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Title
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Permalink

Journal
Manufacturing & Service Operations Management, 8(1)

ISSN
1523-4614

Author
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Publication Date
2006

DOI
10.1287/msom.1050.0089

Peer reviewed

Sale Timing in a Supply Chain: When to Sell to the Retailer

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A fundamental decision for any manufacturer is *when* to sell to a downstream retailer. A manufacturer can sell either *early*, i.e., well in advance of the selling season, or *late*, i.e., close to the selling season. This paper examines the impact of information asymmetry, retailer sales effort, and contract type on the manufacturer's sale-timing decision. We find that if information is symmetric, demand is not influenced by sales effort, and the contract specifies that the price paid is linear in the order quantity, the manufacturer prefers to sell late. This result extends to the case where the retailer exerts sales effort during the selling season. However, if the retailer exerts sales effort prior to the selling season or has superior information about market demand, the manufacturer may prefer to sell early. We characterize the manufacturer's sale-timing preference in these settings, providing clear conditions under which the manufacturer prefers to sell either early or late. We show that the retailer, manufacturer, and total system may be *hurt* by the retailer's having *higher*-quality information.

Key words: supply-chain contracting; pricing; sales effort; asymmetric information
History: Received: November 21, 2003; accepted July 18, 2005. This paper was with the author 9 months for 3 revisions.

1. Introduction

A fundamental decision that any firm faces is *when* to sell its product. Consider a product where the production leadtime is long relative to the selling season and where market demand is stochastic and retail-price dependent. Products that share these characteristics include toys, fashion apparel, and technology products. To ensure that the product is available when the selling season begins, the upstream manufacturer must determine the production quantity well in advance of the selling season. The manufacturer has two alternatives regarding when to sell to a downstream retailer: (1) The manufacturer can sell in advance of the selling season, i.e., *sell early*, and push off the demand risk onto the retailer; (2) the manufacturer can sell close to the selling season, i.e., *sell late*, and bear the demand risk.

Should the manufacturer sell early or late? If market conditions turn out to be strong, then the retailer will be able to set a high retail price and sell a large quantity. Consequently, the late-selling manufacturer will be able to charge a high price to the retailer, and the manufacturer will tend to benefit from selling late. On the other hand, if market conditions are weak,

then the price the late-selling manufacturer will be able to charge the retailer will be relatively low. Further, ex post it may be clear that the manufacturer incurred production costs for units that, given the weak condition of the market, it is not profitable for the manufacturer to sell. Hence, the manufacturer will tend to do poorly by selling late. On balance, which effect should dominate? Under what circumstances would the manufacturer prefer to sell early or late?

This paper examines the impact of information asymmetry, retailer sales effort, and contract type on the manufacturer's sale-timing decision. We find that if information is symmetric, demand is not influenced by sales effort, and the contract specifies that the price paid is linear in the order quantity, the manufacturer prefers to sell late. This result extends to the case where the retailer exerts sales effort during the selling season. However, if the retailer exerts sales effort prior to the selling season or has superior information about market demand, the manufacturer may prefer to sell early. We characterize the manufacturer's sale-timing preference in these settings, providing clear conditions under which the manufacturer prefers to sell either early or late. Given the widespread use of

contracts where the price paid is linear in the order quantity (so called, wholesale price-only contracts), we devote particular attention to this contract form. However, firms may also employ more general contracts, where the price paid is a general function of the quantity ordered. We report the manufacturer's sale-timing preference under this more general contractual form; the analysis and results are of the most interest when there is asymmetric information. Overall, two general themes emerge regarding the manufacturer's sale-timing preference: First, low (high) production costs tend to make late (early) selling attractive. Second, general contracts tend to favor selling early more strongly than do linear contracts.[1] Finally, we also consider the impact of the quality of the retailer's private demand information on the profits of the firms. We show that the retailer, manufacturer, and total system may be *hurt* by the retailer's having *higher*-quality information.

Restricting attention to the decision to sell either early or late facilitates a comparison between two simple alternatives. This is also in line with industries such as the toy industry, where products are frequently either manufactured without any firm order commitments or produced only to meet firm orders (Woelbern 2001). However, a manufacturer may do better by selling at both times. Focusing on linear contracts, Erhun et al. (2001) examine the profit improvement resulting from selling both early and late rather than just early. Anand et al. (2001) show that a manufacturer that sells at two points in time prefers not to commit to a future wholesale price.

Cvsa and Gilbert (2002) and Gilbert and Cvsa (2003), collectively referred to as CG, examine the manufacturer's wholesale price commitment decision and issues related to sale timing. Cvsa and Gilbert (2002) focus on a setting with two competing retailers, while Gilbert and Cvsa (2003) focus on a setting with a single retailer that makes early investments in cost reduction. Our analysis differs from CG in three important ways. First, a crucial aspect of our problem

formulation is that the production lead time is long and demand is uncertain; consequently, production must occur before demand uncertainty is resolved. In contrast, CG assume that production occurs after uncertainty is resolved; consequently, the late-selling manufacturer simply produces to order rather than speculatively building inventory in advance (this remark also applies to Erhun et al. 2001 and Anand et al. 2001). Second, CG assume that the late-buying retailer prices optimally, but the early-buying retailer does not (the early-buying retailer always sells her entire quantity due to operational inflexibility); we assume optimal behavior throughout. Third, we consider general demand curves, rather than restricting attention to linear demand curves, and information asymmetry. In a recent paper, Biyalogorsky and Koenigsberg (2004) consider the impact of inventory responsibility on manufacturer and retailer profits, assuming a linear demand curve and a two-point distribution for uncertain demand. They show that the manufacturer prefers to be responsible, which is analogous to our finding that the manufacturer prefers to sell late. Their approach emphasizes the retailer's preference regarding inventory responsibility and focuses on issues such as the pass-through rate. Our approach employs a more general formulation and focuses on how the manufacturer's preference is impacted by information asymmetry and retailer sales effort.

Other researchers consider contracts that expand the terms of trade beyond the wholesale price. Padmanabhan and Png (1997) examine full-credit returns, Emmons and Gilbert (1998) examine partial-credit returns, and Cachon and Lariviere (2005) examine revenue sharing. *Full-credit returns* is similar to selling late in that the demand risk resides entirely with the manufacturer. Padmanabhan and Png show that, depending on the exogenous parameters, manufacturer profit may be higher or lower under full-credit returns. All of the preceding papers suppose that the retail price is endogenous.

In practice, typically, market demand is sensitive to the retail price and retailers have substantial discretion in setting retail prices. A retailer's pricing decision depends on her inventory, her ability to purchase additional units (and on what terms), and

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[1] More precisely, with a general contract the manufacturer weakly prefers to sell early in all scenarios with a single exception, and in this case (asymmetric information, low production cost), the manufacturer prefers to sell late regardless of whether the contractual form is general or linear.

the state of the market. The manufacturer's whole-sale pricing decision, in turn, depends on the retailer's subsequent retail pricing decision (e.g., the price the retailer is willing to pay depends on the price sensitivity of market demand). Consequently, the retailer's role in setting the retail price has important implications for the manufacturer's sale-timing decision. Several papers consider issues related to sale timing when the retail price is instead exogenous: Lariviere and Porteus (2001) examine the early-selling manufacturer's wholesale price decision. Ferguson (2003) and Ferguson et al. (2005) examine the issue of sale timing in a two-firm supply chain with demand forecast updating. Assuming the retail price is exogenous leads Ferguson (2003) and Ferguson et al. (2005) to pursue a substantially different focus and approach. In particular, when the retail price is exogenous and the demand forecast update provides full information, the wholesale-price-setting manufacturer's sale-timing preference is relatively straightforward to ascertain. Consequently, they instead focus on partial information updating and comparing the preferences of the firms when the wholesale price is set by the retailer, is set by the manufacturer, or is set to equalize the firms' profits. Our approach differs in that we consider price-sensitive retail demand, sales effort, and asymmetric information. Özer et al. (2003) examine the profit improvement resulting from selling both before and after a forecast update rather than selling only before the update. Cachon (2004) identifies the Pareto set of wholesale price contracts when the manufacturer sells early and/or late.

A number of papers in the supply chain literature consider asymmetric information (e.g., Cachon and Lariviere 2001, Corbett 2001, and Ha 2001) and sales effort effects (e.g., Chu 1992, Desai and Srinivasan 1995, Lariviere and Padmanabhan 1997, and Taylor 2002). Cachon (2003) and Chen (2003) provide comprehensive reviews of the supply chain contracting literature. Van Mieghem and Dada (1999) consider a single firm, rather than a supply chain, and examine the profit improvement resulting from postponing price and production decisions.

The paper is organized as follows. Section 2 presents a model that examines the manufacturer's sale-timing decision in a basic setting. Se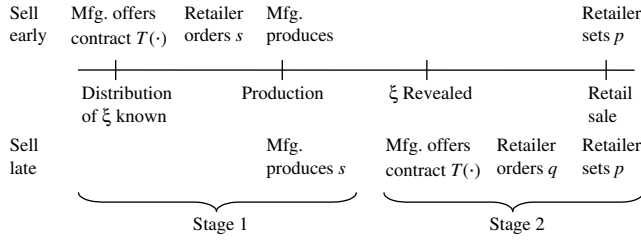ction 3 explores the setting in which retailer sales effort influences demand. Section 4 explores the setting in which information is asymmetric. Section 5 provides concluding remarks.

2. Model

We focus on settings where the manufacturer is powerful relative to the retailer; in particular, the manufacturer is assumed to have all the bargaining power. Regardless of when the manufacturer sells to the retailer, the sequence of events is as follows: The manufacturer makes a take-it-or-leave-it offer of a contract to the retailer. The contract specifies the payment $T(q)$ to be paid to the manufacturer as a function of the retailer's order quantity q. The retailer responds by either ordering or rejecting the contract (ordering zero). The manufacturer then fulfills the order. After market uncertainty is resolved, the retailer sets the retail price and sells to consumers. The retail price when q units are sold in market state ξ is $p(q, \xi)$, assumed to be continuous and strictly increasing in ξ and twice differentiable and decreasing in q. The resulting revenue to the retailer is $\pi(q, \xi) \equiv p(q, \xi)q$, assumed to be concave in q. All functions described as concave or convex are strictly so. The market state, ξ, is a random variable having mean μ, distribution $\Phi(\cdot)$, and density $\phi(\cdot)$, where $\phi(\xi) > 0$ for $\xi > 0$ and $\phi(\xi) = 0$ for $\xi \le 0$; similar results are obtained when the support is $[l, h]$, where $0 \le l < h \le +\infty$.

Because the production lead time is long, production must occur before the market state is known. The cost of producing is linear in the quantity with per-unit cost c, the salvage value of unsold units is zero, and $\pi_q(q, \xi)q$ is concave in q, where the subscript indicates a partial derivative;[2] these assumptions can be relaxed, as discussed below. Exclude the uninteresting case in which the cost of production is sufficiently high that the manufacturer's expected profit is negative for all positive production quantities; similarly, exclude the case where the cost is sufficiently

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[2] If $p(q, \xi)$ is three times differentiable in q, this last condition is equivalent to $4p_q(q, \xi) + 5p_{qq}(q, \xi)q + p_{qqq}(q, \xi)q^2 < 0$, which is satisfied if $p(\cdot, \xi)$ is not too convex and is decreasingly so. This and the other assumptions of our model are satisfied by the $p(q, \xi)$ functions associated with the commonly used demand curves $D(p, \xi) = a + \xi - bp$, $D(p, \xi) = (a - bp)\xi$, $D(p, \xi) = a\exp(-bp)\xi$, and $D(p, \xi) = ap^{-d}\xi$, where $a, b > 0$ and $d > 1$.

Figure 1   Sequence of Events

Sell early | Mfg. offers contract $T(\cdot)$ | Retailer orders s | Mfg. produces | Retailer sets p

Distribution of ξ known | Production | ξ Revealed | Retail sale

Sell late | Mfg. produces s | Mfg. offers contract $T(\cdot)$ | Retailer orders q | Retailer sets p

Stage 1 | Stage 2

low that the manufacturer's profit is unbounded. Figure 1 depicts the sequence of events.

We begin by assuming that all parameters are commonly known. In particular, in Stage 1 both firms know the distribution of ξ, and in Stage 2 both firms know the realization of ξ. Stage 2 corresponds roughly to the beginning of the retail selling season. At this time, firms have observed consumer interest in and demand for samples and similar products, and consequently, in many settings, much of the market uncertainty has been resolved.

When the manufacturer sells early, he specifies the terms of the contract, produces to meet the retailer's order, and delivers the order before market uncertainty is resolved. When the manufacturer sells late, he produces speculatively prior to resolution of market uncertainty; after market uncertainty is resolved, he specifies the terms of the contract and delivers the ordered quantity. This modeling approach allows for a clean comparison of the manufacturer's alternatives of building-to-order (selling early) versus building-to-stock (selling late).

In one particular form of contract that is widely used, the transfer payment is a linear function of the quantity purchased:

$$T(q) = wq. \tag{1}$$

We consider this linear contract as well as the more general contract $T(q)$. Results are stated for the case of general $T(q)$, except as noted.

2.1. Manufacturer Sells Late
We begin by considering linear contracts (1); we discuss the results for general contracts at the end of §2.3. This subsection examines the manufacturer that sells close to the selling season. When the manufacturer sells late, the sequence of events is as follows: In

Stage 1, the manufacturer determines his production quantity and produces s. In Stage 2, the market state, ξ, is revealed, the manufacturer sets the wholesale price, w, and the retailer orders q and sets the retail price, p.

At the end of Stage 2, the retailer faces a known ξ. The retailer will, of course, only order units she plans to sell. Hence, the retailer's retail pricing and ordering problems collapse into the single problem

$$\max_{q \geq 0} \{\pi(q, \xi) - wq\},$$

where, recall, $\pi(q, \xi) = p(q, \xi)q$. The second order condition for the retailer's profit maximization is satisfied because the retailer's revenue is concave in q. Assume $w \leq \pi_q(0, \xi)$, noting that $\pi_q(0, \xi) = p(0, \xi)$, so that the retailer's optimal order quantity $\hat{q}(w)$ is the unique solution to the first order condition:

$$\pi_q(q, \xi) = w.$$

At the beginning of Stage 2, the manufacturer's wholesale pricing problem is, given the state of the market ξ and the quantity s he produced earlier, to choose a wholesale price that maximizes his revenue:

$$\max_{\{w \geq 0,\, \hat{q}(w) \leq s\}} w\hat{q}(w). \tag{2}$$

Because there is a one-to-one mapping between the wholesale price and the quantity sold, the manufacturer's wholesale pricing problem can be parameterized as the quantity-setting problem

$$\max_{q \in [0,\, s]} \pi_q(q, \xi)q.$$

In Stage 1, the manufacturer's expected profit from s units is

$$\widehat{m}(s) = \int_0^\infty \left[\max_{q \in [0,\, s]} \pi_q(q, \xi)q \right] d\Phi(\xi) - cs,$$

and the manufacturer's production problem is

$$\max_{s \geq 0} \widehat{m}(s). \tag{3}$$

Because $\pi_q(q, \xi)q$ is concave in q, $\widehat{m}(s)$ is concave in s and has a unique maximizer, denoted $\hat{s}$. Let $\widehat{M}$ and $\widehat{R}$ denote the manufacturer's and retailer's expected profit when the manufacturer sells late.

2.2. Manufacturer Sells Early

This subsection examines the manufacturer that sells well in advance of the selling season. When the manufacturer sells early, the sequence of events is as follows: In Stage 1, the manufacturer sets the wholesale price, the retailer orders, and the manufacturer produces and fulfills the order. In Stage 2, the market state is revealed, and retailer sets the retail price.

In Stage 2, the retailer's pricing problem is, given the state of the market ξ and the quantity s she purchased earlier, to choose a retail price that maximizes her revenue. Because there is a one-to-one mapping between the retail price and the quantity sold, the retailer's retail price-setting problem can be parameterized as the quantity-setting problem

$$\max_{q\in[0,\,s]} \pi(q,\xi).$$

Consequently, in Stage 1 the retailer's expected profit when she orders s is

$$\int_0^\infty \left[\max_{q\in[0,\,s]} \pi(q,\xi) \right] d\Phi(\xi) - ws.$$

It is easy to check that the second order condition is satisfied. Assume $w \le \int_0^\infty \pi_q(0,\xi)\,d\Phi(\xi)$, so that the retailer's optimal order quantity $\tilde{s}(w)$ is the unique solution to the first order condition:

$$\int_0^\infty \frac{\partial}{\partial s} \left[\max_{q\in[0,\,s]} \pi(q,\xi) \right] d\Phi(\xi) = w. \qquad (4)$$

Equation (4) equates the retailer's expected marginal revenue for the sth unit with the marginal cost. Let $\mathcal{R}(w)$ denote the early-buying retailer's expected profit under wholesale price w; recall that $\hat{R}$ is the late-buying retailer's expected profit. If the retailer refuses to purchase early, the manufacturer is compelled to sell late (a threat to do otherwise is not credible). Consequently, the early-selling manufacturer must set his wholesale price such that $\mathcal{R}(w) \ge \hat{R}$ to ensure the retailer's participation.[3] At the beginning of Stage 1, the manufacturer's wholesale pricing problem is

$$\max_{\{w\ge 0,\,\mathcal{R}(w)\ge \hat{R}\}} (w-c)\tilde{s}(w). \qquad (5)$$

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[3] All of the paper's main results hold when the assumption that the early-buying retailer's profit is weakly greater than the late-buying retailer's expected profit is relaxed. The only qualitative insight that would change with this relaxation is that the retailer will sometimes strictly prefer to buy late.

2.3. Comparison of Selling Early vs. Late

Theorem 1 compares the manufacturer's profit when he sells early and late. Let M denote the profit of the early-selling manufacturer; recall that $\widehat{M}$ is the profit of the late-selling manufacturer.

THEOREM 1. *Suppose the manufacturer employs a linear contract.*

$$\widehat{M} \ge M,$$

where the inequality is weak if and only if

$$\pi_q(q,\xi) + q\pi_{qq}(q,\xi) \ge 0 \qquad (6)$$

for all ξ and all $q \le \hat{s}$.

In other words, the manufacturer's expected profit is always greater when he sells late. The theorem provides a readily interpretable condition, which indicates when the late-selling manufacturer's profit is *strictly* greater. Equation (6) holds in the relevant range if and only if it is always optimal for the late-selling manufacturer, regardless of the state of the market, to sell all of the units that he produced earlier to the retailer. The proof of Theorem 1, as well as those of the other main results, is given in the appendix.

The manufacturer prefers to sell late because in doing so he is able to exercise greater control. In particular, when the manufacturer sells early he only determines his production quantity. When the manufacturer sells late, he effectively determines both his production quantity and (through his wholesale price decision) the retail price. The early-selling manufacturer sells his entire production quantity. In contrast, the late-selling manufacturer may be able to increase his revenue, at least in some states, by withholding stock from the retailer (setting a higher wholesale price). Thus, in postponing his pricing decision, the late-selling manufacturer essentially buys an option.[4] His downside loss is capped for low market realizations, but he is able to adjust his price upward to exploit high market realizations. Because of the advantage conferred to the late-selling manufacturer by the option to withhold stock, the manufacturer's expected revenue from any given production quantity is greater when he sells late. The result that the

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[4] The author thanks the senior editor for this interpretation.

manufacturer prefers to sell late is robust to several of our assumptions. It continues to hold when the production cost is any nonlinear function, units have a salvage value that is contingent on the realized market state ξ, and $\pi_q(q, \xi)q$ is not concave in q (see Taylor 2005).

To gain insight into when the manufacturer strictly prefers to sell late, we consider a specific demand curve. In particular, for the remainder of the paper we focus on linear demand curves:

$$D(p, \xi) = a + \xi - bp; \tag{7}$$

$a + \xi$ can be interpreted as the market size. Taylor (2005) discusses the results for other commonly used demand curves. Applying Theorem 1 yields that $\widehat{M} \geq M$, where the inequality is weak if and only if

$$c \geq \mu/b.$$

That is, the manufacturer does strictly better by selling late only if the production cost is sufficiently small. When the production cost is large, the optimal production quantity is sufficiently small that it is always optimal for the late-selling manufacturer to sell all the units he produced. Conversely, if the production cost is small, then the production quantity is sufficiently large that it is optimal for the manufacturer to withhold stock when market conditions are poor. Taylor (2005) provides numerical evidence that selling late substantially increases the manufacturer's expected profit when production is inexpensive and market uncertainty is pronounced. However, in these cases selling late reduces the retailer's profit. In contrast, when market uncertainty is small, the manufacturer's gain from selling late does not come at the expense of the retailer, so selling late results in a Pareto-dominant outcome.

When the manufacturer employs a linear contract, there is a loss in system efficiency due to double marginalization (Spengler 1950). Because the manufacturer and retailer each receive only a portion of the total system profit margin, the retailer underorders and the manufacturer underproduces relative to the production quantity for the integrated system. This loss in system efficiency is eliminated if the manufacturer employs a properly designed nonlinear contract $T(q)$. This is trivial to verify: It is sufficient to

restrict attention to a price-quantity contract (t, q) in which the manufacturer offers a specific quantity q for a price t. If the late-selling manufacturer produces the integrated system optimal quantity and after observing the market state offers this quantity for a price equal to the revenue generated by this quantity, the manufacturer will capture the integrated system expected profit. If the early-selling manufacturer offers the integrated system optimal production quantity for a price equal to the integrated system expected revenue, the manufacturer will capture the integrated system expected profit. Consequently, the manufacturer that employs a general contract $T(q)$ is indifferent to the timing of his sale. This demonstrates that, as one would expect, the manufacturer's sale-timing preference depends on the form of the contract.

We have demonstrated that a manufacturer that employs a linear contract prefers to sell late and a manufacturer that employs a general contract is indifferent to the time of his sale. The next section examines the extent to which these results continue to hold when the retailer exerts sales effort.

3. Retailer Sales Effort

In many settings, retailer sales effort is important in influencing demand. For example, a retailer can stimulate demand by developing marketing and advertising campaigns, investing in attractive physical stores, hiring sales personnel, and providing training. Typically, such sales effort decisions must be made well in advance of the selling season. Alternately, the retailer can stimulate demand by exerting sales effort during the selling season: advertising at the point-of-sale, providing attractive shelf space, and guiding purchases with sales personnel. For products such as fashion apparel, where shelf space and in-season advertising are important, sales effort exerted during the selling season may be particularly significant. For technology products, where trained sales personnel are important (especially for corporate clients), sales effort decisions made in advance of the selling season may be particularly significant. To the extent that the manufacturer's product comprises the bulk of the retailer's offerings, early sales effort decisions may be particularly important. This section examines the manufacturer's sale-timing decision when the retailer

exerts sales effort either prior to or during the selling season.

We generalize the linear demand curve introduced in (7) to allow for effort to influence demand in an additive fashion.

$$D(e, p, \xi) = a + e + \xi - bp \qquad (8)$$

is the quantity demanded when the retailer exerts effort e under retail price p in market state ξ. Consequently, the retailer's sales revenue when she exerts effort e and sells q units is

$$\Pi(e, q, \xi) = (a + e + \xi - q)q/b.$$

The retailer's cost of effort is quadratic

$$V(e) = \alpha e^2;$$

α can be interpreted as the costliness of effort. The demand and effort-cost assumptions can be relaxed, as discussed below. Because it is difficult for a third party to verify the retailer's effort level, effort is noncontractible. We continue to assume that at the beginning of the retail selling season that both firms observe the condition of the market, although this now reflects sales effort investments made before this time. That is, at the beginning of Stage 2, both firms observe the market size $a + e_1 + x$, where e_1 is the sales effort exerted prior to the selling season. Our formulation allows for both the case where the manufacturer, by the start of the selling season, is able to directly observe the retailer's prior sales effort (e.g., investments in renovating stores) and the case where the manufacturer has a good sense of market demand, but is unable to attribute the degree to which this demand is due to the retailer's earlier investments. Our formulation imposes no restrictions on whether the manufacturer observes sales effort exerted during the selling season. Because the analysis of sales effort during the selling season is more straightforward, we consider it first.

3.1. Sales Effort in Stage 2

This subsection examines the sale-timing decision when the retailer exerts sales effort during the selling season, i.e., in Stage 2. The sequence of events follows that described in §2, with the single modification that in Stage 2, when the retailer sets her retail price, she also sets her effort level.

Although the introduction of retailer sales effort may appear to represent a significant change in the problem setting, we show that under relatively mild assumptions, the essential aspects of the sale-timing issue are unchanged, and hence the results carry through from the previous section. This insight stems from the fact that regardless of whether the manufacturer sells early or late, the retailer determines her effort level *after* the manufacturer sets his wholesale price. The retailer's effort decision is relevant to the manufacturer only in that it influences the retailer's order quantity. The retailer's order quantity depends fundamentally on the revenue generated by a given quantity, and accordingly, we focus on this quantity. For technical convenience, we assume $P(\xi = \infty) = 0$ almost surely. Recall that $\Pi(e, q, \xi)$ is the retailer's sales revenue when she exerts effort e and sells q units. The retailer's revenue net of the effort cost in market state ξ when she sells q units and exerts the optimal effort is

$$\pi(q, \xi) = \max_{e \geq 0}\{\Pi(e, q, \xi) - V(e)\}.$$

It is straightforward to show that $\alpha > (2b)^{-1}$ implies that $\pi(q, \xi)$ is concave in q and $\pi_q(\infty, \xi) \leq 0$ almost surely; this ensures that the manufacturer's profit is finite. Consequently, the quantity and effort problem reduces to a quantity-only problem of the type studied in §2. Thus, the result from §2 that the manufacturer employing a linear contract prefers to sell late extends to the case in which the retailer exerts sales effort during the selling season. This result holds more broadly than for the specific demand and effort-cost model considered here. In particular, the assumption of quadratic effort cost is inessential. Further, the result generalizes to settings where effort has a multiplicative, rather than additive, effect on demand, and the demand curve is not linear.[5] In general, a sufficient condition for the manufacturer to prefer to sell

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[5] In terms of the cost of effort function, all that is required is that $V(e)$ be increasing, twice differentiable, and satisfy $V(0) = 0$ and $V_{ee}(e) > (2b)^{-1}$. In terms of other demand functions, the result holds, for example, when $D(e, p, \xi) = ae\xi \exp(-bp)$ and $V(e) = \alpha e^u$, where $a, \alpha, b > 0$ and $u > 1$. Similarly, the result holds when $D(e, p, \xi) = e\xi - bp$ and $V(e) = V_{ee}(e) > h^2/(2b)$, where h is the finite upper limit on the support of $\phi(\xi)$.

late is that the demand curve and cost of effort function result in a retailer net revenue function that is concave in the quantity sold. The result from §2, that with general contracts the manufacturer is indifferent to the timing of his sale, also extends to this case.

In the next subsection we observe that the retailer's effort decision depends importantly on whether the manufacturer's contract offer precedes the retailer's effort decision. When the retailer exerts sales effort during the selling season, the temporal ordering of these decisions does not depend on the manufacturer's sale timing. In contrast, when the retailer exerts sales effort prior to the selling season, the temporal ordering of these decisions *does* depend on the sale timing; this leads to sharply different results for the manufacturer's sale-timing preference.

3.2. Sales Effort in Stage 1
This subsection examines the sale-timing decision when the retailer exerts sales effort prior to the selling season, i.e., in Stage 1. The sequence of events follows that described in §2, with the single modification that the retailer exerts effort prior to the revelation of ξ.

Consider the late-selling manufacturer. In Stage 1, the retailer chooses her effort level and the manufacturer determines his production quantity. In Stage 2, the retailer's ordering and retail price problem can be parameterized as the quantity problem

$$\max_{q \geq 0}\{\Pi(e, q, \xi) - wq\}.$$

Given the retailer's optimal order quantity, the manufacturer's wholesale pricing problem at the beginning of Stage 2 is (2). The resulting manufacturer's revenue is a function of the manufacturer's production quantity s, as well as the retailer's effort e and the market state ξ because these impact the retailer's optimal order quantity. In Stage 1, the manufacturer's and retailer's expected profit are

$$\widehat{m}(s, e) = \int_0^{4s-a-e} \frac{(a+e+\xi)^2}{8b} \, d\Phi(\xi)$$
$$+ \int_{4s-a-e}^{\infty} \frac{(a+e+\xi-2s)s}{b} \, d\Phi(\xi) - cs$$

and

$$\hat{r}(s, e) = \int_0^{4s-a-e} \frac{(a+e+\xi)^2}{16b} \, d\Phi(\xi)$$
$$+ [1 - \Phi(4s-a-e)]\frac{s^2}{b} - V(e).$$

The manufacturer's production quantity and the retailer's sales effort constitute a Nash equilibrium if

$$\hat{s} = \arg\max_{s \geq 0} \widehat{m}(s, \hat{e})$$

and

$$\hat{e} = \arg\max_{e \geq 0} \hat{r}(\hat{s}, e).$$

Lemma 1 establishes that there is a unique Nash equilibrium. In specifying the equilibrium, the following definitions are useful: Let $k(x)$ denote the unique solution to

$$\int_k^{\infty} \frac{\xi - k}{b} \, d\Phi(\xi) = x,$$

and let $K = k(c)$.

LEMMA 1. *When the manufacturer sells late under a linear contract, the unique Nash equilibrium in the manufacturer's production quantity and the retailer's effort is*

$$\hat{s} = \frac{1}{4}\left(a + K + \frac{1}{16\alpha b - \Phi(K)}\int_0^K (a+\xi) \, d\Phi(\xi)\right) \quad (9)$$

$$\hat{e} = \frac{1}{16\alpha b - \Phi(K)}\int_0^K (a+\xi) \, d\Phi(\xi). \quad (10)$$

Consider the early-selling manufacturer. In Stage 1, the retailer chooses her order quantity and effort level. In Stage 2, the retailer's retail price problem can be parameterized as the quantity problem

$$\max_{q \in [0, s]} \Pi(e, q, \xi).$$

In Stage 1, the retailer's expected profit as a function of her effort and order quantity is

$$r(s, e) = \int_0^{2s-a-e} \frac{(a+e+\xi)^2}{4b} \, d\Phi(\xi)$$
$$+ \int_{2s-a-e}^{\infty} \frac{(a+e+\xi-s)s}{b} \, d\Phi(\xi) - ws - V(e),$$

and her effort and ordering problem is

$$\max_{e \geq 0, \, s \geq 0}\{r(s, e)\}.$$

Lemma 2 specifies the retailer's optimal effort and order quantity.

LEMMA 2. *When the manufacturer sells early under a linear contract, the retailer's optimal order quantity and effort are*

$$\tilde{s}(w) = \frac{1}{2}\left(a + k(w) + \frac{a + E\min(k(w), \xi)}{4\alpha b - 1}\right)$$

$$\tilde{e}(w) = \frac{a + E\min(k(w), \xi)}{4\alpha b - 1}. \quad (11)$$

Given the retailer's optimal order quantity as a function of the wholesale price, the early-selling manufacturer's wholesale pricing problem at the beginning of Stage 1 is (5), where the early-buying retailer's expected profit is $\mathcal{R}(w) = r(\tilde{s}(w), \tilde{e}(w))$.

Clearly the manufacturer's profit is a function of the retailer's effort level, with manufacturer profit increasing in the effort. To gain insight into whether the manufacturer is better off selling early or late, first consider how the timing of the manufacturer's sale influences the retailer's effort investment. When the manufacturer sells late, he sets the wholesale price *after* the retailer chooses her effort level. Anticipating that the manufacturer will charge a high wholesale price when she exerts a high level of effort, the retailer instead chooses a lower effort level. This is the celebrated *hold-up* problem: Anticipating that ex post the manufacturer will appropriate the gains associated with the retailer's ex ante relationship-specific investment in sales effort, the retailer underinvests. In contrast, when the manufacturer sells early, he sets the wholesale price *before* the retailer chooses her effort level. Here the effect of the hold-up problem is mitigated because the manufacturer commits to the terms of trade prior to the retailer's relationship-specific investment. Because the manufacturer's profit is increasing in the retailer's effort, this suggests that by alleviating the hold-up problem, selling early may result in higher profit to the manufacturer. Theorem 2 formalizes this intuition. Let e^* denote the retailer's effort when the manufacturer sells early.

Theorem 2. *Suppose the manufacturer employs a linear contract. If $c < \mu/b$, then there exist $\bar{\alpha}$ and $\underline{\alpha}$ such that $\bar{\alpha} \geq \underline{\alpha} > (4b)^{-1}$; if $\alpha > \bar{\alpha}$, then*

$$\widehat{M} > M;$$

if $\alpha < \underline{\alpha}$, then $\widehat{M} < M$ and $\hat{e} < e^*$. If $c > \mu/b$, then

$$\widehat{M} < M$$

and $\hat{e} = 0 < e^*$.

Theorem 2 demonstrates that the manufacturer's sale-timing preference depends on the costliness of sales effort and production, and Figure 2 summarizes this relationship. First consider the case where the production cost is low ($c < \mu/b$). If sales effort is

Figure 2 Manufacturer's Sale-Timing Preference with Sales Effort in Stage 1 and a Linear Contract

		Sales effort cost	
		Low	High
Production cost	High	Early	Early
	Low	Early	Late

sufficiently cost effective, the manufacturer prefers to sell early; if sales effort is expensive, the manufacturer prefers to sell late. The intuition is that for any fixed effort level, the manufacturer prefers to sell late because of the advantage conferred by the option to withhold a portion of the stock (Theorem 1). However, if the manufacturer sets the wholesale price after the retailer's effort decision, then, as discussed above, the retailer will underinvest in effort. Hence, there is a trade-off to the manufacturer in selling late between the advantage conferred by the option to withhold stock and the disadvantage from undermining the retailer's incentive to invest in sales effort. If effort is cheap, then the impact of effort on profit will be large, and consequently, the disadvantage from underinvestment will dominate. Conversely, if effort is expensive, then its effect will be small, whether the retailer sells early or late. Consequently, the disadvantage resulting from underinvestment will be outweighed by the advantage from the option to withhold stock.

If the production cost is high ($c > \mu/b$), then the manufacturer is strictly better off selling early. The intuition is that when the production cost is sufficiently high, the retailer anticipates that the late-selling manufacturer will always price to sell out regardless of the realization of the market condition. In this case, the manufacturer captures all the gains from retailer effort, and anticipating this, the retailer invests nothing in sales effort. On the contrary, when the manufacturer sells early, the hold-up problem is alleviated, resulting in greater profit for the manufacturer. (In the limiting case, as sales effort becomes prohibitively costly $\alpha \to \infty$, the manufacturer's gain from selling late disappears $(M - \widehat{M}) \to 0$, so Theorem 2 is consistent with Theorem 1.) In a distinct but related setting, Gilbert and Cvsa (2003) find that the manufacturer that produces and sells late prefers

to commit to the wholesale price in advance when demand uncertainty is low.

There is some evidence that Theorem 2 is consistent with industry practice in the toy industry. The typical industry practice for so-called "exclusive products" requires that the retailer place a binding order well in advance of the selling season (i.e., the manufacturer sells early). In contrast, for nonexclusive products, retailers, for the bulk of their purchases, typically do not place binding orders until relatively close to the selling season (i.e., the manufacturer sells late). Retailer sales effort is considerably more important for exclusives than for other products. "A key difference with exclusives is that the retailer takes on the onus of the advertising, where for other products the manufacturer takes on the onus of the marketing and advertising," said David Trenteseaux, senior vice president of supply chain for Hasbro (Trenteseaux 2004). Admittedly, our model does not capture manufacturer sales effort. However, in our model, settings where retailer effort is unimportant correspond to large α ($\alpha = \infty$ corresponds to the case where effort is irrelevant), while settings where effort is important correspond to small α. To the extent that retailers exert sales effort for exclusives prior to the selling season, selling early alleviates the hold-up problem. Exclusives differ from nonexclusives in other dimensions besides the role of retailer sales effort (e.g., exclusives cannot be sold at other retailers without modification), and these dimensions may also play a role in the manufacturer's sale-timing decision.

Theorem 2 characterizes the manufacturer's sale-timing preference when $c > \mu/b$, and when $c < \mu/b$ and $\alpha < \underline{\alpha}$ or $\alpha > \bar{\alpha}$. However, the theorem is silent as to whether $\underline{\alpha} = \bar{\alpha}$. To shed light on this issue, we conducted a numerical study of the region where $c < \mu/b$ and $\alpha > (4b)^{-1}$. Suppose that ξ is a Normal(μ, σ^2) random variable, truncated such that its probability mass is distributed over $\xi \geq 0$ and that $a = 0$. We considered the 400 problems defined by the following parameters: $b = \{1/2, 1\}$, $\mu = \{4, 6, 8, 10\}$, $\sigma = \{1, 3, 5, 7, 9\}$, and $c = \{0, 0.1\mu/b, 0.2\mu/b, \dots, 0.9\mu/b\}$. In every problem, we observed that $\underline{\alpha} = \bar{\alpha}$; that is, there exists a single threshold such that the manufacturer prefers to sell early if and only if α is less than the threshold.

Finally, we note that a manufacturer that employs a general contract prefers to sell early. This is trivial to establish: By the same argument in the no-effort case, with a properly designed contract the early-selling manufacturer captures the integrated system expected profit. When the manufacturer sells late, the hold-up problem leads the retailer to exert zero effort. Consequently, the manufacturer is strictly better off selling early.

To summarize the implication of contract type on the manufacturer's sale-timing preference, general contracts favor selling early more strongly than do linear contracts. In all the settings considered thus far, the manufacturer that employs a general contract prefers (at least weakly) to sell early. This contrasts sharply with the results under a linear contract, where the retailer often strictly prefers to sell late (Theorems 1 and 2). The intuition for this divergence in results is that what makes selling late attractive with a linear contract—the ability to withhold stock—confers no advantage with a general contract.

Although in all the settings considered thus far the manufacturer that employs a general contract prefers to sell early, the next section demonstrates that when the parties are asymmetrically informed, this result may be reversed.

4. Asymmetric Demand Information

This section explores the implications of asymmetric demand information for the manufacturer's sale-timing decision. In the base-case model of §2, well in advance of the selling season both firms share the same distribution of the market state. In reality, the retailer's rich knowledge of consumer preferences and buying patterns might provide the retailer with better information about demand well in advance of the selling season.[6] We focus on the case where the retailer has a better forecast of demand, but both firms share the same information regarding the market state after uncertainty is resolved early in the selling season. This is appropriate when, early in the selling season, the manufacturer as well as the retailer observe consumer interest in and demand for samples and similar products (e.g., the manufacturer may receive such

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[6] Although, in principle, the manufacturer could have better information about demand, we believe that typically when there is asymmetric demand information, the retailer is better informed, and we focus on this case.

information from other retailers that share point-of-sale information, or from its own captive retailers). To the extent that the manufacturer is far removed from the retail market at the beginning of the selling season, the retailer's informational advantage may persist into the selling season; our model does not address this case.

To examine the implications of asymmetric demand information well in advance of the selling season, we modify the informational assumptions of §2 by making a single change. As before, both firms know the distribution of ξ in Stage 1, and both firms observe the realization of ξ in Stage 2. To capture that the retailer has better information well in advance of the selling season, assume that in Stage 1 the retailer observes a signal $v \in \{v_1, v_2, \ldots, v_N\}$ that conveys more accurate information about the distribution of the market state. The probability of observing signal v_i is f_i, where $f_i > 0$ for all i and $\sum_{i=1}^{N} f_i = 1$. The demand distribution conditioned on observing v_i is $\Phi(\xi \mid v_i)$. Observing a signal with a larger index corresponds to a more favorable demand distribution. $\Phi(\xi \mid v_{i+1})$ dominates $\Phi(\xi \mid v_i)$ under first order stochastic dominance:

$$\Phi(\xi \mid v_{i+1}) \leq \Phi(\xi \mid v_i) \quad (12)$$

for $i = 1, \ldots, N - 1$. The signal provides some information:

$$\Phi(\xi \mid v_i) \neq \Phi(\xi \mid v_j) \quad (13)$$

for at least one (i, j). The unconditional distribution of demand is $\Phi(\xi) = \sum_{i=1}^{N} f_i \Phi(\xi \mid v_i)$. The demand curve is given by (7).

We begin by considering general contracts $T(q)$ and discuss the results for the linear contract (1) subsequently. We place no a priori restrictions of the form of $T(q)$; rather, the manufacturer chooses a contract among all possible contracts to maximize his expected profit. One alternative is to make the payment a step function of the quantity ordered:

$$T(q) = \begin{cases} 0 & \text{if } q = 0 \\ t_i & \text{if } q \in (q_{i-1}, q_i] \\ \infty & \text{if } q > q_\eta, \end{cases} \quad (14)$$

where $0 \equiv q_0 < q_1 \leq q_2 \leq \cdots \leq q_\eta$ and $t_1 \leq t_2 \leq \cdots \leq t_\eta$. If $\eta = 1$, this corresponds to a price-quantity contract: The retailer essentially can purchase q_1 for a price

of t_1. When $\eta > 1$, this corresponds to a menu of price-quantity contracts $(t_i, q_i)_{i=1,\ldots,\eta}$. The contract in (14) provides just one way of many to implement such a menu.

Consider the late-selling manufacturer. Observe that no informational asymmetry exists at the time the manufacturer offers the contracts. To motivate the manufacturer's optimal decisions, consider the integrated system of manufacturer and retailer when the retailer observes no signal in Stage 1. Suppose that the manufacturer sets his production quantity equal to the optimal production quantity of the integrated system; after observing the market state, the manufacturer offers a price-quantity contract in which the quantity is the optimal selling quantity for the integrated system and the price is the resulting revenue for the retailer. By doing so, the manufacturer maximizes the profit of the total system, where the retailer's Stage 1 information is ignored, and appropriates it entirely. Clearly, the late-selling manufacturer can do no better.

Consider the early-selling manufacturer. From the revelation principle, we can without loss of generality restrict attention to a menu of price-quantity contracts. The early-selling manufacturer's contract design problem can be written

$$\max_{(t_i, q_i)_{i=1,\ldots,N}} \sum_{i=1}^{N} f_i[t_i - cq_i] \quad (15)$$

subject to $\quad r_i(q_i) - t_i \geq 0 \quad \text{for } i = 1, \ldots, N \quad (16)$

$$r_i(q_i) - t_i \geq r_i(q_j) - t_j$$

$$\text{for } i = 1, \ldots, N, j = 1, \ldots, N, \text{ and } j \neq i, \quad (17)$$

where $r_i(q)$ is the expected revenue generated by q units when the retailer prices optimally, under demand distribution $\Phi(\xi \mid v_i)$. The manufacturer offers a menu of N price-quantity contracts, where contract (t_i, q_i) is intended for the retailer that has observed signal $v = v_i$. The optimal menu of contracts may involve pooling. For example, if $(t_i, q_i) = (t_{i+1}, q_{i+1})$, then the manufacturer intends that the retailer that has observed signal v_i and the retailer that has observed signal v_{i+1} select the same contract. There are N individual rationality constraints (16); these ensure the retailer's participation in buying early.

There are $N \times (N-1)$ incentive compatibility constraints (17); these ensure that the retailer selects the intended contract.

Recall that under symmetric information, the manufacturer that employs a general contract always (weakly) prefers to sell early. The next theorem demonstrates that when information is asymmetric, this result may be reversed. The intuition is that with asymmetric information, selling early places the manufacturer at an informational disadvantage relative to the retailer, which makes selling early less attractive. The theorem demonstrates that the manufacturer's sale-timing preference depends on the cost of production. As before, M and $\widehat{M}$ denote the early- and late-selling manufacturer's profit.

THEOREM 3. *There exist $\bar{c}$ and $\underline{c}$ such that $\bar{c} \geq \underline{c} > 0$; if $c < \underline{c}$, then*

$$\widehat{M} > M;$$

if $c > \bar{c}$, then $\widehat{M} < M$.

In other words, if production is inexpensive, the manufacturer prefers to sell late; if production is costly, the manufacturer prefers to sell early. Selling early with a menu of contracts allows the manufacturer to make production decisions with better information about market demand. However, eliciting this information from the retailer is costly to the manufacturer (the manufacturer must cede information rents to the retailer). If the production cost is high, then knowledge of the signal of market demand at the time of production is of significant value to the manufacturer. This value outweighs the cost of eliciting the information, and the manufacturer prefers to sell early. Conversely, if the production cost is low, then knowledge of the market signal at the time of production is of little value to the manufacturer and, consequently, the manufacturer prefers to sell late.

Anecdotal evidence suggests that, indeed, one motivation for selling well in advance of the selling season is to obtain better information about market demand. In the ski-apparel industry, well in advance of the selling season retailers often possess better information about demand than the manufacturer. With the explicit objective of eliciting this information from its retailers, apparel manufacturer Sport Obermeyer initiated its Early Write program, in which it sells to retailers well (eight months) in advance of the selling season (Fisher et al. 1994). (Sport Obermeyer also sells to retailers subsequent to the Early Write program.) Similarly, the traditional practice in the athletic footwear industry is to sell well in advance of the selling season. The dominant manufacturer, Nike, sells almost entirely through such advanced orders. A primary motivation is to obtain demand information from retailers prior to investing in production. "Nike would never know how much of a production line to put in unless it gets advanced orders," said Wells Fargo Van Kasper analyst John Shanley (Hirsch 2001).

Theorem 3 characterizes the manufacturer's sale-timing preference when $c < \underline{c}$ or $c > \bar{c}$, but is silent as to whether $\underline{c} = \bar{c}$. To shed light on this issue, we conducted a numerical study. There are $N = 2$ possible signals. With probability f_i, the retailer observes that ξ is a Normal(μ_i, σ^2) random variable, truncated such that its probability mass is distributed over $\xi \geq 0$, for $i = 1, 2$. We considered the 450 problems defined by the following parameters: $a = 0$, $b = \{1/2, 1\}$, $f_1 = \{1/5, 1/3, 1/2, 2/3, 4/5\}$, $\mu_1 = \{4, 6, 8\}$, $\mu_2 = \{10, 12, 14\}$, and $\sigma = \{1, 3, 5, 7, 9\}$. In every problem, we observed that $\underline{c} = \bar{c}$; that is, there exists a single threshold such that the manufacturer prefers to sell late if and only if c is less than the threshold.

Theorem 3 holds when the assumption that the demand curve is linear is relaxed.[7] In addition, Theorem 3 extends to the case where the contract is linear (Taylor 2005 provides the proof for general demand curves). Thus, the structure of the manufacturer's sale-timing preference under asymmetric demand information, as reflected in Theorem 3, does not depend on the contract form. This contrasts with observations that when information is symmetric or demand is influenced by retailer effort, the manufacturer's sale-timing preference depends on the contract form.

As in previous sections, we have assumed that the manufacturer offers product for sale at only one point in time. The manufacturer might be able to increase his expected profit by offering product for sale at

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[7] The theorem holds for very general demand curves. All that is required are the mild, technical assumptions that $p(0, \xi)$ and $\sup_{q \geq 0} \pi(q, \xi)$ are finite for all ξ. Taylor (2005) provides the proof for general demand curves.

more than one point in time (e.g., so that a retailer that has observed one of a particular set of signals buys early, but otherwise postpones purchasing). Because our focus is on comparing selling early versus late, we leave this topic for future research.

Impact of Quality of Demand Information on Profit

In the remainder of this section we focus on the impact of the quality of the retailer's private information on the firms' profits. Because the retailer's private information does not affect the firms' profits when the manufacturer sells late, we focus on the case where the manufacturer sells early. We also highlight the impact of information quality on the manufacturer's sale-timing decision. We begin by considering general contracts $T(q)$ and discuss the results for the linear contract (1) at the end of this section.

For the remainder of this final section we consider a simple special case of the demand signal model described above. In Stage 1, the retailer observes one of $N = 2$ possible signals. The probability of observing v_1 is $f_1 = (\theta + \lambda - 1)/(2\theta - 1)$; $f_2 = 1 - f_1$. The unconditional and conditional demand distributions are

$$\Phi(\xi) = \begin{cases} 0 & \text{if } \xi < L \\ \lambda & \text{if } \xi \in [L, H) \\ 1 & \text{if } \xi \geq H, \end{cases} \qquad \Phi(\xi \mid v_1) = \begin{cases} 0 & \text{if } \xi < L \\ \theta & \text{if } \xi \in [L, H) \\ 1 & \text{if } \xi \geq H, \end{cases}$$

$$\Phi(\xi \mid v_2) = \begin{cases} 0 & \text{if } \xi < L \\ 1 - \theta & \text{if } \xi \in [L, H) \\ 1 & \text{if } \xi \geq H, \end{cases}$$

where $H > L > 0$. Under the unconditional demand distribution, $\xi = L$ with probability λ and $\xi = H$ otherwise.[8] Without loss of generality, assume $a = 0$.

With $v_1 = L$, $v_2 = H$, and $\theta \in [\max(\lambda, 1 - \lambda), 1]$, this model has the following interpretation: The signal $v = \{L, H\}$ that the retailer observes is accurate, i.e., $\xi = v$, with probability θ. If $\theta = 1$, then in Stage 1 the retailer knows the true ξ with certainty; if $\theta = \max(\lambda, 1 - \lambda)$, the signal provides no

information. Hence, θ is a measure of the quality of the retailer's information and the degree of informational asymmetry, with both quantities increasing in θ. We focus on the case where the signal provides some information: $\theta > \max(\lambda, 1 - \lambda)$. We refer to the retailer that has observed the unfavorable signal v_1 (favorable signal v_2) as the low-type (high-type) retailer.

The early-selling manufacturer's contract design problem is given in (15)–(17). Lemma 3 specifies the optimal menu of contracts. In specifying the menu, the following definitions are useful: $\Gamma(q, \xi) \equiv (\xi - \min(q, \xi/2)) \min(q, \xi/2)/b$ and $Z \equiv \theta(3 + \lambda - 3\theta) - 1$. $\Gamma(q, \xi)$ is retailer's revenue in market state ξ when she has q units and sets her retail price optimally.

LEMMA 3. *The early-selling manufacturer's optimal menu of contracts is*

$$t_1^* = \theta\Gamma(q_1, L) + (1 - \theta)\Gamma(q_1, H), \tag{18}$$

$$t_2^* = \theta\Gamma(q_2, H) + (1 - \theta)\Gamma(q_2, L) - (2\theta - 1)$$
$$\cdot [\Gamma(q_1, H) - \Gamma(q_1, L)],$$

$$q_1^* = \max(1_{\{Z>0\}}[H/2 - (\theta + \lambda - 1)bc/(2Z)],$$
$$(L - bc)/2 + Z(H - L)/[2(\theta + \lambda - 1)], 0), \tag{19}$$

$$q_2^* = \max(H/2 - bc/(2\theta), [\theta H + (1 - \theta)L - bc]/2, 0).$$

The manufacturer offers a contract to the low-type retailer ($q_1^* > 0$) *if and only if the production cost* $c < [L + Z(H - L)/(\theta + \lambda - 1)]/b$.

The manufacturer offers a high-price, high-quantity contract and a low-price, low-quantity contract: $t_2^* > t_1^*$ and $q_2^* > q_1^*$. The expected profit of the low-type retailer is zero while the expected profit of the high-type is positive, i.e., the high-type retailer captures information rents. Assuming $c < [\theta H + (1 - \theta)L]/b$ ensures that the manufacturer's expected profit is positive. When the retailer has observed the high-demand signal, the quantity the manufacturer produces, q_2^*, is the optimal production quantity of the integrated system. However, when the retailer has observed the low-demand signal, the quantity that the retailer purchases, q_1^*, is less than the optimal production quantity of the integrated channel: The manufacturer's optimal contract distorts the production quantity downward to ameliorate the costs of

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[8] Chu (1992), Lariviere and Padmanabhan (1997), and Padmanabhan and Png (1997) assume similar two-point distributions with linear demand curves.

asymmetric information. If the production cost is sufficiently high, the manufacturer only offers a contract to the high-type retailer.

We now consider the sensitivity of the firms' profits to the quality of the retailer's early information about demand, beginning with the manufacturer. Recall that θ denotes the accuracy of the signal the retailer observes, and hence is a measure of the quality of the retailer's information. If $\theta = \max(\lambda, 1 - \lambda)$, then the signal provides no information and we say that the retailer is *uninformed*; if $\theta > \max(\lambda, 1 - \lambda)$, then the retailer is *informed*. Let M_U denote the early-selling manufacturer's expected profit when he sells to an uninformed retailer; let M_I denote the analogous quantity when the retailer is informed. It is easy to check that the expected profit of the early- and late-selling manufacturer are identical when the retailer is uninformed: $M_U = \widehat{M}$. Thus, an immediate implication of Theorem 3 is that the early-selling manufacturer prefers to sell to an uninformed (informed) retailer if the production cost is low (high): if $c < \underline{c}$, then

$$M_U > M_I; \qquad (20)$$

if $c > \bar{c}$, then $M_U < M_I$. The intuition is similar to the reasoning described above: If the retailer is informed, the manufacturer can make production decisions with better information about market demand. However, selling to an informed retailer puts the manufacturer at a relative informational disadvantage; the manufacturer must cede rents to the retailer to ensure the retailer's participation. As before, if the production cost is low, then the value of knowing the market signal at the time of production is of little value relative to the cost of eliciting the information from an informed retailer; consequently, the manufacturer prefers to sell to an uninformed retailer.

We now turn to the sensitivity of the retailer's profits to the quality of her information. The retailer's profit is solely due to information rents. If the retailer has no informational advantage (i.e., $\theta = \min(\lambda, 1 - \lambda)$), then her profit is zero. As she gains an informational advantage, her profit increases: R increases as θ increases from $\min(\lambda, 1 - \lambda)$. One might conjecture that the retailer's profit is always increasing (at least weakly) in the quality of her information. Theorem 4 shows that this conjecture is false, providing sufficient conditions under which the retailer's

profit is strictly decreasing in the quality of her information θ. Recall that the R denotes the early-buying retailer's expected profit. Let $\tilde{c} = [L - (1 - \lambda)H]/(b\lambda)$ and $\check{c} = [L + \min(1 - \lambda, (7 + 4\sqrt{3})\lambda - 3 - 2\sqrt{3}) \cdot (H - L)]/b$. Also, $2 - \sqrt{3} \simeq 0.27$.

Theorem 4. *Suppose $c \geq \tilde{c}$. There exists $\underline{\theta} < 1$ such that*

$$R \text{ is decreasing in } \theta \text{ on } \theta \in [\underline{\theta}, 1].$$

If $\lambda \leq 2 - \sqrt{3}$ or $c \geq \check{c}$, then $R = 0$. Otherwise, there exists $\bar{\theta} \in (\underline{\theta}, 1]$ such that

$$R \text{ is strictly decreasing in } \theta \text{ on } \theta \in [\underline{\theta}, \bar{\theta}], \qquad (21)$$

and $R = 0$ for $\theta \in [\bar{\theta}, 1]$.

If the production cost is high, then the retailer's profit decreases in the quality of her information, for high levels of information quality.

To see the intuition for why better information can lead to lower profit, it is helpful to separate two effects that information quality has on retailer profit. First, for any fixed menu of contracts, the retailer's profit is increasing in the quality of her information. Intuitively, having better information puts the retailer in a superior position in selecting a contract, and this results in larger profit. On the other hand, information quality also influences the optimal menu of contracts offered by the manufacturer. The main intuition behind Theorem 4 is that this second effect can have a negative impact that outweighs the positive impact of the first effect. The manufacturer takes the retailer's information quality into account when designing the menu of contracts, and better information quality may lead to reduced information rents for the retailer under the optimal menu.

Consider the manufacturer's decision in constructing a menu of contracts. If the quality of the signal that the retailer receives is poor, there is little to distinguish a retailer that has observed a favorable or unfavorable signal, and the contracts the manufacturer offers will be quite similar. However, if the manufacturer knows that the retailer has an accurate forecast of its demand, the manufacturer can offer radically different contracts. More specifically, as θ increases, in the optimal menu of contracts q_2^* increases and q_1^* decreases. The high-type retailer's information rent is equal to the profit she receives if she selects the contract intended for the low-type retailer. The retailer's

expected profit can be written as a function of her information quality and the quantity in the contract intended for the low-type retailer:

$$R = f_2(2\theta - 1)[\Gamma(H, q_1^*) - \Gamma(L, q_1^*)].$$

The first term, f_2, is the probability that the retailer observes the favorable demand signal, and the remainder is the high-type retailer's information rent. In setting the optimal q_1^*, the manufacturer trades off improving systemwide efficiency, which favors a high q_1^*, versus ameliorating the cost of asymmetric information, which favors a low q_1^*. Note that

$$(\partial/\partial\theta)R = \underbrace{[\Gamma(H, q_1^*) - \Gamma(L, q_1^*)]}_{>0} + \underbrace{(\theta - \lambda)}_{>0}$$
$$\times \underbrace{(\partial/\partial q)[\Gamma(H, q_1^*) - \Gamma(L, q_1^*)]}_{>0} \times \underbrace{(\partial/\partial\theta)q_1^*}_{<0}.$$

When the production cost and information quality are high, q_1^* is small, so that the contribution from the first term is small and the negative contribution from the remaining terms dominates: $(\partial/\partial\theta)R < 0$.

Figure 3 illustrates this result. The retailer's profit is increasing and then decreasing in the quality of her information. When the retailer has a small informational advantage, it is optimal for the manufacturer in designing contracts to cede informational rents (which are small due to the retailer's small advantage) in exchange for contracts that favor systemwide efficiency. When the retailer has a larger informational advantage, the manufacturer's optimal menu emphasizes ameliorating the cost of asymmetric information, at the cost of systemwide inefficiency.

Figure 3 **Expected Profit for Early-Buying Retailer, Early- and Late-Selling Manufacturer, and Total System Under Selling Early as a Function of the Retailer's Information Quality**
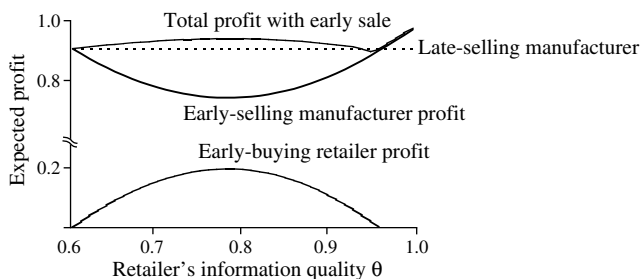
Note. System parameters are $H = 4$, $L = 2$, $\lambda = 0.6$, $c = 0.9$, and $b = 1$.

Figure 3 also demonstrates that the total system can be hurt by the retailer's having better information. The intuition is that the payment of information rents from the manufacturer to the retailer involves transfers within the supply chain, but does not change the total system profit. System profits are solely a function of the degree the manufacturer is willing to tolerate systemwide inefficiency to ameliorate information rents, and at high levels of information quality the manufacturer is willing to tolerate inefficiency to reduce rents.

These results have implications for both retailers and manufacturers. By investing in forecasting, test marketing, etc., retailers can improve the quality of their (private) information about market demand well in advance of the selling season (Lariviere 2002). Theorem 4 suggests that retailers should be wary in making such investments: Even when the cost of obtaining such information is ignored, better information can lead to lower expected profit. Although a retailer always (weakly) benefits from having a small information advantage over the manufacturer, the retailer may be hurt by having too much of an information advantage, especially when the production cost is high.

Figure 3 demonstrates the manufacturer's profit need not be monotone in the retailer's information quality. This suggests that manufacturers should not blindly seek out retailers that have strong forecasting capabilities. Given a choice between two informed retailers, the manufacturer sometimes prefers to sell to the retailer with better information and sometimes prefers to sell to the retailer with worse information. The analytical result above (20) provides guidance when the choice between retailer information quality is extreme.

Figure 3 also provides insight into the impact of information quality on the manufacturer's sale-timing preference. The result in Figure 3 is representative of a much larger numerical study[9] in which we observed results that are consistent with the following refinement of Theorem 3: The manufacturer prefers to sell early if and only if his production cost and

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[9] We considered the 480 problems defined by the parameters: $H \in \{3.0, 4.0, 5.0\}$, $L \in \{0.5, 1.0, 2.0, 2.5\}$, $\lambda \in \{0.2, 0.4, 0.6, 0.8\}$, $b = 1$, and $c \in \{0, 0.1\mu/b, 0.2\mu/b, \ldots, 0.9\mu/b\}$.

the retailer's information quality are sufficiently high. When production costs are high, selling early benefits the manufacturer because he is able to make his production decisions with better information about market demand, and the manufacturer benefits when the quality of that information is high. When production costs are low, such information is of little value and the manufacturer prefers to sell late—regardless of the retailer's information quality.

The main insight of the second half of this section is that the manufacturer, retailer, and total system can be helped or *hurt* by the retailer's having better information. Although this result was shown for general contracts, the result continues to hold when the contract is linear; Taylor (2005) provides examples where firm and system profit are nonmonotone in the retailer's information quality. We conclude by reminding the reader that our results rely on the assumptions that the manufacturer has all the bargaining power and knows (or can infer) the quality of the retailer's information; managers in contexts where these assumptions are not appropriate should be cautious in interpreting our results. Further research is required to address if and how the results change when these assumptions are relaxed.

5. Conclusion

The objective of this paper is to provide guidance to the manufacturer that is addressing the fundamental decision of when he should sell his product. We provide clear guidance in a variety of settings as to when the manufacturer should sell well in advance of the selling season (early) and when he should sell close to the selling season (late). When information is symmetric, retailer sales effort is unimportant, and the contract is linear, the manufacturer prefers to sell late. This result extends to the case in which the retailer exerts sales effort during the selling season. However, when retailer exerts effort in advance of the selling season, selling late introduces a hold-up problem. Hence, if effort is sufficiently cheap (or the production cost is sufficiently high), then the manufacturer prefers to sell early. The result that the manufacturer prefers to sell late extends to the case where the retailer has superior information about demand prior to the selling season, provided that the production cost is low; otherwise, the manufacturer prefers

to sell early; this result extends to the case of general contracts.

Our analysis of retailer sale effort assumed that both firms shared the same information regarding the costliness and effectiveness of effort. In many contexts it is likely that the retailer would have a better understanding of the costliness and effectiveness of her effort than the manufacturer. Further, if the retailer has better information about sales effort, she may also have better information about the demand distribution. Taylor (2005) shows that our sale-timing preference results extend to the case where the retailer is privately informed about both demand and the costliness of effort.

One theme that cuts across the various settings is that low (high) production costs tend to make selling late (early) attractive. The intuition is that selling late is attractive when the late-selling manufacturer has wide latitude in pricing in response to market conditions. High production costs make speculative production uneconomical for the late-selling manufacturer, constraining his subsequent pricing decision and limiting his profit. In contrast, low production costs translate into wide flexibility in pricing for the late-selling manufacturer. This suggests that manufacturers may benefit by changing their sale-timing practices over the course of a product's life cycle. When the product is in its first few selling seasons and production costs are high, the manufacturer may sell well in advance of the selling season. As the product matures, learning curve effects that cause production costs to fall will tend to make it attractive for the manufacturer to delay selling until closer to the start of the retail selling season. A second theme that emerges is that general contracts favor selling early more strongly than do linear contracts. Thus, manufacturers that replace linear contracts with more sophisticated contracts may further benefit by changing the timing of their contract offering—in particular, by selling farther in advance of the selling season.

Although sales effort and asymmetric information are two factors that shape the manufacturer's sale-timing preference, there are a number of other factors that may also play an important role. We have considered a bilateral monopoly where the manufacturer has all the bargaining power, the firms are risk neutral, and they do not face cash constraints. Manufacturer

risk aversion or cash constraints will tend to make selling early more attractive. If the retailer has all the bargaining power, then for trade to occur the retailer must commit to purchase in advance, and the manufacturer will sell early. Assessing the impact of other factors (e.g., exchange-rate fluctuations, manufacturer competition) remains for future research.

Perhaps our most surprising result is that the retailer, manufacturer, and total system can be hurt by the retailer's having better information. This issue is worthy of a more comprehensive treatment, and we hope that future work will follow.

Acknowledgments

Appendix

PROOF OF THEOREM 1. There is a one-to-one mapping between the early-selling manufacturer's wholesale price and the retailer's order quantity (see (4)). Consider the equivalent problem in which the early-selling manufacturer's wholesale price is parameterized as a function of the retailer's order quantity. The early-selling manufacturer's expected profit when he chooses a wholesale price that induces retailer order quantity s is

$$m(s) = \int_0^\infty \frac{\partial}{\partial s}\left[\max_{q\in[0,\,s]} \pi(q, \xi)\right] d\Phi(\xi) \times s - cs.$$

The manufacturer's problem is

$$\max_{\{s\geq 0,\, r(s)\geq \hat{R}\}} m(s),$$

where $r(s)$ denotes the early-buying retailer's expected profit when the manufacturer sells s. Note that $\widehat{m}(s) \geq m(s)$ where the inequality is weak if and only if (6) holds for all ξ and all $q \leq s$. Thus, $\widehat{M} \geq M$. Because $\widehat{m}(\cdot)$ is concave and $\widehat{m}(s) \geq m(s)$, $\widehat{M} = M$ implies $\widehat{m}(\hat{s}) = m(\hat{s})$, which in turn implies that (6) holds for all ξ and all $q \leq \hat{s}$. With a little effort, one can verify that if (6) holds for all ξ and all $q \leq \hat{s}$, then $\widehat{m}(\hat{s}) = m(\hat{s})$ and $r(\hat{s}) = \widehat{R}$. Thus, $M \geq m(\hat{s}) = \widehat{M}$. □

PROOF OF LEMMA 1. First, we characterize the firms' best-response functions. Because $(\partial^2/\partial s^2)\widehat{m}(s, e) = -4[1 - \Phi(4s - a - e)]/b < 0$, the manufacturer's best response, $\hat{s}(e)$, is the unique solution to the first order condition:

$$\int_{4s-a-e}^\infty \frac{a+e+\xi-4s}{b}\, d\Phi(\xi) = c. \tag{22}$$

Because $(\partial^2/\partial s^2)\hat{r}(s, e) = -2\alpha - s\phi([4s - a - e]^+)/(2b) + \Phi(4s - a - e)/(8b) < 0$, the retailer's best response, $\hat{e}(s)$, is the unique solution to the first order condition:

$$\int_0^{4s-a-e} \frac{a+e+\xi}{8b}\, d\Phi(\xi) = 2\alpha e. \tag{23}$$

With the change of variable $y = 4s - a - e$, (22) becomes

$$\int_y^\infty \frac{\xi-y}{b}\, d\Phi(\xi) = c, \tag{24}$$

and (23) becomes

$$\int_0^y \frac{a+e+\xi}{8b}\, d\Phi(\xi) = 2\alpha e,$$

or equivalently,

$$e = \frac{1}{16\alpha b - \Phi(y)} \int_0^y (a+\xi)\, d\Phi(\xi).$$

Because the left-hand side of (24) is strictly decreasing in y, there exists a unique, finite solution to (24): $y = K$. Therefore, (10) is immediate. Reversing the change of variables yields $\hat{s} = (a + K + \hat{e})/4$. □

PROOF OF LEMMA 2. It is straightforward to verify that $r(\cdot, \cdot)$ is jointly concave. The retailer's optimal order quantity and effort satisfy the first order conditions

$$\int_{2s-a-e}^\infty \frac{a+e+\xi-2s}{b}\, d\Phi(\xi) = w \tag{25}$$

$$\int_0^{2s-a-e} \frac{a+e+\xi}{2b}\, d\Phi(\xi) + \frac{s}{b}[1 - \Phi(2s - a - e)] = 2\alpha e. \tag{26}$$

With the change of variable $y = 2s - a - e$, (25) becomes

$$\int_y^\infty \frac{\xi-y}{b}\, d\Phi(\xi) = w \tag{27}$$

and (26) becomes

$$\int_0^y \frac{a+e+\xi}{2b}\, d\Phi(\xi) + \frac{a+e+y}{2b}[1 - \Phi(y)] = 2\alpha e,$$

or equivalently,

$$e = \frac{a + E\min(y, \xi)}{4\alpha b - 1}.$$

The unique solution to (27) is $y = k(w)$. Therefore, (11) is immediate. Reversing the change of variables yields $\tilde{s}(w) = [a + k(w) + \tilde{e}(w)]/2$. □

PROOF OF THEOREM 2. With a little effort one can verify that $\widehat{M} > 0$ if and only if $c < (\mu + a)/b$; similarly, $M > 0$ if and only if $c < (\mu + a)/b$, so we restrict attention to $c < (\mu + a)/b$. First, suppose $c \in (\mu/b, (\mu + a)/b)$. This implies $K = \mu - bc < 0$, so $\hat{e} = 0$ and

$$\widehat{M} = (a + \mu - bc)^2/(8b)$$

$$\widehat{R} = (a + \mu - bc)^2/(16b).$$

Further, $k(w) = \mu - bw < 0$. Now consider the early-selling manufacturer. Let $\mathcal{M}(w) = (w - c)\tilde{s}(w)$ denote the

early-selling manufacturer's profit when he sets price w. Note that

$$\mathcal{M}(w) = (w - c)\frac{2\alpha b(a + \mu - bw)^+}{4\alpha b - 1},$$

which is concave in w and maximized at $w^* = (a + \mu + bc)/(2b)$. So,

$$\mathcal{M}(w^*) = \alpha(a + \mu - bc)^2/(8\alpha b - 2)$$

$$\mathcal{R}(w^*) = \alpha(a + \mu - bc)^2/(16\alpha b - 4).$$

Because $\mathcal{R}(w^*) > \widehat{R}$, $M = \mathcal{M}(w^*)$. Further, $\mathcal{M}(w^*) > \widehat{M}$. Finally, $e^* = \tilde{e}(w^*) = (a + \mu - bc)/(8\alpha b - 2) > 0$.

Second, suppose $c < \mu/b$. Using Lemma 1 and $c = \int_K^\infty (\xi - K)/b \, d\Phi(\xi)$ yields

$$\widehat{M} = [a + \hat{e} + E\min(K, \xi)]^2/(8b)$$

$$\widehat{R} = [a + \hat{e} + E\min(K, \xi)]^2 - \alpha\hat{e}^2,$$

where $\hat{e}$ is given by (10). Now consider the early-selling manufacturer. Because the manufacturer's profit is zero if $w \geq (\mu + a)/b$ or $w = c$, we restrict attention to $w \in (c, (\mu + a)/b)$. Using Lemma 2 and $w = \int_{k(w)}^\infty (\xi - k(w))/b \, d\Phi(\xi)$ yields

$$\mathcal{M}(w) = \frac{w - c}{2}\left(a + k(w) + \frac{a + E\min(k(w), \xi)}{4\alpha b - 1}\right)$$

$$\mathcal{R}(w) = \frac{\alpha[a + E\min(k(w), \xi)]^2}{4\alpha b - 1}.$$

Note that

$$\lim_{\alpha \to (4b)^{-1}} \mathcal{M}(w) = \infty, \quad \lim_{\alpha \to (4b)^{-1}} \mathcal{R}(w) = \infty,$$

$$\lim_{\alpha \to (4b)^{-1}} \widehat{M} < \infty, \quad \text{and} \quad \lim_{\alpha \to (4b)^{-1}} \widehat{R} < \infty;$$

therefore,

$$\lim_{\alpha \to (4b)^{-1}} [\widehat{M} - M] < 0.$$

Note that

$$\lim_{\alpha \to (4b)^{-1}} \tilde{e}(w) = \infty \quad \text{and} \quad \lim_{\alpha \to (4b)^{-1}} \hat{e} < \infty;$$

therefore,

$$\lim_{\alpha \to (4b)^{-1}} [\hat{e} - e^*] < 0.$$

Because $\widehat{M}$, M, $\hat{e}$, and e^* are continuous in α, there exists $\underline{\alpha} > (4b)^{-1}$ such that if $\alpha < \underline{\alpha}$, then $\widehat{M} < M$ and $\hat{e} < e^*$. From Theorem 1,

$$\lim_{\alpha \to \infty} [\widehat{M} - M] > 0.$$

Because $\widehat{M}$ and M are continuous in α, there exists $\bar{\alpha} \geq \underline{\alpha}$ such that if $\alpha > \bar{\alpha}$, then $\widehat{M} > M$. □

PROOF OF THEOREM 3. First we show that if $c = 0$, $\widehat{M} > M$. Suppose $c = 0$. Let I denote the expected profit of the integrated system under demand distribution $\Phi(\xi)$. At $c = 0$,

$\widehat{M} = I = \int_0^\infty (a + \xi)^2/(4b)\,d\Phi(\xi)$. Because $I = \sum_{i=1}^N f_i \int_0^\infty (a + \xi)^2/(4b)\,d\Phi(\xi \mid v_i)$, I is an upperbound on the early-selling manufacturer's profit: $M \leq I$. The proof is by contradiction. Suppose $M = I$. For the early-selling manufacturer to achieve the integrated system expected profit and appropriate it entirely requires that the constraint in (16) binds for all i and

$$r_i(q_j) - t_j \leq 0 \tag{28}$$

for all i and j. Further, it must be that $t_i > 0$ and $q_i > 0$. (12) and (13) imply that for some $\{h, l\} \in \{1, \ldots, N\}$

$$\Phi(\xi \mid v_h) < \Phi(\xi \mid v_l) \tag{29}$$

for some ξ. Because (16) binds for $i = l$,

$$t_l = \int_0^\infty [a + \xi - \min(q_l, (a + \xi)/2)]$$
$$\cdot \min(q, (a + \xi)/2)/b\,d\Phi(\xi \mid v_l). \tag{30}$$

Note that

$$r_h(q_l) - t_l = \int_0^\infty [a + \xi - \min(q_l, (a + \xi)/2)]$$
$$\cdot \min(q, (a + \xi)/2)/b\,d\Phi(\xi \mid v_h) - t_l > 0, \tag{31}$$

where the inequality follows from (29) and (30). However, (31) contradicts (28), and so we conclude $M < I = \widehat{M}$. Because $\widehat{M}$ and M are decreasing in c and $\widehat{M}$ is continuous in c, there exists $\underline{c} > 0$ such that if $c < \underline{c}$, then $\widehat{M} > M$.

For the second part of the proof it is helpful to define $c_\mu = a + \int_0^\infty \xi\,d\Phi(\xi)$ and $c_N = a + \int_0^\infty \xi\,d\Phi(\xi \mid v_N)$. Note that if $c \geq c_\mu$, then $\widehat{M} = 0$. Suppose $c \in [c_\mu, c_N)$. Suppose the early-selling manufacturer offers a single price-quantity contract where the quantity is the optimal quantity for the integrated system when the demand distribution is $\Phi(\xi \mid v_N)$ and the price is the (strictly positive) system expected profit under this quantity and demand distribution. The retailer that has observed signal v_N will accept the contract. The early-selling manufacturer's expected profit under this contract is strictly positive, so the early-selling manufacturer's expected profit under the optimal menu of contracts must be strictly positive: $M > 0$. □

PROOF OF LEMMA 3. The contract design problem is given in (15)–(17). The constraints are

$$r_1(q_1) - t_1 \geq 0, \tag{32}$$

$$r_2(q_2) - t_2 \geq 0, \tag{33}$$

$$r_2(q_2) - t_2 \geq r_2(q_1) - t_1, \tag{34}$$

$$r_1(q_1) - t_1 \geq r_1(q_2) - t_2. \tag{35}$$

Consider the relaxed problem that includes (32) and (34) and excludes (33) and (35). Clearly, (32) must bind, as otherwise one can simultaneously increase t_1 and t_2 by the same amount without violating any constraint. Clearly, (34) must

bind, as otherwise one can increase t_2. The contract design problem simplifies to

$$\max_{\{q_1, q_2\}} \{\eta_1(q_1) + \eta_2(q_2)\},$$

where

$$\eta_1(q_1) = [\theta - f_2(1 - \theta)]\Gamma(L, q_1) + [1 - (1 + f_2)\theta]\Gamma(H, q_1) - f_1 c q_1$$

and

$$\eta_2(q_2) = f_2[(1 - \theta)\Gamma(L, q_2) + \theta\Gamma(H, q_2) - cq_2].$$

Without loss of generality, assume $q_1, q_2 \leq H/2$. $\eta_2(\cdot)$ is concave, and q_2^* is the unique solution to the first order condition. Note that

$$\frac{\partial^2}{\partial q_1^2} \eta_1(q_1) = \begin{cases} \dfrac{-2(\theta + \lambda - 1)}{b(2\theta - 1)} & \text{if } q_1 < \dfrac{L}{2} \\[2mm] \dfrac{-2Z}{b(2\theta - 1)} & \text{if } q_1 > \dfrac{L}{2} \end{cases}$$

and

$$\frac{\partial}{\partial q_1} \eta_1(L/2) = \frac{Z(H - L) - (\theta + \lambda - 1)bc}{b(2\theta - 1)}.$$

If $c < Z(H - L)/[b(\theta + \lambda - 1)]$, then $Z > 0$, $\eta_1(\cdot)$ is concave, and $(\partial/\partial q_1)\eta_1(L/2) > 0$; consequently, $q_1 = H/2 - (\theta + \lambda - 1) \cdot bc/(2Z)$ is the unique solution to the first order condition. If $c \geq Z(H - L)/[b(\theta + \lambda - 1)]$, then the optimal satisfies $q_1 \leq L/2$, and further

$$q_1 = \frac{[(\theta + \lambda - 1)(L - bc) + Z(H - L)]^+}{2(\theta + \lambda - 1)}$$

is optimal. To see this, first note that $(\partial/\partial q_1)\eta_1(L/2) \leq 0$. If $Z > 0$, then $\eta_1(\cdot)$ is concave and the result is immediate. If $Z < 0$, then $\eta_1(q_1)$ is convex on $q_1 \in [L/2, H/2]$; the result follows because $(\partial/\partial q_1)\eta_1(H/2) < 0$. With a little effort, one can show that this characterization of the optimal q_1 is equivalent to (19). Finally, it is straightforward to check that the solution to the relaxed problem satisfies (33) and (35). □

PROOF OF THEOREM 4. From Lemma 3,

$$R > 0 \Leftrightarrow c < \check{c}, \tag{36}$$

where $\check{c} = \{L + Z(H - L)/(\theta + \lambda - 1)\}/b$. It is straightforward to show that either of the following conditions:

$$\lambda \leq 2 - \sqrt{3} \quad \text{and} \quad c \geq \tilde{c}, \quad \text{or}$$
$$\lambda > 2 - \sqrt{3} \quad \text{and} \quad c \geq \check{c},$$

imply that $c \geq \check{c}$, which in turn implies that $R = 0$. So, for the remainder of the proof, we restrict attention to $\lambda > 2 - \sqrt{3}$ and $c \in [\tilde{c}, \check{c})$, noting that $\lambda > 2 - \sqrt{3}$ implies $\tilde{c} < \check{c}$. Define

$$\bar{\theta} = \{(3 + \lambda)H - (2 + \lambda)L - bc + \sqrt{Y}\}/[6(H - L)],$$
$$\tilde{\theta} = \{(3 + \lambda)H - (2 + \lambda)L - bc - \sqrt{Y}\}/[6(H - L)],$$

where $Y = [(3 + \lambda)H - (2 + \lambda)L - bc]^2 - 12(H - L)[H - \lambda L - (1 - \lambda)bc]$. Observe that $c < \check{c}$ implies $c < [L + (7\lambda - 3 - 2\sqrt{3}|1 - 2\lambda|)(H - L)]/b$, which in turn implies $Y > 0$. Note that

$$c < \check{c} \Leftrightarrow \theta \in (\tilde{\theta}, \bar{\theta}).$$

From (36), $\theta \geq \bar{\theta}$ implies $R = 0$. It is straightforward to show that R is continuous in θ with the sole exception that if $c = 0$ and $\lambda \geq 2\sqrt{3} - 3$, then R is discontinuous at $\theta = (3 + \lambda \pm \sqrt{\lambda^2 + 6\lambda - 3})/6$. Therefore, there exists $\underline{\theta} < \bar{\theta}$ such that (21) holds. It remains to show that

$$\bar{\theta} \in (\max(\lambda, 1 - \lambda), 1]. \tag{37}$$

Note that

$$\frac{\partial^2}{\partial c^2}\bar{\theta} = -2b^2(H - L)(1 - 2\lambda)^2 Y^{-3/2} \leq 0,$$

$$\left.\frac{\partial}{\partial c}\bar{\theta}\right|_{c=\tilde{c}} = \frac{b(-1 + 4\lambda - 7\lambda^2 - |1 - 4\lambda + \lambda^2|)}{6|1 - 4\lambda + \lambda^2|(H - L)} < 0,$$

so $\bar{\theta}$ is decreasing on $c \in [\tilde{c}, \check{c})$. Further,

$$\bar{\theta}|_{c=\tilde{c}} = 1.$$

It remains to show that

$$\bar{\theta}|_{c=\check{c}} \geq \max(\lambda, 1 - \lambda).$$

If $\lambda \geq 1/2$, then $\bar{\theta}|_{c=\check{c}} = \lambda$; if $\lambda < 1/2$, then $\bar{\theta}|_{c=\check{c}} = 1 - \lambda + (1 - 2\lambda)/\sqrt{3}$. Thus, we have established (37). □

References

Anand, K., R. Anupindi, Y. Bassok. 2001. Strategic inventories and procurement contracts. Working paper, University of Pennsylvania, Philadelphia, PA.

Biyalogorsky, E., O. Koenigsberg. 2004. Lead time, uncertainty, and channel decision making. Working paper, University of California, Davis, Davis, CA.

Cachon, G. P. 2003. Supply chain coordination with contracts. S. Graves, T. De Kok, eds. *The Handbook of Operations Research and Management Science: Supply Chain Management*. North Holland, Amsterdam, The Netherlands.

Cachon, G. P. 2004. The allocation of inventory risk in a supply chain: Push, pull and advanced-purchase discount contracts. *Management Sci.* **50**(2) 222–238.

Cachon, G. P., M. A. Lariviere. 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Sci.* **47**(5) 629–646.

Cachon, G. P., M. A. Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Sci.* **51**(1) 30–44.

Chen, F. 2003. Information sharing and supply chain coordination. S. Graves, T. De Kok, eds. *The Handbook of Operations Research and Management Science: Supply Chain Management*. North Holland, Amsterdam, The Netherlands.

Chu, W. 1992. Demand signalling and screening in channels of distribution. *Marketing Sci.* **11**(4) 327–347.

Corbett, C. J. 2001. Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Oper. Res.* **49**(4) 487–500.

Cvsa, V., S. M. Gilbert. 2002. Strategic commitment versus postponement in a two-tier supply chain. *Eur. J. Oper. Res.* **141**(3) 526–543.

Desai, P. S., K. Srinivasan. 1995. Demand signalling under unobservable effort in franchising: Linear and nonlinear price contracts. *Management Sci.* **41**(10) 1608–1623.

Emmons, H., S. M. Gilbert. 1998. Note: The role of returns policies in pricing and inventory decisions for catalogue goods. *Management Sci.* **44**(2) 276–283.

Erhun, F., P. Keskinocak, S. Tayur. 2001. Sequential procurement in a capacitated supply chain facing uncertain demand. Working paper, Carnegie Mellon University, Pittsburgh, PA.

Ferguson, M. E. 2003. When to commit in a serial supply chain with forecast updating. *Naval Res. Logist.* **50**(8) 917–936.

Ferguson, M. E., G. DeCroix, P. Zipkin. 2005. Commitment decisions with partial information updating. *Naval Res. Logist.* **52**(8) 780–795.

Fisher, M. L., J. H. Hammond, W. R. Obermeyer, A. Raman. 1994. Making supply meet demand in an uncertain world. *Harvard Bus. Rev.* **72**(3) 83–93.

Gilbert, S. M., V. Cvsa. 2003. Strategic supply chain contracting to stimulate downstream process innovation. *Eur. J. Oper. Res.* **150**(3) 617–639.

Ha, A. 2001. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Res. Logist.* **48**(1) 41–64.

Hirsch, J. 2001. K-Swiss profit slips, warns of more declines. *Los Angeles Times* (Apr 28) C1.

Lariviere, M. A. 2002. Inducing forecast revelation through restricted returns. Working paper, Northwestern University, Evanston, IL.

Lariviere, M. A., V. Padmanabhan. 1997. Slotting allowances and new production introductions. *Marketing Sci.* **16**(2) 112–128.

Lariviere, M. A., E. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing Service Oper. Management* **3**(4) 293–305.

Özer, Ö., O. Uncu, W. Wei. 2003. Analysis of a dual purchase contract. Working paper, Stanford University, Stanford, CA.

Padmanabhan, V., I. P. L. Png. 1997. Manufacturer's returns policies and retail competition. *Marketing Sci.* **16**(1) 81–94.

Spengler, J. 1950. Vertical integration and antitrust policy. *J. Political Econom.* **58** 347–352.

Taylor, T. A. 2002. Supply chain coordination under channel rebates with sales effort effects. *Management Sci.* **48**(8) 992–1007.

Taylor, T. A. 2005. Sale timing in a supply chain: When to sell to the retailer (unabridged). Working paper, Columbia University, New York.

Trenteseaux, D. 2004. Private conversation with Hasbro executive.

van Mieghem, J., M. Dada. 1999. Price versus production postponement: Capacity and competition. *Management Sci.* **45**(12) 1631–1649.

Woelbern, Matthew. 2001. Private conversation with Mattel executive.