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Journal

Journal of Applied Physics, 23(5)

ISSN

00218979

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Publication Date

1952

DOI

10.1063/1.1702246

Peer reviewed

Theory of Jet Formation by Charges with Lined Conical Cavities*

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(Received January 7, 1952)

An article by Birkhoff, MacDougall, Pugh, and Taylor (see reference 1) presented hydrodynamic theories of jet formation and target penetration by explosives with lined conical cavities. However, it was unable to explain satisfactorily why the jets produced are several times as long and, therefore, several times as effective as the steady-state theory predicts. It is shown here that these difficulties are overcome by assuming a variable instead of a constant collapse velocity for the walls of the conical liner. The variability in the collapse velocity produces a surprisingly large change in the process of jet formation.

THE effects produced by high explosive charges depend to a great extent upon the geometry of the charge assembly. Charges with conical cavities which are lined with a metal have probably attracted the most interest because of their usefulness in making holes through thick armor plate or producing deep penetration into other materials. The first fairly complete explanation of these phenomena appeared (in unclassified literature) in the June, 1948, issue of the *Journal of Applied Physics*.¹ By assuming that the pressures and energies involved were so great that the strength of the metals could be ignored and that, therefore, these metals could be treated as perfect fluids, the authors were able to produce mathematical theories, based upon classical hydrodynamics, that quantitatively explained most of the experiments performed upon these phenomena during the late war. They were able to show that a detonation wave sweeping from apex to base along a conical liner collapses the liner into a small diameter jet shooting forward with velocities much greater than that of the highest speed rifle bullet. They were also able to show that when this high speed jet impinged upon the target material it produced pressures close to a million atmospheres which forced the target material to flow plastically out of the path of the jet.

The mathematical theory of penetration showed that the depth of the hole produced is proportional to the length of the jet and, surprisingly, independent of the velocity of the jet, provided the velocity is great enough that the strength of the target can be neglected.

The dependence of the depth of the hole upon the length of the jet explained why deeper holes are produced when the charge with the lined conical cavity is detonated at some distance from the target. It was learned experimentally at an early date that the front end of these jets had higher velocities than the rear and that the velocities varied continuously from front to rear. This gradient in velocity along the jet causes it to lengthen as it travels. Increasing the standoff gives

time and space for the jet to lengthen so that it may produce deeper holes in the target. This benefit from the increased standoff does not continue indefinitely with increasing standoff because of the tendency of the jet to spread out and become less effective.

While the theory of penetration took account of the increasing length of the jet resulting from its velocity gradient, the theory of jet formation did not explain satisfactorily why this velocity gradient exists. In fact, as recognized in the article,² the jets from conical liners are much longer and contain much more liner material than one should expect from the steady-state hydrodynamic theory set forth in that article. While the front of these jets followed the theory quite accurately, the rear of the jets did not. Flash radiographs of the early jet formation verified the predictions of the steady-state hydrodynamic theory. Radiographs of the later stages appeared to show jet emerging from the end of the slug long after the collapse process was completed. This gave rise to the postulate that the front of the jet was formed according to the steady-state theory but that the rear of the jet, called the "after jet," was formed by some entirely different process; possibly by extrusion or ductile drawing from the collapsed slug. The "after jet" had to account for the majority of the penetration at large standoff. If the jet were formed by the steady-state theory only, the depth of penetration would be independent of standoff and, for a conical steel liner penetrating into a steel target, would be equal only to the slant height of the cone. The penetration actually observed is from two to four times this slant height.

The present version of this theory visualizes the whole jet as formed by one single continuous process that is very much like the process previously postulated for the formation of the front part of the jet. The whole process of jet formation can be explained by adding one new assumption to the steady-state theory. According to this new assumption, the velocities with which the various elements of the cone liner collapse when they are struck by the detonation wave depends upon the original position of the element in the cone. The collapse velocities decrease continuously from apex

* This work was performed under a research contract with the Office of the Chief of Ordnance, U. S. Army, and has been released for publication by the Office of Public Information, Department of Defense.

¹ Birkhoff, MacDougall, Pugh, and Taylor, *J. Appl. Phys.* 19, 563 (1948).

² See reference 1, p. 572.

to base.³ The rate of decrease is very gradual near the apex but becomes much more rapid near the base. This is just what one should expect,³ since near the base the belt of explosive surrounding the liner is much thinner and the masses of the liner elements are much greater than near the apex.

The decreasing velocity of collapse of the liner elements as the detonation wave sweeps the liner produces the remarkable effect of greatly increasing both the length of the jet and the total mass of liner material that is formed into it.

A qualitative idea of the effect of these velocity variations can be obtained from Fig. 1, if it is remembered that the velocity of the jet and the proportion of the liner that goes into the jet depend critically upon the angle β that the collapsing liner makes with the axis. As β increases the velocity of the jet decreases, but the proportion of the liner going into the jet increases. Figure 1 shows how the decreasing velocity of collapse increases the angle β .⁴ While the detonation wave sweeps from P to Q along the surface of the cone APQ , the element originally at P collapses⁵ from P to J . The element of cone originally at P' , starting later and collapsing more slowly, arrives at M at the same time as the element from P arrives at J . If the element from P' collapsed with the same velocity as the element from P , the element from P' would reach N when the element from P reached J . Thus with constant collapse velocity the contour of the collapsing cone is conical, i.e., JNQ is a straight line. However, since the velocity of collapse from P' is less than that from P , the collapsing liner takes on the contour JMQ which is not conical. The angle β which the liner makes with the axis at J is greater than the angle β^+ which it would have made with the axis, if the collapse velocity were constant. This, of course, assumes that the liner is thin and that the velocity of each infinitesimal mass element of the

Fig. 1. Cross section showing the collapse process in a conical liner.

³ This assumption is an obvious one and many have suggested it. The flash radiographs, however, were thought to prove that the "after jet" could not be so simply explained. However, a remarkable effect produced by the gradient in the collapse velocities had been overlooked.

⁴ This increase in the angle β as the collapse proceeds towards the base of the cone is shown in the article by J. C. Clark, *J. Appl. Phys.* 20, 363 (1949), Fig. 13.

⁵ The details of a similar collapse process have been treated by Theodore E. Sterne in "A Note on Collapsing Cylindrical Shells," *J. Appl. Phys.* 21, 73-74 (1950).

Fig. 2. Velocity vectors of an element of a collapsing conical liner.

liner is unaffected by its neighbors. The assumption that adjacent liner elements do not affect each other is consistent with the assumption made in the original theory that the liner acted like a perfect fluid. Elements of a perfect fluid cannot exert tensile or shearing forces on each other.

A small decrease in the collapse velocity may produce a relatively large decrease in the jet velocity, because the jet velocity decreases both when the collapse velocity decreases and when the angle β increases. Thus a small gradient in V_0 produces a larger gradient in V_j .

THEORY OF JET FORMATION

The fact that the velocities of collapse of the cone liner decrease instead of remaining constant makes it impossible to choose coordinate systems in which true steady-state conditions exist. This is not a serious handicap, since most of the relations needed were obtained, even in the steady-state theory, by applying the laws of conservation of mass, momentum, and energy to individual zonal elements. Each element can be considered independently in the appropriate constant velocity coordinate system. Because there was some evidence indicating that the detonation velocity might slow down when it reached the thin belt of explosive around the base of the liner, accurate measurements of this velocity were made. No change in the detonation velocity U_D could be detected.

Since the velocity U_D of the detonation wave is constant, the impulse given to the liner by the detonation wave can be studied in a single inertial system. Consider a system of coordinates moving with constant velocity $U = U_D \sec \alpha$ from P to Q in Fig. 1. If the impulse per unit mass given to the liner by the detonation wave were constant, a steady-state would be set up in this coordinate system. As shown in Fig. 2, where the detonation wave is in contact with the liner, the liner would enter this region with a velocity U along the line QP and leave with the same velocity along the line PA . Since the pressures resulting from the detonation wave are everywhere perpendicular to the motion of the liner in these coordinates they can change only the direction and not the magnitude of the liner velocities. In Fig. 2, QJ is drawn parallel to PA and equal in length to QP . If the magnitudes of QP and QJ are equal to U , they represent the velocities in the moving

coordinate system of the liner elements entering and leaving the region around P . The vector $PJ = \vec{V}_0$ is the velocity of the collapsing liner element in a stationary system of coordinates. The arguments given in the earlier paper then are valid. It is evident that the liner element does not move perpendicular to its original surface but, rather, along a line that makes a small angle δ with the normal. From trigonometry in Fig. 2, this angle is given, as in the earlier theory, by

$$\sin \delta = V_0/2U. \quad (1)$$

If the impulse per unit mass given by the detonation wave to the liner is not constant but decreases, the analysis is the same, provided it is assumed that adjacent elements of the liner do not interact with each other. The small angle δ merely becomes smaller according to the above equation. V_0 and δ are variables dependent upon the shape of the charge and the original position of the element in the cone. Referring to Fig. 1, $\delta = (\beta^+ - \alpha)/2$. In the steady-state where V_0 is constant, $\beta^+ = \beta$. If V_0 is considered constant and these relations are substituted for δ in the equations that follow they

FIG. 3. Vector velocity relations at the moving junction.

will be found to be identical with the equations in the earlier paper.¹

Where a ring element of the collapsing cone intersects the axis of symmetry, the analysis is more difficult because a coordinate system traveling with this junction is an accelerated system. From the analysis in the steady-state system¹ it is known that each element of mass dm splits at this junction into two parts; one part of mass dm_j goes into the jet and the other of mass dm_s goes into the slug. For each element there are four unknown quantities dm_j/dm , dm_s/dm , V_j , and V_s , where V_j and V_s are, respectively, the velocities of dm_j and dm_s in the stationary system of coordinates. From symmetry it is obvious that these velocities must be parallel to the axis. To find the four unknown quantities there are three relations available; the equations of conservation of mass, energy, and momentum. One more relation is needed. In the steady-state system this additional equation was supplied by the Bernoulli equation which cannot properly be applied to a single element and which is valid in the steady-state but not in an accelerated system.

Choose a coordinate system moving with a constant velocity V_1 equal to the instantaneous velocity of the

junction when the element considered intersects the axis. At this junction the element of mass dm divides into two elements of mass dm_j and dm_s which move in opposite directions along the axis. Let these two elements have velocities with respect to the moving coordinates of v_j and v_s , respectively, where the direction of V_1 is considered positive. Then $V_j = V_1 + v_j$ and $V_s = V_1 + v_s$. The choice of coordinate system causes the liner element dm to move in toward the junction with a relative velocity that is parallel to the surface of the liner element. Let the absolute value of this relative velocity be v . If V_1 and β were the same for all elements, as in the steady-state system,¹ Bernoulli's theorem could be used to show that $v_s = -v$. Let us assume that $v_s = -v$ in this case also, since it is the most reasonable assumption that can be made for these quantities. It merely assumes that the liner element changes the direction but not the magnitude of its velocity as it goes past the junction in the moving coordinate system. Conservation of energy now demands that $v_j = v$; thus

$$v_j = -v_s = v. \quad (2)$$

The geometrical relations at the moving junction are shown in Fig. 3. The axis of the cone is along JR , and OJ is the element of the liner which is moving toward the axis. This element has a velocity $\vec{OR} = \vec{V}_0$ in stationary coordinates and a velocity of $\vec{OJ} = \vec{v}$ in moving coordinates. The velocity of the moving coordinates is $\vec{JR} = \vec{V}_1$. By the law of sines from Fig. 3

$$v = V_0 \cos(\alpha + \delta) / \sin \beta \quad (3)$$

and

$$V_1 = V_0 \cos(\beta - \alpha - \delta) / \sin \beta. \quad (4)$$

In fixed coordinates the velocities of the jet and slug elements are, respectively,

$$V_j = V_1 + v_j = V_1 + v$$

and

$$V_s = V_1 + v_s = V_1 - v.$$

Substituting Eqs. (3) and (4) in these and making some well-known trigonometric simplifications, the velocities of jet and slug elements can be written

$$V_j = V_0 (\csc \beta / 2) \cos(\alpha + \delta - \beta / 2) \quad (5)$$

$$V_s = V_0 (\sec \beta / 2) \sin(\alpha + \delta - \beta / 2), \quad (6)$$

respectively.

In the special case, treated in the earlier article, where the velocity of liner collapse V_0 is constant, the angle β is also constant and is given by $\beta = \beta^+ = \alpha + 2\delta$. In this special steady-state case then, Eqs. (5) and (6) become

$$V_j = V_0 (\csc \beta / 2) \cos(\alpha / 2) \quad (5a)$$

and

$$V_s = V_0 (\sec \beta / 2) \sin(\alpha / 2). \quad (6b)$$

These two equations are identical with Eqs. (2) and (3) in the earlier article, though in somewhat simpler form.

Equation (1) can be used to eliminate δ from Eqs. (5) and (6)

$$V_j = V_0 (\csc \beta / 2) \cos(\alpha - \beta / 2 + \sin^{-1} V_0 / 2U) \quad (7)$$

$$V_s = V_0 (\sec \beta / 2) \sin(\alpha - \beta / 2 + \sin^{-1} V_0 / 2U). \quad (8)$$

Equations (7) and (8) are valid either in the steady-state case when V_0 is the same for all liner elements or in the nonsteady case where V_0 is different for each liner element. In the steady-state case, however, β could be expressed in terms of α , U and V_0 and need not appear in the equation.

Two of the four unknowns, dm_j/dm and dm_s/dm , remain to be calculated. As in the steady-state theory these can be calculated by combining the conservation of mass equation

$$dm = dm_j + dm_s$$

with the conservation of momentum equation

$$-dm v \cos \beta = dm_j v_j + dm_s v_s$$

or, since $v_j = -v_s = v$,

$$dm \cos \beta = dm_s - dm_j.$$

The resulting equations,

$$dm_j/dm = \sin^2 \beta / 2 \quad (9)$$

and

$$dm_s/dm = \cos^2 \beta / 2 \quad (10)$$

are identical with those obtained in the steady-state theory which were labeled (4) in the first paper.¹

Equations (7), (8), (9), and (10) describe the way in which each element of the cone liner is divided into jet and slug and give the velocities of these jet and slug elements. They depend upon the cone angle 2α and the detonation velocity $U_D = U \cos \alpha$, both of which are easily measured constants; and upon the collapse velocity V_0 and the angle β , both of which are assumed

to be variable. The collapse velocity V_0 depends upon the magnitude of the impulse given to the liner element by the detonation wave. No relation has yet been given for determining β . In the steady-state case previously assumed, the collapsing liner remained conical in shape but with an apex angle larger than that of the original cone. Thus the contour of the liner in the cross section shown in Fig. 1 was the straight line JQ and the angle β^+ could be determined by simple trigonometry. However, with V_0 different for each element of the liner, the contour of the collapsing liner is not straight but curved⁶ like the line JMQ in Fig. 1. Thus β depends upon the shape of the collapsing liner in the immediate neighborhood of J . Let the cylindrical coordinates of M in Fig. 1 be (r, z) and the coordinates of the original position P' of M in the liner be $(x \tan \alpha, x)$. Then,

$$z = x + V_0(t - T) \sin A \quad (11)$$

and

$$r = x \tan \alpha - V_0(t - T) \cos A, \quad (12)$$

where t is the time elapsed since the detonation wave passed the apex of the cone, $T = x/U_D = x/U \cos \alpha$ and $A = \alpha + \delta$. The slope of the contour of the collapsing liner at any given time t can be obtained by differentiating r with respect to z while t is kept constant. Then from (12), since $\partial T / \partial z = (\partial T / \partial x) \partial x / \partial z = (1/U_D) \times \partial x / \partial z$,

$$\partial r / \partial z = (\partial x / \partial z) \{ \tan \alpha - V_0'(t - T) \cos A + V_0 \cos A / U + V_0 A'(t - T) \sin A \} \quad (13)$$

where

$$V_0' = \partial V_0 / \partial x \text{ and } A' = \partial A / \partial x.$$

To eliminate $\partial x / \partial z$ from (13) differentiate (11) with respect to z .

$$1 = (\partial x / \partial z) \{ 1 - V_0'(t - T) \sin A - V_0 \sin A / U_D + V_0 A'(t - T) \cos A \}. \quad (14)$$

Dividing (13) by (14)

$$\frac{\partial r}{\partial z} = \frac{\tan \alpha - V_0'(t - T) \cos A + V_0 \cos A / U_D + V_0 A'(t - T) \sin A}{1 + V_0'(t - T) \sin A - V_0 \sin A / U_D + V_0 A'(t - T) \cos A} \quad (15)$$

which is the slope of the contour of the collapsing liner at any given time t . The time when a given element reaches the axis can now be obtained by substituting $r=0$ in Eq. (12) which gives

$$t - T = \frac{x \tan \alpha}{V_0 \cos A} \quad (16)$$

Substituting (16) into (15) gives $(\partial r / \partial z)_{r=0} = \tan \beta$. However, before making this substitution it is desirable to

make some further simplifications. Equation (1) gives $2 \sin \delta = V_0 / U = (V_0 \cos \alpha) / U_D$. Differentiating this with respect to x gives

$$2\delta' \cos \delta = V_0' / U \text{ or } \delta' = V_0' / (2U \cos \delta),$$

and since

$$A = \alpha + \delta, A' = \delta' = (V_0' / 2U \cos \delta) = (V_0' / V_0) \tan \delta \quad (17)$$

When (1), (16), and (17) are substituted into (15) the following simplified form is obtained.

$$\tan \beta = \frac{\sin \alpha + 2 \sin \delta \cos A - x \sin \alpha (1 - \tan A \tan \delta) V_0' / V_0}{\cos \alpha - 2 \sin \delta \sin A + x \sin \alpha (\tan A + \tan \delta) V_0' / V_0}.$$

⁶ This curvature can be observed in the original negatives of flash radiographs of collapsing cones. See reference 3.

Since β^+ is the value of β that would be obtained if V_0' were zero, from Fig. 1, $2\delta = \beta^+ - \alpha$ and $2A = \beta^+ + \alpha$ and the last equation can be further simplified as follows,

$$\tan\beta = \frac{\sin\beta^+ - x \sin\alpha(1 - \tan A \tan\delta)V_0'/V_0}{\cos\beta^+ + x \sin\alpha(\tan A + \tan\delta)V_0'/V_0}. \quad (18)$$

The quantities enclosed in parentheses in both the numerator and the denominator of Eq. (18) are positive unless the cone angle 2α is very much larger than is usually used. Therefore, when the collapse velocity decreases from apex to base, i.e., when V_0' is negative, the angle β is greater than β^+ . From Eqs. (7) and (9) it can be seen that the increase in the angle β decreases the jet velocity and increases the proportion of the mass of the liner flowing into the jet.

It is now possible to check experimentally the validity of the assumptions made in deriving this extension of the original theory. Eqs. (1), (7), (10), and (18) have been developed from four independent concepts. The quantities U and α are easily determined constants. The quantities δ , V_0 , V_j , β , and dm_s/dm are all functions of x . If all five of these variables⁷ could be determined experimentally, each of these four equations could be checked independently. At this writing, only V_j and dm_s/dm have been determined accurately enough on

⁷ Since $\beta^+ = \alpha + 2\delta$ and $A = \alpha + \delta$, these five are the only variables in the four equations.

any given charge design to provide a reliable check. This leaves three unknown variables δ , V_0 , and β in the four independent equations. Equations (1) and (7) can be used to eliminate δ and V_0 , leaving two independent equations, (10) and (18) in the single variable β . In an experimental paper following this one, it is shown that the two sets of values of β , determined by Eqs. (10) and (18), agree well with each other. This provides an excellent check on the theory.

It is now possible to see why the radiographs of the later stages of jet formation appeared to show jet issuing from the slug long after the collapse process was completed. This illusion is created by the fact that the last formed jet element travels at the same speed as the last formed slug element and by the fact that the velocity gradients stretch out all of the jet elements to great lengths. The last formed jet and slug elements come from liner elements near the base of the cone. In this region $V_0 \ll 2U$ and $V_0 \ll xV_0'$ for the charge design under consideration. Using these approximations in Eqs. (1), (7), (8), and (18), the following approximate relations are obtained: $\beta^+ \cong \alpha$, $\beta \cong 90^\circ + \alpha$, and $V_j \cong V_s \cong V_0$. Under these conditions, the last jet and slug elements that are formed travel at the same speed.

Thus the formation of the entire jet (including the "after jet") produced by charges with lined conical cavities is explained by this simple extension of the steady-state hydrodynamic theory.

The authors are indebted to Colonel C. H. M. Roberts for many valuable suggestions.

Summer Courses

A special program in Infrared Spectroscopy will be given from June 16 to June 27 during the 1952 Summer Session at the Massachusetts Institute of Technology.

The program, to be offered jointly by the Institute's Spectroscopy Laboratory and Department of Chemistry, is designed for those who wish an introduction to infrared instrumentation and laboratory methods and for those interested in the use of infrared spectra in the solution of chemical problems.

The Summer Laboratory Course in Techniques and Applications of the Electron Microscope will be given again this summer from June 16 to June 28, 1952, by the Laboratory of Electron

Microscopy in the Department of Engineering Physics of Cornell University. The course, under the direction of Dr. Benjamin M. Siegel, will have Dr. James Hillier of the RCA Laboratories, Princeton, New Jersey, and Dr. C. E. Hall of M.I.T., Cambridge, Massachusetts, as guest lecturers this year.

The course is designed for those research workers, institutional and industrial, who have recently entered the field of electron microscopy or who are now planning to undertake research problems involving applications of this instrument. Further inquiries should be addressed to Dr. Benjamin M. Siegel, Department of Engineering Physics, Rockefeller Hall, Cornell University, Ithaca, New York.