, A. Nohtomi, T. Sakae, M. Matoba, M. Nakano, T. Maki, and N. Koori, Single-particle strengths measured with $^{48}\text{Ca}(d,p)^{49}\text{Ca}$ reaction at 56 MeV, *Nuclear Physics A* **576**, 123 (1994).
- [16] M. Matoba, O. Iwamoto, Y. Uozumi, T. Sakae, N. Koori, T. Fujiki, H. Ohgaki, H. Ijiri, T. Maki, and M. Nakano, $^{40}\text{Ca}(p,d)^{39}\text{Ca}$ reaction at 65 MeV, *Phys. Rev. C* **48**, 95 (1993).
- [17] L. L. Lee and J. P. Schiffer, Experimental evidence for j dependence of the angular distribution from (d,p) reactions, *Phys. Rev. Lett.* **12**, 108 (1964).
- [18] R. H. Fulmer and W. W. Daehnick, Spin Dependence of (d,t) $l_n = 1$ Angular Distributions in Ni and Fe, *Phys. Rev. Lett.* **12**, 455 (1964).
- [19] R. Sherr, E. Rost, and M. E. Rickey, j dependence of the angular distributions from (p,d) reactions at 28 MeV, *Phys. Rev. Lett.* **12**, 420 (1964).
- [20] C. A. Whitten, E. Kashy, and J. P. Schiffer, j -dependence in the $^{56}\text{Fe}(p,d)^{55}\text{Fe}$ reaction, *Nuclear Physics* **86**, 307 (1966).
- [21] C. Glashauser and M. E. Rickey, Studies of j -dependence in neutron pickup reactions, *Phys. Rev.* **154**, 1033 (1967).
- [22] R. L. Kozub, (p,d) reaction on $N = Z$ Nuclei in the $2s - 1d$ shell, *Phys. Rev.* **172**, 1078 (1968).
- [23] H. Ohnuma and J. Yntema, Study of the j -dependence in the $(d,^3\text{He})$ reaction, *Phys. Rev.* **178**, 1654 (1969).
- [24] B. Mayer, J. Gosset, J. Escudie, and H. Kamitsubo, (p,d) reactions induced by a polarized proton beam, *Nuclear Physics A* **177**, 205 (1971).
- [25] C. Maples and J. Cerny, High-spin assignments in the $1p$ -shell obtained with the j -dependent (p,α) reaction, *Physics Letters B* **38**, 504 (1972).
- [26] J. E. Kim and W. W. Daehnick, Evidence for j dependence in $^{60}\text{Ni}(^3\text{He},d)^{61}\text{Cu}$ cross sections, *Phys. Rev. C* **24**, 1461 (1981).
- [27] K. Hirota, Y. Aoki, N. Okumura, and Y. Tagishi, Deuteron elastic scattering and (d,p) reactions on ^{208}Pb at $E_d = 22$ mev and j -dependence of T20 in (d,p) reaction, *Nuclear Physics A* **628**, 547 (1998).
- [28] E. Henley and T. Kelley, A general comment concerning the j dependence of direct reaction cross sections, *Physics Letters* **22**, 495 (1966).
- [29] J. Alty, L. Green, G. Jones, and J. Sharpey-Schafer, The dwba and spin-orbit effects in the $^{46,48}\text{Ti}, ^{50,52}\text{Cr}(d,p)$ reactions, *Nuclear Physics A* **100**, 191 (1967).
- [30] R. C. Johnson and F. D. Santos, Effects of the deuteron d state and j dependence in (d,p) and (p,d) reactions, *Phys. Rev. Lett.* **19**, 364 (1967).
- [31] B. Robson, j -dependence for $2p$ neutron transfer in (d,p) reactions, *Physics Letters B* **26**, 501 (1968).
- [32] S. Jongile, N. Kheswa, P. Papka, O. Sorlin, A. Lemasson, M. Wiedeking, C. Brits, S. Mthembu, B. Kheswa, L. Makhathini, F. Nemulodi, B. Maqabuka, and M. Sithole, Encapsulated Sulfur targets for light ion beam experiments, *EPJ Web Conf.* **229**, 03004 (2020).
- [33] R. Neveling, H. Fujita, F. Smit, T. Adachi, G. Berg, E. Buthelezi, J. Carter, J. Conradie, M. Couder, R. Fearick, S. Förtsch, D. Fourie, Y. Fujita, J. Görres, K. Hatanaka, M. Jingo, A. Krumbholz, C. Kureba, J. Mira, S. Murray, P. von Neumann-Cosel, S. O’Brien, P. Papka, I. Poltoratska, A. Richter, E. Sideras-Haddad, J. Swartz, A. Tamii, I. Usman, and J. van Zyl, High energy-resolution zero-degree facility for light-ion scattering and reactions at iThemba LABS, *Nucl. Instr. and Meth. A* **654**, 29 (2011).
- [34] S. Jongile, The structure of ^{33}Si , ^{35}S and the magicity of the $N = 20$ gap at $Z = 14, 16$, Ph.D. thesis, University of Stellenbosch (2021).
- [35] J. Chen, J. Cameron, and B. Singh, Nuclear Data Sheets for $A = 35$, *Nucl. Data Sheets* **112**, 2715 (2011).
- [36] J. D. Harvey and R. C. Johnson, Influence of breakup channels on the analysis of deuteron stripping reactions, *Phys. Rev. C* **3**, 636 (1971).
- [37] P. D. Kunz and E. Rost, The Distorted-Wave Born Approximation, in *Computational Nuclear Physics 2*, edited by K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer, New York, NY, 1993).
- [38] J. J. H. Menet, E. E. Gross, J. J. Malanify, and A. Zucker, Total-Reaction-Cross-Section Measurements for 30-60-MeV Protons and the Imaginary Optical Potential, *Phys. Rev. C* **4**, 1114 (1971).
- [39] R. C. Johnson and P. J. R. Soper, Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering, *Phys. Rev. C* **1**, 976 (1970).
- [40] F. D. Becchetti and G. W. Greenlees, Nucleon-Nucleus Optical-Model Parameters, $A > 40$, $E < 50$ MeV, *Phys. Rev.* **182**, 1190 (1969).

- [41] G. R. Satchler, Adiabatic Deuteron Model and the $^{208}\text{Pb}(p, d)$ Reaction at 22 MeV, *Phys. Rev. C* **4**, 1485 (1971).
- [42] J. Cameron, J. Chen, B. Singh, and N. Nica, Nuclear Data Sheets for $A = 37$, *Nucl. Data Sheets* **113**, 365 (2012).
- [43] B. M. Preedom, E. Newman, and J. C. Hiebert, Distorted wave predictions of observed J dependence in the $(d, ^3\text{He})$ reaction, *Physics Letters* **22**, 657 (1966).
- [44] M. H. Ohnuma, T. Suehiro and S. Yamada, (p, d) reactions at 52 MeV, *J. Phys. Soc. Jpn.* **36**, 1236 (1974).
- [45] D. Robson, Isobaric Spin in Nuclear Physics, *Annu. Rev. Nucl. Sci.* **16**, 119 (1966).
- [46] G. Delic and B. Robson, j -dependence in $l = 3$ transitions in (d, p) and (p, d) reactions, *Nuclear Physics A* **193**, 510 (1972).
- [47] G. R. Satchler, Direct Nuclear Reactions (Clarendon Press, 1983).
- [48] I. J. Thompson and F. M. Nunes, Nuclear Reactions for Astrophysics: Principles, Calculation and Applications of Low-Energy Reactions (Cambridge University Press, 2009).
- [49] N. Timofeyuk and R. Johnson, Theory of deuteron stripping and pick-up reactions for nuclear structure studies, *Progress in Particle and Nuclear Physics* **111**, 103738 (2020).
- [50] R. Johnson and P. Tandy, An approximate three-body theory of deuteron stripping, *Nuclear Physics A* **235**, 56 (1974).
- [51] F. M. Nunes and A. Deluva, Adiabatic approximation versus exact Faddeev method for (d, p) and (p, d) reactions, *Phys. Rev. C* **84**, 034607 (2011).
- [52] N. Otuka, E. Dupont, V. Semkova, B. Pritychenko, A. Blokhin, M. Aikawa, S. Babykina, M. Bossant, G. Chen, S. Dunaeva, R. Forrest, T. Fukahori, N. Furutachi, S. Ganesan, Z. Ge, O. Gritzay, M. Herman, S. Hlavač, K. Katō, B. Lalremruata, Y. Lee, A. Makinaga, K. Matsumoto, M. Mikhaylyukova, G. Pikulina, V. Pronyaev, A. Saxena, O. Schwerer, S. Simakov, N. Soppera, R. Suzuki, S. Takács, X. Tao, S. Taova, F. Tárkányi, V. Varlamov, J. Wang, S. Yang, V. Zerkín, and Y. Zhuang, Towards a More Complete and Accurate Experimental Nuclear Reaction Data Library (EXFOR): International Collaboration Between Nuclear Reaction Data Centres (NRDC), *Nuclear Data Sheets* **120**, 272 (2014).
- [53] C. D. Pruitt, J. E. Escher, and R. Rahman, Uncertainty-quantified phenomenological optical potentials for single-nucleon scattering, *Phys. Rev. C* **107**, 014602 (2023).
- [54] L. J. Titus, A. Ross, and F. M. Nunes, Transfer reaction code with nonlocal interactions, *Comput. Phys. Commun.* **207**, 499 (2016).
- [55] A. Koning and J. Delaroche, Local and global nucleon optical models from 1 keV to 200 MeV, *Nuclear Physics A* **713**, 231 (2003).
- [56] L. Lalanne, O. Sorlin, A. Poves, M. Assié, F. Hammache, S. Koyama, D. Suzuki, F. Flavigny, V. Girard-Alcindor, A. Lemasson, A. Matta, T. Roger, D. Beaumel, Y. Blumenfeld, B. A. Brown, F. D. O. Santos, F. Delaunay, N. de Séréville, S. Franchoo, J. Gibelin, J. Guillot, O. Kamalou, N. Kitamura, V. Lapoux, B. Mauss, P. Morfouace, M. Niikura, J. Pancin, T. Y. Saito, C. Stodel, and J.-C. Thomas, Structure of ^{36}Ca under the coulomb magnifying glass, *Phys. Rev. Lett.* **129**, 122501 (2022).
- [57] S. Khan, T. Kihm, K. Knöpfle, G. Mairle, V. Bechtold, and L. Freidrich, The mass of ^{35}P and spin-parity assignments for excited ^{35}P states, *Physics Letters B* **156**, 155 (1985).
- [58] J. French and M. MacFarlane, Isobaric-spin splitting of single-particle resonances, *Nuclear Physics* **26**, 168 (1961).
- [59] C. Nesaraja and E. McCutchan, Nuclear Data Sheets for $A = 41$, *Nucl. Data Sheets* **133**, 1 (2016).
- [60] G. Kramer, H. Blok, and L. Lapikás, A consistent analysis of $(e, e'p)$ and $(d, ^3\text{He})$ experiments, *Nuclear Physics A* **679**, 267 (2001).
- [61] P. Doll, G. Wagner, K. Knöpfle, and G. Mairle, The quasihole aspect of hole strength distributions in odd potassium and calcium isotopes, *Nuclear Physics A* **263**, 210 (1976).
- [62] S. Banks, B. Spicer, G. Shute, V. Officer, G. Wagner, W. K. hopf, L. Qingli, C. Glover, D. Devins, and D. Friesel, The $^{48}\text{Ca}(d, ^3\text{He})^{47}\text{K}$ reaction at 80 MeV, *Nuclear Physics A* **437**, 381 (1985).
- [63] H. Ong, I. Tanihata, A. Tamii, T. Myo, K. Ogata, M. Fukuda, K. Hirota, K. Ikeda, D. Ishikawa, T. Kawabata, H. Matsubara, K. Matsuta, M. Mihara, T. Naito, D. Nishimura, Y. Ogawa, H. Okamura, A. Ozawa, D. Pang, H. Sakaguchi, K. Sekiguchi, T. Suzuki, M. Taniguchi, M. Takashina, H. Toki, Y. Yasuda, M. Yosoi, and J. Zenihiro, Probing effect of tensor interactions in ^{16}O via (p, d) reaction, *Physics Letters B* **725**, 277 (2013).
- [64] S. Kawase, T. Uesaka, T. L. Tang, D. Beaumel, M. Dozono, T. Fukunaga, T. Fujii, N. Fukuda, A. Galindo-Uribarri, S. Hwang, N. Inabe, T. Kawabata, T. Kawahara, W. Kim, K. Kisamori, M. Kobayashi, T. Kubo, Y. Kubota, K. Kusaka, C. Lee, Y. Maeda, H. Matsubara, S. Michimasa, H. Miya, T. Noro, Y. Nozawa, A. Obertelli, K. Ogata, S. Ota, E. Padilla-Rodal, S. Sakaguchi, H. Sakai, M. Sasano, S. Shimoura, S. Stepanyan, H. Suzuki, T. Suzuki, M. Takaki, H. Takeda, A. Tamii, H. Tokieda, T. Wakasa, T. Wakui, K. Yako, J. Yasuda, Y. Yanagisawa, R. Yokoyama, K. Yoshida, K. Yoshida, and J. Zenihiro, Exclusive quasi-free proton knockout from oxygen isotopes at intermediate energies, *Progress of Theoretical and Experimental Physics* **2018**, 021D01 (2018).
- [65] H. Dussan, M. H. Mahzoon, R. J. Charity, W. H. Dickhoff, and A. Polls, Elastic nucleon-nucleus scattering as a direct probe of correlations beyond the independent-particle model, *Phys. Rev. C* **90**, 061603 (2014).
- [66] E. Caurier, F. Nowacki, and A. Poves, Merging of the islands of inversion at $N = 20$ and $N = 28$, *Phys. Rev. C* **90**, 014302 (2014).

- [67] A. Revel, O. Sorlin, F. M. Marqués, Y. Kondo, J. Kahlbow, T. Nakamura, N. A. Orr, F. Nowacki, J. A. Tostevin, C. X. Yuan, N. L. Achouri, H. Al Falou, L. Atar, T. Aumann, H. Baba, K. Boretzky, C. Caesar, D. Calvet, H. Chae, N. Chiga, A. Corsi, H. L. Crawford, F. Delaunay, A. Delbart, Q. Deshayes, Z. Dombrádi, C. A. Douma, Z. Elekes, P. Fallon, I. Gašparić, J.-M. Gheller, J. Gibelin, A. Gillibert, M. N. Harakeh, W. He, A. Hirayama, C. R. Hoffman, M. Holl, A. Horvat, A. Horváth, J. W. Hwang, T. Isobe, N. Kalantar-Nayestanaki, S. Kawase, S. Kim, K. Kisamori, T. Kobayashi, D. Körper, S. Koyama, I. Kuti, V. Lapoux, S. Lindberg, S. Masuoka, J. Mayer, K. Miki, T. Murakami, M. Najafi, K. Nakano, N. Nakatsuka, T. Nilsson, A. Obertelli, F. de Oliveira Santos, H. Otsu, T. Ozaki, V. Panin, S. Paschalis, D. Rossi, A. T. Saito, T. Saito, M. Sasano, H. Sato, Y. Satou, H. Scheit, F. Schindler, P. Schrock, M. Shikata, Y. Shimizu, H. Simon, D. Sohler, L. Stuhl, S. Takeuchi, M. Tanaka, M. Thoennessen, H. Törnqvist, Y. Togano, T. Tomai, J. Tscheuschner, J. Tsubota, T. Uesaka, Z. Yang, M. Yasuda, and K. Yoneda (SAMURAI21 collaboration), Extending the southern shore of the island of inversion to ^{28}F , *Phys. Rev. Lett.* **124**, 152502 (2020).
- [68] J. Kahlbow, T. Aumann, O. Sorlin, Y. Kondo, T. Nakamura, F. Nowacki, A. Revel, N. L. Achouri, H. Al Falou, L. Atar, H. Baba, K. Boretzky, C. Caesar, D. Calvet, H. Chae, N. Chiga, A. Corsi, F. Delaunay, A. Delbart, Q. Deshayes, Z. Dombrádi, C. A. Douma, Z. Elekes, I. Gašparić, J.-M. Gheller, J. Gibelin, A. Gillibert, M. N. Harakeh, A. Hirayama, M. Holl, A. Horvat, A. Horváth, J. W. Hwang, T. Isobe, N. Kalantar-Nayestanaki, S. Kawase, S. Kim, K. Kisamori, T. Kobayashi, D. Körper, S. Koyama, I. Kuti, V. Lapoux, S. Lindberg, F. M. Marqués, S. Masuoka, J. Mayer, K. Miki, T. Murakami, M. Najafi, K. Nakano, N. Nakatsuka, T. Nilsson, A. Obertelli, N. A. Orr, H. Otsu, T. Ozaki, V. Panin, S. Paschalis, D. M. Rossi, A. T. Saito, T. Saito, M. Sasano, H. Sato, Y. Satou, H. Scheit, F. Schindler, P. Schrock, M. Shikata, K. Shimada, Y. Shimizu, H. Simon, D. Sohler, L. Stuhl, S. Takeuchi, M. Tanaka, M. Thoennessen, H. Törnqvist, Y. Togano, T. Tomai, J. Tscheuschner, J. Tsubota, T. Uesaka, H. Wang, Z. Yang, M. Yasuda, and K. Yoneda (SAMURAI21-NeuLAND Collaboration), Magicity versus superfluidity around ^{28}O viewed from the study of ^{30}F , *Phys. Rev. Lett.* **133**, 082501 (2024).
- [69] E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves, and A. P. Zuker, The shell model as a unified view of nuclear structure, *Rev. Mod. Phys.* **77**, 427 (2005).
- [70] V. Somà, Self-consistent Green's function theory for atomic nuclei, *Front. in Phys.* **8**, 340 (2020).
- [71] V. Somà, T. Duguet, and C. Barbieri, *Ab initio* self-consistent Gorkov-Green's function calculations of semimagic nuclei: Formalism at second order with a two-nucleon interaction, *Phys. Rev. C* **84**, 064317 (2011).
- [72] V. Somà, C. Barbieri, and T. Duguet, *Ab initio* self-consistent Gorkov-Green's function calculations of semimagic nuclei: Numerical implementation at second order with a two-nucleon interaction, *Phys. Rev. C* **89**, 024323 (2014).
- [73] A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, B. D. Carlsson, C. Forssén, M. Hjorth-Jensen, P. Navrátil, and W. Nazarewicz, Accurate nuclear radii and binding energies from a chiral interaction, *Phys. Rev. C* **91**, 051301 (2015).
- [74] W. G. Jiang, A. Ekström, C. Forssén, G. Hagen, G. R. Jansen, and T. Papenbrock, Accurate bulk properties of nuclei from $a = 2$ to ∞ from potentials with Δ isobars, *Phys. Rev. C* **102**, 054301 (2020).
- [75] P. Arthuis, K. Hebeler, and A. Schwenk, Neutron-rich nuclei and neutron skins from chiral low-resolution interactions, (2024), [arXiv:2401.06675 \[nucl-th\]](https://arxiv.org/abs/2401.06675).
- [76] V. Somà, C. Barbieri, T. Duguet, and P. Navrátil, Moving away from singly-magic nuclei with Gorkov Green's function theory, *Eur. Phys. J. A* **57**, 135 (2021).
- [77] V. Somà and T. Duguet, On the calculation and use of effective single-particle energies: the example of the neutron $1d_{3/2}$ - $1d_{5/2}$ splitting along $N = 20$ isotones, *Phil. Trans. A. Math. Phys. Eng. Sci.* **382**, 20230117 (2024).
- [78] S. R. Stroberg, A. Gade, J. A. Tostevin, V. M. Bader, T. Baugher, D. Bazin, J. S. Berryman, B. A. Brown, C. M. Campbell, K. W. Kemper, C. Langer, E. Lunderberg, A. Lemasson, S. Noji, F. Recchia, C. Walz, D. Weisshaar, and S. J. Williams, Single-particle structure of silicon isotopes approaching ^{42}Si , *Phys. Rev. C* **90**, 034301 (2014).
- [79] T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi, Evolution of nuclear shells due to the tensor force, *Phys. Rev. Lett.* **95**, 232502 (2005).