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Authors
Friedman, Henry L
Heinle, Mirko Stanislav

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Influence activities, coalitions, and uniform policies: Implications for the regulation of financial institutions

Henry L. Friedman and Mirko S. Heinle*

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Abstract

We examine a setting where agents can form lobbying coalitions to influence a policy-maker. Policy uniformity causes agents to free ride on each other’s lobbying and gives them an incentive to form lobbying coalitions. We investigate when coalitions are formed by similar or dissimilar agents and show that endogenous coalition formation causes the effects of policy uniformity and lobbying costs on aggregate lobbying activity and policy strength to be non-monotonic. Our model suggests that increased competition in the market for coalition-facilitating lobbyists can lead to less lobbying. We discuss implications for the regulation of financial institutions.

Keywords: lobbies, coalitions, one-size-fits-all, regulation

JEL Codes: D72, G38, L51, M40

*Henry Friedman is at UCLA and can be reached at henry.friedman@anderson.ucla.edu. Mirko Heinle is at the University of Pennsylvania and can be reached at miheinle@wharton.upenn.edu. We are grateful to Jeremy Bertomeu, Jonathan Glover, Jack Stecher, Michael Kosfeld, Anna Rohlffing, Oliver Schenker, and participants at Frankfurt School of Finance & Management, Goethe University Frankfurt, University of Pittsburgh, UCLA, the 2016 Purdue Accounting Theory Conference, the 2016 Junior Accounting Theory Conference, and the 2017 INSEAD Accounting Symposium for helpful suggestions.
1 Introduction

Regulatory policies are usually set with a degree of uniformity, applying similar treatments to diverse agents. These agents, in turn, often have the ability to influence the policies they are exposed to and can organize into coalitions or lobbies that coordinate their influence activities. In this paper, we explore the agents’ choices of how to organize into lobbies, and how agents’ ability to organize influences their lobbying activities and a regulator’s choices.

In our model, a regulator (or enforcement agency) chooses the policies that apply to a set of heterogeneous agents. Each agent can take an action that is privately beneficial but socially harmful, and stronger regulatory policies reduce the probability that the agents succeed at taking their socially inefficient actions. The regulator is interested in minimizing the welfare losses, but can be influenced by the agents, who lobby for weaker policies. Although the agents in our model differ in the socially-harmful but privately-beneficial actions they can take, the regulator is constrained to treat the agents similarly (though not necessarily equivalently). With lobbying, exposing different agents to similar policies causes an externality of one agent’s lobbying on other agents’ policies, which, in turn, results in a lobbying-related free-rider problem among agents (Friedman and Heinle, 2016). To overcome the free-rider problem and, thus, to more effectively reduce the extent of regulation or enforcement, agents can organize into lobbying coalitions, i.e., lobbies.

As a motivating example, we consider the regulation of financial institutions. Regulators such as the Federal Reserve write capital rules, run stress tests, and perform on-site examination reviews to help ensure the stability of the financial system. These regulatory actions reduce financial institutions’ opportunities to pursue strategies that impose systemic risk on the economy (e.g., excess leverage and risk-taking, insufficient capital buffers). Financial institutions differ in their systemic importance, e.g., due to size, complexity, or connectedness. While federal laws recognize this heterogeneity, they also prescribe a degree of homogeneous treatment. For example, all bank holding companies (BHC’s) are subject to the Fed’s safety and soundness regulation. But, with the passage of the Dodd-Frank Act,
BHC’s with more than $50 billion in assets were made subject to additional prudential requirements. Congress’s passage of Dodd-Frank set the $50 billion threshold and the nature of the prudential requirements while delegating to the Fed’s Board of Governors authority to set the specific regulations (see Section 165 of the Dodd-Frank Act). A commitment to partial regulatory uniformity manifests in this example via the Fed’s statutory charge to set prudential standards for all BHC’s, with enhanced standards for large BHC’s.¹

Financial institutions lobby both individually and in coalitions. Public lobbying filings list several multi-firm lobbying organizations, including the American Bankers Association, the Credit Union National Association, the Independent Community Bankers of America, and the Financial Services Roundtable. Aside from forming their own lobbying organizations, different financial institutions can also retain the services of the same lobbyist to facilitate coordination. For example, Citigroup, Goldman Sachs, Hartford Financial Services Agents, and other financial institutions retained the services of Subject Matter, a lobbying services firm.² Through meetings with Fed staff and other regulators (e.g., the SEC, FDIC, and OCC) several financial institutions and lobbying coalitions have attempted to influence rule-making in the wake of Dodd-Frank.

Our model consists of three agents, which can represent specific financial institutions (e.g., Citigroup, Goldman Sachs, and Wells Fargo) or sets of firms (e.g., systemically important BHC’s, other large banks, and regional credit unions). We assume that there is a cost of forming a coalition and that the cost increases in the size of the coalition. Additionally, we assume that each coalition needs a specialist (or lobbyist) to enable within-coalition coordination. Lobbyists cannot force agents to join coalitions, but they extract a fraction of

¹Regulatory oversight via a patchwork of regulators provides another example of partially uniform regulation. For instance, in the U.S., the Office of the Comptroller of the Currency (OCC) regulates nationally chartered banks, the Federal Reserve, Federal Deposit Insurance Corporation (FDIC), and state-level banking departments oversee state-chartered banks. Finally, the Federal Reserve and Financial Stability Oversight Council (FSOC) regulate bank holding companies and Systemically Important Financial Institutions (SIFIs), which can include large insurers and asset managers. In other words, the US Congress, through laws that set up different regulators for different sets of financial institutions and overlapping oversight responsibilities, has enacted a degree of partial uniformity in the regulation of financial firms in the US.

²Unless otherwise noted, lobbying data comes from https://www.opensecrets.org/.
the net coalition gains from coalition formation.

We focus on the implications of endogenously-formed coalitions. To define an equilibrium coalition, we introduce the notion of offer-stable coalitions. When a set of coalition members receive an offer to deviate, offer-stable coalitions can prevent these members from leaving by making a successful counteroffer. As such, offer-stability is closely related to notions based on the core and bargaining sets (e.g., Von Neumann and Morgenstern, 1944; Aumann and Maschler, 1964; Ray and Vohra, 1999). In our setting with heterogeneous agents, externalities, and not necessarily superadditivity, offer-stability predicts at least one stable coalition structure for any feasible set of parameters, while other equilibrium notions may not. When offer-stability predicts multiple stable coalition structures, we allow the lobbyist to “break the tie” and form the coalition that provides her with the greatest benefit.\(^3\) Interestingly, we find that this coalition formation mechanism can support the grand coalition, a coalition of the two agents with the largest potential for private benefits, or a coalition between the agents with the smallest and the largest potential for private benefits. The only coalition that is never stable is the one that includes the two agents with the smallest potential for private benefits. In other words, offer-stable coalitions can be between either similar agents (i.e., medium and large), between dissimilar agents (small and large), or between all agents.

We show that regulatory uniformity has non-monotonic effects on aggregate lobbying and policy strength when coalitions form endogenously. This occurs because uniformity, by promoting free-riding between lobbying agents, encourages them to form coalitions. That is, uniformity tends to decrease lobbying and strengthen regulatory policies, as in Friedman and Heinle (2016), but also causes agents to coalesce into new or larger lobbying coalitions. Coalition formation and expansion cause discrete jumps in the agents’ ability to overcome free-rider problems, leading to increased lobbying and, in turn, weaker policies.\(^4\)

Additional comparative statics show similar non-monotonicities related to the costs of

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\(^3\)The unique coalition predicted by offer-stability and the lobbyist’s preferences is in the core when the core is non-empty. Additionally, offer-stable equilibrium coalitions exist even when the core is empty.

\(^4\)This result should also hold in a setting with a continuous mass of firms when the number of coalitions is discrete and endogenous.
lobbying and the costs of coalition formation. Higher lobbying costs directly lead to less lobbying and less distorted policy, which in turn reduces the free-riding problem that motivates coalition formation. However, in our setting, one agent benefits when the two other agents form a coalition, because the coalition members increase their lobbying and this tends to weaken the regulation imposed on the non-coalition agent. Typically, higher lobbying costs lead to smaller coalitions, but there are sets of parameters for which higher lobbying costs can cause the coalition structure to transition from two-agent to three-agent, through the non-proportional influence on coalition-member and non-coalition agents. Changes in coalition costs, through similar mechanisms, can have non-monotonic effects on coalition structures, and, through their influence on the coalition structure, have non-monotonic effects on lobbying and policy strength. In contrast, an increase in the lobbyists’ share of the gains from coalition formation, i.e., a decrease in competition in the unmodeled lobbying sector, always causes a shift towards larger coalitions. This tends to increase total lobbying and decrease average regulatory strength. Restricting the supply of lobbyists (e.g., through laws against revolving doors), can thus have a negative effect on regulatory strength and can increase the degree to which policies are influenced by lobbying.

Beyond the regulation of systemically important financial institutions, the central tensions in our model carry over to several settings. These key tensions arise from a set of agents who can form coalitions, differ in their abilities to take actions that are socially harmful (e.g., through negative externalities, lower consumer surplus, or tax burdens imposed on unrelated parties), and face at least partially homogeneous regulations (e.g., safety standards or activity-based subsidies). The importance of coalitions in particular is highlighted by recent popular press articles, noting that “Corporate America can’t seem to get enough of the ad hoc coalitions that are formed to put muscle and money behind a lobbying push, 

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5Donelson et al. (2016) examine potential determinants of regulatory preferences for rules-based versus principles-based standards. Interestingly, they capture the degree to which a standard is rules-based with both the number of bright-line thresholds, which tend towards one-size-fits-all, and the amount of scope and legacy extensions, which relate more to individualized regulations. Their construct definition suggests that rules-based standards can be either more uniform or less uniform than principles-based standards.
and hardly a week passes in Washington without a new group appearing on the scene.” (Bogardus, 2013).

1.1 Related literature

The foundation for our study is the literature on lobbying and policy choice in economics, which shows how lobbying and regulatory capture can cause regulators to choose non-welfare-maximizing rules and transfers (e.g., Stigler, 1971; Grossman and Helpman, 1996). Our model is most closely related to Friedman and Heinle (2016), who present a two-agent model involving a regulator who can probabilistically prevent a privately costly but socially wasteful action through regulation but is subject to regulatory capture via lobbying pressure. Similarly, Rodrik (1986) analyzes trade-offs between industry-wide tariffs and firm-specific subsidies in a setting in which industry-wide tariffs promote free-riding on firms’ tariff-seeking. However, the models in Friedman and Heinle (2016) and Rodrik (1986) preclude the formation of lobbying coalitions, leaving agents no way to overcome the free-riding problem generated by uniform policies. Similarly, Bebchuk and Neeman (2010) investigate a model in which different groups lobby the regulator over the level of investor protection in a perfectly uniform regulatory regime. Although this is a regime in which coalitions would be most valuable (as we show in our model), lobbying coalitions are assumed impossible. In Bertomeu and Magee (2011, 2014, 2015), regulatory outcomes are chosen by a combination of a majoritarian vote by firms and the standard setter’s bliss point. In their model, as in most that feature voting as the policy selection tool, voter collusion (e.g., via trading or selling votes) is excluded by assumption.

Several studies have examined the regulation of financial institutions, especially banks. Focal issues include deposit insurance to prevent inefficient bank runs (Diamond and Dybvig, 1983), financial contagion and systemic risk (Acharya, 2009), risk-taking incentives and

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6Bogardus (2013) and Ho (2015) provide examples of: FedEx, Nike, and Verizon joining coalitions lobbying for tax reform; Intel, Microsoft, Qualcomm, and Texas Instruments lobbying through a coalition for immigration reform; and 3M, Caterpillar, and GE lobbying through the Coalition for 21st Century Patent Reform for revisions to laws covering patents.
capital regulation (Kim and Santomero, 1988), banking unions and supra-national banking regulation (Foarta, 2018), bank branching restrictions (Kroszner and Strahan, 1999), financial reporting (Acharya and Ryan, 2016), and disclosure of information such as inspection and stress test results (Goldstein and Sapra, 2014). Studies of these issues typically examine whether particular regulatory interventions can address specific frictions arising in financial institutions.

Our study instead focuses on regulatory influence, which has received significant interest from empirical researchers. This recent interest has been facilitated by novel large-sample data capturing firms’ and coalitions’ lobbying activities and expenditures. Lambert (2018) find that banks that lobby are significantly less likely to face enforcement actions. Igan and Mishra (2014) and Igan et al. (2012) suggest that the financial industry’s lobbying succeeded in preventing or softening regulation prior to the financial crisis. Johnson and Kwak (2011) provide an overview of the financial industry’s lobbying and its plausible effects on financial regulation in the wake of and leading up to financial crises. Outside of banking, Blanes i Vidal et al. (2012), Bertrand et al. (2014), and Kang and You (2018) suggest that lobbyists use their connections to politicians and are able to extract monetary premiums for their connections. Cooper et al. (2010) examine cross-sectional associations between corporate campaign contributions and future stock returns. Despite the interest in the effects of lobbying and the ubiquity of lobbying coalitions, few studies have explicitly considered coalition formation.7 Our study suggests that such consideration might yield novel insights.

Crucially, our study examines the interplay between regulatory uniformity, lobbying, and coalition formation. Recent studies of banking regulators in particular have highlighted how the structure of banking regulators can lead to more or less uniformity. Agarwal et al. (2014) show that treatment varies depending on whether bank examiners come from state or federal agencies. Similarly, Gopalan et al. (2017) documents that the presence of a more local OCC

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7A notable exception is Bombardini and Trebbi (2012), who relate the proportion of lobbying expenditures undertaken via cooperative channels (e.g., industry lobbies) to measures of competition and product differentiation at the industry level.
field office is associated with less risky choices by OCC-chartered banks. If local offices apply more tailored bank supervision than less local offices, our model suggests that closing local field offices may lead to the formation of larger regional lobbying coalitions. Foarta (2018) models a banking union with cross-country transfers of capital for bailouts but a national distribution of these funds. While a banking union between countries improves welfare when the national governments are benevolent, their own rent-seeking behavior can make a union inefficient. Foarta (2018) suggests that a full union, which also decides about the allocation of funds, is more efficient. Our model suggests that a full union may lead to larger lobbying coalitions that may undo the benefits of the union.

Because we investigate coalition formation, our analysis is related to the literature on cooperative game theory, which we discuss in Appendix B. Coalition formation also appears in several applied settings. In studies of trade policy, lobbies can influence regulation involving socially inefficient tariffs and subsidies. Lobbies in different industries compete for trade subsidies, generating negative externalities, in contrast to the lobbies in our model, which have positive externalities on each other. Mitra (1999), for example, allows firms to form lobbies to coordinate their efforts on lobbying for trade protections or subsidies. Lobbies pay an exogenous cost of organizing, but only firms within the same industry can organize with each other and the regulator has to treat all firms within an industry identically. The focus of Mitra (1999) is on which types of industries get organized, rather than which types of parties join together when not all firms need to participate. Additionally, Mitra (1999) considers firms in “disorganized” industries as too small to lobby individually, in contrast to our paper in which individual agents can lobby successfully. Pecorino (1998) considers symmetric firms that repeatedly play a stage game in which they lobby and a tariff is chosen. The schedule that transforms aggregate lobbying into tariffs is exogenously given, and Pecorino (1998) explores trigger strategies (i.e., reversion to non-cooperative play) that can

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8Beyond lobbying, endogenous coalitions are important in theories explaining the formation and actions of clubs (e.g., Ellickson et al., 1999 or Nordhaus, 2015), cartels, customs unions, and research and public goods coalitions (e.g., Yi, 1997).
sustain cooperation amongst lobbying firms. Pecorino (1998) shows that sustaining lobbying cooperation can be either more difficult or less difficult as the number of firms in the industry grows. Magee (2002) extends Pecorino (1998) by endogenizing the schedule that maps contributions to tariffs.

Beyond trade, Damania and Fredriksson (2000) explore coalition formation in a repeated game with two firms that jointly lobby the government for lower pollution taxes and also compete in a product market. Damania and Fredriksson (2000) focus on how collusion in the product market and the firms’ discount rates influence the firms’ incentives to form coalitions. Finally, Drazen et al. (2007) focus on the implications of contribution caps on bargaining between interest groups and politicians. The model builds on Grossman and Helpman (1994), exploring lump-sum taxes that cover a subsidy provided to capital owners who lobby in favor of the subsidy. As in Mitra (1999), all firms within a sector can pay a fixed cost to form a lobby. Drazen et al. (2007) exogenously assume a number of competing sectors that are organized. Introducing contribution caps can push some lobbies out, which makes the remaining lobbies better off.

Building on the prior literature, we focus on the interaction between regulatory uniformity and coalition formation. In contrast to prior work, we allow for degrees of uniformity and are interested in which agents form a like-minded coalition, while much of the prior literature assumes that either all-firms or no-firms in a given industry form a coalition. In contrast to much of the trade literature, we focus on a setting in which all agents lobby for a similar type of policy, so there is no competition between lobbying groups. Our agent heterogeneity can capture both within-industry and across-industry varieties, and we focus on which firms, if any, form coalitions.
2 Model

In the model, there are five risk neutral actors: three agents, a policy-setter/regulator, and a lobbyist. Agents are indexed by \( i \in \{s, m, l\} \) for small, medium, and large, respectively. Each agent can take a privately-beneficial action, for which she gains \( D_i \), where \( 0 < D_s < D_m < D_l \). The action associated with \( D_i \) imposes a social cost of \((1 + \lambda) D_i\), where \( \lambda > 0 \).\(^9\)

The privately-beneficial action is therefore socially inefficient and imposes a net welfare loss of \( D_i \lambda > 0 \). Our assumption of \( D_i > 0 \) implies that each agent always prefers to take the privately-beneficial action. For ease of exposition, we refer to agents with higher \( D_i \) as larger and agents with lower \( D_i \) as smaller, where size relates to the impact of the privately-beneficial action and can reflect, for instance, systemic importance or complexity.

Regulation limits each agent’s opportunity to take the privately-beneficial action (e.g., through site inspections, audits, and restrictions on short-term debt or credit exposures). Specifically, we model the intensity of regulation governing each agent \( i \) as the probability, \( \pi_i \), that the agent is unable to take the action. Finally, before the regulator specifies the regulatory intensities, each agent can exert effort \( B_i \) to lobby the regulator to relax the regulatory intensity she faces.

Agents benefit only from the privately-beneficial action. Each agent incurs a personal cost of lobbying the regulator, \( \frac{c}{2} B_i^2 \). The parameter \( c > 0 \) captures the ability of agents to effectively lobby. A higher value of \( c \) reflects a less severe problem related to the lobbying that facilitates inefficient regulatory policies. Each agent’s expected utility is given by

\[
U_i = (1 - \pi_i) D_i - \frac{c}{2} B_i^2.
\] (1)

With probability \( (1 - \pi_i) \), the agent is able to take the privately-beneficial action and consume \( D_i \). Agents always bear the cost of lobbying because they lobby the regulator before the action is taken. We assume that agents cannot commit to “share the spoils” with the

\(^9\) Agents in our model are heterogeneous in the impact of their actions, although they are homogeneous in the proportional costs of their actions, \( 1 + \lambda \).
regulator to extract regulatory concessions.

When the regulator chooses regulatory intensity, the costs of lobbying, \( \frac{c}{2} B_i^2 \), are sunk. Therefore, the aggregate utility that can be influenced by the regulator is given by the expected losses from the agents’ actions:

\[
L(\pi, D, \lambda) = -\lambda \sum_{i \in \{s,m,f\}} D_i (1 - \pi_i). \tag{2}
\]

The welfare-interested regulator is only concerned about the actions because of the welfare loss, \( \lambda D = \lambda \sum_{i \in \{s,m,f\}} D_i \), that the actions impose on society. This welfare loss occurs with probability \( (1 - \pi_i) \), for each agent \( i \). The regulator wants to minimize this welfare loss subject to the costs of regulation.\(^{10}\)

Regulation is costly for three reasons. First, regulation is costly in and of itself, with a convex cost of regulation, \( \frac{1}{2} \pi_i^2 \), for regulation covering each agent. More stringent regulation and enforcement, e.g., more frequent bank inspections, naturally require larger staffs and potentially more costly training. Second, each agent can influence the regulator through lobbying activity \( B_i \), which increases the cost of regulatory intensity by \( B_i \pi_i \).\(^{11}\) Third, defining a different regulatory intensity for different agents imposes additional costs, which we model as \( \frac{k}{2} \sum_i \left( \pi_i - \frac{1}{2} \sum_{i' \neq i} \pi_{i'} \right)^2 \). We interpret the parameter \( k \) as the degree of regulatory uniformity; when \( k = 0 \), the regulator is free to choose individualized regulation without incurring any penalty whereas the regulator sets the same regulatory intensity for all agents as \( k \to \infty \), enacting a one-size-fits-all uniform regime. Crucial for our results is that the regulator is committed to apply somewhat similar regulatory policies to different agents.

\(^{10}\)We assume that the regulator cares only about the welfare loss. Glaeser et al. (2001) model judges and regulators as alternative enforcement mechanisms and assume that regulators, while easier to motivate, can also be overzealous and prefer excessively-strict policies. In such a setting, lobbying can be beneficial as a counterweight to regulator’s overzealousness. See Friedman and Heinle (2016) for further discussion of interpretations of \( c, \lambda \), and \( \pi_i \).

\(^{11}\)Much of the related literature (building on Grossman and Helpman (1996)) uses a menu-auction approach in which the agents contribute an amount in exchange for an amount of lobbying. Note that this is a technical difference, rather than one of substance, as the underlying economic forces are similar. See Gregor (2011) for a recent review of the literature on lobbying.
The total cost of regulation is given by

\[
C(\pi, B, k) = \sum_{i \in \{s, m, \ell\}} \left( \frac{\pi_i^2}{2} + B_i \pi_i + \frac{k}{2} \left( \pi_i - \frac{1}{2} \sum_{i' \neq i} \pi_{i'} \right)^2 \right),
\]  

(3)

and the regulator’s objective function is

\[
U_R = L(\pi, D, \lambda) - C(\pi, B, k).
\]

(4)

Agents, in turn, can form coalitions to coordinate their lobbying efforts. A coalition is defined as a set of agents, denoted by \( l_j \equiv \{i : i \text{ is a member of coalition } j\} \), where \( j \) indicates a particular coalition. Each coalition also includes a lobbyist (described below). We impose an increasing cost to forming larger lobbies, defined as \( \chi_{|l_j|} \), where \( |l_j| \) is the number of agents in the lobby, with \( \chi_1 = 0 < \chi_2 < \chi_3 \). These costs prevent the payoff structure from generally being superadditive. Aumann and Dreze (1974) discuss how difficulties associated with collaboration, such as transaction costs of side payments or within-coalition monitoring, would give rise to such size-based coalition costs.\(^{12}\) Additionally, these costs could be affected by institutional mechanisms that combat coalition formation.

There are five possible coalition structures, as shown in Table 1.

<table>
<thead>
<tr>
<th>Structure name</th>
<th>Notation</th>
<th>Coalition structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent agents</td>
<td>( I )</td>
<td>{{s}, {m}, {\ell}}</td>
</tr>
<tr>
<td>Small-medium lobby</td>
<td>( sm )</td>
<td>{{s, m}, {\ell}}</td>
</tr>
<tr>
<td>Small-large lobby</td>
<td>( sl )</td>
<td>{{s, \ell}, {m}}</td>
</tr>
<tr>
<td>Medium-large lobby</td>
<td>( ml )</td>
<td>{{s}, {m, \ell}}</td>
</tr>
<tr>
<td>Grand lobby</td>
<td>( G )</td>
<td>{{s, m, \ell}}</td>
</tr>
</tbody>
</table>

| Table 1  
| Coalition Structures |

Let \( U_{i,l_j} \) be the expected utility before coalition costs, \( \chi_{|l_j|} \), generated by agent \( i \) in

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\(^{12}\)Allowing the two-agent coalition costs to depend on which firms are in the coalition would potentially make the \( sm \) coalition feasible at the cost of added complexity. For the sake of parsimony, we assume that coalition costs only depend on the number of coalition partners.
coalition structure $l_j \in \{I, sm, sl, m\ell, G\}$. For example, $U_{i,l}$ is the expected utility generated by agent $i$ if all agents lobby independently, $U_{s,m\ell}$ is the expected utility generated by agent $s$ when agents $m$ and $\ell$ form a coalition together, and $U_{m,G}$ is the expected utility generated by agent $m$ in the grand coalition, $G$. Furthermore, let $U_{ij} = \sum_{i \in l_j} U_{i,l_j}$ be the cumulative utility generated by all members of a coalition, e.g., $U_G = \sum_i U_{i,G}$. We define the (potentially negative) net gains from coalition formation as

$$\Delta U_{ij} = U_{ij} - \chi_{|l_j|} - \sum_{i \in l_j} U_{i,l}.$$ 

The gain from coalition formation reflects the additional expected utility for the agents in the coalition net of the coalition costs, prior to splitting any net gains with the lobbyist.

The lobbyist, who is necessary for coalition formation, can extract an exogenous fraction of $\Delta U_{ij}$. We assume that the agents keep a fraction $\alpha \in (0, 1]$ of these gains and that the lobbyist keeps the remaining $(1 - \alpha)$.$^{13}$ The share that the lobbyist retains captures the competitiveness or specialization in the lobbying sector, as, in our model, only one coalition at most will form. The lobbyist’s utility is given by

$$U_L (l_j) = (1 - \alpha) \Delta U_{ij}. \quad (5)$$

This assumption reflects the idea that the lobbyist owns a technology necessary for coalition formation.$^{14}$ We interpret $\chi_{|l_j|}$ as a monitoring cost of ensuring compliance with the coalition’s strategy, rather than a fee paid to the lobbyist. Essentially, the lobbyist is necessary for the coalition to exist (in a binary sense), and her fee is a fraction of the gains from coalition formation net of within-coalition monitoring costs. In some sense, the lobbyist

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$^{13}$We cover the case of $\alpha = 0$ in the Appendix following the solution for low $\alpha$, and obtain similar qualitative results for $\alpha = 0$ as for low values of $\alpha$.

$^{14}$Essentially, only the lobbyist owns the technology to facilitate collusion on lobbying between insiders. Because there can be no more than one multi-agent coalition in our three-agent setting, we assume that there is only one lobbyist. This abstracts from explicit competition between lobby-forming specialists, but substantially simplifies the problem.
is a monitoring device that helps ensure that the coalition members pursue strategies that maximize the coalition’s net gain.

Within a lobby, utility is transferable. This is a convenience assumption, as our interest is in the effects of coalitions on lobbying activity and policy choice, rather than within-coalition allocations of utility. When agents choose their lobbying activities, whether individually or jointly, the coalition costs, $\chi_{l_j}$, are sunk. So, the agents who are joined in a particular lobby choose lobbying efforts to maximize $\sum_{i \in l_j} U_{i,l_j}$, which maximizes the total amount split between the agents and the lobbyist.

Table 2 shows the timeline.\textsuperscript{15} We leave the issue of coalition formation open for now and return to it after Section 3, where we derive the regulatory policies and lobbying strategies conditional on coalition structures. Throughout the analysis, we use the terms lobby and coalition interchangeably. We solve for the Perfect Bayesian Nash Equilibrium using backward induction.

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
<th>$t = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coalition formation</td>
<td>Agents choose influence activities</td>
<td>Regulator chooses regulatory intensities</td>
<td>Privately-beneficial actions may occur</td>
</tr>
<tr>
<td>$l_j$</td>
<td>$B_i$</td>
<td>$\pi_i$</td>
<td>Payoffs</td>
</tr>
</tbody>
</table>

Table 2
Timeline

\textbf{2.1 Discussion of main assumptions}

The model features three agents who differ in the effects of their privately beneficial, socially costly actions. If the potential harm to society is proportional to assets, we could interpret

\textsuperscript{15}In practice many lobbying efforts are coordinated through long-lived institutions (or firms). Simply repeating the one-shot game depicted in the timeline would allow for the coalition formation cost $\chi$ to be founded on the probability of observing deviations from coalition strategies and associated punishments as in Damania and Fredriksson (2000). Additionally, agents may form coalitions prior to knowing exactly what the potential new regulation is going to be. In our model, we could allow $\lambda$ to be drawn from a commonly known distribution at $t = 2$. In our linear-quadratic structure, lobbying strategies would be unaffected but the gains from coalition formation may depend on the uncertainty over $\lambda$, in part because of risk-sharing agreements within coalitions as in Wilson (1968).
agent ℓ as JPMorgan Chase & Co. (~$2.5 trillion in assets), agent m as US Bancorp (~$450 in assets) and agent s as Suntrust (~$200 billion in assets).\textsuperscript{16} Alternatively, individual agents could represent types of financial institutions or the degree of interconnectedness; it may be the case that large banks or exchanges (ℓ) are more systemically important than large insurers or brokers (m) and asset managers (s).

While we capture the regulator’s commitment with the \( k \)-based cost, alternative ways of modeling commitment generate similar forces. For instance, we could assume that the regulator receives a benefit from more similar regulation, which may reflect a regulator’s preference for comparability (see (Ray, 2017)). Alternatively, we could assume that the regulator maximizes utility subject to some constraints that force regulatory similarity, such as \( \pi_g - \frac{1}{\kappa} \leq \pi_i \leq \pi_g + \frac{1}{\kappa} \), with the commitment to uniformity captured by \( \kappa > 0 \). One of the main economic consequences of uniformity is the free rider problem induced by agent i’s lobbying affecting \( \pi_g \). In a setting where the commitment comes from similarity bounds given by \( \kappa \) rather than a cost of dissimilarity given by \( k \), agent i’s lobbying efforts will affect \( \pi_g \) as long as some \( \kappa \)-based constraints bind.\textsuperscript{17}

Given that heterogeneous regulation plausibly requires greater care in drafting and increased expenditures in enforcement (e.g., staff costs), \( k \) can be interpreted as a technical constraint on the regulator. Alternatively, \( k \) could be an institutional commitment (for example, a legislative mandate) to regulate different firms in a similar fashion, such as the buckets for financial firms on either side of the $50 billion threshold discussed above.\textsuperscript{18} Notably, rules-based policies applying to multiple firms is ubiquitous, potentially due to cost savings and the potential for rules (e.g., limited discretion) to mitigate agency issues within the regulator. In the setting of the regulation of financial institutions, higher \( k \) might reflect a single regulator covering a broader set of firms (e.g., the Federal Reserve supervising system-

\textsuperscript{16}Amounts based on annual reports dated December 31, 2016.

\textsuperscript{17}The similarity-bounds and cost-constraint can be linked if the regulator can, at an earlier stage, pay a \( k \)-based cost to reduce \( \kappa \).

\textsuperscript{18}Discussing a recently-passed Senate bill, Ackerman (2018) notes that “Dodd-Frank limits the Fed’s reach because it spells out that all banks above $50 billion in assets must face stricter rules.”
ically important financial institutions whether they are banks, insurers, or asset managers), explicit moves towards uniformity, consistency across bank regulators, or centralization of tasks or offices. Lower values of \( k \) might reflect a regulatory environment in which there are several regulators, each with a different domain (e.g., the OCC, state regulators, and FDIC covering different types of banks). Agarwal et al. (2014) provide evidence that different regulators apply different standards, suggesting that having the same regulator leads to more uniform treatment than having different regulators (who might, essentially, face different regulatory cost functions). The closing of the Office of Thrift Supervision and the transfer of its oversight responsibilities to the Fed and OCC could be used as a shock to de facto regulatory uniformity (see Granja and Leuz, 2017). Similarly, recent proposed increases of the enhanced supervision threshold from $50 billion would provide another potential shock to the buckets within which firms face more uniform regulation (Ackerman and Tracy, 2018; Ackerman, 2018).

Even within the Fed, different regional offices may have different policies.\(^{19}\) The Fed can take actions to promote uniformity across offices or allow offices to decide on enforcement priorities more independently. A recent federal report (GAO, 2017) noted that “implementation of LISCC [Large Institution Supervisory Coordinating Committee] policies has been inconsistent across reserve banks. To some extent, this inconsistency may be related to the structure of the Federal Reserve; specifically, all 12 Reserve Banks operate semi-independently from one another,” and that “as a result of inconsistencies, the OC [LISCC Operating Committee] endorsed the development of a LISCC Program Manual” to implement policies more consistently. Carletti et al. (2016) describe agency issues within bank supervisor hierarchies. Centralized bank supervision implies greater uniformity while local bank supervision allows for more regulatory heterogeneity.

There are other institutional settings that are similar in spirit. In the European Union, some bank oversight responsibilities fall under the European Central Bank (ECB) while other

\(^{19}\) Additionally, the Fed may apply different standards in individual firms’ stress tests.
parts are the responsibilities of country-specific central banks. ECB-designated oversight is likely to be more uniform than oversight activities undertaken separately by different countries’ central banks. Goyal et al. (2013) discuss the potential move to a banking union in the Euro area, which would homogenize treatment of banks across member countries. In China, the government recently announced a plan to consolidate the separate bank and insurance regulators into one agency (Wong, 2018).

We use a continuous \( k \) rather than a binary indicator for complete uniformity to allow for smoother transitions across regimes and comparative statics analysis with finer regime gradations. While we take \( k \) as an exogenous parameter, Friedman and Heinle (2016) examine how a system designer, such as a legislature, would optimally set \( k \) in a setting with two agents who cannot form a coalition. For instance, the legislature could decide to mandate centralized decision-making or to let each OCC office or FRB have its own policies.

## 3 Regulation and influence activities

Given lobbying efforts, the regulator chooses regulatory intensities in period \( t = 2 \) as

\[
\arg \min_{\pi_i, i \in \{s, m, t\}} \sum_{i} \left[ \lambda (1 - \pi_i) D_i + \frac{\pi_i^2}{2} + B_i \pi_i + \frac{k}{2} \left( \pi_i - \frac{1}{2} \sum_{i' \neq i} \pi_{i'} \right)^2 \right].
\]

The first-order-conditions (FOC) are a set of three equations that imply,

\[
\hat{\pi}_i = \frac{4 (\lambda D_i - B_i) + 9k \left( \lambda \frac{\hat{B}}{3} - \frac{\hat{B}}{3} \right)}{4 + 9k},
\]

where \( \hat{B} = \sum_{i \in \{s, m, t\}} B_i \). In what follows, we derive the optimal lobbying efforts, conditional on the coalition structure. We assume throughout that the exogenous parameters are such that regulation is defined by (6) and \( \hat{\pi}_i \in (0, 1) \) \( \forall i \).

In each setting, a coalition (including a one-agent coalition) chooses lobbying to maximize the expected utility of its members, which is \( \sum_{i \in l_j} U_i |_{\pi_i = \hat{\pi}_i} \), where \( U_i \) is defined in (1) and \( \hat{\pi}_i \) is defined in (6). We do not specify transfers within lobbies, restricting them only to be
feasible (i.e., the sum of the individual utilities of the lobby members equals the total utility of the members of the lobby).

To develop the optimal lobbying, regulatory strengths, and expected agent and lobbyist utilities for each of the coalition structures, we introduce additional notation. Specifically, to represent agents within coalition structures, we use the subscripts \( \{g, h, i\} \) to denote different agents, where \( g, h, i \in \{s, m, \ell\} \) and \( g \neq h \neq i \). This allows us to introduce flexible terms, like \( \bar{D}_{gh} = D_g + D_h \) as a stand-in for each of \( \bar{D}_{sm}, \bar{D}_{st}, \) and \( \bar{D}_{ml} \). Additionally, let \( \bar{D}^2_{gh} = D^2_g + D^2_h \), and \( \bar{D}^2 = \sum_i D^2_i \). The proof of the following Lemma exploits this notation.

**Lemma 1** There exists a unique equilibrium \( \{\hat{B}_{i,j}, \hat{\pi}_{i,j}\} \) for each coalition structure \( j \).

Whenever the regulator is at least somewhat constrained to enact uniform regulation, i.e., whenever \( k > 0 \), the free-riding problem between agents provides an opportunity for gains from forming coalitions. Whenever agents form coalitions, they lobby more because they internalize the effect of their lobbying on the other agents(s) in the coalition. This reduces regulatory strength for all agents (that is, \( \hat{B}_{i,I} \leq \hat{B}_{i,ig} \leq \hat{B}_{i,G} \) and \( \hat{\pi}_{i,I} \geq \hat{\pi}_{i,ig} \geq \hat{\pi}_{i,G} \)). Furthermore, regulatory uniformity implies that the non-coalition member (e.g., agent \( i \) in the presence of the \( gh \) coalition) faces weaker regulation even though that agent lobbies as if all agents were independent (\( \hat{B}_{i,gh} = \hat{B}_{i,I} \) and \( \hat{\pi}_{i,I} \geq \hat{\pi}_{i,gh} \)).

Although lobbying efforts are greater, each agents’ expected utility is also greater, and not just for the members of the coalition. Note that absent coalition costs, the combined expected utility of all coalitions is greatest for the three-agent coalition, and lowest when each agent lobbies independently, as

\[
\hat{U}_G > \hat{U}_{i,gh} + \hat{U}_{gh,gh} > \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I},
\]  

where \( \hat{U}_{i,I} = (1 - \hat{\pi}_{i,I}) D_i - \frac{1}{2c} \left( \frac{4 + 3k}{4 + 9k} \right)^2 D^2_i \). Finally, the gains from forming coalitions are increasing in the degree of regulatory uniformity, \( k \), as \( \frac{d}{dk} (\Delta U_{i,I}) > 0 \).

To facilitate the exposition, we introduce “effective coalition formation costs,” \( \chi^E_2 = \)
\( \chi_2 \frac{2c(4+9k)^2}{9k^2} \) and \( \chi_3^E = \chi_3 \frac{2c(4+9k)^2}{9k^2} \). These effective coalition formation costs incorporate the parameters associated with the net costs and benefits of lobbying, \( c \) and \( k \), that affect the net gains from coalition formation. That is, \( c \) and \( k \) only affect the equilibrium coalition structure through \( \chi_2^E \) and \( \chi_3^E \). Additionally, we let \( \Delta_I^G = \left( \hat{U}_G - \sum_i \hat{U}_{i,I} \right) \chi_3^E / \chi_3 = 2 \left( \frac{D^2 + D_s D_m + D_s D_t + D_m D_t}{D_I^2} \right) \). There is a net benefit to the formation of the grand coalition if \( \Delta_I^G - \chi_3^E > 0 \) and a net benefit to the formation of the \( m \ell \) coalition if \( \frac{D_I^2}{D_{m \ell}} - \chi_2^E > 0 \).\(^{20}\)

Similarly, let \( \Delta_{gh}^G = \frac{2c(4+9k)^2}{9k^2} \left( \hat{U}_G - \hat{U}_{i,gh} - \hat{U}_{gh,gh} \right) \), which captures the difference between the expected utility from the grand coalition and the expected utility of all players given a two-agent coalition structure \( gh \).

\[ \] 4 Coalition formation and equilibrium

4.1 Offer-stability

Necessarily, coalitions that emerge must be stable in that no set of agents (or individual agent) prefers to deviate and form a new coalition (or go off on her own). We formalize this requirement and introduce “offer-stability” as our coalition stability concept below. We begin by defining feasible allocations of utility across agents for a given structure, \( X \), where the possible coalition structures are listed in Table 2.

Definition 1 (Allocation) An allocation \( A \) is a vector of utilities \( u_i \) for each agent \( i \in \{s, m, \ell\} \). An allocation is feasible if \( \sum_{i \in I} u_i = \sum_{i \in I} U_{i,I} + \alpha \Delta U_{i,I} \) for each \( i \) given the coalition structure, \( X \).

A feasible allocation allows utility transfers within a coalition, but not across coalitions. Transfers across coalitions imply a measure of cooperation across coalitions, which is inconsistent with our assumption that there are costs to forming coalitions that are borne by the coalitions themselves.\(^{21}\)

\(^{20}\)The grand coalition has a net benefit when \( \hat{U}_G - \sum_i \hat{U}_{i,I} > \chi_3 \), which we can rewrite as \( \frac{2c(4+9k)^2}{9k^2} \left( \hat{U}_G - \sum_i \hat{U}_{i,I} \right) > \chi_3^E \) or \( \Delta_I^G > \chi_3^E \).

\(^{21}\)Many lobbying organizations, such as the American Bankers Association (ABA), charge membership fees and limit their membership. The limited membership direct the activities of the organization, including the
**Definition 2 (Offer-stability)** A feasible allocation-structure pair \((A, X)\) is blocked if there is a nonempty coalition in which each member is made weakly better off and one member is made strictly better off in an alternative feasible allocation-structure pair \((A', X')\). A structure is offer-stable if, for every alternative feasible allocation-structure pair, \((A', X')\), with \(X' \neq X\), there is an allocation-structure pair \((A'', X)\) that is not blocked by \((A', X')\).

Our notion of offer-stability is based on the idea that if agent \(i\) considers leaving a given coalition, then the remaining agents in the coalition could offer agent \(i\) any amount up to the amount that those agents gain from being members of the coalition including \(i\). The logic extends naturally if we replace agent \(i\) with a set of agents. We use the term offer-stability because it is based on the idea that coalitions can use counteroffers to prevent being blocked. Specifically, \(A''\) is a counteroffer that prevents \(X\) from being blocked by an allocation-structure pair that includes an alternative structure, \(X'\). Our concept of offer-stability is somewhat different from other coalition stability criteria as defined in Von Neumann and Morgenstern (1944) and Aumann and Maschler (1964) (see also Ray and Vohra, 2014). We contrast offer-stability with concepts of stability based on the core and bargaining sets in Appendix B, where we also define core-stability and provide a description of stability based on bargaining sets.

To illustrate the notion of offer-stable coalitions, consider the grand coalition. If \(\Delta_i^G > \chi_3^E\), then the gain from forming the grand coalition, relative to not forming any coalition, outweighs the costs. The grand coalition is then preferred to each agent lobbying independently and the grand coalition is partially stable because agents \(i\) and \(k\) can offer up to \(\frac{9k^2(\Delta_i^G - \chi_3^E)}{2c(4+9k)^2}\) to agent \(i\) to prevent her from leaving. However, when \(\chi_2\) is not too large, two members of the grand coalition might prefer to form a two-agent coalition over the grand coalition. That is, it may be that the extra costs of having a three-agent coalition (relative to a two-agent coalition) outweigh the benefit. For example, with \(\chi_2 = 0\) and \(\chi_3^E > \Delta_i^G\), a two-agent coalition may be optimal. Clearly, there are gains to be had from a two-agent coalition, but we have not yet shown whether any two-agent coalition is offer-stable.

topics on which they lobby and the positions they endorse. The Chamber of Commerce also collects membership fees that pay for services that are meant to directly benefit its members. For both the ABA and the Chamber, there are potential lobbying spillovers (e.g., to other businesses or smaller financial institutions).
4.2 Coalition formation protocol and equilibrium definition

In this subsection we define a protocol for coalition formation. In our setting, there is always at least one offer-stable coalition. When there is more than one, our protocol provides for a reasonable selection mechanism. Such protocols are common in cooperative games.\footnote{Ray and Vohra (1999), focusing on endogenous coalition formation, label some players as proposers, who by proposing coalition structures and within-coalition transfers play essentially the same role as the lobbyist here. Bloch (2002) similarly presents a sequential coalition formation game featuring proposers.}

Our protocol breaks the coalition-formation period \((t = 0)\) into two subperiods, \(i\) and \(ii\). In the first subperiod, the agents can come to the lobbyist and propose a coalition to be formed. If an offer-stable coalition is proposed, the game proceeds to \(t = 1\). If no coalition is proposed in subperiod \(i\), the lobbyist can make a take-it-or-leave-it offer to a set of agents to form an offer-stable coalition in subperiod \(ii\). For each region of the parameter space we find at least one offer-stable coalition. In regions with multiple offer-stable coalitions, our protocol in which the lobbyist chooses allows for clean predictions. An equilibrium in our model is defined as follows:

**Definition 3 (Equilibrium)** An equilibrium is defined as a septuple consisting of the coalition structure, lobbying efforts, and regulatory policies, \(\{X, \hat{B}_{i,j}, \hat{\pi}_{i,j}\}\), such that:

1. The regulator chooses optimal regulatory policies conditional on its objective function and agents’ lobbying.
2. Agents choose optimal lobbying efforts conditional on the coalition structure and the regulator’s anticipated strategy.
3. The coalition structure is determined following the protocol and consists of offer-stable coalitions conditional on anticipated lobbying strategies and the regulator’s anticipated strategy.

4.3 Coalition formation

An offer-stable two-agent coalition, as defined, cannot be broken up by the agent who is not a member of the coalition. However, offer-stability implies that the outsider’s utility in the presence of a two-agent coalition is relevant for determining the surviving coalition. For this
reason, the agents’ share of coalition gains, $\alpha$, plays a role in determining offer stability. In the following theorem, we develop the offer-stable coalition structures when agents keep a sufficiently high fraction of the coalition gains, $\alpha > \frac{D_\ell(D_s+D_m)}{D^2_{m\ell}+D^2_\ell}$. For notational convenience, let $\Delta^G_{m\ell} = 2D^2 - (D_m - D_\ell)^2 - 2 \left(1-\frac{\alpha}{\ell}\right)D_sD_{m\ell}$ and $\Delta^G_{sl} = 2D^2 - (D_s - D_\ell)^2 - 2 \left(1-\frac{\alpha}{s}\right)D_mD_{sl}$.

These $\Delta^G_{m\ell}$ and $\Delta^G_{sl}$ terms capture “effective” benefits to the agents of forming the grand coalition relative to being in the $m\ell$ or $sl$ coalition structure, respectively, much as the $\chi^E$ terms capture effective coalition costs.

**Theorem 1** When $\frac{D_\ell(D_s+D_m)}{D^2_{m\ell}+D^2_\ell} < \alpha \leq 1$,

1. $\chi^E_2 \geq \frac{D^2_{m\ell}}{D^2_{m\ell}+D^2_s}$, and

   (a) $\chi^E_3 > \Delta^G_{m\ell}$, then no coalitions will form and all agents will lobby individually;

   (b) $\chi^E_3 \leq \Delta^G_{m\ell}$, then the offer-stable coalition structure consists of the grand lobby;

2. $\chi^E_2 \in \left(\frac{D^2_s}{D^2_{m\ell}}, \frac{D^2_{m\ell}}{D^2_{m\ell}+D^2_s}\right)$, and

   (a) $\chi^E_3 - \chi^E_2 > \Delta^G_{m\ell}$, then the offer-stable coalition structure consists of the $m\ell$ lobby;

   (b) $\chi^E_3 - \chi^E_2 \leq \Delta^G_{m\ell}$, then the offer-stable coalition structure consists of the grand lobby;

3. $\chi^E_2 \leq \frac{D^2_s}{D^2_{sl}}$, and

   (a) $\chi^E_3 - \chi^E_2 > \Delta^G_{sl}$, then the offer-stable coalition structure consists of the $sl$ lobby;

   (b) $\chi^E_3 - \chi^E_2 \leq \Delta^G_{sl}$, then the offer-stable coalition structure consists of the grand lobby.

Note that when the costs to forming a two-agent coalition are sufficiently low, $\chi^E_2 \leq \frac{D^2_s}{D^2_{sl}}$, and the extra cost of forming the grand coalition is sufficiently large, then the only offer-stable coalition is the $sl$ lobby (part 3.a). Under these conditions, the $m\ell$ lobby is not stable because the $m$ agent’s gain from the $sl$ lobby is relatively high. For that reason, the $s$ agent’s gain from joining the lobby exceeds the $m$ agent’s loss from leaving the lobby and the $sl$ lobby persists. However, if $\chi^E_2 \in \left(\frac{D^2_s}{D^2_{sl}}, \frac{D^2_{m\ell}}{D^2_{m\ell}+D^2_s}\right)$, then it is no longer profitable for the $s$ and $\ell$ agents to form a coalition together, as their net coalition gains are negative. For $\chi^E_2$ in this
region, the net coalition gains to the \( m\ell \) coalition are positive, implying that this coalition is offer-stable when the negative coalition gains to \( s\ell \) make that coalition infeasible.

When \( \alpha \) is not sufficiently high, i.e., \( \alpha \leq \frac{D_s(D_m+D_{m\ell})}{D_{m\ell}^2+D_I^2} \), agents do not capture sufficient gains from coalition formation to make them prefer to be inside the coalition rather than outside. Recall that coalition members’ net gain is increasing in \( \alpha \), while the benefit to the non-coalition participant, i.e., agent \( s \) in the presence of \( m\ell \), is independent of \( \alpha \). Similarly, the gain to the grand coalition is increasing in \( \alpha \), meaning that a lower \( \alpha \) makes it less desirable for the 2-agent coalition outsider to join the three-agent coalition.

Among two-agent coalitions, when \( \alpha \) is too low, each agent prefers for the other two agents to form a coalition. This arises due to the positive externalities that coalition formation have on the non-coalition member, as shown by \( \hat{U}_{i,gh} = \hat{U}_{i,I} + \frac{9k^2D_I\hat{D}_{gh}}{c(4+9k)^2} \). When each agent prefers to be the outsider, the agents are in a sort of 3-way prisoners’ dilemma, in that each player prefers to be the outsider in the presence of a two-agent coalition, but is worse off if no coalitions are formed at all. In such a standstill, any 2-agent coalition structure can form. Once a two-agent coalition forms, it will be stable, as the outsider will not wish to make a deviation-offer. However, prior to a two-agent coalition forming, no agent will willingly join a coalition as long as the option to wait for a coalition that does not include them to form remains.\(^{23}\) Given our protocol, the lobbyist emerges as an equilibrium selection mechanism: if \( \alpha \leq \frac{D_s(D_m+D_{m\ell})}{D_{m\ell}^2+D_I^2} \), then the lobbyist chooses the coalition that is in her best interests in sub-period \( b \). As the coalition gain from the \( m\ell \) coalition is the largest, the lobbyist will choose this coalition. Additionally, it may be that this is the only cost-effective coalition.

Theorem 2 follows.

**Theorem 2** When \( 0 < \alpha \leq \frac{D_s(D_m+D_{m\ell})}{D_{m\ell}^2+D_I^2} \),

1. \( \chi_2^E > \frac{D_{m\ell}^2}{D_{m\ell}^2+D_I^2} \), and

(a) \( \chi_3^E > \Delta I^G \), then no coalitions will form and all agents will lobby individually;

---

\(^{23}\)This is essentially a 3-way Mexican standoff, wherein each player wants someone else to act (shoot) first. Laver and Shepsle (1990) discuss the issue of the Mexican standoff in the context of coalition governments.
(b) \( \chi_3^E \leq \Delta_j^G \), then the offer-stable coalition structure consists of the grand lobby;

2. \( \chi_2^E \leq \frac{D_{m\ell}^2}{c_3} \), and

(a) \( \chi_3^E - \chi_2^E > \Delta_{m\ell}^G \), then the offer-stable coalition structure consists of the m\ell lobby;

(b) \( \chi_3^E - \chi_2^E \leq \Delta_{m\ell}^G \), then the offer-stable coalition structure consists of the grand lobby.

Essentially, inaction among agents with respect to forming two-agent coalitions eliminates the possibility for the \( s\ell \) coalition to form.

We present the solution to the knife-edge case of \( \alpha = 0 \) in Appendix A, directly following the proofs to Theorems 1 and 2. The possible coalition structures with \( \alpha = 0 \) are identical to those with low \( \alpha \) in Theorem 2. However, even though the lobbyist chooses the coalition structure, the parameter values that lead to a particular coalition formation are slightly different between the low \( \alpha \) and \( \alpha = 0 \) cases. The difference is driven by the fact that the \( s \) agent potentially benefits from joining \( m \) and \( \ell \) to form the grand coalition when \( \alpha > 0 \), but gains nothing when \( \alpha = 0 \).

Figures 1(a) and 1(b) summarize Theorems 1 and 2 by mapping regions of coalitions as functions of effective coalition formation costs, \( \chi_2^E \) and \( \chi_3^E \). The area in the bottom-right is not feasible, as it is defined by \( \chi_2^E > \chi_3^E \leftrightarrow \chi_2 > \chi_3 \). The remaining area, in the upper-left of the figure, is divided into regions in which different coalition structures exist in equilibrium. These regions correspond to the regions described in Theorems 1 and 2. Note that the areas of the regions can change, but the shapes that define the regions hold generally.

In addition to these regions, each figure features two rays that each start at a gray dot and proceed up and to the right. These rays are useful for thinking about how the coalition structure changes when \( k \), \( c \), or \( \chi \) change. Specifically, each ray traces out the nexus of points defined by \( \chi_3^E = \frac{\chi_2^E}{\chi_2} \chi_2 \) for \( \chi_2^E \geq 18c\chi_2 \). Overall, the \( \chi \)'s determine the slope of the ray, while \( c \) determines the point closest to the origin, and \( c \) and \( k \) jointly determine the relevant point on the ray that defines the equilibrium coalition structure, i.e., where we fall on the line segment. For a given \( c \), the lowest and left-most point on the ray, at the gray
Coalition structures as functions of effective coalition-formation costs, $\chi^E_2$ and $\chi^E_3$. The grey line segment in each subfigure indicates a nexus of points such that $\chi^E_3 = \chi^E_2 \frac{\chi^E_3}{\chi^E_2}$. The grey dot at the lower-left end of each line segment is at $(\chi^E_2, \chi^E_3) = (18c\chi_2, 18c\chi_3)$. On the vertical axes, $\Delta_{m\ell}^G = 2(D_s D_m + D_s D_t + D_m D_{\ell})$, $\Delta_{m\ell}^G = 2D^2 - (D_m + D_{\ell})^2 - 2\left(\frac{1-c}{a}\right) D_s D_{m\ell}$, and $\Delta_{m\ell}^G = 2D^2 - (D_s + D_{\ell})^2 - 2\left(\frac{1-c}{a}\right) D_m D_{m\ell}$.

dot, is defined by the point $(\chi^E_2, \chi^E_3) = (18c\chi_2, 18c\chi_3)$, because $\lim_{k \to \infty} \frac{2c(4+9k)^2}{9k^2} = 18c$. If $c$ is very small, the gray dot is close to the origin, but if $c$ is large, the gray dot is far. Large values for the cost of lobbying, $c$, imply that effective coalition costs are large because agents do not lobby much. As $c \to 0$, the gray dot approaches the origin for any $(\chi_2, \chi_3)$ pair, because low $c$ implies extensive lobbying and high benefits to forming the grand coalition. Via the term in the $\chi^E$'s, $c$ and $k$ can be thought of as determining the location on the ray. Increasing $k$ or decreasing $c$ causes a shift down and to the left. Decreasing $k$ or increasing $c$, in contrast, cause shifts in the other direction.

Figure 1

Coalition structures as functions of effective coalition-formation costs, $\chi^E_2$ and $\chi^E_3$. The grey line segment in each subfigure indicates a nexus of points such that $\chi^E_3 = \chi^E_2 \frac{\chi^E_3}{\chi^E_2}$. The grey dot at the lower-left end of each line segment is at $(\chi^E_2, \chi^E_3) = (18c\chi_2, 18c\chi_3)$. On the vertical axes, $\Delta_{m\ell}^G = 2(D_s D_m + D_s D_t + D_m D_{\ell})$, $\Delta_{m\ell}^G = 2D^2 - (D_m + D_{\ell})^2 - 2\left(\frac{1-c}{a}\right) D_s D_{m\ell}$, and $\Delta_{m\ell}^G = 2D^2 - (D_s + D_{\ell})^2 - 2\left(\frac{1-c}{a}\right) D_m D_{m\ell}$.

dot, is defined by the point $(\chi^E_2, \chi^E_3) = (18c\chi_2, 18c\chi_3)$, because $\lim_{k \to \infty} \frac{2c(4+9k)^2}{9k^2} = 18c$. If $c$ is very small, the gray dot is close to the origin, but if $c$ is large, the gray dot is far. Large values for the cost of lobbying, $c$, imply that effective coalition costs are large because agents do not lobby much. As $c \to 0$, the gray dot approaches the origin for any $(\chi_2, \chi_3)$ pair, because low $c$ implies extensive lobbying and high benefits to forming the grand coalition. Via the term in the $\chi^E$'s, $c$ and $k$ can be thought of as determining the location on the ray. Increasing $k$ or decreasing $c$ causes a shift down and to the left. Decreasing $k$ or increasing $c$, in contrast, cause shifts in the other direction.
5 Analysis

5.1 Coalitions

When the lobbyist captures a sufficient amount of the net gains from coalition formation, i.e., for $\alpha \leq \frac{D_s(D_s+D_m)}{D_m^2+D^2_l}$, the two possible equilibrium coalitions are $m \ell$ and $G$. In either coalition, agents group by similarity. That is, either no agents, the higher types, or all agents form a coalition. This result is similar to much of the prior literature on endogenous lobbying (e.g., Mitra, 1999), in which only the most similar agents organize into coalitions. In contrast, when the agents capture a sufficient amount of the net gains from coalition formation, i.e., with $\alpha > \frac{D_s(D_s+D_m)}{D_m^2+D^2_l}$, the $s \ell$ coalition is also possible. This resulting coalition structure contrasts with much of the prior literature on endogenous lobbying, in which only the most similar agents organize into coalitions, but is similar in spirit to the results of Baccara and Yariv (2016), who find potential for polarization or similarity of membership in peer-selected groups organized to produce public goods. Our result is summarized in the following proposition.

**Proposition 1 Lobbyist power**

*If agents capture more of the net gains from coalition formation, i.e., as $\alpha$ increases, coalitions are weakly smaller and are more likely to be polarized, featuring dissimilar agents.*

An increase in $\alpha$ indicates that the agents’ share of the net gains increases. As a comparison between Figures 1(a) and 1(b) shows, two transitions can occur when $\alpha$ crosses the threshold $\frac{D_s(D_s+D_m)}{D_m^2+D^2_l}$. Specifically, for $\chi_2 < \frac{D^2_s}{D_m^2+D^2_l}$ and $\chi_3^E - \chi_2^E > \Delta^G_{m \ell}$, low values of $\alpha$ lead to the $m \ell$ coalition. However, when $\alpha$ increases from below $\frac{D_s(D_s+D_m)}{D_m^2+D^2_l}$ to above $\frac{D_s(D_s+D_m)}{D_m^2+D^2_l}$, the only offer-stable coalition structure is $s \ell$. Similarly, when $\chi_2 < \frac{D^2_s}{D_m^2+D^2_l}$ and $\Delta^G_{s \ell} < \chi_3^E - \chi_2^E < \Delta^G_{m \ell}$, low values of $\alpha$ lead to the grand coalition whereas high values of $\alpha$ lead to the $s \ell$ coalition. For all other parameter values, changes in $\alpha$ do not have an effect on the equilibrium coalition structure.
Next, we turn to the influence of coalition costs, $\chi$, the cost to agents of lobbying efforts, $c$, and the degree of regulatory uniformity, $k$, on coalition formation, lobbying, $B$, and regulatory strength, $\pi$. Several of the comparative statics below rely on transitions driven by changes in parameters. These transitions can be understood graphically from Figure 1. Additionally, we provide Figure 2. The edges indicate boundaries between coalition structures. In the caption, we algebraically characterize the conditions under which parameter changes lead to transitions across coalition structures and link these conditions to the labeled edges.

**Proposition 2  Coalition costs**

1. An increase in $\chi_3$ causes weakly smaller lobbies.
2. An increase in $\chi_2$ can cause lobbies to grow or to disband.
3. Concurrent proportional increases in $\chi_2$ and $\chi_3$ can lead to larger or smaller lobbies.

In Figures 1 and 2 an increase in $\chi_3$ corresponds to an upward shift or a steepening of the gray rays, which can cause a transition from the grand coalition to either the $s\ell$ lobby (edge 7), the $m\ell$ lobby (edge 4), or to no lobby (edge 2). Not surprisingly, higher costs can lead to smaller lobbies, as is always the case with the cost of the three-agent coalition, $\chi_3$. An increase in $\chi_2$ corresponds to a rightward shift or a flattening of the gray rays in Figures 1 and 2 (e.g., a transition from the dashed gray ray to the solid gray ray). Holding $c$ and $k$ constant, this can cause the coalition shifts described in part 2 of Proposition 2 along edges 1, 3, 4, 5, 6, or 7. The $m\ell$ coalition can become unstable with an increase in $\chi_2$, causing a transition either to the grand coalition or to no coalitions as $\chi_2^{E}$ moves from below to above $D_2^{2} + D_m^{2}$.

Graphically, a proportional increase in both $\chi_2$ and $\chi_3$ can be interpreted as a shift up and to the right along any of the four gray rays in Figures 1. For example, in Figure 1(b) (Figure 2), a shift up and to the right along the gray dashed line can cause transitions from $G$ to $s\ell$ (edge 7), from $s\ell$ to $G$ (edge 6), from $G$ to $m\ell$ (edge 4), from $m\ell$ to $G$ (edge 3), or from $G$ to $I$ (edge 2). Much of this non-monotonicity occurs because the non-coalition member’s
Numerically labeled boundaries between coalition structures. On the vertical axes, $\Delta_i^G = 2\left(\overline{D}^2 + D_s D_m + D_s D_\ell + D_m D_\ell\right)$, $\Delta_{mt}^G = 2\overline{D}^2 - (D_m + D_\ell)^2 - 2\left(\frac{1-a}{a}\right) D_m \overline{D}_{st}$, and $\Delta_{st}^G = 2\overline{D}^2 - (D_s + D_\ell)^2 - 2\left(\frac{1-a}{a}\right) D_m \overline{D}_{st}$. Transitions between coalitions correspond to the following: for $\chi_3^E > \Delta_i^G$, decreasing $\chi_2^E$ from above to below $\overline{D}_{mt}$ yields a shift from $I$ to $mt$ (edge 1). For $\chi_2^E > \overline{D}_{mt}^2$, decreasing $\chi_3^E$ from above to below $\Delta_{mt}^G$ yields a shift from $I$ to $G$ (edge 2). For $\Delta_i^G > \chi_3^E > \Delta_{mt}^G + D_\ell^2 + D_m^2$, decreasing $\chi_2^E$ from above $\overline{D}_{mt}^2$ to below $\overline{D}_{mt}^2$ yields a shift from $G$ to $mt$ (edge 3). For $\chi_2^E < \overline{D}_{st}^2$, increasing $\chi_3^E$ from below to above $\Delta_{mt}^G + \chi_2^E$ or decreasing $\chi_2^E$ from above to below $\chi_3^E - \Delta_{mt}^G$ yields a shift from $G$ to $mt$ (edge 4). The same transition occurs additionally, for $\chi_2^E < \overline{D}_{st}^2$ and $\alpha \leq \frac{D_i(D_s+D_m)}{D_{st}^2+D_{st}^2}$, when $\chi_2^E$ increases from below to above $\Delta_{mt}^G + \chi_2^E$ or when $\chi_2^E$ decreases from above to below $\chi_3^E - \Delta_{mt}^G$ (edge 4). For $\chi_3^E > \Delta_{mt}^G + \chi_2^E$ and $\alpha > \frac{D_i(D_s+D_m)}{D_{st}^2+D_{st}^2}$, $\chi_2^E$ decreasing from above $\overline{D}_{st}^2$ to below $\overline{D}_{st}^2$ yields a transition from $mt$ to $st$ (edge 5). For $\Delta_{st}^G + \chi_2^E < \chi_3^E < \Delta_{mt}^G + \chi_2^E$, and $\alpha > \frac{D_i(D_s+D_m)}{D_{st}^2+D_{st}^2}$, decreasing $\chi_2^E$ from above $\overline{D}_{st}^2$ to below $\overline{D}_{st}^2$ yields a transition from $G$ to $st$ (edge 6). For $\alpha > \frac{D_i(D_s+D_m)}{D_{st}^2+D_{st}^2}$ and $\chi_2^E$ below $\overline{D}_{st}^2$, increasing $\chi_3^E$ from below to above $\Delta_{st}^G + \chi_2^E$ or decreasing $\chi_2^E$ from above to below $\chi_3^E - \Delta_{G}^G$ yields a transition from $G$ to $st$ (edge 7).
utility is important for determining the offer-stable coalition. Specifically, the non-coalition member gains from the other agents forming a coalition (relative to the non-coalition case) because the coalition members are able to overcome the free-rider problem and each agent’s lobbying decreases all agents’ regulatory strength. This externality of the two-agent lobby can make it profitable for one of the agents to leave the grand coalition.

Proposition 3 Lobbying costs and uniformity

Increases in lobbying costs, $c$, and decreases in regulatory uniformity, $k$, have similar effects as proportional increases in both coalition costs, $\chi_2$ and $\chi_3$, and can lead to larger or smaller lobbies.

Note that $\frac{\chi_3^E}{\chi_3} = \frac{\chi_2^E}{\chi_2} = \frac{2c(4+9k)^2}{9k^2}$, and that lobbying costs, $c$, and regulatory uniformity, $k$, only influence coalition structures through their influence on $\chi_2^E$ and $\chi_3^E$. As such, any change in $c$ or $k$ causes a proportional change in both $\chi_2^E$ and $\chi_3^E$, just as a proportional change in both $\chi_2$ and $\chi_3$ would.

Graphically, lobbying costs, $c$, and regulatory uniformity, $k$, determine the relevant location on a given $\chi_3^E = \chi_2^E \chi_3/\chi_2$ ray; the particular gray lines in in Figure 1 are examples. First, higher lobbying costs decrease the benefit of lobbying, and thereby decrease the benefits from forming a coalition that helps overcome the free-rider problem on lobbying effort. At one extreme, as $c \to 1$, agents have no reason to form lobbies. At the other extreme, as $c \to 0$, agents exert significant lobbying effort, making the grand coalition highly desirable. At intermediate levels of $c$, increasing $c$ causes a shift up and to the right along a given gray ray, which can cause various transitions between coalition structures.

Increases in regulatory uniformity, $k$, exacerbate the free-rider problem on lobbying and, thus, tend to promote the formation of lobbies. When $k$ is very low, the lobbying externalities are insignificant, giving agents little incentive to bear the costs of forming coalitions. Note that $\lim_{k \to 0} \chi_j^E \to \infty$ which corresponds to locations on the gray rays in Figure 1 that are in the “No Lobbies” region. Increasing $k$ causes a shift down and to the left along a given ray. When $k$ is sufficiently high, agents can benefit from coordinating their lobbying efforts, which tends to favor larger lobbies, but can locally cause transitions to smaller lobbies as
well. Even with high $k$, though, lobbying can be too costly for coalition formation to be beneficial.

5.2 Lobbying activity and regulatory strength

We turn to the effects of parameter changes on lobbying efforts and regulatory strength.

**Corollary 1 **Lobbyist power
An increase in the fraction of the net gains from coalition formation that are retained by the agents, i.e., higher $\alpha$, causes weakly lower lobbying, $B$, higher regulatory strength, $\pi$, and lower expected welfare losses from agents’ privately beneficial activities.

As described in Proposition 1, and illustrated in comparisons between Figures 1(a) and 1(b), higher $\alpha$ makes the $s\ell$ coalition feasible in regions of parameter-space that would otherwise be characterized by either the $m\ell$ coalition or the grand coalition. Higher $\alpha$ in these regions thus leads to smaller coalitions that pursue less total lobbying, which lessens the agents’ influence on equilibrium regulatory strength, allowing the regulator to set stronger policies, i.e., higher $\pi$. This in turn deters the privately beneficial action more frequently, lowering the expected welfare losses.

Similar to $\alpha$, changes in $\chi_2$ and $\chi_3$ only affect lobbying efforts and regulatory strengths through their effects on the equilibrium coalition structure. These are discussed next. Changes in $c$ and $k$, discussed below, affect lobbying and regulatory strength both directly (through agents’ choice of lobbying and the regulator’s choice of regulatory strength conditional on coalition structure) and indirectly (through their effects on the endogenous coalition structures).

**Corollary 2 **Coalition costs

1. An increase in $\chi_3$, all else equal, causes weakly lower lobbying, $B$, higher regulatory strength, $\pi$, and lower expected welfare losses from agents’ privately beneficial activities.

2. An increase in $\chi_2$, all else equal, can cause either weakly lower lobbying, $B$, higher regulatory strength, $\pi$, and lower expected losses from diversion; or greater lobbying, $B$, lower regulatory strength, $\pi$, and greater expected welfare losses from agents’ privately beneficial activities.
Corollary 2 arises from the results in Proposition 2 and because larger coalitions lobby more. More lobbying in turn weakens regulation and increases agents’ chances to inefficiently take the privately beneficial actions. Coalition formation costs influence lobbying and regulatory strength indirectly, that is, only through their influence on the coalition structure. In a regression of total lobbying on formation costs and observed coalition structures, for instance, formation costs should have no explanatory power because their explanatory power is completely subsumed by the coalition structures that emerge.

As noted above, unlike coalition formation costs, both lobbying costs and regulatory uniformity have direct effects on equilibrium lobbying behavior and regulatory strengths. In fact, as shown in Friedman and Heinle (2016), absent lobbying coalitions, $k$ and $c$ have monotonic effects on lobbying and regulatory strength. Absent coalitions, more regulatory uniformity and higher lobbying costs both lead to less lobbying and stronger regulatory policies, all else equal. In the presence of endogenously-formed coalitions, both regulatory uniformity and lobbying costs influence whether and which agents organize into lobbying coalitions, causing the effects of $c$ and $k$ to be non-monotonic.

**Corollary 3 Lobbying costs and uniformity**

An increase in lobbying costs, $c$, or regulatory uniformity, $k$, all else equal, can cause either weakly lower lobbying, $B$, and higher regulatory strength, $\pi$; or greater lobbying, $B$, and lower regulatory strength, $\pi$.

Corollary 3 combines the results of Proposition 3 with the monotonic effects of lobbying costs and regulatory uniformity shown in Friedman and Heinle (2016). The results described in Corollary 3 are illustrated in Figure 3, which plots total lobbying, $\bar{B}$, and average regulatory strength, $\pi_{\text{ave}} = \frac{1}{3} \sum \pi_i$, as functions of regulatory uniformity, $k$, for two sets of parameters that differ only in $\chi_2$. The solid gray curves have $\chi_2 = 1$, while the dashed black curves have $\chi_2 = 2$. In both cases, $\chi_3 = 2.65$.24

When $\chi_2 = 2$, coalition formation costs are concave, corresponding to a relatively flat $\chi_3^E = \chi_2^E \chi_3 \chi_2$ solid ray in Figure 1(b) and the solid plots in Figure 3. Starting from $k = 0$,

24Other parameters are set as $\alpha = 1$, $D_s = 1$, $D_m = 15$, $D_t = 30$, $c = 1$, and $\lambda = 1.2$. 

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increasing $k$ tends to reduce lobbying and increase regulatory strength. These continue monotonically in $k$ until we reach a threshold level of $k$ that makes the benefit of forming a three-agent coalition sufficiently large (edge 2 in Figure 2). At this point, as the coalition is formed, we see a discrete jump in total lobbying and a drop in average regulatory strength. Further increases in regulatory uniformity have no more effects, as $k$ is moot in the presence of the grand coalition and increases in $k$ maintain the dominance of the grand coalition.

When $\chi_2 = 1$, coalition costs are convex, corresponding to a steeper $\chi_3^E = \chi_2^E \frac{\chi_3}{\chi_2}$ dashed ray in Figure 1(b) and the dashed plots in Figure 3. Total lobbying and average regulatory strength mostly behave as they do when coalition costs are concave. There is, however, a key difference in the intermediate range of $k \in (0.06, 0.08)$. In this region, as we increase $k$ from 0.06, we first see a transition from the grand coalition to the $m\ell$ coalition around $k = 0.061$. This first transition occurs when the $m\ell$ coalition becomes feasible, as $\chi_2^E$ drops below $D_{\ell}^2 + D_m^2$ (edge 3 in Figure 2(b)). As $k$ continues to increase, the $m\ell$ coalition remains optimal, but total lobbying decreases and average regulatory strength increase, as the free-rider problem between the $m\ell$ coalition and agent $s$ worsen. As $k$ increases past about 0.07, $\chi_3^E$ becomes less than $\chi_2^E + 2D_{\ell}^2 - D_{m\ell}^2 + D_m D_{\ell}$, and, as the gain from the grand coalition starts to dominate the gain from the $m\ell$ coalition, we see a shift back to the high lobbying

Figure 3
Total lobbying and average regulatory strength as functions of regulatory uniformity, $k$, when $\alpha > \frac{D_{\ell}(D_s + D_m)}{D_{s}^2 + D_{m}^2}$. Parameters are set as $\alpha = 1$, $D_s = 1$, $D_m = 15$, $D_{\ell} = 30$, $\chi_3 = 2.65$, $c = 1$, and $\lambda = 1.2$. For the solid gray line, $\chi_2 = 1$. For the dotted black line, $\chi_2 = 2$. 

\begin{itemize}
\item \textbf{Figure 3}
\item Total lobbying and average regulatory strength as functions of regulatory uniformity, $k$, when $\alpha > \frac{D_{\ell}(D_s + D_m)}{D_{s}^2 + D_{m}^2}$. Parameters are set as $\alpha = 1$, $D_s = 1$, $D_m = 15$, $D_{\ell} = 30$, $\chi_3 = 2.65$, $c = 1$, and $\lambda = 1.2$. For the solid gray line, $\chi_2 = 1$. For the dotted black line, $\chi_2 = 2$. 
\end{itemize}
and low regulatory strength associated with the grand coalition (edge 4). (Recall that when costs are convex, \( x_3^E \) will decrease faster in \( k \) than \( x_2^E \) will.) Next, as \( k \) increases further, the \( sl \) coalition becomes feasible with \( x_2^E \) dipping below \( D_s^2 + D_s^2 \). At \( k \approx 0.074 \), the \( sl \) coalition is preferable to the grand coalition (edge 6), but at \( k \approx 0.076 \), the grand coalition again becomes preferable, as \( x_3^E \) drops below \( x_2^E + 2D_s^2 - D_s^2 + D_sD_t \) (edge 7). With \( k \) between 0.074 and 0.076, increases in regulatory uniformity again cause decreases in lobbying and increases in regulatory strength, as the free-rider problem between the \( sl \) coalition and agent \( m \) gets worse. As \( k \) increases beyond 0.076, the grand coalition is again optimal, and further increases in regulatory uniformity cease to play a significant role.

Results, though not plotted, are similar for changes in lobbying costs, \( c \), as increases in \( c \) have similar effects as decreases in \( k \) as described in Proposition 3 and Corollary 3 and illustrated graphically with shifts along the gray rays in Figures 1(a) and 1(b). Figure 3 shows that increases in \( k \) have non-monotonic effects on lobbying and regulatory strength when agents can organize into lobbies to overcome free-rider problems on lobbying. Almost everywhere, the effects of \( k \) on lobbying (regulatory strength) are locally negative (positive), but these effects can be significantly outweighed by discrete jumps or drops as changes in \( k \) cause agents to change how they organize into coalitions.

6 Discussion and implications

In this section we relate our results to our motivating example of the regulation of financial institutions. Our main results relate to the interactions between regulatory uniformity, coalition formation, and lobbying. Existing evidence on coalition formation is sparse, so our empirical implications come mainly as suggestions for potential studies rather than interpretations of existing evidence.\(^{25}\)

Overall, empirical studies related to the results of our model require proxies for the central constructs. For lobbying costs and changes in lobbying costs, we recommend: judicial

\(^{25}\)An exception is Bombardini and Trebbi (2012), discussed in footnote 7.
decisions that changes how easily money can be raised and channeled to politicians (e.g., the Citizens United case that paved the way for corporate Political Action Committees); policies affecting how quickly a regulator can take a job in a related industry (i.e., revolving doorstops); the extensiveness of requirements for lobbying expenditure disclosures; campaign finance rules and reforms (e.g., the 2002 McCain-Feingold Bipartisan Campaign Reform Act); and anti-corruption rules that can limit de facto influence activities (e.g., the Foreign Corrupt Practices Act and its 1998 and 1998 amendments). To capture lobby membership, researchers can hand-collect membership rolls of active lobbying groups and trade associations, such as the Financial Services Roundtable (FSR) and American Bankers Association (ABA), or exploit overlapping lobbyists. Specifically, firms that hire or employ the same lobbyist might be considered members of a lobbyist-facilitated coalition. Data on which lobbying firms work for which firms is available in federal forms collected and disseminated by the Center for Responsive Politics (CRP). Lobbying activities can include both expenditures captured in the CRP data as well as non-pecuniary influence activities, such as meetings with regulators, which are often logged by the regulators.\footnote{Logs of Fed meetings, including attendees, dates, and summaries of the topics discussed are available from the Fed’s website at https://www.federalreserve.gov/regreform/communications-with-public.htm.} Finally, proxies for regulatory uniformity could depend on the degree of regulatory responsibilities at higher levels in a regulatory hierarchy (e.g., at the ECB versus country-level central banks), whether a set of firms are overseen by the same regulator or office, and whether firms fall into the same characteristic-based buckets (e.g., which side of the $50 billion Dodd-Frank cutoff). Office closures (Gopalan et al., 2017), regulatory consolidation (Granja and Leuz, 2017; Wong, 2018), alternating supervisory responsibilities (Agarwal et al., 2014), and legislative changes to prudential thresholds (Ackerman, 2018) each provide specific examples of shocks to regulatory uniformity that we expect would affect incentives to form coalitions and influence regulators.

Within a given set of policies, there may also be considerable variation in the degree of uniformity. Minimum capital requirements may be applied to firms uniformly, while
firm-specific stress tests could be applied with significant heterogeneity. We might therefore expect large bank lobbies, like the ABA, to focus more on capital requirements, while firms’ individualized lobbying might focus more on individualized stress testing.

As an example, in 2006, the Lobbying Disclosure Act of 1995 was amended to require that lobbyists register with congressional officials. Arguably, this registration requirement imposes a cost on becoming a lobbyist and, thus, creates a barrier to entry that restricts the supply of lobbyists. We expect that the 2006 amendment of the Lobbying Disclosure Act has increased the bargaining power of the remaining lobbyists. In our model, this translates into a reduction in $\alpha$. Our model (Proposition 1 and Corollary 1) predicts that following the 2006 amendment, lobbying coalitions are more likely to be formed by larger financial institutions and that total lobbying should increase as a result. Consistent with this prediction, in the 1998 to 2016 period, the number of unique registered lobbyists who have actively lobbied on behalf of the Finance, Insurance & Real Estate sector peaked in 2007 and has steadily decreased every year since, with the exception of a slight uptick in 2013. During the same period, the total lobbying spending by firms in the Finance, Insurance & Real Estate sector increased each year from 2008 to 2016, with the exception of a slight reduction in 2015. While our prediction is consistent with the secular trend, empirical studies could examine whether explicit and implicit coalition structures changed around the 2006 amendment and other shocks to lobbyists’ bargaining power (e.g., Bertrand et al., 2014).

The 2008 to 2016 period has also seen an increase in the prevalence of lobbying coalitions, as highlighted in the popular press (e.g., Bogardus, 2013; Ho, 2015), and the dollar amount of political contributions, particularly soft money contributions, made by the financial sector. These trends are consistent with the potential effects of changes in lobbying costs and regulatory uniformity brought about by recent changes in the institutional environment introduced by the Supreme Court and Congress. Ho (2015) provides examples of coalitions featuring both similar and dissimilar members.

Turning to regulatory uniformity, Title 1 of Dodd-Frank established the Financial Sta-
bility Oversight Council as a coordinator of the oversight and regulation of systemically important financial institutions. Additionally, Section 312 of the Dodd-Frank Act specified that the Office of Thrift Supervision be closed down and its oversight responsibilities reallocated to the Federal Reserve, FDIC, and OCC. Both changes tend towards regulatory policies that treat financial large financial institutions with a great degree of homogeneity and differences in treatment between large and small banks. Potentially in response to this change in regulatory uniformity, the FSR recently announced a reduction in its membership to focus more on issues affecting the largest banks.\textsuperscript{27} This is consistent with our model’s prediction (Proposition 3) that decreased uniformity across types of banks can lead to the splintering of coalitions.

Overall, one of the main messages of our paper is that changes in the underlying parameters that affect the coalition structure can go in the opposite direction of changes that do not affect the coalition structure. Empirical analyses of lobbying and regulatory outcomes might find no effects for uniformity or muted effects for changes in lobbying costs if coalition effects are ignored. Observing coalition effects in the data requires measuring coalition structures and examining whether these change around shocks to underlying parameters of interest. For instance, a small increase in uniformity might lead to a reduction in lobbying, while a further increase in uniformity might lead to an increase in lobbying because a new coalition formed.

\section{Conclusion}

In this study we investigate the propensity of agents to form coalitions that lobby against stricter policies. For example, financial firms can take privately-beneficial actions that impose systemic risks on the financial system and lobby against policies that reduce their ability

\textsuperscript{27}Specifically, the FSR board recently voted to limit its membership to banks with more than $25B in assets and very large payments companies (e.g., MasterCard and Visa). This restructuring led to a more homogeneous set of members via the exclusion of smaller financial institutions, insurers, and asset managers (Guida, 2018). The FSR also recently merged with the policy arm of The Clearing House, another large bank coalition (Clozel, 2018).
to take such actions. In our model, agents form coalitions because regulation is, at least partly, uniform across agents. This uniformity, in turn, implies that one agent’s lobbying has effects on all other agents’ regulation, such that a free-rider problem on lobbying arises among agents. The benefit to forming coalitions emerges because we assume that agents within a coalition are able to overcome the free-rider problem, which increases the lobbying efforts of all agents in the coalition. As the free-rider problem is caused by uniformity, a more uniform regulatory policy increases the benefits to forming a lobbying coalition.

We assume that a lobbyist is necessary to form a coalition, and allow the lobbyist’s bargaining power, i.e., the fraction of the net coalition benefit she extracts, to vary. In this setting, the degree of lobbyist power influences the potential lobbies that can arise in equilibrium, with dissimilar agents potentially joining together only when agents retain a high fraction of the net coalition gains. We further find that endogenous coalition formation causes the effects of regulatory uniformity and lobbying costs on total lobbying and average policy strength to be non-monotonic. Increasing the degree of uniformity both: (i) increases the free-rider problem, which decreases lobbying and increases average policy strength; and (ii), increases the benefit of forming coalitions, which increases lobbying and decreases average policy strength. In an environment with a fixed coalition structure, only the first effect is present. However, since the coalition structure reacts to changes in policy uniformity, we find non-monotonic effects of increasing the extent of uniformity. Similarly, increasing the cost agents bear personally for lobbying has a direct effect that decreases lobbying but has an indirect effect on the benefit of forming a lobby. Surprisingly, we find that lobbying decreases (and regulatory quality increases) when competition in the lobbyist market increases. The reason is that when agents retain a higher share of the benefits to forming coalitions, they tend to form smaller coalitions which results in less lobbying. As a result, increasing barriers to entry in the lobbyist market, e.g., through anti-revolving-door policies, can lead to more lobbying for policies that benefit private interests.
References


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8 Appendix A

8.1 Proof of Lemma 1

Independent agents. When the three agents act independently, lobbying, regulatory strengths, and agents’ utility are given by

\[
\hat{B}_{i,I} = \frac{4 + 3k}{c(4 + 9k)} D_i, \quad (8)
\]

\[
\hat{\pi}_{i,I} = \frac{\lambda c(4 + 9k) - (4 + 3k)}{c(4 + 9k)^2} (4D_i + 3k\bar{D}) , \text{ and} \quad (9)
\]

\[
\hat{U}_{i,I} = (1 - \hat{\pi}_{i,I}) D_i - \frac{1}{2c} \left(\frac{4 + 3k}{4 + 9k}\right)^2 D_i^2. \quad (10)
\]

Two-agent lobbies. We first characterize the results for all possible two-agent lobbies and then summarize them using generic notation.

Small-medium agent coalition: When the small (s) and medium (m) agents form a lobby together, lobbying is as follows:

\[
\hat{B}_{s,sm} = \hat{B}_{s,I} + \frac{3kD_m}{c(4 + 9k)} = \frac{4D_s + 3k\bar{D}_{sm}}{c(4 + 9k)},
\]

\[
\hat{B}_{m,sm} = \hat{B}_{m,I} + \frac{3kD_s}{c(4 + 9k)} = \frac{4D_m + 3k\bar{D}_{sm}}{c(4 + 9k)},
\]

\[
\hat{B}_{t,sm} = \hat{B}_{t,I} = D_t \frac{4 + 3k}{c(4 + 9k)}, \text{ and}
\]

\[
\hat{B}_{sm} = \hat{B}_I + \frac{3k\bar{D}_{sm}}{c(4 + 9k)}.
\]
where \( \bar{D}_{sm} = D_m + D_s \). Regulatory strengths are:

\[
\hat{\pi}_{s,sm} = \frac{4 \left( \lambda D_s - \bar{B}_{s,I} - \frac{3kD_m}{c(4+9k)} \right) + 3k \left( \lambda \bar{D} - \bar{B}_I - \frac{3k\bar{D}_{sm}}{c(4+9k)} \right)}{4 + 9k},
\]

\[
\hat{\pi}_{m,sm} = \hat{\pi}_{s,I} - \frac{3k \left( 4D_m + 3k\bar{D}_{sm} \right)}{c(4+9k)^2}, \quad \text{and}
\]

\[
\hat{\pi}_{t,sm} = \hat{\pi}_{t,I} - \frac{9k^2D_{sm}^2}{c(4+9k)^2}.
\]

The utilities are:

\[
\hat{U}_{sm,sm} = \frac{c(9k+4) \left( (9k+4)\bar{D}_{sm} - \lambda \left( 3k\bar{D}_{sm}\bar{D} + 4\bar{D}_{sm}^2 \right) \right) + 3k\bar{D}_{sm}\bar{D} (3k+4) + 8\bar{D}_{sm}^2}{c(9k+4)^2}
\]

\[
= \hat{U}_{s,I} + \hat{U}_{m,I} + \frac{9k^2\bar{D}_{sm}^2}{2c(4+9k)^2} \quad \text{and}
\]

\[
\hat{U}_{t,sm} = \frac{D_t \left( 4D_t \lambda - 4 - 2c(9k+4)(3k(\lambda \bar{D} - 3)) + D_t (3k+4)^2 + 12k(3k+2)\bar{D}_{sm} \right)}{2c(9k+4)^2}
\]

\[
= \hat{U}_{t,I} + \frac{9k^2D_t\bar{D}_{sm}}{c(4+9k)^2},
\]

where \( \bar{D}_{sm}^2 = D_s^2 + D_m^2 \).

**Small-large agent coalition:** When the small (\( s \)) and large (\( t \)) agents form a lobby together, lobbying is as follows:

\[
\hat{B}_{s,lt} = \hat{B}_{s,I} + \frac{3kD_t}{c(4+9k)} = \frac{4D_s + 3k\bar{D}_{sl}}{c(4+9k)},
\]

\[
\hat{B}_{m,lt} = \hat{B}_{m,I} = \frac{4 + 3k}{c(4+9k)},
\]

\[
\hat{B}_{t,lt} = \hat{B}_{t,I} + \frac{3kD_s}{c(4+9k)} = \frac{4D_t + 3k\bar{D}_{st}}{c(4+9k)}, \quad \text{and}
\]

\[
\bar{B}_{sm} = \hat{B}_I + \frac{3k\bar{D}_{st}}{c(4+9k)}.
\]
where $\bar{D}_{s\ell} = D_s + D_\ell$. Regulatory strengths are:

\[
\tilde{\pi}_{s,s\ell} = \tilde{\pi}_{s,I} - \frac{3k \left( 4D_\ell + 3k\bar{D}_{s\ell} \right)}{c(4 + 9k)^2},
\]

\[
\tilde{\pi}_{m,s\ell} = \tilde{\pi}_{m,I} - \frac{9k^2\bar{D}_{s\ell}}{c(4 + 9k)^2}, \text{ and}
\]

\[
\tilde{\pi}_{\ell,s\ell} = \tilde{\pi}_{\ell,I} - \frac{3k \left( 4D_s + 3k\bar{D}_{s\ell} \right)}{c(4 + 9k)^2},
\]

and utilities are:

\[
\hat{U}_{s,s\ell} = \frac{c(9k + 4) \left( (9k + 4)\bar{D}_{s\ell} - \lambda \left( 4\bar{D}_{s\ell}^2 + 3k\bar{D}_{s\ell}\bar{D}_s \right) \right) + 8\bar{D}_{s\ell}^2 + 3k\bar{D}_{s\ell}\bar{D}_s(3k + 4)}{c(9k + 4)^2}
\]

\[
= \hat{U}_{s,I} + \hat{U}_{\ell,I} + \frac{9k^2\bar{D}_{s\ell}^2}{2c(4 + 9k)^2} \text{ and}
\]

\[
\hat{U}_{m,s\ell} = \frac{D_m \left( -2c(9k + 4)(3k(\lambda\bar{D}_s - 3) + 4D_m\lambda - 4) + 12k(3k + 2)\bar{D}_{s\ell} + D_m(3k + 4)^2 \right)}{2c(9k + 4)^2}
\]

\[
= \hat{U}_{m,I} + \frac{9k^2D_m\bar{D}_{s\ell}}{c(4 + 9k)^2},
\]

where $\bar{D}_{s\ell}^2 = D_s^2 + D_\ell^2$.

Medium-large agent coalition: When the medium ($m$) and large ($\ell$) agents form a lobby together, lobbying is as follows:

\[
\hat{B}_{s,m\ell} = \hat{B}_{s,I} = D_s\frac{4 + 3k}{c(4 + 9k)},
\]

\[
\hat{B}_{m,m\ell} = \hat{B}_{m,I} + \frac{3kD_\ell}{c(4 + 9k)} = \frac{4D_m + 3k\bar{D}_{m\ell}}{c(4 + 9k)},
\]

\[
\hat{B}_{\ell,m\ell} = \hat{B}_{\ell,I} + \frac{3kD_m}{c(4 + 9k)} = \frac{4D_\ell + 3k\bar{D}_{m\ell}}{c(4 + 9k)}, \text{ and}
\]

\[
\bar{B}_{sm} = \bar{B}_I + \frac{3k\bar{D}_{m\ell}}{c(4 + 9k)},
\]

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where $\bar{D}_{m\ell} = D_m + D_{\ell}$. Regulatory strengths are:

\[
\hat{\pi}_{s,m\ell} = \hat{\pi}_{s,I} - \frac{9k^2 \bar{D}_{m\ell}}{c(4+9k)^2},
\]

\[
\hat{\pi}_{m,m\ell} = \hat{\pi}_{m,I} - \frac{3k(4D_{\ell} + 3k\bar{D}_{m\ell})}{c(4+9k)^2}, \text{ and}
\]

\[
\hat{\pi}_{\ell,m\ell} = \hat{\pi}_{\ell,I} - \frac{3k(4D_m + 3k\bar{D}_{m\ell})}{c(4+9k)^2},
\]

and utilities are:

\[
\hat{U}_{m,m\ell} = \frac{c(9k+4)\left((9k+4)\bar{D}_{m\ell} - \lambda \left(4\bar{D}_{m\ell}^2 + 3k\bar{D}_{m\ell}\bar{D}\right)\right) + 8\bar{D}_{m\ell}^2 + 3k\bar{D}_{m\ell}\bar{D}(3k+4)}{c(9k+4)^2}
\]

\[
= \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2\bar{D}_{m\ell}^2}{2c(4+9k)^2}
\]

and

\[
\hat{U}_{s,m\ell} = \frac{D_s(2c(9k+4)(3k(\lambda\bar{D} - 3) + 4D_s\lambda - 4) + 12k(3k+2)\bar{D}_{m\ell} + D_s(3k+4)^2)}{2c(9k+4)^2}
\]

\[
= \hat{U}_{s,I} + \frac{9k^2D_s\bar{D}_{m\ell}}{c(4+9k)^2},
\]

where $\bar{D}_{m\ell}^2 = D_m^2 + D_{\ell}^2$.

**Generic Notation:** Generally, in the two-agent setting, if agents $g$ and $h$ form a lobby or coalition, leaving agent $i$ out, we have the following: $\hat{B}_{i,gh} = \hat{B}_{i,I} = D_i \frac{4+3k}{c(4+9k)}$; $\hat{B}_{g,gh} = \hat{B}_{g,gh} = \hat{B}_{g,I} + \frac{3kD_h}{c(4+9k)^2} = \frac{4D_h + 3k\hat{B}_{gh}}{c(4+9k)}$; and $\hat{B}_{gh} = \hat{B}_{I} + \frac{3k\hat{B}_{gh}}{c(4+9k)}$. Regulatory strengths are: $\hat{\pi}_{i,gh} = \hat{\pi}_{i,I} - \frac{9k^2\hat{D}_{gh}}{c(4+9k)^2}$ and $\hat{\pi}_{g,gh} = \hat{\pi}_{g,I} - \frac{3k(4D_h + 3k\hat{D}_{gh})}{c(4+9k)^2}$. The coalitions’ expected utilities are: $\hat{U}_{gh,gh} = \hat{U}_{g,gh} + \hat{U}_{h,gh} + \frac{9k^2\bar{D}_{gh}^2}{2c(4+9k)^2}$ and $\hat{U}_{i,gh} = \hat{U}_{i,I} + \frac{9k^2D_i\bar{D}_{gh}}{c(4+9k)^2}$, where and $\hat{U}_{gh,gh}$ is the total utility of the agents and the lobbyist in the $gh$ coalition. Note that $\hat{U}_{i,gh} \geq \hat{U}_{i,I}$, implying that the non-coalition member benefits from the other agents forming a coalition.

**Grand coalition/three-agent lobby.** When all agents join together in the three-agent lobby (the grand coalition), lobbying is $\hat{B}_{i,G} = \frac{4D_i + 3k\hat{D}}{c(4+9k)}$ for each agent, which implies that
total lobbying is given by $\tilde{B}_G = \frac{\tilde{D}}{c}$. Regulatory strengths are:

$$\hat{\pi}_{i,G} = \frac{4D_i \left( \lambda - \frac{4}{c(4+9k)} \right) + 3k\tilde{D} \left( \lambda - \frac{1}{c} - \frac{4}{c(4+9k)} \right)}{4 + 9k} \tag{11}$$

and the total expected utility of the grand coalition is

$$\hat{U}_G = \hat{U}_{s,I} + \hat{U}_{m,I} + \hat{U}_{\ell,I} + \frac{9k^2 \left( \frac{D^2 + D_s D_m + D_s D_{\ell} + D_{m} D_{\ell}}{c(4+9k)^2} \right)}{c(4+9k)^2}. \tag{12}$$

### 8.2 Proof of Theorems 1 and 2: Offer-stable coalitions

To help develop coalition stability among two-agent coalitions, we define a maximum deviation offer.

**Definition 4 (Two-agent maximum deviation offer)** If agents $g$ and $h$ are members of the $gh$ coalition, the maximum deviation offer from agent $i$ to agent $g$, $MOD_{i,g}^{gh}$, is the greatest amount that agent $i$ would offer agent $g$ to leave the $gh$ coalition and join the new $ig$ coalition.

We can calculate the maximum deviation offers that a given agent is willing to make as

$$MOD_{i,g}^{gh} = \hat{U}_{i,ig} + \hat{U}_{g,ig} - \hat{U}_{L,ig} - \hat{U}_{i,gh} \tag{13}$$

$$= \hat{U}_{i,I} + \hat{U}_{g,I} + \alpha \left( \frac{9k^2 D^2_{ig}}{2c(4+9k)^2} - \chi_2 \right) - \left( \hat{U}_{i,I} + \frac{9k^2 D_i D_{gh}}{c(4+9k)^2} \right) \tag{14}$$

$$= \hat{U}_{g,I} + \frac{9k^2 \left( \alpha \left( \frac{D^2_{ig} - \chi_2}{c(4+9k)^2} \right) - D_i D_{gh} \right)}{2c(4+9k)^2}, \tag{15}$$

which is the utility that the agents in the $ig$ coalition achieve minus the utility that agent $i$ expects to achieve in the presence of the $gh$ coalition. The $MOD_{i,g}^{gh}$ is the maximum offer, because agent $i$ in coalition $ig$ faces a budget constraint (in terms of transferable utility) of $\hat{U}_{i,ig} + \hat{U}_{g,ig} - \hat{U}_{L,ig}$. Furthermore, agent $i$’s outside option is to remain independent in a structure featuring the $gh$ coalition, meaning that she should be willing to offer agent $g$ no
more than the gain achieved from \( g \)'s deviation from \( gh \) to \( ig \). For example,

\[
MDO_{s,\ell}^{m} = \hat{U}_{\ell,1} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \left( \overline{D}_{st}^2 - \chi_2 \right) - 2D_s \overline{D}_{mt} \right) \quad \text{and} \\
MDO_{m,\ell}^{s} = \hat{U}_{\ell,1} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \left( \overline{D}_{mt}^2 - \chi_2 \right) - 2D_m \overline{D}_{st} \right).
\]

(16) 

(17)

The following definition illustrates the offer-stability concept within the set of two-agent coalitions.

**Lemma 2 (Offer stability for a two-agent coalition)** A two-agent coalition \( gh \) is offer-stable relative to other two-agent coalitions if \( MDO_{i,g}^{gh} < MDO_{h,g}^{gi} \) and \( MDO_{i,h}^{gh} < MDO_{g,h}^{hi} \), i.e., if: 1) agent \( i \)'s maximum deviation offer to \( g \) conditional on coalition \( gh \) is lower than agent \( h \)'s maximum deviation offer to \( g \) conditional on coalition \( ig \); and 2) agent \( i \)'s maximum deviation offer to \( h \) conditional on coalition \( gh \) is lower than agent \( g \)'s maximum deviation offer to \( h \) conditional on coalition \( ih \). A coalition structure containing a stable two-agent coalition is two-agent stable.

Lemma 2 allows us to prove Theorems 1 and 2. First, \( MDO_{s,\ell}^{sm} > MDO_{m,\ell}^{st} \):

\[
\begin{align*}
MDO_{s,\ell}^{m} & > MDO_{m,\ell}^{s} \\
\alpha D_s^2 + \alpha D_{\ell}^2 - 2D_s D_m - 2D_s D_{\ell} & > \alpha D_m^2 + \alpha D_{\ell}^2 - 2D_m D_s - 2D_m D_{\ell} \\
2D_m D_{\ell} - 2D_s D_{\ell} & > \alpha D_m^2 - \alpha D_s^2 \\
2D_{\ell} (D_m - D_s) & > \alpha (D_m + D_s) (D_m - D_s) \\
2D_{\ell} & > \alpha (D_m + D_s),
\end{align*}
\]

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which implies that agent $s$ can make a credible offer to agent $\ell$ to leave the $m\ell$ coalition. Note that the $\alpha \chi_2$ terms on both sides cancel. Second, $MDO_{\ell,s}^{sm} > MDO_{m,s}^{s\ell}$:

$$
MDO_{\ell,s}^{sm} > MDO_{m,s}^{s\ell}
$$

$\hat{U}_{s,I} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \frac{D_{\ell s}^2}{\alpha D_{\ell s} - 2 D_{\ell} D_{ms}} \right) > \hat{U}_{s,I} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \frac{D_{ms}^2}{\alpha D_{ms} - 2 D_{ms} D_{\ell s}} \right)
$

$$
\alpha D_{\ell}^2 + \alpha D_{s}^2 - 2 D_{\ell} (D_m + D_s) > \alpha D_{s}^2 + \alpha D_{m}^2 - 2 D_{m} (D_{\ell} + D_s)
$$

$$
\alpha (D_{\ell} + D_m) > 2D_s,
$$

which is true for $\alpha > \frac{2D_s}{D_{\ell} + D_m}$, and false for $\alpha < \frac{2D_s}{D_{\ell} + D_m}$. When $\alpha < \frac{2D_s}{D_{\ell} + D_m}$, $MDO_{\ell,s}^{sm} < MDO_{m,s}^{s\ell}$, implying that agent $m$ can make a credible offer to agent $s$ to leave the $s\ell$ coalition. Finally,

$$
MDO_{\ell,m}^{sm} > MDO_{s,m}^{m\ell} \iff
$$

$\hat{U}_{m,I} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \frac{D_{m\ell}^2}{\alpha D_{m\ell} - 2 D_{\ell} D_{ms}} \right) > \hat{U}_{m,I} + \frac{9k^2}{2c(4+9k)^2} \left( \alpha \frac{D_{sm}^2}{\alpha D_{sm} - 2 D_{s} D_{m\ell}} \right) \iff
$

$$
\alpha (D_{\ell} + D_s) > 2D_m.
$$

So, $\alpha > \frac{2D_m}{D_{\ell} + D_s} \Rightarrow MDO_{\ell,m}^{sm} > MDO_{s,m}^{m\ell}$, implying that $\ell$ can successfully motivate $m$ to leave the $sm$ coalition. But $\alpha < \frac{2D_m}{D_{\ell} + D_s} \Rightarrow MDO_{\ell,m}^{sm} < MDO_{s,m}^{m\ell}$, implying that $s$ can successfully motivate $m$ to leave the $m\ell$ coalition.

Comparing the thresholds, we have

$$
\frac{2D_s}{D_{\ell} + D_m} < \frac{2D_m}{D_{\ell} + D_s}
$$

$$
2D_s (D_{\ell} + D_s) < 2 D_m (D_{\ell} + D_m)
$$

$$
2D_s D_{\ell} + 2D_s^2 < 2 D_m D_{\ell} + 2D_m^2.
$$
If $\alpha > \frac{2D_m}{D_t + D_s}$, then $\alpha > \frac{2D_s(D_m + D_\ell)}{D^2_t + D^2_s}$, as

$$\frac{2D_m}{D_t + D_s} > \frac{2D_s(D_m + D_\ell)}{D^2_t + D^2_s}$$

$$2D_mD^2_\ell + 2D_mD^2_s + 4D_mD_\ell D_s > 2D^2_sD_\ell + 2D_mD^2_s + 2D_sD^2_\ell + 2D_mD_sD_\ell$$

$$D^2_\ell(D_m - D_s) + D_\ell D_s(D_m - D_s) > 0.$$ 

If $\alpha > \frac{2D_m}{D_t + D_s}$, then $\alpha > \frac{2D_m(D_\ell + D_s)}{D^2_m + D^2_\ell}$ may or may not be true, as

$$\frac{2D_m}{D_\ell + D_s} > \frac{2D_m(D_s + D_\ell)}{D^2_m + D^2_\ell}$$

$$2D_m(D^2_m + D^2_\ell) > 2D_m(D_s + D_\ell)^2$$

$$D^2_m + D^2_\ell > D^2_s + D^2_\ell + 2D_sD_\ell$$

$$\frac{D_m + D_s}{D_s + D_\ell}(D_m - D_s) > D_\ell.$$

Overall, we have the following for regions of parameter-space defined by $\alpha$:

1. $\alpha > \frac{2D_m}{D_t + D_s} \Rightarrow MDO^{ml}_{s,\ell} > MDO^{ml}_{m,\ell}, MDO^{sm}_{\ell,s} > MDO^{ml}_{m,s},$ and $MDO^{sm}_{\ell,m} > MDO^{ml}_{s,m}$. $s\ell$ is offer-stable.

2. $\frac{2D_s}{D_t + D_m} < \alpha < \frac{2D_m}{D_t + D_s} \Rightarrow MDO^{ml}_{s,\ell} > MDO^{ml}_{m,\ell}, MDO^{sm}_{\ell,s} > MDO^{ml}_{m,s},$ and $MDO^{sm}_{\ell,m} < MDO^{ml}_{s,m}$. $s\ell$ is offer-stable.

3. $\alpha < \frac{2D_m}{D_t + D_m} \Rightarrow MDO^{ml}_{s,\ell} > MDO^{ml}_{m,\ell}, MDO^{sm}_{\ell,s} < MDO^{ml}_{m,s},$ and $MDO^{sm}_{\ell,m} < MDO^{ml}_{s,m}$. $sm$ is hypothetically offer-stable.

In the following subsections we derive coalition structures in each region of the parameter-space.

8.2.1 $\chi_2 < \frac{D^2_s}{D^{2}_{ml}}$

If $MDO^{ml}_{s,\ell} > 0$, then the $s$ agent is better-off in a pairing with $\ell$ than as the outsider when $ml$ form. To be able to form a pairing with $\ell$, $s$ must be able to offer $\ell$ more than what $\ell$
would gain from being the outsider in the presence of the \( sm \) lobby, meaning \( \alpha \left( \frac{D^2_{\ell s}}{D^2_{s} + D^2_{\ell} - \chi_2} \right) - 2D_{\ell}D_{sm} > 0 \iff \alpha > \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} \), which in turn implies

\[
\alpha > \frac{2D_{\ell}(D_s + D_m)}{D^2_{\ell} + D^2_{s} - \chi_2} \Rightarrow \alpha (D^2_{s} + 2D_s D_{\ell}) - \alpha \chi_2 > 2D_{\ell}D_s (1 + \alpha) + 2D_{\ell}D_m
\]

\[
\Rightarrow \alpha (D_{\ell} + D_s) (D_{\ell} + D_s) - \alpha \chi_2 > +2D_{\ell}D_m + 2D_{m}D_s + 2D_{\ell}D_s (1 + \alpha) - 2D_m D_s
\]

\[
\Rightarrow \alpha > \frac{2D_m}{D_{\ell} + D_s} + \frac{2D_s (D_{\ell} - D_m) + 2\alpha D_{\ell} D_s + \alpha \chi_2}{(D_{\ell} + D_s)^2} > \frac{2D_m}{D_{\ell} + D_s}
\]

\[
\Rightarrow \alpha > \frac{2D_m}{D_{\ell} + D_s}.
\]

It is possible, that for \( \alpha < \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} \), the \( \ell \) agent will turn down the \( s \)-agent’s offer to form a coalition, preferring to wait for the \( sm \) coalition to form or willing to form an \( m\ell \) coalition. The \( \ell \) agent will prefer to form a coalition with \( m \) rather than waiting for the \( sm \) coalition to potentially form if

\[
\alpha > \frac{2D_{\ell} (D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2}.
\]

So, if \( \alpha \in \left( \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2}, \frac{2D_{\ell}(D_\ell + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} \right) \), then the \( \ell \) agent will be willing to form an \( m\ell \) coalition but unwilling to form an \( s\ell \) coalition. In this range, the \( m \) agent will be willing to form the coalition with \( \ell \) rather than wait as long as

\[
\alpha > \frac{2D_m (D_s + D_{\ell})}{D^2_{m} + D^2_{\ell} - \chi_2},
\]

which is implied by \( \alpha > \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} \) as \( \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} > \frac{2D_m (D_s + D_{\ell})}{D^2_{m} + D^2_{\ell} - \chi_2} \). For \( \alpha \in \left( \frac{2D_{\ell}(D_s + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2}, \frac{2D_{\ell}(D_\ell + D_m)}{D^2_{m} + D^2_{\ell} - \chi_2} \right) \), \( \alpha > \frac{2D_m}{D_{\ell} + D_m} \) is true and implies that \( s\ell \) is offer-stable. So, \( s \) can make a successful deviation offer to \( \ell \) to leave the \( m\ell \) coalition. The problem is that \( \ell \) would rather be the outsider in the presence of the \( sm \) coalition than be in the \( s\ell \) coalition, but would rather be in the \( m\ell \) coalition.
coalition than be the outsider. This means that the following hold:

\[ MDO_{\ell,s}^{em} < 0, \]
\[ MDO_{\ell,m}^{em} > 0, \text{ and} \]
\[ MDO_{s,\ell}^{m} > MDO_{m,\ell}^{s} > 0. \]

So, this arrangement should result in the \( s\ell \) lobby forming as well, via the following sequence:

1) \( m \) forms a coalition with \( \ell \); 2) \( s \) makes a successful deviation-offer to \( \ell \).

If \( \alpha < \frac{2D_D(D_s+D_m)}{D_2^s + D_2^m = \chi_2} \) (i.e., \( MDO_{\ell,m}^{em} \)), then the \( \ell \) agent will prefer to wait rather than join a coalition with \( s \) or \( m \). In this case, the \( sm \) coalition will emerge if \( s \) and \( m \) both prefer joining the coalition to waiting, i.e., if

\[ \alpha > \frac{2D_s(D_m + D_s)}{D_2^s + D_2^m - \chi_2} \quad \text{and} \quad \alpha > \frac{2D_m(D_s + D_\ell)}{D_2^s + D_2^m - \chi_2} \]

Now, \( \alpha > \frac{2D_m(D_s+D_\ell)}{D_2^s + D_2^m - \chi_2} \) \( \Rightarrow \alpha > \frac{2D_s(D_m+D_\ell)}{D_2^s + D_2^m - \chi_2} \), so we can focus only on the \( \alpha > \frac{2D_m(D_s+D_\ell)}{D_2^s + D_2^m - \chi_2} \) condition. It is not possible to have \( \alpha \in \left( \frac{2D_m(D_s+D_\ell)}{D_2^s + D_2^m - \chi_2}, \frac{2D_s(D_m+D_\ell)}{D_2^s + D_2^m - \chi_2} \right) \), i.e., this range is empty, as \( \frac{2D_m(D_s+D_\ell)}{D_2^s + D_2^m - \chi_2} > \frac{2D_s(D_m+D_\ell)}{D_2^s + D_2^m - \chi_2} \). To show this, let \( D_m = D_s + d_m \) and \( D_\ell = D_s + d_m + d_\ell \) with \( d_\ell > 0 \) and \( d_m > 0 \). Then \( \frac{2D_m(D_s+D_\ell)}{D_2^s + D_2^m - \chi_2} < \frac{2D_s(D_m+D_\ell)}{D_2^s + D_2^m - \chi_2} \) implies

\[
\begin{align*}
2(D_s + d_m)(D_s + D_s + d_\ell)((D_s + d_m)^2 + (D_s + d_\ell)^2 - \chi_2) \\
< 2(D_s + d_\ell)(D_s + D_s + d_m)(D_s^2 + (D_s + d_m)^2 - \chi_2) \\
\Rightarrow 0 > 2(D_s + d_m)(D_s + D_s + d_m + d_\ell)((D_s + d_m)^2 + (D_s + d_m + d_\ell)^2 - \chi_2) \\
-2(D_s + d_m + d_\ell)(D_s + D_s + d_m)(D_s^2 + (D_s + d_m)^2 - \chi_2) \\
\Rightarrow 0 > 2 \left( 5D_s d_m^3 + 4D_s^3 d_m + D_s d_\ell^3 + 2D_s^3 d_\ell + d_m d_\ell^2 + 3d_m^3 d_\ell + d_m^4 + 8D_s^2 d_m^2 \\
+4D_s^2 d_\ell^2 + 3d_m^3 d_\ell^2 + 7D_s d_m d_\ell + 10D_s d_m d_\ell + 10D_s^2 d_m d_\ell + \chi_2 D_s d_\ell \right)
\end{align*}
\]

Therefore, the \( m \) agent will prefer to wait rather than join with \( s \). So, for \( \alpha < \frac{2D_D(D_s+D_m)}{D_2^s + D_2^m - \chi_2} \),
neither the \( m \) nor \( \ell \) agents are willing to join a coalition and we have a standstill. Overall, for 
\[ \alpha > \frac{D_l(D_s+D_m)}{D_m^2+D_l^2}, \]
we should see the offer-stable \( s\ell \) coalition. For \( \alpha < \frac{D_l(D_s+D_m)}{D_m^2+D_l^2} \), the lobbyist will choose the \( m\ell \) coalition.

When \( \alpha > \frac{D_l(D_s+D_m)}{D_m^2+D_l^2} \) and \( \chi_2 < \bar{D}_{s\ell}^2 \), the agents will rationally form a 3-agent lobby if

\[
\alpha \left( \frac{9k^2 \left( D^2 + D_s D_m + D_s D_\ell + D_m D_\ell \right)}{c(4+9k)^2} - \chi_3 \right) \geq \hat{U}_{s\ell,s\ell} - \chi_2 + \hat{U}_{m,s\ell} - \chi_3 \\
\geq \alpha \left( \frac{9k^2 \bar{D}_{s\ell}^2}{2c(4+9k)^2} - \chi_2 \right) + \frac{9k^2 D_m \bar{D}_{s\ell}}{c(4+9k)^2} \\
2\bar{D}^2 - (D_s - D_\ell)^2 - 2 \left( \frac{1 - \alpha}{\alpha} \right) D_m \bar{D}_{s\ell} + \chi_2^E \geq \chi_3^E.
\]

When \( \alpha < \frac{D_l(D_s+D_m)}{D_m^2+D_l^2} \) and \( \chi_2 < \bar{D}_{s\ell}^2 \), the agents will rationally form a 3-agent lobby if

\[
\alpha \left( \frac{9k^2 \left( D^2 + D_s D_m + D_s D_\ell + D_m D_\ell \right)}{c(4+9k)^2} - \chi_3 \right) \geq \hat{U}_{m\ell,m\ell} - \chi_2 + \hat{U}_{s,m\ell} - \chi_3 \\
\geq \alpha \left( \frac{9k^2 \bar{D}_{m\ell}^2}{2c(4+9k)^2} - \chi_2 \right) + \frac{9k^2 D_s \bar{D}_{m\ell}}{c(4+9k)^2} \\
2\bar{D}^2 - (D_m - D_\ell)^2 - 2 \left( \frac{1 - \alpha}{\alpha} \right) D_s \bar{D}_{m\ell} + \chi_2^E \geq \chi_3^E - \chi_2^E.
\]

8.2.2 \( \chi_2 \in \left( \bar{D}_{s\ell}^2, \bar{D}_{m\ell}^2 \right) \)

If \( \chi_2^E > \bar{D}_{s\ell}^2 \), then the \( s \) and \( \ell \) agents will choose to disband the lobby, even though it is stable relative to other two-agent coalitions. If \( \chi_2 \in \left( \bar{D}_{s\ell}^2, \bar{D}_{m\ell}^2 \right) \), then both the \( s\ell \) and \( sm \) lobbies would each impose a net cost on the its member agents, but the \( m\ell \) lobby would result in a net gain to its members. Therefore, the \( m\ell \) lobby is the only two-agent coalition that would feasibly form, regardless of the value of \( \alpha \), as \( \bar{D}_{s\ell}^2 - \chi_2 > 0 \iff \alpha \left( \bar{D}_{s\ell}^2 - \chi_2 \right) \geq 0 \forall \alpha \in [0,1] \).

Suppose the \( m\ell \) coalition has formed. Then, if agent \( \ell \) defects and joins the \( s\ell \) coalition, we know that the instant later, that coalition will fall apart. However, if \( \chi_2^E < \bar{D}_{m\ell}^2 \), then agent \( m \) can offer a portion of the positive net coalition gain to agent \( \ell \), which is better than what
agent \ell \) would obtain from defecting to the doomed \( s\ell \) coalition.

The agents will rationally form a 3-agent lobby if

\[
\alpha \left( \frac{9k^2 (D^2 + D_sD_m + D_sD_t + D_mD_t)}{c(4+9k)^2} - \chi_3 \right) \geq \alpha \left( \frac{9k^2 D_{m\ell}}{2c(4+9k)^2} - \chi_2 \right) + \frac{9k^2 D_s D_{m\ell}}{c(4+9k)^2} \\
2D^2 - (D_m - D_{\ell})^2 - 2 \left( \frac{1 - \alpha}{\alpha} \right) D_s D_m \geq \chi_3 - \chi_2.
\]

8.2.3 \( \chi_2 > \frac{D_{m\ell}^2}{c} \)

If \( \chi_2 > \frac{D_{m\ell}^2}{c} \), then no two-agent lobby benefits its members, so none will form. A 3-agent lobby will form when

\[
\frac{9k^2 (D^2 + D_sD_m + D_sD_t + D_mD_t)}{c(4+9k)^2} - \chi_3 > 0,
\]

or \( D^2 + D_sD_m + D_sD_t + D_mD_t > \chi_3^E \).

8.3 Lobbyist captures all coalition gains: \( \alpha = 0 \)

Finally, we investigate the case where \( \alpha = 0 \), which implies that agents are indifferent across coalition structures, as the lobbyist extracts the full net gain from coalition formation. Since agents are indifferent, the lobbyist determines the coalition structure. The lobbyist, maximizing (5), always prefers to form the \( m\ell \) lobby over either the \( s\ell \) or the \( sm \) lobby. This implies that the lobbyist will only form either the grand or the \( m\ell \) lobby. The equilibrium lobbies are given by the following theorem.

**Theorem 3 (Lobbyist-based coalitions)** When \( \alpha = 0 \),

1. \( \chi_2^E > D_{m\ell}^2 \), and

   (a) \( \chi_3^E > \Delta_1^G \), the lobbyist will not form a coalition;

   (b) \( \chi_3^E < \Delta_1^G \), the lobbyist will optimally form the grand lobby;

2. \( \chi_2^E \leq D_{m\ell}^2 \), and

   (a) \( \chi_3^E - \chi_2^E > \Delta_1^G - D_{m\ell}^2 \), the lobbyist will optimally form the \( m\ell \) coalition;

   (b) \( \chi_3^E - \chi_2^E \leq \Delta_1^G - D_{m\ell}^2 \), the lobbyist will optimally form the grand lobby.
Note that the only difference between the results in Theorems 2 and 3 are the thresholds in parts 2.a and b. That is, the feasible coalition structures are the same once 
\[ \alpha < \frac{D_1(D_a + D_m)}{D_m + D_f^2}, \]
but when exactly the \( m \ell \) coalition is formed depends on whether the firms retain any share of the gains from coalition formation. Relative to the grand lobby, the \( m \ell \) coalition is more stable when agents retain some share of the gains because the increased utility of the non-member agent enhances the stability of the 2-member coalition.

Appendix B

In this Appendix we discuss literature involving cooperative game theory and contrast our offer-stability concept with notions of stability based on the core and bargaining sets.

Two important features of our model limit our ability to use well-known solution techniques or to directly apply the results of earlier studies. First, the externalities of lobbies in a policy regime with any degree of uniformity imply that the value of a given coalition to its members depends on the overall coalition structure. Therefore, we cannot write a function for the value of a coalition that depends only the characteristics of that coalition and ignores the overall coalition structure (i.e., a characteristic function). As such, solutions based on characteristic functions are not applicable (e.g., those based on the core and Shapley value; see Myerson (2013)). Thrall and Lucas (1963) and Myerson (1978) analyze games in partition function form where the value to a coalition depends on the entire coalition structure. Thrall and Lucas (1963) present results primarily for 2- and 3-player games, and allow for transfers across coalitions, which we prohibit. Myerson (1978) explores the role of commitments to threat strategies, although these threats may not be sequentially rational ex post. Kóczy (2007), Ray and Vohra (1997, 1999), and Yi (1997) also explore games with externalities in which, by definition, a player’s utility is influenced by the coalition structure as long as the coalition structure influences other players’ equilibrium actions. Yi (1997) develops rules for stable coalition structures in the presence of positive externalities, but
assumes (Yi’s condition P.2) that per-member payoffs are decreasing in coalition size. In our model, the payoff structure emerges as a function of the regulatory environment, and is not in general characterized by per-member payoffs that decrease in coalition size. Instead, absent coalition costs, larger coalitions yield larger per-member payoffs because coalitions are able to eliminate the free-rider problem between all members.

Second, by the nature of our focus on the importance of regulatory uniformity, it is crucial for the players, i.e., the agents in our model, to be heterogeneous. In a model with homogeneous agents, restricting the regulator to any degree of regulatory uniformity is free, as the regulator optimally desires to set homogeneous regulation across the cross-section of agents. The lack of homogeneity means that we care about which agents are members of which lobbies or coalitions, and cannot simply use coalition size as an outcome variable of interest, as in the model of Bloch (2002). A benefit of allowing for heterogeneity is that we can derive predictions on which agents find it optimal to associate with each other, and which associations can be sustained in equilibrium (as in Baccara and Yariv, 2016).

Traditional notions of the core (e.g., Von Neumann and Morgenstern, 1944) and extensions into cooperative games with externalities (e.g., Kóczy, 2007) rely on the concept of blocking (Ray and Vohra, 1997, 2014). An allocation-structure pair \((A, X)\) is blocked if there is an alternative allocation-structure pair \((A', X')\) that makes a subset of players better off, assuming that the subset of players can cause a deviation from \((A, X)\) to \((A', X')\). An allocation-structure pair is said to be in the core if it is not blocked by any achievable alternative. Whether \((A', X')\) makes the deviating subset of players better off depends on how the other players, the residual players, react to the deviation to \((A', X')\). Various definitions of cores have been put forth, with different assumptions about how the residual players behave (i.e., whether the residual players are punitive towards the deviators, supportive towards the deviators, or optimizers, as well as whether and how the residual players rearrange themselves; see Kóczy (2007) for a concise discussion). Our offer-stability differs from the stability notion used in Ray and Vohra (1997), in which subsets of \(l_j\) can deviate,
but deviations always make the coalition structure finer, as the deviants are precluded from joining with insiders who were not initially in $l_j$. In contrast, we allow for the deviating insiders to form coalitions with new partners.

A well-known problem with cores is that they can be empty or non-unique. Furthermore, in games with superadditive payoff structures and fully transferable utility (i.e., in which larger coalitions are always better), there is no loss in only considering how the spoils from the grand coalition are allocated. Absent superadditivity, determining the coalition structures that are in the core is a non-trivial problem. Our setting features non-superadditive payoffs and admits an empty core in some cases (as described below). To avoid these core-related problems, we introduce offer-stability as a means of determining agent-formed coalition structures. Before proceeding, we define core stability.

**Definition 5 (Core stability)** A feasible allocation-structure pair $(A, X)$ is blocked if there is a nonempty coalition in which each member is made weakly better off and one member is made strictly better off in an alternative feasible allocation-structure pair $(A', X')$. An allocation-structure pair $(A, X)$ is in the core and the structure $X$ is core-stable if $(A, X)$ is not blocked by any alternatives.

Under core stability, a given allocation has to be robust to all possible alternatives. That is, the same allocation has to prevent any possible deviation if that allocation-structure pair is in the core. Under offer-stability, the structure must be consistent, but we allow the allocation within a structure to adjust to prevent blocking by different alternatives. For example, suppose we are in the $ml$ structure, with allocation $A = \{u_s, u_m, u_\ell\} = \{1, 2, 2\}$. Now, if the $sm$ coalition formed, assume that any allocation $A' = \{u'_s, 3.5 - u'_s, 0.6\}$ is feasible, implying that $sm$ have a budget of 3.5 to split between them. In this example, allocation $A$ is blocked by $A' = \{1.1, 2.4, 0.6\}$ because $A'$ makes agents $s$ and $m$ better off.

Assume $\chi_3 \to \infty$, so we only consider 2-agent coalitions. Clearly, the allocation provided by $I$ can be dominated by allocations in any of the two-agent coalition structures. In any two-agent coalition, we must have $u_s > 0.9$, $u_m > 1$, and $u_\ell > 1.1$. Now, any allocation in $X = sm$ is blocked, since either $3.5 - u'_s \leq 1 \Rightarrow m$ unilaterally deviates, or $u'_s \leq 2.5$,
Table 3
Payoff Structure for a sample game

<table>
<thead>
<tr>
<th>$X$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>${-\infty, -\infty, -\infty}$</td>
</tr>
<tr>
<td>$sm$</td>
<td>${u'_s, 3.5 - u'_s, 1.2}$</td>
</tr>
<tr>
<td>$sl$</td>
<td>${u'_s, 1.1, 4 - u''_s}$</td>
</tr>
<tr>
<td>$ml$</td>
<td>${1, u'_m, 4.5 - u'_m}$</td>
</tr>
<tr>
<td>$I$</td>
<td>${0.9, 1, 1.1}$</td>
</tr>
</tbody>
</table>

in which case $s$ and $\ell$ can deviate to, for example, $(sl, \{2.6, 1.1, 1.4\})$. Therefore, $sm$ is not core-stable. An allocation in $sl$ must have $u'_s > 0.9$, implying $u_\ell < 4 - 0.9 = 3.1$. But, in this case, $m$ and $\ell$ can deviate to $(ml, \{1, 1.3, 3.2\})$, implying that $sl$ is not core-stable. For $ml$, $u'_m > 1$ prevents unilateral deviation of $m$, but $m$ can also get up to 2.5 for deviating with $s$ to $sm$. Similarly, $\ell$ can get up to 3 for deviating with $s$ to $sl$. It is impossible for an allocation in $ml$ to provide both $u'_m \geq 2.5$ and $u_\ell = 4.5 - u'_m \geq 3$ implying that any allocation in $ml$ is blocked by either $sm$ or $sl$. Therefore, the core to the game above (again, with $\chi_3 \to \infty$) is empty, and there is no core-stable coalition structure, as every allocation is dominated by another allocation with an alternative structure. If, instead, the set of feasible allocations in $ml$ were $\{1, u'_m, 5.7 - u'_m\}$, then the core would consist of $(ml, \{1, x, 5.7 - x\})$, with $x \in (2.5, 2.7)$.

Despite the lack of a core-stable coalition structure, there is an offer-stable coalition structure, $ml$. The $ml$ structure is offer-stable because: any deviation from $ml$ to $sl$ can be prevented by giving $\ell$ at least 3 and still leaving $m$ better off with at least the 1.1 he would receive under $sl$; and any deviation to $sm$ can be prevented by giving $m$ at least 2.5 while leaving $\ell$ with more than the 1.2 he would receive under $sm$. Clearly, these cannot both be accomplished, but the can be accomplished as counteroffers to proposals received individually. Our nomenclature of offer-stability comes directly from the idea that a coalition structure is offer-stable if coalition partners can be given counteroffers that prevent deviations. That is, if $s$ proposes an allocation to $m$ that involves a move to the $sm$ structure,
we allow \( \ell \) to make a counteroffer to \( m \) to keep him in the coalition.

Myerson (2013) provides a concise and intuitive presentation of the bargaining set solution to cooperative games introduced by Aumann and Maschler (1964). The discussion here borrows from Myerson (2013). To set the stage, consider an \( N \)-player game in which the value available to a coalition \( S \) in structure \( X \) is \( v(S, X) \). Consider an initial payoff-structure pair \((A, X_I)\), where the initial allocation is \( A \) and the initial coalition structure is \( X_I \). The bargaining set is based on objections to allocations and counterobjections to those objections. An objection by player (or set of players) \( i \) against another player (or set of players) \( g \) is an allocation-coalition-structure triple, \( (A_0, S, X_0) \), such that \( A_0 \in \mathbb{R}^N \), \( S \subseteq N \), \( i \in S \), \( g \notin S \), 
\[
v(S, B_S) = \sum_{k \in S} a'_k,\]
and \( A' >_S A \), where \( >_S \) indicates that the players in \( S \) prefer allocation \( A' \) to allocation \( A \) and \( a'_k \) is the payoff to player \( k \) in allocation \( A' \). A counterobjection to \( i \)'s objection \( (A', S, X_S) \) against \( g \) and \( A \) is similarly a triple \( (A'', T, B_T) \) such that \( A'' \in \mathbb{R}^N \), \( T \subseteq N \), \( g \in T \), \( i \notin T \), \( T \cap S \neq \emptyset \), 
\[
v(T, B_T) = \sum_{k \in S} a''_k,\]
and \( A'' >_{T \cap S} A' \). In the counterobjection, player \( g \) can form a coalition \( T \) that takes away some of \( i \)'s partners in the objection (but not \( i \)) and makes them at least as well off as in the objection; thus, \( g \) can restore himself/themselves and the other members of \( T \) to payoffs at least as good as they had in \( x \). Additionally, all the players in \( T \) weakly prefer the allocation under \( A'' \) to the allocation under \( A' \). A payoff-structure pair \((A, X)\) is stable if there is a counterobjection to each possible objection.

Offer-stability relies on offers and counteroffers, which are essentially synonymous with objections and counterobjections. The primary difference is that with offer-stability, we weaken the bargaining set to ensure only that player (or set of players) \( g \) is at least as well off under the counteroffer or counterobjection as he (they) would be if the original offer or objection were accepted. That is, we replace \( A'' >_T A \) with \( A'' >_T A' \).