An experimental investigation of design parameters for pico-hydro Turgo turbines using a response surface methodology

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Abstract
Millions of off-grid homes in remote areas around the world have access to pico-hydro (5 kW or less) resources that are undeveloped due to prohibitive installed costs ($/kW). The Turgo turbine, a hydroelectric impulse turbine generally suited for medium to high head applications, has gained renewed attention in research due to its potential applicability to such sites. Nevertheless, published literature about the Turgo turbine is limited and indicates that current theory and experimental knowledge do not adequately explain the effects of certain design parameters, such as nozzle diameter, jet inlet angle, number of blades, and blade speed on the turbine’s efficiency. In this study, these parameters are used in a three-level (3^4) central composite response surface experiment. A low-cost Turgo turbine is built and tested from readily available materials and a second order regression model is developed to predict its efficiency as a function of each parameter above and their interactions. The effects of blade orientation angle and jet impact location on efficiency are also investigated and experimentally found to be of relatively little significance to the turbine. The purpose of this study is to establish empirical design guidelines that enable small hydroelectric manufacturers and individuals to design low-cost efficient Turgo Turbines that can be optimized to a specific pico-hydro site. The results are also expressed in dimensionless parameters to allow for potential scaling to larger systems and manufacturers.

1. Introduction

The Turgo turbine is a hydroelectric impulse turbine that has gained renewed attention in research because of its potential application to millions of off-grid 5 kW-or-less pico-hydro sites and to energy recovery of discharged water at public water systems [1]. Generally, pico hydro systems are run of the river, which means that impoundment is not necessary. Such a scheme diverts water from the river as needed, feeding it down a steep slope through a penstock, a nozzle and then the turbine, after which the effluent water rejoins the river or is used for irrigation and other community purposes.

In developing countries alone, where 1.6 billion people live without electricity, a recent study showed that there are 4 million potential pico-hydro sites [2]. Furthermore, a World Bank study in 2006 showed that pico-hydropower is the most competitive off-grid power technology on a levelized cost of electricity (LCOE) basis ($/kWh), as shown in Fig. 1 [3,4]. In Rwanda for example, the national utility retail price for electricity in 2009 was $0.24/kWh [5], yet pico-hydro is estimated to cost less than $0.20/kWh. Nevertheless, the installation cost ($/kW) of pico-hydro systems can become cost prohibitive. In 2011, Meier reported a study of 80 Indonesian villages that revealed capital costs per kW exponentially increase with smaller sized systems, potentially surpassing $10,000/kW for low-head systems less than 5 kW [6]. Within the United States it is currently understood that sites with less than 100 kW of electrical potential are “best left undeveloped” because of extremely high installation costs ($59,000/kW on average) [7].

To reduce capital costs, standardized off-the-shelf turbines, as opposed to turbines that are customized to a specific site, are sold commercially by manufacturers worldwide, including the United States, Canada, and China [8–13]. The cost of these pico-hydro turbines can range from $125/kW to over $1200/kW. The

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mechanical efficiency in a laboratory testing environment can be upward of 82%, which is typical of a well-manufactured Turgo machine [14]. However, because standardized turbines are not customized to a specific site, they are prone to be less efficient in the field, some reporting only 40% water-to-wire efficiency [8]. Furthermore, off-the-shelf systems shipped abroad are more difficult and costly to repair and maintain in-country due to the system’s proprietary design, the dependency upon imported spare parts with lengthier lead times, which results in longer power outages, and taxes levied by customs, which have been shown to increase equipment costs by 40% on pico-hydro systems [2].

Simpler do-it-yourself turbines reduce installation costs, but these turbines usually suffer from poor efficiencies due to a lack of design guidelines, fabrication facilities, or technical expertise. An example of a simple turbine built in Rwanda is shown in Fig. 2. Meier’s assessment of pico-hydropower in Rwanda estimated that simple improvements in turbine design could increase efficiency by 20% with no additional equipment cost [6].

Therefore, this study aims not to build a standardized off-the-shelf turbine, but to develop a set of standardized design equations for optimizing the most important parameters of a Turgo turbine based on a site’s available head and flow. This approach can facilitate the custom design and local manufacture of low-cost, yet efficient Turgo turbines. In the present study, a Turgo turbine is built from materials of low cost and a set of empirical design equations is established for optimizing the Turgo turbine’s most significant parameters. Specifically, the nozzle diameter, \( d \), jet inlet angle, \( \alpha \), and jet velocity, \( v \), are considered.
angle, $\alpha$, number of blades, $Z$, dimensionless blade speed, $4$, jet impact location, and blade orientation angle, are investigated to determine if and how they influence the turbine’s efficiency, $\eta$.

2. Pico-hydropower background

The suitability of a hydroelectric turbine to a particular site depends on the site’s head, $H$, and water flow rate, $Q$. Impulse turbines, such as the Pelton and Turgo, are suited for sites with high head and low flow, while reaction turbines operate best at low head and high flow conditions. The difference between the Pelton and Turgo turbines is illustrated in Fig. 3 [15]. Water strikes the bucket of a Pelton wheel in the same plane as the wheel, then splits in half, reverses, and discharges at both sides. The jet of a Turgo turbine is angled to one side of the turbine’s plane and the discharged water exits out the other side after being reversed by the blades.

The available hydraulic power, $P_H$, of a hydraulic turbine is a function of the head and flow:

$$P_H = \rho g H Q$$

(1)

Meanwhile, the turbine’s mechanical power output, $P_M$, is the product of the torque, $\tau$, and rotational velocity, $\omega$:

$$P_M = \tau \omega$$

(2)

A turbine’s experimental efficiency is:

$$\eta = \frac{P_M}{P_H}$$

(3)

As a momentum transfer machine, the Turgo turbine’s theoretical efficiency can be calculated by the change in the water’s velocity between the inlet and outlet of the blade. Fig. 4 depicts the velocity vectors as a jet of water impacts a single Turgo blade [16]. The reference direction is that of the blade velocity, $u$. The water enters with an absolute velocity, $v_1$, an angle, $\alpha_1$, a relative velocity, $R_1$, with respect to the blade’s reference frame, and a relative angle $b_1$. The relative inlet angle of the water is assumed to be parallel to the blade’s inlet edge for smooth entry in order to prevent hydraulic shock, or water hammer [17]. When the nozzle and blade are oriented this way the condition is known as the “no-shock” condition, $\alpha_{\text{no-shock}}$.

The absolute inlet velocity can be calculated by:

$$v_1 = c_v \sqrt{2gH}$$

(4)

where $c_v$ is a constant called the coefficient of velocity. The relative inlet angle can be expressed as:

$$\beta_1 = \tan^{-1} \left( \frac{\sin \alpha_1}{\cos \alpha_1 - \phi} \right)$$

(5)

where $\phi$ is the speed ratio, defined as:

$$\phi \equiv \frac{u}{v_1} = \cos \alpha_1 - \frac{\sin \alpha_1}{\tan b_1}$$

(6)

The Turgo blade causes the fluid to change direction and exit at a relative angle $\beta_2$, which is equal to the blade’s discharge angle. The relative effluent velocity is $R_2$ and its absolute velocity is $v_2$.

Using the velocity triangles, the theoretical hydraulic efficiency can be expressed as [18]:

$$\eta_H = 2c_v^2 \left( \phi \cos \alpha_1 - \phi^2 + \frac{k\phi \sin \alpha_1 \cos \beta_2}{\sin \beta_1} \right)$$

(7)

where the constant $k$ is a friction coefficient equal to the ratio of relative exit velocity to relative inlet velocity ($R_2/R_1$). $k$ is typically 0.90–0.95 for a well-designed blade [17].

Using Equation (7), efficiency is plotted as a function of speed ratio in Fig. 5, for jet inlet angles ranging from $10^\circ$ to $40^\circ$, $c_v = 1$ and $\beta_2 = 15^\circ$. For every value of $\alpha$, there exists an optimum speed ratio, $\phi_{\text{opt}}$, called the Best Efficiency Point (BEP) of the turbine, and a corresponding relative jet inlet angle, $\beta_{1,\text{opt}}$, which is theoretically...
equal to the blade’s inlet angle in order to satisfy the no-shock condition. The converse is true as well: for fixed inlet and exit blade angles there exists a theoretical \( \phi_{\text{opt}} \) and \( \alpha_{\text{no-shock}} \), which is theoretically equal to \( \alpha_{\text{opt}} \). These can be calculated when the derivatives of Equations (7) and (6) are substituted and solved for each other. Upon inspecting Fig. 5, an approximation of \( \alpha_{\text{no-shock}} \) for small angles, which Gibson explains in detail [17], is simply,

\[
\alpha_{\text{no-shock}} = \frac{\beta_1}{2}
\]

(8)

In the special case of a Pelton wheel, \( \alpha_1 = 0 \), and the BEP is at \( \phi_{\text{opt}} = 0.5 \). However, because Turgo turbines have a non-zero inlet angle, the theoretical BEP depends upon \( \alpha_1 \) and \( \beta_2 \). Note that the \( \beta_1 \) term from Equation (7) can be expressed in terms of \( \alpha_1 \) and \( \phi \) by rearranging and substituting Equation (6).

3. Previous work

Unlike the Pelton, relatively little experimental research for the Turgo has been documented [16]. The Turgo turbine is usually treated the same as the Pelton Wheel, despite their differences [19,20]. While the Pelton has a slightly higher theoretical efficiency, the Turgo has several distinct advantages. The Turgo Turbine’s Pitch to Center Diameter (PCD) can be half the diameter of the Pelton, resulting in less wind resistance and higher RPMs, which is better for generator matching [15,21,22]. The Turgo is also known for its flat efficiency curve, meaning its performance is less sensitive to changes in flow rate, which is most pertinent to run-of-the-river pico-hydro systems [14,23].

Several key experimental findings regarding the Turgo turbine and the literature’s limitations are described below.

3.1. Jet inlet angle

While theory states that the optimum angle of attack is \( \alpha_{\text{no-shock}} \) in practice, \( \alpha_{\text{opt}} \) is usually a few degrees less than \( \alpha_{\text{no-shock}} \) [18]. However, if the difference is too large then hydraulic shock can occur and energy is lost due to eddies circulating counter to the flow.

Furthermore, a theoretical expression for \( \alpha_{\text{opt}} \) can be formulated by taking the derivative of Equation (7) with respect to \( \alpha \). By substituting Equation (6), \( \beta_1 \) drops out and \( \alpha_{\text{opt}} \) becomes a function of speed ratio and relative exit angle only. However, several recent studies have suggested that \( \alpha_{\text{opt}} \) depends on other factors such as the number of blades and the nozzle diameter, but these interactions have yet to be quantitatively characterized. A 2013 study on pico-hydro Turgo turbines by Williamson [16] revealed that the optimum inlet angle depends on the nozzle diameter. For example, for a 20 mm nozzle, \( \alpha_{\text{opt}} = 10^\circ \) but for a 30 mm nozzle, \( \alpha_{\text{opt}} = 20^\circ \), which suggests that a larger nozzle diameter requires a larger jet angle. In 2012, Cobb [20] reported \( \alpha_{\text{opt}} \) to be 18–20° for a Turgo of 28 blades and comparable PCD to Williamson’s, which used only 9 blades. Furthermore, in 2011, Koukouvinis et al. [24] reported an optimum jet inlet angle of 30°, with a 7–10 point drop in efficiency at angles ±10°. Other published sources have assumed \( \alpha_{\text{opt}} = 25^\circ \) [21, 20° [23], and 30° [25] but without an explanation as to what factors lead to such different values.

3.2. Blade exit angle and best efficiency point

According to Equation (7), the relative exit angle, \( \beta_2 \), should be as small as possible; however, if the angle is too small the effluent water will interfere with the oncoming blade. Experiments indicate that a discharge angle of 15–20° performs the best [17,26,27]. Furthermore, as stated previously, according to theory, \( \phi_{\text{opt}} \) is a function of \( \alpha_1 \) and \( \beta_2 \) only. However, it is known that the actual BEP is usually less than the theoretically predicted \( \phi_{\text{opt}} \) [16]. Therefore, Equation (7) does not adequately explain the factors that contribute to \( \beta_2 \text{opt} \) and \( \phi_{\text{opt}} \).

3.3. Effect of nozzle diameter

The blade width to nozzle diameter ratio (w/d) has an important influence on the way water flows over a curved surface [28]. As this ratio decreases there is increasing turbulence and re-circulation, which disrupts the flow. For a Pelton, the w/d ratio is usually between three and five [17,18,29]. A study in 2007 suggested w/d = 1.45 for the Turgo, but without justification [21].

In his review of the Turgo turbine’s history, Wilson stated that another dimensionless ratio, D/d, decides all the main characteristics of an impulse turbine [15]. While a smaller D/d is desirable for faster speeds, it becomes too small and it could result in flow limitations [30]. Several sources report that the ratio of D/d for a Pelton wheel should be no less than 10:1 [15,17,18,27,29]. The Turgo turbine is capable of cutting this ratio in half, at least [15,17,24,31].

In a recent Turgo turbine study, Williamson [16] tested five nozzle diameters: 10, 15, 20, 25, and 30 mm. The associated turbine efficiencies are plotted in Fig. 6. The best performing design was d = 20 mm, which corresponds to 1.75 w/d and 7.5 D/d. The efficiency dropped drastically for larger nozzles, by up to 30% with the

![Fig. 5. Efficiency curves of a Pelton and an impulse turbine of changing jet inlet angle.](image)

![Fig. 6. Effect of nozzle diameter on efficiency (used with permission from Elsevier)](image)
30 mm nozzle (w/d = 0.86 and D/d = 5). In Cobb’s 2013 study the nozzle diameter did not significantly affect the efficiency, but the D/d ratio remained greater than 10 and the ratio w/d was always 2.25 or greater, both of which are large values.

3.4. Effect of multiple blades

The velocity triangle calculations only apply to a single Turgo blade and do not capture the effect of multiple blades. The optimum number of blades for a Pelton turbine is given by Tyggun’s empirical formula [25,32]:

\[ Z = 15 + \frac{D}{2d} \] (9)

In addition to citing Tyggun’s formula, Kadambi [29] and Thake [27] have published a chart for determining the best number of blades on a Pelton wheel based on the ratio D/d.

While these design equations exist for the Pelton, no empirical formula or suggested optimum number of blades exists for the Turgo turbine, to the authors’ knowledge. The number of blades investigated for the Turgo turbine.

3.5. Jet’s axial impact location

Williamson also investigated the effect of the jet’s axial impact location on Turgo efficiency [16]. “Axial” refers to the location along the width of the blade from entry to exit. The results, shown in Fig. 7, indicate that the best efficiency occurred at an impact location 6 mm inside the entry edge of the blade, which corresponds to roughly one-sixth the width of the blade. Misaligning the jet by 4–6 mm (11%–17% of the width) resulted in a 10% drop in efficiency.

3.6. Turbine power specific speed

The non-dimensional power specific speed, \( \Omega_{sp} \), is a parameter that assists engineers in selecting the best type of turbine for a site, independent of the turbine’s size [18]. For example, if the specific speed is between 0.094 and 0.15, then a single jet Pelton wheel is appropriate, whereas a low head Francis turbine would be suited to specific speeds between 0.34 and 2.3. The power specific speed is given by:

\[ \Omega_{sp} = \left( \frac{\sqrt{\frac{1}{2}} M}{\rho^{1/2} (gH)^{5/7}} \right)^{1/2} \] (10)

and is evaluated at the maximum efficiency point. This is equivalent to the less common form [30]:

\[ \Omega_{sp} = \left( 2\sqrt{2} \pi \eta_{\text{max}} \right)^{1/2} \frac{d}{\varphi D} \] (11)

3.7. Contribution to the literature

First, this study sets out to quantitatively determine the effect of blade number, jet angle, nozzle diameter, and speed ratio on a low-cost Turgo turbine’s efficiency. By means of regression analysis, a prediction equation will provide a more complete picture of how these factors affect efficiency and how they interact with one another. The prediction equation will also enable each parameter to be optimized for a site’s flow characteristics. Dimensionless parameters, such as d/s, w/d and the specific speed will be considered to help establish design guidelines for the Turgo. Furthermore, this study will probe the sensitivity of the UC Davis Turgo turbine’s efficiency to variations in the jet’s axial impact location and the blades’ orientation angle (inlet and outlet angles).

Table 1 compares the design parameters of the UC Davis Turgo turbine to that of pico-hydro Turgo turbines used in recent literature by Cobb [20] and Williamson [16].

4. Methodology

A low-cost Turgo turbine, shown in Fig. 8, was built at UC Davis. The cost of the turbine’s materials was approximately $30. Tablespoons were chosen for blades because they are common, inexpensive, made of stainless steel for rust and pitting resistance, and have dimensional similarity to other Turgo blades, such as those used by Cobb and Williamson. The spoon’s angle of curvature was measured to be \( \delta = 82^\circ \), as shown in Fig. 9. The blade was rotated 30’ from the hub’s face, resulting in a blade inlet angle of \( \theta_1 = 79^\circ \) and exit angle of \( \theta_2 = 19^\circ \), which is assumed to be equal to \( \beta_2 \). With these inlet and exit angles and a friction factor of \( k = 0.9 \), the theoretical hydraulic efficiency of the turbine (Equation (7)) is approximately \( \eta_H = 90\% \), and using Equations (6) and (7), \( \delta_{\text{shock}} = 42^\circ \), and, \( \delta_{\text{psi}} = 0.62 \).

An adjustable nozzle mount allowed for adjustments to the nozzle angle, height and distance away from the turbine (Fig. 10). Water was supplied from the 150-foot tall domestic water tower at UC Davis. An analog pressure gauge was placed directly before the nozzle and the dynamic pressure was held constant at 50 ± 0.5 psi for every test because turbine efficiency and BEP have been shown to be independent of the water pressure [20]. Three nozzles were used and the coefficient of discharge, \( C_D \), was taken to be 0.95 for all tests [35]. The nozzle dimensions and test conditions are shown in Table 2.

An 18 cm water-brake dynamometer from Land & Sea, Inc. was used for turbine load control and measurement of the turbine’s torque and rpm. The turbine produced power ranging upward of 1.4 kW (1.9 Hp) at 1650 rpm down to 0.06 kW (0.082 Hp) at
Discharge angle, \( \theta_2 \) (deg) 19

Curvature radius (mm) 30

Inlet angle, theta1 (deg) 79

Discharge angle, theta2 (deg) 19

Head (m) 35

**Dimensionless ratios**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>UC Davis (Gaiser et al.)</th>
<th>Cobb et al. [20] (169 mm)</th>
<th>Cobb et al. [20] (131 mm)</th>
<th>Williamson et al. [16]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head (m)</td>
<td>18–28</td>
<td>17–25</td>
<td>0.5–3.5</td>
<td></td>
</tr>
<tr>
<td>( \frac{d}{s} )</td>
<td>5.3–2.04</td>
<td>2.25–3.6</td>
<td>2.25–3.6</td>
<td>2–0.86</td>
</tr>
</tbody>
</table>

**4.2. Experiment design and procedure**

A response surface method is a common statistical technique used to empirically model the relationship between a response variable (output, such as efficiency) and several independent parameters (input variables), with the objective of optimizing the response. Through a carefully constructed design of experiment, such as a central composite design (CCD), the cost of optimization can be reduced when compared to more complex methods such as CFD analysis. A three-level face-centered CCD [37,39] is used to develop a complete second order four-factor regression equation:

\[ \eta = f\left( \alpha, Z, d, \frac{d}{s}, \theta_1, \theta_2, v, w, V, \rho, g, \mu \right) \] (13)

On the left side of the equation, \( \eta \), efficiency, is the response variable. On the right side, \( \frac{d}{s} \) is the speed ratio, \( \frac{d}{s} \) is the Reynolds number, and \( \frac{d}{s} \) is the Froude number. By Equation (11) the product of terms three and four, \( \frac{d}{s} \times \frac{d}{s} \), is directly proportional to the specific speed of the turbine, and the product of \( Z \times \frac{d}{s} \) is proportional to the ratio \( d/s \).

To decrease the number of variables for testing, the spoon volume, \( V \), turbine diameter, \( D \), and water pressure, represented by velocity, \( v \), were held constant. Turbine performance is typically insensitive to the Reynolds number, which makes this parameter of little interest [18]. Similarly, the Froude number is not explored. Finally, \( \alpha \) is replaced with \( \sin(\alpha) \) to account for the trigonometric nature of the velocity vector triangles. This results in a function with four independent factors:

\[ \eta = f\left( \sin \alpha, Z, d, \frac{d}{s} \right) \] (14)

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\[ \eta = b_0 + b_1 X_{11} + b_2 X_{21} + b_3 X_{31} + b_4 X_{41} + b_{11} X_{11}^2 + b_{22} X_{21}^2 + b_{33} X_{31}^2 + b_{44} X_{41}^2 + b_{12} X_{11} X_{21} + b_{13} X_{11} X_{31} + b_{14} X_{11} X_{41} + b_{23} X_{21} X_{31} + b_{24} X_{21} X_{41} + b_{34} X_{31} X_{41} \] (15)

In Equations (15), \( X_1 \) is jet angle, \( X_2 \) is number of blades, \( X_3 \) is nozzle diameter, \( X_4 \) is speed ratio, “i” indicates the replicate number, and the “b”-terms are the coefficients to be determined by

360 rpm, for various turbine and nozzle configurations.

**4.1. Dimensional analysis**

Dimensional analysis can be used to express the functional relationship between the efficiency of a hydroelectric turbine and its independent design variables [36,37]. Fourteen independent design parameters were identified:

\[ \eta = f\left( \alpha, Z, d, D, \omega, \theta_1, \theta_2, v, w, V, \rho, g, \mu \right) \] (12)

In this case, the number of blades, \( Z \), is used rather than the spacing between blades, \( s \), although they are related by the turbine’s circumference, \( \pi D \). \( V \) is the blade’s volume and \( \mu \), the viscosity of water. A non-dimensional relationship with fewer dimensionless independent parameters can be obtained according to the Buckingham Pi theorem. Using \( D, v, \) and \( \rho \) as control variables, the dimensionless Equation (13) was developed by application of the Iensen method, as outlined by Gray [38].
experimentation. The null hypotheses for each term in Equation (15) state that the parameter’s b-term is zero, which would relegate the parameter insignificant. A confidence level of 95% is used to guard against the incorrect rejection of a true null hypothesis. SAS software is used to conduct a series of t-tests on each parameter and to check for multicollinearity within the data. Stepwise selection methods were employed and the $C_p$, $R^2_{adj}$, and Predicted Sum of Squares (PRESS) values, all of which assess the fit of a regression line, were used to select the best model.

To reduce multicollinearity, the three levels are evenly spaced and converted to coded units: +1, −1 and 0, such that, for example, $X_1$ is a function of $\alpha$. The levels for each factor are summarized in Table 3 and are chosen so that a wide range of values relevant to previous literature is tested (see Table 1).

Five additional replicates are performed at each of the points (0, 0, 0, 0), (0, 0, 0, −1), and (0, 0, 1, 0), in order to have a good estimate of the pure error [40]. The total number of data points was $3^4 + 15 = 96$; however, the factor $X_4$, speed ratio (i.e., RPM), was decreased continuously during each experiment, so in effect 31 randomized experiments were performed. As the RPM decreased, the torque was recorded and Equations (1)–(3) were used to calculate efficiency and Equation (16) to calculate speed ratio.

$$\varphi = \frac{u}{v} = \frac{\pi D (\text{RPM})}{60 \sqrt{2gH}}$$

(16)

Fig. 11 is an example where $X_1$, $X_2$, and $X_3$ are zero. A best-fit

![Table 3](image_url)

<table>
<thead>
<tr>
<th>Level</th>
<th>Coded variable</th>
<th>−1</th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td>Jet angle, $\theta$</td>
<td>X1</td>
<td>10</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>Number of blades, $Z$</td>
<td>X2</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Nozzle diameter, d (mm)</td>
<td>X3</td>
<td>7.13</td>
<td>12.85</td>
<td>18.60</td>
</tr>
<tr>
<td>Speed ratio, $\varphi$</td>
<td>X4</td>
<td>0.25</td>
<td>0.35</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sin \alpha$, $X1$</td>
<td>X1</td>
<td>0.174</td>
<td>0.423</td>
<td>0.643</td>
</tr>
<tr>
<td>$Z$, $X2$</td>
<td>X2</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>$d/D$, $X3$</td>
<td>X3</td>
<td>0.055</td>
<td>0.099</td>
<td>0.143</td>
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<tr>
<td>Speed ratio, $\varphi$, $X4$</td>
<td>X4</td>
<td>0.25</td>
<td>0.35</td>
<td>0.45</td>
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</tbody>
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![Table 2](image_url)

<table>
<thead>
<tr>
<th>Name</th>
<th>Diameter (±0.013 mm)</th>
<th>Diameter (±0.0005 in)</th>
<th>Measured flow rate @ 50 psi (L/s)</th>
<th>Hydraulic power (kW)</th>
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<tbody>
<tr>
<td>1/8” nozzle</td>
<td>18.593</td>
<td>0.7320</td>
<td>7.13 (113.0 GPM)</td>
<td>2.455</td>
</tr>
<tr>
<td>1/4” nozzle</td>
<td>12.852</td>
<td>0.5050</td>
<td>3.42 (54.2 GPM)</td>
<td>1.185</td>
</tr>
<tr>
<td>1/2” nozzle</td>
<td>7.125</td>
<td>0.2805</td>
<td>1.05 (16.7 GPM)</td>
<td>0.366</td>
</tr>
</tbody>
</table>
2nd order polynomial is fitted to the data and the efficiencies at the three predetermined levels of speed ratio ($\phi = 0.25, 0.35$ and $0.45$), shown by the vertical dotted lines, are extracted.

5. Results and discussion

5.1. Multiple linear regression analysis

Of the 96 trials, three data points had standard deviations greater than 2.5 from the predicted value. These points were taken with a faulty pressure gauge, which was replaced in subsequent tests. The outliers were removed and the Mean Standard Error (MSE) noticeably improved.

The four independent variables, blade angle ($X_1$), number of blades ($X_2$), nozzle diameter ($X_3$) and speed ratio ($X_4$) were used in SAS to determine the coefficients in the regression Equation (15). The removal of $X_1$ results in the best regression fit in which all the parameters are significant at a 95% confidence level, as indicated by the p-values in the “Pr > |t|” column of Table 4. The parameter estimates in Table 4, the prediction equation is written in its coded form:

$$\hat{\eta} = 57.68 + 3.64X_2 + 5.66X_3 + 1.43X_4 - 4.17X_2^2 - 2.04X_3^2$$
$$- 8.15X_3^2 - 4.94X_4^2 + 3.42X_1X_2 + 0.98X_1X_3 - 1.56X_1X_4$$
$$- 0.75X_2X_3 + 2.95X_2X_4 + 2.98X_3X_4$$

(17)

All four factors have a significant second-order effect on the efficiency of the turbine, with the nozzle diameter ($X_3$) having the largest effect. All of the interaction terms are significant too. It is more useful to express $X_1, X_2, X_3$ and $X_4$ in terms of dimensionless parameters, $\sin(\phi), Z, d/D$ and $\phi$, respectively. Substituting results in Equation (18), which is valid within the domains outlined in Table 3.

$$\hat{\eta} = -42.077 + 32.772\sin{\alpha} + 0.243Z + 732.42\frac{d}{D} + 232.25\phi$$
$$- 75.786\sin^2{\alpha} - 0.08162 - 4191.2\left(\frac{d}{D}\right)^2 - 494\phi^2$$
$$+ 2.916Z\sin{\alpha} + 94.770\frac{d}{D}\sin{\alpha} - 66.505\phi\sin{\alpha}$$
$$- 3.406\frac{d}{D}Z + 5.94\phi Z + 675.61\frac{d}{D}\phi$$

(18)

Fig. 11. Test result for (0,0,0), with a best-fit curve.

5.2. Residual analysis

Linear regression assumes that the error terms are normally distributed, independent and identically distributed. Fig. 12 shows residual analysis plots for the data. A histogram of the residuals and a normal QQ plot both confirm the normality of the error terms and a plot of the R-studentized residuals versus predicted value confirms that the residuals are independently distributed with no significant outliers. The chronologal plot indicates the residuals are sufficiently random with respect to time.

5.3. Model validation

To validate the model, fourteen new experiments were conducted at various levels of each factor, including new angles of 15 and 30°. The value of $\phi$ that corresponded to the experimental BEP was used. To measure the predictible ability of the selected model, the mean squared prediction error is calculated, as defined by Kuentz [40]. The validation cases are shown in Table 5.

Table 4: ANOVA and parameter estimates for the best regression model.

<table>
<thead>
<tr>
<th>Model</th>
<th>DF</th>
<th>Sum of squares</th>
<th>Mean square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>79</td>
<td>381.08</td>
<td>4.8238</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected total</td>
<td>92</td>
<td>7188.86</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Root MSE 2.1963
R-Square 0.9470
Adj R-Sq 0.9383

5.4. Optimization and comparison to literature

Due to the second-order nature of the prediction equation, the predicted optimum value of each factor can be obtained. The partial derivative of Equation (18) is taken, set equal to zero and solved with respect to the factor under consideration. This results in four unique design equations, which are used to express the optimum value of each factor in terms of the other three factors.

$$\alpha_{opt} = \sin^{-1}\left(0.0192Z + 0.624\frac{d}{D} - 0.4388\phi + 0.2162\right)$$

(19)
By substitution, this system of equations is solved to find the optimum values for the turbine:

\[ \frac{D}{d} = 0.113 \sin \alpha - 0.0004Z + 0.0806\phi + 0.0874 \]  

\[ Z_{\text{opt}} = 17.87 \sin \alpha - 20.8 \frac{d}{D} + 36.15\phi + 1.49 \]  

\[ d_{\text{opt}} = 0.0113 \sin \alpha - 0.0004Z + 0.0806\phi + 0.0874 \]  

\[ \phi_{\text{opt}} = -0.0673 \sin \alpha + 0.0060Z + 0.689 \frac{d}{D} + 0.2351 \]  

The predicted efficiency of the turbine under these conditions is \( \eta_{\text{opt}} = 63\% \), lower than the theoretical efficiency of 89\%. This discrepancy is likely due to the relatively large skin friction and eddy formation inherent to using low cost tablespoons [17]. The optimum value of \( D/d \) is 8.5, which is consistent with literature. Note that the optimum blade number of 25 is outside of the experimental domain and because it is extrapolated it cannot be
completely trusted. However, $Z_{opt}$ can be, and often is, within the tested domain (10–20), depending on the values of the other parameters.

Fig. 13 is a photograph depicting the physical flow of the water through the turbine at low speed. The water enters from the right, splits between two blades, reverses direction and exits to the left. Some water inefficiently interacts with the blades due to edge effects and deviation of the jet’s entrance angle from the no-shock condition. Clearly, the number of blades interacting with the flow will affect the turbine’s performance.

In the following sections, the interaction terms in Equation (18) are explored. Throughout the analysis, $d/D$ is replaced with $d$ for simplicity; the results are equivalent.

$\alpha$ and $Z$

According to Equation (19), $\alpha_{opt}$ scales with $Z$, which means that as the number of blades increases and the other parameters remain constant, the incident angle of the nozzle should increase to maintain optimum performance. Surface plots are used to illustrate this interaction between $\alpha$ and $Z$. In Fig. 14, $\alpha$ and $d$ are plotted against efficiency and three plots are overlaid, each corresponding to a different $Z$. As $Z$ increases, $\alpha_{opt}$ shifts to the right. Furthermore, the turbine’s efficiency is at its lowest when there are few blades and a large incident angle. This suggests that the number of blades, as seen from the cross-sectional viewpoint of the jet, is important to fully capturing the water. The greater the angle, the more space there is for water to flow unimpeded. The converse is also true by Equation (20): $Z_{opt}$ scales with $\alpha$.

In general, the turbine is mostly unaffected by misalignments in $\alpha$. Under most circumstances, if $\alpha$ strays 10° in either direction of $\alpha_{opt}$, the turbine’s efficiency can be expected to drop by only 2%.

d and $Z$

In comparison to previous literature, Equation (20) is the Turgo equivalent to Tygun’s formula for the Pelton turbine, shown by Equation (9), verifying that the number of blades impacts the Turgo’s efficiency and that the optimum number of blades depends on $\alpha$, $d$ and $\phi$. Equations (20) and (21) indicate that the optimum blade number and nozzle diameter are inversely related to each other; however, Figs. 14 and 15 show that this interaction is minimal. For example, increasing the nozzle diameter from 7 mm to 19 mm results in only a slight decrease in the optimum number of blades (Fig. 15). Conversely, doubling the number of blades from 10 to 20 results in a slight decrease in the optimum nozzle diameter (Fig. 14).

This is not to say that the ratio of nozzle diameter to blade spacing, $d/s$ (spacing, $s$, is just the reciprocal of $Z$ multiplied by the turbine’s circumference, a constant), is not important (see section 3.4); indeed, it is. In Fig. 14, for $\alpha = 25°$, a 7 mm nozzle with 10 blades ($d/s = 0.17$) has an efficiency of 37%, while the same nozzle with 20 blades ($d/s = 0.35$) is 46% efficient. Furthermore, a 19 mm nozzle with 20 blades ($d/s = 0.91$) yields an efficiency of 56%. In Fig. 16, efficiency versus $d/s$ ratio is plotted for every trial. The trend suggests that there must be a sufficient number of blades ($d$)

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Fig. 13. Photo of water splitting between two blades.

Fig. 14. Jet angle ($\alpha$) and nozzle diameter ($d$) with blade number ($Z$) overlaid. $\phi = 0$.

Fig. 15. Jet angle ($\alpha$) and blade number ($Z$) with nozzle diameter ($d$) overlaid. $\phi = 0$. 
s > 0.45) in order to fully capture and reverse the flow of water.

There is a large spread in efficiencies for each value of d/s because each turbine speed is plotted (ρ = 0.25, 0.35 and 0.45). In reality, once d/s is set by site characteristics and turbine design, a turbine operator has the flexibility to fine-tune \( \varphi \) to the BEP. Therefore, for each d/s ratio tested, the efficiency that corresponds to the experimental BEP (the peak in Fig. 12, for example) is plotted in Fig. 17.

### \( \alpha \) and \( d \)

Recalling Section 3.1, theory states that \( \alpha_{opt} \) is a function of \( \varphi \) and \( \beta_2 \) only. From Equation (19), it is clear that the optimum inlet angle also depends on the number of blades and the nozzle diameter. Equation (19) also confirms Williamson’s findings, which indicate that \( \alpha_{opt} \) scales with \( d \). If the nozzle diameter increases, the incident angle of the nozzle should increase as well. The converse is also true: \( \alpha_{opt} \) scales with \( a \), as shown by Equation (21).

While the interaction between \( \alpha_{opt} \) and \( d \) is statistically significant, Fig. 15 shows that \( \alpha_{opt} \) remains relatively unchanged. At \( Z = 10 \), a change from a 7 mm–19 mm nozzle correlates to \( \alpha_{opt} \) increasing from 16.8° to 20.1°, not nearly as substantial as the 10° shift observed by Williamson. In this sense, while the trends are similar, the results of the UC Davis turbine indicate that the optimum jet inlet angle is much less sensitive to changes in nozzle diameter.

Moreover, Massey [18] states that \( \alpha_{opt} \) is generally a few degrees less than \( \alpha_{no-shock} \). For the spoon's inlet angle of 79°, a theoretical calculation gives \( \alpha_{no-shock} \) to be 42°, so the experimental values of \( \alpha_{opt} \) = 35° is in agreement with Massey.

### \( \alpha \) and \( \varphi \); \( d \) and \( Z \)

Equations (19) and (22) show that \( \alpha \) and \( \varphi \) are inversely related to each other. Fig. 18 shows that at higher speeds, the turbine performs best with a small jet angle and as the speed decreases, \( \alpha_{opt} \) increases. Conversely, by Equation (22), the smaller the jet angle, the larger \( \alpha_{opt} \) will be. This is because at small jet angles the velocity vector, \( v \cos \alpha \), will be larger, requiring the turbine to spin faster in order for all the water to be captured. Equation (22) also highlights that \( Z \) and \( d \) affect \( \varphi_{opt} \), and not just \( \alpha \) and \( \beta_2 \), as described in Section 3.2.

### \( \varphi \) and \( Z \)

Fig. 18 also highlights a major interaction between \( Z \) and \( \varphi \). At fast turbine speeds more blades are necessary. For example, at \( \varphi = 0.45 \) and \( \alpha = 25° \), the turbine with 10 blades is 45.5% efficient and with 20 blades it is 59.0% efficient. At slow turbine speeds whether fewer blades are necessary depends on the value of \( a \).

#### 5.5. Effect of axial impact location on efficiency

Three levels of axial impact location were tested. The locations were –8 mm (toward the leading edge), 0 mm (the center of the blade), and 8 mm (toward the discharge edge). Three replicates were collected at each level. A Tukey multiple comparison test was performed and the analysis revealed that 8 mm and 0 mm are not significantly different from each other, but that –8 mm was slightly lower in efficiency (Fig. 19). The impact location affects the performance of the UC Davis turbine by only a couple of percentage points, making it relatively robust against misalignment of the jet’s axial impact location.

#### 5.6. Effect of blade orientation angle on efficiency

The purpose of this experiment was to understand the turbine's sensitivity to changing the blade's orientation angle and to find the best orientation angle. As stated in Section 3.2, the discharge angle, \( \beta_2 \), is typically set to 15°–20° to prevent effluent water from interfering with the oncoming blade. However, in this experiment, to change the discharge angle, the blade’s orientation angle had to be changed because the curvature of the spoon was constant. Therefore, the experiment cannot distinguish whether an effect is due to
the discharge angle or the blade’s inlet angle, only the orientation angle. Discharge angle is only used for reference.

The orientation angle, which was held constant at 30° for the Central Composite Design experiment, was also tested at 25°, 35°, and 40°. The equivalent discharge angles were 9°, 15°, 19°, and 24°. The jet inlet angle was kept constant at 30°. Three replicates were performed at each level, and for each experiment the peak efficiency and the corresponding best operating speed ratio were recorded as response variables.

First, a box plot of efficiency versus discharge angle is shown in Fig. 20. While the discharge angle appears to have a parabolic trend that peaks in efficiency near 19°, Duncan and Tukey tests indicate that angles 14°, 19°, and 24° are not significantly different from each other at a 95% confidence level.

Secondly, the best operating speed ratios for each discharge angle are plotted in Fig. 21 and a Duncan Multiple Range test confirms that the values of φ are significantly different from each other. The best operating speed ratio decreases linearly with increasing discharge angle, which is what theory predicts, except the experimental φopt is smaller and has a greater slope (see Table 6). The theoretical calculations assume a friction factor of k = 0.9. In reality, the friction factor should be larger for this turbine, accounting for the discrepancy between experiment and theory.

In summary, a blade discharge angle of 19° is recommended. Straying from this angle by ±5° does not result in any significant change in turbine efficiency, but will affect the best operating speed ratio slightly.

6. Conclusions and future work

Using a face-centered central composite experiment, a regression equation was developed to predict the efficiency of a low-cost Turgo turbine as a function of jet inlet angle, number of blades, nozzle diameter, and the rotational speed of the turbine. Significant second-order effects were noticed. The nozzle diameter and the turbine’s speed ratio played the most prominent roles in influencing the turbine’s efficiency, as did the interaction between jet angle and number of blades, number of blades and speed ratio, and nozzle diameter and speed ratio. The prediction equation had an $R^2_{adj}$ equal to 0.93, indicating that the second order model is sufficient and that higher order terms would not likely improve the model.

Due to the second order nature of the prediction equation, a global optimum point was calculated. The optimum jet angle was 35°, or 7° less than the no-shock angle, which is relatively large due to the spoons’ shallow curvature compared to typical turgo blades. The optimum speed ratio was 0.425, the optimum number of blades was 25, and the optimum jet diameter was 15.4 mm, which corresponds to a ratio of $d/s = 0.94$. These optimum values will vary based on a site’s head and flow, but the optimization equations developed in this study enable the designer to calculate the best values of $a$, $Z$, $d$ and $\phi$.

Of particular interest, the equation for optimum jet angle depends heavily on the number of blades, moderately on the speed ratio, and slightly on the nozzle diameter. The results also indicate that the turbine performs best when the ratio $d/s$ is greater than 0.45. Furthermore, the results indicate that variations in the jet’s axial impact location on the blade and the blade’s orientation angle do not severely degrade the turbine’s performance. The turbine performed the best when the jet was aimed at the center of the blade and at an orientation of about 30°.

This particular tablespoon turbine could be a more economical alternative to off-grid villages than hydroelectric turbines that require casting and pattern-made molds or that are purchased commercially and imported. The turbine’s materials, including shaft and bearings, were locally available and repurposed and the
cost of new parts totaled to $35. Furthermore, the design guidelines produced by this study can be used to rapidly design commercial Turgo Turbines with geometrically similar blades for a specific site's head and flow, thereby improving mechanical efficiency.

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