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RADIATIVE PION DECAY INTO ELECTRONS

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ABSTRACT

The possibility of distinguishing the pion structure-dependent radiation from the conventional inner bremsstrahlung radiation in the radiative decay of pions into electrons is discussed. Calculation of the photon energy spectrum and angular correlation shows that evidence for intermediate-state structure would be obtained if any photons of energy less than 70 Mev were detected in 180° coincidence with π-decay electrons. The probability of such events per unit solid angle is approximately 0.6×10^{-7} relative to ordinary $\pi \rightarrow \mu + \bar{\nu}$ decay. Two simple assumptions are made relating the absolute radiative decay rate to the π^0 lifetime.

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I. INTRODUCTION

The universal V-A form of the Fermi interaction has in recent years been suggested by the evidence in β and μ decay. The other weak interactions are then, in principle, consequences of strong couplings together with the universal Fermi interaction. In π decay of π mesons into electrons, where the momentum transfer is large, evidence on the decay mechanism can be obtained,[1,2] in principle, by observing the associated radiative decay $\pi \rightarrow e + \nu + \gamma$. In this paper we amplify the calculation by Vaks and Ioffe[1] and discuss the possibility of distinguishing structure-dependent effects from less interesting structure-independent effects. We supplement the electron spectrum already presented[1,3] by calculating the photon spectrum, which may be more easily observed experimentally.

The diagrams for the radiative decay are given in Fig. 1. Diagrams (a) and (b), when defined in a gauge-invariant way, give rise to the inner bremsstrahlung by a decelerated or accelerated charge or magnetic moment. The matrix element for this is proportional to $eGm/\sqrt{k}$, where e and G are the electromagnetic and Fermi coupling constants, m is the electron (or muon) mass, and k the photon energy. Diagrams (c) and (d) of Fig. 1 are structure-dependent, since here the emission of a photon depends on the nature of the "black box." The matrix elements for these diagrams are proportional to $eG\sqrt{k}\,(\mu/M)$, where μ is the pion mass and M a mass or energy typical of the intermediate states involved in the "black box." The two processes--inner bremsstrahlung and "black box" (or structure-dependent) radiation--are coherent, but the interference term is negligible in $\pi \to e + \bar{\nu} + \gamma$ decay. (In $\pi \to \mu + \bar{\nu} + \gamma$ the reverse is the case: because of the small momentum transfer involved, the structure-dependent radiation is small compared with the inner bremsstrahlung, and the interference term dominates the square of the structure-dependent matrix element. For this reason radiative π-μ decay, although more frequent by several orders of magnitude than radiative π-e decay, reveals nothing indicative of the pion decay structure.) The interesting question is not whether radiative $\pi^{\pm}$ decay occurs, but whether the interesting structure-dependent effects can be disentangled from the ordinary quasi-classical bremsstrahlung. We find that a unique proof of structure to the π-decay mechanism can be obtained if any photons of energy less than $k_{max} = 70$ Mev are detected in 180° correlation to the direction of the decay electron. The probability of such a decay per unit solid angle per π decay is, however, approximately 0.6×10^{-7}.

II. INNER BREMSSTRAHLUNG (IB)

The matrix element for the inner bremsstrahlung is defined as the gauge-invariant part of diagrams (a) and (b) of Fig. 1. On invariance grounds this is of the form

$$M_{IB} = e(m/\sqrt{k}\ )\, f_A(\mathcal{P}^2)\, \bar{\psi}_e \{ (p\cdot\epsilon/p\cdot k - p\cdot\epsilon/\mathcal{P}\cdot k) + i\, \sigma_{\mu\nu} F_{\mu\nu}/4p\cdot k \}\, \psi_\nu ,$$

where $f_A(\mathcal{P}^2)$ is the amplitude for the nonradiative decay, $\mathcal{P}$ is the pion four-momentum, p the electron four-momentum, ϵ the photon polarization four-vector, $F_{\mu\nu} = \epsilon_\mu k_\nu - \epsilon_\nu k_\mu$, and ψ_e and ψ_ν are respectively the electron (or muon) and neutrino field operators. The two terms in M_{IB} correspond to emission of radiation by the accelerated charge and magnetic moment respectively.

This matrix element leads to the differential transition probability,

$$d^3W_{IB} = W_{e+\bar{\nu}} \frac{\alpha}{(2\pi)^3} \frac{1}{\mu^2} \{ \mu^2(\underset{\sim}{p}^2 - (\underset{\sim}{p}\cdot\underset{\sim}{k})^2/k^2)$$

$$+ 2(Ek - \underset{\sim}{p}\cdot\underset{\sim}{k})(\mu k - Ek + \underset{\sim}{p}\cdot\underset{\sim}{k}) \} \frac{d^3p\, d^3k}{[Ek - \underset{\sim}{k}\cdot\underset{\sim}{p}]^2} \delta(\mu - E - E_{\bar{\nu}} - k), \quad (1)$$

where $\alpha = e^2/4\pi$ is the fine-structure constant, E the electron energy, $E_{\bar{\nu}}$ the neutrino energy, and $W_{e+\bar{\nu}}$ the nonradiative decay rate. The electron energy spectrum resulting from this expression has been given previously[1,3] and is not repeated here.

We suspect that, because of the overwhelming background of $\pi \to e + \bar{\nu}$ and $\pi \to \mu \to e$ electrons, the photon radiation (or at least the hard component in which we are interested) may be more easily distinguished than the spectrum of electrons. The spectrum of photons into solid angle $d\Omega = 2\pi \sin\theta\, d\theta$, obtained by integrating Eq. (1) over electron energies, is

$$d^2W_{IB} = W_{e+\bar{\nu}} \frac{\alpha}{2\pi^2} \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right) \frac{(x - 1)^2 + 1}{[2 + x(\cos - 1)]^2} \frac{dx}{x} d\Omega , \quad (2)$$

where $k = xk_{max}$, and θ is the angle between the electron and the photon. For $\theta = 180^\circ$, according to the kinematics, the electron, photon, and neutrino must all be collinear, and either $k = k_{max}$ or $E = E_{max}$. Now if the electron radiates at all it must have less than maximum energy E_{max}. Thus at 180° the only inner bremsstrahlung photons possible are those of energy $k = k_{max}$.

Integrating Eq. (2) for photon energies greater than some low-energy cutoff δ, we obtain the electron-photon angular correlation,

$$dW_{IB}/d\Omega = W_{e+\bar{\nu}}(\alpha/2)(2\pi)^{-2}\lambda^{-3}\{\lambda + (1-\lambda)(1-2\lambda^2)\log(1-\lambda) + 2\lambda^2(1-\lambda)[\log(1/x_{min}) - 1]\}, \tag{3}$$

where $\lambda = \sin^2\theta/2$ and $x_{min} = 2\delta/\mu$. The rate of $\pi \rightarrow e + \bar{\nu} + \gamma$ decay per unit solid angle with e and γ at 180° to each other is 0.05 sec^{-1}.

Equation (3) agrees with Eq. (21) of Vaks and Ioffe, since when the minimum photon energy is δ the maximum electron energy is approximately $\mu/2 - \delta(1-\lambda)$, so that y_{max} in Vaks and Ioffe equals $1 - (1-\lambda)x_{min}$ above.

III. STRUCTURE-DEPENDENT (SD) RADIATION

Out of the pseudoscalar pion field operator ϕ and the electromagnetic field operator A_μ only two vectors $a\phi F_{\mu\nu}\rho_\nu$ and $b\phi \tilde{F}_{\mu\nu}\rho_\nu$ can be constructed in a gauge-invariant manner. Here $F_{\mu\nu} = A_\mu k_\nu - A_\nu k_\mu$ and $\tilde{F}_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu\lambda\rho}F_{\lambda\rho}$ is the tensor dual to $F_{\mu\nu}$, and a and b are functions of $\rho\cdot k$, which must be, assuming PC invariance, relatively real. The gauge-invariant contribution of diagrams (c) and (d) of Fig. 1 must therefore be of the form

$$\begin{aligned} M_V &= -i\sqrt{\alpha}\; G_V a\phi\tilde{F}_{\mu\nu}\rho_\nu\, \bar{\psi}_e\gamma_\mu\frac{1+\gamma_5}{2}\psi_\nu, \\ M_A &= i\sqrt{\alpha}\; G_A b\phi F_{\mu\nu}\rho_\nu\, \bar{\psi}_e\gamma_\mu\frac{1+\gamma_5}{2}\psi_\nu. \end{aligned} \tag{4}$$

Writing $q_\mu = p_\mu + p_\mu^{\bar\nu}$, $B_\mu = \bar\psi_e \gamma_\mu \frac{1+\gamma_5}{2} \psi_\nu$, $G_{\mu\nu} = q_\mu B_\nu - q_\nu B_\mu$, one finds that Eqs. (4) take the form

$$M_V = -\frac{i}{2} \sqrt{\alpha}\ G_V a\, \phi \tilde{F}_{\mu\nu} G_{\mu\nu} ,$$

$$M_A = \frac{i}{2} \sqrt{\alpha}\ G_A b \phi F_{\mu\nu} G_{\mu\nu} . \tag{5}$$

(a) Assumption Relating M_V to π^0 Decay.

The matrix element for the decay of pseudoscalar π^0 mesons is, on invariance grounds,[4]

$$M_{\pi^0} = -\frac{i}{2}\ \alpha\, c \phi \tilde{F}_{\mu\nu} F_{\mu\nu} , \tag{6}$$

from which the rate of π^0 decay is

$$W_{\pi^0} = (\alpha^2/4)(2\pi)^{-8}\ \mu^3\ c^2 . \tag{7}$$

In lowest-order perturbation theory (where the "black box" in Fig. 1 stands for a nucleon-antinucleon loop, each of mass M, coupled to the pion field via $(g/\sqrt{2})\ \bar\psi_N \gamma_5 \underset{\sim}{\tau} \psi_N \cdot \underset{\sim}{\phi})$, we have

$$c = a = 4(\pi)^{5/2}\ g/M . \tag{8}$$

This relation between the electromagnetic decay of the π^0 and the vector radiative decay of the $\pi^\pm$ holds to all orders if the Feynman-Gell-Mann principle of conservation of the weak vector current[5] is assumed.

(b) Assumption Relating M_A to M_V.

In lowest-order perturbation theory b equals a.[6] This result is suggestive of all orders since for every diagram (c) containing the weak vector vertex γ_μ there occurs an otherwise equivalent diagram (d) containing the axial vector $\gamma_\mu \gamma_5$. Since we go to lowest order in e and G, only these diagrams are always linear in ϵ_μ and B_μ and differ by the presence of an odd γ_5 in the V case. It is this extra γ_5 that leads to the alternating symbol distinguishing the vector and axial vector currents $a\phi \tilde{F}_{\mu\nu} \mathcal{P}_\nu$ and $b\phi F_{\mu\nu} \mathcal{P}_\nu$. It is suggestive (but not proven) that the numerical coefficients a and b collected from higher-order diagrams will also be equal.

For a = b (and also taking $G_A = G_V = G$) the matrix element for radiative π^+ decay can be written

$$M_V + M_A = \frac{i}{2} \sqrt{\alpha}\ Ga\phi(F_{\mu\nu} - \tilde{F}_{\mu\nu})G_{\mu\nu} ,$$

or, in the pion rest frame,

$$M_V + M_A = \sqrt{\alpha}\ Ga\phi\psi_e^+ \underset{\sim}{\sigma}\cdot(E + i\ \underset{\sim}{H})\psi_\nu ,$$

where $\psi_\nu = \gamma_5 \psi_\nu$. The significance of the assumption a = b is that only right circularly polarized structure-dependent photons are emitted in π^+ decay. If this were indeed the case, the products of $\pi^\pm$ decay would all be (in the approximation of zero electron mass) longitudinally polarized.

The assumptions (a) $a = c = (2/\alpha)(2\pi)^4 \mu^{-3/2} \sqrt{W_{\pi^0}}$ and (b) a = b will be made in the remainder of this paper. The first assumption determines the over-all rate of $\pi^\pm$ decay, while the second assumption fixes the details of the photon spectrum and angular correlation.

(If assumption (b) is relaxed, giving $bG_A/aG_A \neq 1$, then the more general formulae of Vaks and Ioffe should be applied.)

(c) Photon Spectrum and Angular Correlation.

The differential transition probability[7] obtained from Eq. (4) or Eq. (5) is

$$d^3W_{SD} = 4(G^2/\alpha)(2\pi)^{-5}\,\mu^{-2}W_{\pi^0}\,k(1-\beta\cos\theta)(1+\cos\phi)d^3p\,d^3k\;\;\delta(E+E_\nu+k-\mu), \qquad (9)$$

where β is the electron velocity, and $\phi = \langle(\underset{\sim}{p}_\nu\,\underset{\sim}{k})$. From Eq. (9) we obtain the spectrum angular distribution of photons,

$$d^2W_{SD} = (1/\alpha)(2\pi)^{-4}\,G^2\,\mu^4\,W_{\pi^0}\,(1-\cos\theta)^2\;\frac{x^3(1-x)^4}{[2+x(\cos\theta-1)]^4}\,dx\,d\Omega\,. \qquad (10)$$

According to this equation the photons emitted by the pion decay structure are typically of energy ~ 35 Mev and in the direction $\theta \sim \pi$. The inner bremsstrahlung photons are, according to Eq. (2), predominantly of very low energy and in the forward direction, $\theta \sim 0$. Upon integration of Eq. (10) over solid angle, the spectrum of structure-dependent photons,

$$dW_{SD} = (G^2/6\alpha)(2\pi)^{-3}\,\mu^4\,W_{\pi^0}\;\;x^3(1-x)dx\,, \qquad (11)$$

is obtained. The angular correlation between photon and electron obtained from Eq. (10) is

$$dW_{SD}/d\Omega = (G^2/\alpha)(2\pi)^{-4}\,\mu^4\,W_{\pi^0}\,[\,\lambda^2/4 + (1-\lambda)\lambda^{-6}(-\lambda^3 + 15\lambda^2 - 45\lambda + 35)\log(1-\lambda)$$
$$+ (1-\lambda)(12\lambda^5)^{-1}\,\{\,3\lambda^3(\lambda^3 + \lambda^2 + \lambda + 1)$$
$$+ 10(5\lambda^2 - 33\lambda + 42)\}\,] \quad . \tag{12}$$

Equations (3) and (12) are plotted in Fig. 2, with the lower limit[8] assumed to be $0.5 \times 10^{16}\ \mathrm{sec}^{-1}$ for W_{π^0}. The electron-photon angular distribution is the superposition of two noninterfering mechanisms: (a) the inner bremsstrahlung, Eq. (3), and (b) the photon emission by the intermediate states in π^+ decay, Eq. (12). In calculation of the latter, the A and V currents have been related to the rate of π^0 decay by the two assumptions of Section III. Because π^0 decays faster than the present experimental limit, the structure-dependent photons will be even more prominent than indicated.

IV. DISCUSSION

Kinematically, when photon and electron are in 180° anticoincidence, either the photon or the electron must have maximum energy, k_{max} or E_{max}, depending on the direction of the unobserved neutrino. In the process of inner bremsstrahlung (Eq. (2)) the electron must lose energy, $E < E_{max}$, and at $\cos\theta = -1$ only photons of $k = k_{max}$ can be emitted. For structure-dependent radiation, on the other hand, at $\cos\theta = -1$ only $E = E_{max}$ and $k < k_{max}$ are dynamically possible (Eq. (10)). Consequently detecting any photons of energy $k < k_{max}$ in 180° anticoincidence with electrons constitutes unambiguous proof of structure mediating the π decay. The number of such decays per π decay into unit solid angle of about 180° is $\gtrsim 0.6 \times 10^{-7}$, according to Eq. (12).

If these photons are not observed one will have to conclude either (a) that the vector current is not conserved, and that if the pion decays through intermediate baryons at all, the typical energies of the intermediate states involved probably exceed the nucleon rest mass, or (b) that the pion decay should be regarded as primary. In either case applying the idea of the universal Fermi interaction to other weak decays will have practically lost its attractiveness.

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6. This result agrees with that obtained by N. Cabibbo (to be published). Burkhardt, Cassels, Rigsby, Wetherell, and Wormald, Proc. Phys. Soc. (London) 72, 144 (1958), on the contrary, obtain $b = \frac{1}{3} a$.
7. We disagree with Vaks and Ioffe for the electron spectrum in the structure-dependent case; we obtain

$$dW_{SD}/dy = G^2(\alpha/3)(2\pi)^{-11}\, 2^{-7}\, \mu^7\, y^2\, [6(1 - 2y)(b - a)^2 + y^2(7(a^2 + b^2) - 10ab)].$$

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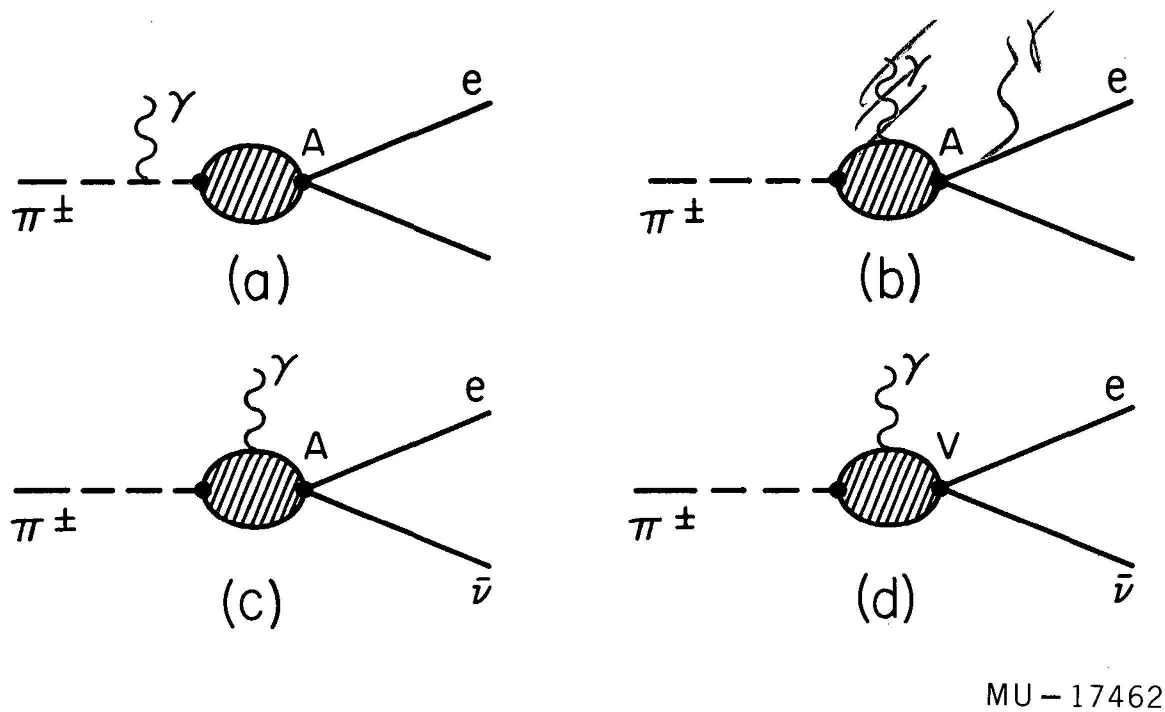

Fig. 1. The possible diagrams for the radiative electron decay of the pion.

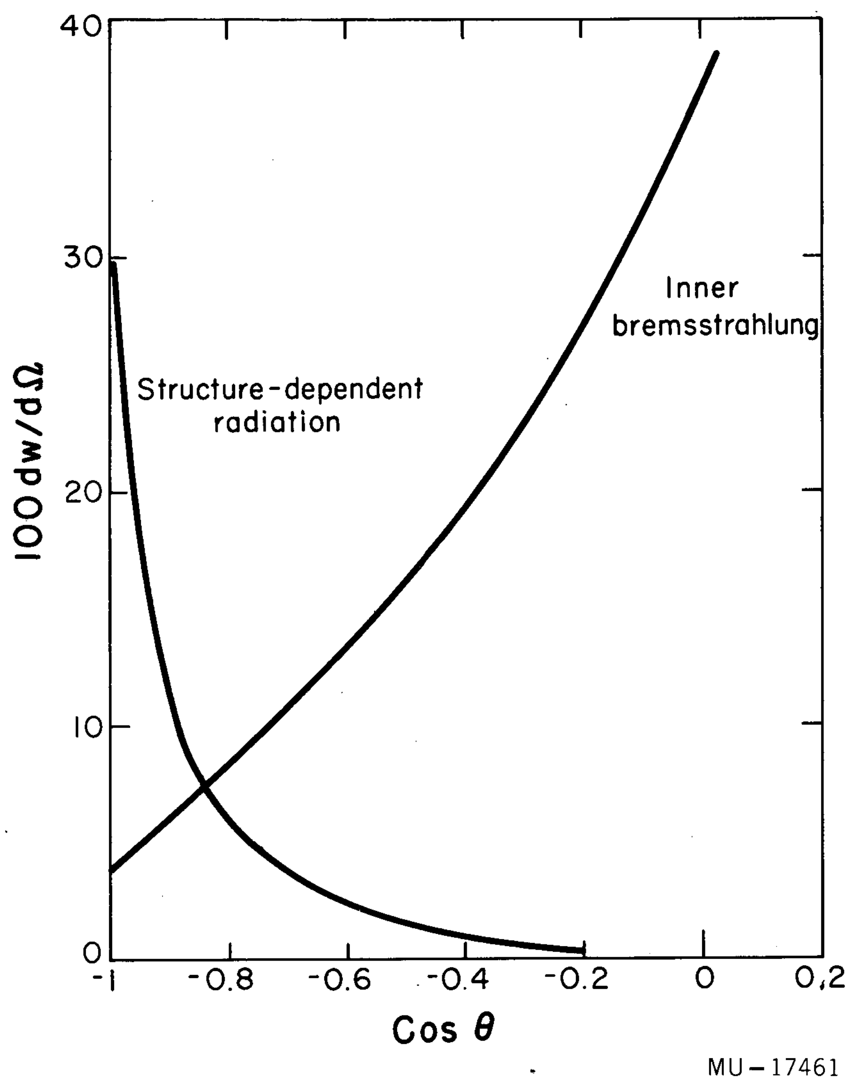

Fig 2. Plot of the angular distribution between the photon and electron for inner bremsstrahlung and for structure-dependent radiation.