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Investigating the Relationship Between Student Mathematical Talk, Mathematical Student Roles, and Teacher Discourse Around Student Behavior in the Classroom

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Investigating the Relationship Between Student Mathematical Talk, Mathematical Student Roles, and Teacher Discourse Around Student Behavior in the Classroom

A dissertation submitted in partial satisfaction of the

requirements for the Degree of Philosophy

in Education

by

Cecilia Henríquez Fernández

2017

ABSTRACT OF THE DISSERTATION

Investigating the Relationship Between

Student Mathematical Talk, Mathematical Student Roles,

and Teacher Discourse around Student Behavior in the Classroom

By

Cecilia Henríquez Fernández

Doctor of Philosophy in Education

University of California, Los Angeles, 2017

Professor Noreen M. Webb, Chair

The purpose of this study was to understand the relationship between teacher discourse around social norms, student mathematical roles, and student mathematical explanations in the classroom. Mathematical explanations are important for learning mathematics and professional organizations encourage their use in the classroom. This study sought to understand aspects of the social environment that support explanations in the classroom. Using a mixed-methods approach, the researcher attempted to make sense of how student roles, explanations, and teacher discourse practices were related in four elementary classrooms. One-hour videos of one to two lessons per classroom were coded for the enactment of student roles, mathematical talk (explanations and non-explanations), student participation structures, and teacher follow-up.

Quantitative analyses using chi-square tests examined student patterns of explanations and roles across different participation structures in the classroom. Results indicated Classroom A had significantly more explanations than Classrooms B, C, and D because students enacted the role of Sharer of Details of Problem-Solving Strategy more often; this role was always associated with students giving explanations. In addition, students in Classrooms B, C, and D enacted the roles of Agreer/Disagreer and Comparer which were not associated with explanations.

Qualitative analyses indicated that the teachers used seven different discourse practices across the classrooms in different ways. All teachers used Directives and General Announcements, but in some classrooms these practices elicited the role of Sharer of Details of Problem-Solving Strategy, while in other classrooms, they elicited the roles of Comparer or Agreer/Disagreer. Teachers used Voting Exercises and Modeling of Comparing Behavior to elicit the Comparer role; teacher practices for the Agreer/Disagreer role included Voting Exercises as well as Validation Statements to students who enacted this role. Finally, the Classroom A teacher used Modeling Behavior to model Sharer of Details of Problem-Solving Strategy.

This study corroborates previous work findings that teacher follow-up is important to elicit explanations in the classroom; without it, certain discourse practices did not result in the explanations that would have occurred otherwise. This study extended previous work by looking at how students enacted mathematical roles across multiple classroom settings, and how these roles were supported.

The dissertation of Cecilia Henriquez Fernandez is approved.

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2017

DEDICATION

For my parents, José Alberto and Luz María, who taught me the power of an education: "Una educación, nadie te la puede quitar."

To my beautiful children, Daniel Alberto, Belén Paola, and Diego José. May you always have the curiosity that you have now in learning and discovering yourself, the world, and how you relate to the world.

To David, for always believing in me, and the never-ending support you have provided.

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Gutierrez, K., Ali, A., & Henríquez, C. (2009). Syncretism and Hybridity: Schooling, Language, and Race and Students from Non-dominant Communities. In M.W. Apple, S. J. Ball, & L.A. Gandin. (Eds.), *The Routledge International Handbook of the Sociology of Education*. (358-369) New York, NY: Routledge.

Henriquez, C. & Robles, J. (2010). "I Challenge You": How Students Develop and Maintain Expert Identities and Expertise in Game Play. Paper presented at the *American Educational Research Association Annual Meeting*. Denver, CO.

Henriquez, C. (2011). The Negotiation and Formation of Elementary School Children's Social Identities in Game Playing. Paper presented at the *American Educational Research Association Annual Meeting*. New Orleans, LA.

Henríquez Fernández, C**.** (2011). Latinos/as Mathematical Experiences: A Review of Latinos/as and Mathematics Education. *Journal of Urban Mathematics Education,* 4(2), 131-139

Fernández, C.H., Cook, M.S., Wong, J., Monroe, S. (2012). Beyond the Transcript: Toward Innovative Methods for Video-Based Classroom Interaction. Paper presented at the *American Educational Research Association Annual Meeting*. Vancouver, Canada.

Chan Turrou, A., & Henríquez Fernández, C. (2013). Mathematical Proficiency and Perseverance in Action: The Case of Maria and Andrew*. *Journal of Mathematics Education at Teachers College*, 3(2), 67-72

Fernández, C.H. (2014). Student Identity and Participation in Mathematics Classroom. Poster Presented at the *Latina Researchers Networking Conference.* New York, NY.

Webb, N.M., Franke, M.L., Ing, M., Wong, J., Fernandez, C.H., Shin, N., & Turrou, A.C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices and learning. *International Journal of Educational Research*, 63, 79-93

Franke, M.L., Turrou, A.C., Webb, N.M., Ing, M., Wong, J., Shin, N., & Fernandez, C.H. (2015). Student's Engagement with Other's Mathematical Ideas: The Role of Teacher Invitation and Support Moves. *The Elementary School Journal*, 116(1), 126-148

Chapter 1: Introduction

Mathematics as a Social and Cultural Activity

Because mathematics has long been recognized as a social and cultural activity, sociocultural theory has been applied to investigations of mathematics classrooms in order to make sense of how students learn mathematics (Cobb, 1994; Cobb, Yackel, & Wood, 1992; Greeno, 1998; Lerman, 2001; Saxe & Guberman, 1998; Wedege, 2010). Using sociocultural theory indicates that studying classrooms involves looking at the social, historical, and cultural contexts of mathematics in order to make sense of how people learn mathematics. Defining a classroom as a community of practice to investigate learning is one example of how sociocultural theory can be applied to classrooms because Wegner (1998, 2000) used social participation guided by the participants of a community to theorize about and understand the process of learning. In mathematics, investigating how students and teachers make mathematical meaning includes understanding how the classroom community establishes what it means to have competence in mathematics, what rules for interaction are used by the classroom community, and what shared repertoires of how to do mathematics exist in the classroom (Forman, 2003). This approach is useful because it allows researchers to understand what it means to do mathematics in the classroom by investigating the social aspects of how students and teachers jointly construct and interpret mathematics in a joint social space. Thus, examining how students learn mathematics in the classroom using sociocultural theory involves not only looking at the classroom community as a whole to investigate the relationship between the multiple social components of a classroom community, but also looking at the individual participants who make up the community and the experiences they bring to the classroom.

Purpose of the Study

In this study, I follow in the tradition of sociocultural researchers who have investigated mathematical learning by examining three social components of mathematics classrooms: mathematical participation, the social rules of mathematical participation, and the individual experiences that students bring into the classroom. I aim to make sense of what it means to do mathematics in four elementary classrooms by focusing my study on mathematical participation in the classroom and its relationship to the normative social mathematical behavior as established by teacher discourse practices. Specifically, I study the relationship between student mathematical roles that students take up in the classroom during mathematical activity, explanations, and how these two aspects of participation are elicited by teacher discourse practices. In this way, I hope to understand how both the students and teachers in four elementary classrooms construct what it means to do mathematics in the classroom.

Research Questions

I sought to understand how classroom social norms as communicated by the teacher and the mathematical roles adopted by students are related to the way students provide explanations in elementary classrooms. In order to investigate these relationships, I used the following questions to guide my study:

1. How did classrooms differ in terms of explanation-related mathematical talk in the classroom?
2. What mathematical roles (related to the mathematical activities in the observed classrooms) did students enact in the classroom?
    a. How were the mathematical roles students enacted in the classroom associated with explanation-giving by students?

3. What discourse practices and student activities did teachers use to elicit observed student mathematical roles and explanations in the classroom?
    a. How did the teacher discourse relate to/support student roles and explanations?

Significance

With this study, I strove to contribute to the current understanding of the relationship between student mathematical talk in the classroom, teacher discourse practices around student mathematical behaviors in third and fourth grade elementary school classrooms. More specifically, from this study, I attempted to show how teacher discourse around expected student behaviors contributes to students' mathematical participation (e.g., mathematical explanations); the variety of ways in which teacher discourse can be targeted towards students in the classroom (e.g., one-on-one directives versus classroom voting); and how students' mathematical roles relate to the ways students explain in the classroom. A deeper understanding of the relationships between these three constructs is important, and can be used to guide teacher professional development that focuses on the design of social spaces in classrooms. This knowledge can also be used to empower teachers to learn how they can elicit particular kinds of behavior that are important for learning mathematics through the norms they communicate to students. Lastly, this understanding can be used to motivate teachers to consider how student roles in relation to classroom activities can influence students' in-class participation.

Chapter 2: Literature Review

Student Explanations in Mathematics Classrooms

In this study, I defined student participation as giving mathematical explanations in the classroom. I used student explanations as a marker of classroom participation because they have been recognized for promoting student mathematical learning (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). For example, the National Council of Teachers of Mathematics (NCTM, 1991), an organization that focuses on equitable and high-quality learning of mathematics, recommended in their Principles and Standards for School Mathematics that teachers encourage students to "talk about their mathematical ideas and justify their problem-solving strategies" (p.57). Additionally, empirical research across a variety of content areas has positively linked student explanations to student achievement (Webb & Palincsar, 1996). Specifically in mathematics education, giving elaborated explanations has been positively linked to student achievement (Fuchs et al., 1997; Nattive, 1994; Slavin, 1987; Veenman, Denessen, Van Den Akker, & Van Der Rijt, 2005). Since giving explanations in the classroom is important for student achievement, and achievement is one way of measuring learning, we can infer that explanations in the classroom are important for the learning of mathematics. But how exactly are explanations and learning related?

There are several thoughts on how explanations in the classroom are hypothesized to support mathematical learning. First, the opportunity to give explanations is considered important for metacognitive processes because it allows people to organize and make sense of information they gather during their classroom experiences (Wittrock, 1989). In classroom settings, students encounter multiple sources of information: their teachers, their peers, classroom activities, tools and artifacts of the classroom. As such, giving explanations in the classroom is an important facet of the classroom experience because students have the chance to make sense of the information they encounter in the classroom. For example, in a series of studies on student explanations, students who were pushed to elaborate their explanations displayed signs of improved comprehension because they were pushed to mediate their own thinking as they elaborated (King, 1992, 1999). Thus, in giving explanations, students can clarify their own understanding of the information they encounter in the classroom because they are pushed to organize this information for themselves. Moreover, doing so in a public way offers students the opportunity to collect additional information that they might need in order to make sense of what they are trying to explain.

Another way that explanations may support mathematical learning in the classroom is in how they may potentially benefit those who hear them. Receiving explanations has been shown to be beneficial in help-seeking scenarios, when explanations are adequately elaborated to help students who are having difficulty with the content apply the explanations to their current activities (Webb & Mastergeorge, 2003). This likely occurs because, through peer interaction, the structure of the presented information is streamlined in a way that facilitates learning (Sweller, 1989). Thus, both giving and receiving explanations are hypothesized to support mathematical learning because they help students process information received in the classroom in multiple ways (organizing and streamlining information).

Thus, it seems that encouraging students to give explanations in the classroom is an important consideration for mathematics educators and researchers because explanations seem to support student learning and are positively related to student achievement in mathematics. However, although giving explanations and engaging in discussions around mathematical ideas are desired forms of student participation, they are not necessarily the normative behavior in many classrooms. In fact, opportunities for students to engage in discussions around their mathematical ideas and strategies are lacking in U.S. classrooms (Hiebert, 2013; Hiebert et al., 2005; Stigler & Hiebert, 2009).

This study investigated two possible influences on the extent to which students give explanations in the classroom: (a) the norms around explaining that the teacher communicates to students in the classroom, and (b) student enactment of participation roles during classroom mathematical activities.

Establishing Norms Around Participation

In order to participate in a social setting, people frequently need to read and make sense of the social settings in which they find themselves: "appropriate social behavior from moment to moment require[s] knowing what context one is in and when contexts change as well as knowing what behavior is considered [appropriate] in each of these contexts" (Erickson & Schultz, 1997, p. 147). In the classroom, being able to discern the norms of that setting is theoretically crucial in determining the amount and level of participation a student will "do" there. Understanding the norms of the classroom is hypothesized to "make or break" a student's academic career. For example, Erickson (2004), in his research on discourse practices in the classroom, found that elementary children who could not pick up on the social norms of interacting in kindergarten had become withdrawn by third grade from the social interactions required to be successful in the classroom.

Research on the influences of norms on student participation has focused on three activities: (a) identifying normative behavior in mathematics classrooms that is meaningful for mathematical learning and development; (b) describing ways in which normative behavior is established in the classroom; and (c) testing intervention programs meant to establish particular norms in the classroom.

Researchers have identified a number of norms related to participation. Teachers can work with classrooms to build the understanding that students are expected to help each other and listen to each other's ideas during collaborative problem solving. Moreover, teachers can emphasize that students should persist when working on challenging problems, share explanations with the class, and engage in meaningful activity (Cobb, Wood, Yackel, & McNeal, 1992; Cobb, Yackel, et al., 1992; Kazemi, 1998). Teachers can also work with students to build an understanding of what constitutes acceptable mathematical explanations and justifications, what marks mathematical difference and sophistication, what is mathematical argumentation and mathematical agreement. In addition, teachers can lead students to think about the importance of using multiple strategies to solve problems, exploring relationships between strategies, and using errors as opportunities to re-conceptualize of problems or examine contradictions (Hershkowitz & Schwarz, 1999; Kapur & Bielaczyc, 2012; Kazemi, 1998; Kazemi & Stipek, 2001; Yackel & Cobb, 1996; Yackel, Rasmussen, & King, 2000). While these norms address different dimensions of participation in the classroom, both types of norms are important for supporting student explanations in the classroom because these dimensions are not mutually exclusive. For example, one cannot successfully share mathematical explanations in the classroom without understanding what an acceptable mathematical explanation is. Additionally, no evidence supports the assertion that any one of these dimensions is more important than the other for student explanations. In the present study, I chose not distinguish between these kinds of norms in the classroom; rather, I chose to look at how teachers use discourse practices to communicate norms to the students.

The research on establishing classroom norms for desired student behavior has focused on showing how powerful the role of teacher practice is for establishing and negotiating classroom norms that build classroom culture. Teachers have been shown to facilitate positive student dispositions towards mathematics and learning through the classroom environments they build (Walshaw & Anthony, 2008). Teachers can elicit desired forms of student participation in various ways. One way that teachers establish classroom norms and improve social behavior in the classroom is through discourse with the whole class. By explicitly communicating to students how they should engage with each other during mathematical activity, teachers can express to students what normative social practices should be, such as sharing ideas with their partners (Mercer, 1996; Webb, 2009). Likewise, teachers can support desired mathematical behavior by explicitly asking students to engage with each other in ways that promote desired mathematical practices, such as engaging in mathematical argumentation, requesting strategy details from students, and communicating what is an acceptable mathematical explanation (Forman, Mccormick, & Donato, 1997; Kazemi, 1998; Kazemi & Stipek, 2001; Yackel, 2002). The teacher, then, can explicitly support desired student social and mathematical behavior by constantly communicating to students his or her expectations during classroom activities.

Another teacher discourse practice that seems to play an important role in establishing desired behavior is intervention during small group activities. For example, teachers can use probing questions during small group activities to encourage students to give detailed explanations (Webb et al., 2009). Pressing students for elaboration has yielded their participation during small group activities consisting of detailed forms of mathematical argumentation (Kazemi, 1998; Webb et al., 2008). By contrast, students in classrooms where teachers do not press for elaborated explanations have shown less frequent mathematical participation during small group activities (Webb, Nemer, & Ing, 2006). Teacher discourse practices, then, play a large role in establishing, promoting, and validating the social norms of student participation.

Because students come into the classroom with preconceived ideas of what constitutes mathematics in the classroom, they contribute to establishing classroom norms through student-teacher interactions. As such, another important discourse practice for teachers is to listen actively to students as they express their mathematical ideas in order to signify to students acceptable behavior in the classroom (Yackel & Cobb, 1996; Yackel et al., 2000). This practice helps teachers identify when and how to engage in the discourse practices mentioned above in order to establish classroom norms with students through student-teacher interactions.

Intervention Programs to Help Teachers Engage in Productive Discourse Practices

Many researchers have designed intervention programs that encourage student explanations in the classroom. Most of these interventions focus on students' desired behaviors. For example, in a series of studies designed to facilitate student understanding via elaboration of the material, King (1989, 1990, 1992) designed a set of generic question stems based on Bloom's (1956) taxonomy of thinking to help students engage with the material by developing their own questions and answers in relation to the material being learned in class.

However, very few intervention studies have focused on the discourse that teachers themselves should use when interacting with students to elicit explanations. For example, in a series of studies, Gillies (2004, 2006; Gillies & Boyle, 2008) trained teachers in a 2-day workshop on the value of small group processes of collaborative learning, and how to embed collaborative learning into the class curriculum. Part of the workshop included training teachers in communication skills such as: probing and clarifying student thinking, acknowledging and validating students ideas, confronting discrepancies and clarifying options, and giving suggestions. During the workshop, the teachers were given opportunities to practice with each other through role-playing activities and discuss hypothesized student behavior in relation to the application of these communication skills.

Likewise, Mercer and colleagues (Mercer, Dawes, Wegerif, & Sams, 2004; Mercer & Littleton, 2007; Mercer & Sams, 2006; Mercer, Wegerif, & Dawes, 1999; Mercer et al., 2004) have developed and evaluated programs designed to improve the quality of talk between middle school science and math students. In these programs, teachers participated in a 1-day workshop as a research team led the introduction of a set of nine lessons meant to promote and establish "ground rules" that the researchers defined as important for generating student talk. These ground rules included: sharing all relevant information; providing reasoning for opinions, judgments, and statements; asking for reasoning when appropriate; and reaching group agreement. The teachers were encouraged to support the lesson activities by asking thought-provoking questions in interaction with the students and modeling the kinds of talk they expected of students. Thus, the few programs targeting teacher discourse practices have worked on developing teacher communication skills that support student discourse, but have not looked specifically at how teachers used these discourse practices to target student behavior.

Linking Classroom Norms to Student Explanations

In summary, the research on teacher discourse practices[1] for setting up norms around student explanations has focused on two important facets of teacher discourse practices. Gillies and Mercer focused on researching communication skills for student-teacher interaction that can

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[1]Teacher discourse practices are not the only source of norm establishment in the classroom. Students also play an important role in determining classroom norms as each student brings his or her own ideas of what mathematics should be. While I understand that norm establishment works both ways, for the purpose of this study and the sake of simplicity, I focused solely on the norms as enacted by teacher discourse practices in the classroom.

elicit desired student behavior. Kazemi and Cobb, Yackel, and Wood focused on the kinds of teacher discourse that elicits specific details of students' mathematical explanations. However, a third facet of teacher discourse practices has yet to be investigated. In this study, I investigated the relationship between classroom norms as communicated by teacher discourse practices on student behavior by examining the importance of the targeted recipient of the norms. In other words, does it matter if teacher discourse practices around classroom norms are targeted to the class in general, to specific students, or to both? It is important to study this distinction for several reasons. First, while the teacher generally plans lessons for the class as a whole, she also takes into consideration the students' individual needs. Likewise, in communicating the classroom norms, the teacher needs to communicate them to the entire class so that all students know what is expected; however, she also needs to communicate the norms to students at the individual level in order to address individual student participation needs. Second, the teacher interacts with students at multiple levels: at the whole group level, at a small group level, and at an individual level. For example, a teacher can interact with the whole class by leading class discussions during a lesson, but in working individually or in small groups, she can also direct student thinking in personal interactions with individual students. Thus, it is important to investigate the vital role of teacher discourse on multiple levels of student-teacher interaction to communicate to students the desired norms of classroom participation.

Chapter 3: Methods

In this study, I investigated the relationship between student mathematical explanations, teacher discourse around the social norms of the classroom, and students' mathematical roles by conducting a secondary analysis of data that were collected by the UCLA Mathematics Teaching and Learning Group as part of a larger research project focusing on the relationship between student explanations and student achievement. I joined the research project team during data analysis, and through this experience gained familiarity with what kinds of information were available in the data set. I intended to build on the initial analyses by seeking to understand the social contexts of the classroom that drive student mathematical explanations as they learn. More specifically, I was interested in looking at the role that teacher discourse on classroom norms and student mathematical roles plays in encouraging mathematical explanations given by students in the classroom.

Data Collection: Observations in Classrooms

Setting/Participants. The data for this project were collected at the UCLA Laboratory School during Spring 2008 and Spring 2009. The UCLA Laboratory School is designed to meet the needs of students from Pre-K through Grade 6, and its admission procedures (stratified by ethnicity and income) ensure that the student population represents the ethnic and socioeconomic diversity of California's schools. In 2009, the student population in the UCLA Laboratory school was 38% White, 19% Latino, 7% Asian, 6% African American, and 30% Multiethnic; 23% of families had annual incomes below $50,000 (13% below $35,000); 18% of students qualified for free or reduced meals. The school was not designed to serve the children of the UCLA community; only a small percentage of students are children of UCLA faculty or students. The UCLA Laboratory School describes itself as a learning environment that values diversity, encourages creativity and innovation, supports disciplined inquiry, involves families and their communities, and makes a commitment to meeting the needs of the whole child. The school is divided into multi-age groups (called levels): the early childhood level includes 4- to 6-year olds (Pre-K, K), the primary level has 6- to 8-year olds (Grades 1-2), the intermediate level has 8- to 10-year-olds (Grades 3-4), and the upper level has 10- to 12-year-olds (Grades 5-6).

The UCLA Mathematics Teaching and Learning Group chose the Laboratory School as the ideal site for data collection for three reasons. First, the student population is diverse, with a variety of ethnic backgrounds and a range in income, as described above. Second, a school-wide goal is to teach for mathematical proficiency (as defined by *Adding it Up,* National Research Council, 2001), with teachers asking students about their mathematical thinking, supporting mathematical discussion, and focusing on problem solving. Third, and most important to the present study, even though all teachers worked toward the same goals, previous observations showed variability within and across grades in how teachers engaged students in discussions about mathematics.

Observation data collection procedures. Prior to formal data collection, the UCLA Mathematics Teaching and Learning Group spent six months becoming familiar with the school classrooms by observing classrooms, taking field notes, talking with teachers, and practicing videotaping. At the end of the 6 months, the research team carried out formal observations during regular mathematics lessons. Data collection involved videotaping of two (1-hour) lessons per classroom in 14 classrooms, in which we tracked the participation of 12 to 20 students per classroom. One stationary video camera with two flat microphones captured the ongoing flow of the class and the interaction of up to two groups of students. Four flip cameras captured the interaction of the remaining students, operated by research team members to follow students. Additionally, six digital audio recorders on students' desks captured student-to-student talk that was not recorded by the cameras. Video and audio were converted into a whole class video and multiple table videos for different student groups.

All observation data were collected during regular mathematics instruction. Teachers were asked to teach what they would consider a typical mathematics lesson. Although the mathematics content of the lessons varied across grades (e.g., whole number operations, fractions, measurement), all teachers knew that the focus of the data collection was on engagement of students' thinking around the mathematics and student participation.

Study sample. For the purpose of this secondary analysis, I selected four of the fourteen classrooms to code and analyze. Three of the four classrooms were a combined Grade 3 and Grade 4 classroom. In the combined classrooms, the number of students in each classroom ranged from 17 to 26. The fourth classroom consisted of only Grade 4 students, and in this classroom there were 11 students. The four classrooms were chosen because they consisted of students from the same grade level, but the classrooms varied in terms of student participation and classroom norms.

Coding of Classroom Observation Data

The coding of student participation, teacher discourse practices, and student roles was based on video timelines created for each lesson. First, for each classroom on each occasion, the project team created a movie for the whole class that followed the teacher during the entire class period and captured all student interactions with the teacher. In addition, for each class, separate movies were created that captured each small group's discussion on every problem. Each movie was made by merging the best video and audio sources at each point in time. Whole class and small group movies were combined into a single movie for coding. The software StudioCode was used to create a timeline for each movie and codes (StudioCode Business Group, 2011). The timelines allowed for moment-to-moment coding of all interactions occurring in the movie.

Figure 1 gives a screenshot of a coded timeline for a whole movie, while Figure 2 gives a close-up view of the coded 7 minutes of the whole timeline in Figure 1. In Figure 1, we can see the multiple layers of codes that the project team applied to the videos in order to capture multiple dimensions of the classrooms. For example, we can see participation structures coded (Whole Class, Pair Share), the number of problems coded, and student participation variables. In each row are numerous "instances" marked by the rectangular markings in each row. We also see a picture of the movie to which the timeline is connected. In Figure 2, we see how the instances of codes capture the interplay of moves made by both the teacher and the students. For example, for explanation instance 15 (minute 26:00 to minute 27:30), we see how the explanation began with an answer instance (the green row located right below the explanation row), and then the teacher followed up with an explanation five times (five instances coded "follow-up" in the purple row in Figure 2.).

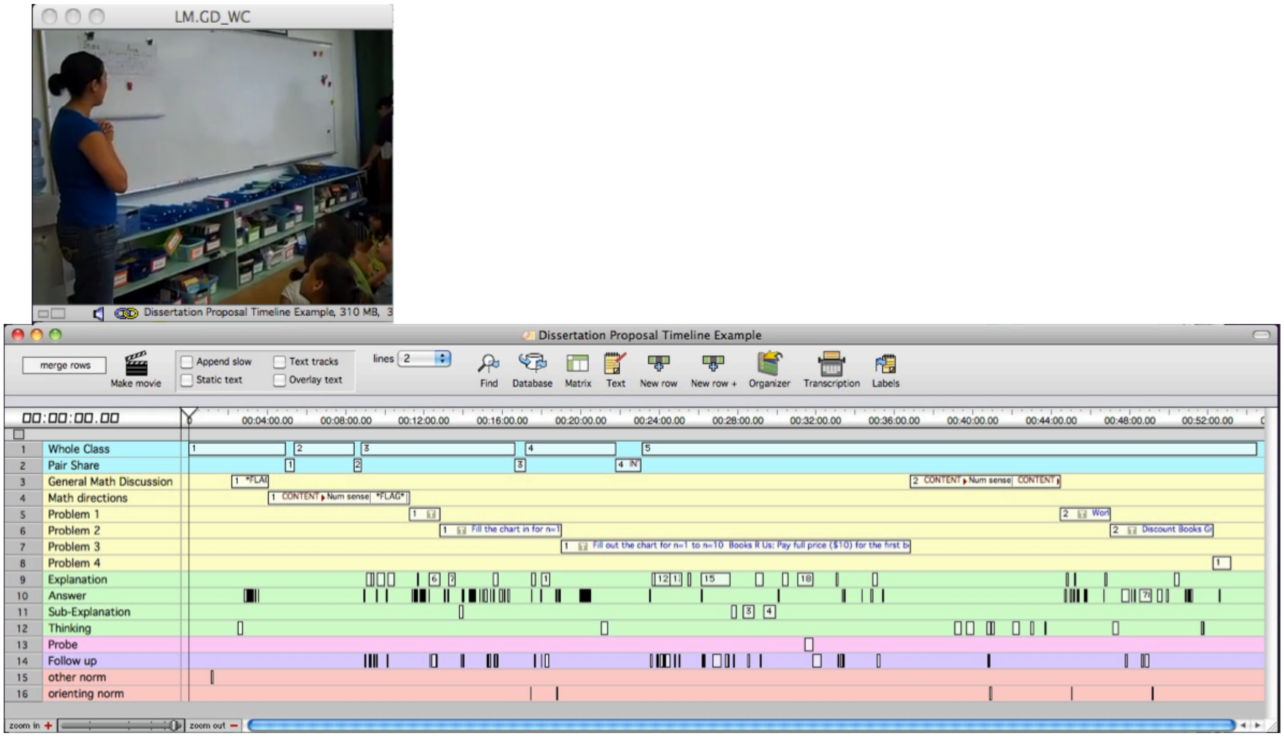

Figure 1. Whole movie coded timeline

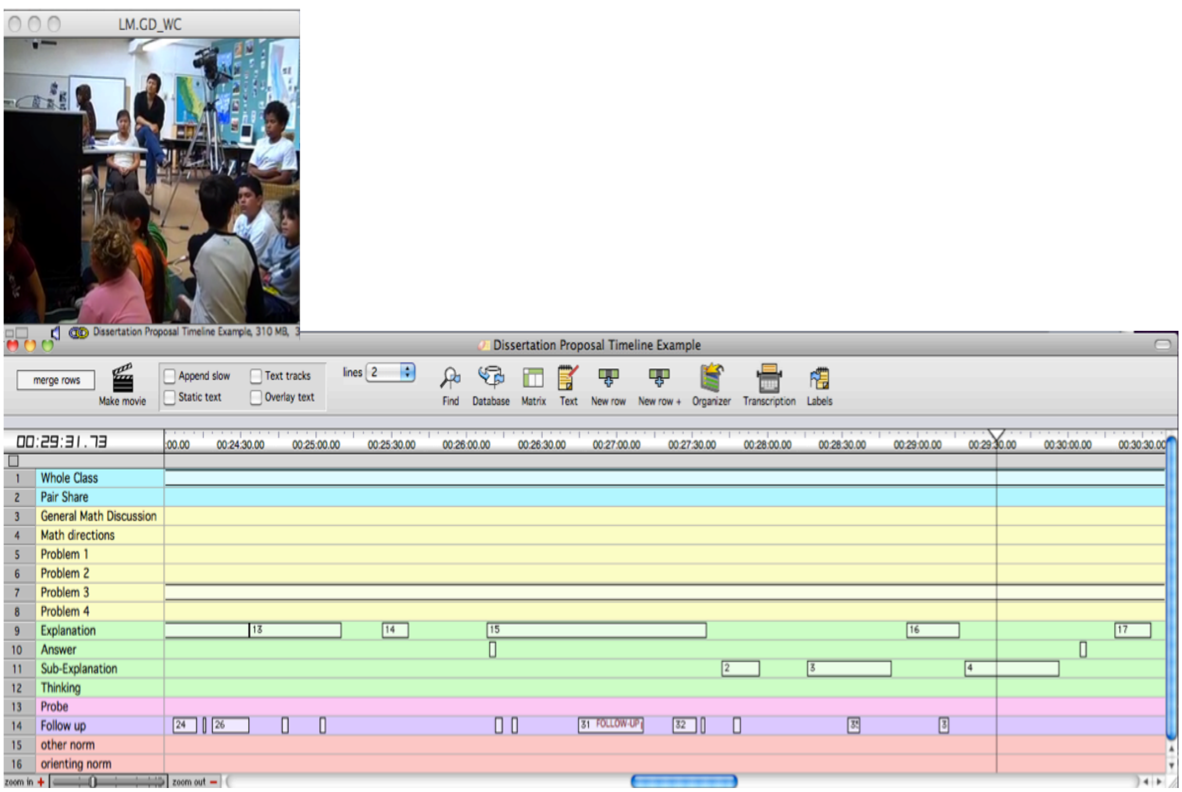

Figure 2. 6.5 minutes of whole movie coded

Unit of Analysis and Code Application. The unit of analysis for all coding was an intact conversation about a problem. Conversations could happen between students, or between a student (or students) and the teacher. For each conversation, multiple layers of codes were applied for each of the following: the student mathematical talk—specifically whether they gave an explanation or not (and the type of explanation; see coding information as described below), the role (or roles) that students enacted during the conversation, and the teacher follow-up (if it occurred during that time). Depending on the participants of a conversation, multiple types of mathematical talk could occur during a conversation, for example, if a student gave an explanation, and his/her partner followed up with a mathematical comment (a non-explanation) while enacting a particular role, then multiple kinds of mathematical talk could occur during this time.

For example, during the Pair Share time, students might have five minutes to discuss a problem, however, several conversations could occur during this time for one group of students. Once a student began a conversation, I would identify the kind of mathematical talk that student engaged in. I would also code for the role(s) that student(s) enacted. Students typically enacted only one role during a conversation, but occasionally, particularly in Classroom B, a student could enact multiple roles. All of the enacted roles would be coded for within a conversation about a problem. Teacher follow-up was coded when it occurred as well, and could at times occur multiple times within one conversation.

During Whole Class, a mathematical conversation around a problem was typically led by the teacher. That is, the teacher usually initiated a conversation with a student. During conversations that occurred during Whole Class, a student might be invited to interact with the teacher, and the student typically responded. The student would either provide an explanation or not while enacting whatever role was elicited, the teacher would follow-up if she/he felt necessary, and/or involve other students. Once the student-teacher conversation around a problem was over, the teacher may or may not start another conversation (around the same problem) with another student. Typically, during Whole Class, a conversation only occurred between one student and the teacher, but occasionally other students were invited to participate in the conversation. For any classrooms in which Individual time occurred, the mathematical conversations that occurred were coded the same way in which the conversations were coded during Whole Class. In other words, typically these conversations occurred between a teacher and student only. During Individual time, it was rare for a second student to be involved in a teacher-student conversation.

For one conversation, there was usually one coding of mathematical talk (explanation or non-explanation). There was typically only one code of a mathematical role applied, but occasionally, especially in Classroom B, multiple role codes were applied. While Teacher follow-up was typically coded for more than once as the original coding differentiated for different kinds of follow-up, I was only interested in whether it occurred or not.

Student explanation-related codes. The project team coded student explanation-related participation along multiple dimensions. These codes focused on the mathematical work that students were asked to complete, and capture the various ways in which students were participating *mathematically* in the classroom. For the purpose of this study I used the following codes: Explanation, Sub-Explanation, and Non-explanation[2] (called Thinking in previous work by the project team), as described in Table 1.

Table 1

Student Explanation Codes

Explanation Code	Definition/Description	Example
Explanation	Students articulate mathematical strategies to solve problems.	So first I did 6 plus 6 19 times. You can see it up there. Then I did 6+6+6+6+6+6. And it all equaled 12. But something right here (*pause*) is that I had a leftover 6. So then I did 12 plus 12 and that equaled 24. Then I added another 12. Then I figured out that I should add the 12 to the 6 and I got 18. Then 24 plus 24 equals 48. Then I did 48 plus 48 and that equals 96. Then I did 96 plus 18 and that got me 114."
Sub-Explanation	A student gives an explanation to a sub-problem (posed by either the teacher or a student) rather than the whole problem that is being worked on.	T: Which number is bigger, 1/2 or 1/3? What do you have to do first? S1: Find the common denominator. It is six, because two times three is six.
Non-explanation mathematical thinking	A student articulates additional mathematical detail but is not giving an explanation or an answer to a problem.	Problem: How did you know how to get from 96 to 102? S3: 6, 12, 18, 24, 30. And the next one is 36. And you see how it's 6?

[2] The original coding of mathematical participation included other non-explanations including answers and written work. However, I chose to focus on what was previously coded as thinking because I wanted to include other mathematically substantive dialogue that occurred in the classroom.

I limited my coding of student participation to these three codes because they documented different ways in which the students publicly presented mathematical strategies.

In addition, explanations and sub-explanation codes contained labels that helped to differentiate the kinds of explanations that occurred in the classroom. These included: (a) "full-detail" indicating a student had verbally detailed step-by-step how to solve a problem; (b) "some detail" that indicated a student had verbally detailed some of the steps necessary to solve the problem but not detailed every single step; and (c) "wrong-detail" that indicated a student verbally expressed detail but it was mathematically incorrect. An explanation variable was created by aggregating explanation and sub-explanation codes into one category, and non-explanation consisted of all other non-mathematical talk coded as thinking.

Student role codes. While student role codes were central to the present study, they were not the focus of the original research conducted by the UCLA Teaching and Learning Group. Therefore, I created my own codes of student roles in the classroom. In order to come up with the coding scheme for student roles in the classroom, I initially selected a few minutes of video from each of the four classrooms to construct an initial list of codes for roles that students enacted in the classroom. I randomly selected excerpts from multiple participation structures to ensure my initial list encompassed as many different student roles as possible in a variety of settings. I also went back to the literature to identify what kinds of roles are considered important for mathematical participation in the classroom (e.g., Sharer of Details of Strategy, Comparer, Teacher, Helper, etc.). I then organized these examples and codes into a table. Next, I went through the complete corpus of videos and applied the codes to the corresponding student behaviors captured in the videos. If I encountered student behavior that did not fit into the initial set of codes but appeared to be an important role, I marked it with a blank code, and revisited my initial list of codes to see if I needed to modify the definitions of the codes or consider a new code. I went through the coded timelines multiple times to ensure that the codes were accurately defined and I had exhaustively captured all relevant mathematical student roles in the classroom. Once I was satisfied with my coding, I approached an independent researcher who was familiar with the original project and asked her code clips from the videos using my coding scheme. After she did so, we discussed discrepancies or problems with definitions. I then modified the list of codes and reviewed the corpus of videos again to update the coding.

The final set of codes are presented in Table 2. The table lists the codes, definition, and notes examples of the various ways in which this behavior could occur. For example, the role code "Comparer" captures when students examined mathematical strategies for similarities or differences. This role could be enacted while giving or not giving an explanation. Thus a student could state similarity and explain how or why her strategy was the same of different, or a student could simply state similarity/difference without giving an explanation. For each code, there are short transcript examples illustrating what enactment of this role could look like. Variables for analysis were created by simply aggregating the number of times a role occurred in the classroom.

Table 2

Student Role Codes and Examples

Student Role Code	Behavior	Mathematical Talk	Example
Comparer	Examines mathematical strategies for similarities or differences	Explanations Example	T: Well, I did it the same, except I did it with 12. Um, I broke up six into 2 and 4. And same as M, I know that 6 plus 4 is 10, so I added the 4 and the 6 and that was 10. I had 2 left, so I added that and got 12.
		Non-Explanations	M: I had a different way that I did it in my head
Agreer/Disagr-eer	Publicly shares whether they agree or disagree with another student's strategy	Explanations	I think all of them are right because they are basically the same thing except if in different forms.
		Non-Explanations	J (to teacher): E agrees with me!
Sharer of Details of Problem-Solving Strategy	Shares the details of how to solve a problem: it could be that student's own strategy, the strategy of another student, or the strategy of a pair/group.	Explanations	P: I did the t-chart. First I made the chart, and on this side I did one, two, three, four, five, six, seven, eight, nine…all the way to nineteen. And on the other side I did 6, 12, 18,…,114.
Sharer of Details of Mathematical Claim	Shares the details of a mathematical claim, it could be that student's own claim or the claim of another student.	Explanations	B: 0 is a whole because you have to start at some place. Because, look, it's a whole, because you start at 0 and there's a 1, 2, 3, 4. And if there is a negative number, then that would be a whole. Because you're counting into the negatives, that would still be a…
		Non-Explanations	N: I think zero is a whole number.

Table 2 (continued)

Student Role Code	Behavior	Mathematical Talk	Example
Pattern Finder	Identifies/uses patterns in a mathematical activity	Explanations	E: Yeah, in the ones place, there is a pattern, you always add the same in each column.
		Non-Explanations	S1: I got a pattern.
Multiple Strategist	Uses and shares multiple strategies to solve a problem	Explanations	F: Well what I did, I did 6 19s and I added them and I got 114; and I did the t-chart from 1 to 19, which I got 114; and I did circles and tallies.
		Non-Explanations	S1: I had a third strategy, but I didn't do it.
Connection Maker	Makes connections between lessons or other situations	Explanations	A: In problem 1 we were doing, [we] were giving them out apples. Now [in this problem] we already know how many apples each get, but we don't know how many people there are.
		Non-Explanations	C2: This problem is like the counting by sixes!
Helper	States they will help another student	Explanations	C: This one is wrong. F: Look, one, two, three, four, five, six, seven, eight, nine. You only did nine. You're supposed to do nineteen. Just do it on a separate paper, I'll help you.
		Non-Explanations	E: Let me help you.
Teacher	Offers to teach/or teaches another student a mathematical idea	Explanations	M: Let's do the Algorithm! A: But I don't know how to do the algorithm in multiplication. M: I'll show you. You just do it diagonally, look. 9 and then ninetee—uh 6 and 19. Six.
		Non-Explanations	S2: I can teach you how to do the algorithm.

Teacher discourse: follow-up codes. Because I was interested in understanding the influence the teacher had on the mathematical talk in which students engaged in the classroom, I also used the original coding of teacher follow-up to determine when a teacher engaged in students' mathematical talk. The code "follow-up" captured when a teacher responded to a student's initiation of mathematical talk with a mathematical comment. While the original coding contained distinctions for the different kinds of mathematical follow-up that occurred in the classroom, for this study I was only interested in whether it occurred or why it did not occur. When follow-up did not occur, I documented why it did not occur. The three scenarios included:

- the teacher did not follow up because she was not present.
- the teacher did not follow up, but she was present and engaged, however, interactions with students during a conversation were limited to invitations to share mathematical thinking.
- the teacher did not follow-up because she was present but not engaged (simply observed).

Teacher discourse: elicitation practices. I then turned to understanding what the teacher discourse around expected behavior looked like in each classroom, and what was the potential influence of this discourse on eliciting the observed student roles and explanations in the classroom. I began by creating a list of known teacher discourse practices that I believed influenced the students' behavior in the classroom, namely, directives, general announcements, voting exercises, and evaluations. I returned to the literature on teacher classroom discourse practices around social norms and attempted to validate that these teacher practices were expected to occur in a classroom and be practiced often. Moreover, I looked at the literature for any practices I may not have considered in my initial list, but once I read about it, I could recollect an example of such a practice. Once, I felt I had an adequate list of teacher practices, I went through each classroom to create the classroom narrative. I began by selecting the student enactment of a frequently occurring role during an early phase, and identifying the teacher discourse practice that elicited that role. I transcribed both the teacher discourse example and the student enactment of a role, and analyzed the conversation in order to identify how the teacher discourse influenced student roles and explanations (or lack thereof) in the classroom. I then continued through the rest of the phases, looking for different ways in which the teacher elicited the same (or different frequently occurring) roles. If the practice was different, I transcribed both the teacher practice and examples of students enacting the elicited role and then analyzed the conversation.

As I finished one classroom, I turned to the others and used the same process, identifying the variety of teacher discourse practices that elicited the frequently occurring roles in each phase. I transcribed both the teacher practices and student enactment of roles in order to analyze how the teacher discourse led to these observed roles and explanations. As I went through the different classrooms, I sometimes added teacher discourse practices that were not in the original list, e.g., validation statements (Classroom D only). Table 3 presents the final list of teacher discourse practices used during the observed lessons in these classrooms, the definitions of these practices, and examples observed during the lessons.

Table 3

Teacher Discourse Practices Used to Elicit Student Roles and Explanations in the Classroom

Teacher Discourse Practice	Definition	Example
Directives	The teacher directed students to behave in certain ways. Usually targeted (a) specific student(s).	N, can you share with the rest of the group what you just shared with me?
General Announcements	The teacher announced to students which behaviors he/she expected the students to engage in. Usually directed to the entire class.	Okay, so N is going to come up to share how R solved the problem, and everyone needs to be ready to say whether you agree or disagree.
Behavior Models	The teacher modeled for students the particular behaviors he/she wanted the students to engage in.	Oh, so N, you did the problem like M, but with different numbers.
Voting Exercises	The teacher often provided students with options that students had to select. The options were often limited.	Raise your hand if you think B is right. Okay, raise your hand if you think M is right. Okay.
Evaluations	The teacher provided feedback, usually positive, to students that enacted desired mathematical behaviors.	Great job, E!
Validations	The teacher provided statements that affirmed a student's mathematical behavior (but did not evaluate it).	It's okay to disagree with N, everyone is entitled to their opinion.

Classroom participation structures. Because the social context of the classroom plays an important role in how students and teachers behave in the classroom, this study investigated whether the patterns of student behavior differed by social settings in the classroom, especially if the teacher communicated different norms during the different participation structure settings. As such, I decided to include the codes originally developed to capture student participation structures in the classroom. Table 4 identifies the three codes used to make the different participation structures that occurred in the classroom. Application of these codes was related to orienting statements made by the teacher that indicated what and how students were expected to interact with each other.

Table 4

Classroom Participation Structures

Code	Definitions	Example
Whole Class	Indicates a participation structure in which everyone is expected to be paying attention to the same thing (usually the teacher or whatever the teacher is showing to the students) and most public comments are meant for all to hear. It is understood that some peer-to-peer talk may take place in whole-class conversations, but it is not the expected or invited form of participating.	T: And now I want all pencils down for the next four or five minutes while people are talking. So I'm waiting on A to put hers down. And J, I need you to really focus. R is busy working her little heart out. And we're going to let her continue doing that while we finish talking. So the order I want to go in is M1, M2, and then R. And then I want you [the class]—your job to do is I want you to see if you agree or disagree with their thinking. You all understand what we're doing?
Pair Share	Indicates any participation structure in which multiple groups of students are meant to interact with each other during the time for which the code is applied. This can include work in pairs (even quick "turn to your partner and share" moments), work in small groups or tables, and work conducted in centers. Students are expected to be paying attention mostly to other students, although the teacher may come by and interact with them, and in some groups one may not actually see much math-related sharing. The idea is that the intended participation structure invites peer-peer conversations.	T: One two three, eyes on me. S: One two three, eyes on you. T: Even if you're not done with your second strategy. You should still be able to talk to your partner and tell them, or your partners, and tell them where you are headed. So I want you to. Okay. Share with your partners. Okay or your partner. Your first strategy, one strategy, and then your partner shares one strategy. And then if you got a second one or if you're working on a second one, then you share your second strategy and then they share their second strategy. Okay. But make sure that each one of you shares one strategy at a time. And as a partner, if you see where you can help your neighbor understand something, I want you to go ahead and give suggestions. Say, "I see what you did, let me show you what I did." Maybe you guys have something similar. Okay? Go ahead.
Individual	When students are supposed to work alone: students are explicitly directed to be silent, keep eyes on own paper, etc. (such as in a texting context) or under conditions in which it seems to be universally assumed that students should not work together. If there is some doubt as to whether a condition is PS or I, choose PS.	I'm going to have a question for you—I'd like each person to answer individually. So I'm going to put the question on the board. And just the way we normally do, you copy it down. Don't talk to your neighbor. Figure it out yourself"

Chapter 4: Overall Classroom Results

Overall Findings Across the Classrooms

In this section, I present my overall findings in order to summarize the patterns observed across the classrooms. First, I examine patterns in the kind of mathematical talk (explanations and non-explanations) that took place across the classrooms. Second, I focus on the roles that students enacted across classrooms and the relationship between the roles that students assumed and the mathematical talk that occurred. Finally, I look at the patterns between student roles and explanations in relation to classroom participation structure and teacher follow-up within each classroom.

Variation across classrooms in frequencies of different kinds of mathematical talk. Table 5 presents information on types of mathematical talk given by students in the classroom that related to the mathematical activities in which they engaged. It breaks down student mathematical talk as explanations and non-explanations. In addition, explanations are broken down into kinds based on the detail given during the explanation, such as full-detail, some-detail, and wrong-detail explanations. Full-detail explanations indicate students provided an explanation in which they completely verbalized step-by-step how they solved the problem. Some-detail explanations occurred when a student only verbalized some of the steps on how she solved the problem, but did not verbalize every single step. Wrong-detail explanations occurred when the detail given by the students was mathematically incorrect. Non-explanations included talk that was still mathematical in nature and about the problems students were working on, but were not explanations. For example, non-explanations could be observations (e.g., “I did it like Carl.”) or a mathematical claim (e.g., “I think zero is a whole number.”).

The first comparison tested was whether classrooms differed in mathematical talk (explanations vs. non-explanations) across the classrooms. A chi-square test[3] of independence was conducted to test the relation between classrooms and explanations vs. non-explanations. The relation between these variables was significant $\chi^2(3, N = 365) = 59.31$, $p < 0.0001$, indicating there was a difference in the observed frequency of explanations and non-explanations across classrooms. Classroom A had the highest incidence of explanations (compared to non-explanations); Classrooms B, C, and D had lower rates of explaining than Classroom A; and there were no significant differences between Classrooms B, C, and D in the incidence of explanations.

In addition to investigating the patterns of distribution of explanations versus non-explanations across the classrooms, I investigated whether there was a relationship between kinds of explanations (full-detail, some-detail, and wrong-detail) and classroom. The majority of explanations across the classrooms were some-detail (Range: 28-64), while full-detail explanations ranged from 6 to 23, and wrong-detail explanations ranged from 2 to 12. A chi-square test of independence was conducted and indicated no association between kinds of explanations and classroom $\chi^2(6, N = 245) = 9.21$, $p = 0.1621$. There was no evidence that the kinds of explanations differed by classroom.

In summary, when it came to explanations vs. non-explanations, Classroom A had significantly higher incidences of explanations than non-explanations, and a higher incidence of explanations than the other classrooms. There were no significant differences across Classrooms B, C, and D. In addition, while there were no differences in the distributions of kinds of explanations across the classrooms, I do want to highlight that some-detail explanations occurred

[3]http://vassarstats.net

most frequently in all of the classrooms. The distinctions in the kinds of mathematical talk observed in Classroom A raises the question of why these differences existed in Classroom A. The next section investigates variation in roles that students took on as a way to explain the difference in explanation rates across classrooms.

Table 5

Mathematical Talk by Classroom

Classroom	Explanations: Full-Detail	Explanations: Some Detail	Explanations: Wrong Detail	Explanations: Total	Non-Explanations	Total[c]
A	23 (23%)[a] [23%][b]	64 (63%) [65%]	12 (12%) [12%]	99 (97%) [100%]	3 (3%)	102 (100%)
B	19 (22%) [37%]	28 (33%) [55%]	4 (5%) [12%]	51 (60%) [100%]	34 (40%)	85 (100%)
C	19 (19%) [33%]	34 (33%) [60%]	4 (4%) [7%]	57 (56%) [100%]	45 (44%)	102 (100%)
D	6 (8%) [16%]	30 (39%) [79%]	2 (3%) [5%]	38 (50%) [100%]	38 (50%)	76 (100%)
Total	67 (18%) [27%]	156 (43%) [64%]	22 (6%) [9%]	245 (67%) [100%]	120 (32%)	365 100%

[a]This percentage (and all in parentheses) represents the frequency in that cell divided by all mathematical talk given in that classroom.
[b]This percentage (and all in brackets) represents the frequency in that cell out of the total number of explanations.
[c]The total represents the total number mathematical conversations that occurred in the classroom.

Variation across classrooms in frequencies of student roles. Table 6 gives, for each classroom, the number of times a student enacted a particular role during a conversation with the teacher or with other students. A student could enact a role verbally, nonverbally or both. For example, for the role Comparer, a student could enact the role verbally by saying "My strategy is the same as D's" during a conversation with her peers during Pair Share time. Or a student could enact the role of Comparer during a class conversation when the teacher asked students to point to a strategy that was similar to theirs and a student pointed to said strategy.

Inspection of Table 6 indicates that the students in Classroom A appeared to take on different roles from the students in Classrooms B, C, and D. While the role Sharer of Details of Problem-Solving Strategy (or SDPSS) occurred in all four classrooms, this role occurred most frequently in Classroom A, with 97% (n = 99) of observed enactment of student roles in Classroom A consisting of the role SDPSS. This percentage was more than twice as many as in Classroom B (42%, n = 36), and more than three times as many as in Classrooms C (29%, n = 30) and Classroom D (25%, n = 19).

Table 6 also shows that the role Comparer was common to only two classrooms, Classrooms B and C. Comparer appeared to occur with the same amount of frequency in both classrooms, occurring 14% (n = 12) in Classroom B and 17% (n = 17) in Classroom C.

The role Agreer/Disagreer was also only observed to occur in two classrooms: Classrooms C and D. However, unlike the role Comparer, Agreer/Disagreer did not occur with the same frequency in these classrooms. In Classroom C, the role Agreer/Disagreer made up 17% (n = 18) of the observed enactment of student roles. Yet, in Classroom D, Agreer/Disagreer was observed to occur almost three times as often as in Classroom C, making up 53% (n = 40) of

Table 6

Student Roles by Classroom

Student Roles	Classroom A	Classroom B	Classroom C	Classroom D
Sharer of Details of Problem-Solving Strategy	97%[a] (n = 99)[b]	42% (n = 36)	29% (n = 30)	25% (n = 19)
Sharer of Details of Mathematical Claim	0% (n = 0)	0% (n = 0)	22% (n = 22)	3% (n = 2)
Comparer	0% (n = 0)	14% (n = 12)	17% (n = 17)	0% (n = 0)
Agreer/Disagreer	0% (n = 0)	0% (n = 0)	17% (n = 18)	53% (n = 40)
Sharer of Details of Problem-Solving Strategy and Comparer	0% (n = 0)	8% (n = 7)	0% (n = 0)	0% (n = 0)
Multiple Strategist	0% (n = 0)	0% (n = 0)	3% (n = 3)	0% (n = 0)
Helper	1% (n = 1)	13% (n = 11)	0% (n = 0)	3% (n = 2)
Connection Maker	1% (n = 1)	7% (n = 6)	12% (n = 12)	9% (n = 7)
Teacher	0% (n = 0)	1% (n = 1)	0% (n = 0)	0% (n = 0)
Pattern Finder	1% (n = 1)	14% (n = 12)	0% (n = 0)	8% (n = 6)
Total	102[c]	85	102	76

[a] This percentage represents the observed frequency of a student enacted role divided by the total number of observed roles in the classroom.
[b] This entry represents the observed frequency of a role during all of the student-teacher or student-student conversations that took place in a classroom
[c] This entry represents all of the roles that were observed during all of the student-student and teacher-student conversations that took place in the classroom.

the role enactments in Classroom D. This also indicated that the role Agreer/Disagreer was the most frequently occurring role in Classroom D.

A final role that appeared to occur frequently was the role of Sharer of Details of Mathematical Claim (SDMC), accounting for 22% (n = 22) of the roles that were observed in Classroom C. However, the role SDMC was only observed to occur in Classroom C.

A few other roles were observed to occur across the four classrooms, but they did not occur frequently during these observed class sessions. The role Connection Maker, as can be seen in Table 6, occurred in all four classrooms. However, it only accounted for 1% (n = 1) of the roles that were seen to occur in Classroom A. Connection Maker accounted for only 7% (n = 6) of the observed enactment student roles in Classroom B; 12% (n = 12) of the observed enactment of student roles in Classroom C; and 9% (n = 7) in Classroom D.

The role Pattern Finder was observed in all but one classroom (Classroom C), but like the role Connection Maker, it accounted for a very small portion of the observed student roles. In Classroom A, for example, it only accounted for 1% (n = 1) of the observed enactment of roles. In Classroom D, it only accounted for 8% (n = 6) of the observed enactment of student roles. In Classroom B, Pattern Finder did account for more student roles than in Classrooms A and D, but only for 14% (n = 12) of the observed roles.

Likewise, the role Helper occurred in three of the four classrooms, but again did not occur frequently. In Classroom A, for example, this role only accounted for 1% (n = 1) of the observed enactment of student roles, and in Classroom D, this role accounted for 3% (n = 2) of the observed student roles. In Classroom B, the role was observed to occur more often, but it still only accounted for 13% (n = 11) of the observed student roles.

Finally, Classroom B had some distinctions that made it unique from the other classrooms. First, the role Teacher, which was only observed to occur once in Classroom B, accounted for 1% of the observed student roles. Second, the roles SDPSS and Comparer appeared to occur together a few times (n = 7), and accounted for 8% of the observed roles in the classroom.

In summary, the roles that were most frequent in each classroom were:

- Classroom A: Sharer of Details of Problem-Solving Strategy
- Classroom B: Sharer of Details of Problem-Solving Strategy; Comparer
- Classroom C: Sharer of Details of Problem-Solving Strategy; Comparer; Agreer/Disgreer; Sharer of Details of Mathematical Claim
- Classroom D: Sharer of Details of Problem-Solving Strategy; Agreer/Disagreer

Kinds of mathematical talk associated with different student roles. The results given above showed that Classroom A differed from the other classrooms in many ways. First, more explanations were given in Classroom A than in Classrooms B, C, and D. Second, in Classroom A, 97% of the roles students enacted in the classrooms consisted of the role Sharer of Details of Problem-Solving Strategy, which was a much higher incidence than in the other classrooms. In addition, the roles Comparer and Agreer/Disagreer, which occurred often in Classrooms B, C, and D, were not observed in Classroom A. Did the different roles students enacted in the classroom account for the different kinds of mathematical talk that were observed in the classrooms? The following paragraphs show how student roles and mathematical talk were related in the classroom.

Table 7 gives a breakdown of roles by mathematical talk for all four classrooms. Some roles were associated with much explaining, the role Sharer of Details of Problem-Solving

Strategy by far yielded the most explanations (100%, n = 184 of 184 conversations) across all four classrooms. Another role that yielded a high percentage of explanations (100%, n = 24 of 24 conversations) was the role of Sharer of Details of Mathematical Claim. Likewise, the combination role of Sharer of Details of Problem-Solving Strategy and Comparer yielded 100% (n = 7 of 7 conversations) of explanations. Finally, the role Teacher also yielded 100% (n = 1 of 1 conversations) of explanations (however, it was observed only once).

Other roles produced few explanations. The role Agreer/Disagreer, for example, only yielded 3% (n = 2 of 58 conversations) explanations across the classrooms. Similarly, the role Comparer yielded 10% (n = 3 of 29 conversations) explanations across the four classrooms. The role Multiple Strategist also did not yield many explanations, with 0% (n = 0 of 3 conversations) explaining when students enacted this role. Finally, the role Helper also did not yield many explanations, 14% (n = 2 of 14 conversations).

Table 7

Student Roles vs. Mathematical Talk Across Classrooms

Roles	Explanations	Non-Explanations	Total
Sharer of Details of Problem-Solving Strategy	100%[a] (n = 184)[b]	0% (n = 0)	100% (n = 184)
Share of Details of Mathematical Claim	100% (n = 24)	0% (n = 0)	100% (n = 24)
Comparer	10% (n = 3)	90% (n = 26)	100% (n = 29)
Agreer/Disagreer	3% (n = 2)	97% (n = 56)	100% (n = 58)
Sharer of Details of Problem-Solving Strategy and Comparer	100% (n = 7)	0% (n = 0)	100% (n = 7)
Multiple Strategist	0% (n = 0)	100% (n = 3)	100% (n = 3)
Helper	14% (n = 2)	86% (n = 12)	100% (n = 14)
Connection Maker	37% (n = 11)	63% (n = 15)	100% (n = 26)
Teacher	100% (n = 1)	0% (n = 0)	100% (n = 1)
Pattern Finder	61% (n = 11)	39% (n = 8)	100% (n = 19)
Total	245[c]	120	365

[a] The percentage represents the observed frequency in b divided by the total number of type of mathematical talk (explanation or non-explanation) observed across all of the classrooms.
[b] n represents the frequency in which a role resulted in an explanation during all observed conversations across all of the classrooms.
[c] The total number of explanations, non-explanations or mathematical talk that were observed across all of the classrooms.

Still other roles generated a moderate amount of explaining.   For example, the role Connection Maker yielded explanations in 37% of episodes (n = 11 of 26 conversations) across all of the classrooms. The role Pattern Finder yielded explanations in 61% of episodes (n = 11 of 19 conversations). Neither of these roles were observed frequently in all of the classrooms.

The results described above suggest that some roles students engaged in resulted in more explaining than other roles across classrooms. Does this then explain why Classroom A had significantly more explanations occurring in the classroom than in Classrooms B, C, and D? Or perhaps, could the differences be accounted for because the roles found to be common across the four classrooms yielded different amounts of explanations within the different classrooms? Some roles occurred more often in some classrooms than in others—does this account for more observed explanations in Classroom A? To answer these questions, Table 8 displays the relationship between roles and mathematical talk, but (in contrast to Table 7) takes into account the different classrooms.

Table 8 indicates that for the roles that were predominant in Classrooms A, B, C, and D, there appear to be no differences in terms of whether they led to or did not lead to explanations in the classroom. For example, Sharer of Details of Problem-Solving Strategy always led to explanations, regardless of classroom. Similarly, the roles Comparer and Agreer/Disagreer rarely led to explanations, regardless of the classroom in which these roles were observed.

Table 8

Student Roles vs. Mathematical Talk by Classroom

Role	Classroom A		Classroom B		Classroom C		Classroom D	
	Exp	Non-E	Exp	Non-E	Exp	Non-E	Exp	Non-E
Sharer of Details of Problem-Solving Strategy	100%[a] (n = 99)[b]	0% (n = 0)	100% (n = 36)	0% (n = 0)	100% (n = 30)	0% (n = 0)	100% (n = 19)	0% (n = 0)
Sharer of Details of Mathematical Claim					100% (n = 22)	0% (n = 0)	100% (n = 2)	0% (n = 0)
Comparer			17% (n = 2)	83% (n = 10)	6% (n = 1)	94% (n = 16)		
Agreer/Disagreer					0% (n = 0)	100% (n = 18)	5% (n = 2)	95% (n = 38)
Sharer of Details of Problem-solving Strategy and Comparer			100% (n = 7)	0% (n = 0)				
Multiple Strategist					0% (n = 0)	100% (n = 3)		
Helper	0% (n = 0)	100% (n = 1)	0% (n = 0)	100% (n = 11)			100% (n = 2)	0% (n = 0)
Connection Maker	0% (n = 0)	100% (n = 1)	0% (n = 0)	100% (n = 6)	33% (n = 4)	67% (n = 8)	100% (n = 7)	0% (n = 0)
Teacher			0% (n = 0)	100% (n = 1)				
Pattern Finder	0% (n = 0)	100% (n = 1)	50% (n = 6)	50% (n = 6)			100% (n = 6)	0% (n = 0)
Total	99[c]	3	51	34	57	45	38	38

[a] The percentage represents the observed frequency in b divided by the total number of explanations or non-explanations observed within each of the classrooms. [b] n represents the frequency in which a role resulted in an explanation during all observed conversations within each of the classrooms. [c] The total number of explanations or non-explanations that were observed in each classroom.

Roles and explanations summary. The findings given above indicate that what seemed to matter in eliciting explanations from students were the roles students enacted. The role Sharer of Details of Problem-Solving Strategy always resulted in explanations regardless of classroom. Other roles, such as Comparer and Agreer/Disagreer, did not result in explanations from students, regardless of the classroom in which they occurred. Classroom A had the highest incidence of Sharer of Details of Problem-Solving Strategy; Classrooms B, C, and D had much lower incidences of this strategy. Classrooms B, C, and D had much higher incidences of the roles that did not lead to explanations (especially Comparer and Agreer/Disagreer).

Breaking down the patterns even further: Exploring contextual information. In order to further clarify how students participated in the classroom and why they participated as they did, I explored contextual information. In the following tables, I broke down the roles and explanation data while including student participation structures and teacher follow-up. The information captured during the whole class structure occurred in the context of conversations with the teacher; in pair share and individual sessions student roles and mathematical talk largely occurred without the teacher being present. In addition, I included information on when the teacher followed up on student-initiated mathematical talk in order to understand better the teachers' discursive moves during the qualitative analysis on how and when teacher discourse and student roles resulted in explanations in the classroom. Tables 9-12 break down the explanation data by classroom, participation structures, teacher follow-up, and student roles. These tables are limited to the most frequently occurring roles in each classroom.

Table 9 shows when students enacted the role of Sharer of Details of Problem-Solving Strategy in Classroom A, they always provided explanations, whether the participation structure was whole class or pair share, and whether or not there was teacher follow up.

Table 9

Classroom A Student Roles and Teacher Follow-up

Student Roles		Whole Class		Pair Share			
		No Teacher Follow-up[4]	Teacher Follow-up	No Teacher Follow-up			Teacher Follow-up
				1	2	3	
Sharer of Details of Problem-Solving Strategy	Total	N = 9	N = 8	N = 58	N = 1	N = 10	N = 13
	Exp	n = 9	n = 8	n = 58	n = 1	n = 10	n = 13
	% Exp	100%	100%	100%	100%	100%	100%

- Sharer of Details of Problem-Solving Strategy = A student who shares the details of how to solve a problem; it could be that student's own strategy, the strategy of another student, or the strategy of a pair/group.
- Whole Class = Teacher-led classroom conversation, where teacher interacts with one student in front of class.
- Pair Share = Student-led conversations, where student pairs or groups are encouraged to discuss or work on mathematical problems
- Teacher Follow-up = Teacher followed up on initial student mathematical idea.
- Explanation (Exp) = Students articulate a mathematical strategies or mathematical detail (related to the problem) that solves problems presented by the teacher or sub-problems presented by the teacher or another student.
- No Teacher Follow-up 1: The teacher did not follow up because he/she was not present.
- No Teacher Follow-up 2: The teacher was present and engaged, but this was limited to initial invitations to participate.
- No Teacher Follow-up 3: The teacher was present, but did not engage with students (just simply observed).

[4]During Whole Class, No Teacher Follow-up indicates that the teacher was always present and engaged, but this was limited to initial invitations to participate.

Table 10 shows the patterns of explanations given by the students in Classroom B when they enacted the roles Sharer of Details of Problem-Solving Strategy, Comparer, and Sharer of Details of Problem-Solving Strategy combined with Comparer during the different participation structures of this classroom. The Sharer of Details of Problem-Solving Strategy role (whether combined with Comparer or not), always led to explanations, regardless of participation structure or teacher follow up. Comparer almost never led to explanations, regardless of participation structure or teacher follow up. Interestingly, when the teacher followed up on students' initial mathematical ideas, students rarely adopted the Comparer role (1 of 18 conversations). When there was no teacher follow up of students' ideas, students were more likely to adopt the Comparer role (11 of 37 conversations). This was the case whether the participation structure was Whole Class or Pair Share. This finding raises the question about whether the teacher follow up may have influenced the role that students adopted, for example, by changing what was initially a Comparer role into a Sharer of Details of Problem-Solving Strategy role by the end of the conversation. This possibility will be explored in a later section.

Table 11 shows the patterns of explanations given by the students in Classroom C. Classroom C had no Pair Share time. When students enacted the roles of Sharer of Details of Problem-Solving Strategy and Sharer of Details of Mathematical Claim, the students shared explanations, regardless of teacher follow-up. Similarly, when students enacted the roles of Comparer or Agreer/Disagreer, they almost never offered explanations, regardless of teacher follow up. As in Classroom B, the roles that students enacted in Classroom differed depending on whether there was teacher follow up. With teacher follow up, students most often adopted the roles of Sharer of Details of Problem-Solving Strategy and Sharer of Details of Mathematical

Claim (which led to explanations). Without teacher follow up, students often adopted the roles of Comparer or Agreer/Disagreer (which rarely led to explanations).

Table 12 illustrates the student participation patterns in Classroom D. In contrast to the other classrooms, there was relatively little teacher follow up in Classroom D. Thus, most student conversations in this classroom occurred without teacher follow up. Consistent with Classrooms B and C, when there was no teacher follow up, students were quite likely to adopt a role that generated few explanations, in this case the Agreer/Disagreer role. In Classroom D, when there was no teacher follow up, students enacted the Agreer/Disagreer role in 23 of 34 conversations in the Whole Class, and in all 14 conversations in Pair Share. During Individual Work, the pattern was somewhat different because during the three documented conversations the teacher invited students to explain their thinking. Interestingly, the two episodes of Agreer/Disagreer that *did* involve teacher follow up (these occurred in the Whole Class) led to explanations.

Table 10

Classroom B Explanations, Student Roles, and Teacher Follow-up by Participation Structure

Student Roles		Whole Class: No Teacher Follow-Up[5]	Whole Class: Teacher Follow-Up	Pair Share: No Teacher Follow-Up 1	Pair Share: No Teacher Follow-Up 2	Pair Share: No Teacher Follow-Up 3	Pair Share: Teacher Follow-up	Individual: No Teacher Follow-up	Individual: Teacher Follow-Up
Sharer of Details of Problem-Solving Strategy	Total	N = 6	N = 9	N = 9	N = 4	N = 0	N = 5	N = 0	N = 3
	Exp	n = 6	n = 9	n = 9	n = 4		n = 5		n = 3
	% Exp	100%	100%	100%	100%		100%		100%
Comparer	Total	N = 4	N = 0	N = 4	N = 2	N = 1	N = 1	N = 0	N = 0
	Exp	n = 0		n = 1	n = 0	n = 0	n = 1		
	% Exp	0%		25%	0%	0%	100%		
Sharer of Details of Problem-solving Strategy and Comparer	Total	N = 1	N = 0	N = 3	N = 3	N = 0	N = 0	N = 0	N = 0
	Exp	n = 1		n = 3	n = 3				
	% Exp	100%		100%	100%				

- Sharer of Details of Problem-Solving Strategy = A student who shares the details of how to solve a problem; it could be that student's own strategy, the strategy of another student, or the strategy of a pair/group.
- Comparer = A student who examines mathematical strategies for similarities or differences.
- Whole Class = Teacher-led classroom conversation, where teacher interacts with one student in front of class.
- Pair Share = Student-led conversations, where student pairs or groups are encouraged to discuss or work on mathematical problems.
- Individual = Students work/create individually strategies for problems.
- Teacher Follow-up = Teacher followed up on initial student mathematical idea.
- Explanation (Exp) = Students articulate a mathematical strategies or mathematical detail (related to the problem) that solves problems presented by the teacher or sub-problems presented by the teacher or another student.
- No Teacher Follow-up 1: The teacher did not follow-up because he/she was not present.
- No Teacher Follow-up 2: The teacher was present and engaged, but this was limited to initial invitations to participate.
- No Teacher Follow-up 3: The teacher was present, but did not engage with students (just simply observed)

[5]During Whole Class, No Teacher Follow-up indicates that the teacher was always present and engaged with students, but this was limited to initial invitations to participate.

Table 11

*Classroom C Explanations, Student Roles, and Teacher Follow-up Whole Class**

Student Roles		No Teacher Follow-Up[6]	Teacher Follow-Up
Sharer of Details of Problem-Solving Strategy	Total	N=16	N=14
	Exp	n=16	n=14
	% Exp	100%	100%
Share of Details of Mathematical Claim	Total	N=13	N=9
	Exp	n=13	n=9
	% Exp	100%	100%
Agreer/Disagreer	Total	N=17	N=1
	Exp	n=0	n=0
	% Exp	0%	0%
Comparer	Total	N=17	N=0
	Exp	n=1	
	% Exp	6%	

*Note: Only one explanation was captured during individual time—student role: connection maker with follow-up

- Sharer of Details of Problem-Solving Strategy = A student who shares the details of how to solve a problem; it could be that student's own strategy, the strategy of another student, or the strategy of a pair/group.
- Agreer/Disagreer = Student who takes a position relative to another student's mathematical solution (i.e., agrees or disagrees).
- Comparer = A student who examines mathematical strategies for similarities or differences
- Sharer of Details of Mathematical Claim = A student that shares the details of a mathematical claim; it could be that student's own claim or the claim of another student.
- Whole Class = Teacher-led classroom conversation, where teacher interacts with one student in front of class.
- Pair Share = Student-led conversations, where student pairs or groups are encouraged to discuss or work on mathematical problems.
- Individual = Students work/create individually strategies for problems.
- Teacher Follow-up = Teacher followed up on initial student mathematical idea.

[6] In Classroom C, No Teacher Follow-up meant the teacher was present and engaged with students, but engagement was limited to initial invitation to participate.

Table 12

Classroom D Explanations, Student Roles, and Teacher Follow-up by Participation Structure

Student Roles		Whole Class		Pair Share				Individual			
		No Teacher Follow-up[7]	Teacher Follow-up	No Teacher Follow-up			Teacher Follow-up	No Teacher Follow-up			Teacher Follow-up
				1	2	3		1	2	3	
Sharer of Details of Problem-Solving Strategy	Total	N = 11	N = 3	N = 0	N = 0	N = 0	N = 0	N = 0	N = 4	N = 0	N = 1
	Exp	n = 11	n = 3						n = 4		n = 1
	% Exp	100%	100%						100%		100%
Agreer/Disagreer	Total	N = 23	N = 2	N = 12	N = 2	N = 0	N = 0	N = 1	N = 0	N = 0	N = 0
	Exp	n=0	n=2	n=0	n=0	n=0		n=0			
	% Exp	0%	100%	0%	0%	0%		0%			

- Sharer of Details of Problem Solving Strategy = A student who shares the details of how to solve a problem; it could be that student's own strategy, the strategy of another student, or the strategy of a pair/group.
- Agreer/Disagreer: A student who takes a position relative to another student's mathematical solution (i.e. agrees or disagrees).
- Whole Class = Teacher-led classroom conversation, where teacher interacts with one student in front of class.
- Pair Share = Student-led conversations, where student pairs or groups are encouraged to discuss or work on mathematical problems.
- Individual = A time during class.
- Teacher Follow-up = Teacher followed up on initial student mathematical idea.
- Explanation (Exp) = Students articulate a mathematical strategies or mathematical detail (related to the problem) that solves problems presented by the teacher or sub-problems presented by the teacher or another student.
- No Teacher Follow-up 1: The teacher did not follow up because he/she was not present.
- No Teacher Follow-up 2: The teacher was present and engaged, but this was limited to initial invitations to participate.
- No Teacher Follow-up 3: The teacher was present, but did not engage with students (just simply observed).

[7] In Classroom D, No Teacher Follow-up meant the teacher was present and engaged with students, but engagement was limited to initial invitation to participate.

During Pair Share in Classroom D, 100% (14 of 14 conversations) of the enacted student roles were accounted for by the role Agreer/Disagreer, and these roles never led to explanations in Classroom D. In addition, these conversations occurred when the teacher did not follow up on student-initiated mathematical talk. Finally, during Individual Time, 20% (1 of 5 conversations) of the roles enacted in the classroom were accounted by Agreer/Disagreer and did not lead to explanations when the teacher did not follow up.

Differences across classrooms summary. In summary, classrooms differed the most during the conversations when the teacher did not follow up on student-initiated mathematical talk in the classroom:

- In Classroom A, when the teacher did not follow up on mathematical conversations, students always enacted the role of Sharer of Details of Problem-Solving Strategy; they never adopted the roles of Comparer or Agreer/Disagreer.
- In Classroom B when the teacher did not follow up on mathematical conversations during Pair Share, 27% of the adopted student roles were of the role Comparer. When the teacher did not follow up conversations in Whole Class, 36% of the adopted student roles were of the role Comparer.
- In Classroom C, in Whole Class, when the teacher did not follow up on mathematical conversations, students adopted the role of Agreer/Disagreer (27% of Whole Class conversations) and Comparer (27% of Whole Class conversations).
- In Classroom D, when the teacher did not follow up on mathematical conversations in both Whole Class and Pair Share, students adopted the role of Agreer/Disagreer (68% of Whole Class conversations included the role Agreer/Disagreer, and 100% of Pair Share conversations included the role Agreer/Disagreer).

Statistical tests comparing classrooms showed that Classroom A differed significantly from the other classrooms in terms of the roles that students adopted when the teacher did not follow up[8].

Dispersion of Student Participation. While information presented in this chapter so far demonstrated what roles students enacted in the classroom and which roles were associated with explanations in the classroom, information on how many students in each classroom enacted these roles and how many students in each classroom gave explanations would help provide a more detailed picture of who participated in the classroom. Table 13 provides this information across all four of the classrooms. That is, Table 13 indicates how many students participated in each of the classrooms, and in which ways. This information is important in order to understand which roles of the classroom were made available to which students, and also to understand whether the differences observed in terms of explanations could be simply explained by the roles that students enacted in the classroom.

Table 13 provides information that helps make sense of who enacted the observed mathematical behaviors in the classroom. While the previous information indicated that Classroom A differed from the rest of the classrooms, the information of who enacted which roles and who provided explanations allows for the interpretation that the classrooms really weren't that different in terms of explanation-giving in the classroom. Most (if not all) of the students in Classrooms A, B, C, as is indicated in Table 13, had the opportunity to provide explanations in the classroom (not only did they have the opportunity—the majority of the students did provide explanations). Across the three classrooms, anywhere from 84% of the students observed to 100% of the students observed provided at least one explanation. However,

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[8] Fisher's Exact Test (http://vassarstats.net) and post-hoc analyses using the Goodman Procedure (1963, 1969, 1971) to make pairwise comparisons.

in Classroom D, only 34% on Day 1 and 59% on Day 2 of the students observed provided an explanation. That is, less students actively participated via giving explanations in Classroom D, than in Classrooms A, B, and C. Table 13 also indicates that the same finding holds true for the enactment of the role of Sharer of Details of Problem-Solving Strategy.

In addition, the information presented in Table 13 also indicates that while the students in Classrooms B, C, and D enacted additional roles, only a few of the students present in each individual class enacted these roles. That is, that although it could be interpreted that students in Classrooms B, C, and D enacted the role of Sharer of Details of Problem-Solving Strategy less often because they were enacting other roles, only a few distinct students enacted these "other" roles in Classrooms B, C, and D. However, those few students, particularly in Classroom D, enacted these roles that were not associated with explanations frequently in the classroom.

Thus, Table 13 indicates that while almost all of the students in Classroom A participated, they only participated in one particular way during the lesson. Classroom B had similar participation patterns in that almost all of the students that participated during class time had the opportunity to provide explanations, but in addition some students also had the opportunity to participate through the enactment of the role of Comparer. In Classroom C, not only did almost all of the students provide explanations, they also often participated in numerous roles that sometimes led to explanations (such as the role of Sharer of Details of Mathematical Claim) or sometimes they did not (such as the roles of Comparer and/or Agreer/Disagreer). While in in Classroom D, students predominantly enacted two kinds of roles, these behaviors were attributed to very few students. As a result, very few students actively participated in Classroom D compared to students in the other classrooms.

Table 13

Dispersion of Student Participation: Explanations and Roles

Participation		Classroom A	Classroom B	Classroom C Day 1	Classroom C Day 2	Classroom D Day 1	Classroom D Day 2
Students who gave explanations	**n**[a]	n = 26	n = 14	n = 11	n = 16	n = 8	n = 13
	%[b]	96%	93%	100%	84%	34%	59%
	N[c]	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22
Students who enacted Sharer of Details of Problem-Solving Strategy	**n**	n = 26	n = 14	n = 9	n = 12	n = 7	n = 9
	%	96%	93%	82%	63%	30%	40%
	N	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22
Students who enacted Comparer	**n**	n = 0	n = 7	n = 4	n = 4	n = 0	n = 0
	%	0%	47%	36%	21%	0%	0%
	N	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22
Students who enacted Sharer of Details of Mathematical Claim	**n**	n = 0	n = 0	n = 9	n = 6	n = 1	n = 1
	%	0%	0%	82%	32%	4%	5%
	N	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22
Students who enacted Agreer/Disagreer	**n**	n = 0	n = 0	n = 5	n = 2*	n = 6	n = 10
	%	0%	0%	45%	11%	26%	45%
	N	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22
Students who enacted Sharer of Details of Problem-Solving Strategy & Comparer	**n**	n = 0	n = 8	n = 0	n = 0	n = 0	n = 0
	%	0%	53%	0%	0%	0%	0%
	N	N = 27	N = 15	N = 11	N = 19	N = 23	N = 22

*There were 13 instances of Agreer/Disagreer where the camera did not capture which student was participating.
[a]n represents how many different students gave an explanation or enacted a role during the observed lesson.
[b]% represents the percentage of students that gave an explanation or enacted a role of the total number of students present during the observed lesson.
[c]N represents the total number of students present during the observed lesson.

Overall Summary

The major findings in this chapter are as follows:

1. Almost all of the students in Classrooms A, B, and C gave at least one explanation and enacted the observed roles at least once, while the observed participation in Classroom D (both in the explanations given and observed roles) were limited to a few students.
2. Classroom A had the most explaining of all four classrooms—significantly more than the other classrooms. Furthermore, every student in this classroom (except one) provided explanations.
3. Classrooms B, C, and D were very similar in terms of explaining, and all had much less explaining than Classroom A. However, a closer inspection of the classrooms indicated that almost every single student in Classrooms B and C provided an explanation, whereas explanations were given by a limited few students in Classroom D.
4. The role Sharer of Details of Problem-Solving Strategy was associated with high incidences of explaining, and Classroom A had many more instances of this role than what was observed in the other classrooms. However, in Classrooms A, B, and C, pretty much all of the students enacted this role at least once, whereas in Classroom D, only a few students enacted this role.
5. The role of Sharer of Details of Mathematical Claims was also associated with explaining, but this role was only observed to occur in Classroom C and was not frequent. Not all of the students in Classroom C enacted this role.

6. The combination of the roles Sharer of Details of Problem-Solving Strategy and Comparer was always associated with explaining; however, this combination was only observed to occur in Classroom B, was not frequent, and was not enacted by all of the students.
7. The role Comparer was associated with non-explanations, and this role occurred often in Classrooms B and C, but was never observed to occur in Classroom A. In Classrooms B, about half of the students enacted this role, whereas in Classroom C, even fewer students enacted this role.
8. The role Agreer/Disagreer was also associated with non-explanations, and this role occurred often in Classrooms C and D, but was never observed to occur in Classroom A. However, the observed instances of the role were enacted by only a few students in the classroom.
9. The roles that students enacted without follow up from the teacher differed across classrooms. In Classroom A, students enacted the role Sharer of Details of Problem-Solving Strategy when the teacher did not engage in follow up of students' ideas. In the other Classrooms, students quite often enacted the roles of Comparer and/or Agreer/Disagreer when the teacher did not engage in follow up of their ideas. These roles allowed students to engage in each other's mathematical ideas without giving explanations.

The next chapters analyze each classroom in depth to provide insights into how students enacted different roles, why some roles did or did not lead to explaining, and the teacher's role in the observed differences of student behavior in the classroom.

Chapter 5: Classroom A Narrative

Classroom A Explanation and Role Findings Summary

The previous section showed that (a) the most frequently occurring role in Classroom A was the role of Sharer of Details of Problem-Solving Strategy (SDPSS); (b) this role occurred more frequently in Classroom A than in the other classrooms; and (c) in all classrooms, this role was always accompanied by student explaining. Consequently, students in Classroom A produced more explanations than students in the other classrooms. Close inspection indicates that students were likely to enact the role of Sharer of Details of Problem-Solving Strategy when the teacher did not follow up on ideas or suggestions that students made.

The purposes of this section are (a) to show what the teacher did to encourage students to enact the role of SDPSS and (b) to illustrate how explaining emerged when students adopted the role of SDPSS in Classroom A. I describe how the teacher designed the social participation structures to promote SDPSS by looking at: (a) the goals of student participation in the classroom as communicated by the teacher, and (b) the details of the teacher's specific directions, procedures, and evaluation statements that promoted SDPSS. Specifically, I will show that:

- The teacher's goals of student participation in Classroom A were that students should:
    - Solve math problems (by themselves or with their partners), and
    - Share with each other the strategies they came up with.
- The teacher communicated the importance of sharing the details of problem-solving strategies by directing students to solve problems during Pair Share, requesting that students share their strategies during Whole Class, modeling the kind of information students should share, explicitly requesting explanations from students after they

shared their answer/solution with the class, and providing positive evaluations to students who shared an explanation of their problem-solving strategy.

Classroom overview. The teacher's goal of student participation in Classroom A was that students should solve math problems (by themselves or with their partners) by creating strategies and share the details of problem-solving strategies with their peers. These goals were promoted through a cycling of student participation structures throughout the day where the teacher instructed students to: (a) solve problems and (b) talk to their peers as they solved them. These same instructions were observed during both Pair Share and Whole Class, but in different ways. A student would typically encounter the problem first during Pair Share, where students were observed to always take on the role of SDPSS. During Whole Class, the teacher would directly ask a student to share with the class his or her solution to a problem and/or to share how he or she solved a problem. The teacher in Classroom A structured the social environment in a way that promoted the role of SDPSS. She directly communicated to students that she wanted them to enact these roles in a variety of ways. In doing so, she encouraged student explanations.

Physical Description of Classroom A. Classroom A's physical environment was designed to support student sharing of explanations. At the front of the classroom there was a white board, which the teacher used often to center discussions around the mathematical work at hand. To the right of the whiteboard there was a SMARTboard, an interactive board which can be connected to a laptop/computer, for use during lessons and presentations. There were five large tables set up in a U shape in the classroom, with 4-7 male and female (mixed) students sitting at each table. This set-up of the student desks allowed for students to have a view of the board, which often was the center of classroom discussions during whole class time. In the middle of the U set up there was a teacher's desk, with a laptop connected to an Elmo projector (a projector that uses a camera to project documents) that projected onto the SMART board. The teacher used the Elmo to project student work on the smartboard, which allowed students to share the details of their strategies around their mathematical work. In addition, the teacher provided the students with small whiteboards for the students to work together on a problem. The whiteboards were introduced halfway through the class, and are discussed in more detail in the narrative that follows.

Classroom participation structures. The teacher in Classroom A provided students with two main participation structures: Pair Share and Whole Class. Classroom A was the only classroom across the four classrooms that did not have individual time for students to work on problems on their own. Typically, these structures occurred in the classroom as follows: the teacher would present a problem to the students and instructed them to work on it together during Pair Share time. After Pair Share time, the teacher would call on one student or a pair of students to share their strategy during Whole Class. Occasionally, additional students were selected to share during this time. Students were typically selected on a first-come first-serve basis—meaning that the teacher called on students who raised their hands. However, the teacher also often selected students who had not had an opportunity to share in the Whole Class setting yet. In this way, the teacher gave all students an opportunity to share during Whole Class. It is also interesting to note that the participation structures appeared to be linked in cycles. In other words, the students would do their initial mathematical work during Pair Share, then would be called on by the teacher to present their work during Whole Class.

In order to understand that the goals and purposes of student participation in Classroom A were to get students to create and share their strategies with others, one needs to have an understanding of what the teacher communicated to students during these different participation structures. The following sections describe the instructions that the teacher gave to students during Pair Share and Whole Class in order to highlight how the teacher communicated to students which behaviors were important in the classroom. Specifically, I show that during Pair Share, the teacher instructed the students to create and share their strategies with each other, while during Whole Class, the teacher instructed students to share their strategies/solutions with the class, sometimes by themselves and sometimes with their partner, and praised students who presented their solutions. In this way, during both participation structures, the teacher communicated to students that the most valued student participation was the sharing of their strategies with the rest of the class.

Classroom timeline of participation structures. Table 14 depicts the chronological timeline of the participation structures in Classroom A for the one day that was recorded. Each dark bullet point states the participation structure that occurred and the length of time spent in that participation structure. In addition to the structure and length of time spent, under each participation structure I list the problem(s) that students worked on during that time. This chapter draws examples from multiple phases of this timeline.

Student roles and explanations by phase. Table 15 shows the frequency with which SDPSS was enacted in each phase during the lesson. In addition, I indicate whether the teacher followed up on a student's mathematical idea, and whether the enactment of SDPSS resulted in an explanation in the classroom. During Pair Share, there were three different scenarios for why the teacher did not follow up. The first scenario of no teacher follow-up occurred because the teacher was not present during the conversation; the second scenario occurred when the teacher was present and engaged during small group discussion, but this engagement was limited to initial invitations to participate; the third scenario occurred when the teacher was present but did not engage with students. During Whole Class, the teacher was always present and engaged with students, but no teacher follow-up indicates that the teacher did not go beyond initial invitations to participate. Table 15 also indicates that students always enacted the role of SDPSS regardless of whether or not the teacher followed up during their conversation with their peers and/or teacher.

Classroom Materials in Classroom A. In Classroom A, the whiteboard played an important role during both Whole Class and Pair Share discussions. The whiteboards helped to facilitate discussions in both participation structures, and was used by the teacher and student participants to mediate mathematical discussions in the classroom.

During Whole class discussion, the whiteboard in the front of the classroom often served as a tool that facilitated student enactment of the role SDPSS. Students were often selected by the teacher to share the way they solved the problems in the class, and the teacher would either put up student work on the whiteboard or ask the student to describe how they solved a problem, while the teacher wrote what the student said. Both ways served as a way to publicly display the student work that allowed for the rest of the class to be able to follow as the student enacted the role of SDPSS. More details are provided below in the analytical narrative below.

During some of the Pair Share conversations (starting at Phase 8), the students were allowed to use small individual whiteboards as a shared workspace. The introduction of the whiteboards appeared to change the way in which students participated in the classroom in the sense that students often enacted the role of SDPSS and produced explanations jointly rather than independently in the classroom. This was because in sharing a workspace, student partners needed to verbalize their thoughts in order to be able to come up with a strategy to solve the problem. More details are provided in the analytical commentary of Phase 8.

Table 14

Chronological Timeline for Participation Structures in Classroom A

Day 1

• Phase 1: Whole Class (5 min 4 s)
  ○ Housekeeping
  ○ Counting by tenths from 1/10 to 20 tenths
  ○ Converting 11/10 into decimal
• Phase 2: Pair Share (50 s)
  ○ Converting 11/10 into decimal – discuss what Claire did
• Phase 3: Whole Class (1 min 5 s)
  ○ Converting 11/10 into decimal—explain what Claire did
• Phase 4: Pair Share (36 s)
  ○ Why is 11/10 not written as .11?
• Phase 5: Whole Class (2 min 52 s)
  ○ Why is 11/10 not written as .11?
  ○ Write out mixed number for 12/10 and convert to decimal
  ○ Count from 1.3 to 2.0
• Phase 6: Pair Share (1 min 1 s)
  ○ 4/10 plus seven tenths
• Phase 7: Whole class (1 min 45 s)
  ○ 4/10 plus 7/10
• Phase 8: Pair Share (1 min 38 s)
  ○ Introduction of Whiteboards
  ○ 1.6 plus 1.7
• Phase 9: Whole Class (10 min 53 s)
  ○ 1.6 plus 1.7 (4 groups)
  ○ Counting by hundredths
  ○ Convert 4/100 to decimals
  ○ Convert 15/100 to decimals
• Phase 10: Pair Share (3 min 58 s)
  ○ 92/100 + 12/100
• Phase 11: Whole class (7 min 22s)
  ○ 92/100 + 12/100
  ○ Convert result into decimals
• Phase 12: Pair Share (42 s)
  ○ Compare 3/10 or 4/5
• Phase 13: Whole Class (8 min 27 s)
  ○ What do you have to do to compare 3/10 and 4/5 (and compare)
  ○ Convert 2, 0.95, 1.8, and 1.36 into fractions (HW)
  ○ How many hundredths in 8/10?
  ○ Compare the four numbers
• Phase 14: Pair Share (56s)
  ○ What do you notice between the work we did on the right vs left (place value chart)
• Phase 15: Whole Class (4 min 38 s)
  ○ What do you notice?

Table 15

Classroom A Explanations and Student Roles by Phase

Student Roles		Phase 1 (WC) NTF[9]	Phase 1 (WC) TF	Phase 2 (PS) NTF 1	Phase 2 (PS) NTF 2	Phase 2 (PS) NTF 3	Phase 2 (PS) TF	Phase 3 (WC) NTF	Phase 3 (WC) TF	Phase 4 (PS) NTF 1	Phase 4 (PS) NTF 2	Phase 4 (PS) NTF 3	Phase 4 (PS) TF
Sharer of Details of Problem-Solving Strategy	Total Exp % Exp	N = 1 n = 1 100%	N = 0	N = 10 n = 10 100%	N = 0	N = 1 n = 1 100%	N = 1 n = 1 100%	N = 1 n = 1 100%	N = 0	N = 8 n = 8 100%	N = 0	N = 2 n = 2 100%	N = 1 n = 1 100%

Student Roles		Phase 5 (WC) NTF	Phase 5 (WC) TF	Phase 6 (PS) NTF 1	Phase 6 (PS) NTF 2	Phase 6 (PS) NTF 3	Phase 6 (PS) TF	Phase 7 (WC) NTF	Phase 7 (WC) TF	Phase 8 (PS) NTF 1	Phase 8 (PS) NTF 2	Phase 8 (PS) NTF 3	Phase 8 (PS) TF
Sharer of Details of Problem-Solving Strategy	Total Exp % Exp	N = 1 n = 1 100%	N = 1 n = 1 100%	N = 7 n = 7 100%	N = 0	N = 0	N = 3 n = 3 100%	N = 0	N = 1 n = 1 100%	N = 14 n = 14 100%	N = 0	N = 1 n = 1 100%	N = 0

[9] In Classroom A, during Whole Class, No Teacher Follow-up indicates that the teacher was always present and engaged with students, but this was limited to initial invitations to participate.

Table 15 (continued)

Student Roles		Phase 9 (WC) NTF	Phase 9 (WC) TF	Phase 10 (PS) NTF 1	Phase 10 (PS) NTF 2	Phase 10 (PS) NTF 3	Phase 10 (PS) TF	Phase 11 (WC) NTF	Phase 11 (WC) TF	Phase 12 (PS) NTF 1	Phase 12 (PS) NTF 2	Phase 12 (PS) NTF 3	Phase 12 (PS) TF
Sharer of Details of Problem-Solving Strategy	Total	N = 3	N = 1	N = 6	N = 1	N = 1	N = 5	N = 0	N = 3	N = 11	N = 0	N = 4	N = 0
	Exp	n = 3	n = 1	n = 6	n = 1	n = 1	n = 5		n = 3	n = 11		n = 4	
	% Exp	100%	100%	100%	100%	100%	100%		100%	100%		100%	

Student Roles		Phase 13 (WC) NTF	Phase 13 (WC) TF	Phase 14 (PS) NTF 1	Phase 14 (PS) NTF 2	Phase 14 (PS) NTF 3	Phase 14 (PS) TF	Phase 15 (WC) NTF	Phase 15 (WC) TF
Sharer of Details of Problem-Solving Strategy	Total	N = 3	N = 2	N = 2	N = 0	N = 1	N = 2	N = 0	N = 0
	Exp	n = 3	n = 2	n = 2		n = 1	n = 2		
	% Exp	100%	100%	100%		100%	100%		

Exp = Explanation
NTF 1= No Teacher Follow-up: the teacher did not follow up because he/she was not present.
NTF 2 = No Teacher Follow-up: the teacher was present and engaged with students, but this was limited to invitations to participate.
NTF 3 = No Teacher Follow up: the teacher was present, but did not engage with students.
TF= Teacher Follow-up
WC = Whole Class
PS = Pair Share

The phase-by-phase description and analysis in the subsequent sections explains in detail how the teacher's instructions, directions, and evaluations encouraged students to always enact the role of SDPSS in the Classroom. I will also show how she encouraged students to give explanations in the classroom in the context of students enacting SDPSS.

Classroom A Description and Analysis

Phase 1: Counting, and explaining wholes during Whole Class. During the first Whole Class session, the teacher communicated to the students that the goal of their participation was for them to share how they solved a problem by:

1. directly requesting from the students that they share their mathematical thinking with the class; and
2. modeling what information she was interested in by helping one student simultaneously solve a problem and share the strategy with her classroom peers.

The following discussion details how the teacher specifically communicated using the above-listed teacher practices to students in Classroom A with the goal of student participation. Specifically, the first example illustrates how the teacher asked a student to share why he called out one whole rather than ten tenths; the second example illustrates how a student did not provide the requested answer, but was led by the teacher through a series of questions to help her answer, while also modeling to the rest of the class the kind of information that the teacher expected students to share when providing their mathematical thinking.

The teacher began class by asking students to help her count by tenths all the way to 2 in choral fashion, and then asked the students to give the decimals for the fractions (again in choral fashion). The teacher led the choral counting while standing in the front of the classroom in front of the whiteboard. Students mostly appeared to be facing forward and counting chorally with the teacher. When the students got to ten tenths, several students called out one whole; when they got to twenty tenths, they called out two wholes. The teacher provided a positive evaluation for doing this conversion and asked two students to explain why they did this:

1. T: "So, I love that you guys changed to wholes. Tell me why. Why here (*several students raised their hands)* did you choose not to say ten tenths, why did you say one? Who can tell me, S?
2. S: Because ten tenths is like, like ten out of ten, so you make it one whole.
3. T: you make it one whole.

The teacher in the conversation above directly asked one student, S, to share his mathematical thinking for why he had chosen to say one whole over ten tenths by repeating "why" multiple times. The student responded by sharing not just why, but also how he had converted ten tenths into a whole. Thus, S enacted the role of SDPSS in the classroom when he shared with the teacher and his peers why he said one whole rather than ten tenths, along with how he determined it was one whole.

The teacher continued the discussion and called on one student (C) to give the decimal for the fraction eleven tenths. Essentially, the teacher was looking for an answer for the decimal representation of eleven tenths. However, C did not answer, prompting the teacher to lead C through a serious of questions that modeled to the rest of the class the details she expected students to share in an explanation. C, then, was able to enact the role of SDPSS, but only with the support of the teacher in the following excerpt:

1. T: Okay one, all right. Okay. Very nice. What about here? (*points to 11/10 written on board*). I'm going to call on someone (*several students raise their hands*). C, what do I write here?
2. C: um—I—uh…
3. T: Ah-ha! Okay, don't worry. Guess what C? The reason I called on you is because I know that you are good with helping me pose complex things. Okay? Here's the thing, C. Let's think of this (*teacher grabs marker and circles11/10 on board*) as a mix number. Okay. So you know how right now it's eleven tenths, it's an improper fraction, right? (*C nods head. C's gaze was towards whiteboard*) It's got a larger numerator than a denominator. How can I make this a mixed fraction, a whole

number and a fraction? Is there a whole number inside of eleven tenths? (*several students raised their hands)*

4. C: (*whispering*) Ten tenths?
5. T: Huh? (*C repeated ten tenths*) There's ten tenths, right? (*students put their hands down*) So we take this and we say ten tenths plus what?
6. C: one.
7. T: plus one tenth. Right, so what mixed number is that? This means what?
8. C: one whole.
9. T: one whole. (*teacher wrote on board, while students gazed at board)*
10. C: one tenth.
11. T: One tenth. Okay, so we're going to make that—I'm going to write forever aren't I—one and one tenth, yeah? Okay so now, C, now that you know (*teacher turned to face C)* that it's one and one tenth, how do I write the decimal?
12. C: One point one. (*teacher nodded and wrote 1.1 on board)*
13. T: one point one. Okay.

First, the teacher requested that C give the decimal representation of the fraction eleven tenths. C did not provide an answer, and the teacher then led a series of questions that pushed C to provide some details explaining a strategy that could be used to convert eleven tenths into a decimal. While there was no immediate evidence that the teacher would have asked C to detail her strategy had she given the correct answer, the teacher always appeared to ask the students in Classroom A to share the details of their thinking, such as in the previous example and in examples that will be seen in future phases. The questions the teacher asked of C to help her arrive at the correct answer and the work the teacher wrote on the board modeled for the students the kinds of detail she valued in an explanation. The teacher's modeling along, then, had the potential to inform the students on how they should participate in the future, particularly since students appeared to be following the teacher's interaction with C. In the next phase, several students enacted the role of SDPSS, and the details they provided were similar to the ones that the teacher had requested from C.

Phase 2: Pair Share discussion on how to convert 11/10 into a decimal. During this first Pair Share session, the teacher communicated that the goal of student participation during

Pair Share was for students to share their strategies to solve a problem with their peers. The teacher communicated this goal when she:

1. directed students to talk to their peers about the process of converting 11/10 into a decimal. She wanted them to discuss how the teacher and C had solved the problem; and
2. provided a positive evaluation to a student who enacted the role of SDPSS while providing an explanation. In this way, the teacher communicated to the other students in the group that this was a form of desired student behavior.

The following discussion highlights the details of the teacher's directions to the whole class during Pair Share, and also how she communicated the positive evaluation to the students in one group.

The teacher used the interchange with C in Phase 1 as the impetus for Pair Share in Phase 2. The teacher was not sure that the class understood what C had done, so she asked students to talk about it with their peers at their table. Specifically, she said:

1. Teacher: Okay, do you guys understand how C did this?
2. Student: yeah!
3. Teacher: Hmmmm, I'm not convinced. So do me a favor, turn and talk at your table. So you can—so you can [see] the process of how this happened.

During this phase, students adopted the role of SDPSS without any follow-up from the teacher. The following example shows how the teacher's directions to talk with their peers about the *process* of converting 11/10 into a decimal may have prompted them to enact the role of SDPSS:

1. E: Um, guys....

2. B1[10]: Well, all she did, basically, it's really easy. 'Cause you just take eleven. And it's just, [the first number, the tens place is the one that's before the decimal, and then the ones place is the one that's after the decimal. So…
3. E: (*started talking in the middle of B1's explanation*) [It's the tenths place because the one is the ten. 'Cause there's a ten below it. And then you have a one whole (*B1 stopped talking*) so you have a one whole, and you just put (*inaudible*) ten (*inaudible*) [[so you just put the one right there. It's like eleven sorta.
4. B2: (*started talking in the middle of E's explanation*) [[Because we just got to one. and then you put the decimal and then you put the one.

In the above example, three students, E, B1, and B2, enacted the role of SDPSS after the teacher directed the students to talk about how she and C converted eleven tenths into a decimal. The three students enacted the role of SDPSS by verbalizing how they thought the teacher and C converted the decimal. All three included the same details that the teacher had elicited from C during the previous Whole Class session.

While the students appeared to be talking over each other, there are clues that students were listening to each other's ideas during this interaction, and even using each other's details to create their own explanation. For example, E started her explanation when B1, had just started explaining the conversion process with the number 11. E said the same next step that B1 was discussing, in a slightly different way, but at the same time. The overlapping speech of the same detail indicates that E was attending to B1's explanation, and was able to enact the role of SDPSS and provided details for an explanation. In addition, when B2 enacted the role of SDPSS, it is clear that he was also attending to the explanation given by E and B1. B2 gives the same details that B1 gave when finishing E's explanations, but did so in a different way. Thus the three students were clearly attending to each other's explanations in ways that allowed the students to enact SDPSS while providing what appears to be one shared explanation.

[10] Left Brackets such as "[" indicate overlapping speech. The same number of brackets in a different line indicate the speech that overlapped. For example in line 2 "[the first number" overlapped with "[It's the tenths" in line 3.

The teacher also signaled the importance of sharing the details of problem-solving strategies by giving positive evaluations to students who did so (that is, providing positive reinforcement). In the following example, the teacher listened to A providing an explanation, and smiled and nodded throughout (Line 3). In this case, the reinforcement was nonverbal.

1. A: It's one and one tenths, so you just—
2. J: (*A turned to face J*) Don't say the wrong thing people, (*whispered*) it's filming!
3. A: uh—Okay….Well since its uh eleven tenths (*all the students in the group turn to face A, and were leaning in towards the middle of the table*), it has one (*teacher approached table and stood next to A*) tenth and ten tenths—that's a whole (*teacher leaned into A*). And then just put the one in front (*teacher smiled at A and nodded head*) and then point one. (*teacher walked away).*

The teacher's behavior provided a signal to A and the rest of the students at the table that she wanted them to explain their thinking. The students during this interaction were engaged with A's explanation, as their gaze was on her, and their bodies leaned in towards A while she spoke. As such, for one particular student at the table, E, the teacher's positive evaluation likely impacted her participation in Phase 3, as seen below.

Phase 3: Whole Class explanation on how to convert 11/10 into a decimal. During the second episode of Whole Class, the teacher communicated to the students that the goal of student participation during Whole Class would be to enact the role of SDPSS by sharing how they solved a problem during Pair Share. The teacher communicated this goal when:

1. The teacher directed only one student, E, to share the details of the problem-solving strategy that C and the teacher had used to convert eleven tenths into a decimal in Phase 1.

Essentially, in Phase 3, the teacher communicated to students that they might get selected to share with the rest of the class the strategies discussed during Pair Share. Several students during this time raised their hands (including E). The following example indicates that the student enacted the role of SDPSS after the teacher directed her to do so.

1. <u>T: E, can you explain to us what we did? C and I?</u>
2. E: Well, what you guys did
3. T: Mmmhmmm
4. E: First of all, you know that ten tenths is in eleven tenths.
5. T: Okay
6. E: So. When you broke it up, to first find the whole in it. So its tenth tenths plus one tenth.
7. T: Uh-huh
8. E: And the ten tenths equals to one whole. And plus one whole—it would be a mixed fraction, so it's one whole and it's a mixed number, so one and one tenth, and put it into decimal. It's one point one.
9. T: Okay.

In this example, E volunteered to and enacted the role of SDPSS after the teacher requested that she explain the strategy discussed in Phase 1. This signaled to the student, E, that she should share the details of the strategy with the class. The teacher did not provide an intervention during E's explanation, and E repeated the details that the teacher had encouraged when she modeled the explanation with C.

Phases 4-14: Repetition of Phases 2 and 3. For Phases 4 through 14, the pattern observed in Phases 2 and 3 repeated. The teacher continued to promote the role of SDPSS by requesting from the students that they share with the class (during Whole Class) or with their peers (during Pair Share) how they solved a problem, and provided positive evaluations to students who enacted these roles. As a result, the students continued to enact the role of SDPSS while providing explanations during these phases.

While the above examples illustrate how students often shared the details of their problem-solving strategy alone, the following example, from Phase 8, illustrates how students sometimes enacted the role of SDPSS in collaboration with their peers to produce an explanation.

Students during Phase 8 and afterwards were able to jointly create explanations and enact the role of SDPSS because the teacher introduced the use of shared whiteboards as a way that students could work together in the classroom. In addition to introducing the whiteboards, the teacher paired this new work tool with instructions that emphasized collaboration:

> "Okay, can I just get crazy on you. I'm going to do something that is not even on the board. Alright. Lets see If I already have—its kind of funny because the whole time that week--I have been up here I have been trying to think, what would I have in tenths. I can't do food because I don't like my food that small. So I don't know, we'll see. So if I already have one point six. Okay. And I add. <u>So now if you would like, you can use your dry erase boards.</u> If I have one point six. Or sixteen tenths. (*students started talking loudly for 6.0 s*) They're already working on it and I haven't even given them the other number yet. <u>Good thing its partnerships. Someone is excited for math today.</u> (*clap, clap, clap, clap, clap*). <u>You have enough boards for your partnerships I hope. Does that one have enough? Thank you guys, you guys manage really well.</u> Okay if I have sixteen tenths or one point six. (*4.0s*) Okay. If I have sixteen tenths. And I give you another seventeen tenths. What will you have altogether? Sixteen tenths plus seventeen tenths. Give it up."

As in the previous examples, the teacher had presented the students with a problem and encouraged them to talk about it with their peers during Pair Share. In addition, the teacher encouraged students to enact the role of SDPSS jointly by describing the behavior with the word "partnerships" repeatedly which indicated to students that it was acceptable for them to create strategies together. Finally, the teacher noted that the students appeared "excited" in her instructions. This interpretation of the student behavior had the potential of creating a positive emotional environment for students, as it appeared to occur with one group:

(*All three students were hunched over the whiteboard, which was located in front of A.)*

1. A: Okay, so, well—
2. S: (*reached out to E, who was holding marker*) Wait can I see this?
3. E: yeah. (*handed over marker*)
4. S: [So sixteen—First you sixteen plus— (*S wrote. All three have gaze on whiteboard*)
5. A: [Two—Two point eleven—no! Two point thirteen (*nods head once*)
6. S: No—um…
7. E: seven—thirteen

8. A: two point three! (*nods head*) Two point three!
9. S: three….three point three
10. A: No, its two!
11. S: Because its one plus one—it's one whole—
12. E: Thirty three.
13. A: No, you're wrong! Dude! You're wrong!
14. S: plus one whole. yeah. It's three point three!
15. E: Let's double check it
16. S: No, Look, look, look (*leans in closer to whiteboard, students continued gaze at whiteboard*) Because, it's—one point six plus one point seven (*S's hand moves over whiteboard. He appeared to be pointing to what was written.*)
17. E: see.
18. S: (inaudible) over there. So three point three. There's our answer. (*the three students stand up and move away from the whiteboard in unison*)
19. E: Hold it up! (*A and E lift whiteboard up in the air as teacher walked by. S walked back to his chair.*)
20. T: I love it! I love it, okay don't erase it, I love it. (*E & A smile*)
21. E: Let's all hold it! S! Grab one side! (*S walked back to A & E and put his hand on the whiteboard as the three continued to hold it up*)
22. S: We're a good team.

In the example above, the three students enacted the role of SDPSS by together creating and sharing the details of how they solved one point six plus one point seven, in lines 4-18. The three students continued to add to the explanation by responding to each other's mathematical ideas, as evidenced throughout the student conversation. Throughout the interaction, the students displayed signs of attending to each other, constantly leaning in to each other and always being hunched over the whiteboard, which mediated their conversation. Thus, together, the students enacted the role of SDPSS by sharing how they solved the problem by vocalizing their strategy as they solved it. In addition, the students appeared to internalize the positive attribution that the teacher gave to partnerships in her instructions, as one of the students provided a positive comment on how they worked together as a team (line 22).

Conclusion

This section showed that the teacher promoted the role of Sharer of Details of Problem-Solving Strategy to her students by directing them to discuss the processes of solving various types of problems during Pair Share, requesting that students share this information with the rest of the class during Whole Class, modeling the kinds of details she was interested in, and giving positive evaluations to students who shared an explanation. The teacher explicitly directed her requests to individual students during Whole Class, although occasionally she also directed her requests to groups.

In addition, the teacher created participation structures that focused on supporting students' opportunities to enact the role of SDPSS and provide explanations in the classroom. During Whole Class, the teacher often had students share with the entire class the strategy they had come up with, and used the class whiteboard to mediate their explanation. In addition, during Pair Share, the teacher created a space that allowed students to verbalize an explanation while enacting SDPSS. During this time, students also had access to a shared workspace, small whiteboards shared by the students, that mediated their conversations. These whiteboards allowed some of the students to come up with an explanation and enact the role of SDPSS jointly. As a result, students frequently enacted the role of Sharer of Details of Problem-Solving Strategy in the classroom when the teacher did not follow up on a conversation, and while doing so provided detailed explanations on how they solved the problems given by the teacher.

Chapter 6: Classroom B Narrative

Classroom B Explanations and Role Findings Summary

In Chapter 4, the results indicated that students in Classroom B predominantly enacted the role of Sharer of Details of Problem-Solving Strategy (SDPSS) in the classroom. In addition, students in Classroom B enacted the role of Comparer as well as that role in conjunction with the role of SDPSS, although not as frequently as they enacted the role of SDPSS alone. Students enacted the role of Comparer by stating whether their strategy was the same or different from that of their peers. The role was considered to result in an explanation when the students shared details on how their strategy was similar to their peers, however this did not occur often. While the students of Classroom B enacted the role of SDPSS less often than those in Classroom A did, however, this was not significantly different. Students also produced fewer explanations than those in Classroom A did. The difference was mostly due to students often enacting the role of Comparer or that role in conjunction with SDPSS when there was no teacher follow-up.

This section now explores the social dimensions that promoted the role of Comparer and SDPSS in the classroom to understand why students chose to enact the role of Comparer or Comparer with SDPSS with no teacher follow-up. Specifically, I show that:

- The teacher's goals of student participation in Classroom B were:
  - students should create their own strategies during individual time;
  - students should share their own strategies with their peers during Pair Share and/or Whole Class;
  - students should compare their own strategies with their peers by identifying whether their strategies were similar or different and/or sharing how they were similar or different.

- The teacher in Classroom B communicated to students these goals through directions, procedures, and modeling of expected behaviors.
    - Specifically, the teacher made general announcements that detailed how students were expected to share with their partners, directed the students to create strategies during individual time that would be shared later, instructed students to share with their peers during pair share, and requested that students share during Whole Class.
    - The teacher modeled comparing behaviors during Pair Share, made general announcements that encouraged students to make comparisons, and engaged students in activities for which they had to compare their own strategy with that of another.

It is important to note that the teacher mostly communicated to students that they should compare their strategies while also telling them at the same time that they should share/explain their strategies. As a result, the role of Comparer often led students to enact the role of Sharer of Details of Problem-Solving Strategy. The Classroom Analysis and Narrative below shows how the teacher bundled these goals in the classroom.

Physical Description of Classroom B. Classroom B's physical set-up was similar to the physical set-up to Classroom A, and was meant to support student engagement with each other's ideas. Like Classroom A, the teacher arranged the classroom tables in large clusters throughout the classroom in the shape of a large U, with a projector situated in the middle of the U. Each cluster of tables hosted two to four students, and the students were all situated at the tables such that they faced the white board and Smart Board. While each table hosted two to four students, students in general had a partner or two, thus at one table there could be more than one group.

While the physical set-up of the classroom was the same Classroom A's, the set-up of the classroom in conjunction with the participation structures (as described below) afforded the students the ability to engage in two main student roles in the classroom, one of which encouraged students to engage with each other's ideas.

Classroom participation structures. The students in Classroom B were encouraged to function mostly in three participation structures: Whole Class, Pair Share, and Individual. These three structures were linked in a cycle that allowed the students to talk about particular problems across multiple structures. Typically, through these structures, students were introduced to a problem and asked to think/solve it individually. During this time, the teacher was usually located either at the center of the room (at the projector) while she introduced the problems, then she would walk around the room and observe students (occasionally interacting) while students worked on the problems. The students would then be asked to share their strategies with their partners and compare and help their partners with their strategies. During this Pair Share time, the teacher would again roam the classroom, and listen/interact with the students while selecting multiple students to place their work on the board. During Whole Class, the teacher had students share their work with the rest of the class, and during this time, the teacher would either be located at the board or at the center of the class.

Materials used in Classroom B. In Classroom B, the teacher had the students work in their individual notebooks when they had to work on problems during Individual Time. These notebooks often were used by the students to mediate their conversations during Pair Share time, when students were encouraged to enact roles other than Sharer of Details of Problem Solving Strategy. Finally, during Whole Class the teacher encouraged the use of two different tools to mediate the class conversation: the whiteboard and the student notebook (which was projected onto the whiteboard by an Elmo).

During Phase 3, the teacher invited students one by one during the class discussion to share how they did the problem, and often herself wrote on the board what students described. During Phase 5, the teacher selected several students to the board prior to Phase 6, and requested that they write their strategy on the board, which they would later explain. The whiteboard during Phase 6 then served the purpose of mediating Whole Class discussions around student work. This is described in more detail in the analytical narrative below.

Classroom B timeline of participation structures. Table 16 lists the student participation structure, the amount of time spent in each structure, and the problems students worked on during that participation structure.

Table 17 lists the roles and explanations given within each phase and participation structure. It shows that during Pair Share time, particularly Phase 5, students enacted the role of Comparer or Comparer with SDPSS just as often as SDPSS alone when the teacher did not follow up. The following discussion provides a detailed narrative and analysis of how the teacher promoted the various roles during each phase in the classroom, and why students enacted Comparer and Comparer with SDPSS when the teacher did not follow up.

Table 16

Chronological Timeline for Participation Structures in Classroom B

Day 1
• Phase 1 Whole Class (2 min 32 s) ○ What are we counting by? ○ Counting by sixes past 100 • Phase 2 Pair Share (2 min 21 s) ○ How did you know how to get from 96 to 102? What did you do? • Phase 3 Whole Class (18 min 51s) ○ What strategy did you use to know how to get from 96 to 102? ○ Counting by sixes from 102 to 198 ○ How did you know to go from 198 to 204? • Phase 4 Individual (6 min 49 s) ○ 19 students fold cranes to send to Japan. If each student folds six cranes, how many cranes will they fold in all? Come up with two strategies. • Phase 5 Pair Share (3 min 57s) ○ 19 students fold cranes to send to Japan. If each student folds six cranes, how many cranes will they fold in all? Come up with two strategies. • Phase 6 Whole Class (15 min 6 sec) ○ 19 students fold cranes to send to Japan. If each student folds six cranes, how many cranes will they fold in all? Come up with two strategies.

Table 17

Student Roles and Explanations by Phase in Classroom B

Student Roles		Phase 1 (WC) NTF 2	Phase 1 (WC) TF	Phase 2 (PS) NTF 1	Phase 2 (PS) NTF 2	Phase 2 (PS) NTF 3	Phase 2 (PS) TF	Phase 3 (WC) NTF 2	Phase 3 (WC) TF
Sharer of Details of Problem-solving Strategy	Total Exp % Exp	N = 0	N = 0	N = 3 n = 3 100%	N = 3 n = 3 100%	N = 0	N = 1 n = 1 100%	N = 3 n = 3 100%	N = 5 n = 5 100%
Comparer	Total Exp % Exp	N = 0	N = 0	N = 1 n = 0 0%	N = 1 n = 0 0%	N = 0	N = 0	N = 0	N = 0
Sharer of Details of Problem-solving Strategy and Comparer	Total Exp % Exp	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 1 n = 1 100%	N = 0

WC = Whole Class
PS = Pair Share
I = Individual
Exp = Explanations
TF = Teacher Follow-up
NTF 1 = The teacher did not follow up because he/she was not present.
NTF 2 = The teacher was present and engaged, but this was limited to initial invitations to participate.
NTF 3 = The teacher was present, but did not engage with students (just simply observed).

Table 17 (continued)

Student Roles		Phase 4 (I) NTF	Phase 4 (I) TF	Phase 5 (PS) NTF 1	Phase 5 (PS) NTF 2	Phase 5 (PS) NTF 3	Phase 5 (PS) TF	Phase 6 (WC) NTF 2	Phase 6 (WC) TF
Sharer of Details of Problem-solving Strategy	Total Exp % Exp	N = 0	N = 3 n = 3 100%	N = 6 n = 6 100%	N = 1 n = 1 100%	N = 0	N = 4 n = 4 100%	N = 3 n = 3 100%	N = 4 n = 4 100%
Comparer	Total Exp % Exp	N = 0	N = 0	N = 3 n = 1 33%	N = 1 n = 0 0%	N = 1 n = 0 0%	N = 0	N = 4 n = 0 0%	N = 0
Sharer of Details of Problem-solving Strategy and Comparer	Total Exp % Exp	N = 0	N = 0	N = 3 n = 3 100%	N = 3 n = 3 100%	N = 0	N = 0	N = 0	N = 0

WC = Whole Class
PS = Pair Share
I = Individual
Exp = Explanations
TF = Teacher Follow-up
NTF 1 = The teacher did not follow up because he/she was not present.
NTF 2 = The teacher was present and engaged, but this was limited to initial invitations to participate.
NTF 3 = The teacher was present, but did not engage with students (just simply observed).

Classroom B Description and Analysis

Phase 1: Warm-up choral counting during Whole Class. The teacher used this time to have students count chorally by sixes from 6 to 102. She directed one student to identify the counting pattern.

Phase 2: Pair Share discussion on how to know to get from 96 to 102. During the first Pair Share conversation in Classroom B, the teacher communicated to students that they should share their strategies and compare their strategies with their peers by:

a. making a general participation announcement that directed students to share their strategy with their peers;
b. directing individual students to share with their peers while she interacted with them;
c. providing positive evaluations to students who enacted the role of Sharer of Details of Problem-Solving Strategy (SDPSS); and
d. modeling comparing behaviors when she compared the strategies of two students with who she privately interacted.

Prior to transitioning to a new participation structure, the teacher announced how she expected students to engage with each other, specifically wanting them to share their strategies with their peers:

> Let's take a pause break right there. I heard someone go "oh!" because maybe they discovered something in their brain. Okay. <u>I want you to turn</u>—hold on—<u>and talk to your partner</u>—hold on F, I didn't tell you what you're going to talk about yet. I heard everybody seem to be able to count by sixes. <u>I want you to turn</u>—actually let's take a minute and think in your brain what you did to know how to get from this number to this number—oh please don't shout out, think in your brain. <u>How did you know how to get from ninety-six to one hundred two. Think about it right now. In your brain. In your brain. What did you do? In your brain. Okay. Now turn to your partner and share what you did.</u>

The teacher's language of "turn and talk to your partner," "how you know," and "Share what you did" emphasized to students that they needed to enact the role of SDPSS by verbally sharing their strategies with their partners. After the teacher made this announcement, seven conversations with the role of SDPSS were observed; only one conversation included teacher follow-up. In addition to the general announcement she made to students, the teacher interacted with one set of students and promoted the role of SDPSS and Comparer to that specific group:

1. Teacher: [*(approached N, and asked N to move to M and R's table.)*
2. M: [I broke up six into four and two and added the four to ninety-six…
3. T: M, I heard you say something really neat that R didn't get to hear. Can you say it again? You said…(*teacher leaned in to M. N and R gaze at M*)
4. M: I said (*M shifted gaze from board to teacher)* I broke the six from uh four and two, and added the four to ninety-six and I got one hundred because I know that six plus four is ten and then I added the two again and that's one hundred two.
5. T: oh, that was a lot of thinking, but you got it pretty fast. (*R leaned back shifted gaze towards board, then back to teacher)* All that went down in your brain at one time?
6. M: uh-huh
7. T: Wow. (R raised hand and leaned in to teacher) What about you, what did you do? What was your strategy?
8. R: *(M and N shifted gaze toward R.)* Well, I did six to twelve. So I know that four plus six is ten. But four plus two is six, so there is an extra two, so that makes twelve.
9. T: Ah, so kind of (*pointed pencil at M*), kind of like M's but with a different number. So we can see…
10. M: Well twelve in hundreds would be 102, so it makes sense.
11. T: mm-hmm, mm-hmm

In the conversation above, the teacher promoted the role of SDPSS by directly requesting M and R to share their strategies with their partner (lines 3 and 7). In addition, the teacher provided M with positive evaluations, first by referring to his strategy as "really neat," next by commenting on how fast he figured out the strategy, and finally by stating "wow." These evaluative statements encouraged the role of SDPSS among the three students because they provided others with an example of how she expected students to share their strategy with their peers while giving explanations.

In addition, the student's attended to each other's explanations due to the teacher's second request of M. She specifically requested from M to repeat his explanation such that R would have the opportunity to listen. R's body language then indicated that he attended M's explanation, and R used similar details as M when he enacted SDPSS and shared an explanation.

After R shared his strategy, the teacher promoted the role of Comparer when she compared M's strategy to R's (line 9). She commented that their strategies were similar and used her pencil to point back and forth between M and R, but that both students had used different numbers. This comment then modeled to the students how to enact the role Comparer in the classroom: by making a comparative statement and noting differences. The effects of this comment were seen later in Phase 3, when R shared his strategy with the class and stated that he used a similar strategy to M.

Phase 3: Whole Class discussion students share how they knew to get to 102 from 96. During Phase 3, the teacher used similar methods as in Phase 2 to promote the role of Sharer of Details of Problem-Solving Strategy during the Whole Class conversation. She used positive evaluations to describe the conversations she had heard during Pair Share (specifically positive evaluations towards students enacting the role of SDPSS) and she directed students to share their strategies with the class. As a result, eight conversations with the role of Sharer of Details of Problem-Solving Strategy were observed, five with teacher follow-up that involved the teacher requesting additional details. In addition, one student, R, enacted the role of Comparer as a result of the teacher modeling behaviors in Phase 2.

Phase 4: Individual student work: Students solve crane problem. Unlike Classroom A, students in Classroom B were encouraged to work individually on a problem first before talking/sharing it with their partners. The purpose of working individually was so that students could later share strategies with their partners. During this Phase, three conversations with the role of SDPSS were observed because the teacher interacted privately with one student. During this time, the teacher encouraged the student to write down his mathematical strategy so he could share it with his partner in the near future. As such, during this private conversation, the teacher continued to promote the role of SDPSS for this individual student.

Phases 1-4 predominantly mimicked the pattern of Classroom A in that the students predominantly enacted the role of SDPSS because the teacher encouraged the students to do so in the classroom. However, in Phase 5, student participation shifted, and students enacted the role of Comparer just as often as SDPSS. The following description of Phase 5 focuses on explaining why and how students enacted both roles in the classroom.

Phase 5: Pair Share discussion of 19 students folding six cranes—Sharing and Comparing strategies. Phase 5 differed from the other phases in that the role of Comparer and SDPSS often happened together during one conversation. In Classroom B, the role of Comparer promoted the role of SDPSS because the teacher encouraged the students to both share and compare their strategies. Thus, during Phase 5, students enacted the role of Comparer just as often as the role of SDPSS. Close inspection of all the student groups revealed that in some groups, some students enacted both the role of SDPSS and Comparer, and while students in other groups enacted either the role of SDPSS or Comparer. The student was coded for the role he/she enacted in conversation with either peers or peers and teacher. It was possible for a student to have several conversations within one phase, and as a result, students could enact multiple roles within one participation structure. Two aspects of the social environment communicated to students that they needed to enact the role of SDPSS and/or Comparer. These social aspects included:

a. teacher instructions that communicated to students that they were expected to share their strategies with their peers and/or compare their strategies with their peers.

While the students were encouraged to enact the role of SDPSS as in Phase 2 through the teacher's instructions, the teacher bundled additional instructions to encourage students to enact the role of Comparer. These instructions directed students to make suggestions and/or statements of similarities around the strategies given by their peers. As a result, students enacted the roles of SDPSS and Comparer. In the following discussion, I present the teacher instructions and three examples of student conversations to illustrate how the instructions supported the roles of SDPSS and Comparer in student conversations among all student groups.

The teacher used a bundled general announcement during Phase 5 to promote the roles of SDPSS and Comparer in the classroom by encouraging students to make statements of similarities as they shared their problem-solving strategies with their peers. The transcript below depicts the exact language that the teacher used in the announcement:

> Even if you're not done with your second strategy. *You should still be able to <u>talk to your partner and tell them, or your partners, and tell them where you are headed.</u>* So I want you to. Okay. *<u>Share with your partners. Okay or your partner. Your first strategy, one strategy, and then your partner shares one strategy. And then if you got a second one or if you're working on a second one, then you share your second strategy and then they share their second strategy. Okay. But make sure that each one of you shares one strategy at a time. And as a partner, if you see where you can help your neighbor understand something, I want you to go ahead and give suggestions. Say, "I see what you did, let me show you what I did." Maybe you guys have something similar.</u>* Okay? Go ahead.

In this Pair Share announcement, unlike the announcement for Phase 2, the teacher promoted two roles: SDPSS and Comparer. She communicated to the students how they should enact the role of SPDSS by "telling your partner where you are headed," "share your first strategy," and "share your second strategy." In addition, the teacher directed students to "make suggestions to their partners" because they "might have something similar." Finally, the teacher emphasized sharing the details of a student's own strategy when she modeled how she expected them to respond to their peers: "Say I see what you did, let me show you what I did." Thus, while the first set of instructions directed students to share the details of their strategies, the middle part of the instructions directed them to identify whether their strategies were similar to those of their peers. The emphasis in the directions, however, was in sharing the details of each student's own strategy, as the teacher modeled in the instructions.

After the teacher made this announcement, the students enacted the roles of SDPSS and/or Comparer during their conversations in Pair Share. In some groups, during one conversation, one student from the group/pair would enact the role of SDPSS, and the partner/peer would enact the role of Comparer. In other groups, during one conversation, the same student who enacted the role of SDPSS was the student who enacted the role of Comparer. Finally, during one conversation, a student would enact either the role of SDPSS or the role of Comparer by itself. The following examples illustrate how these different scenarios occurred in the classroom. The first example illustrates how one student could enact both the role of Comparer and SDPSS within one conversation; the second illustrates how one student enacted the role of SDPSS and his/her partner enacted the role of Comparer; and the third illustrates how one student solely enacted the role of Comparer. These examples show how the teacher's instructions promoted the roles, and how the role of Comparer did not lead to an explanation while the combination of Comparer and SDPSS did lead to an explanation.

Student Conversation Example 1: One student enacted both the role of SDPSS and Comparer. The following transcript is of the first conversation between E, C, and A. In this conversation, once the teacher delivered her directions, E enacted both the role of Comparer and

SDPSS. Notice how he followed the teacher instructions, specifically giving a suggestion based on having "something similar":

1. E: C! (*looked at his paper, then turned to face A*) What did you get? One hundred fourteen?
2. A: I got two different answers.
3. E: What. Okay, one thing. You could do that. This is what me and C did and we got the same (*pointed to his notebook*).
4. A: What did you guys get?
5. E: one hundred fourteen.
6. A: I got one hundred and nineteen.
7. E: Just do this: nineteen plus nineteen plus nineteen on top, like this (*pointed to his paper*). Then thirty-eight and thirty-eight and thirty-eight—
8. C: Want me to tell you?
9. E: No, she already knows what to do. We got the same right? *(pushed notebook towards C)* Did we get the same?
10. C: (*looked at E's notebook*) Why did you write "same"? *(shifted gaze back to his notebook. A and C went back to writing on their papers quietly while E looked around the classroom*)

In the above transcript, E enacted two roles. First, he enacted the role of Comparer in line 3, when he told A that he and C did the same thing ("this is what we did") and confirmed that they "got the same." While he did not verbalize how he determined whether A's strategy was legitimate, nor did he verbalize how he knew that C and E had the same strategy, he was able to reach these two conclusions. First, in order to reach the conclusion that A's strategy was legitimate, he had to be able to quickly review the process she chose to solve the problem, and understand how to follow the steps she had taken (despite that there was an error) to solve the problem in order to conclude that she should arrive at the same solution as C and E did. Second, he also had to be able to follow C's strategy and identify the similarities between his method and C's method for solving the problem in order to conclude that they did "the same." He did all this intellectual work mentally in order to make these statements to A.

He then enacted the role of SDPSS in line 7, and shared an (incomplete) explanation that detailed his strategy. In lines 1-7, then, E had followed the teacher's directions to both share his strategy and make a suggestion to his neighbor by enacting the roles of Comparer and SDPSS. E continued to follow the teacher instructions, when in line 9, E continued to enact the role of Comparer by asking C to confirm they got the same. In this example, the role of Comparer in conjunction with the role of SDPSS occurred because E was following the teacher's directions to help his neighbor, A. While C did not give an explanation while enacting the role of Comparer, he had to do significant work in order to arrive at the mathematical conclusions he did during this interaction.

In the next example, E and C had a second conversation that was initiated by the arrival of the Teaching Assistant (TA) to the group. During this conversation, the TA reminded the group of the teacher's instructions, which prompted one student to enact the role of SDPSS and his partner/peer to enact the role of Comparer.

Example 2: One student enacted the role of SDPSS and another enacted the role of Comparer.

(*C and A were writing on their papers. The TA then approached the student group*)

1. E: (*slid notebook towards TA*) I got right on both answers.
2. TA: Did you guys all have the right answer?
3. E: No, she got the wrong answer.
4. A: No, I got it! I got it.
5. TA: Okay, talk about how you got your answer and why you got your answer. (*TA walked away*)
6. E: Okay, well first one I did six plus six plus six plus six.
7. C: How many times?
8. E: Nineteen times. Then I did six plus six is twelve. For all of them. Then twelve plus twelve plus twelve plus twelve. And then twenty-four, twenty-four, twenty-four and then an eighteen?
9. C: How'd you get an eighteen?
10. E: Because I did a spare six and I did twelve plus six equals eighteen.
11. C: Yup.
12. E: and then the eighteen. You kind of twenty-four plus twenty-four equals forty-eight. And then ninety-six.
13. C: How did you get ninety-six?
14. E: Forty-eight plus forty-eight.
15. C: Oh, and then you got a ninety-six? Ninety-six plus eighteen?

16. E: And then I got ninety-six plus eighteen would be one hundred and fourteen. And I did a little algorithm to help.
17. C: Okay. <u>That's exactly what I did</u>. But I did it after because I had nothing to do.

The TA's arrival prompted E to initiate another conversation with his peers. In line 6, the TA promoted the teacher's instructions by encouraging them to "talk about how" and "why you got your answer" before she walked away. These directions then appeared to remind E to share his strategy with his peers, as in lines 7-17 he enacted the role of SDPSS and gave an explanation. During this explanation, C asked E detailed questions that prompted E to share several details. These details, then, appeared to provide the information that C needed in order for him to enact the role of Comparer in line 18, where he stated that he did the same strategy. Thus, in this second conversation, it was the teacher's directions, re-verbalized by the TA that, again, promoted the combination role of SDPSS and Comparer among this group.

While C enacted the role of Comparer, and did not provide an explanation, it is important to note the deep intellectual work that C had to do during his interaction with E in order to support E, while E enacted the role of SDPSS such that E's explanation could be thorough enough for C to make a comparative statement at the end. That is, the very specific questions C asked (lines 7, 9, 13, and 15) provide insight into the details that C needed to conclude that he and E used the same strategy. In addition, the same questions are telling of how C listened carefully to E's explanations, by asking for the right details that would help detail out E's explanation. Thus, in this example, while C did not provide an explanation, the role C played was important in supporting E's ability to provide a very detailed and accurate explanation of what he did so that he could enact the role of Comparer at the end of the conversation without having to provide an explanation.

In the previous two examples, it was the teacher's instructions that promoted the combination of the roles of SDPSS and Comparer. These emphasized sharing the details of one's strategy, and as such, students predominantly spent their Pair Share time sharing their own strategies, with comparative statements occurring after at least one student had shared his/her strategy. However, not all student groups enacted a combination of the roles during a conversation. Some student groups enacted either the role of SDPSS or Comparer, but not both. The following example illustrates one such case in which a student enacted solely the role of Comparer.

Example 3: One Student enacted solely the role of Comparer. After the teacher shared the student directions, G engaged in three separate conversations during Pair Share time. First, G and J jointly enacted the role of SDPSS and shared an explanation based on J's work. Then G went back to work on her own strategy (she did not appear to be done). The TA then approached, and G initiated a conversation with the TA and enacted the role of SDPSS, but this time alone and with her own strategy. Some time after the TA left the group, the teacher approached G and J, prompting G to initiate a third conversation, this time with the teacher. In this conversation, G enacted the role of Comparer and did not provide an explanation:

1. *G and J worked quietly in their own notebooks. The teacher approached G and J, and leaned in towards J to look at her notebook*
2. G: We—We checked (*teacher switched gaze towards G*) both of our answers (*teacher switched gaze back to J's notebook, and leaned closer to J*) and <u>we both got the same. In both strategies.</u>
3. J: I'm just finishing my work.
4. T: (*to J*) How did you do this? (*pointed to J's notebook*)
5. J: Huh?
6. T: How did you do this?
7. J gave an explanation. And the teacher requested from J that she write her solution on the board.

In line 1, when the teacher approached, G enacted the role of Comparer when she told her that they “got the same. In both strategies.” It appeared then, that the teacher’s presence reminded G of the instructions to look for something similar, which prompted her to enact the role of Comparer. It is not clear how G determined that the strategies were the same; perhaps it was from her earlier conversation with J or when she looked at J’s notebook, using J’s work to determine this. However, the teacher’s instructions did not include directions to share how or why the strategies were similar, and so G simply stated that the strategies were the same. In addition, the teacher did not follow up with G’s statement, but rather started a separate conversation with J around J’s strategy. As such, G simply made a comparative statement and did not share an explanation.

While it was not clear during this part of the interaction how G determined that the strategies that she and J used were the same, in order for her to arrive at this conclusion, G had to do significant work in order to determine that their strategies were the same. She likely engaged in this work during their earlier interaction, in which J and G discussed the details of one of J’s strategies. During this time, they were observed going over the individual details of J’s strategy, where G then likely was able to decide whether every single step J took was the same as her own. As such, once the teacher arrived, G did not need to review J’s work in order to indicate that they had the “same” as she had already done so previously when she worked through J’s strategy, and observed whether it was like her own.

The teacher’s instructions promoted that the students to enact the roles of SDPSS and Comparer in conjunction with each other. When the roles appeared together in a student conversation, they always led to explanations because the teacher directions emphasized sharing the details of one’s own strategy rather than explaining the comparisons made. While the teacher did direct students to compare, the example in her directions focused on explaining one's own strategy if one had something "similar." In addition, when the teacher and TA interacted with the students, they also emphasized the importance of sharing the details of one's strategy. Comparing entailed only making statements of similarity.

While the instructions from the teacher promoted that students enact the roles of SDPSS and Comparer in conjunction with each other, this did not always occur in all groups. In some groups, the students enacted the roles of SDPSS and Comparer by themselves. While it is unclear why some students chose to enact the role of SDPSS by itself, it is possible that the teacher's emphasis in her directions (on sharing the details), along with the promotion of the role of SDPSS in previous phases, drove students to focus on this role. In fact, in one of the student groups whose members only enacted the role of SDPSS, one student was heard on the recording saying, "We're supposed to explain our strategies."

For those students who enacted the role of Comparer alone, it is unclear why students did so. A possible explanation might be that students relied on visual inspections of their notebooks to make such claims. While they were sometimes observed looking at each other's notebooks, this explanation could be verified had students been observed pointing to their notebooks as they made statements of similarity or difference, and/or if students had been heard referencing each other's work when they made statements of similarity—for example, had a student said something along the lines of "I did it like you, I also added six nineteen times, like you did here (while pointing to notebook)," or "I see you wrote nineteen six times in your notebook like me," or when the teacher approached and said, "See, X and I did the same thing (while showing teacher notebooks)." This type of evidence would suggest that the students relied on visual inspections of their notebooks to make the claims of similarities in their strategies.

Another possible explanation for why students solely enacted the role of Comparer could be that they relied on previous conversations with their peers to make such observations. For example, in Example 3 above, G had spent some time looking at J's strategies for solving the problem, and even helped her check the work. Thus, from that conversation, G might have had enough information to determine that her strategy was the "same" as J's. Evidence that might verify this explanation would be a student referencing previous conversations with peers. For example, had G said, "J and I both shared our strategies with each other and we both did the same thing. We both added 6 nineteen times," then it would be clear that previous conversations with her partner I influenced her statements.

Phase 6: Whole Class sharing of student strategies for crane problem and comparisons. During Phase 6, the teacher communicated to students the goal of comparing the strategies on the board by:

a. announcing that students presenting on the board had similar or different strategies than those sitting at their desks;
b. directly asking students to make comparisons between strategies; and
c. engaging students in voting exercises that directed students to compare their own strategy with one that was on the board.

The teacher promoted the role of Comparer during the Phase 6 Whole Class conversation in these three ways: a combination of announcements, direct requests from students, and voting exercises that elicited the role of Comparer from the students. In addition, the teacher used the same methods from Phase 3 to encourage the role of SDPSS; that is, she directed individual students to enact the role of SDPSS by asking them to share their strategies with the rest of the class. As a result, in Phase 6, six students enacted the role of Sharer of Problem-Solving

Strategy (in which all were explained) and four students enacted the role of Comparer. The examples below detail how the teacher promoted the role of Comparer in the classroom.

> Teacher: Thank you. Okay. Friends are still sharing with each other their strategies. And I have some friends, that are sharing sss—um, putting some up here because they had strategies that were similar to something that you did—that some of you did that we can talk about. And also um that were different that—a strategy that maybe nobody did that I would like to talk about.

In the above announcement, the teacher communicated to the students that they should be prepared to engage in comparing behaviors by specifically stating/warning them that the strategies they would hear from their peers might be similar or different to what they did on their own. This helped to prepare students mentally to engage in the role of Comparer during the rest of the Whole Class discussion.

In addition, as the teacher directed students to share their strategies with the class, she used the time between student presentations of strategies to direct the conversation around comparisons. Specifically, she encouraged the role of Comparer by directly requesting students to compare two strategies that had just been presented on the board:

1. T: Okay, so he [E] got the same answer as A, but did he [E] do it the same way?
2. S1: No!
3. T: What did E have up here?
4. S2: He had the same thing as A.
5. T: What did he do? A had six groups of nineteen, but what did E do?
6. C: Nineteen groups of six.

After A and E had both shared their strategies, the teacher promoted the role of Comparer by asking the class to compare the strategies presented by these two students. Her question of "Did he do it the same way?" limited the kind of answer students could provide (hence no explanation), but did force them to make comparisons between the two strategies in order to determine their similarity or difference. In order for the students to be able to make the comparisons (lines 2 and 4), the students needed to be able to identify how and what made the two strategies used by the students different. A had used the strategy of six groups of nineteen, while E had used the strategy of nineteen groups of 6; and both had been written on the board. The students in Classroom B needed to be able to look at the written work of A and E, and determine that the groupings were different, and that the counters within each group was different in order to determine that the strategies that the students used were different. Thus, despite the fact that the teacher did not encourage the students to provide an explanation, they had to engage in complicated intellectual mathematical work in order to be able to determine that the two strategies written on the board were different.   The teacher then went on to ask a student to explain E's strategy, encouraging the student to enact the role of SDPSS when he explained E's strategy (lines 3-6).

A third way the teacher promoted the role of Comparer during Phase 6 was engaging students in a voting exercise where they compared their own strategies with one on the board, and expressed their comparisons by raising their hands. In the following example, the teacher promoted the role of Comparer by encouraging the students to share whether their strategy was similar to C's:

1. T: Okay, so looking at C's how many of you did a strategy similar to C's?
2. Students raise hands.
3. T: E, R, G. Um, so what C did, E just explained, right? At the board.

In this excerpt, the teacher asked students to engage in the role of Comparer by asking them to compare their own method of solving the problem to what C had written on the board. Specifically, she asked "how many," prompting students to express their comparison by raising their hands (line 2). The teacher further supported these students by calling out the names of those who had a strategy similar to C's in line 3.

While again, the students in this example did not share explanations, they had to do a significant amount of intellectual work in order to determine that their own strategies were similar to C's strategies. For example, the students needed to be able to look at C's and determine whether the method he used (groupings of six) was the same method they used, and whether the details within the grouping coincided with the details in their own grouping. While I do not have information on how each of the students created their mathematical representation (for example, did the students write out 6 nineteen times versus did the students draw 19 circles with six lines in each circle), they had to be able to decipher that the strategy they used was the same despite possibly using different representations.

Summary

In Classroom B, students enacted the roles of SDPSS and Comparer because the teacher structured the social environment to promote these roles. She did this by creating linked cycles of participation structures—(a) Pair Share-Whole Class and (b) Individual-Pair Share-Whole Class—in which the goals of participation were that students create strategies for a problem individually that would later be shared and/or compared with those of their peers. During Phases 1-4, the teacher communicated these goals to the students by making general announcements during each different participation structure that communicated how she expected them to participate, by instructing students to work individually in their own notebooks, by directing students to share their strategies when the teacher interacted with student pairs, and by modeling comparing behaviors after two students shared their strategies. As a result, in Phases 1-4, students predominantly enacted the role of SDPSS and provided explanations in the classroom.

During Phase 5, the teacher provided directions that bundled behaviors she expected from the students during Pair Share time. She directed students to, first and foremost, share the details of their own strategies with their peers, and make suggestions by sharing their own strategy if they had something similar. As such, during Phase 5, students often combined the roles of SDPSS and Comparer during one conversation. When this occurred, students always provided explanations because they were sharing the details of their own strategy as the teacher had encouraged. The example the teacher provided during her Phase 5 instructions simply included a statement of similarity, and as such, when a student solely enacted the role of Comparer during a conversation, an explanation did not occur. Students simply stated whether their strategies were similar. Some students, during Phase 5, solely enacted the role of SDPSS or Comparer alone during a conversation. While it is unclear why the students chose to enact SDPSS alone, it was perhaps the emphasis of the teacher's instructions to focus on the details of one's own strategy plus the promotion of SDPSS in previous phases that led these students by default to enact the role of SDPSS by itself. Those who enacted the role of Comparer by itself did so after the teacher (as in Example 3) or TA approached the student group. The TA sometimes prompted the role of Comparer when she asked the students if they had compared answers, but sometimes the students were not verbally prompted (see Example 3).

During Phase 6, the teacher encouraged the role of Comparer by making a general announcement that identified the strategies written by the students as being similar or different, instructing them to make statements of similarity or difference between strategies on the board, and by engaging them to compare their own strategies through votes during Whole Class. These methods, then, did not lead students to share explanations when they enacted the role of Comparer because students did not receive the opportunities to go beyond making statements of similarity or difference in the classroom.

However, it is important to note that despite the fact that there were not explanations associated with the role of Comparer, the unspoken intellectual work that students had to do in order to engage in the ideas of others was important mathematical work that afforded students to give explanations in future scenarios. For example, in the second example in Phase 5, the student C asked the right questions in order to be able to enact the role of Comparer, and later on in Phase 6, was able to verbalize in a succinct way a description of the strategy that E used (because it was similar to his) and provided an explanation. In this way, the role of Comparer was an important piece in supporting student explanations in the classroom.

In addition, the role of Comparer functioned to allow students to extrapolate the strategies they used so that students could see how their strategies could be used in different ways, and therefore provided yet another way for students to give explanations in the future. For example, during the warm-up activity (Phase 3) a student enacted the role of Comparer in order to identify that he used the same strategy as another peer, but with a different set of numbers. He was able provide an explanation, and explain how the details were different, but at the same time the same (different numbers, same steps) in order to solve the problem. Thus, in Classroom B, the role of Comparer afforded the students of the class to employ intellectual work that allowed them to engage in each other's mathematical ideas.

Chapter 7: Classroom C Narrative

Classroom C Explanations and Role Findings Summary

In Classroom C, the most frequently occurring roles were Sharer of Details of Problem-Solving Strategy (SDPSS), Comparer, Agreer/Disagreer, and Sharer of Details of Mathematical Claim (SDMC). As a reminder, students who enacted the role of SDPSS shared the details of their strategy, whether their own strategy, the strategy of another student, or the strategy of a pair/group. Comparer was enacted when a student stated whether her/his strategy was similar to another student's strategy. Students who enacted the role of Agreer/Disagreer did so by stating agreement or disagreement with another student's strategy. Finally, students who enacted the role of SDMC shared details that supported their stance on a mathematical claim. In addition, this section shows that when the teacher did not follow up with students during Whole Class, students often enacted the roles of Agreer/Disagreer or Comparer without giving explanations in the classroom.

This section also looks at how the teacher designed the social environment to promote the roles of Agreer/Disagreer, Comparer, and SDMC among the students when she did not follow up on student mathematical ideas, and why the roles of Agreer/Disagreer and Comparer did not lead to explanations in the classroom. To explain this, I show that:

- The teacher's goals of student participation in Classroom C were:
    - students were to create their own strategies during individual time;
    - selected students were to write their strategies on the board (and possibly explain them);
    - students were to identify which strategy on the board was most similar to their own (and possibly explain it);

    - students were to state whether they agreed or disagreed with their peers' strategies and/or explanations; and
    - students were to defend their stance on a mathematical claim.
- The teacher communicated to the students the above goals through:
    - general announcements that encouraged students to be prepared to make comparisons in the class;
    - requests from students to share their strategies with the class;
    - directing students to state agreement/disagreement with student explanations;
    - requests from students to nonverbally identify (through pointing or hand raising) which strategies written on the board their own strategies were similar to;
    - directing students to explain the strategies that were similar to their own; and
    - directing students to defend their position on a mathematical claim.

Physical Description of Classroom C. The physical set-up of Classroom C differed from that of Classroom A and B. Rather than the U shape cluster of tables observed in Classrooms A and B, the tables in Classroom C were arranged in a grid-like fashion (4 rows and 3 columns) that faced forward towards the white board. Each table held 2 students. Posted above the white board were five large white poster boards with black writing on them that had what appeared to be student expectations written in paragraph form:

> "Room C is a good learning community. Students have a positive and ( ) attitude. They make good decisions and are active participants...help each other…[don't speak] out of turn. All in all, they are...complete their work…"

In this classroom document, the phrases "good learning community," "active participant," "help each other, and "complete their work" all communicate to students how they are expected to behave in the classroom in a public way. From just looking at these posters, we don't know what these phrases mean, but in the rest of the chapter, I will show that students were expected to engage in each other's ideas by enacting a variety of roles that afforded many students the ability to provide explanations in the classroom.

Student participation structures. Classroom C had only two participation structures: Individual and Whole Class time. Unlike the other classrooms, the students in Classroom C did not participate in Pair Share time. Typically, these participation structures occurred in the classroom when the teacher presented a new problem to students that was to be worked on individually by the student. During this time, students worked on their own papers, and the teacher walked around and checked in on students. The teacher then selected which students should write their strategies on the board. Afterward, she transitioned students to Whole Class, where she led the discussion by having students explain either their strategies, another student's strategy, or their alignment with another student's mathematical idea. During the Whole Class discussion, students were also asked to state agreement/disagreement with another's ideas or to compare their strategies with those on the board. For each new problem, the teacher cycled through these two participation structures the same way each time. Because all of the observed student participation in Classroom C occurred during the Whole Class participation structure, the teacher was always present during discussions and engaged with the students. As such, no teacher follow-up means that the teacher was present and engaged with the students, but this engagement was limited to initial invitations to the students to participate.

Materials used in Classroom C. Because Classroom C had only two participation structures, conversations during Whole Class typically centered around the class whiteboard, and the work that was presented there. The teacher often selected students ahead of Whole Class discussions to present work on the board, but as different students engaged in the mathematical ideas presented, these students would come up to the board and used the work on the board to mediate the discussions they had. In addition, the board served as a symbol of authority in the classroom. Students who often wanted to share their thoughts (regardless of who's strategy it was) often came to the board to direct their verbalizations to the rest of the class. In the analytical narrative that follows, I will demonstrate how the white board played an important function in facilitating the various student roles that students took on in the classroom.

Classroom C timeline of participation structures. Table 18 depicts how the time was spent in Classroom C over two days. It depicts the cycling of participation structures, the problem(s) that the students engaged in during each cycle of participation structures, and the length of time spent in that participation structure and on that problem. This timeline guides how the enactment of student roles unfolded in the classroom.

Tables 19 and 20 show how often the roles of SDPSS, Agreer/Disagreer, Comparer, and SDMC were observed in each phase. Each cell represents the number of times the role occurred within each phase, whether the role involved an explanation, and whether the teacher followed up on the students' mathematical thoughts when they enacted that role during a conversation

Table 18

Chronological Timeline for Participation Structures on Days 1 and 2 in Classroom C

Day 1 (56 minutes 19 seconds total)	Day 2 (64 minutes 35 seconds total)

• Phase 1 Whole Class (1 min 23 s)
  ○ Counting Backwards by thirds from 4
• Phase 2 Individual Time (3 min 24 s)
  ○ Counting Backwards by thirds from 4
• Phase 3 Whole Class (17 min 38 s)
  ○ Write out counting by thirds from 4 and circle wholes
  ○ Does Zero count as a whole?
• Phase 4 Individual (9 min 13 s)
  ○ Draw and label 5/3 liter of Soda
• Phase 5 Whole Class (13 min 49s)
  ○ Draw and Label 5/3 Liter of Soda
• Phase 6 Individual (6 min 41 s)
  ○ Draw and label 5 parts of ¼ pizza
• Phase 7 Whole Class (4 min 52 s)
  ○ Draw and label 5 parts of ¼ pizza
	• Phase 1 Whole Class (3 min 35 s)
	  ○ Housekeeping
	• Phase 2 Individual (3 min 40 s)
	  ○ Count by 3s up to 48 and guess how many threes you will be able to take out of 48 (write it out)
	• Phase 3 Whole Class (4 min 21 s)
	  ○ Count by 3s up to 48 (out loud) and guess how many threes you will be able to take out of 48
	• Phase 4 Individual (6 min 35 s)
	  ○ 12 pears. Each person gets 3 pairs. How many people get pears? (Draw and use the special bracket)
	• Phase 5 Whole Class (35 min 10 s)
	  ○ 12 pears. Each person gets 3 pairs. How many people get pairs?
	• Phase 6 Individual (7min 3s)
	• Write three equations that show what this problem is about
	• Phase 7 Whole Class (4 min 11 s)

Table 19

Classroom C Explanations and Student Roles by Phase on Day 1

Student Roles		Phase 1 (Whole Class)		Phase 2 (Individual)		Phase 3 (Whole Class)		Phase 4 (Individual)		Phase 5 (Whole Class)		Phase 6 (Individual)		Phase 7 (Whole Class)	
		TF	NTF	TF	NTF	TF	NTF	TF	NTF	TF	NTF	TF	NTF	TF	NTF
Sharer of Details of Problem-Solving Strategy	Total Exp % Exp	N = 0	N = 1 n = 1 100%	N = 0	N = 0	N = 2 n = 2 100%	N = 2 n = 2 100%	N = 0	N = 0	N =3 n = 3 100%	N = 1 n = 1 100%	N = 0	N = 0	N = 3 n = 3 100%	N = 0
Sharer of Details of Mathe-matical Claim	Total Exp % Exp	N = 0	N = 0	N = 0	N = 0	N = 7 n = 6 86%	N = 8 n = 8 100%	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0
Agreer/ Disagreer	Total Exp % Exp	N = 0	N = 2 n = 0 0%	N = 0	N = 0	N = 0	N = 3 n = 0 0%	N = 0	N = 0	N = 1 n = 0 0%	N = 6 n = 0 0%	N = 0	N = 0	N = 0	N = 0
Comparer	Total Exp % Exp	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 6 n = 0 0%	N = 0	N = 0	N = 0	N = 5 n = 0 0%

TF = Teacher Follow-up
NTF = No Teacher Follow-up[11]
Exp = Number of Explanations
% Exp = Percentage of Explanations

[11]In Classroom C, No Teacher Follow-up means the teacher was present and engaged with students, but this engagement was limited to initial invitations to participate.

Table 20

Classroom C Explanations and Student Roles by Phase on Day 2

Student Roles		Phase 2 (Individual)		Phase 3 (Whole Class)		Phase 5 (Whole Class)		Phase 6 (Individual)		Phase 7 (Whole Class)	
		TF	NTF	TF	NTF	TF	NTF	TF	NTF	TF	NTF
Sharer of Details of Problem-Solving Strategy	Total	N = 0	N = 0	N = 1	N = 2	N = 4	N = 10	N = 0	N = 0	N = 0	N = 1
	Exp			n = 1	n = 2	n = 4	n = 10				n = 1
	% Exp			100%	100%	100%	100%				100%
Sharer of Details of Mathematical Claim	Total	N = 0	N = 0	N = 0	N = 0	N = 2	N = 5	N = 0	N = 0	N = 0	N = 0
	Exp					n = 2	n = 5				
	% Exp					100%	100%				
Agreer/Disagreer	Total	N = 0	N = 0	N = 0	N = 1	N = 0	N = 5	N = 0	N = 0	N = 0	N = 0
	Exp				n = 0		n = 0				
	% Exp				0%		0%				
Comparer	Total	N = 0	N = 0	N = 0	N = 1	N = 0	N = 5	N = 0	N = 0	N = 0	N = 0
	Exp				n = 0		n = 2				
	% Exp				0%		40%				

TF = Teacher Follow-up
NTF = No Teacher Follow-up[12]
Exp = Number of Explanations
% Exp = Percentage of Explanations

[12]In Classroom C, No Teacher Follow-up means the teacher was present and engaged with students, but this engagement was limited to initial invitations to participate.

between the student(s) and the teacher. Tables 19 and 20 show that on both days, the frequent observation of the role of Agreer/Disagreer when the teacher did not follow up was due to Phase 5 of each day, respectively. In addition, the observation of the role of Comparer when the teacher did not follow up was due to Phases 5 and 7 on Day 1 and Phase 5 on Day 2. Finally, the observed frequency of the role of SDMC when the teacher did not follow up was due to Phase 3 on Day 1 and Phase 5 on Day 2. The following discussion provides an analytical narrative that explains and illustrates the typical ways in which the role played out in these phases based on how the teacher communicated to the students what roles they were supposed to enact.

Analysis of Social Dimensions That Promoted Classroom Roles

Day 1, Phases 1-2: Counting backwards by thirds and promoting SDPSS and Agreer/Disagreer roles. During Phase 1 on Day 1, the teacher communicated her goals of creating and sharing a strategy and agreeing/disagreeing with another student's strategy when she:

a. requested an individual student to share how she would solve the problem with her peers, and
b. directed students to express agreement/disagreement with S's solution.

As a result, one student, S, enacted the role of SDPSS and gave an explanation (with no teacher follow-up). In addition, because the teacher directed the students to express agreement/ disagreement, two students enacted the role of Agreer/Disagreer but did not provide explanations (with no teacher follow-up). The example below illustrates how the teacher promoted the role of SDPSS by inviting one student to share how she would do the problem. It also illustrates how she directed students to enact the role of Agreer/Disagreer after the first student enacted the role of SDPSS.

Example 1: Count Backwards from 4 by thirds. How do you write 4 wholes in thirds?

1. T: …from four by thirds. So you are going to start at four (*wrote 4 on the board*), right? (*2.0s*) and you are going to count backwards from four. So S, what do you think that you are going to start with? (*placed index finger on 4, gazed towards S*)
2. S: So with four wholes in thirds.
3. <u>T: So, how would we write that?</u>
4. S: um
5. T: This is tricky.
6. S: Twelve thirds.
7. N: ooh! Three. Two Thirds!
8. <u>T: Why 12 thirds? Do you agree?</u>
9. Student 1: No.
10. Student 2: Um. Well, twelve thirds….
11. Student 3: I don't know.
12. T: <u>S, why did you say twelve thirds?</u> (*wrote 12/3 on the board*)
13. S: Um because 4, uh. Because 3 times 4 is 12. And um yeah, that's (*inaudible)* three times four.
14. T: N?
15. S: I know that three times four is twelve.
16. N: Um, I think that it'd be three and two thirds.
17. T: So you also could do it as…
18. N: Yes! And then…
19. T: Well, no we're saying for four.
20. N: or twelve thirds.
21. <u>T: So you also agree with S?</u>
22. N: Yeah.
23. T: Okay, so you are either going to do it. You can do it N's way. Which is, the next number would be what?
24. N: Three two thirds.
25. T: Or you can do it S's way. Or you can do it both. Okay? (*teacher walked away from board and students worked quietly.*)

After stating the problem of counting backwards from four by thirds to the class, the teacher invited one student, S, to share how she would start the problem. Once she responded, the teacher encouraged S to share the details of her strategy with her peers by asking: (a) how the first number should be written (line 3), and (2) why four should be written as twelve thirds (lines 8 and 12). With this request, the teacher encouraged S to enact the role of SDPSS and provide details of her strategy that would result in an explanation (lines 13 and 15). While the teacher was interacting with S, N tried to participate in the conversation. While the camera was not on N while he spoke, it appeared he knew the teacher was not interacting with him directly, but his excitement for participating in math class could not be contained. Line 7 then depicts N's excitement to participate and interact with the teacher around the mathematical problem, despite the fact that she had made it clear that she was interacting with another student (by calling on that student directly).

In addition, the teacher directed the students to enact the role of Agreer/Disagreer twice during this conversation, first when she followed up her initial request for an explanation with a request for agreement from the class (line 8), and second when she directly requested from N whether he agreed with S's statements. After both conversations, both N and another student enacted the role of Agreer/Disagreer by verbally stating their agreement (N) or disagreement (Student 1) with S. The students did not explain why they agreed or disagreed with S, nor did the teacher ask them to explain. However, N had to do some intellectual work in order to arrive at the same conclusion as the teacher: that he agreed with S. In line 16, N had already moved onto the next number in the series, when the teacher asked him to go back to the original question. While he was trying to do the counting for the class, he also needed to attend to S's answer and explanation in order to know that he agreed with her statements. Thus, in this example, although we do not see N share an explanation as to why he agrees with S, we see that he is able to conclude that he agrees with S's answer (and therefore had to pay attention to what S was saying when she interacted with the teacher).

After the teacher conversation with S and N, the students shifted into the Individual participation structure during Phase 2. While the teacher did not direct the students to work individually, based on what the students did, it appeared that it was the norm for them to work individually on their own papers when first introduced to the problem. As students worked individually, the teacher walked around and selected which students would write their work on the board.

Phase 3: Whole Class discussion counting backwards by thirds and does zero count as a whole? Phase 3 differed from the other phases in the class because students enacted the role of SDMC. SDMC was enacted when the students shared mathematical details in support of their mathematical claim. In Classroom C during Phase 3, the teacher encouraged her goal of defending one's mathematical claim by:

a. directly asking students to defend their claim when they expressed it, and
b. asking students to respond to each other.

As a result, the students enacted the role of SDMC 15 times, both when the teacher followed up and when she did not follow up, and the majority of these led to explanations. The following example shows how the teacher moderated the discussion and supported the students in enacting the role of SDMC in the classroom.

Phase 3 began with students counting out loud backwards from four by thirds. When the students reached zero thirds, a student posed the question of whether zero was a whole number. The teacher turned the question back to the class. The example below illustrates how the teacher encouraged the students to enact the role of SDMC while providing explanations.

Example 2: Is zero thirds a whole number?

1. T: S (*pointed and nodded to S*).
2. S: Um, um. B, B was like, um, and me and C and um, B were arguing about we're like—
3. C: —B said that one whole
4. S: Oh. I wanted to actually kind of (*stood up and walked to the front of the classroom*) come up and say something.
5. S: Um, B said that zero thirds is a whole. (*turned to face B*) B, it's not. It's not a whole.
6. <u>T: Can you give him any evidence of that?</u>

7. S: Well, it's not a whole because it's not, none of the, like part of the, like yeah, yeah (*grabbed marker and turned to white board and began to draw a vertical rectangle below three thirds*) like for three thirds (*drew three partitions in the rectangle*).
8. C: Three thirds of one whole. The whole is shaded in.
9. S: And all of it is shaded in (*shaded in the three pieces*). But nothing is shaded in this one (*pointed to rectangular drawn by C and pointed to it*).
10. C: Yeah.
11. T: But S, what is this? Three parts of what? (*Pointed to rectangular drawing that S put on board*) Let's make this a real thing. It's easier to talk about.
12. S: um—three pieces of choco—a brownie.
13. T: So this is candy bar—oh okay a tray of brownies. So you're saying these three pieces, this is a whole. A whole Brownie that's been what?
14. S: Cut into three pieces.
15. <u>T: Okay, B, do you have something to say?</u>
16. B: Yeah.
17. T: I thought so. (B went up to the board and enacted the role SDMC after stating his position)

S shared her claim in lines 2-5 that she did not think zero was a whole. In order to assert her stance, S made her way to the white board and spoke directly to B from this position. This physical move by S was important because of the authority that the white board traditionally gives teachers. The whiteboard in Classroom C (and often in other classrooms) represented the authority of the classroom because when someone lead a discussion from the whiteboard, the orientation of the desks/tables naturally forces the attention of everyone else in the classroom on the speaker at the whiteboard. However, this power move by S was not sufficient for S to make her case in the view of the teacher. As a result, for the rest of the conversation, the teacher guided the students participating in this interaction to engage in each other's ideas.

The teacher in line 6 asked S to defend her claim that zero was not a whole number by asking her to provide B with "evidence." This prompted S to enact SDMC in lines 7-9, when she drew a picture to help her provide the evidence that the teacher requested for her claim that zero was not a whole number. S then used the whiteboard as a tool to help her provide the details in the explanation that was requested by the teacher. The teacher followed up with questions (lines

11 and 13) that would help other students in the class understand S's justification and explanation. Finally, after S had finished, the teacher in line 15 asked B to respond to S's comments by asking B if he had "something to say." This then prompted B to go to the board (again as a way to establish his authority on the subject) and share his stance that zero was a whole number and defend it while providing an explanation.

In addition, another student C, demonstrated intense emotional interest in stating his position on the problem "is zero a whole number," as he continuously tried to participate in the interaction between S and the teacher. C frequently interjected S's explanation with additional detail about why he (and S) argued with C in regards to zero being a whole number (lines 3, 8, 10). In this way, C supported S's argument (S also used some of C's work to give her explanation).

Finally, an interesting aspect of this interaction is that the teacher did not provide either of the students with any indication that either of them are right. The teacher's role in this discussion between students appeared to be solely to moderate the conversation, and to make sure that the students provided each other with information in order to create mathematical arguments. The lack of evaluative comments was very different from all of the other classrooms, which had less diverse enactment of student roles than Classroom C.

Phase 3 differed from the other phases in Classroom C because the teacher encouraged students to argue over a mathematical idea (whether zero was a whole) that was posed by one of the students. She did so by encouraging students to respond to each other, take a position on a mathematical claim, and defend it by providing evidence to their peers for why they took their position. This then led to students providing explanations while they justified their stance on whether zero was a whole number.

Phase 4: Individual participation: Draw and label 5/3 liters of soda. During Phase 4, the teacher introduced a new problem: "Draw and label 5/3 liters of a soda." Like Phase 2, students worked quietly and on their own, while the teacher walked around and selected which students would write their work on the board.

Phase 5: Whole Class discussion of draw and label 5/3 liters of soda—Encouraging the roles of Agreer/Disagreer and Comparer in the classroom. During Phase 5, the teacher communicated her student participation goals of (a) identifying which strategy was similar to their own and explaining it, and (b) stating whether students agreed or disagreed with another student's strategy by:

a. making a general announcement that directed students to be prepared to explain the strategy that was similar to their own;
b. directly asking students which strategy written on the board was similar to their own;
c. directing students to identify nonverbally which strategy on the board they agreed with;
d. requesting that students explain another student's strategy; and
e. making a general announcement that directed students to be prepared to state agreement/disagreement.

As a result of these announcements, instructions, and requests, students were observed to enact the role of Comparer six times, and did not lead to explanations when the teacher did not follow up; the role of Agreer/Disagreer was also observed to occur six times and did not lead to explanations when the teacher did not follow up. In addition, the role of SDPSS was enacted by three students, all of which resulted in explanations with no teacher follow-up. The excerpts below show how the teacher promoted the role of Comparer, encouraged students to enact

SDPSS around someone else's strategy, and promoted the role of Agreer/Disagreer during the Whole Class setting.

The teacher first encouraged the roles of SDPSS and Comparer by making a general announcement that encouraged students to identify which strategy on the board they could explain. Her exact language was as follows:

> You need to look and see which one, if you can explain CM's. Or if you can explain D's, or if you have something different. So be ready to explain, and help us to understand what exactly this is saying.

The above announcement accomplished two aims: (a) it encouraged students to enact the role of Comparer by having the students identify which of the strategies on the board was similar to their own; and (b) it promoted the role of SDPSS by encouraging students to be prepared to provide an explanation. This announcement then communicated to the students that in the near future, they might be selected to enact the role of SDPSS and give an explanation. Thus, during that time, students needed to enact the role of Comparer by identifying which strategy on the board was similar to their own. There were no immediate observations of students enacting either of these roles, but this announcement supported these roles during the rest of Phase 5.

As the students finished writing their solutions on the board, the teacher walked around and checked in with them individually. As she did, she continued to promote the role of Comparer when she directly asked individual students to state which solution was similar to their own, as shown in the example below:

Example 3: Promoting Comparer with drawings of 5/3 liters of soda

1. <u>T: Is yours the same as CM's?</u>
2. S1: (inaudible)
3. Teacher walked to student sitting next to S1.
4. <u>T: So, is yours looking more like CM's or D's?</u>
5. S2: D's.
6. T walked to the front of the classroom.

In lines 1 and 4, the teacher specifically asked students S1 and S2 to state which strategy was like their own. While we could not hear S1's responses, given that the teacher appeared satisfied with the inaudible response, it appeared that S1 enacted the role of Comparer in the classroom at this time. As a result, students S1 and S2 both enacted the role of Comparer when they stated whose strategy was most similar to their own. However, neither provided an explanation for how or why their strategy was similar to the one they identified as similar to their own, nor did the teacher follow up to ask them why their strategy was similar to CM's or D's strategy.

However, despite the fact that both S1 and S2 did not articulate an explanation, it is important to recognize the mental work they both had to do in order to arrive at the conclusion that their strategy was the same as one of the strategies that was written on the board. While the problem here was different from the problems in Classroom B, children's representational drawings could still differ, but be the same. For example, the way the students chose to draw the representation of a soda bottles might differ, and so students would need to be able to determine whether the partitions and shapes were the same mathematically in order to determine which representation was like their own.

After the teacher returned to the front of the room, she asked D to explain his strategy. This student enacted the role of SDPSS and shared an explanation with the class. The teacher then promoted the role of Agreer/Disagreer and the role of Comparer after D shared his strategy with the class. The example below illustrates how the teacher asked the students to: (a) enact the role of Comparer nonverbally, (b) enact the role of SDPSS by explaining someone else's strategy, and (c) enact the role of Agreer/Disagreer by announcing to one student that he needed to be prepared to agree or disagree.

Example 4: Non-verbal Comparer and SDPSS for drawing 5/3 liters of soda

1. T: (*nodded*) all the pieces. Do you agree with him [D]? He's labeling all the things that we have been learning about in fractions.
2. S1: No!
3. S2: I have something different! Ms. C, I have something different.
4. <u>T: You have a different picture? Let's first—point to—If you have a picture that looks like one of the pictures up here, will you point to it? Thanks D, have a seat. All right, so we need someone—C, you want to give it a try? CM's?</u>
5. C: I think that.
6. T: Wait, honey, come up here. <u>CE, are you paying attention? You want to see if you agree with her.</u>

In lines 1-3, the teacher used the same methods as in Phase 1 to promote the role of Agreer/ Disagreer. The students S1 and S2 appear to display signs of emotional investment, as they both spoke without direct invitation from the teacher and enthusiastically shared their mathematical thoughts of D's representation. In line 4, the teacher then promoted the role of Comparer by asking students to identify which strategy on the board they agreed with nonverbally. She did so by directing students to point to the strategy on the board that looked like their own. While the video did not show students pointing to the board (as it is focused on the teacher), the teacher then selected a student to explain CM's strategy—presumably from one of the students who had pointed to CM's strategy as written on the board.

While the students who enacted the roles Comparer and Agreer/Disagreer did not give explanations, the students still had to do significant intellectual work in order to come to the conclusion that they agreed/disagreed with the drawings on the board or that their representations were like one of the ones on the board. They needed to determine whether things were the same regardless of whether the shape they used might have been different, or even if they used equivalent (but different) partitions. And while they might not have verbalized why they disagreed, the students still needed to identify what they disagreed with in order to come to the conclusion that they disagreed. As such, despite the fact that the students did not verbalize explanations when they enacted these roles, the mathematical work they did in order to arrive to these conclusions was important work that leads to argumentation skills needed for more complex mathematical problems.

In line 4, the teacher encouraged the role of SDPSS by asking C to try to explain CM's strategy (which she presumably identified as similar to her own). Finally, the teacher supported the role of Agreer/Disagreer again by directing CE to determine whether he agreed/disagreed with C's explanation of CM's strategy. C then enacted the role of SDPSS in the classroom and provided an explanation. After C's explanation, CE enacted the role of Agreer/Disageer and stated that he did not agree with her explanation of CM's strategy.

Phases 6-7: Draw and label five parts of ¼ slices of pizza—Repetition of Phases 4 and 5. During Phase 6, the teacher introduced the problem and directed students to work on it individually at their desks. Then, the teacher used the same techniques as in Phase 5 to encourage the roles of SDPSS and Comparer in the classroom in Phase 7. Specifically, the teacher made a general announcement that directed students to be prepared to identify which strategy on the board was similar to their own and to point to which strategy was most similar to their own, she also selected students to enact the role of SDPSS by requesting them to explain the strategies they had identified as similar to their own.

Day 2, Phases 1-2: Housekeeping, counting by 3s up to 48 during individual time. Day 2 began with some housekeeping. The teacher and students discussed how long homework should take and also what to do if they had a hard time. The teacher then promoted the role of Comparer by:

a. making a general announcement told students that they were going to be comparing during the lesson.

While students did not enact any roles during Phases 1 and 2, this general announcement from the teacher at the beginning of the lesson provided the students with information about how she would expect them to participate throughout the lesson. The exact language she used was:

> So, you remember when Mrs. G was here before, and she taught us about apples. And we all knew the answer, didn't we. That's going to happen again in today's lesson. Today's lesson is going to be, I'll write the problem, <u>we'll work on it, then its rules and comparing. Today is all about comparing.</u> This lesson and that lesson. So today's question that everyone is going to write on the paper that you put the date on—the date on the upper left-hand corner—and now I am is going to write the question.

In the general announcement above, the teacher provided the students with directives of what actions and behaviors she expected them to engage in during the lesson of the day. Specifically, she directed them to "work on the problem" (after she wrote it down) and "then it's rules and comparing." She emphasized that the day's lesson was "all about comparing." This message then promoted the role of Comparer to the students in advance of the activities they would be doing that day. The teacher presented the problem of "counting by 3s up to 48." Students proceeded to work quietly on their own during Individual time.

Phase 3: Choral counting and repeat of methods to solicit SDPSS and Agreer/ Disagreer in the classroom. Students counted chorally from 3 to 48. The teacher then used the same methods as in Day 1, Phase 1 to promote the role of SDPSS; that is, she requested students to share how they figured out how many 3s were in 48. As such, three students enacted the role of SDPSS and provided explanations for how they knew how many 3s were in 48. In addition, the teacher used the same methods as in Day 1, Phase 1 to promote the role of Agreer/Disagreer; that is, she asked the class in general if they agreed/disagreed with one of the students who had enacted the role of SDPSS. As a result, one student enacted the role of Agreer/Disagreer without providing an explanation in the classroom. Finally, one student enacted the role of Comparer without any teacher prompting. This student was likely influenced by the general announcement made by the teacher earlier in the day. This student also did not provide an explanation.

Phases 4 and 5: Introduction of pear problem—Repetition of Day 1 techniques for the promotion of SDMC, SDPSS, Agreer/Disagreer, and Comparer roles. During Phase 4, the teacher introduced the problem, "There are twelve pears, each person gets three pears. How many people get pears?" Students then worked on the problem individually at their desks while the teacher walked around.

During Phase 5 of Day 2, the teacher used the same methods that she used throughout Day 1 to promote the roles of SDPSS, Agreer/Disageer, Comparer, and SDMC in the classroom. She began Phase 5 with a general announcement reminding the students that the goal was to engage in comparing during the class. The teacher promoted the role of SDPSS during Phase 5 by directing students to approach the board and share an explanation of their strategy, similar to Day 1, Phase 1. To promote the role of Agreer/Disagreer, the teacher used the technique of directly asking the class and/or single students if they agreed with a strategy one of their peers had shared (as in Day 1, Phase 1). Similarly, the teacher promoted the role of Comparer by directly asking individual students if they had something similar to their peers (as in Phase 5 of Day 1). Finally, the teacher promoted the role of SDMC by directing students to respond to each other and share details in support of a mathematical claim about the correct equation for a group of three stacked pears. As a result, students predominantly enacted the role of SDPSS when the teacher did not follow up (10 observed episodes of the SDPSS) on student conversations and provided explanations. The roles of Agreer/Disagreer (with no explanations), Comparer (with very little explanation), and SDMC (with explanations) each resulted in five observed instances of each role during the conversations that occurred in Phase 5 of Day 2.

Phase 6, Individual time: Write three labeled equations. During Phase 6, students worked individually to write three labeled equations for the pear problem. Students worked quietly while the teacher walked around and selected students to write their equations on the board. No student roles were observed during this time.

Phase 7: Three labeled equations written on the board. While several students wrote their labeled equations on the board during Phase 6, only one student enacted the role of SDPSS with an explanation. This was due to the teacher spending some time explaining what she meant by "label" and also the class running out of time. The teacher encouraged this one student to enact the role of SDPSS by directing him to share and explain only one of his labeled equations with the class (as in Day 1, Phase 1).

Summary

Classroom C students predominantly enacted four different roles in the classroom: SDPSS, SDMC, Agreer/Disagreer, and Comparer. In Classroom C, both SDPSS and SDMC always led to explanations, but the role of Comparer hardly led to explanations and the role of Agreer/Disagreer never led to explanations. The role SDPSS always led to explanations because the teacher directed students to share with their peers their strategies for solving the problem during the Whole Class sessions. The teacher specifically used such language as "why" when instructing students that encouraged them to give detailed explanations when sharing how they solved the problems.

The role of SDMC also always led to explanations, regardless of whether the teacher followed up or not. The teacher promoted this role by directing students to respond to each other around a mathematical claim and support their position on a claim with detailed mathematical thoughts. When students were encouraged by the teacher to provide their peers with evidence on their position around a mathematical claim (such as zero is/is not a whole number), they were more apt to provide an explanation because they had to justify and explain their particular view in order to convince their peers of their position.

The role of Agreer/Disagreer never led to explanations because students were also not encouraged to go beyond stating whether they agreed or disagreed with another student's strategy. The teacher would often ask the students to state their agreement as a general question to the entire class after a student gave an explanation, and several students would exclaim whether they agreed or not with the explanation given. The teacher would also encourage the students to enact this role nonverbally, and have them raise their hands if they agreed with a student's explanation. Finally, the role was also promoted through general announcements that prepared students to state agreement or disagreement with a student's explanation of a strategy. As such, the teacher never promoted the students to explain why they agreed or disagreed with another student's strategy explanation.

The role of Comparer rarely led to explanations because the teacher mostly encouraged the students to enact this role in order to select who would explain their strategies. She encouraged this role through general announcements and directions to the class as a whole and to individual students, allowing them to identify which strategies were similar to their own. However, the teacher never encouraged students to explain how or why their strategies were similar to others. She simply would select a student who identified with a particular strategy and have him or her explain that strategy.

Classroom C differed from the other classrooms in that both the roles of Comparer and Agreer/Disagreer were promoted in the classroom when the teacher did not follow up. However, as in Classrooms B and D, these roles hardly ever led to explanations. Classroom C also differed from the other classrooms in that the role of SDMC was uniquely observed there and it always led to explanations, regardless of whether the teacher followed up or not. SDMC was uniquely observed in Classroom C because the teacher encouraged students to discuss mathematical questions that differed from solving a problem. Finally, as with the other classrooms, the role of SDPSS was observed to occur in Classroom C and always led to explanations, regardless of teacher follow-up.

While it was rare for the roles of Agreer/Disagreer and Comparer to result in explanations in Classroom C, it is interesting to note that students enacted these roles just as often as the roles of SDPSS and SDMC. The diverse roles that students enacted in Classroom C then, really allowed students to engage in each other's ideas in the classroom because these roles pushed students to think about each other's mathematical ideas, and also to think about their own mathematical ideas in relation to that of others. The students then had to identify details that made their strategies or representations similar or different from those of others. In addition, students had to have reasons to agree or disagree with other students' mathematical ideas, and while they did not verbalize them, they still had to be able to think about what details they disagreed or agreed with.

Another interesting note about Classroom C is how emotionally invested the students were in the mathematical work they did. This was evident throughout the both of the observed periods, but in particular the discussion around zero as a whole number, student investment stood out. While the teacher moderated the conversation, and ensured that all students who wanted to speak had an opportunity to speak, she did not herself engage with students and their mathematical ideas much during this time. Rather, the teacher during this conversation helped students engage in thoughtful and detailed mathematical arguments with each other. The different roles that the teacher encouraged students to take on seemed to allow them to really invest themselves emotionally in the mathematical discussions at hand. That is, because students were afforded the ability to agree/disagree with their peers and also to compare their own mathematical ideas with those of others, this allowed students to become emotionally invested, and thus passionate about the mathematical work they did in the classroom. This emotional involvement then has the potential to create positive dispositions about mathematics in general, and can allow the students to desire to participate frequently in mathematics classrooms (Gresalfi & Cobb, 2006).

Chapter 8: Classroom D Narrative

Classroom D Explanation and Role Findings Summary

The two most frequent roles in Classroom D were Sharer of Details of Problem-Solving Strategy (SDPSS) and Agreer/Disagreer. The role of Agreer/Disagreer was enacted by students when they expressed agreement or disagreement with another student's strategy. When the teacher did not follow up on student participation, students were more apt to take on the role of Agreer/Disagreer rather than any other role in the classroom. This role was rarely associated with students giving explanations.

This section explores the classroom social factors that promoted the role of Agreer/ Disagreer among the students of Classroom D, and illustrates how the Agreer/Disagreer role rarely produced explanations—specifically, why students enacted the role of Agreer/Disagreer when the teacher did not interact with them to follow up on their suggestions. I look at three different social factors of the classroom (goals of student participation, the ways the teacher communicated these goals, and the tools the students used in the classroom) to understand how the role of Agreer/Disagreer was promoted in this classroom. Specifically, I will show that:

- The goal of student participation in Classroom D was that those students needed to be able to create a representation of a problem by themselves, then decide if they agreed or disagreed with the solutions presented on the board by their peers. That is, it was expected that a student would compare her/his written solution with what was written on the board (essentially enact the role of Agreer/Disagreer) in order to determine and express agreement with another student's strategy.
- The teacher communicated the above goals to students via announcements, instructions, voting activities, and validation statements that encouraged students to

express agreement or disagreement with another student's strategy in the classroom. The given announcements, instructions, and voting activities always directly communicated to students that he wanted them to agree or disagree with written mathematical work on the board. The teacher also encouraged students to express agreement/disagreement by making validating statements that communicated to students that these behaviors were acceptable. The teacher rarely encouraged students to go beyond expressing agreement/disagreement to give an explanation.

Physical Description of Classroom. In some ways, Classroom D was setup in a way similar to Classroom C. The tables all faced forward towards the chalk board, and the tables were arranged into three rows and two columns. Each table held four students, and the students worked in pairs. The physical set up of the classroom allowed students to do their work individually, to have quick conversations with their partners during the few times that pair share occurred, and also to be able to view the chalkboard, which played a crucial role for Whole Class discussions. In the following sections, I will show how this set-up afforded the students the opportunity to enact the roles of Sharer of Details of Problem Solving Strategy and Agreer/Disagreer.

Student participation structures. There were three participation structures in Classroom D: Individual, Pair Share, and Whole Class. The three structures typically functioned as follows. Students were presented with a problem and expected to work on it individually. During this time, the teacher would select one or two students to write their work on the board. When the teacher called the students to Whole Class, one or two students would be asked to explain the work they wrote on the board, and the remaining time was spent asking students to take a personal position relative to the strategies on the board (whether they agreed or disagreed).

Likewise, during Pair Share time, students were asked to agree or disagree with work that other students presented on the board. The work done in individual time was then meant to support the mathematical discussions that the teacher moderated during Whole Class as well as the student discussions during Pair Share. Overall, then, the goal of student participation in Classroom D was for students to develop their own strategies individually and then decide whether they agreed or disagreed with the work that was selected to go on the board.

The whiteboard was used by the teacher and students during Whole Class discussion. In Classroom D, the teacher held the authority of the whiteboard in the classroom. The teacher used the whiteboard to write the problems on the board, always selected who would display their work on the board prior to whole class discussions, and called on the students to come to the board to explain their work. As such, the board, like in Classroom C, represented a place of authority. In the analytical narrative that follows, whenever the whiteboard was used during an interaction, I will discuss how the whiteboard served to mediate the interactions that occurred, and also how only a few limited students were able to gain the authority to use the board (and therefore participate) in the classroom.

Classroom D timeline. Table 21 depicts the chronological timeline of the participation structures in Classroom D for both recorded days. Each participation structure is depicted with the amount of time spent in that participation structure as well as the problems that the students worked on during that time. This timeline serves to guide this chapter.

Tables 22 and 23 depict how the roles of SDPSS and Agreer/Disagreer played out in each of the phases of classroom participation. Each cell represents the number of times the role occurred within each phase, whether the role involved an explanation, and whether the teacher followed up when the student enacted the role within a conversation. The narrative in the following pages describes how the goals of participation, teacher communication, and the tools used by the students promoted the roles and explanations that were observed, and why students enacted the role of Agreer/Disagreer when the teacher did not follow up with them in the classroom.

Table 21

Chronological Timeline for Participation Structures on Days 1 and 2 in Classroom D

Day 1 (Total Time 51 min)	Day 2 (Total Time 62 min)

• Phase 1: Whole Class (7 min 44 sec)
  ○ Housekeeping
  ○ Problem: Who said the rule yesterday?
• Phase 2: Pair Share (44 s)
  ○ What was the rule from yesterday?
• Phase 3: Whole Class (7 min 32 s)
  ○ What do you remember from yesterday
• Phase 4: Individual Time (11 min 1 s)
  ○ Bill & Sally Problem
• Phase 5: Whole Class (8 min 56)
  ○ Bill & Sally Problem, Matt, Max, Rose
• Phase 6: Pair Share (47 s)
  ○ What disagree with Max
• Phase 7: Whole Class (4 min 22 s)
  ○ What disagree with max
• Phase 8: Individual Time (4 min 18 s)
  ○ Counting Bill and Sally's side using 1/3 on Bill's side and ¼ on Sally's side
• Phase 9: Whole Class (6 min 24 s)
  ○ Count together
  ○ Come up with a rule
	• Phase 1: Whole Class (5 min 26 s)
	• Phase 2: Individual Time (7 min 22 s)
	  ○ What does 1/10 meter look like?
	  ○ How many ways can you count from 0 to 1 by tenths?
	• Phase 3: Whole Class (3 min 22 s)
	  ○ What does 1/10 meter look like
	    ▪ Voting , Lewis, India, Uma
	• Phase 4: Pair Share (58 s)
	  ○ how many ways can you count from 0 to 1 by tenths
	    ▪ Students discuss which one they vote for
	• Phase 5: Whole Class (5 min 40 seconds)
	  ○ How many ways can you count from 0 to 1 by tenths
	    ▪ Students share publicly the vote discussed during PS
	• Phase 6: Individual Time (1 min 49 seconds)
	  ○ Update notes
	• Phase 7: Whole Class (4 min 42 s)
	  ○ What is the probability of each event happening?
	    ▪ Introduce Problem (students don't work on it)
	• Phase 8: Individual Time (6 min 34 s)
	  ○ Probability & show relationship
	• Phase 9: Whole Class (5 min 33 s)
	  ○ Probability & Relationship
	    ▪ Jackson shares his solution
	    ▪ Agree/disagree w/Jackson
	• Phase 10: Individual (6 min 2 s)
	  ○ Come up with description for scale
	• Phase 11: Whole Class (5 min 34s)
	  ○ Come up with description for scale
	    ▪ Sharing different labels for scale
	  ○ Probability of events
	• Phase 12: Individual (1 min 55s)
	  ○ Write a rule about today
	• Phase 13: Whole class (1 min 56)
	  ○ Sharing of Rules

Table 22

Classroom D Explanations and Student Roles by Phase on Day 1

Student Roles		Phase 5 (WC)		Phase 6 (PS)				Phase 7 (WC)		Phase 8 (I)				Phase 9 (WC)	
		NTF[13]	TF	NTF 1	NTF 2	NTF 3	TF	NTF[1]	TF	NTF 1	NTF 2	NTF 3	TF	NTF[1]	TF
Sharer of Details of Problem-Solving Strategy	Total	N = 1	N = 3	N = 0	N = 0	N = 0	N = 0	N = 1	N = 0	N = 0	N = 1	N = 0	N = 1	N = 1	N = 0
	Exp	n = 1	n = 3					n = 1			n = 1		n = 1	n = 1	
	% Exp	100%	100%					100%			100%		100%	100%	
Agreer/ Disagreer	Total	N = 9	N = 0	N = 4	N = 1	N = 0	N = 0	N = 3	N = 0	N = 0	N = 0	N = 0	N = 0	N = 2	N = 0
	Exp	n = 0		n = 0	n = 0			n = 0						n = 0	
	% Exp	0%		0%	0%			0%						0%	

TF = Teacher Follow-up
NTF = No Teacher Follow-up
Exp = Explanation

[13] During Whole Class, the teacher was always present and engaged with students, but this engagement was limited to initial invitations to participate in the classroom dialogue.

Table 23

Classroom D Explanations and Student Roles by Phase on Day 2

		Phase 2 (I)				Phase 3 (WC) [14]		Phase 4 (PS)				Phase 5 (WC)		Phase 6 (I)		Phase 7 (WC)	
Student Roles		NTF			TF	NTF	TF	NTF			TF	NTF	TF	NTF	TF	NTF	TF
		1	2	3				1	2	3							
Sharer of Details of Problem-Solving Strategy	Total	N = 0	N = 1	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 2	N = 0	N = 0	N = 0	N = 0	N = 0
	Exp		n = 1									n = 2					
	% Exp		100%									100%					
Agreer/ Disagreer	Total	N = 0	N = 0	N = 0	N = 0	N = 2	N = 2	N = 8	N = 1	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0	N = 0
	Exp					n = 0	n = 2	n = 0	n = 0								
	% Exp					0%	100%	0%	0%								

TF = Teacher Follow-up
NTF = No Teacher Follow-up
Exp = Explanation

[14] During Whole Class, NTF indicated that the teacher was always present and engaged with students, but this engagement was limited to initial invitations to participate in the classroom dialogue.

Table 23 (continued)

Student Roles		Phase 8 (I)		Phase 9 (WC)		Phase 10 (I)				Phase 11 (WC)		Phase 12 (I)		Phase 13 (WC)	
		NTF	TF	NTF[15]	TF	NTF 1	NTF 2	NTF 3	TF	NTF[3]	TF	NTF	TF	NTF[3]	TF
Sharer of Details of Problem-Solving Strategy	Total	N = 0	N = 0	N = 2	N = 0	N = 0	N = 2	N = 0	N = 0	N = 3	N = 0	N = 0	N = 0	N = 1	N = 0
	Exp			n = 2			n = 2			n = 3				n = 1	
	% Exp			100%			100%			100%				100%	
Agreer/ Disagreer	Total	N = 0	N = 0	N = 3	N = 0	N = 1	N = 0	N = 0	N = 0	N = 3	N = 0	N = 0	N = 0	N = 1	N = 0
	Exp			n = 0		n = 0				n = 0				n = 0	
	% Exp			0%		0%				0%				0%	

[15]During Whole Class, the teacher was always present and engaged with students, but this engagement was limited to initial invitations to participate in the classroom dialogue.

Analysis of Social Dimensions That Promoted the Roles of Sharer of Details of Problem-solving Strategy and Agreer/Disagreer in the Classroom

Day 1, Phases 1-4: Logistical housekeeping and individual work in Classroom D. During Phase 4, the teacher communicated to the students that the goal of participation was for the students to be able to create a representation/solution of a problem by themselves and share their strategy:

a. The teacher made a general announcement before presenting the problem to the students that they should work individually and he would select students to share their ideas with the class.

The following announcement was typical:

> I'm going to have a question for you—<u>I'd like each person to answer individually.</u> So I'm going to put the question on the board. And just the way we normally do, you copy it down. <u>Don't talk to your neighbor. Figure it out yourself</u>…. I will walk around, I'll figure out who we need to come up or who would be helpful to the class to come up. And it could be anybody.

Phase 5: Whole Class discussion of the Bill and Sally problem. During Phase 5, the teacher communicated two goals. First, he asked a few students to come to the board and share their thinking about the solution they had written there (thereby casting them as SDPSS). Second, he asked the rest of the class to (a) decide whether they agreed or disagreed with the students sharing their thinking, and (b) share whether they agreed or disagreed with another student's mathematical thoughts both verbally and nonverbally in voting exercises (thereby casting them as Agreer/Disagreer).

The example below highlights the teacher's language in promoting the Agreer/Disagreer role in the classroom:

1. T: And now I want all pencils down for the next four or five minutes while people are talking. So I'm waiting on A to put hers down.
2. Student: And J.

3. T: And J, I need you to really focus. R is busy working her little heart out. And we're going to let her continue doing that while we finish talking. So the order I want to go in is M1, M2, and then R. And then I want you [the class]—your job to do is I want you to see if you agree or disagree with their thinking. You all understand what we're doing?

The teacher specified who would be sharing their strategies and in what order, and also directed the rest of the class to determine whether they agreed or not (but did not appear to encourage students to express this agreement/disagreement). In addition, it is important to note that the teacher spent significant time in his announcement telling the students what kinds of behaviors (non-mathematical behaviors) he expected from them. He wanted "pencils down," and students to "focus." After his announcement, the teacher then requested M1, M2, and R, in turn, to share their strategies, which they did. R made an error in her strategy (she wrote that 10/4 was one whole) and the teacher asked her to select someone to respond to her strategy. R selected A, who came up and pointed out R's error and explained the correct concept of a fraction representation of a whole number (thereby enacting the role of SDPSS):

1. A: Um, It's, it's when it's double the numbers, it's not when it's ten—it's because (*A walks up to the chalkboard and points to 4/4)* when both of the numbers are the same (*moves finger up and down between the two fours in 4/4*) is when it's a whole. So this (*places finger firmly on 4/4 and looks at teacher*) is a whole.
2. T: Four fourths is a whole?
3. A: Yeah.

After A gave his explanation, the teacher repeated an important detail in what A had said, and then asked a number of students (N, I, A2—see below) whether they agreed with this detail. The teacher's actions placed those students in an Agreer/Disagreer role. Most importantly, the teacher did not ask for any elaboration of student thinking, only whether students agreed with what A had said.

4. T: (*teacher walked closer to A and R and raised hand in front of them*) okay everybody step back for a second. J, I need you to focus or I'm going to change your seat. Could you guys step back? So everybody can see. What he's, what A, is

saying is (*teacher circled 4/4 on the board*) that four fourths is a (*drew a line from circle up vertically*) whole (*teacher wrote "whole" on the board*) or one (*teacher wrote "1" above the word "whole"*).

5. Student: The matching numbers.
6. T: that's what he says.
7. R: Okay….
8. Teacher: <u>What do you say to that? What do you say to what he said? (*R looked at teacher and shrugged.*) Um. N, do you think four fourths is a whole or not?</u>
9. N: Yes, I do.
10. T: <u>I, what do you think?</u>
11. I: Yeah, I do.
12. T: <u>So some people are agreeing that four fourths is a whole. A2, what do you think?</u>
13. A2: It's a whole.
14. Teacher: It is a whole?
15. A2: Yeah.

There are a few important things to note in this interaction between the teacher and the students. First, the teacher communicated to one student that he expected him to pay attention (to the teacher) or that he would have to change his seat (a negative consequence). This kind of direction was rarely seen in any of the other classrooms (however it did occur frequently in Classroom D.) Second, the teacher noted that he wanted the students to have viewable access to him. He then used the whiteboard as part of the classroom authority—that is he used the whiteboard to control the attention of the class. R's response to the teacher appeared to be neutral, but once the teacher pressed R for a response, R simply shrugged. While it is unclear why R shrugged (perhaps R simply did not understand the teacher's question), there appeared to be a lack of interest in R to engage in A's mathematical thoughts on R's own strategy. The teacher then specifically asked other students for their opinion on A's statement (lines 10 and 12). Once several students stated agreement, the teacher repeated the agreement to the rest of the class.

After concluding the discussion around R's strategy, the teacher returned the conversation to the strategies that M1 and M2 had shared. The teacher led students through additional voting exercises on who agreed with M1 and M2's strategy (as written on the board). These exercises entailed the teacher presenting several positions for the students to take ("Who agrees with M1's strategy?"; "How many agree with M2's strategy?"; and "How many do not agree with M1's strategy?"), and having students "vote" by physically raising their hands to express their alignment with the strategies on the board.

Phase 6: Pair Share discussion of M2's strategy on the Billy and Sally problem. The teacher continued to communicate the student goal of comparing the students' own mathematical thoughts with the strategies of others by:

a. modeling Agreeing/Disagreeing behaviors, and
b. directing students to identify an element in M2's strategy with which the teacher disagreed.

In the following example, the teacher modeled how he expected students to interact around another student's strategy when he stated that he disagreed with M2's strategy, and directed students to talk with their partners about the teacher's disagreement with M2's strategy. As in previous cases, the teacher's instructions used language that explicitly focused on comparing—in this case, the repeated use of the word "disagree."

1. Teacher: So, does anybody disagree with M2 besides me? I—there is something I disagree with him on (*one student raised hand*), but uh, it's nothing personal, believe me, but there is something I disagree with him on. M3.
2. M3: Yeah?
3. Teacher: There is something I disagree with him on.
4. Student: I disagree. (*with raised hand*)
5. Teacher: I hope you're paying attention back there (*another student raised hand*). S, what do you do—do you disagree with him?
6. S: Yeah.
7. Teacher: Okay, everybody look and see what Mr. G might disagree on, and talk to your partner about it. (*students began to talk*) There is something that M2 made a slight error on. (*students continued to talk*)
8. Student: Oh I know.
9. Teacher: Slight!

10. Student A: (*to teacher*) I see.
11. Teacher: Do you see it?
12. Student A: (*to teacher*) It's the answer.
13. Student B: Yeah, it's the answer.
14. Student A: (*shrugged*) I don't know.
15. (*Several students stated "I see what he disagrees on" but no one gave an explanation.*)

The teacher stated three times that he disagreed with something in M2's strategy, asked whether anyone else also disagreed with M2's strategy. In line 5, the teacher had to ask students again, as in the previous transcript example, to "pay attention." This appears to indicate that students were not necessarily engaged or interested in the work that was taking place during this Whole Class interaction. The teacher thus asked students to identify what he (the teacher) disagreed with, and transitioned into a rare Pair Share participation structure.

When carrying out their conversations with their partners, students were mostly not able to identify what the teacher disagreed with (S's side was drawn in thirds rather than fourths*)*. As such, they simply stated that they knew what was wrong and did not provide any explanation, for example, about why M2's work was wrong or about the concept involved. Thus, students did not seem to interpret the teacher's direction to "talk to your partner about it" as a directive to explain their thinking about why M2's work was wrong. Or perhaps the students simply were not engaged in the work that was taking place (given the teacher comment earlier). While some of the students did express that they had identified what the teacher disagreed with, some of them showed lack of interest by shrugging and stating that they "didn't know" (line 14). In addition, the students near the teacher did not engage each other during this Pair Share moment, but rather spoke to the teacher. While some students did in fact enact the student roles of Agreer/Disagreer, in general students in Classroom D during this Pair Share moment participated quite differently from students in Classrooms B and C, in that they were not engaged with the original student's mathematical thoughts.

Phase 7: Whole Class voting. Phase 7 repeated the voting activities discussed in Phase 5. That is, one student was directed to share mathematical thoughts, and the rest of the class was directed to enact the role of Agreer/Disagreer through voting exercises ("How many agree with A?"; "How many do not agree with A?").

Phases 8 and 9: Repeated patterns observed in Phases 4 and 5 Individual and Whole Class sessions. Phases 8 and 9 repeated the observed patterns of Phases 4 and 5. Specifically, the teacher asked the students to work individually during Phase 8 in order to solve a problem. (However, unlike Phase 4, two episodes of the role SDPSS were observed in student conversations because the teacher directed two students who were sitting next to each other to have a discussion about how they did the problem.) Phase 9 repeated the patterns of Phase 5, and included directions to students to share the rule as well as a statement students created to describe what they learned that day and to state agreement or disagreement with the rules that students proposed.

Day 2, Phases 1-2: Housekeeping and individual work. The class began with housekeeping and introduction to the research team. The teacher then introduced the lesson's topic (Probability). During Phase 2, he asked students to work individually on a problem and selected five students to write their strategies on the board.

Phase 3-9: Repetition of patterns observed on Day 1. Phases 3, 4, 5, 8, and 9 repeated the patterns observed on Day 1 in which the teacher led students through voting exercises ("How many people think N is right?"; "How many people think B is right?"; "How many people think neither of them is right?"; "How many people think they are both right?"), directed students to work individually, made general announcements that directed students to be prepared to make comparisons, directed students to share the details of their explanations, and gave voting exercises. Phase 6 did not result in student participation because students were directed to update their notes individually during this time. Afterwards, the teacher introduced the idea of Probability by discussing how good students were at guessing the weather in the morning before school.

Phase 10: Teacher validations of students who state agreements/disagreements as a way to promote Agreer/Disagreer in the classroom. A final way that the teacher communicated to the students that the role of Agreer/Disagreer was desirable was when he:

a. provided validation statements to students who enacted the role of Agreer/Disagreer.

Validations statements by the teacher communicated that he valued the students' enactment of the role of Agreer/Disagreer. The following example illustrates how the teacher validated a student's unprompted enactment of the role of Agreer/Disagreer in the classroom.

The teacher introduced a new problem: developing labels for the scale, a line with multiple tick marks, that the students had used earlier to rank the probability of three events ("I will get wet"; "I will throw a six throwing a regular dice"; "Someone will be cross"). This scale was originally simply a line with ticks on it, and the teacher had previously asked a student to rank order the three events (based on the probability that the event would occur). The teacher next asked the students to place the events on the unlabeled line. After the students placed the events on the line, the teacher requested that the students generate labels for the scale. When the teacher finished presenting the problem, a student approached the teacher in order to interact privately with the teacher and stated a disagreement:

1. Teacher: Would everybody do something for me? While we are thinking about this. Put your hands down. Hold on. This little line here needs to be labeled in some way.

We need some words up here, words here. Would everybody figure out what words to put? And then, I will pick some people and we can put our words here. You with me? So everybody figure that out.

2. (*U got up from desk and walked up to teacher. Said something that was not picked up by the microphone.)*
3. T: <u>You can disagree, it's okay. J disagrees too. It's perfectly all right. She is still entitled to her opinion though.</u> (*U walked back to her seat)*

In this brief conversation, the teacher communicated to U that he supported her enactment of the role of Agreer/Disagreer by validating her position. The teacher validated U's enactment of the role of Agreer/Disagreer by: (a) accepting her disagreement, (b) connecting her with other students who shared the same opinion, and (c) validating the original student's opinion. The teacher did not push her to say why she disagreed with the original student's statement. The rest of Phase 10 proceeded like other individual phases where the teacher selected students to write their solutions on the board.

Phases 11-13: Whole Class discussion: Voting on how to label a line. Phase 11 included voting exercises that asked students to agree and/or disagree with the labels written on the board by other students and directed students who disagreed to provide alternative labels. During Phase 12, the teacher directed students to work individually to come up with a "rule" about what they learned during the Probability lesson and write it down in their notebooks. During Phase 13, the teacher directed the students to share the rule they came up with.

Summary

The teacher in Classroom D predominantly encouraged students to align themselves with other students' strategies by communicating that the goals of student participation were for students to develop their own strategies individually; a few students to share their strategies verbally and on the board; and finally, the rest of the students to decide which strategy(ies) they agreed (or disagreed) with. The teacher emphasized agreement/disagreement behaviors by warning students to be prepared to agree/disagree after their peers shared their strategies; engaging students in voting exercises that encouraged them to align with or against another student's strategy; directly requesting from students their alignment about whether they agreed with or disagreed with something presented; modeling how to express agreement or disagreement; and validating students who agreed or disagreed. He rarely asked students to explain why they agreed or disagreed with a suggestion made by someone else. Correspondingly, when the teacher did not follow up with students, they expressed agreement and/or disagreement without explaining their reasons.

In addition, while some students appeared to express agreement/disagreement, the engagement of the students in Classroom D quite was different from Classroom C, where students also enacted the role of Agreer/Disagreer. In Classroom D, while students enact the role of Agreer/Disagreer, students also appeared to be disengaged with the work at hand. There were several instances in which the teacher called the students' to pay attention to the conversations that were taking place. In addition, often, when the teacher called on students to respond to other students (through the roles of Agreement/Disagreement) students at times displayed body language that indicated that they either did not know or they were apathetic (for example, shoulder shrugs or no responses). This was a huge contrast to how students in Classroom C engaged in each other's ideas, often referencing each other's names and details in order to indicate who or what they disagreed with. As such, while students frequently enacted the role of Agreer/Disagreer, the same kind of intellectual work that took place in Classroom C did not appear to occur in Classroom D. Some students did not appear to know what they disagreed/agreed with, and some students simply did not state any opinions.

Chapter 9: Discussion

Summary and Discussion of Major Findings

Summary of study: Purpose and data collection. Mathematics is a sociocultural activity that is defined by social, cultural, and historical factors. As such, communities that use mathematics as part of their daily activities are constantly defining and redefining their interpretation of how to use mathematics within their everyday contexts. In the classroom, this means that both teachers and students come into the classroom with notions about what mathematics is and how they should perform mathematics in the classroom, but as a collective, these notions are defined and re-defined as their attempt to "do" classroom mathematics together. An important way in which students perform mathematics, particularly in classrooms, is through participation in classroom activities. Commonly, a form of student participation in the classroom includes providing explanations to problems that are posed by the teacher. Explanations have been deemed important for mathematical learning, and as such are a practice that are valued and encouraged in the classroom (NCTM, 1991). Professional organizations, such as the National Council of Teachers of Mathematics, have encouraged all teachers across classrooms in the United States to motivate students to provide explanations. Because students and teachers come with preconceived ideas of what participating in the mathematics classroom looks like (including whether participation means providing explanations and what those explanations look like), it is important to understand how participants in a classroom community define mathematics in order to: (a) understand student and teacher behaviors during mathematics lessons that support mathematical explanations in the classroom, and (b) consider how these notions can be modified/need to be modified in order to change behaviors that may impact student explanations in the classroom. Understanding these two aspects of classroom mathematics can give us the knowledge necessary to help all students learn mathematics successfully by providing access to opportunities in which they can give their own explanations in the classroom.

The purpose of this study, then, was to understand what it meant to engage in mathematics[16] in four elementary school classrooms by studying student mathematical participation and teacher discourse practices that were specifically related to students explaining in the classroom. Specifically, I studied the relationship between student roles and explanations, and the teacher discourse practices that supported the enactment of student roles and explanations in the classroom. The research questions that guided this study were:

1. How did classrooms differ in terms of explanation-related mathematical talk in the classroom?
2. What mathematical roles (related to the mathematical activities in the observed classrooms) did students enact in the classroom?
    a. How were the mathematical roles students enacted in the classroom associated with explanation-giving by these students?
3. What discourse practices and student activities did teachers use to elicit observed student mathematical roles and explanations in the classroom?
    a. How does the teacher discourse relate to/support student roles and explanations?

This study was a secondary study on information collected by the UCLA Mathematics Teaching and Learning group at the UCLA Laboratory School. The UCLA Laboratory School serves students from Pre-K through Grade 6, and its admissions procedures ensure that the student population represents the ethnic and socioeconomic diversity of California's schools. The information that was originally collected consisted of videotapes of two (1-hour) lessons per

[16] "To engage in mathematics" indicates how classroom participants occupied themselves with classroom mathematics.

classroom in 14 classrooms at the UCLA Laboratory School. For this study, I selected four classrooms (Grade 3 and 4 classrooms) that varied in terms of student participation and classroom norms. The information was collected using one stationary video camera with two flat microphones, four flip cameras, and six digital audio recorders. Video and audio were combined into a whole class video and multiple table videos for different student groups.

Coding of student participation and student roles was done using the software StudioCode (StudioCode Business Group, 2011). A timeline[17] was created for each classroom, and the software was used to apply codes directly onto the video timeline for student participation and student roles in the classroom. Student participation was captured by coding for students' public representations of their mathematical thoughts and strategies. These codes included explanations (students who verbally articulated their mathematical strategies for problem solving); sub-explanations (students who gave an explanation to a sub-problem rather than the whole problem being worked on); and thinking (talk that was still mathematical and related to the problem, but did not fit the definition of an explanation or sub-explanation).
In addition, a set of student roles were created to account for a second dimension of student participation. It was possible for students to be assigned multiple roles at the same time during one problem-solving conversation, however, this rarely occurred. These roles included:

- Sharer of Details of Problem-Solving Strategy (SDPSS)—a student shared the details of how to solve a problem. It could be her/his strategy, that of another student, or a strategy belonging to the group.
- Sharer of Details of Mathematical Claim (SDMC)—a student shared the details of a mathematical claim.

[17]The "timeline" referred to here is a linear representation of events of the class; however, because of how the information was collected and how the software functioned, simultaneously occurring events were represented in a consecutive manner rather than demonstrating that they occurred simultaneously.

- Comparer—a student examined mathematical strategies for similarities and/or differences.
- Agreer/Disagreer—a student took a position relative to another student's mathematical statement (agreed or disagreed).
- Helper—one student helped another student solve a problem by sharing information with a partner on how to solve the problem.
- Multiple Strategist—a student suggested multiple ways to solve a problem.
- Connection Maker—a student connected the problem he or she was working on with a mathematical activity from another day.
- Pattern Finder—a student identified patterns in order to help solve a problem.
- Teacher—a student offered to "teach" a student a strategy to solve a problem.

Finally, I identified six different teacher discourse practices that the teacher used to elicit one or more of the roles mentioned above. The teacher discourse practices included:

- Follow-up—teacher mathematical response to student-initiated mathematical dialogue.
- Directives—teacher discourse that directed students to behave in certain ways.
- General Announcements—teacher discourse in which teacher announced to students which behaviors he/she expected the students to engage in.
- Behavior Models—teacher molded for the students the particular behaviors he/she wanted the students to engage in.
- Voting Exercises—teacher discourse that created a social space for students to express their mathematical thoughts by selecting which strategy students related to through verbal or nonverbal (hand raising) expressions.

- Evaluations—teacher discourse in which the teacher provided positive feedback to students when they enacted particular mathematical behaviors.
- Validation Statements—teacher affirmed a student's mathematical behavior in the classroom.

In this study, I used a mixed-methods approach to address the research questions. First, to address the questions around student participation, I used the student codes to create variables and identify patterns along two dimensions: the mathematical roles that students enacted in the classroom and the kinds of mathematical talk that students produced in the classroom. I identified patterns by creating multiple contingency tables with different dimensions that showed the kinds of mathematical talk occurring in each of the classrooms, the roles that students enacted, and how these two were related within the context[18] of each classroom.

After identifying and breaking down the student participation patterns across the classrooms, I constructed narratives for each classroom to help explain the observed student behavior. I first identified the context of the classroom by identifying and describing the participation patterns as they occurred in the classroom, materials that were used, and the physical set-up of the classroom. I then went through the classroom videos and identified and transcribed key examples that illustrated the patterns identified during the quantitative analysis described above. That is, I identified examples of the students enacting the frequently occurring roles during the different participation structures. I also identified and transcribed examples of all of the teacher discourse practices that encouraged the observed behavior. I provided analytical commentary that explained how the teacher's language and discourse practice elicited certain roles and explanations (or did not elicit explanations) during that/those particular lesson(s)/day(s). In addition, I provided analytical commentary around student-student

[18]Dimensions included Student Participation Structures and whether Teacher Follow-up occurred.

interactions that focused on the intellectual work that students had to do in order to enact some of the observed roles, but that was not verbally present in their interactions when they engaged in each other's ideas.

Summary of findings

This study looked at four different classrooms and aimed to identify patterns of student participation and teacher practices that promoted student behaviors deemed important for mathematical learning. The results were as follows:

- The roles of Sharer of Details of Problem-Solving Strategy and Sharer of Details of Mathematical Claim were always associated with explanations, and the roles of Comparer and Agreer/Disagreer were not.
- The teacher discourse practices that promoted the student roles of Sharer of Details of Problem-Solving Strategy and Sharer of Details of Mathematical Claim (and hence student explanations) included Directives, Modeling Behavior, and Positive Evaluations.
- The teacher discourse practices that promoted the student roles of Comparer and Agreer/Disagreer (and hence student participation other than explanations) included Directives, Voting Exercises, Validation Statements, Modeling Behavior, and General Announcements.
- Classrooms differed in terms of the student roles and teacher discourse practices that emerged:
    - All of the classrooms had the student role of Sharer of Details of Problem-Solving Strategy and the teacher discourse practice of Directives and General Announcements.

- Some classrooms had the student role of Comparer and the teacher discourse practices of Voting Exercises and Behavior Modeling.
- Other classrooms had the student role of Agreer/Disagreer and the teacher discourse practices of Voting Exercises and Validation Statements.
- Still other classrooms had the student role of Sharer of Details of Mathematical Claim and the teacher discourse practice of Directives.

The findings of this study corroborated previous research on the value of teacher practices that encourage student participation. Studies have shown that teacher practices targeted at students directly had positive relationships between student explanations in the classroom (Ing et al., 2015). Specifically, some of the initial teacher practices in this study (such as directives) that initiate student explanations have been shown to always yield explanations in the classroom (Franke et al., 2015). Likewise, this study corroborated previous research on student behaviors that play an important part in student participation. For example, roles and behaviors such as SDPSS (called other names in other works) in previous research were seen as desirable behaviors for students in mathematics classrooms because they led to desired and accepted forms of participation in the classroom (Bishop, 2012; Wood, 2013). In addition, this study corroborated that certain teacher practices (such as the Voting Exercises seen in this study) limit students' opportunities to engage in explanations. Voting Exercises have been observed to limit participation by not providing students with the opportunity to engage deeply with other's ideas because such teacher discourse practices do not press students to provide details with which they can truly engage in other students' mathematical ideas (Kazemi & Stipek, 2001).

While this study found that teachers encouraged student behaviors similar to other behaviors that have been deemed important for mathematical learning, some interesting differences among student behaviors emerged in this study. For example, some of the roles enacted by students as a result of the teacher's directions and elicitations did not always lead to explanations (whereas in some previous studies, the same roles were found to lead to explanations). For example, engaging in comparing behaviors is considered an important mathematical practice in the classroom because it allows students to understand mathematical relationships between strategies (Yackel & Cobb, 1996). In previous studies, student enactment of this role has led to student explanations in the classroom because of the opportunity to engage with each other's ideas when teachers pressed students for more details in their comparisons (Kazemi & Stipek, 2001; Webb et al., 2014). That is, when teachers followed up on students and encouraged them to share explanations around differences, students often did. However, in the classrooms in this study, when students engaged in comparing behaviors, these behaviors were consistently limited to simply stating whether they were similar or different. This applied to students enacting the role of Agreer/Disagreer as well. Student behavior around agreement/ disagreement was consistently limited to simply stating said agreement or disagreement. Students did not discuss or explain how or why things were similar or different, or why they agreed or disagreed with another student's explanation. This was a missed opportunity for students to use mathematical arguments in order to understand mathematical difference. The findings in this study suggested that the initial teacher moves meant to elicit student roles and explanations were not necessarily sufficient to produce student explaining; teacher follow up was often required to produce fuller student participation. Thus, these findings corroborated previous work that identified teacher moves after student-initiated mathematical dialogue that encourage explanations in the classroom (Franke, Webb, Chan, Ing, Freund, & Battey, 2009). In particular, in the present study, students were not likely to share explanations when enacting the roles of

Comparer and Agreer/Disagreer because their teachers did not follow up on students' suggestions.

However, the fact that these roles in these classrooms did not lead to explanations should not be construed as a negative aspect of the classroom. In fact, Classrooms C and B displayed a rich environment that allowed for students to enact a diverse set of roles that afforded students the ability to engage in each other's ideas. While students did not always verbally express explanations on how they came to their various roles (Agreer/Disagreer or Comparer), it was sometimes easy to follow (based on the commentary that the students did provide) how they engaged with each other's mathematical ideas. As such, while explanations were not observed to occur as frequently in Classroom B and C as in Classroom A, there is tremendous value in the diverse mathematical behaviors that students did enact in these classrooms: students were afforded the opportunity to participate in different ways and students were provided with a way to engage in other's mathematical ideas, an important skill necessary to build mathematical dialogue between students in the classroom.

While more explanations in conjunction with the observed enactment of the roles of Agreer/Disagreer and Comparer would have added to the already observed rich discussions in the classrooms, based on the information collected here, it is not possible to ascertain why the teachers who encouraged comparing and agreement behaviors did not encourage explanations. There are several possibilities for why they did not encourage students to go beyond stating similarity or difference (or agreement/disagreement). Perhaps the teachers were not focused on explanations around difference; rather, they wanted the students to note them and would encourage them in the future to work on explaining differences. Perhaps the teachers simply wanted to focus on students explaining their own strategies. Or perhaps the teachers thought it was simply enough to have students state whether strategies were similar or different and felt it was unnecessary for students to explain why. Unless I had interviewed the teachers themselves, it remains impossible to tell why they made the choices they did when encouraging students to engage in comparing roles. However, these findings do highlight the important role explanations play as part of the different mathematical roles that students enact in the classroom.

These findings also raise questions about how and when teachers encourage students to give explanations. For example, when students who enacted these comparing behaviors did share explanations, the teacher often followed up and provided interventions that occurred after they made initial observations. Specifically, then, one question is: how and when do teachers choose to follow up with students' initial explanations? Another question is: what do teachers think about when choosing which discourse practices to use in the classroom in order to help them achieve the student behaviors they think are important for the lesson at hand? Specifically, how do teachers plan and choose how they elicit student behavior? Why do they make the choices they do about the discourse practices they use in the classroom? How do they ensure that these discourse practices elicit the behaviors they desire? If they are not able to elicit the behaviors they desire from the students, how and when do they choose to modify their teacher discourse practices by intervening in student talk?

Findings Contributions

The findings from this study add to the body of research around participation in math classrooms in two ways: (a) by identifying frequently occurring classroom roles across multiple classrooms that allowed students to participate in ways that are important for mathematical learning, and (2) by identifying how teacher discourse practices elicit these behaviors and influence the way in which students enact the roles in the classroom.

Most studies that look at students performing desired behaviors in the classroom have typically only looked at one to two students in the classroom, and limited their focus on immediate interactions (Bishop, 2012; Wood 2013). While these studies help to identify what behaviors are important for the learning of mathematics, it is also important to see how multiple students enact those same behaviors in different classrooms. This knowledge is important because it can provide a better understanding of what variations exist in how these roles are enacted by students. It can also provide information on why these variations exist and illuminate how teachers can elicit these behaviors in the classroom. Thus, this study expanded on previous work by identifying how these mathematical student behaviors occurred across multiple classrooms and were enacted by multiple students. Specifically, I found that students in classrooms where the teachers promoted only the role of Sharer of Details of Problem-Solving Strategy enacted the role more frequently than in classrooms in which the teachers promoted other roles. In addition, how the roles were supported differed across the classrooms and impacted which roles students enacted in the classroom.

This study also contributed to the body of research on classroom teacher practices that are important for particular kinds of talk in the classroom, particularly in the field of mathematics. Previous research focused on teacher communication skills that support student discourse, but not on how teachers use their own discourse practices to target student behavior (Mercer, Dawes, Wegerif, & Sams, 2004; Mercer, Wegerif, & Dawes, 1999; Mercer et. al 2004). This study added to the body of knowledge because the findings showed how the six teacher practices that were identified elicited various mathematical roles in the classroom. Specifically, the teachers used these practices during different participation structures in varying ways to target students and elicit different behaviors. For example, during Whole Class, the teacher often targeted the entire class by interacting with one student to promote the role of Sharer of Details of Problem-Solving Strategy. During this time, the teachers across the classrooms used Directives to promote Sharer of Details of Problem-Solving Strategy. Thus, Classrooms A and B did not look very different during Whole Class time: Students enacted the role of Sharer of Details of Problem-Solving Strategy and the teacher often followed up on their mathematical thoughts. However, Pair Share time differed across the classrooms, and teachers used different practices to elicit a variety of roles during this time, presumably as the teacher's intent during this time also differed across classrooms. For example, in Classrooms B and D, the teachers wanted students to either compare or position themselves or their strategies in relation to the strategies of others, and so both used Directives and other teacher practices to promote these behaviors. However, Directives and supporting teacher practices limited students from providing explanations; thus, the same practices used by the teachers to promote roles related to explaining in the classroom were also used differently by the teachers to promote roles that were not related to explaining. As a result, the language the teachers used when enacting different practices in the classroom drove how they communicated during the different structures and ensured the participation they desired.

It was interesting to see the teachers use the same practices in different ways that then promoted different roles and kinds of mathematical talk in the classroom. These findings suggest that what is important in understanding student behavior is not simply teacher practices, but rather how the teachers implemented these practices in the classroom and why. This contribution is important because it indicates that teacher practices themselves can be modified to help teachers elicit desired behaviors from students during different participation activities. Thus, this holds important implications for teacher practices to design the social environment in the classroom. Not only does the teacher discourse practice matter, but the teacher's intent for various teacher practices used with different classroom participation structures can impact how students behave when the teacher is not always interacting with them.

Findings in relation to classroom practice. The findings from this study are important for classroom practice because they help us identify ways that teachers can encourage meaningful participation for learning: by encouraging students to produce explanations with a variety of roles and/or by creating a social space in which students can enact a variety of roles that allows them to engage with each other's ideas. For example, this study found that the roles of Sharer of Details of Problem-Solving Strategy and Sharer of Details of Mathematical Claim were always associated with explanations, and the roles of Comparer and Agreer/Disagreer were not. Because explanations are considered important for learning, teachers should consider how the roles they encourage in their classrooms can be promoted to include explanations. That is, rather than not encouraging the roles of Comparer and Agreer/Disagreer in the classroom because there were no explanations, teachers should consider how they can elicit these roles in conjunction with explanations. For example, the role of Comparer was often found to be elicited through Voting Exercises. This is a good practice that allows teachers to understand class strategies as a whole, compared to the original way a student solved a problem. However, beyond that, the teacher obtains little other information from this activity. One way, then, to move towards an explanation would be to ask students to share how/why their strategy was similar or different. This would encourage explanations in the classroom that push students to consider different or similar ways to solving problems as well as what makes the strategies similar or different. These mathematical skills are fundamental for the development of analytical proficiency that prepares students for advanced mathematical argumentation.

Alternatively, while providing multiple opportunities for students to explain in the classroom (such as in Classroom A) is valuable in helping to push student thinking about their own mathematical ideas, there is something to be said about creating opportunities for students to think about and engage in their peer's mathematical ideas. Opportunities for students to engage in each other's mathematical ideas creates a diverse way for students to participate mathematically in the classroom. Students then can choose to participate in ways that are meaningful for themselves. For example, when students enacted the role of Comparer in Classrooms B and C, these students had to do some heavy intellectual work (such as paying attention to the details of another student's explanation or being able to anticipate the next mathematical step that the original student might have skipped during an explanation) in order to simply determine whether their strategies were similar to those of their peers. In the case of the role of Agreer/Disagreer, particularly in Classroom C during the zero argument. The argument was more theoretical in nature, and as such, students had to be able to use the evidence that their peers presented to make sense of their own mathematical thoughts to determine whether they agreed or disagreed. As such, the opportunity to engage in these diverse roles in Classrooms B and C afforded students the opportunity to participate mathematically in a variety of ways that helped build and expand their own abilities to think about mathematical ideas discussed in the classroom.

Thus, teachers may consider how they can elicit explanations in conjunction with these roles from the very beginning, for example, when directing students to engage in comparing behaviors, including instructions that elicit explanations about why strategies are similar or different or consider ways in which they can encourage a variety of roles in the classroom such that more students are able to participate in the classroom. The takeaway message is that, teachers should consider encouraging students to give explanations when enacting those mathematical roles and teachers should provide numerous opportunities for students to participate mathematically in the classroom. Some constraints need to be considered (such as time limits), and careful planning of the teacher's goals for student participation should include which roles teachers want to focus on to promote diverse ways that encourage student participation in the classroom.

Findings in relation to teacher development. These findings are also important for professional development. What was learned here as well as from other similar studies can teach teachers to identify how the language and language practices they use are important for encouraging particular kinds of participation in the classroom. As such, professional development courses should not only consider spending time on the importance of language in the classroom, but also touch on how similar language and teacher discourse practices can lead to very different student behaviors in the classroom as a result of other social factors. In other words, professional development courses need to provide teachers with the skills they need to look at the classroom holistically and understand how multiple factors, including discourse practices, lead students to participate in specific ways in the classroom. For example, in two of the classrooms in this study, the teachers used very similar language to transition to the same participation structure and appeared to encourage the same roles. However, the roles students enacted and the mathematical talk students engaged in were very different between the two classrooms, although a close analysis revealed that the teachers used the same language. While it was unclear why the students enacted different roles at that moment, other social factors were likely the determining key. For example, how teachers encourage students to interpret the language used throughout the school year can influence student behavior. In the two classrooms, one teacher used the same language repeatedly, predominantly elicited explanations by directing students to enact Sharer of Details of Problem-Solving Strategy and share explanations with peers, and supported these behaviors in other ways (such as evaluations) in one class period. By contrast, the other teacher supported other behaviors throughout the observed class periods that were not consistent with what he encouraged, although his language was similar to that of the first teacher. Thus, professional development instructors should help teachers consider all the social factors that can impact how teachers elicit and encourage student mathematical roles and explanations, and how students can be socialized to interpret particular teacher discourse practices in specific ways. This can include discussions around participation structures, the way problems are worded, which behaviors were favorably commented on by the teacher, and so on. Finally, professional development courses can help teachers realize that even implementing particular practices will not guarantee particular results (Hand, 2006). Rather, they need to realize that designing the social classroom context is an ongoing iterative process that gives teachers the skills to identify the need to change their language practices and address the students' social needs in relation to the teacher's desired student participation goals.

Limitations of Study

While these findings provide additional information on how teacher discourse practices influence student roles and participation in the classroom, the study had several limitations that need to be considered which can inform ways to strengthen future studies on the relationship between teacher discourse practices, student behaviors, and mathematical participation in the classroom. First, this study did not obtain enough information on why students interpreted some instructions in particular ways. While I hypothesize that students were socialized to interpret teacher language in particular ways early on in the school year, it is unclear how and why students were socialized this way. Additional information that can illuminate why students interpreted things as they did include: (a) information capturing how norms are established over time, and (b) student interview/survey data on how students interpret teacher instructions in particular ways. The use of additional observations or video collections that look at classrooms at different points in time—earlier in the school year as well as throughout—would capture data on how norms are established over time. In addition, creating student surveys or conducting interviews with students asking how and why they interpret teacher instructions in specific ways would inform our understanding of why students enact the roles they did at times when their teachers did not follow up on initial participation or explicitly communicate which behaviors they were expecting.

Another limitation of this study concerns teacher discourse practices. It would be helpful to have additional information on how teachers communicate to students that they should engage in each other's ideas. While I did not code for this information (in either student participation codes or teacher discourse coding), it would be helpful for providing a more complete picture of how the social facets of the classroom support student roles and participation. Students were often observed engaging in each other's ideas, particularly in Classroom A where the teacher introduced the use of whiteboards for student pairs during Pair Share time as a way to mediate group thinking. However, I did not document how or when the teacher encouraged this kind of participation across all four classrooms, nor did I develop ways to capture how students engaged in each other's ideas. A closer look at how teachers encourage engaging in each other's ideas and how and when students engage in each other's ideas would provide additional perspectives specifically on the influence of student-student conversational interactions on participation and explanations.

Other helpful information would be on how the teachers planned and designed the social environment in the classroom. This would illuminate how teacher discourse practices impact the roles that students enact in the classroom and help educate new teachers to consider purposefully how they shape the classroom environment to influence desired student behavior. With this information, we would understand more completely the social environment of the classroom. Specifically, the information would include: (a) what teachers think their goals are for student participation, (b) how these goals shift over time, (c) how teachers consider mathematical identity plays a role in the students' participation, and (d) whether teachers think they achieve their student participation goals. The data could be obtained through several teacher interviews throughout the school year that document how they think about student participation during that time period. Such interviews and/or surveys would examine, for example, why the teachers chose different participation structures for their classrooms.

Knowing why teachers chose particular participation structures might also clarify the role these structures play in student participation. For example, Classroom A had a stable participation pattern, compared with Classroom D. Students in Classroom A were often presented with a new problem during Pair Share time, and despite using non-explicit language, they often engaged in the role of SDPSS while giving explanations. In Classroom D, Pair Share time only occurred once a day, and although the teacher used the same language as the teacher in Classroom A, students enacted different roles and did not participate in the same way. Perhaps the reliable pattern of participation structures in Classroom A, along with how students were socialized to perceive the teacher's instructions, would provide a deeper understanding of how the students chose to participate. This would support previous research showing that stable participation structures provide students with time to practice the behaviors that the teacher encourages. The repetition of structures in Classroom A may have allowed the students to practice understanding what the teacher wanted them to enact and use more opportunities to act on these encouraged behaviors (Mehan, 1980).

Moreover, information on the teacher's purposeful planning of the participation structures and other social aspects would be useful because previous research has shown that giving students opportunities for cognitive dissonance allows them chances for more explaining (Cole & Griffin, 1983). Knowing that the Classroom A teacher purposely chose to have students encounter a problem for the first time during Pair Share to facilitate student conversations and explanations is a way to understand when certain structures are important for student participation in the classroom.

The question of whether one can truly compare these classrooms, could be considered a limitation of this study. As qualitative analytical results indicated, the classroom social ecology of the classrooms was set up in very different ways. From the participation structures the teachers created, to the problems students engaged in, to the materials used in the classroom, to the ways in which the teachers communicated to the students how they should be participating, the observed results were due to the different ways in which the teachers shaped the classrooms. However, it is precisely due to this diversity that the classrooms should be compared.

The comparisons made at the beginning of the study indicate the lasting impact the social structures of the classroom have on student participation. That is, within a school environment where teachers are using the same curriculum, how students experience mathematics in the classroom can be significantly different simply due to small minute details of how the teacher decides to socially construct the environment of the classroom, and how the students interpret and relate the social environment they are in. Any change in any of the delicate threads of the social environment could produce quite different results, yet the comparisons made in this study, particularly in Chapter 4, help us to understand how the social differences in the classrooms as they were on those days and at those times could impact a student's classroom experience. For example, it appeared that the simple difference in terms of the order of the participation structures in Classrooms A and B changed what roles students were afforded in the classroom. Both Classrooms had the participation structure Pair Share, yet because in one classroom students engaged in Pair Share after they encountered the problem on their own (Classroom B), then students in that classroom were afforded an additional role they could enact in the classroom. These initial comparisons are important to the study because they allow us to dig further into the classrooms, and to ask the right questions about what is observed in order to deeply understand how the social aspects of the classroom impact student participation in the classroom.

Last but not least, a final limitation to this study is that, unfortunately, not all student discussions were audible. As a result, the information on student participation is not complete in all of the classrooms, especially as not all students were captured during Pair Share discussions. While the majority of conversations were captured, it is possible that the information that had not been captured could have changed how the researcher interpreted the data.

Questions for Further Study

As stated earlier among the implications of the findings, this study expanded our knowledge of how teacher discourse practices influence student classroom participation. But it also raises additional questions about the relationship between participation, student behavior, teacher language practices, and student identity in the classroom.

A student's identity can influence how he or she participates in the classroom. It is believed that students enact social identities in the classroom when they display expected behaviors that fit with specific social identities (Wortham, 2004, 2006). That is, students enact the behaviors they deem appropriate in the classroom in order to portray the behaviors they consider important for the identity of "math student" (Nasir & Hand, 2008; Sfard, 2007). Thus, looking at the roles that students take on, and how they display these roles through discursive practices, can help explain student behavior in the classroom and also help us understand how students interpret what mathematical participation should look like (Turner et al., 2013; Wood, 2013).

However, the relationship between identity and student behavior can be reflexive—that is, while that behavior is influenced by the identity students already have, the identity students develop over time is informed by the behavior they enact in the classroom (Wortham, 2004, 2006). For example, in the ground-breaking work of "Figured Worlds," Holland, Lachicotte, Skinner, and Cain (1998) discussed how "identity is an important outcome of participation within a community of practice" (p.57). As such, students participating in their mathematics classrooms and within the social constraints of the classroom are actively enacting behaviors that, in turn, shape their classroom identity—and vice versa. However, it is important to note that enacting particular behaviors in order to display a particular identity does not necessarily mean that the student automatically becomes that identity. As Fields and Enyedy (2013) note, it takes time for someone to become the identity they aspire to be because of the multiple social facets composing identity development, including the acceptance of the identity/role from other participants in the social environment. Thus, examining the relationship between participation and identity would be the goal of future research for further understanding of the social environment of the classroom. Questions that could guide this research would include:

- How does identity influence the participation roles that students enact?
    - How do students associate the roles they enact in the classroom with how they view themselves as mathematical students at large? That is, do students relate the roles they take on in the classroom to their identity as math students in general?
- How do the roles that students take on influence their identity?
- How do student identity and teacher discourse practices together influence the participation roles that students take on?
- How do teachers consider mathematical identity when planning facets of the social environment in the classroom?
- How do teachers consider mathematical identity when considering which language practices to use in their day-to-day interaction with students in the classroom?
    - Which teacher practices are missing that teachers could have used to support the observed student roles?
    - How do teachers socialize students to interpret the language they use to communicate with students in particular ways?
- What additional student roles and teacher discourse language practices contribute to students' mathematical identity development and explanations in the classroom?
- How can teacher development programs develop interventions to help teachers use the discourse practices that will support mathematical roles and student explanations in the classroom?

Studies that examine these questions can further expand our knowledge of how to design social environments that support explanations in the classrooms and provide us with information on the relationship between the roles that students enact in the classroom and student mathematical identity. This information can be used to create positive long-term math identities for young students, and create a culture of explaining in mathematics classrooms.

References

Bishop, J. P. (2012). "She's always been the smart one. I've always been the dumb one": Identities in the mathematics classroom. *Journal for Research in Mathematics Education*, *43*(1), 34-74.

Bloom, B. S., Englehart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: Handbook 1. Cognitive domain.* New York, NY: David McKay.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, *23*(7), 13-20. doi:10.3102/0013189X023007013

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal*, *29*(3), 573-604. doi:10.3102/00028312029003573

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and learning in mathematics classroom situations. *Educational Studies in Mathematics*, *23*(1), 99-122.

Cole, M., & Griffin, P. (1983). A socio-historical approach to re-mediation. *The Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 5*(4), 69-74.

Erickson, F. (2004). *Talk and social theory: Ecologies of speaking and listening in everyday life* (1st ed.). Boston, MA: Polity.

Erickson, F., & Schultz, J. (1997). When is a context? Some issues and methods in the analysis of social competence. In M. Cole, Y. Engestrom, & O. Vasquez (Eds.), *Mind, culture, and activity: Seminal papers from the Laboratory of Comparative Human Cognition*. New York, NY: Cambridge University Press.

Fields, D., & Enyedy, N. (2013). Picking up the mantle of "expert": Assigned roles, assertion of identity, and peer recognition within a programming class. *Mind, Culture, and Activity, 20*, 113-131.

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. J. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 333-352). Reston, VA: National Council of Teachers of Mathematics.

Forman, E. A., McCormick, D. E., & Donato, R. (1997). Learning what counts as a mathematical explanation. *Linguistics and Education*, *9*(4), 313-339. doi:10.1016/S0898-5898(97) 90004-8

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battery, D. (2009). Teacher questioning to elicit mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Franke, M.L., Turrou, A.C., Webb, N.M., Ing, M., Wong, J., Shin, N., & Fernandez, C.H. (2015). Student's Engagement with Other's Mathematical Ideas: The Role of Teacher Invitation and Support Moves. *The Elementary School Journal*, 116(1), 126-148

Franke, T. M., Ho, T., & Christie, C. A. (2012). The chi-square test: Often used, and more often misinterpreted. *American Journal of Evaluation, 33*(3), 448-458. doi:10.1177/109821 4011426594

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N. B., Karns, K., & Dutka, S. (1997). Enhancing students' helping behavior during peer-mediated instruction with conceptual mathematical explanations. *The Elementary School Journal*, *97*(3), 223-249. doi:10.2307/1002198

Gillies, R. M. (2004). The effects of communication training on teachers' and students' verbal behaviours during cooperative learning. *International Journal of Educational Research*, *41*(3), 257-279. doi:10.1016/j.ijer.2005.07.004

Gillies, R. M. (2006). Teachers' and students' verbal behaviours during cooperative and small-group learning. *British Journal of Educational Psychology*, *76*(2), 271-287. doi:10.1348/000709905X52337

Gillies, R. M., & Boyle, M. (2008). Teachers' discourse during cooperative learning and their perceptions of this pedagogical practice. *Teaching and Teacher Education*, *24*(5), 1333-1348. doi:10.1016/j.tate.2007.10.003

Goodman, L. A. (1963). Simultaneous confidence intervals for contrasts among multinomial populations. *The Annals of Mathematical Statistics, 35*(2), 716-725.

Goodman, L. A. (1969). How to ransack mobility tables and other kinds of cross-classification tables. *American Journal of Sociology, 75,* 1-40. doi.org/10.1086/224743

Goodman, L. A. (1971). Partitioning of chi-square: Analysis of marginal contingency tables and estimation of expected frequencies in multidimensional contingency tables. *Journal of American Statistical Association, 66*(134), 339-344.

Greeno, J. G. (1998). The situativity of knowing, learning, and research. *American Psychologist*, *53*(1), 5-26. doi:10.1037/0003-066X.53.1.5

Gresalfi, M. S., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies*, *1*(1), 49-57.

Hand, V. (2006). Operationalizing culture and identity in ways to capture the negotiation of participation across communities. *Human Development*, *49*(1), 36-41.

Hershkowitz, R., & Schwarz, B. (1999). The emergent perspective in rich learning environments: Some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics*, *39*(1/3), 149-166.

Hiebert, J., Stigler, J. W., Jacobs, J. K., Givvin, K. B., Garnier, H., Smith, M., . . . Gallimore, R. (2005). Mathematics teaching in the United States today (and tomorrow): Results from the TIMSS 1999 video study. *Educational Evaluation and Policy Analysis*, *27*(2), 111-132. doi:10.3102/01623737027002111

Hiebert, J. (2013). The constantly underestimated challenge of improving mathematics instruction. In K. R. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 45-56). New York, NY: Springer. doi: 10.1007/978-1-4614-6977-3_3

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds.* Cambridge, MA: Harvard University Press.

Ing, M., Webb, N. M., Franke, M. L., Turrou, A. C., Wong, J., Shin, N., & Fernandez, C. H. (2015). Student participation in elementary classrooms: The missing link between teacher practices and student achievement. *Educational Studies in Mathematics, 90*, 341-356.

Kapur, M., & Bielaczyc, K. (2012). Designing for productive failure. *Journal of the Learning Sciences*, *21*(1), 45-83. doi:10.1080/10508406.2011.591717

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics*, *4*, 410-414.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, *102*(1), 59-80.

King, A. (1989). Effects of self-questioning training on college students’ comprehension of lectures. *Contemporary Educational Psychology*, *14*(4), 366-381. doi:10.1016/0361-476X(89)90022-2

King, A. (1990). Enhancing peer interaction and learning in the classroom through reciprocal questioning. *American Educational Research Journal*, *27*(4), 664-687. doi:10.3102/00028312027004664

King, A. (1992). Facilitating elaborative learning through guided student-generated questioning. *Educational Psychologist*, *27*(1), 111.

King, A. (1999). Discourse patterns for mediating peer learning. In A. O’Donnell & A. King (Eds.), *Cognitive perspectives on peer learning* (pp. 87-116). Mahwah, NJ: Lawerence Erlbaum.

Langer-Osuna, J. M. (2011). How Brianna became bossy and Kofi came out smart: Understanding the trajectories of identity and engagement for two group leaders in a project-based mathematics classroom. *Canadian Journal of Science, Mathematics, and Technology Education, 11*(3), 207-225.

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics*, *46*(1-3), 87-113. doi:10.1023/A:1014031004832

Mehan, H. (1980). The competent student. *Anthropology and Education Quarterly, 11*(3), 131-152.

Mercer, N. (1996). The quality of talk in children's collaborative activity in the classroom. *Learning and Instruction*, *6*(4), 359-377. doi:10.1016/S0959-4752(96)00021-7

Mercer, N., Dawes, L., Wegerif, R., & Sams, C. (2004). Reasoning as a scientist: Ways of helping children to use language to learn science. *British Educational Research Journal*, *30*(3), 359-377. doi:10.1080/01411920410001689689

Mercer, N., & Littleton, K. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. New York, NY: Taylor & Francis.

Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve math problems. *Language and Education*, *20*(6), 507-528. doi:10.2167/le678.0

Mercer, N., Wegerif, R., & Dawes, L. (1999). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal*, *25*(1), 95-111.

Nasir, N. S., & Hand, V. (2008). From the court to the classroom: Opportunities for engagement, learning, and identity in basketball and classroom mathematics. *Journal of Learning Sciences, 17*(2), 143-179.

Nasir, N. S, & Saxe, G. B. (2003). Ethnic and academic identities: A cultural practice perspective on emerging tensions and their management in the lives of minority students. *Educational Researcher, 32*(5), 14-18

Nasir, N. S. (2002). Identity, Goals, and Learning: Mathematics in Cultural Practice. *Mathematical Thinking and Learning,* 4:2-3, 213-247, DOI: 10.1207/S15327833MTL04023_6

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Nattiv, A. (1994). Helping behaviors and math achievement gain of students using cooperative learning. *The Elementary School Journal*, *94*(3), 285-297. doi:10.2307/1001900

Saxe, G. B., & Guberman, S. R. (1998). Studying mathematics learning in collective activity. *Learning and Instruction*, *8*(6), 489-501. doi:10.1016/S0959-4752(98)00037-1

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a cognitive standpoint. *The Journal of the Learning Sciences*, *16*(4), 565-613.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, 34(4), 14-22. doi:10.3102/0013189X034004014

Sharpe, D. (2015). Your chi-square test is statistically significant: Now what? *Practical Assessment, Research, and Evaluation, 20*(8), 1-10.

Slavin, R. E. (1987). Ability grouping and student achievement in elementary schools: A best-evidence synthesis. *Review of Educational Research*, *57*(3), 293-336. doi:10.3102/00346543057003293

Stigler, J. W., & Hiebert, J. (2009). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Simon and Schuster.

StudioCode Business Group. (2011). *StudioCode*. California: Author. Retrieved from http://www.studiocodegroup.com/

Sweller, J. (1989). Cognitive technology: Some procedures for facilitating learning and problem solving in mathematics and science. *Journal of Educational Psychology*, *81*(4), 457-466.

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199-234.

Veenman, S., Denessen, E., Van Den Akker, A., & Van Der Rijt, J. (2005). Effects of a cooperative learning program on the elaborations of students during help seeking and help giving. *American Educational Research Journal*, *42*(1), 115-151. doi:10.3102/00028312042001115

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, *78*(3), 516-551. doi:10.3102/0034654308320292

Webb, N. M. (1982). Group composition, group interaction, and achievement in cooperative small groups. *Journal of Educational Psychology*, *74*(4), 475-484. doi:10.1037/0022-0663.74.4.475

Webb, N. M., & Palincsar, A. S. (1996). Group processes in the classroom. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 841-873). New York, NY: Prentice-Hall.

Webb, N. M., & Mastergeorge, A. (2003). Promoting effective helping behavior in peer-directed groups. *International Journal of Educational Research*, *39*(1-2), 73-97. doi:10.1016/S0883-0355(03)00074-0

Webb, N. M., Nemer, K. M., & Ing, M. (2006). Small-group reflections: Parallels between teacher discourse and student behavior in peer-directed groups. *Journal of the Learning Sciences*, *15*(1), 63-119. doi:10.1207/s15327809jls1501_8

Webb, N. M. (2009). The teacher's role in promoting collaborative dialogue in the classroom. *British Journal of Educational Psychology*, *79*(1), 1-28. doi:10.1348/000709908X380772

Webb, N. M., Franke, M. L., Ing, M., Chan, A., De, T., Freund, D., & Battey, D. (2008). The role of teacher instructional practices in student collaboration. *Contemporary Educational Psychology*, *33*(3), 360-381. doi:10.1016/j.cedpsych.2008.05.003

Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., & Melkonian, D. K. (2009). "Explain to your partner": Teachers' instructional practices and students' dialogue in small groups. *Cambridge Journal of Education*, *39*(1), 49-70. doi:10.1080/03057640802701986

Webb, N.M., Franke, M.L., Ing, M., Wong, J., Fernandez, C.H., Shin, N., & Turrou, A.C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices and learning. *International Journal of Educational Research*, 63, 79-93

Wedege, T. (2010). Sociomathematics: A subject field and a research field. Retrieved from http://dspace.mah.se/handle/2043/10027

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York, NY: Cambridge University Press.

Wenger, E. (2000). Communities of practice and social learning systems. *Organization*, *7*(2), 225-246. doi:10.1177/135050840072002

Wittrock, M. C. (1989). Generative processes of comprehension. *Educational Psychologist*, *24*(4), 345.

Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourth-grade classroom. *Journal for Research in Mathematics Education*, *44*(5), 775-808.

Wortham, S. (2004). The interdependence of social identification and learning. *American Educational Research Journal*, *41*(3), 715-750.

Wortham, S. (2006). *Learning Identity: The Joint Emergence of Social Identification and Academic Learning*. New York, NY: Cambridge University Press.

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behavior*, *21*(4), 423–440. doi:10.1016/S0732-3123(02)00143-8

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458-477. doi:10.2307/749877

Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (pp. 12–21). Reston, VA: National Council of Teachers of Mathematics.

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *The Journal of Mathematical Behavior*, *19*(3), 275-287. doi:10.1016/S0732-3123(00)00051-1