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ELEMENTARY PROBLEMS AND SOLUTIONS

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Send all communications concerning Elementary Problems and Solutions to Howard Eves, Mathematics Department, University of Maine, Orono, Maine. This department welcomes problems believed to be new, and demanding no tools beyond those ordinarily furnished in the first two years of college mathematics. To facilitate their consideration, solutions should be submitted on separate, signed sheets, within three months after publication of problems.

A Divergent Series

E 1578 [1963, 331]. *Proposed by E. O. Thorp, New Mexico State University*

Consider the series $\sum_{n=1}^{\infty} |\sin n|^a$, where a is an arbitrary positive number. For which values of a does the series converge and for which values does it diverge?

Solution by Julius Vogel, The Prudential Insurance Company of America, Newark, N. J. The series diverges for all positive a , because $|\sin n|^a$ does not approach zero with increasing n . In fact, given any integer n_0 , however large, define t as an integer falling in the interval

$$2\pi n_0 + \pi/6 < t < 2\pi n_0 + 5\pi/6.$$

Then $t > n_0$ and $|\sin t|^a > (1/2)^a$.

Also solved by E. R. Barnes, D. I. A. Cohen, Frank Dapkus, Michael Gemignani, Michael Goldberg, Ralph Greenberg, T. J. Grilliot, R. A. Jacobson, Erwin Just and Norman Schaumberger (jointly), Joel Kugelmass, A. E. Livingston, D. C. B. Marsh, D. J. Peterson, Stanton Philipp, D. L. Silverman, Guy Torchinelli, W. C. Waterhouse, and the proposer.

Livingston showed, more generally, that $\sum_{n=1}^{\infty} |\sin nx|^a$ diverges for all real a and $x \not\equiv 0 \pmod{\pi}$.