Title

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Journal
Physical Review C, 81(3)

ISSN
2469-9985

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Publication Date
2010-03-01

DOI
10.1103/physrevc.81.034910

Peer reviewed

On the energy dependence of K/π fluctuations in relativistic heavy ion collisions

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In this note we will discuss the energy dependence of particle ratio fluctuations in heavy ion collisions. We study how the inherent multiplicity dependence of ratio fluctuations is reflected in the excitation function of the dynamical fluctuations. Specifically, we will show that the observed excitation function of dynamical K/π fluctuations is consistent with the expected dependence on the number of accepted pions and kaons in both the STAR and NA49 experiments.

I. INTRODUCTION

Over the last several years event-by-event fluctuations of many observables have been studied in relativistic heavy ion collisions [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. The measurement of these fluctuations may reveal separate event classes, fluctuations associated with phase-transitions etc. (for a recent review see [14]). The first measurement of event-by-event hadron ratio fluctuations has been carried out by the NA49 collaboration, which analyzed the fluctuations of the kaon to pion (K/π) ratio in lead-lead collisions at a center of mass energy of $\sqrt{s} = 17.3\,\mathrm{GeV}$ [1]. This measurement found surprisingly little fluctuations, of the order of a few percent, which could be explained by a combination of Bose-Einstein correlations together with resonance decays [15, 16].

Subsequently, the NA49 collaboration has measured event-by-event ratio fluctuations for several values of the center of mass energy, ranging from $\sqrt{s} = 6.3\,\mathrm{GeV}$ to $\sqrt{s} = 17.3\,\mathrm{GeV}$ [3]. In addition, at the Relativistic Heavy Ion Collider (RHIC) the STAR, PHENIX, and PHOBOS collaborations have measured fluctuations up to center of mass energies of $\sqrt{s} = 200\,\mathrm{GeV}$ so that excitation functions of many fluctuation observables are now available over a wide range of energies. While most excitation functions show only little energy dependence, that for the kaon to pion ratio exhibits a steep increase towards the lower energies. So far this increase could not be reproduced in either thermal model calculations [17] nor with the microscopic transport model UrQMD [3] and thus has sparked quite some interest and speculations concerning the QCD phase transition [18]. Another calculation using the HSD event-generator [19] can describe the general increase of the fluctuations towards lower energies but fails to reproduce the very steep rise exhibited in the NA49 data.

In this note, we will point out that the observable generally used in the discussion of dynamical fluctuations,

$$\sigma^2_{\mathrm{dynamical}} = \sigma^2_{\mathrm{data}} - \sigma^2_{\mathrm{mixed\ events}} \tag{1}$$

has an inherent dependence on the multiplicity of particles that are used in the experimental analysis - these are the particles located within the phase space domain covered by experimental acceptance with particle identification capability. This has some nontrivial consequences for excitation functions extracted with fixed target experiments, such as NA49, where the acceptance changes considerably with beam energy. Specifically, in case of K/π - ratio fluctuations, the dynamical fluctuations depend on the inverse of the *accepted* number of pions and kaons, and since their number decreases with beam energy this may lead to non-negligible corrections, as we shall show. Indeed in [7] a significant centrality dependence of these ratio fluctuations was found at top RHIC energies, consistent with an inverse multiplicity scaling. In addition for a fixed target experiment the actual acceptance changes with beam energy. Since it is the multiplicity of the *accepted* particles which determines the fluctuations, this needs to be taken into account if one studies an excitation function.

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It is the purpose of this note to discuss the multiplicity dependence of particle ratio fluctuations and in particular those of the kaon-to-pion ratio. After a brief review of the underlying formalism governing fluctuations of (particle) ratios, we will discuss several ways to remove the inherent multiplicity dependence in the definition of the dynamical fluctuations, $\sigma^2_{\rm dynamical}$, Eq. 1. We then apply them to the fluctuations of the kaon-to-pion ratio and discuss the energy dependence of this observable in this context.

II. FLUCTUATIONS OF PARTICLE RATIOS

Following [14, 15] the fluctuations of a particle ratio A/B are given to leading order by

$$\sigma^2 = \frac{\left\langle \left(\delta \frac{A}{B}\right)^2 \right\rangle}{\left\langle \frac{A}{B} \right\rangle^2} = \left(\frac{\langle \delta A^2 \rangle}{\langle A \rangle^2} + \frac{\langle \delta B^2 \rangle}{\langle B \rangle^2} - 2 \frac{\langle \delta A \, \delta B \rangle}{\langle A \rangle \langle B \rangle} \right) + \mathcal{O}(\delta^4) \tag{2}$$

with the definition

$$\begin{aligned} \delta A &= A - \langle A \rangle \\ \delta B &= B - \langle B \rangle \end{aligned}$$

Since

$$\begin{aligned} \langle \delta A^2 \rangle &= \langle A^2 \rangle - \langle A \rangle^2 \\ \langle \delta B^2 \rangle &= \langle B^2 \rangle - \langle B \rangle^2 \\ \langle \delta A \, \delta B \rangle &= \langle A \, B \rangle - \langle A \rangle \langle B \rangle \end{aligned}$$

Eq. 2 can also be written as

$$\sigma^2 = \left(\frac{\langle A^2 \rangle}{\langle A \rangle^2} + \frac{\langle B^2 \rangle}{\langle B \rangle^2} - 2 \frac{\langle A \, B \rangle}{\langle A \rangle \langle B \rangle} \right)$$

In absence of any correlation, $\langle \delta A^2 \rangle = \langle A \rangle$ and $\langle \delta B^2 \rangle = \langle B \rangle$, and the scaled variance (Eq. 2) reduces to

$$\sigma^2_{\rm uncorrelated} = \left(\frac{1}{\langle A \rangle} + \frac{1}{\langle B \rangle} \right). \tag{3}$$

The difference between the scaled variance σ^2 and the uncorrelated scaled variance $\sigma^2_{\rm uncorrelated}$ is commonly referred to as the dynamical fluctuations, $\sigma^2_{\rm dynamical}$,

$$\begin{aligned} \sigma^2_{\rm dynamical} &= \sigma^2 - \sigma^2_{\rm uncorrelated} \\ &= \left(\frac{\langle \delta A^2 \rangle - \langle A \rangle}{\langle A \rangle^2} + \frac{\langle \delta B^2 \rangle - \langle B \rangle}{\langle B \rangle^2} - 2 \frac{\langle \delta A \, \delta B \rangle}{\langle A \rangle \langle B \rangle} \right) \\ &= \left(\frac{\langle A \, (A-1) \rangle}{\langle A \rangle^2} + \frac{\langle B \, (B-1) \rangle}{\langle B \rangle^2} - 2 \frac{\langle A \, B \rangle}{\langle A \rangle \langle B \rangle} \right) \\ &= \nu_{\rm dynamical} \end{aligned} \tag{4}$$

where $\nu_{\rm dynamical}$ is the variable usually used by the STAR collaboration. Obviously in the absence of any correlations, $\sigma_{\rm dynamical} = 0$, by construction. Introducing the scaled correlations

$$C_{AB} \equiv \frac{\langle \delta A \delta B \rangle - \delta_{AB} \langle A \rangle}{\sqrt{\langle A \rangle \langle B \rangle}} \tag{5}$$

the dynamical fluctuations, $\sigma^2_{\rm dynamical}$, can be written as

$$\sigma^2_{\rm dynamical} = \left(\frac{1}{\langle A \rangle} C_{AA} + \frac{1}{\langle B \rangle} C_{BB} - \frac{2}{\sqrt{\langle A \rangle \langle B \rangle}} C_{AB} \right) \tag{6}$$

We note that the scaled correlations, C_{AB}, typically do not or only weakly depend on the multiplicity as we will show explicitly in the context of a resonance gas. Consequently, Eq. 6 already suggests, that a simple scaling of $\sigma^2_{\text{dynamical}}$ with number of charged particles may not be sufficient. Depending on which of the scaled correlations dominates, $\sigma^2_{\text{dynamical}}$ may either scale with $1/\langle A\rangle$, $1/\langle B\rangle$ or with $1/\sqrt{\langle A\rangle\langle B\rangle}$ or some combination of those. Furthermore, if the particle abundances differ considerably, say $\frac{\langle A\rangle}{\langle B\rangle} \ll 1$, as it is the case for the kaon to pion ratio, the dynamical fluctuations will be dominated by the least abundant particle, even if the scaled correlations are of the same magnitude. This follows directly from Eq. 6. In this case a scaling with $1/\langle A\rangle$ should work best.

Finally, quantum statistics gives rise to additional correlations [15, 20, 21]

$$\langle \delta A^2\rangle \simeq \langle A\rangle \left(1 \pm \frac{\langle n_A^2\rangle}{\langle n_A\rangle}\right) \tag{7}$$

with

$$\langle n_A^2\rangle = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\,[n_A(p)]^2$$

where $(+)$ stands for Bosons and $(-)$ for Fermions and $n_A(p)\,\mathrm{d}p$ is the number of particles of type A in momentum bin $(p, p+\mathrm{d}p)$. The correction term due to quantum statistics is typically of the order of a $5-10\%$ for systems of consideration, resulting in $\mathcal{O}(1-2\%)$ effects for the dynamical fluctuations [15, 20]. After these general remarks about fluctuations of particle ratios and their scaling with multiplicity let us turn to the specific case of kaon-to-pion ratio fluctuations.

III. MULTIPLICITY SCALING OF K/π FLUCTUATIONS

Let us now turn to the specific case of K/π fluctuations. In this case the scaled variance, Eq. 2, is given by

$$\sigma^2_{K/\pi} = \frac{\left\langle \left(\delta\frac{K}{\pi}\right)^2\right\rangle}{\left\langle \frac{K}{\pi}\right\rangle^2} = \left(\frac{\langle(\delta K)^2\rangle}{\langle K\rangle^2} + \frac{\langle(\delta\pi)^2\rangle}{\langle\pi\rangle^2} - 2\frac{\langle\delta K\,\delta\pi\rangle}{\langle K\rangle\langle\pi\rangle}\right) \tag{8}$$

and the dynamical fluctuations, Eq. 6, are

$$\sigma^2_{\text{dynamical}} = \left(\frac{1}{\langle K\rangle}C_{KK} + \frac{1}{\langle\pi\rangle}C_{\pi\pi} - \frac{2}{\sqrt{\langle K\rangle\langle\pi\rangle}}C_{K\pi}\right)$$

Since $K = K_+ + K_-$ and $\pi = \pi_+ + \pi_-$

$$\begin{aligned}
\langle K\rangle &= \langle K_+ + K_-\rangle \\
\langle \pi\rangle &= \langle \pi_+ + \pi_-\rangle && (9)\\
\langle(\delta K)^2\rangle &= \langle(\delta K_+)^2\rangle + \langle(\delta K_-)^2\rangle + 2\langle\delta K_+\delta K_-\rangle && (10)\\
\langle(\delta\pi)^2\rangle &= \langle(\delta\pi_+)^2\rangle + \langle(\delta\pi_-)^2\rangle + 2\langle\delta\pi_+\delta\pi_-\rangle \\
\langle\delta K\,\delta\pi\rangle &= \langle\delta K_+\delta\pi_+\rangle + \langle\delta K_-\delta\pi_+\rangle + \langle\delta K_+\delta\pi_-\rangle + \langle\delta K_-\delta\pi_-\rangle && (11)
\end{aligned}$$

so that the scaled correlations, Eq. 5, are given by

$$\begin{aligned}
C_{KK} &= \frac{\langle(\delta K_+)^2\rangle + \langle(\delta K_-)^2\rangle + 2\langle\delta K_+\delta K_-\rangle - \langle K_+ + K_-\rangle}{\langle K_+ + K_-\rangle} \\
C_{\pi\pi} &= \frac{\langle(\delta\pi_+)^2\rangle + \langle(\delta\pi_-)^2\rangle + 2\langle\delta\pi_+\delta\pi_-\rangle - \langle\pi_+ + \pi_-\rangle}{\langle\pi_+ + \pi_-\rangle} \\
C_{K\pi} &= \frac{\langle\delta K_+\delta\pi_+\rangle + \langle\delta K_-\delta\pi_+\rangle + \langle\delta K_+\delta\pi_-\rangle + \langle\delta K_-\delta\pi_-\rangle}{\sqrt{\langle K_+ + K_-\rangle\langle\pi_+ + \pi_-\rangle}} && (12)
\end{aligned}$$

We note, that the "diagonal" scaled correlations, C_{KK} and $C_{\pi\pi}$, also contain cross-correlations between the positively and negatively charged kaons and pions respectively. Therefore, correlations introduced by resonances such as the

ϕ-meson and the ρ_0-meson will *enhance* the "diagonal" scaled correlations and thus the dynamical fluctuations. Resonances decaying into a kaon and a pion, such as the K_0^*-meson will contribute to the off-diagonal scaled correlation, $C_{K\pi}$ and will reduce the dynamical fluctuations. This will be different if charge specific ratios such as $\frac{\delta K_+}{\delta \pi_-}$ are considered. In this case only resonances which decay in either two K_+-mesons or two π_--mesons will contribute to the diagonal terms, while the K_0^*-mesons do contribute to the off-diagonal scaled correlation. Consequently, the fluctuations of the charge specific ratio is not necessarily the same as that of the ratio of sums of negative and positive kaons/pions. This is also seen in the data by the STAR collaboration [7].

It may be instructive to discuss the above equations in the context of a simple model system which contains $n_{\pi^\pm}$ charged pions and $n_{K^\pm}$ charged kaons as well as n_{ρ_0} *neutral* rho-mesons, n_ω omega-mesons, n_ϕ phi-mesons, and $n_{K_0^*}$ neutral K_0^* and $n_{\bar{K}_0^*}$ $\bar{K}_0^*$-mesons. Via their decay channels these resonances will give rise to the various correlation terms in Eq. 12. In reality, of course, there are many other resonances contributing to σ and to the scaled correlations. The ones chosen here are the lightest and most abundant ones which lead to correlations and thus should provide a rough idea on how the different terms contribute [27]. A detailed investigation would involve a full study in the hadron resonance gas model, which is not the purpose of this paper. In addition to correlations due to resonances there are effects due to quantum statistics [20, 22], as already discussed. Here, for simplicity we work with classical (Boltzmann) statistics. Noting that the branching ratio $BR\left(\phi \to K_+ + K_-\right) \simeq \frac{1}{2}$ and $BR\left(K_0^* \to K_+ + \pi_-\right) = BR\left(\bar{K}_0^* \to K_- + \pi_+\right) \simeq \frac{2}{3}$ the average particle numbers in our simple model are given by

$$
\begin{aligned}
\langle K \rangle &= \langle K_+ + K_- \rangle = n_{K_+} + n_{K_-} + n_\phi + \frac{2}{3}\left(n_{K_0^*} + n_{\bar{K}_0^*}\right) \\
\langle \pi \rangle &= \langle \pi_+ + \pi_- \rangle = n_{\pi_+} + n_{\pi_-} + 2n_{\rho_0} + 2n_\omega + \frac{2}{3}\left(n_{K_0^*} + n_{\bar{K}_0^*}\right)
\end{aligned}
$$

and we obtain the following expression [15] for the (co)-variances in Eq. 11

$$
\begin{aligned}
\left\langle (\delta K_+)^2 \right\rangle &= n_{K_+} + \frac{1}{2} n_\phi + \frac{2}{3} n_{K_0^*} \\
\left\langle (\delta K_-)^2 \right\rangle &= n_{K_-} + \frac{1}{2} n_\phi + \frac{2}{3} n_{\bar{K}_0^*} \\
\left\langle (\delta \pi_+)^2 \right\rangle &= n_{\pi_+} + n_{\rho_0} + n_\omega + \frac{2}{3} n_{\bar{K}_0^*} \\
\left\langle (\delta \pi_-)^2 \right\rangle &= n_{\pi_-} + n_{\rho_0} + n_\omega + \frac{2}{3} n_{K_0^*} \\
\langle \delta K_+ \delta K_- \rangle &= \frac{1}{2} n_\phi \\
\langle \delta \pi_+ \delta \pi_- \rangle &= n_{\rho_0} + n_\omega \\
\langle \delta K_+ \delta \pi_- \rangle &= \frac{2}{3} n_{K_0^*} \\
\langle \delta K_- \delta \pi_+ \rangle &= \frac{2}{3} n_{\bar{K}_0^*}
\end{aligned}
$$

leading to (see Eq. 11)

$$
\begin{aligned}
\left\langle (\delta K)^2 \right\rangle &= \left\langle \delta K_+^2 + \delta K_-^2 + 2\left(\delta K_+ \delta K_-\right) \right\rangle = \langle K \rangle + n_\phi \\
\left\langle (\delta \pi)^2 \right\rangle &= \left\langle \delta \pi_+^2 + \delta \pi_-^2 + 2\left(\delta \pi_+ \delta \pi_-\right) \right\rangle = \langle \pi \rangle + 2\left(n_{\rho_0} + n_\omega\right) \\
\langle \delta K \, \delta \pi \rangle &= \frac{2}{3}\left(n_{K_0^*} + n_{\bar{K}_0^*}\right)
\end{aligned}
$$

The corresponding scaled correlations, Eq. 5, are

$$
\begin{aligned}
C_{KK} &= \frac{n_\phi}{\langle K \rangle} \\
C_{\pi\pi} &= \frac{2\left(n_{\rho_0} + n_\omega\right)}{\langle \pi \rangle} \\
C_{K\pi} &= \frac{2}{3} \frac{n_{K_0^*} + n_{\bar{K}_0^*}}{\sqrt{\langle K \rangle \langle \pi \rangle}}
\end{aligned}
\tag{13}
$$

We find, that the scaled correlations in our simple model indeed depend only weakly (if at all) on the multiplicity, since both the number of resonances and the number of kaons and pions are expected to scale roughly with the multiplicity

or volume of the system. In a thermal system at fixed temperature they would be constant. Furthermore, as already discussed the correlations introduced via the resonances affect all three scaled correlations. While the K_0^*-mesons control the off-diagonal correlation term, $\langle \delta K\, \delta\pi \rangle$, both the ϕ-meson and the ρ_0 and ω contribute to the diagonal parts. The former reduce the dynamical fluctuations while the latter increase them. Putting everything together, the scaled variance, Eq. 2, is given by

$$\sigma^2_{K/\pi} = \left(\frac{\langle K \rangle + 2n_\phi}{\langle K \rangle^2} + \frac{\langle \pi \rangle + 2\left(n_{\rho_0} + n_\omega\right)}{\langle \pi \rangle^2} - 2\,\frac{\frac{2}{3}\left(n_{K_0^*} + n_{\bar{K}_0^*}\right)}{\langle K \rangle \langle \pi \rangle} \right)$$

leading to

$$\sigma^2_{\text{dynamical}} = \left(\frac{n_\phi}{\langle K \rangle^2} + \frac{2\left(n_{\rho_0} + n_\omega\right)}{\langle \pi \rangle^2} - 2\,\frac{\frac{2}{3}\left(n_{K_0^*} + n_{\bar{K}_0^*}\right)}{\langle K \rangle \langle \pi \rangle} \right).$$

or in terms of the scaled correlations

$$\sigma^2_{\text{dynamical}} = \left(\frac{1}{\langle K \rangle} C_{KK} + \frac{1}{\langle \pi \rangle} C_{\pi\pi} - \frac{2}{\sqrt{\langle K \rangle \langle \pi \rangle}}\, C_{K\pi} \right) \tag{14}$$

Evaluating scaled correlations for our simple model, Eq. 13, for a temperature of $T = 170\,\text{MeV}$ and vanishing chemical potential, we get

$$\begin{aligned} C_{KK} &= 0.1 \\ C_{\pi\pi} &= 0.36 \\ C_{K\pi} &= 0.13 \end{aligned}$$

Obviously, in our simple model all the scaled correlations are of the same order of magnitude. In addition, adding more resonances will likely reduce $C_{\pi\pi}$ as there are many resonances decaying into only one pion, which add to the denominator, $\langle \pi \rangle$, but not the numerator of $C_{\pi\pi}$. While it would be worthwhile to study these scaled correlation coefficients in a full hadron gas model, here instead we want to concentrate on simple phenomenological scaling rules, with a special emphasis on the effect of a varying acceptance.

IV. PHENOMENOLOGICAL SCALING

In this section we want to discuss several ways to scale out the multiplicity dependence of the dynamical fluctuations. Of course, if all the relevant scaled correlations are known and if they, as we argued, depend only weakly on the multiplicity and beam energy, the appropriate scaling is simply given by Eq. 14. This is equivalent to having full understanding of all the sources for the fluctuations in which case this discussion is mute. In general, however, we do not have a full understanding of all the sources contributing to the fluctuations. In this case the multiplicity/energy dependence may provide additional information such as signals for a possible phase transition etc. In order to extract this information, one needs to understand "trivial" dependencies on the multiplicity, as exhibited e.g. in Eq. 14. In the following we will discuss several "trivial" scaling prescriptions which we believe should be applied before conclusions about new phenomena can be drawn. We will focus on the rather interesting energy dependence of the K/π-fluctuations, referring to [7] for a discussion on the centrality dependence. In addition to applying appropriate scaling prescriptions to the dynamical fluctuations, it is essential to realize that the multiplicities which control the fluctuations, such as $\langle K \rangle$ and $\langle \pi \rangle$ are that of the *identified* particles and *not* the extrapolated total multiplicities or mid-rapidity multiplicities. This is especially important when studying the energy dependence of fixed target data, as the acceptance varies with beam energy. We will discuss this point in detail below after we have introduced the various scaling prescriptions.

A. Multiplicity Scaling Prescriptions

As suggested in [15], one way to avoid any scaling of the dynamical fluctuations with the number of accepted particles would be to take the ratio of the measured scaled variance σ^2 over that of mixed events σ^2_{mixed},

$$f \equiv \frac{\sigma^2}{\sigma^2_{\text{mixed}}},$$

instead of the difference as it is done in the definition of $\sigma^2_{\rm dynamical}$, Eq. 1. This, however, has the disadvantage, that correlations and fluctuations due to the the detector do not cancel out. In order to remove the multiplicity dependence in the same fashion, one can simply divide the dynamical fluctuations, $\sigma^2_{\rm dynamical}$, by that of uncorrelated particles, $\sigma^2_{\rm uncorrelated}$, Eq. 3, evaluated for the same number of particles.

$$f_{\rm Poisson} \equiv \frac{\sigma^2_{\rm dynamical}}{\sigma^2_{\rm uncorrelated}}. \tag{15}$$

with

$$\sigma^2_{\rm uncorrelated} = \frac{1}{\langle K\rangle} + \frac{1}{\langle \pi\rangle}$$

for the case at hand. This scaling we will subsequently denote as "Poisson" scaling, as we scale with the scaled variance of a Poisson distribution based on the observed multiplicities. The advantage of this scaling is that it is unbiased in the sense that one does not need to make any assumptions about the relative magnitude of the scaled correlations. Another unbiased scaling would be to scale with the number of particles involved, i.e. $N_p = \langle K\rangle + \langle \pi\rangle$. This is similar in spirit of [7] where a scaling with the number of charged particles was studied. We shall henceforth refer to this as particle number scaling,

$$f_{\rm Particle\,Number} = \left(\langle K\rangle + \langle \pi\rangle\right)\sigma^2_{\rm dynamical} \tag{16}$$

Finally the expression for $\sigma^2_{\rm dynamical}$ in terms of the scaled correlations, Eq. 14, suggests the scaling with the kaon or pion number or with the geometric mean of both, depending on which of the scaled correlations dominates.

$$\begin{aligned} f_{\rm Kaon\,Number} &= \langle K\rangle\,\sigma^2_{\rm dynamical} \\ f_{\rm Pion\,Number} &= \langle \pi\rangle\,\sigma^2_{\rm dynamical} \\ f_{\rm geometric} &= \sqrt{\langle K\rangle\,\langle \pi\rangle}\sigma^2_{\rm dynamical} \end{aligned}$$

Since the number of kaons is much smaller than the number of pions, at least for the lower energies the kaon-number scaling is similar to the Poisson scaling,

$$\sigma^2_{\rm uncorrelated} = \frac{1}{\langle K\rangle} + \frac{1}{\langle \pi\rangle} = \frac{\langle K\rangle + \langle \pi\rangle}{\langle K\rangle\,\langle \pi\rangle} \simeq \frac{1}{\langle K\rangle},$$

for $\langle K\rangle \ll \langle \pi\rangle$. Alternatively the above scaling relations allow to relate the dynamical fluctuations at a given center of mass energy with those at another energy. Specifically, given the dynamical fluctuations at top RHIC energies, $\sqrt{s} = 200\,{\rm GeV}$, the dynamical fluctuations at any other center of mass energy is given by

- *Poisson scaling:*

$$\sigma_{\rm dynamical}\left(\sqrt{s}\right) = \sigma_{\rm dynamical}\left(200\,{\rm GeV}\right)\;\frac{\sqrt{\frac{1}{\langle K\rangle} + \frac{1}{\langle \pi\rangle}}|_{\sqrt{s}}}{\sqrt{\frac{1}{\langle K\rangle} + \frac{1}{\langle \pi\rangle}}|_{200\,{\rm GeV}}} \tag{17}$$

- Particle Number scaling

$$\sigma_{\rm dynamical}\left(\sqrt{s}\right) = \sigma_{\rm dynamical}\left(200\,{\rm GeV}\right)\;\frac{\sqrt{\langle K\rangle + \langle \pi\rangle}|_{200\,{\rm GeV}}}{\sqrt{\langle K\rangle + \langle \pi\rangle}|_{\sqrt{s}}} \tag{18}$$

- *N_K-scaling:*

$$\sigma_{\rm dynamical}\left(\sqrt{s}\right) = \sigma_{\rm dynamical}\left(200\,{\rm GeV}\right)\;\frac{\sqrt{\langle K\rangle}|_{200\,{\rm GeV}}}{\sqrt{\langle K\rangle}|_{\sqrt{s}}} \tag{19}$$

- *N_π-scaling:*

$$\sigma_{\rm dynamical}\left(\sqrt{s}\right) = \sigma_{\rm dynamical}\left(200\,{\rm GeV}\right)\;\frac{\sqrt{\langle \pi\rangle}|_{200\,{\rm GeV}}}{\sqrt{\langle \pi\rangle}|_{\sqrt{s}}} \tag{20}$$

- *Geometric Scaling:*

$$\sigma_{\rm dynamical}\left(\sqrt{s}\right) = \sigma_{\rm dynamical}\left(200\,{\rm GeV}\right)\ \frac{\left(\langle K\rangle\,\langle\pi\rangle\right)^{1/4}|_{200\,{\rm GeV}}}{\left(\langle K\rangle\,\langle\pi\rangle\right)^{1/4}|_{\sqrt{s}}} \tag{21}$$

The resulting *scaled* dynamical fluctuations are shown in Fig. 1 where we find that the energy dependence of $\sigma^{K/\pi}_{\rm dynamical}$ is reasonably reproduced by any of the scaling rules discussed above. The values for the multiplicities of identified particles taken from [3, 7] as well as the result for the rescaled fluctuations according to Eqs. (17-21). The essential point for the success of these scaling rules is that we have used the number of *identified* kaons and pions for the mean values, $\langle K\rangle$ and $\langle\pi\rangle$, entering the scaling formulae, Eqs. 17-21. This leads to an additional energy dependence especially for the NA49 data. Since NA49 is a fixed target experiment, the actual acceptance and thus the fraction of identified particles of the total number of particles may vary considerably with the beam energy. This is illustrated in Fig. 2, where we applied the same scaling formulae, Eqs. 17-21, but used the mid-rapidity yields, $\frac{{\rm d}N}{{\rm d}y}(y=0)$ for the respective mean particle numbers, $\langle K\rangle$ and $\langle\pi\rangle$ [23, 24, 25]. Obviously the energy dependence is not reproduced, especially for the highest SPS energies, $\sqrt{s} = 12.3\,{\rm and}\,17.3\,{\rm GeV}$.

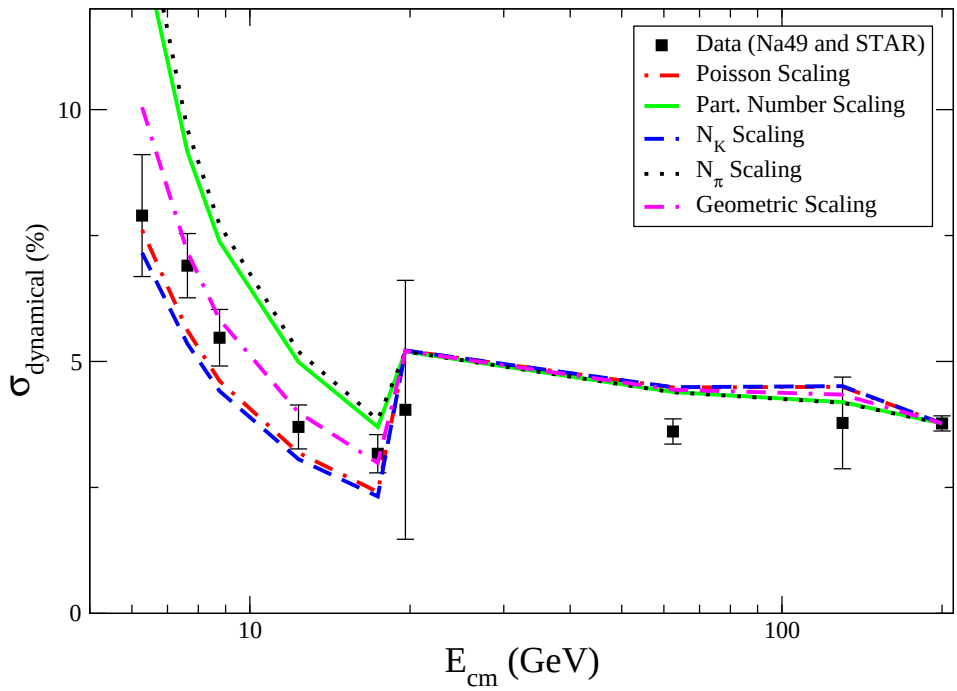

Figure 1: Different scaling scenarios, Eqs. 17-21 based on the $\sqrt{s} = 200\,{\rm GeV}$ data from STAR

Looking more closely at Fig. 1, we see that the STAR data at $\sqrt{s} = 62.4\,{\rm GeV}$, which have a rather small error-bar are not well reproduced. Instead of arguing for new physics in this energy regime, we note that in Ref. [7] the two most central values for $\sigma_{\rm dynamical}$ at this energy do not agree very well with the systematics developed by the STAR collaboration either. As already discussed in section II, since $\langle K\rangle\,/\,\langle\pi\rangle \ll 1$ the dynamical fluctuations are dominated by the kaons. Therefore, the kaon-number or Poisson scaling should work better than pion-number or particle number scaling. This is indeed the case.

One may be tempted to use the quality of agreement of the various scaling prescriptions to draw conclusions about the importance of the various contributions to the dynamical fluctuations. Given the experimental error-bars and the quality of agreement of the various scaling rules the value of such an exercise is not obvious. At least, a global fit based on the various scaling prescriptions needs to be carried out before any more detailed conclusions about the strength of the various contributions can be drawn.

Finally let us comment on the fact that calculations based on the UrQMD model do not reproduce the rise of the dynamical fluctuations as observed by NA49, although the NA49 acceptance has been applied [3]. First of all, while UrQMD seems to do reasonably well for mid-rapidity abundances, it is not clear if UrQMD does reproduce the correct multiplicity of identified particles *within* the acceptance, especially at small center of mass energies. Second, it is not at all obvious, if UrQMD contains indeed all the relevant sources for fluctuations and correlations. For

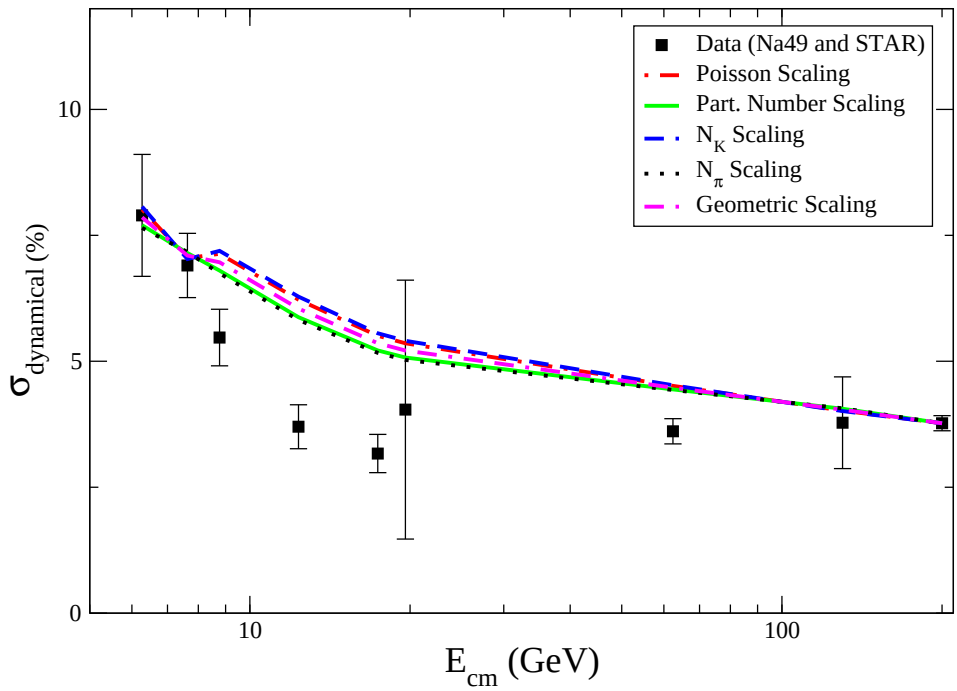

Figure 2: Scaling with $\frac{\mathrm{d}N}{\mathrm{d}y}(y=0)$ instead of the actual number of identified particles contributing to $\sigma_{\mathrm{dynamical}}$. Otherwise same scaling formulae are used as in Fig. 1.

instance quantum statistics is not taken into account. The fact that the above scaling relations are able to connect the dynamical fluctuations over a wide range of beam energies does not say anything about the nature and origin of these fluctuations except that they seem to be the same at all energies. While this considerably weakens the argument for the K/π-fluctuations being a signature for the QCD critical point, the disagreement with UrQMD may very well point to a yet to be discovered new source for fluctuations/correlations. This however, requires that all "trivial" sources, such as e.g. quantum statistics, are systematically taken into account.

Finally, let us conclude this section by proposing an observable, which should, to leading order, be independent of the multiplicity. As already discussed at the beginning of this section, the ratio

$$f \equiv \frac{\sigma^2}{\sigma^2_{\mathrm{mixed}}}$$

proposed in [15] has the disadvantage that correlations and fluctuations due to the detector do not cancel out. We, therefore, propose to study instead the ratio, Eq. 15

$$f_{\mathrm{Poisson}} \equiv \frac{\sigma^2_{\mathrm{dynamical}}}{\sigma^2_{\mathrm{uncorrelated}}}$$

or equivalently $f_{\mathrm{Poisson}}+1$, which has the same limit as f in the absence of correlations. In Fig. 3 we show $f_{\mathrm{Poisson}}+1$ for both the K/π fluctuations as well as for the p/π-fluctuations, which have been measured by NA49 as well. We see a rather weak energy dependence of the K/π-fluctuations, whereas the p/π-fluctuations exhibit a variation with energy. We note, however, that the energy dependence of the p/π-fluctuations in the scaled variable is opposite to that in $\sigma^2_{\mathrm{dynamical}}$. While $\sigma^2_{\mathrm{dynamical}}$ decreases with energy, the magnitude of f_{Poisson} increases, suggesting in increase of the the strength of the correlations with energy, which seems to level off at top SPS energies. This suggests that the correlations, mostly due to baryon resonances, increase with energy up to $\sqrt{s} \simeq 15\,\mathrm{GeV}$. Using the chemical freeze-out parameters of Ref. [26] and assuming that only the delta resonance contributes to the scaled correlation coefficient $C_{p\pi}$ we find indeed an increase of the correlations from AGS up to SPS energies.

V. CONCLUSIONS

In this article we have reviewed the multiplicity dependence of particle ratio fluctuations. We have provided several scaling prescriptions which correct for the inherent dependence of the dynamical fluctuations $\sigma^2_{\mathrm{dynamical}}$ on

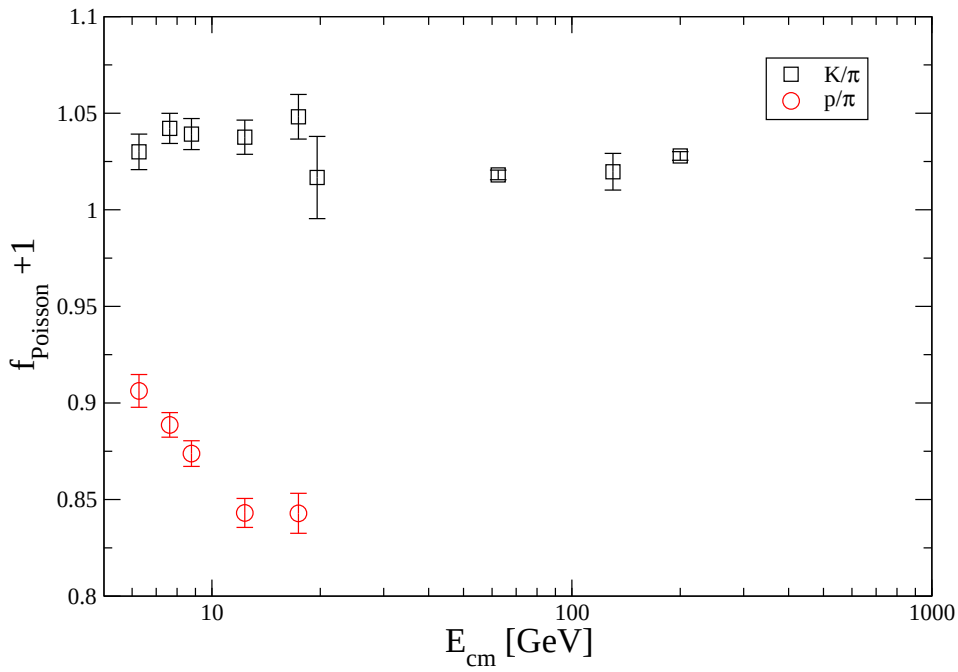

Figure 3: The rescaled fluctuations $f_{\rm Poisson} + 1$ for K/π (black) and p/π-fluctuations (red).

$\sqrt{s}$[GeV]	$\langle\pi\rangle_{ident.}$	$\langle K\rangle_{ident.}$	$\sigma_{\rm dyn.}$[%]	Geom. scaling	N_K-scaling	N_π-scaling	Poisson-scaling	Part. Num. scaling	$f_{\rm Poisson}$(%)
6.27	30.9	5.7	7.89	10.05	7.15	14.11	7.61	13.27	3.0
7.63	66.6	10.2	6.90	7.17	5.35	9.60	5.61	9.15	4.2
8.77	103.3	15.0	5.47	5.83	4.41	7.71	4.61	7.37	3.9
12.3	227.6	31.2	3.70	3.99	3.05	5.19	3.18	4.98	3.8
17.3	416.6	54.2	3.17	2.99	2.32	3.84	2.41	3.69	4.8
19.6	227.6	10.7	4.04	5.21	5.21	5.19	5.21	5.19	1.7
62.4	319.2	14.5	3.61	4.44	4.48	4.38	4.48	4.39	1.8
130	351.9	14.4	3.78	4.34	4.51	4.18	4.49	4.19	2.0
200	432.6	20.6	3.77	3.77	3.77	3.77	3.77	3.77	2.8

Table I: Table of the results for the various scaling scenarios Eqs. (17-21) using the STAR data at $\sqrt{s} = 200\,$GeV as reference. The data for identified multiplicities, $\langle\pi\rangle_{ident.}$ and $\langle K\rangle_{ident.}$, are from [3, 7].

the number of *identified* particles. We have demonstrated that these scaling rules naturally reproduce the trend seen in the energy dependence of the kaon-to-pion fluctuations. Consequently any interpretation of the rise of $\sigma^{K/\pi}_{\rm dynamical}$ towards lower energies in the context of a possible QCD critical point, needs to account for the "trivial" effect due to the multiplicity dependence. We propose that future studies of energy and / or multiplicity dependencies should correct for the "trivial" multiplicity dependencies inherent in $\sigma_{\rm dynamical}$. In our view, the simplest and least biased scaling is the Poisson scaling, $f_{\rm Poisson}$, which could serve as a benchmark. Of course a more complete analysis of the energy dependence would include a thorough global fit of the available data, which we have not carried out in this paper.

We have further applied the scaling to the p/π-fluctuations measured by the NA49 collaboration. We find that the properly scaled observable still exhibits a strong energy dependence, which, however, is opposite to the unscaled one. In this context it would be interesting to include the STAR data, which unfortunately are not available in a final version yet.

Finally we point out that any multiplicity scaling needs to be based on the mean multiplicity of the actual identified particles used for the fluctuation measurement instead of an extrapolated multiplicity such as $\mathrm{d}N/\mathrm{d}y$.

Acknowledgments

The authors would like to thank G. Westfall and R. Stock for useful discussions. This work was initiated during the Program "The QCD Critical Point" at the Institute for Nuclear Theory (INT) in Seattle, WA. The authors would like to thank the INT for the hospitality during this program. This work is supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Divisions of Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. The authors also acknowledge support by the Deutsche Forschungsgemeinschaft (DFG) and the Helmholtz Research School on Quark Matter Studies.

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