Conceptual and procedural distinctions between fractions and decimals: A cross-national comparison

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A B S T R A C T

Previous work has shown that adults in the United States process fractions and decimals in distinctly different ways, both in tasks requiring magnitude judgments and in tasks requiring mathematical reasoning. In particular, fractions and decimals are preferentially used to model discrete and continuous entities, respectively. The current study tested whether similar alignments between the format of rational numbers and quantitative ontology hold for Korean college students, who differ from American students in educational background, overall mathematical proficiency, language, and measurement conventions. A textbook analysis and the results of five experiments revealed that the alignments found in the United States were replicated in South Korea. The present study provides strong evidence for the existence of a natural alignment between entity type and the format of rational numbers. This alignment, and other processing differences between fractions and decimals, cannot be attributed to the specifics of education, language, and measurement units, which differ greatly between the United States and South Korea.

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1. Introduction

1.1. Conceptual and processing differences between fractions and decimals

A major conceptual leap in the acquisition of formal mathematics takes place with the introduction of rational numbers (typically fractions followed by decimals, at least in curricula used in the United States). These are the first formal numbers students encounter that can represent magnitudes less than one. Both fraction and decimal symbolic notations often prove problematic for students. Children, and even some adults, exhibit misconceptions about the complex conceptual structure of fractions (Ni & Zhou, 2005; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler, Thompson, & Schneider, 2011; Stigler, Givvin, & Thompson, 2010). Such difficulties have also been reported in high mathematics-achieving countries such as South Korea (Kim & Whang, 2011, 2012; Kwon, 2003; Pang & Li, 2008). Students also encounter problems in learning to understand decimals (Rittle-Johnson, Siegler, & Alibali, 2001), but generally master the magnitudes of decimals before fractions (Iuculano & Butterworth, 2011).

Fractions and decimals are typically introduced as alternative notations for the same magnitude, other than rounding error (e.g., 3/8 km vs. 0.375 km). For example, the Common Core State Standards Initiative (2014) for Grade 4 refers to decimals as a "notation for fractions". However, psychological research has revealed both conceptual and processing differences between the two notations. Whereas the bipartite (a/b) structure of a fraction represents a two-dimensional relation, a corresponding decimal represents a one-dimensional magnitude (English & Halford, 1995; Halford, Wilson, & Phillips, 1998) in which the variable denominator of a fraction has been replaced by an implicit constant (base 10). Studies have shown that magnitude comparisons can be made much more quickly and accurately with decimals than with fractions (DeWolf, Grounds, Bassok, & Holyoak, 2014; Iuculano & Butterworth, 2011), but that fractions are more effective than decimals in tasks such as relation identification or analogical reasoning, for which relational information is paramount (DeWolf, Bassok, & Holyoak, 2015a). Importantly, various aspects of performance with both fractions and decimals predict subsequent success with more advanced mathematical topics, such as algebra (Booth, Newton, & Twiss-Garrity, 2014; DeWolf, Bassok, & Holyoak, 2015b; Siegler et al., 2011, 2012, 2013).

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1.2. Semantic alignment and the ontology of quantity types

There is considerable evidence that people's interpretation and use of arithmetic operations is guided by *semantic alignment* between mathematical and real-life situations. The entities in a problem situation evoke semantic relations (e.g., tulips and vases evoke the functionally asymmetric "contain" relation), which people align with analogous mathematical relations (e.g., the non-commutative division operation, tulips/vases) (Bassok, Chase, & Martin, 1998; Guthormsen et al., 2015). Rapp, Bassok, DeWolf, and Holyoak (2015) found that a form of semantic alignment guides the use of different formats for rational numbers, fractions and decimals. Specifically, adults in the United States selectively use fractions and decimals to model discrete (i.e., countable) and continuous entities, respectively. Similarly, DeWolf et al. (2015a) demonstrated that American college students prefer to use fractions to represent ratio relations between countable sets, and decimals to represent ratio relations between continuous quantities.

The preferential alignment of fractions with discrete quantities and decimals with continuous quantities appears to reflect a basic ontological distinction among quantity types (e.g., Cordes & Gelman, 2005). Sets of discrete objects (e.g., the number of girls in a group of children) invite counting, whereas continuous mass quantities (e.g., height of water in a beaker) invite measurement. Continuous quantities can be subdivided into equal-sized units (i.e., discretized) to render them measurable by counting (e.g., slices of pizza), but the divisions are arbitrary in the sense that they do not isolate conceptual parts. Even for adults, the distinction between continuous and discrete quantities has a strong impact on selection and transfer of mathematical procedures (Alibali, Bassok, Olseth, Syc, & Goldin-Meadow, 1999; Bassok & Holyoak, 1989; Bassok & Olseth, 1995).

The different symbolic notations for rational numbers, fractions and decimals, appear to have different natural alignments with discrete and continuous quantities (see Fig. 1). A fraction represents the ratio formed between the cardinalities of two sets, each expressed as an integer; its bipartite format (a/b) captures the value of the part (the numerator a) and the whole (the denominator b). A decimal can represent the one-dimensional magnitude of a fraction ($a/b = c$) expressed in the standard base-10 metric system.

The fraction format is well-suited for representing sets and subsets of discrete entities (e.g., balls, children) that can be counted and aligned with the values of the numerator (a) and the denominator (b) (e.g., 3/7 of the balls are red). Also, as is the case with integer representations, the fraction format can be readily used to represent continuous entities that have been discretized—parsed into distinct equal-size units—and therefore can be counted (e.g., 5/8 of a pizza). In contrast, the one-dimensional decimal representation of such discrete or discretized entities seems much less natural (~0.429 of the balls are red; 0.625 of a pizza).

In contrast, the decimal format is well-suited to represent portions of continuous entities, particularly since unbounded decimals capture all real numbers (i.e., all points on a number line). This alignment appears to be especially strong when decimals (base 10) are used to model entities that have corresponding metric units (0.3 m, 0.72 l). When continuous entities have non-metric units (e.g., imperial measures with varied bases such as 12 in. or 60 min), their alignment with decimals may require computational transformations. Given that the denominator of a fraction is a variable that can be readily adapted to any unit base, it is computationally easier to represent non-metric measures of continuous entities with fractions (2/3 of a foot) than with decimals (0.67 ft). Because computational ease likely interacts with the natural conceptual alignment of continuous entities with decimals, metric units are predominantly represented with decimals, whereas imperial units may be represented by fractions (Rapp et al., 2015).
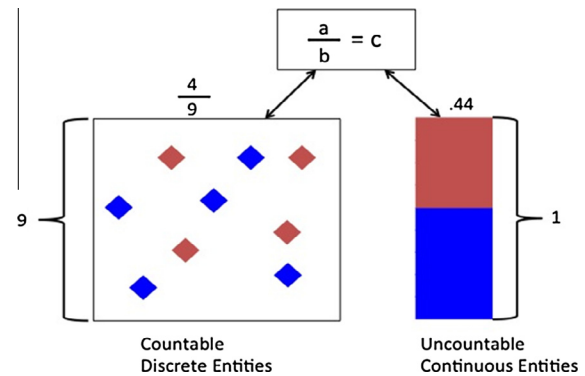

Fig. 1. Hypothesized alignment of fractions and decimals with discrete and continuous entities. Copyright © 2015 by the American Psychological Association. Reproduced with permission from Rapp et al. (2015).

1.3. The need for cross-national comparisons

The conceptual and processing differences between the different notations for rational numbers have been interpreted as reflecting basic representational differences between alternative formats for such numbers. Fractions may be better suited to represent two-dimensional relations (DeWolf et al., 2015a), whereas decimals may be more closely linked to one-dimensional magnitude values (DeWolf et al., 2014). In addition, the mental representations of fractions and decimals may inherently align with discrete and continuous quantities, respectively (Rapp et al., 2015).

However, the interpretation of these findings as reflections of deep representational distinctions remains speculative, as all the phenomena we have reviewed have been demonstrated only with American students. It is well-known that students in the United States lag behind students in various Asian countries (including South Korea, Singapore, and Japan) in their math achievement (OECD, 2012). Perhaps the gaps observed between performance on various tasks (e.g., the superiority of decimals in magnitude comparison, or of fractions in relational reasoning) reflect deficiencies in the knowledge American students have attained about rational numbers. Similarly, the distinction between discrete and continuous entities has linguistic and cultural correlates (Geary, 1995); hence it is possible that non-English-speaking students from a different culture would not align distinct mathematical symbols with distinct types of quantity. Such interpretive issues can be addressed by cross-national and cross-cultural research (cf. Bailey et al., 2015; Hiebert et al., 2003; Richland, Zur, & Holyoak, 2007; Stigler, Fernandez, & Yoshida, 1996). In order to develop general theories in the field of higher cognition, it is critical to distinguish between phenomena that are specific to particular educational practices in specific contexts, and those that reflect representational capacities of the human mind that are not determined by specific educational practices or cultural contexts. The methodological approach of identifying those aspects of cognitive performance that are the same or different across populations varying in culture, language, and educational practices is especially informative in answering these types of basic questions.

1.4. Overview of the present study

Here we report a cross-national comparison of conceptual and processing differences between fractions and decimals. We systematically replicated several studies conducted in the United States that compared performance with the two types of rational numbers, using tasks involving both magnitude comparison and relational reasoning, with samples drawn from college students in South Korea. Several factors make South Korea a particularly

interesting nation to use in a cross-national comparison with the United States. The Korean language is structurally very different from English, and the culture and education system differ with respect to several factors that may impact students' conceptions of rational numbers. First, in comparison to the U.S., South Korea has excelled in mathematics achievement in recent years. According to the 2012 PISA results (OECD, 2012), South Korea ranked 5th in mathematics achievement (compared to the 36th-place standing of the U.S.). There is evidence that much of this superior achievement in Asian countries can be explained by educational techniques that emphasize achieving deeper conceptual understanding and mastery before moving on to more complex concepts (Bailey et al., 2015; Perry, 2000; Stigler et al., 1996). If Korean students have superior skill in whole number division, they will be able to translate fractions to decimals quickly and efficiently. Such a difference in procedural fluency might result in national differences in performance on conceptual tasks (Bailey et al., 2015).

The language differences between Korean and English are another factor that could lead to conceptual differences between different types of rational numbers. In particular, the Korean number-naming system is structurally different from that of English. Whereas the English number system has irregularly formed decade names (e.g., ten, twenty, thirty, and forty) and teen names (e.g., eleven, twelve, thirteen, and fourteen), many Asian languages (including Korean, Chinese, and Japanese) use numerical names that are more consistently organized using the base-10 numeration system. For example, in Korean, the name of the number 11 is literally translated as "ten-one", and 12 as "ten-two". Also, 20 is translated as "two-ten(s)" and 30 as "three-ten(s)" (where plurals are tacitly understood).

The Korean spoken numerals thus inherently represent the base-10 numeration system, which may affect the conceptual representation of numbers. For example, Miura, Okamoto, Kim, Steere, and Fayol (1993) claimed that the transparent base-10 structure of Asian counting systems affects children's understanding of place value. In their study, when first graders were asked to represent numbers using blocks, Chinese, Japanese, and Korean-speaking children used canonical base-10 constructions more often than did English, French, and Swedish-speaking children. For example, to represent the number 42, Asian children used 4 tens blocks and 2 ones blocks, whereas English-speaking children used 42 ones blocks. The base-10 structure of the linguistic system for number names is echoed in the measurement system used in Korea. Base-10 (metric) units are used exclusively in South Korea, whereas non-base-10 (e.g., imperial) units are widely used in the U.S., and are known to affect students' interpretation and use of fractions and decimals (Rapp et al., 2015). These differences might be expected to impact performance with different types of rational numbers.

The fraction-naming systems also differ between Korea and the U.S. In the Korean language, the fractional parts are explicitly represented in fraction names (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Mix & Paik, 2008; Paik & Mix, 2003). For example, the Korean name for "one third" translates as "of three parts, one"; and "two thirds" translates as "of three parts, two". These types of fraction names transparently convey part-whole relations that are not made so obvious in the English equivalents (although it is not clear whether differences in fraction names influence children's performance; Paik & Mix, 2003).

Finally, South Korean curricula place a strong emphasis on the interchangeable use of different rational numbers. For example, the Korean mathematics textbook (Korean Ministry of Education, 2014) formally introduces fractions and decimals in the same chapter for the third-grade level (fractions first, immediately followed by decimals); this is the first time when students are taught about either fractions or decimals. Throughout the elementary-school curricula, students are expected to learn how fractions and decimals are conceptually related and to use them interchangeably. Some word problems use both fractions and decimals to represent quantities of the same type (e.g., 0.5 l and 2/3 l of water). In contrast, in curricula used in the United States, fractions are typically introduced around first grade whereas decimals are introduced only in third grade, after fractions are thought to be well understood. Other than a brief transition explaining the relation between fractions and decimals, the two number formats are typically considered separately (Common Core State Standards Initiative, 2014; Scott Foresman-Addison Wesley, 2011). This difference in pedagogy may have an impact on conceptual understanding and performance with different types of rational numbers, perhaps reducing or eliminating processing differences between fractions and decimals in Korea relative to the United States.

In the present study we used samples of college students in South Korea to replicate several experimental paradigms, previously used in the United States, in which performance with fractions and decimals was compared. Our overarching aim was to determine whether the reported differences in processing between the two types of rational numbers reflect specific characteristics of the samples used in the original studies conducted in the United States, or reflect more fundamental representational differences between alternative formats for rational numbers.

The work reported here examines both magnitude processing and reasoning with fractions and decimals. First, we performed a textbook analysis to establish whether Korean textbooks show an ontological alignment between fractions and decimals and discrete and continuous entities, respectively, as appears to be the case for textbooks used in the United States (Rapp et al., 2015). We then performed a series of experiments investigating whether Korean adults show sensitivity to this alignment. If decimals map more readily than fractions to continuous representations, then one important task that would be expected to show a decimal advantage is magnitude comparison (typically assumed to depend on a continuous mental number line, see Dehaene, 1992). In Experiment 1 we tested whether adults in Korea would show the same advantage in magnitude processing for decimals and integers over fractions as do American adults (DeWolf et al., 2014). Given that Korean students learn fractions and decimals together, and the fraction-naming system in Korean provides a more explicit mapping of symbol to referent, one might expect that Korean students would develop a more unified representation of number types, and hence not show differences in processing efficiency. Indeed, decimals and fractions could both be aligned with a continuous representation, so it is not at all obvious that a performance difference between the two number types would be observed in the Korean population. By focusing instruction on the translation between fractions and decimals, the Korean system may encourage students to map fractions onto continuous representations. On the other hand, if the cognitive representation of decimals is inherently more closely linked to a continuous representation of one-dimensional magnitude than is the representation of fractions, then similar differences in processing efficiency may be observed in the Korean sample.

Additional work was directed at the conceptual nature of fractions and decimals. In particular, we sought to determine whether Korean students, like their American counterparts, honor a natural alignment of fractions with discrete quantities and decimals with continuous ones. Using paradigms adapted from Rapp et al. (2015) and DeWolf et al. (2015a), Experiments 2–4 tested whether Korean college students show the same natural alignment between rational numbers and entity types as do American students, despite differences in language and use of different measurement systems in the two countries (invariably metric in Korea vs. often imperial in the United States). Finally, Experiment 5 investigated whether Korean college students show differences in reasoning

about fractions in a task designed to highlight the advantages of the bipartite fraction format over the unidimensional decimal format when aligned with various discrete and continuous representations (DeWolf et al., 2015a).

2. Textbook analysis

We examined the possible alignment of fractions to discrete quantities versus decimals to continuous ones using the standard Korean textbook for grades 3–6 in which fractions and decimals are formally introduced. Rapp et al. (2015) argued that this selective alignment may be due in part to how these quantities are used to represent entities in the real world. In an analysis of textbook word problems, they found that educators in the U.S. are more likely to create problems in which continuous entities are represented with decimals and discrete entities are represented with fractions. Given that learning history may affect subsequent alignment, we examined a set of word problems in a comparable Korean mathematics textbook to assess whether this alignment also holds for problems that Korean mathematics educators present to students as situation models of rational numbers.

2.1. Method

We examined the Korean mathematics textbook 2014 series from 3rd through 6th grade (Korean Ministry of Education, 2014). This textbook series is a national standard used in elementary schools in South Korea. Each grade consists of two semesters, in which one main textbook and one workbook are used. The 3rd to 6th grade levels were selected because they cover the main introduction and use of rational numbers in math curricula prior to the start of formal algebra in both the United States and South Korea. Although in the U.S. curricula fractions initially appear around the first grade, the main introduction of both fractions and decimals begins in the third grade (Common Core State Standards Initiative, 2014; Scott Foresman-Addison Wesley, 2011), as is also the case in Korea. To allow a comparable analysis, we focused only on the 3rd through 6th grade levels from Rapp et al. (2015). In South Korea, neither fractions nor decimals are covered at all prior to the 3rd grade.

We analyzed all the problems that involved either fraction or decimal numbers (but not both); problems that included both fractions and decimals were excluded from the analysis. Such problems comprised only 9.5% of the total problems (the analogous problems excluded from analyses by Rapp et al. comprised 2% of the total). This criterion yielded a total of 274 problems for analysis (159 with fractions and 115 with decimals).

2.2. Problem coding

We used a coding scheme that was developed by Rapp et al. (2015). Problems were categorized by their number type (fraction vs. decimal) and entity type (continuous vs. countable). Problems were classified as *fraction* or *decimal* based on the number type that appeared in the problem text or was called for in the answer. Problems were classified as *continuous* or *countable* based on the entities in the problems. Continuous problems involved entities that are referred to with mass nouns (e.g., weight, volume, length), whereas countable problems involved either discrete (e.g., marbles, balloons) or explicitly discretized entities (e.g., an apple cut into slices). Two researchers coded all of the problems separately using the above coding schemes. Each coder was blind to the other coder's judgments. The two coders agreed on 263 (96%) of the textbook problems. A third researcher, who was blind to the first two coders' judgments, then coded the 11 problems on which the first two

coders had differed. These problems were then placed into whichever category to which it was assigned by two of the three coders.

2.3. Results and discussion

The distributions of the textbook problems are shown in Fig. 2. Of the 274 problems, 115 used decimals and 159 used fractions. Continuous entities comprised a large majority of the decimal problems (96%), but this percentage dropped drastically in fraction problems (63%). In a complementary way, countable entities appeared more often in fraction problems (37%) than in decimal problems (4%). A chi-square test of independence between number type and continuity confirms that the two factors are significantly associated, $\chi^2(1) = 40.01$, $p < .001$, $phi = .382$ ($phi = .334$ in U.S.). The pattern of results and effect sizes closely matched across the U.S. and Korean samples.[1]

This Korean textbook analysis thus yielded a pattern qualitatively similar to that observed in the United States for the same grade levels, 3–6 (Rapp et al., 2015), such that the preference for continuous entities is greater in decimals and the preference for countable entities is greater in fractions. While the decimals showed an almost exclusive alignment with continuous entities, fractions were applied to countable entities far more often than were decimals in both Korean and American textbooks.

3. Experiment 1

In Experiment 1, we examined the performance of Korean college students on a magnitude-comparison task. College students in the United States make magnitude comparisons much more quickly and accurately with decimals and integers than with fractions, and show an exaggerated distance effect (i.e., decrease in response time as a function of numerical magnitude) with fractions (DeWolf et al., 2014). These differences have been interpreted as evidence that magnitude representations for fractions are accessed more slowly and are less precise. If the slower magnitude-comparisons for fractions relative to decimals observed in American students reflect relative inexperience with calculating fractions magnitudes, or poor understanding of fractions, then we would expect that such a difference would diminish or even disappear in Korea. If, however, we observe similar patterns of responses in Korean and American students, the results would lend support to the view that these performance differences reflect deep conceptual differences in the representations of different types of rational numbers - the more natural alignment of decimals than of fractions with the continuous number-line representation of magnitude.

3.1. Method

3.1.1. Participants

A total of 50 undergraduate students (male = 15; mean age = 21.82) from Yonsei University in South Korea participated

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[1] Because the experiments were conducted separately between the two nations and sample sizes varied, it would have been inappropriate to make direct statistical comparisons between the Korean and U.S. samples. There was also one major design difference: for Experiment 1, a within-subjects design was adopted for the Korean sample, whereas a between-subjects design was used for the U.S. sample. Because of these methodological and design differences, we report effect sizes for both samples in order to better compare performance across the two nations. Overall, the magnitudes of effect sizes were highly similar for the two nations across the textbook analyses and the five experiments we report. The biggest difference in magnitude of effect size was found in Experiment 5, where the Korean students showed much higher accuracy for the continuous display type for decimals (relative to fractions), and much longer response times for the countable display types for decimals (relative to fractions). Nonetheless, the qualitative patterns of the results were still consistent between the U.S. and Korean samples.
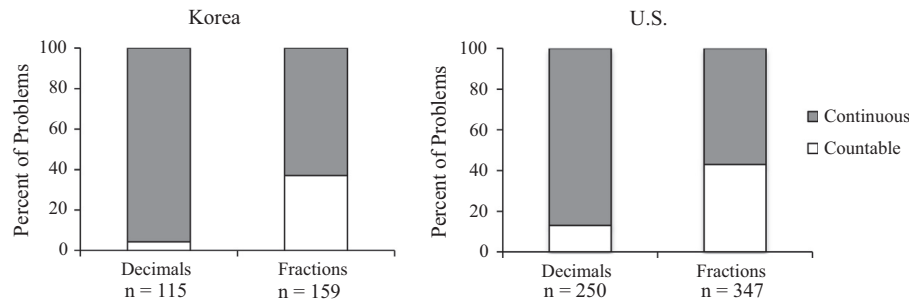
Fig. 2. Percentage of decimal and fraction problems that were continuous or countable in a textbook analysis (grades 3–6) for South Korea (left panel), and in a comparable analysis of textbooks in the United States (grades 3–6 only from Rapp et al., 2015; right panel).

in the study for course credit. The U.S. sample used for comparison consisted of 95 undergraduates from the University of California, Los Angeles (UCLA, male = 23; mean age = 21), who received course credit (Experiment 2, DeWolf et al., 2014).[2]

3.1.2. Design, materials, and procedure

Participants completed magnitude comparisons across different types of numbers. Experimental materials were adapted from Experiment 2 of DeWolf et al. (2014), which was itself modeled closely on a study reported by Schneider and Siegler (2010). The study was conducted as a within-subjects design with three types of numbers: fractions, decimals, and three-digit integers. These three types of numbers were presented as separate blocks, and the order of the three blocks was counterbalanced across participants. Each block started with four practice comparisons, followed by the target comparisons. Order of problems was randomized within each block.

All of the comparisons were done against the reference value of 3/5 for fractions, 0.613 for decimals, and 613 for integers. The target numbers ranged between 20/97 and 46/47 for fractions. Decimals were the magnitude equivalents of the fractions. The decimals were all rounded to three digits to make the decimals consistent in length regardless of their fraction equivalent. In addition, using three digits helped to standardize the number of digits participants had to process across number type (the fractions had between two and four digits). Integers were simply 1000 times the value of the decimals. The complete set of fractions, decimals, and integers used in Experiment 1 are listed in DeWolf et al. (2014, p. 76, Table 2).

On each trial, the target number was displayed at the center of the screen and participants were asked to select the *a* or *l* key, respectively, to indicate whether the number was smaller or larger than the reference value. Written reminders appeared on the screen ("smaller than 3/5", or the reference value appropriate to the number type, on the left side; "larger than 3/5" on the right side). Participants were told to complete the comparison as quickly and accurately as possible. No fixed deadline was imposed on time to reach a decision.

3.2. Results and discussion

Fig. 3 shows the mean error rates for each target, for each of the three number types. Error rates were much higher for fractions
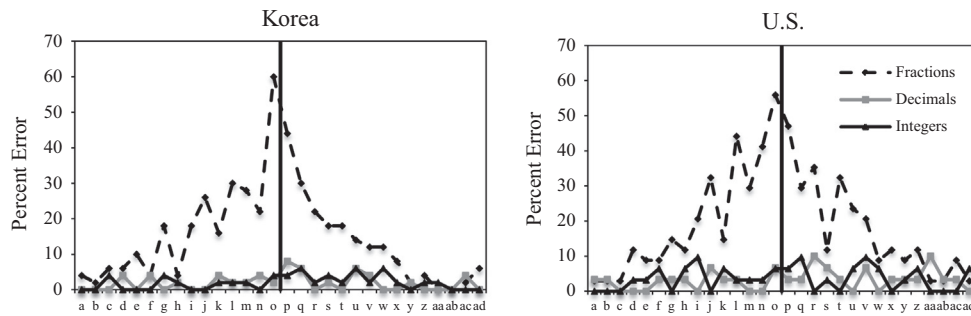
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[2] Across the five experiments we reported, the Korean samples consisted of college students at Yonsei University (Experiments 1–5), whereas the U.S. samples were either from UCLA (Experiments 1, 4, and 5) or the University of Washington (Experiments 2 and 3). According to the U.S. News & World report rankings (as of 2015), Yonsei University is ranked 5th in Korea, and UCLA and University of Washington are ranked 23rd and 52nd in the U.S., respectively. More detailed comparisons across the schools are not possible because there is no standardized exam used across American and Korean schools. Also, Yonsei University does not make admissions information public. However, in qualitative terms, students at all three schools are relatively high-achieving relative to their respective countries.

($M = 14.87$, $SD = 8.96$) than for decimals ($M = 1.87$, $SD = 2.53$) or integers ($M = 1.93$, $SD = 2.44$). A one-way within-subjects ANOVA revealed that differences among number types were highly reliable, $F(2,98) = 95.46$, $p < .001$, $\eta_p^2 = .661$ ($\eta_p^2 = .471$ in U.S.). The fraction number type showed significantly higher error rates than the decimal number type, $t(49) = 9.67$, $p < .001$, $d = 1.97$ ($d = 1.71$ in U.S.), and the integer number type, $t(49) = 10.43$, $p < .001$, $d = 1.97$ ($d = 1.68$ in U.S.), but there was no significant difference between the decimal and integer number types, $t(49) = 0.16$, $p = .875$, $d = 0.02$ ($d = 0.15$ in U.S.). The comparable effect sizes across the samples indicate that the pattern and magnitude of the differences across the two nations were highly similar.

The pattern of response times (RTs) across all three number types is depicted in Fig. 4. RTs were considerably slower for fractions than for decimals and integers. Collapsing over all targets, RTs were slower for fractions ($M = 3.48$, $SD = 2.51$) than for decimals ($M = 0.89$, $SD = 0.20$) or integers ($M = 0.79$, $SD = 0.16$). A one-way within-subjects ANOVA revealed reliable overall differences among number types, $F(2,98) = 59.22$, $p < .001$. $\eta_p^2 = .547$ ($\eta_p^2 = .560$ in U.S.). The fraction number type yielded significantly longer times than the decimal number type, $t(49) = 7.53$, $p < .001$, $d = 1.45$ ($d = 1.98$ in U.S.), and the decimal number type took significantly longer than the integer number type, $t(49) = 3.72$, $p = .001$, $d = 0.55$ ($d = 0.06$ in U.S.), although the difference was only 0.1 second between these two number types. Even for relatively common fractions such as 1/4, 1/2, and 2/3 (coded in Fig. 4 as b, h, and s, respectively), response times for their decimal counterparts were considerably faster than fraction response times (1/4 vs. .250: 2.39 s vs. 0.91 s, $t(48) = 2.87$, $p = .006$; 1/2 vs. .500: 2.71 s vs. 0.71 s, $t(46) = 6.19$, $p < .001$; 2/3 vs. .667: 3.85 s vs. 1.04 s, $t(39) = 5.66$, $p < .001$).
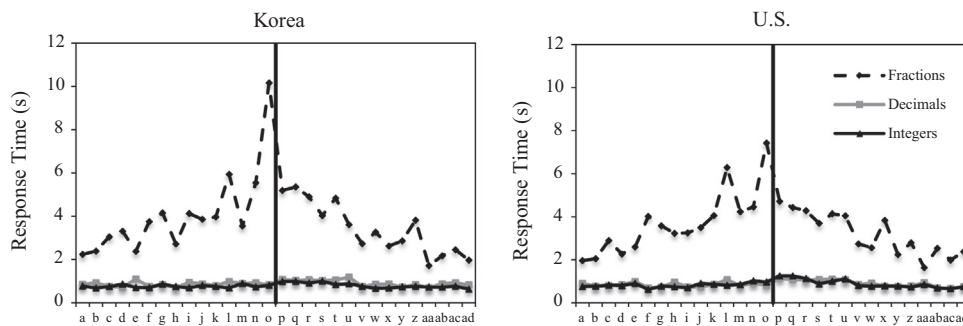
As shown in Fig. 3, error rates for comparisons of fractions showed a clear distance effect, whereas errors for the other two number types were uniformly low. Because accuracy was at ceiling for both decimals and integers, we focused on distance effects based on the RT measure (see Fig. 4). In order to assess the functional form of the distance effect, we conducted regression analyses for response times based on the logarithmic distance measure (i.e., log (|target-reference|), which we will abbreviate as "log Dist" (Dehaene, Dupoux, & Mehler, 1990; Hinrichs, Yurko, & Hu, 1981; Schneider & Siegler, 2010). As in DeWolf et al. (2014), log Dist accounted for a significant amount of variance in response time for all three number types: 75% for fractions, 18% for decimals, and 31% for integers (for fractions, beta = −.87, $t(28) = 9.30$, $p < .001$; for decimals, beta = −.45, $t(28) = 2.68$, $p = .01$; for integers, beta = −.58, $t(28) = 3.72$, $p = .001$).

Overall, the results of Experiment 1 were consistent with those of the comparable study conducted in the United States (DeWolf et al., 2014). Korean students showed lower overall error rates than the American students (15% vs. 19% for fractions; 2% vs. 3% for decimals; 2% vs. 4% for integers). However, both groups consistently

Fig. 3. Distributions of percent errors across target magnitudes for fractions, decimals, and integers in South Korea (left panel) and the U.S. (right panel). The solid vertical line marks the reference value. The U.S. results are from DeWolf et al. (2014, Ex. 2). See Table 2 in DeWolf et al. (2014, p. 76) for identities of targets indicated by code on x-axis.

Fig. 4. Distribution of mean response times across target magnitudes for fractions, decimals, and integers for South Korea (left panel) and the U.S. (right panel). The solid vertical line marks the reference value. The U.S. results are from DeWolf et al. (2014, Ex. 2). See Table 2 in DeWolf et al. (2014, p. 76) for identities of targets indicated by code on x-axis.

showed far more difficulty for comparisons of fractions than comparisons of decimals or integers, and the distance effect was much more pronounced for fractions as shown in the regression analyses for response times. As was the case for adults in the United States, Korean students produced the most similar response patterns for decimals and integers, even though fractions and decimals are more similar in the sense of representing rational numbers.

The present results are based on a wide range of fraction and decimal values. One might expect that complexity and familiarity of numbers could affect the differences among number types. For example, if simple fractions like 20/100 and 9/10 had been used, it is possible that performance in the fraction condition would have been considerably better, reducing the processing differences among the three number conditions. Yet, as was the case in the American population, even magnitudes of simple fractions such as 1/4, 1/2, and 2/3 were more difficult to process than the magnitudes of equivalent decimals. Thus, despite the greater overall math expertise of Korean students, and other cultural, instructional, and linguistic differences, the same basic differences in magnitude comparisons for fractions and decimals were observed in South Korea as in the United States.

4. Experiment 2

Rapp et al. (2015) found that American students show the same alignments as found in the textbook analysis when asked to generate word problems. Experiment 2 was designed to test whether Korean college level adults also show the same pattern of alignments. If Korean students do show the same alignments as US students, this lends support to the hypothesis that these alignments are based on a deeper representational difference between fractions and decimals rather than an artifact of the education or cultural system specific to the given country. In order to test this hypothesis, we asked Korean undergraduate students to

generate word problems that contained either fractions or decimals, and examined the entities (continuous vs. countable) they described in their generated problems. Experimental materials were adapted from Experiment 1 of Rapp et al. (2015).

4.1. Method

4.1.1. Participants

A total of 71 undergraduate students (male = 25; mean age = 21.39) from Yonsei University participated in the study for course credit. A randomly-selected half of these participants generated decimal word problems and the other half generated fraction word problems. The U.S. sample used for comparison consisted of 156 undergraduates from the University of Washington (male = 84; mean age = 19), who received course credit (Experiment 1, Rapp et al., 2015).

4.1.2. Design, materials, and procedure

The study was a between-subjects design with one factor: number type (fraction vs. decimal). Participants were given a single sheet of paper with three examples of simple word problems provided at the top. The three examples involved one countable entity (30 marbles), one continuous entity (5 kilometers), and one discretized mass entity (four 2-kilogram bags of sugar). All of these examples were presented with whole numbers. Participants were then asked to generate two word problems with their own numbers. Depending on the condition, they were told that the numbers in their problems had to be fractions (e.g., 1/4, 1^1/$_2$, 5/2), or decimals (e.g., 0.25, 1.5, 2.5). Participants completed the study using paper and pencil. There was no time limit.

4.2. Results and discussion

There were a total of 142 problems constructed (70 decimals, 72 fractions). The constructed problems were coded using the

classification scheme developed by Rapp et al. (2015). Problems were classified as fraction or decimal based on the number type that appeared in the problem text. Problems were classified as continuous or countable (i.e., discrete) based on the entities that appeared in the constructed problems, using the same criteria as were used for the textbook analysis.

The left panel of Fig. 5 shows the distribution of countable and continuous problems in the decimal and fraction problems. Overall, students generated more continuous problems with decimals than fractions. As in the textbook analysis, continuous entities appeared more often in decimal problems (91%), but this percentage dropped drastically in fraction problems (63%). In a complementary way, countable entities appeared more often in fraction problems (36%) than in decimal problems (9%). A chi-square test confirmed that number type (decimal vs. fraction) and continuity (continuous vs. countable) were significantly associated, $\chi^2(1)$ = 15.42, $p < .001$, phi = .330 (phi = .381 in U.S.). Thus, the size of the effects was both strong across the Korean and U.S. samples. For comparison, the right panel shows the results from U.S. undergraduates (Experiment 1 of Rapp et al., 2015).

Overall, there was a consistent pattern of alignment across the two nations in that students tend to use decimals to represent continuous entities and fractions to represent discrete or countable entities. However, Korean students showed an overall preference towards using continuous rather than countable quantities. Specifically, Korean students used continuous quantities more often than countable quantities when creating fraction word problems (64% vs. 36%). Despite this difference, Korean students, like their American counterparts, used countable quantities more often with fractions than decimals and continuous quantities more often with decimals.

5. Experiment 3

Experiment 3 was designed to rule out the possibility that the alignments observed in word problems (textbook analysis and Experiment 2) had more to do with cultural conventions than with a conceptual distinction between fractions and decimals as representations of discrete and continuous variables, respectively. We assessed this possibility using a task introduced by Rapp et al. (2015, Experiment 2). We manipulated whether fractions and decimals were paired with continuous units (e.g., *kilometer*) or with discrete units (e.g., *sandwich*). We then asked participants to choose either a continuous or discrete graphical representation to depict the given quantity. This paradigm makes it possible to test whether number type, unit type, or both contribute to alignments with discrete or else continuous representations.

5.1. Method

5.1.1. Participants

A total of 57 undergraduate students (male = 14; mean age = 21.12) from Yonsei University participated in the study for course credit. The U.S. sample used for comparison consisted of 157 undergraduates from the University of Washington (male = 42; mean age = 19.4), who received course credit (Experiment 2, Rapp et al., 2015).

5.1.2. Design, materials, and procedure

The study was a 2 (number type: fraction vs. decimal) × 2 (entity type: continuous vs. countable) within-subjects design. There were two trials of each experimental condition, resulting in a total of eight trials per participant. Experimental materials were constructed by adapting the materials used in Experiment 2 of Rapp et al. (2015). Translations of the English versions were created by two Korean-English bilingual researchers, and then back-translated into English to ensure accuracy. Because imperial units (pound, mile) are seldom used in Korea, these were replaced with metric units (liter, degree in Celsius).

Each participant saw eight different expressions, each including either a fraction or a decimal and either a countable (pens, sandwich, books, banana) or continuous (kilometer, liter, degree in Celsius, kilogram) entity type. Four fractions were used (3/4, 5/8, 4/9, 2/7), and their magnitude-equivalent decimals (.75, .63, .44, .29). For example, a participant might see "3/4 kilometer" or ".75 sandwich".[3] Assignments of entity type and number type were counterbalanced so that half of the participants received a fraction with a particular entity (e.g., 3/4 sandwich) and half received the equivalent decimal with that same entity (e.g., .75 sandwich). Thus, each participant saw eight of the 16 possible pairings of number and entity type.

The dependent variable was whether participants selected a continuous circle representation or a discrete dots representation for the number type-entity type expressions (see Fig. 6). Critically, the representation options were the same for all of the statements. Both of the representations depicted the value of 1/2 (.50), which was not used in any of the fractions or decimals given in the statements. The choice of representation type thus could only be guided by its abstract form (continuous or discrete), rather than by matches of specific values. Participants were given eight expressions that paired number type and entity type. For each expression participants were instructed to choose which type of diagram (circle or dots) they would prefer to use to represent it.

5.2. Results and discussion

The left panel of Fig. 7 shows the percentage of total times the continuous representation (circle) versus discrete representation (dots) was chosen for a given combination of entity type and number type. Collapsing across entity type, for decimal expressions participants selected the continuous representation (circle) 64% of the time, whereas for fraction expressions participants chose the continuous representation (circle) 46% of the time. A 2 × 2 within-subjects ANOVA was performed on data coded as the proportion of trials on which the continuous representation (circle) was selected. For simplicity, we report the preference for continuous only. There was a significant main effect of number type, $F(1,56)$ = 8.84, p = .004, η_p^2 = .136 (η_p^2 = .134 in U.S.), indicating that the continuous representation was selected more frequently for decimals than for fractions. There was no main effect of entity type ($F < 1$), nor any reliable interaction effect between number and entity type, $F(1, 56)$ = 1.55, p = .219, η_p^2 = .027 (η_p^2 = .022 in U.S.). The magnitude of the effect size was nearly identical between the Korean and U.S. samples.

For comparison, the right panel of Fig. 7 shows the comparable data from American students (based solely on items using metric units, to maximize compatibility with the items used in Korea). As in Experiment 2, Korean students showed the same basic pattern of alignments as had been found for American students. However, Korean students chose continuous versus discrete representations almost equally often when representing fractions, whereas the U.S. students chose discrete representation more often than continuous representation. Korean students thus showed an overall preference for continuous representations. Despite this, the overall pattern of results closely mirrors the results of the

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[3] In Korean, the translation for a number-continuous entity pair takes the form of "# _____", whereas that for a number-countable entity pair takes the form of "of _____, #". For example, "3/4 kilometer" literally translates as "3/4 kilometer" whereas ".75 sandwich" roughly translates as "of sandwich, .75."
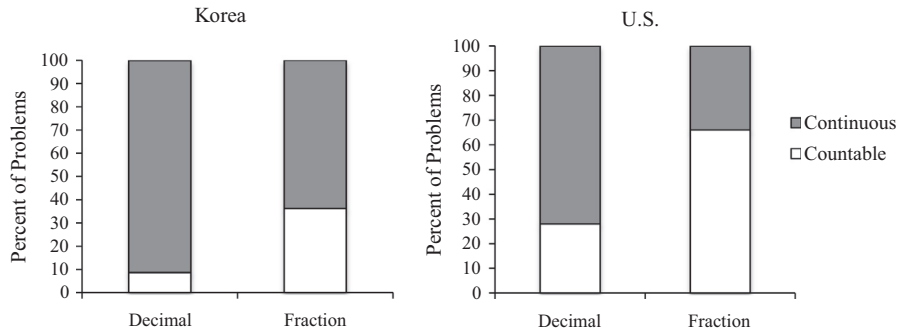
Fig. 5. Distribution of countable and continuous problems in decimal and fraction problems for students in South Korea (left panel) and the U.S. (right panel). The U.S. results are from Rapp et al. (2015, Experiment 1).
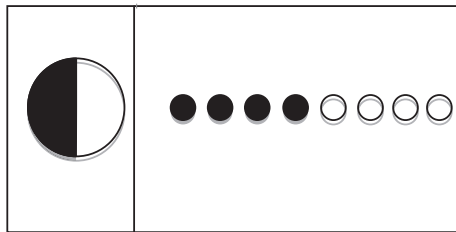

Fig. 6. Options provided to represent continuous (circle) and discrete (dots) representations in Experiment 3. Copyright © 2015 by the American Psychological Association. Reproduced with permission from Rapp et al. (2015).

U.S. results in that a continuous representation was preferred for decimals and a discrete representation was preferred for fractions.

6. Experiment 4

Experiment 3 showed that participants' preferences for representation types varied depending on the type of rational number used. In Experiment 4 we tested for alignment in the opposite direction, adapting a task employed by DeWolf et al. (2015a, Experiment 1). College students were asked to choose either a fraction or decimal for different types of displays that depicted ratio relations. If Korean students show the same alignments as do American students, then they should prefer fractions for discrete displays but decimals for continuous displays. Importantly, in this task students were asked to make a conceptual judgment about the alignment of the representation and the rational number. In Experiment 5, by contrast, participants performed a task that required both procedural and conceptual alignments. To isolate the role of

the conceptual preference for quantity type and rational number type, Experiment 4 tested the alignment using a forced-choice task that did not require any calculation or mathematical procedure. The goal of the experiment is to establish a conceptual link between a conceptual ontology of quantities and types of rational numbers.

6.1. Method

6.1.1. Participants

A total of 60 undergraduate students (male = 18; mean age = 21.08) from Yonsei University participated in the study for course credit. Participants were randomly assigned in equal numbers to two between-subjects conditions (part-to-part vs. part-to-whole ratio; see below). The U.S. sample used for comparison consisted of 48 undergraduates from the UCLA (male = 11; mean age = 20.4), who received course credit (Experiment 1, DeWolf et al., 2015a).

6.1.2. Design, materials, and procedure

The study was a 2 (relation type: part-to-part vs. part-to-whole ratios) × 3 (display type: continuous, discretized, discrete) design, where relation type was a between-subjects factor, and display type was a within-subjects factor. A part-to-part ratio (PPR) is the relation between the sizes of the two subsets of a whole, whereas a part-to-whole ratio (PWR) is the relation between the size of one subset and the whole.

Fig. 8 depicts examples of the three display types. The discrete items were displays of circles, squares, stars, crosses, trapezoids, and cloud-like shapes. The continuous items were displays of rectangles that could differ in width, height and orientation (vertical or horizontal). The discretized items were identical to the continuous
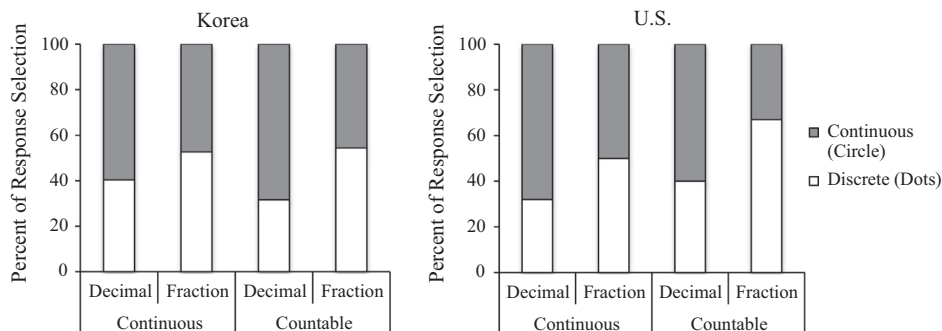

Fig. 7. Percentage of response selection by number type for trials with continuous entities and countable entities in Experiment 3 in South Korea (left panel) and the U.S. (right panel). The U.S. results are from Rapp et al. (2015, Experiment 2, metric units only).

Fig. 8. Examples of continuous, discretized and discrete displays used in Experiment 4. Copyright © 2015 by the American Psychological Association. Reproduced with permission from DeWolf et al. (2015a).

displays except that the rectangles were divided into equal-sized units by dark lines. For the stimuli used in test trials, red[4] and green were used to demarcate the two different subsets. The displays varied which color represented the larger subset versus the smaller subset.

Participants were given instructions for either the part-to-part ratio (PPR) or part-to-whole ratios (PWR) condition. They were given a Korean translation of the following instructions for the PPR condition: "In this experiment, you will see displays that show various part to part relations. In the display below [display with 1 orange circle and 2 blue crosses] this would be the number of orange circles relative to the number of blue crosses. Such relations can be represented with fractions (e.g., 3/4) or with decimals (e.g., .75). For each display your task is to choose which notation is a better representation of the depicted relation—a fraction or a decimal. Note that the specific values (i.e., 3/4 and .75) are just examples and do not match the values in the displays." For the PWR condition, the instructions were identical except for the description of the relations. In this condition the part-to-whole relation was defined using the example of the number of orange circles relative to the total number of blue crosses and orange circles. The relation type (PPR vs. PWR) was manipulated between subjects; thus participants in the PPR condition were only told about PPRs and participants in the PWR condition were only told about PWRs. Participants were shown examples of the continuous and discretized displays, in addition to the discrete display, and were told that displays could appear in any of those formats.

The task was simply to decide whether the relationship should be represented with a fraction (3/4) or a decimal (.75). In order to assess this preference on a conceptual level, the specific fraction and decimal shown to participants (3/4 and .75) were held constant across all trials, and never matched the number of items in the pictures. Thus, no mathematical task needed to be performed. There was therefore no requirement for accuracy, nor was any speed pressure imposed. Since the quantity shown in a display never matched the particular fraction and decimal values provided as response options, there was no real need to even determine the specific value represented in a display. The paradigm of Experiment 4 was thus intended to investigate participants' conceptual representations for fractions and decimals, in a situation in which mathematical procedures were not required.

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[4] For interpretation of color in Fig. 8, the reader is referred to the web version of this article.

Stimuli were displayed on a computer screen and participant responses were recorded. Participants were given the instructions described above for either the PPR condition or the PWR condition. Participants were told to select the *z* key for decimals and the *m* key for fractions. Participants completed 60 test trials (20 for each display type). A fixation cross was displayed for 600 ms between each trial. Display types were shown in a different random order for every participant.

6.2. Results and discussion

Because participants were forced to choose either a fraction or a decimal for each trial, the preference for each is complementary. For simplicity, we report the preference for fractions. The proportion of trials in which participants selected the fraction notation was computed for each display type for each participant. The left panel of Fig. 9 shows the proportion of trials that participants chose either fractions or decimals for each display. A 2 (relation type: PPR vs. PWR) × 3 (display type: discrete, discretized, continuous) ANOVA was performed to assess differences in notation preference. There was a significant main effect of display type, $F(2,116) = 30.88$, $p < .001$, $\eta_p^2 = .347$ ($\eta_p^2 = .305$ in U.S.). Planned comparisons showed that preference for fractions was significantly greater for discretized displays than discrete displays, $t(59) = 2.23$, $p = .029$, $d = .386$ ($d = .303$ in U.S.), which in turn was greater than continuous displays, $t(59) = 4.94$, $p < .001$, $d = 1.055$ ($d = 1.039$ in U.S.). There was no interaction between relation type and display type, $F(2,116) = 1.17$, $p = .314$, $\eta_p^2 = .020$ ($\eta_p^2 = .010$ in U.S.), and no main effect of relation type, $F < 1$. Thus the pattern and magnitude of the effect size were nearly identical between the Korean and U.S. samples.

These results reveal that Korean students preferred to represent both PPR and PWR ratio relationships with fractions when a display showed a partition of countable entities, but with decimals when the display showed a partition of continuous mass quantities. Participants picked the number format that provided the best conceptual match to either continuous or discrete displays.

No mathematical task needed to be performed, and the specific quantities depicted in the displays did not match the numerical values of the fractions and decimals provided as choice options; hence our findings demonstrate that the preferential association of display types (discrete or continuous) and rational number formats (fractions or decimals) has a conceptual basis for Korean as well as American students (DeWolf et al., 2015a). This result closely aligns with the results of Experiments 2–3, in that college-educated adults show a preference for using continuous displays to represent decimals and countable displays to represent fractions. The patterns of results were consistent between Korea and the U.S. Experiment 4 thus provides strong support for the hypothesis that the natural alignment of different symbolic notations with different quantity types has a conceptual basis.

7. Experiment 5

Experiment 4 established a conceptual correspondence between quantity types and symbolic notations for rational numbers. In Experiment 5, adapted from a paradigm introduced by DeWolf et al. (2015a, Experiment 2), we examined whether this conceptual correspondence also makes one or the other symbolic notation more effective in a relational reasoning task, which also requires procedural thinking. College students were asked to evaluate ratio relationships using fractions or decimals, given different types of entities being represented. If there is a conceptual and procedural advantage to using fractions with discrete quantities in such a relational reasoning task, then students should perform better with
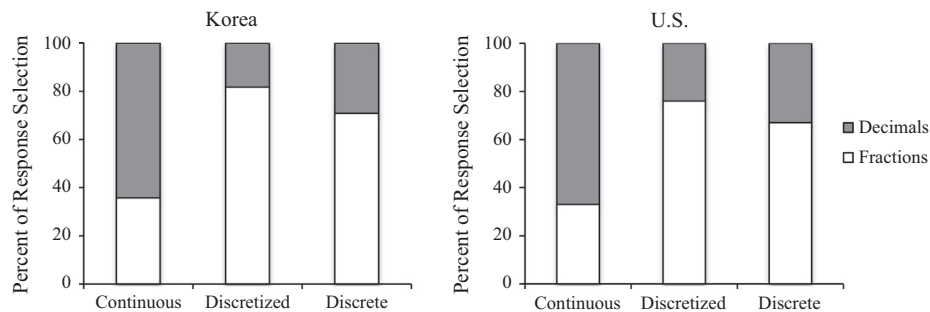
Fig. 9. Percentage response selection for each display type in which either a fraction or decimal were chosen in Experiment 4 in South Korea (left panel) and the U.S. (right panel). The U.S. results are from DeWolf et al. (2015a, Experiment 1).

fractions than decimals for discrete displays. DeWolf et al. found that American students were able to identify relations more effectively using fractions than decimals, at least for discrete displays.

7.1. Method

7.1.1. Participants

A total of 50 undergraduate students (male = 15; mean age = 21.82) from Yonsei University participated in the study for course credit. Participants were assigned in equal numbers to the two between-subjects conditions (fractions vs. decimals; see below). The U.S. sample used for comparison consisted of 58 UCLA undergraduates (male = 9; mean age = 20.4), who received course credit (Experiment 2, DeWolf et al., 2015a).

7.1.2. Design, materials, and procedure

The study was a 2 (symbolic notation: fractions vs. decimals) × 2 (relation type: part-to-part vs. part-to-whole ratios) × 3 (display type: continuous, discretized, discrete) mixed-subjects design. Symbolic notation was a between-subjects factor, and relation type and display type were within-subjects factors.

The displays were similar to those used in Experiment 4 (see Fig. 8). The magnitudes of fractions and decimals were matched. The values of the fractions and decimals were always less than one, and decimals were shown rounded to two decimal places. The values of the rational number presented on each trial represented one of two ratio relationships within the display: part-to-whole ratio (PWR) or part-to-part ratio (PPR). These were the same relationships used in Experiment 4, but the task in Experiment 5 explicitly required participants to identify on each trial which of the two relationships matched a presented number. Thus, a number was paired with the display that specifically matched one of the relationships. For example, Fig. 10 shows an example of a PWR trial with a display with 9 red units out of a total of 10. The number specified is 9/10 (or .90 in a matched problem using decimals), thus corresponding to a PWR. For the corresponding PPR problem, the number would be 1/9 (or .11 in decimal notation). The smaller subset would be the numerator in this case, so that the overall magnitude was always less than one.

Stimuli were presented electronically using the E-prime 2.0 software (Psychology Software Tools, 2012), and response times and accuracy were recorded. Participants received Korean translation of the following instructions: "In this experiment, you will see a display paired with a value. You need to identify which of the two following relationships is shown." Below this, there were two different displays showing the PWR and PPR relations, which were simply referred to as "Relation 1" and "Relation 2". The assignment of the labels was counterbalanced for all participants such that half was told Relation 1 was PPR and the other half was told Relation 1 was PWR. The PPR display contained 1 circle and 2 crosses. For the
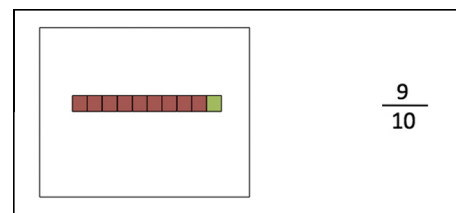
Fig. 10. Example of a ratio identification problem used in Experiment 5. Copyright © 2015 by the American Psychological Association. Reproduced with permission from DeWolf et al. (2015a).

fractions condition this was labeled as "1/2 amount of circles per amount of crosses;" for the decimals condition it was labeled as ".50 amount of circles per amount of crosses." The PWR was represented by a display of 2 circles and 3 crosses. For the fractions condition this was labeled as "2/5 of the total is the amount of circles;" for the decimal condition it was labeled as ".40 of the total is the amount of circles."

The first of these explanations of the PPR and PWR relations was shown with discrete items. The subsequent screen showed the same values paired with discretized displays. A third screen showed the same values paired with continuous displays. Half of the participants were told to select the z key for Relation 1 and to select the m key for Relation 2; the other half received the reverse key assignments.

After this introduction, participants were given an example problem and asked to identify the relation. After they made their judgment, an explanation was shown to participants about why the example showed the correct relation. The explanation also stated what the numerical value would be for the problem if it had shown the alternative relation. Participants were then given another example using the other relation, with the same explanation process. A series of practice trials were then administered. Participants had to complete at least 24 practice trials (four for each of the six within-subjects conditions). If they scored at least 17 correct (i.e., about 70%) they were able to move on to the test trials. If they did not score above this threshold, they continued with additional practice trials until they reached the threshold percentage correct. All of the practice trials were different from those used in the test trials. Feedback was given for correct trials, in the form of a green "O" on the screen and for incorrect trials, in the form of a red "X" on the screen. After the practice trials had been completed, a screen was displayed informing participants that the actual test trials were beginning. Participants were told to try to respond as quickly as possible without sacrificing accuracy. Participants completed 72 test trials (12 for each of the 6 within-subjects conditions). Feedback was continued for incorrect trials. Relation types and display types were shown in a different random order for every participant.

7.2. Results and discussion

Accuracy and mean response time (RT) on correct trials were computed for each condition for each participant. A 2 (symbolic notation: fractions vs. decimals) × 2 (relation type: part-to-part vs. part-to-whole ratios) × 3 (display type: continuous, discretized, discrete) mixed-subjects ANOVA was performed to assess differences in accuracy and RT. As the three-way interaction was not reliable, all analyses are reported after collapsing across the factor of relation type, which was not theoretically important given the goals of the study.

Fig. 11 displays the pattern of accuracy, which exceeded chance level (50%) for all conditions. There was a significant interaction effect between the display type and number type, $F(2,96)$ = 78.27, $p < .001$, $\eta_p^2 = .620$, ($\eta_p^2 = .342$ in U.S.). Planned comparisons indicated that accuracy was higher for fractions than decimals in the discretized condition, 88% vs. 72%; $t(48) = 6.77$, $p < .001$, $d = 1.913$ ($d = 0.979$ in U.S.), and in the discrete condition, 93% vs. 80%; $t(48) = 4.43$, $p < .001$, $d = 1.252$ ($d = 1.277$ in U.S.). However, an opposite pattern was observed in the continuous condition such that accuracy was higher for decimals than fractions, 83% vs. 70%; $t(48) = 4.86$, $p < .001$, $d = 1.373$ ($d = 0.324$ in U.S.). The pattern and size of the effects for the two countable displays were consistent across the U.S. and Korean samples. However, the Korean students showed an even higher reversal in accuracy for the continuous condition for decimals.

Fig. 12 displays the pattern of mean correct RTs across conditions. As for the accuracy analysis, a significant interaction was obtained between display type and symbolic notation, $F(2,96)$ = 45.06, $p < .001$, $\eta_p^2 = .484$, ($\eta_p^2 = .093$ in U.S.). Planned comparisons indicated that RTs were faster with fractions than decimals for the discretized condition, 3.22 s vs. 6.03 s; $t(48) = 4.15$, $p < .001$, $d = 1.174$ ($d = 0.201$ in U.S.) and for the discrete condition, 3.80 s vs. 6.77 s; $t(48) = 5.62$, $p < .001$, $d = 1.589$ ($d = 0.407$ in U.S.).

For the continuous conditions, however, RTs did not reliably differ, 3.95 s vs. 3.58 s; $t(48) = 0.72$, $p = .478$, $d = 0.202$ ($d = .574$ in U.S.). The pattern of results was replicated across the Korean and U.S. results, however the size of the effects was much stronger for Korean students for both of the countable display types.

Overall performance was somewhat more accurate for the Korean students (overall $M = 81.43$, $SD = 8.07$) than for the U.S. students (overall $M = 69.07$, $SD = 14.18$). Nonetheless, the results of Experiment 5 were consistent with the U.S. results in that there was an advantage for identifying ratio relationships in displays when these ratios were represented by fractions rather than decimals. When displays conveyed countable entities (sets of discrete objects, or continuous displays parsed into units of measurement), ratios were evaluated more accurately when the notation was a fraction rather than a decimal. In contrast, when the display showed continuous quantities, Korean students were more accurate when the notation was a decimal rather than a fraction. This finding suggests not only that students in both Korea and the U.S. show a conceptual preference for specific alignments between rational number types and quantity types, but also that this alignment impacts performance on a task that requires students to link their conceptual preference to a procedure that can be used to perform the calculations necessary for the task.

8. General discussion

The pattern of results across a textbook analysis and five experiments conducted in South Korea revealed a natural alignment between decimals and fractions to continuous and discrete entities, respectively, very similar to findings from comparable studies conducted in the United States. The magnitude of effect sizes was also remarkably similar between the United States and South Korea. Experiment 1 replicated the findings of DeWolf et al.
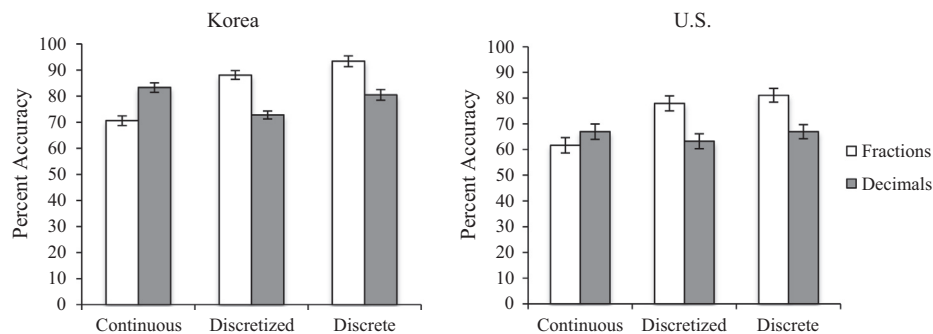

Fig. 11. Mean accuracy of relation identification using factions and decimals across different types of displays in South Korea (left panel) and the U.S. (right panel). The U.S. results are from DeWolf et al. (2015a, Experiment 2). Error bars indicate standard error of the mean.
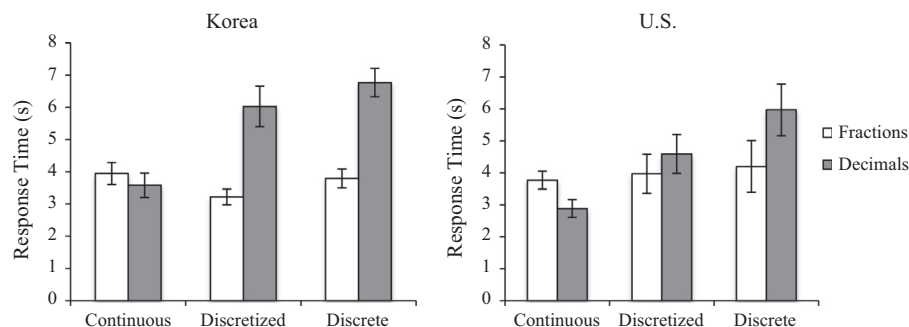

Fig. 12. Mean response time for relation identification using factions and decimals across different types of displays in South Korea (left panel) and the U.S. (right panel). The U.S. results are from DeWolf et al. (2015a, Experiment 2). Error bars indicate standard error of the mean.

(2014) in the U.S., showing that magnitude comparisons with fractions were slower than magnitude comparisons with both decimals and whole numbers. Whole-number and decimal magnitude comparisons, on the other hand, were virtually indistinguishable. Our cross-national comparison thus supports the conclusion that the advantage of decimals over fractions for magnitude comparison is due to inherent representational differences (decimals directly express a continuous one-dimensional magnitude, whereas fractions are two-dimensional), rather than being restricted to American students, who tend to have lower overall math expertise than South Korean students.

Experiments 2–4 provided more direct evidence that whereas decimals are typically used to represent continuous entities, fractions were more likely to represent discrete than continuous entities. Continuity versus discreteness is a basic ontological distinction that affects children's understanding of integers through counting of discrete entities, and (later on) through measurement of continuous entities that have been parsed into discrete units (e.g., Gelman, 1993, 2006; Mix, Huttenlocher, & Levine, 2002a, 2002b; Nunes, Light, & Mason, 1993; Rips, Bloomfield, & Asmuth, 2008). The distinction between continuity and discreteness is preserved throughout the mathematical curriculum. As in the initial cases of counting and measurement, discrete concepts are always taught before their continuous counterparts (e.g., first arithmetic progressions, then linear functions).

Although the overall patterns of results in our experiments were consistent between the U.S. and South Korea, Korean students (Experiments 2 and 3) showed a general preference for using continuous entities and representations of rational numbers. One possible explanation of this preference relates to the exclusive use of the metric measurement system in Korea, as metric units naturally align with continuous quantities (Rapp et al., 2015). As we have discussed, fractions do not align conceptually with continuous entities as well as do decimals. Further studies may investigate the impact that differences in measurement systems may have on fraction and decimal understanding.

The conceptual distinction between fractions and decimals has implications for performance in certain reasoning tasks that use these notations. Experiment 5 replicated the results of DeWolf et al. (2015a) who found that people are better able to distinguish bipartite relations between discrete sets when such relations are denoted with fractions than with decimals. Fractions maintain the mapping of distinct countable sets onto the numerator and the denominator, whereas decimals obscure this mapping.

The close correspondence between the findings from textbook analyses conducted in South Korea and the United States also provides strong evidence that fractions and decimals are preferentially used to model discrete and continuous entities, respectively. These findings lend support to the hypothesis of a natural conceptual alignment between quantity types and number types. Of course, it is possible that textbook writers (and subsequently their students) respond to their respective learning histories of selective use of fractions and decimals as models of discrete or continuous entities. In this case, the alignment may simply reflect automatic responses of selective association (Rothkopf & Dashen, 1995). An alternative interpretation is that textbook writers and mathematics educators adopt semantic alignment in order to ensure that learning makes real-world sense. If in everyday life fractions are more frequently used to represent relations between two sets, and decimals are more frequently used to measure quantities, then an important educational objective will be to provide students with realistic mathematical situations that demonstrate potential domains of application. Future studies should investigate why the alignment between entity type and number type appears in textbooks, and if there are any pedagogical advantages in adopting this alignment.

The results of the current study strongly support the basic hypothesis first proposed by DeWolf et al. (2014, 2015a, 2015b): fractions have a unique advantage over other rational number types, such as decimals, when they are used to represent relations between countable sets (e.g., the ratio of boys to girls in a classroom). Conversely, decimals (which correspond to one-dimensional magnitude representations) are very strongly linked to continuous quantities, which also serve as representations of one-dimensional magnitudes (e.g., the volume of water in a beaker). The present set of studies indicate that this distinction is not simply an artifact of the American educational system or cultural context, but rather reflects important representational differences in how students store, manipulate, and think about rational numbers and the types of quantities they naturally model in the real world.

While many previous studies have focused on the widespread difficulties students have with fractions compared to other mathematical concepts (Ni & Zhou, 2005; Siegler et al., 2013; Stafylidou & Vosniadou, 2004; Stigler et al., 2010), their focus has been solely on how students understand the magnitudes of fractions. The present set of studies highlights representational differences between fractions and other number types that may shed new light on the source of students' difficulties in understanding fractions. In particular, magnitude estimation tasks may be ill-suited for assessing fraction understanding. Fractions, with their bipartite structure, are literally and mentally used to represent relations, whereas decimals are better-suited to represent magnitudes. DeWolf et al. (2015b) found evidence of that these two aspects of mathematical understanding make differential contributions to acquisition of higher-level mathematical concepts, such as those involved in algebra. The present set of studies suggest that representational differences between types of rational numbers are not specific to cultural or educational variations.

The semantic alignment between number type and entity type may reflect computational simplicity. When representing relations between countable sets, it is possible to directly construct a fraction using numbers derived from counting (e.g., 3/4 to represent three girls in relation to a set of four girls and boys), whereas to represent such a relation with a decimal (e.g., 0.75) an additional transformation process (division) is required. Likewise, when measuring quantities, it is often possible to simply read the numbers (in decimal format) provided by measurement tools, whereas a transformation is required in order to represent such a measurement using a fraction. Due to such differences in cognitive simplicity, people may align decimals with continuous quantities and fractions with discrete entities. If this is the case, then students who are computationally proficient in translating between fractions and decimals may show a reduced tendency to honor semantic alignment. Future studies will need to investigate how alignment changes as learners acquire greater expertise in converting rational numbers between alternative formats.

Taken as a whole, the results of the present study provide strong evidence that a natural alignment holds between entity type and different formats of rational numbers. This alignment cannot be attributed to the specifics of education, language, and measurement units, which (as described in the Introduction) differ greatly between the United States and South Korea. Given evidence that students are particularly prone to misconceptions with rational numbers (Ni & Zhou, 2005; Siegler et al., 2013; Stafylidou & Vosniadou, 2004; Stigler et al., 2010), making use of this natural alignment may help students to use their knowledge of entities in the real world to bootstrap their knowledge of rational numbers. The substantial congruence between patterns of performance on multiple tasks using fractions versus decimals across college students in the United States and South Korea suggests that these patterns depend on fundamental representational

differences between number types, rather than specific aspects of education, language or culture. More generally, cross-national comparisons provide a method of exploiting existing variation in order to assess the generality of apparent differences in mental representations.

Acknowledgments

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