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LETTERS

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On the nature of bursting in transport and turbulence in drift wave–zonal flow systems

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The predictions of the extended predator–prey model of the coupled spectral dynamics of drift wave–zonal flow turbulence are presented. The model exhibits three possible types of time-dependent solutions, depending on system parameters, which are: (1) quasiperiodic bursting of the transport and turbulence intensity levels; (2) oscillatory relaxation to a stationary state, and in the collisionless limit; (3) an intensity pulse followed by saturation of zonal flow. These solutions are consistent with the time dependent behavior recently observed in the global gyrokinetic simulations.
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The pressure gradient driven instabilities [such as the ion temperature gradient instability (ITG) of drift waves[1]] in magnetic confinement devices have been the subject of great concern because of their capability to drive transport. However, after a decade of intensive research work, summarized, e.g., in Ref. 2, and especially due to significant recent advances in computer simulations (see, e.g., Ref. 3), a consensus seems to be reached that the problem may be very well self-regulating. The optimism stems from the nonlinear development of the drift waves leading to the excitation of random sheared flows across the gradient. These flows (often called zonal) are not only intrinsically incapable of driving transport but also reduce the intensity of the parent drift waves and thus the transport associated with them.[4–7]

The almost universally accepted mechanism of the zonal flow generation is the modulational (as given, e.g., in Ref. 8) or parametric (see, e.g., Refs. 9 and 10) instability of the Reynolds stress of the drift waves. Based on this concept a model for description of the coupled drift wave (DW)–zonal flow (ZF) turbulence has been suggested in Ref. 8. This model has a characteristic form of a "predator–prey" system in which the population of the DW quanta (prey) growing via linear (ITG) instability, generates ZF (predator) through the Reynolds stress. Concomitantly, the zonal flow growth regulates the prey (DW) population. Starting from the modulational instability of drift waves, this formalism yields a coupled system for the drift wave and zonal flow spectra. The ZF reduces the DW population by random shearing thus making the entire system distinctively self-regulating. A different but related example of the parametric zonal flow generation that has been given recently in Ref. 10, is based on the toroidal coupling of the drift wave triads. Some further possible mechanisms for zonal flow generation and saturation are still debated. For example, the authors of Ref. 11 argue that they may be generated by the (secondary) Kelvin–Helmholtz-type instability of radial streamers occurring in the ITG unstable primary drift wave. Another interesting suggestion[12] invokes the geodesic acoustic modes at the edge transition region.

The steady state solutions of the predator–prey system have been analyzed in Refs. 8, 13, and 14. Such systems are, however, dynamical systems that, along with the equilibrium solutions, often display strong nonlinear oscillations in population densities caused by their mutual self-regulation. Precisely this behavior has indeed been observed in global gyrokinetic simulations.[3,15] However, since such large codes have only been run for a few oscillation periods, the precise nature of the burst phenomena is not understood. It is also important to note that in this model both the populations "live" in the wave vector space rather than in coordinate space. This is crucial since the mechanism by which the ZF "kills" DW is somewhat indirect, namely the DWs are "expelled" by random shearing to the region of high radial wave numbers k_r where they are (linearly) damped. "Shearing" is thus just another word for the three wave interaction generating small radial scales (high k_r) in the drift wave turbulence. It is clear, however, that this type of dynamics cannot be captured by reduced models with only a few modes. Indeed, recent analyses of the gyro-kinetic simulations[6,15] point at the excitation of an extended k_r spectra of the DWs that broadens during

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the periods of enhanced ZF flow activity which is indicative of shearing.

In this Letter we study bursting solutions of the predator–prey model for the purpose of detailed comparison with the simulation results. As in our previous steady state analyses, we shall focus on the dependence of the solutions on ion collisionality. This is important because the ion-ion collisions are shown to be the only secular (nontransient) ZF damping process.[16] However, in contrast to the steady state studies we shall focus on the parameter regime close to marginality, where bursting is usually observed.[15] Also, collisional friction has been accurately treated in simulations[15] so that now detailed comparisons can be made, including the examining the causal relation between ZF and DW turbulent component (phase lag) and the limit of zero collisionality (Dimits shift regime[7]). As we argued earlier,[14] these phenomena cannot be properly addressed within the steady state treatment.

The drift wave evolution in random zonal flow field is described by the following equation for the spatially averaged number of quanta density $\langle N(k_r, k_\theta)\rangle$:[8]

$$\frac{\partial\langle N\rangle}{\partial t}-\frac{\partial}{\partial k_r}D_{\mathbf{k}}\frac{\partial\langle N\rangle}{\partial k_r}=\gamma_{\mathbf{k}}\langle N\rangle-\frac{\Delta\omega_{\mathbf{k}}}{N_B}\langle N\rangle^2. \tag{1}$$

Here the diffusivity in radial wave number k_r is related to the ZF intensity $|\Phi_q|^2$ through $D_{\mathbf{k}}=\Sigma_q k_\theta^2 q^4\rho_s^2 c_s^2 R(\mathbf{k},\mathbf{q})|\Phi_{\mathbf{q}}|^2$, where $\rho_s^2=T_e/M\omega_{ci}^2=c_s^2/\omega_{ci}^2$. The ZF potential is taken to be $\Phi(r,t)=\Sigma_q\Phi_q\exp(iqr-i\Omega_q t)$. The choice of the response function R depends, generally speaking, on the turbulence regime; for the case of relatively weak self-nonlinearity $\Delta\omega_{\mathbf{k}}$ of the DWs ($\Delta\omega_{\mathbf{k}}\langle N\rangle/N_B\ll\gamma_{\mathbf{k}}$, where N_B is the background turbulence level[14]), we may set $R=\pi\delta(\Omega_q-qV_{\mathrm{gr}})$. Here $V_{\mathrm{gr}}=\partial\omega_{\mathbf{k}}/\partial k_r$ is the radial component of the drift wave group velocity. For a stronger DW nonlinearity, namely, when the two terms on the right-hand side of Eq. (1) are approximately in balance, the response function R broadens to $R=\gamma_{\mathbf{k}}/[\gamma_{\mathbf{k}}^2+(\Omega_q-qV_{\mathrm{gr}})^2]$, so that the linear result can also be recovered by taking the limit $\gamma_{\mathbf{k}}\to 0$.[8]

Generation and damping of zonal flow is, in turn, governed by the following equation:

$$(\partial_t+\gamma_d)|\Phi_q|^2=2\Im\Omega|\Phi_q|^2. \tag{2}$$

Here γ_d is the collisional damping rate. The calculation of the zonal flow excitation rate $\Im\Omega$ may be conveniently approached by considering the instability of the DW packet that has the quanta density number modulation with respect to its perturbation of the form $\tilde{N}\sim\exp(iqr-i\Omega_q t)$. The corresponding dispersion equation reads[8,17]

$$1+\frac{qc_s^2}{2V_*}\int\frac{k_\theta d\mathbf{k}}{\Omega-qV_{\mathrm{gr}}}\frac{\partial\langle N\rangle}{\partial k_r}=0. \tag{3}$$

Here the group velocity is $V_{\mathrm{gr}}=-2V_* k_r k_\theta\rho_s^2(1+k_\perp^2\rho_s^2)^{-2}$ and the drift velocity $V_*=-(cT_e/eB)d\ln n_0/dr$. It was shown recently[18] that unless the spectrum $\langle N\rangle$ is monochromatic in *both* k_r and k_θ, there is an amplitude threshold for the zonal flow generation. It is, however, still reasonable to assume that there is a distinct $k_\theta=k_{\theta 0}$ [prescribed by $\gamma_{\mathbf{k}}$ in (1)], since random refraction acts only on k_r. It is convenient to use V_{gr} as a new variable in place of k_r which we denote $V=V_{\mathrm{gr}}(k_r,k_{\theta 0})$. An important role is then played by the caustics $k_r=\pm k_c$, where $\partial V/\partial k_r=0$. This can be seen from Eq. (3) after transforming it to the following form:

$$1-\frac{k_{\theta 0}c_s^2}{2V_*}\int_{-V_m}^{V_m}\frac{dV}{V-C}\frac{\partial}{\partial V}\mathcal{N}=0, \tag{4}$$

where $C=\Omega/q$ and we have denoted $\mathcal{N}(V)=N^+-N^-$. Here $N^\pm$ are the values of N taken at the same V but at different k_r, namely inside ($|k_r|<k_c$) the region between the two caustics and outside of it ($|k_r|\geqslant k_c$). The integral in Eq. (4) is taken between the minimum and the maximum of the function $V(k_r)$. It is reasonable to assume[18] that $N(-k_r)=N(k_r)$, so that $\mathcal{N}(-V)=\mathcal{N}(V)$ in Eq. (4). To better understand the solutions of Eq. (4), using an obvious constraint $\mathcal{N}(\pm V_m)=0$, we first use, for the purpose of clarity, the parabolic approximation, $\mathcal{N}(V)=\mathcal{N}_0(1-V^2/V_m^2)$. The last equation then can be written as

$$\Omega=\frac{2iqV_m}{\pi+\alpha-i\ln\left|\dfrac{1-C/V_m}{1+C/V_m}\right|}\frac{\mathcal{N}_0-V_*V_m/2c_s^2k_{\theta 0}}{\mathcal{N}_0}, \tag{5}$$

where $\alpha=\arg(1-C/V_m)-\arg(1+C/V_m)$. This is an exact, though implicit, solution to Eq. (4). We may assume that $|C|\ll V_m$ and thus drop α and the ln term in denominator. The solution then becomes explicit and the threshold nature of the instability is clearly given by the second factor in Eq. (5). In addition, this consideration shows that being interested in the case of small C/V_m, we may generalize the last result for an arbitrary $\mathcal{N}(V)$. A simple evaluation of Eq. (4) yields

$$\Omega=\left.\frac{iqV}{\pi\partial\mathcal{N}/\partial V}\right|_{V=0}\left[\int_{-V_m}^{V_m}\frac{dV}{V}\frac{\partial\mathcal{N}}{\partial V}+\frac{V_*}{c_s^2k_{\theta 0}}\right]. \tag{6}$$

Now we may obtain the evolution equation for $D_{\mathbf{k}}$ in Eq. (1),

$$(\partial_t+\gamma_d)D_{\mathbf{k}}=\bar{\gamma}_{\mathbf{k}}D_{\mathbf{k}}, \tag{7}$$

with the spectrum averaged growth rate $\bar{\gamma}_{\mathbf{k}}$ given by

$$\bar{\gamma}_{\mathbf{k}}=2\frac{\Sigma_q k_\theta^2 q^4\rho_s^2c_s^2R(\mathbf{k},\mathbf{q})|\Phi_{\mathbf{q}}|^2\Im\Omega}{\Sigma_q k_\theta^2 q^4\rho_s^2c_s^2R(\mathbf{k},\mathbf{q})|\Phi_{\mathbf{q}}|^2}. \tag{8}$$

Since the zonal flow growth rate $\Im\Omega$ increases with q, the main contributions to the integrals over q in the last formula should come from the short scale cutoff $q=q_{\max}$ of the zonal flow spectrum $|\Phi_q|^2$. The latter can be estimated from the restriction that the zonal flow instability time scale be longer than the geodesic acoustic mode (GAM) period, i.e., $\Im\Omega\lesssim c_s/R$ (see Ref. 19), where R is the major radius of the torus. This requirement differentiates the shear layers considered here from GAMs. Therefore, in Eq. (8) we may set $\bar{\gamma}_{\mathbf{k}}=2\Im\Omega(q_{\max})$, where $q_{\max}$, as we mentioned, comes from the relation $\Im\Omega=c_s/R$. Now Eqs. (1) and (7) form a closed system that may be easily analyzed.

To understand the dynamics of coupled DW–ZF turbulence, we integrate the system formed by Eqs. (1) and (7) in

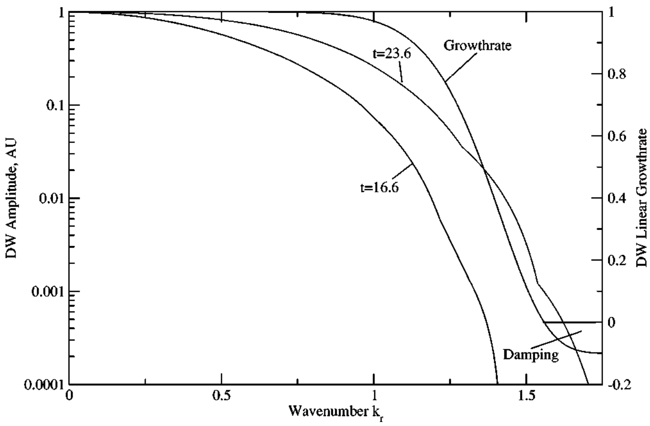

FIG. 1. The linear growth rate of the drift waves and their spectra just before the onset of the zonal flow ($t=16.6$) and at its developed regime ($t=23.6$, see also Fig. 2).

time for a simple model of the DW linear growth rate $\gamma_{\mathbf{k}}$ shown in Fig. 1 together with two DW k_r-spectra scanned at the times indicated in Fig. 2. There are a number of other parameters involved in these equations and we leave the scan of parameters space out of the scope of this Letter. One may identify, however, the most critical parameters that control the selection of one of the three types of behavior:

(1) Bursty relaxation to a steady state;
(2) quasiperiodic bursting;
(3) one pulse of the DW followed by the ZF saturation.

One of the most critical system parameters is the ZF damping γ_d, and the only situation in which (3) occurs is where $\gamma_d=0$. This is clearly a very special case so that it is convenient to start from the general one, namely, $\gamma_d>0$.

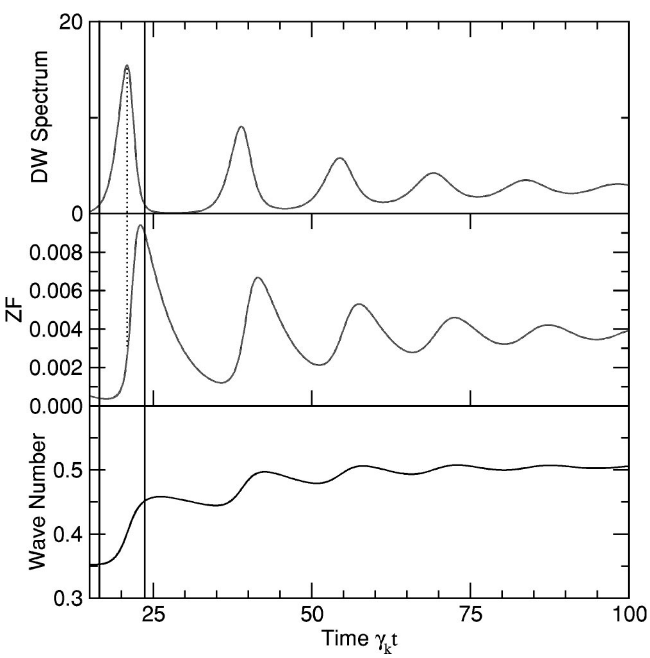

FIG. 2. The sequence of turbulence pulses that time asymptotically converges to a fixed point rather than a limit cycle. The maximum growth rate of the zonal flow coincides with the maximum of the DW intensity, as illustrated by the vertical dotted line. Two other lines indicate times at which the k_r-specra shown in Fig. 1 are scanned.

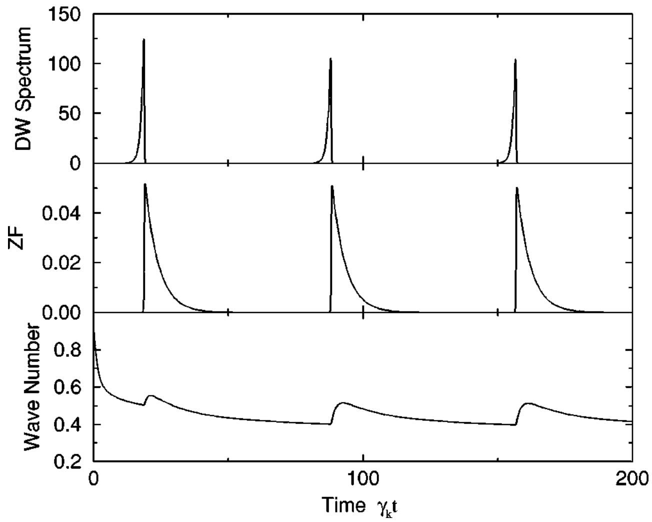

FIG. 3. Almost strictly periodic sequence of bursts (limit cycle) with only the first one slightly stronger than subsequent ones due to the transient effects caused by initial conditions. In this case the initial spectrum was taken to be significantly broader than in Fig. 2, which does not influence the time asymptotic behavior.

The type (1) dynamics (bursty relaxation to a fixed point, Fig. 2) is perhaps the generic one. It occurs rather robustly for sufficiently strong linear and/or nonlinear DW damping and if the threshold of ZF generation in Eq. (6) (the second term in the brackets) is sufficiently small.

The type (2) behavior is shown in Fig. 3. For this to occur the amplitude threshold of the ZF generation is critical. If we (artificially) suppress it, so that the ZF is generated starting from zero DW intensity (as in the monochromatic case[17,18]), the type (2) dynamics occurs only if min $\gamma(k_r)\geqslant 0$ *and* $\Delta\omega=0$. In other words, there must be *no linear and nonlinear dissipation* of DWs, which is clearly unrealistic. Thus, quasiperiodic bursting is unlikely.

In both these cases there exists a causal relation (phase lag) between the DW and the ZF pulses, so that the latter follows the former, precisely as observed in simulations.[15] Note that similar time delay of ZFs has been observed in numerical studies of the resistive ballooning turbulence sys-

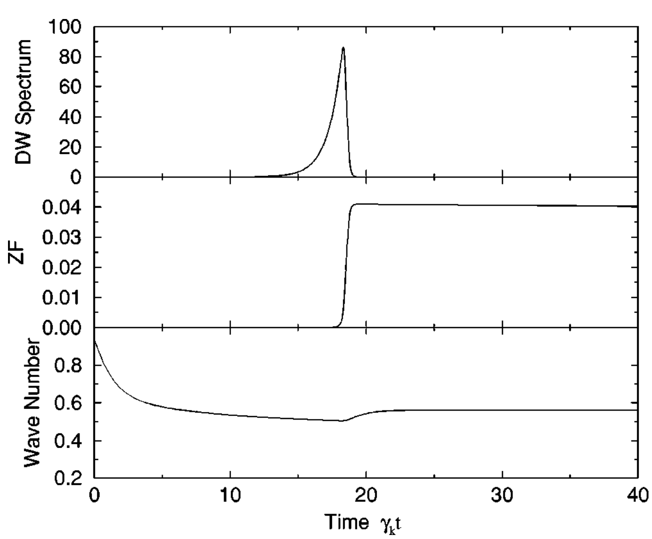

FIG. 4. The system dynamics in the limit $\gamma_d\rightarrow 0$ (Dimits shift regime).

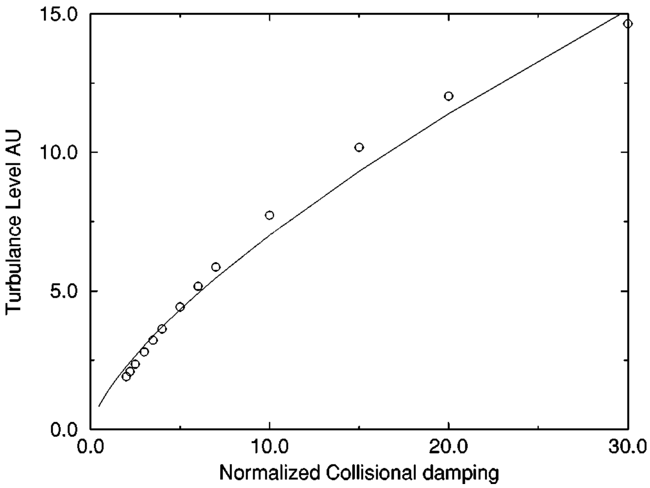

FIG. 5. The scaling of the drift wave intensity level with the zonal flow collisional damping (circles) shown together with the $\gamma_d^{0.75}$ scaling obtained in Ref. 15.

tem in Ref. 20. Similar relationship between the transport driving fluctuations and the ZF intensity have been demonstrated in recent experiments with the H-1 heliac[21,22] and in the subsequent data analysis.[23] The maximum of the DW intensity coincides with the maximum of the ZF growth, Fig. 2. The decay time of ZF flow is set by γ_d. The interval between the bursts, however, while growing with γ_d^{-1},[15] also depends on other parameters, such as $\Delta\omega$. In all cases the onset of the ZF results in broadening of the DW spectrum in k_r, so as its tail spreads over the domain of linear stability, Fig. 1. This ultimately forces DW damping.

Based on the above remark, the case (3) ($\gamma_d=0$) may be interpreted as a special case of the quasiperiodic dynamics (1) or (2) with an infinite period. Indeed, only the first DW burst occurs whereas the subsequently generated ZF never decays, so that the next DW burst never follows due to the prohibitively high shearing rate in Eq. (1). Therefore, in this regime the DWs are completely suppressed by the residual ZF. When the system reaches this steady state, it may be identified with the Dimits shift regime,[7] as shown in Fig. 4.

Finally, one should note that in both cases (1) and (2), one can determine either the time averaged or time asymptotic level of the DW intensity as a function of system parameters. The DW intensity, or equivalently, the ion radial transport has been studied in terms of its dependence upon the collisional damping of the ZF, γ_d in the recent gyrokinetic simulations.[15] The approximate scaling $\langle N\rangle \propto \gamma_d^{0.75}$ has been obtained. Figure 5 shows a fit to this scaling obtained from our simplified model. Although no careful optimization by varying other system parameters such as $\Delta\omega$ or γ has been made, the model fits the simulation data reasonably well (note the latter only varied ion collisionality).

In conclusion, the variability (bursting) phenomena frequently observed in the coupled drift wave–zonal flow systems may be satisfactorily understood in the framework of the dynamic predator–prey model.

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