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GAMOW-TELLER ELECTRON CAPTURE STRENGTH DISTRIBUTIONS IN STARS: UNBLOCKED IRON AND NICKEL ISOTOPES

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Abstract: Results of shell-model calculations of the Gamow-Teller strength distributions for the (n, p) or electron capture direction are presented for $^{54}\text{Fe} \rightarrow ^{54}\text{Mn}$, $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$, $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$, $^{60}\text{Fe} \rightarrow ^{60}\text{Mn}$, and $^{64}\text{Fe} \rightarrow ^{64}\text{Mn}$. In the cases of ^{56}Fe , ^{60}Fe , and ^{64}Fe calculations were performed on both the ground and first excited states. These and similar transitions are characterized by large amounts of Gamow-Teller strength at low excitation energy in the daughter nuclei. The question of quenching in the (n, p) direction remains open. It is expected that the late stages of presupernova stellar evolution (before neutron shell blocking) will depend on the character and distribution of Gamow-Teller strengths for electron capture transitions such as these.

1. Introduction

The basis for understanding electron capture during the late stages of stellar evolution in terms of the shell model was set down by Bethe *et al.*¹⁾ (1979; herein referred to as BBAL). These authors used the reaction $^{56}\text{Fe}(e^-, \nu)^{56}\text{Mn}$ with a zero-order shell model to point out for the first time that considerable Gamow-Teller strength should be present at low excitation energy, i.e. a few MeV, in the daughter nucleus ^{56}Mn , and thus the continuum electron capture rate should be quite fast in the presupernova environments where electron Fermi energies may be in excess of 10 MeV. Fuller, Fowler and Newman²⁻⁵⁾ (herein referred to as FFN I, II, III, IV), with a similar shell-model technique, made estimates of Gamow-Teller resonance strengths and energy centroids which they employed with the available experimentally determined energy levels [see ref.⁶⁾] and *ft* values to compute stellar nuclear reaction rates for many intermediate-mass nuclei over a broad range of stellar conditions.

The FFN calculations were done for both the "electron capture direction", including continuum electron capture and β^+ decay, and the "positron capture

direction", including both continuum positron capture and β^- decay. The generic "electron capture direction" will hereafter be referred to as the (n, p) direction or sometimes as the $T_< \rightarrow T_>$ transition. We consider the " (n, p) direction" as a high-energy spin-isospin transfer reaction of the (n, p) type which is mediated by a Gamow-Teller operator $\sigma\tau^+$, while $T_<$ and $T_>$ refer to the ground-state isospins of parent and daughter, respectively. Similarly the "positron capture direction" is hereafter referred to as the (p, n) direction, or sometimes as a $T_< \rightarrow T_>$, which is a Gamow-Teller transition of the type $\sigma\tau^-$. Note that in what follows we use the nuclear physics isospin convention (t_z (neutron) $\equiv +\frac{1}{2}$) which is opposite to FFN.

Both the BBAL and FFN calculations gave electron capture rates for $^{56}\text{Fe}(e^-, \nu)^{56}\text{Mn}$ which were up to 300 times faster than the non-shell-model electron capture estimates⁷⁾. There were substantial electron capture rate increases over the previous work for most of the fp-shell nuclei (unblocked nuclei) at high temperature and density in the FFN calculations. These increases in neutronization rate can translate into important changes in the initial presupernova stellar core as Weaver, Woosely and Fuller⁸⁾ have found. In particular, more rapid electron capture due to low-lying Gamow-Teller strength can help lead to collapse at a lower entropy per baryon and with a smaller initial iron core mass. Both of these effects increase the likelihood of the success of the core bounce supernova mechanism: the lower initial entropies imply fewer free nucleons and hence smaller neutronization rates via electron capture on free protons and thus a speedier onset of neutron shell blocking of heavy nuclei; the smaller initial cores imply smaller amounts of material for an emerging shock to photodisintegrate resulting in a possibly more viable shock. The reader is referred to Brown, Bethe and Baym⁹⁾, Burrows and Lattimer¹⁰⁾, Weaver, Woosley and Fuller⁸⁾, and Arnett¹¹⁾ for an overview of these points.

In any case there is considerable astrophysical interest in making shell-model estimates of the distribution and character of the Gamow-Teller strength in intermediate-mass nuclei. The intent of this paper is to exhibit the detailed shell-model calculations for a few unblocked cases representative of those that were considered in BBAL and FFN. Preliminary studies of this problem were reported in ref.¹²⁾. Strength distributions for the Gamow-Teller reaction mode have been computed with the Livermore VMC[vector method code, see ref.¹²⁾ and below] for $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$, $^{60}\text{Fe} \rightarrow ^{60}\text{Mn}$, and $^{64}\text{Fe} \rightarrow ^{64}\text{Mn}$ with the collective GT states built on both the ground and first excited states (see below). In addition, further calculations to test sensitivity to the size of the model space and the extent of ground-state correlations were performed for the ground states of $^{54}\text{Fe} \rightarrow ^{54}\text{Mn}$, and $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$. Overall the results, as will be shown, are in rough agreement with the zero-order estimates of FFN and BBAL, except for the now-recognized quenching phenomenon^{13,14)}. However, since there are no (n, p) measurements of Gamow-Teller strength comparable to those available for (p, n) , we can make no reliable estimate of the extent of the quenching. A more detailed discussion of this point is given in sect. 4.

2. Strength distribution calculations

As noted above the (n, p) direction transitions considered here are equivalent to $T_{<} \rightarrow T_{>}$ transitions since the parent nuclei are neutron-rich to begin with. Gamow-Teller transitions in this mode have not been extensively considered by nuclear physicists, since terrestrial or laboratory β^+ or electron capture reactions rarely proceed via these modes. (The classification of different types of Gamow-Teller decays of interest in astrophysics is discussed in some detail in FFN.) This transition direction is considerably simpler than the (p, n) direction because the Gamow-Teller operator produces only $T_{>}$ isospin states in the daughter nucleus. In contrast the (p, n) direction decay of a neutron-rich nucleus (via e.g. negatron decay) in principle produces both $T_{>}$ and $T_{<}$ states¹⁴). Of course, there is no Fermi transition in the (n, p) mode, as there is in the (p, n) mode.

The method used to compute the Gamow-Teller strength distributions requires first the calculation of a correlated ground-state or first-excited-state wave function for each parent nucleus. These states are designated the parent or $|P\rangle$ state vectors. The Gamow-Teller operators are taken as

$$GT^+ = \sum_{ij} \langle i | \sigma \tau^+ | j \rangle a_j^+ a_i, \quad (1)$$

where a_j^+ and a_i are the appropriate shell-model creation and annihilation operators, σ the Pauli spin matrix, and τ^+ the nucleon isospin-raising operator, where this isospin convention has $\tau_p = -\frac{1}{2}$, $\tau_n = +\frac{1}{2}$. The one-body Gamow-Teller operator is applied to the $|P\rangle$ state vector producing the Gamow-Teller collective state vector which we call $|CGT\rangle$. The equation for this is

$$GT^+ |P\rangle = |CGT\rangle. \quad (2)$$

The total Gamow-Teller strength, which we call S_+ , is found by summing over the complete set of the daughter nucleus eigenstates $|i\rangle$. This yields

$$S_+ = \sum_i \langle i | CGT \rangle^2 = \langle CGT | CGT \rangle, \quad (3)$$

which was evaluated in each case. The overall β sum rule [Ikeda *et al.*¹⁵)] must be satisfied,

$$S_- - S_+ = 3(N - Z). \quad (4)$$

In order for this rule to be satisfied, the $GT^\pm$ operator must be unconstrained with regard to the configurations it is allowed to generate in the daughter model space; otherwise an inconsistency could develop between the bases for the daughter (D) and parent (P) vector spaces. Such a circumstance could lead to an unphysical redistribution of the strength distribution [see refs. ^{16,17}]. In this work, the GT operators were always unconstrained and thus we could, and did, use the sum rule to check the results for our various model spaces for consistency with eq. (4).

There are three different possible $T_{<} \rightarrow T_{>}$ Gamow-Teller single-particle transitions for neutron-rich nuclei: no spin flip or zero spin transfer (ZST), forward spin

flip or negative spin transfer (NST), backward spin flip or positive spin transfer (PST). Examples of these three possibilities are illustrated in the zero-order shell-model diagrams for $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ in figs. 1a and b. Note that the f-orbit ZST and NST transitions, as well as the equivalent p-orbit transitions, cannot take place in the simple zero-order ground state shown in fig. 1a since they require some additional particle excitations, as shown in fig. 1b.

The strength distribution or strength function, as we shall call it, of the collective Gamow-Teller state vector $|\text{CGT}\rangle$ is found via the approximate diagonalization method based on the Lanczos algorithm¹⁶⁻¹⁸). As explained in refs.^{16,18}) this algorithm leads to a relatively small set of approximate eigenvectors, typically between 9 and 21 in the cases described in the next section, which subsumes both the complete Gamow-Teller strength and the complete eigenvector excitation spectrum. This strength function will give the envelope of the microscopically detailed result which one would get from an orthodox shell-model diagonalization, if this were feasible (see below for more details). The strength function will of course exhibit the dispersion of the GT strength amongst the eigenvectors of the model hamiltonian due to the fact that the $\sigma\tau^+$ operator does not commute with the hamiltonian. This is shown and discussed in the next section for the particular cases studied here. It is perhaps relevant to note that, were the Lanczos procedure carried to completion, one would then have done an orthodox shell-model diagonalization wherein every possible eigenstate with the proper spin, isospin, and parity is calculated along with the GT strength contained therein. Since this could involve the calculation of hundreds to many thousands of states we call this the microscopic limit, as already noted above. Such a procedure is neither practical or necessary for most applications and certainly not for the intended astrophysical application of interest here. In the present work we used the PMM hamiltonian¹⁹) which is essentially the Kallio-Koltveit interaction²⁰) converted to a Yukawa form factor. This same interaction has been used in several other GT strength-function calculations with fairly good results; see refs.^{16,18}).

The total strength in each approximate eigenstate can be found by taking its overlap with the CGT state vector,

$$S_i^+ = |\langle i|\text{CGT}\rangle|^2. \quad (5)$$

The energy width of the state is finite, since it is not an exact eigenstate of the hamiltonian. This width is given by the Lanczos algorithm along with the (mean) energy¹⁸). In the procedure adopted here each approximate eigenstate is associated with a gaussian distribution with the energy and width calculated as just described and an integrated area proportional to eq. (5). The sum of all the gaussians so constructed for all the approximate eigenstates then yields the total strength function.

We have already mentioned the spurious effects which follow from truncating the one-body operator, in this case the GT operator. We should also mention that one can achieve the same effects by simply truncating the basis available to any

Fig. 1a. The zero-order diagram for the NST (negative spin transfer) GT transition, $1f_{7/2} \rightarrow 1f_{5/2}$, in $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$. The value of the single-particle matrix element (squared) is $\frac{1}{2}$. The corresponding ZST (zero spin transfer) transition, $1f_{7/2} \rightarrow 1f_{7/2}$, for which the matrix element (squared) is $\frac{2}{3}$, is not possible in this case because all the $1f_{7/2}$ neutron states are blocked.

Fig. 1b. The first-order diagram for the same NST transition as shown in 1a. Now one of the several $2p_{2h}$ ground-state correlations, i.e. core-polarization states, is included which leads to coherent contributions from PST (positive spin transfer) transitions such as $1f_{5/2} \rightarrow 1f_{7/2}$ and ZST (zero spin transfer) transitions as well as the zero-order NST transition of 1a.

Fig. 2. Experimental and theoretical energy spectra for ^{54}Mn . The data for the experimental spectrum (leftmost column) is taken from ref. ⁶⁾. The 1p3h theoretical spectrum is based on the configuration $(\pi f_{7/2})^{-3}(\nu 2p_{3/2}, \nu 1f_{5/2}, \nu 2p_{1/2})$. The interaction is described in sect. 2 and other details are given in sect. 3. The 1⁺ levels shown in the 2p4h spectrum are not converged states (see sect. 2) and therefore include more 1⁺ states than shown. See the discussion in sect. 2. Also see figs. 3a, b and c.

operator, including for example the hamiltonian. This is of course a more drastic procedure and is subject to the same kind of difficulties in that it will produce a spurious distribution of the GT strength as well as not satisfying the sum rule, eq. (4) [refs. ^{16,18}]. Neither type of truncation was employed in this work. In this limited sense the calculations presented here are full-basis calculations.

3. Results

In this section we discuss our calculated results and compare them with experiment where appropriate and possible. We use the notation GT^+ to denote the $T_z \rightarrow T_z + 1$ transition as described above.

3.1. $^{54}\text{Fe} (GT^+) ^{54}\text{Mn}$

In fig. 2 we show two theoretical energy level spectra for the daughter nucleus ^{54}Mn as well as the experimental level spectrum up to excitation energies of 3 to 4 MeV. The simplest theoretical spectrum corresponds to the 0p2h “zero-order” model for the parent nucleus ^{54}Fe , i.e. $((\pi f_{7/2})^{-2}(\nu f_{7/2})^8)_{0^+}$. The isospin of this state is, of course, $T = 1$, and its dimension in the occupation number representation is 4 basis vectors (or Slater determinants). The application of the GT^+ operator to this state produces $((\pi f_{7/2})^{-3}(\nu f_{7/2})^8(\nu f_{5/2}))_{1^+}$ and has a total strength of $B(GT)_{\text{tot}} = 10.29$. (We use the customary GT units wherein the total free neutron Gamow–Teller decay strength is 3.) If we wish to include in our model space all the possible 1p3h states in order to get a realistic result for the dispersal of the GT strength by the hamiltonian we must add all the $2p_{3/2}$ and $2p_{1/2}$ particle states to the $1f_{5/2}$ particle states, which brings our total dimension to 68. We have gone to some pains to elucidate how the basis is expanded in this relatively simple case in order to illustrate not only how the unrestricted application of the GT operator must add new configurations to the parent basis but also how the configurations which are *not* produced by the GT operator but still belong to the specified configuration space, in this case 1p3h, will serve to disperse the GT strength. Because of the simple structure of this model it is possible to carry out a complete diagonalization and get a microscopic strength distribution. This is shown in fig. 3a. There are actually six 1^+ configurations in this simple model and, as we see, the prediction is that most of the strength is clumped around excitations of 2.0 and 3.5 MeV. This is even better seen in fig. 3b where we have artificially broadened the resolution of these six lines to widths of 0.4 MeV (from essentially zero). The purpose of this broadening is not only to illustrate the gross distribution but also to make it easier to compare with the results from our next calculation which is 1p1h more complicated, in the parent and the daughter, than our zero-order model. In this case the 1p3h parent (^{54}Fe) has dimensions of 328 basis vectors and the daughter (^{54}Mn) has dimensions of 2665. From the theoretical level spectrum of fig. 2 for this case we see that for the levels below an

excitation of ≈ 1 MeV the fit between experiment and theory is altered only a little from that of the simple 1p3h spectrum, perhaps slightly for the better. This indicates that the extra configuration admixtures affect the low-lying spectrum very little, exactly as expected if we are using a realistic effective interaction [see ref. ¹⁶) and the discussion in sect. 2].

We do not show any levels in the excitation range 1 to 1.9 MeV for the 2p4h spectrum because above 1 MeV we calculated only the GT strength function, which means we limited this range not only to 1^+ states but also only to such states as happened to carry significant GT strength. In this case $B(\text{GT})_{\text{tot}} = 9.12$, a 10% reduction from the zero-order case (see above). Shifting our attention to a comparison of fig. 3b, the broadened 1p3h strength function, and fig. 3c, the 2p4h strength function, we see that in fact the addition of the extra configurations has shifted the strength downwards by perhaps as much as an MeV and in addition has dispersed the upper peak into the excitation region of 8 MeV to 12 MeV. Also it is noteworthy that although the more complex theory still predicts a 1^+ level at 1.9 MeV it now has 130x the strength predicted for the same level in the 1p3h approximation. Although we pursued the GT strength-function calculation for only 10 iterations, meaning we got 10 approximate 1^+ state vectors subsuming the entire GT strength (see sect. 2), we can see from fig. 3c that for our purposes we have indeed extracted the information of critical interest to us here and it does signal a significant change in the GT strength distribution as a function of configuration mixing although the total strength has changed relatively little ($\approx 10\%$). If one were interested in more

Fig. 3a. Theoretical $S(\text{GT})$, Gamow-Teller strength function, in the $0p2h \rightarrow 1p3h$ model for the ground-state reaction $^{54}\text{Fe}(e, \nu)^{54}\text{Mn}$. The distribution is per MeV and is normalized to unity. See sect. 2 for details. In this model there are only six 1^+ states with very little strength in the first two states. The total strength is $B(\text{GT})_{\text{tot}} = 10.29$.

Fig. 3b. Same as (a) except the six 1^+ states have been artificially broadened to have widths of 0.4 MeV in order to facilitate comparison with (c); see below and fig. 2.

Fig. 3c. Theoretical $S(GT)$ in the $1p3h \rightarrow 2p4h$ model for the $^{54}\text{Fe}(e, \nu)^{54}\text{Mn}$ ground-state reaction. In this case there are many more 1^+ states, all subsumed in the three broad states shown in fig. 2. See sect. 3 for details and discussion. The total strength is $B(GT)_{\text{tot}} = 9.12$.

detailed information one could of course expend more computer time and get many more state vectors. Our experience indicates that the general form of fig. 3c would not change much however. We might mention that a calculation was made in the $2p4h$ approximation of the parent nucleus ^{54}Fe which turned out to have a dimension of 10 620 basis vectors. At the present time it is not possible to create the corresponding untruncated basis in ^{54}Mn in order to get the GT strength function.

One could ask what change in the total strength and its distribution would be produced by configuration mixing if we were to allow excitations of protons and

neutrons out of the fp shell and into the gd shell. We could not do that here for reasons of calculation size and the well-known difficulty of shell-model calculations which have model spaces covering two or more major shells [see ref. ¹⁸]. For a reaction like $^{54}\text{Fe} \rightarrow ^{54}\text{Mn}$ or $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ we can guess that gd-shell admixtures will make little difference to the total GT strength and its distribution, since the fp-shell strength is so large and dominant. However, for blocked nuclei, where the fp-shell strength is very weak, small admixtures of gd-shell configurations could greatly increase the small amount of GT strength available for electron capture in stars. In ref. ²¹) (Fuller, 1982) this effect is called configuration-mixing unblocking. Its high-temperature cousin is called thermal unblocking.

Recently Cooperstein and Wambach ²²) (1983) used an RPA approach to calculate the GT strength distribution for blocked nuclei at high temperature. The RPA calculation can accommodate both the fp and gd shells and shows more unblocking at high temperature than the simple model of ref. ²¹). In general for unblocked nuclei, where there is a relatively large amount of GT strength at low excitation energy, a shell-model approach is preferred because of its ability to treat the microscopic characteristics with reasonable fidelity. At the other bound, for blocked nuclei, the RPA approach can of course treat the much larger model spaces required for studying the much higher excitations where the GT strength is now concentrated, but with the usual approximations inherent in the RPA method.

3.2. $^{56}\text{Fe}(\text{GT}^+)^{56}\text{Mn}$

In figs. 4a and b, we show the calculated strength functions for GT^+ for the two parent states, the ground state (0^+) and first excited state (2^+) of ^{56}Fe . The model space for the parent states was $(1f_{7/2})^{-2}(2p_{1/2}, 2p_{3/2}, 1f_{5/2})^2$ with a dimension of 200; see the discussion above for $^{54}\text{Fe}(\text{GT}^+)^{54}\text{Mn}$. The daughter nucleus, produced by the action of the GT^+ one-body operator, will then have up to $3p3h$ and has a dimension of 1038 basis vectors. The spill-over into negative energies of the strength distributions shown in figs. 4a and b, is an artifact of the approximation of the overall strength function as a sum of (in this case) 15 gaussian distributions corresponding to the 15 approximate eigenvectors produced in the Lanczos procedure; see sect. 2 and ref. ¹⁸). The total strengths $B(\text{GT})_{\text{tot}}$ for each of the two parent states are

	Ground state, 0^+	First excited state, 2^+
$B(\text{GT})_{\text{tot}}$	10.003	9.546
		3.930 ($J=3$)
		4.313 ($J=2$)
		1.304 ($J=1$)

where we have also shown the division of the total strength into the three possible final spin states corresponding to the GT^+ mode beginning with the 2^+ state. The

Fig. 4. (a) Theoretical $S(\text{GT})$ in the $2p2h \rightarrow 3p3h$ model for $^{56}\text{Fe}(\text{g.s.})(e, \nu)^{56}\text{Mn}$ reaction. The normalization and distribution units are as in figs. 3a, b and c. The spike at 0.24 MeV is a single state which carries only 2.4% of the total strength. See sect. 3. The centroid of the strength is at 2.8 MeV. The total strength is $B(\text{GT})_{\text{tot}} = 10.00$. (b) Theoretical $S(\text{GT})$ in the $2p2h \rightarrow 3p3h$ model for $^{56}\text{Fe}(1\text{st } 2^+)(e, \nu)^{56}\text{Mn}$ reaction. See the caption for fig. 4a and sect. 3. The centroid of the strength is at 4.3 MeV. The total strength is $B(\text{GT})_{\text{tot}} = 9.55$.

spike shown in fig. 4a corresponds to the excitation of a 1^+ state at a calculated energy of 0.25 MeV. The GT strength of the state is relatively weak (0.026), however it is separated by ~ 1.5 MeV from the next 1^+ state which carries almost ten times as much strength. In such a situation the convergence of the lowest state, in the Lanczos algorithm, will be much more rapid than the higher-lying state leading to the isolated spike of fig. 4a. In comparing fig. 4a and fig. 4b we note the following: the centroid in fig. 4a is at 2.8 MeV whereas for fig. 4b it is at 4.3 MeV, a difference of 1.5 MeV. The calculated excitation of the first 2^+ state is 1.01 MeV (experiment is 0.847 MeV). Furthermore the widths for the distributions shown in figs. 4a and b, are, roughly speaking, the same: ≈ 4.5 MeV. Thus, in a crude fashion the GT distributions built on each state are alike except for the displacement in excitation, which is somewhat larger than the excitation energy difference. However this situation is very different as the neutron excess increases, as we show below.

3.3. $^{56}\text{Ni}(\text{GT}^+)^{56}\text{Co}$

The interesting case of the super-retarded β^+ decay of ^{56}Ni has been the subject of speculation for some years now. The suggestion of Goode and Zamick²³⁾ was that the main part of the state at 1.72 MeV in ^{56}Co , the only 1^+ level energetically accessible from the ground state of ^{56}Ni , was of a 2p2h character and therefore inaccessible to the one-body transition from the (mainly) 0p0h parent ^{56}Ni . In their calculation they diagonalized in a 1p1h plus 2p2h space for ^{56}Co and a 0p0h plus 2p2h space for ^{56}Ni and they did find that the lowest 1^+ level was largely of a 2p2h character with a $\log ft$ of 5.8. This is to be compared to the experimental value of 4.4 and the single-particle value of 2.46. However the exclusion of the 3p3h configurations in ^{56}Co will block the major part of the GT transition strength due to the 2p2h correlations in the parent ^{56}Ni and furthermore lead to a violation of the β -decay sum rule (see sect. 2). Therefore we included the necessary 3p3h correlations generated in ^{56}Co by the application of the GT^+ operator to the parent state calculated in the 2p2h approximation for ^{56}Ni , i.e. $(1f_{7/2})^{-2}(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^2$. We also did the $(0p0h) \rightarrow (1p1h)$ calculation and show the calculated level spectra for both models as well as the experimental level spectrum for ^{56}Co in fig. 5. As can be seen in this figure the unique 1^+ level in the 1p1h model, at 3.6 MeV, is in very bad agreement with the experimental level energy of 1.72 MeV, not to mention the experimental 1^+ doublet at 2.7 MeV. On the other hand the levels with higher spins at energies < 1.5 MeV agree fairly well with experiment, where data is available. The 3p3h calculation was of course much larger than the 1p1h calculation (12612 basis vectors versus 12) and we accordingly carried out a more limited calculation for spins higher than 1. Nonetheless as in the case of ^{54}Mn we find not too much change for the spins > 1 but the 1^+ levels (we show only the three lowest of these) represent a radical change, agreeing much better with experiment than the single and obviously highly inaccurate description for these states available in the 1p1h picture. There

Fig. 5. Experimental and theoretical energy spectra for ^{56}Co . The experimental data is from ref. ⁶). The 1p1h spectrum is based on the configuration $(\pi 1f_{7/2})^{-1}(\nu 2p_{3/2}, \nu 1f_{5/2}, \nu 2p_{1/2})$. See sect. 3 for other details. Note that the 1p1h spectrum has only one 1^+ state at 3.59 MeV. The 3p3h spectrum shows only three 1^+ states. The third 1^+ state (3.8 MeV) is actually a broad state containing many more 1^+ states than just one. A similar situation obtained for ^{54}Mn (see figs. 2 and 3). See fig. 6 and the discussion in sect. 2.

Fig. 6. Theoretical $S(GT)$ for the $^{56}\text{Ni}(e, \nu)^{56}\text{Co}$ ground-state reaction in the $2p2h \rightarrow 3p3h$ model. See sect. 2 and fig. 5. The normalization and units are as in figs. 2a, b and c. It is noteworthy that the zero-order $1p1h$ model had only one 1^+ state in which all of the strength was concentrated whereas the $3p3h$ strength is distributed in a highly nonuniform way from 2 MeV to 22 MeV. The total strength is $B(GT)_{\text{tot}} = 2.46$.

is also a radical change in the GT strength distribution accompanied however by only a 0.3% change in the *total* strength. We shall reserve comment on this last point until later. Now we note that only 0.20 of the strength is in the lowest-lying 1^+ level at 2.20 MeV corresponding to a $\log ft$ of 3.15 whereas over half the strength is localized in the 1^+ level at 2.89 MeV. As shown in fig. 6 the rest of the strength is mainly dispersed in a band of a few MeV beginning at an excitation of 3.6 MeV except for an interesting albeit inaccessible peak at an excitation energy of 22 MeV with a strength corresponding to a $\log ft$ of 3.75. Although we have not succeeded in matching either the experimental spectrum or the observed $\log ft$ values very well it is clear that the expansion of the model space by $1p1h$ has done much to improve

things. Furthermore it is reasonable to expect that a further extension of the model space to include 2p2h correlations (in the parent state vector $|P\rangle$) would improve things even more, although computer limitations do not as yet allow such a calculation.

As to the essentially unaltered GT strength alluded to earlier; what has happened here is that the blocking produced by the excitation of particles into the previously unoccupied orbits is now replaced by rather strong *hole* transitions in the $f_{7/2}$ shell. The role of the p-orbits in this effect is also important since they contribute nothing to the blocking of the very strong $f_{7/2} \rightarrow f_{5/2}$ transitions but do absorb a significant fraction of the 1p1h and 2p2h excitations from the $f_{7/2}$ shell.

Because ^{56}Ni is doubly magic it provides one of the few fast (n, p) type transitions in the fp shell which can be terrestrially observed. The strong, albeit hindered, $\log ft = 4.38$ transition is fast but unfortunately contains only a small fraction of the total GT strength which is, of course, the reason for the hindrance. Thus it would be difficult, as yet, to use the ground-state transition to estimate the quenching factor since the hindrance factor has to be accurately known first.

In passing we might mention that there is as yet no definitive experimental estimate of the degree of GT quenching in the (n, p) direction. The (t, ^3He) investigation of Ajzenberg-Selove *et al.*²⁴⁾ is the best effort in this area and the result here is consistent with the quenching observed in the (p, n) direction, roughly 40%; see ref.¹⁴⁾. The astrophysical rates of FFN [ref.⁵⁾] recommend this value for the quenching when the stellar electron Fermi energy is large enough so that the main shell-model strength dominates.

3.4. $^{60}\text{Fe}(\text{GT}^+)^{60}\text{Mn}$

In figs. 7a and b, we show the calculated strength functions for GT^+ for the two parent states, the ground state (0^+) and first excited state (2^+) of ^{60}Fe . Qualitatively this is somewhat like the results for ^{56}Fe shown in figs. 4a and b, especially for the strength functions based on the 2^+ states. The shell model for the parent, ^{60}Fe , was $(1f_{7/2})^{-2}(2p_{3/2})^4(2p_{1/2}, 1f_{5/2})^2$; see the discussion above for $^{54}\text{Fe}(\text{p}, \text{n})^{54}\text{Mn}$. Although this is a rather small basis it still served rather well in reproducing the first few low-lying levels in ^{60}Fe , as we show below:

Excitation energy [MeV]		Spin-parity
theory	experiment	theory and experiment
0.0	0.0	0^+
1.1	0.8	2^+
2.2	2.0	0^+
2.4	2.3	2^+

The size of the model space for the daughter nucleus ^{60}Mn was also small, 292 basis vectors. Unfortunately aside from the spin-parity assignment of 3^+ for this nucleus,

Fig. 7. (a) Theoretical $S(GT)$ for the $^{60}\text{Fe}(\text{g.s.})(e, \nu)^{60}\text{Mn}$ reaction. Units and normalization are as in figs. 3a, b and c. The peak at 0.5 MeV contains 36% of the total strength which is $B(GT)_{\text{tot}} = 9.47$. See sect. 3 for further discussion. (b) Theoretical $S(GT)$ for the $^{60}\text{Fe}(1\text{st } 2^+)(e, \nu)^{60}\text{Mn}$ reaction. See sect. 3 for further discussion. The total strength is $B(GT)_{\text{tot}} = 8.19$.

which our calculation reproduced, nothing regarding the spectrum is yet known. The total GT strength distribution for each of the two parent states is shown below:

	Ground state, 0^+	First excited state, 2^+
$B(\text{GT})_{\text{tot}}$	9.47	8.19
		3.63 ($J = 3$)
		3.56 ($J = 2$)
		1.00 ($J = 1$)

We note that there is a close correspondence between these results and the same results for ^{56}Fe . However it is no longer true that there is a qualitative resemblance between the strength function built on the ground state and that built on the first excited state. Indeed the resemblance between the strength functions built on the first excited states for ^{56}Fe and ^{60}Fe is very close but the ground-state strength functions are qualitatively very different. Particularly note the 1^+ peak at 0.5 MeV in ^{60}Mn . This peak carries 36% of the total strength. In a case like this one could not say that the GT distribution for the excited state can be even qualitatively inferred from that of the ground state by simply displacing the ground state strength distribution by the excitation energy. In fact the most “standard” distribution would appear to be the one pertaining to the excited states. It might well be that in fact the excited state distributions, or some average of several of them in any particular nucleus, would be the most appropriate ones to use in making *average* predictions e.g. for the purpose of doing astrophysical calculations. However in the averages performed in FFN [ref. ³)] the results of the present shell-model calculations gave astrophysical rates very similar to those using the FFN zero-order shell-model procedure, a result of the Fermi energy for the electrons being larger than the excitation energy differences considered here.

3.5. $^{64}\text{Fe}(\text{GT}^+)^{64}\text{Mn}$

In figs. 8a and b, we show the calculated strength functions for GT^+ for the two parent states, the ground state (0^+) and first excited state (2^+) of ^{64}Fe . In this case it is clear there is very little similarity with any of our previously calculated GT strength distributions. The parent model space was $(1f_{7/2})^{-2}(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^{-2}$ which, like ^{56}Fe (see above), had a dimension of 200. However unlike ^{56}Fe the dimension of the daughter ^{64}Mn was 68, corresponding to the 4-quasiparticle model space $(1f_{7/2})^{-3}(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^{-1}$. The dimension of the daughter space for ^{56}Fe was 1038, corresponding to a much larger 6-quasiparticle configuration. The strong Pauli blocking of the valence neutrons accounts for the very different character of the strength distributions in these two cases. Particularly notable is the fact that 93% of the GT strength is in the ground state of Mn [ref. ⁶⁴]]. Accordingly we show, in fig. 8a, only the excited state strength distribution. On the other hand, the distribution starting from the first excited state of ^{64}Fe , with spin-parity of 2^+ , is

Fig. 8. (a) Theoretical $S(GT)$ for $^{64}\text{Fe}(\text{g.s.})(e, \nu)^{64}\text{Mn}$ reaction. Units and normalization are as in figs. 3a, b and c. In this case we have suppressed the peak in the lowest-lying 1^+ state which absorbs 93% of the GT strength; $B(GT)_{\text{tot}} = 1.16$. In our model this is the ground state of ^{64}Mn . See sect. 3 for further discussion. (b) Theoretical $S(GT)$ for $^{64}\text{Fe}(1\text{st } 2^+)(e, \nu)^{64}\text{Mn}$ reaction. See sect. 3 for other details. The total strength is $B(GT)_{\text{tot}} = 0.9406$.

more dispersed, as shown in fig. 8b and in consonance with the observations concerning similar phenomena in ^{60}Fe . The total GT strength distributions for the different spins and isospins are shown below:

	Ground state, 0^+	First excited state, 2^+
$B(\text{GT})_{\text{tot}}$	1.16	0.9406
		0.533 ($J = 3$)
		0.195 ($J = 2$)
		0.213 ($J = 1$)

Again this demonstrates the greater dispersion with higher excitation.

4. Summary and conclusions

The results presented here provide detailed support for the original suggestions based on zero-order nuclear models ¹⁻⁵) that in general the largest fraction of the (n, p) or electron capture Gamow-Teller strength should be concentrated in the first few MeV of excitation in the daughter nucleus, providing the initial or parent nucleus is in its ground state or close to it. The implications of this finding for understanding the process of stellar collapse as it occurs in supernovas are described in sect. 1. On the other hand, our results show that even the qualitative features of the strength function are profoundly affected by the onset of Pauli blocking, the effect being not only such as to reduce the *total* strength but also to produce a much more contracted distribution, shifting the distribution either higher or lower, depending on whether the parent state is the ground state or the first excited state, using the case of ^{54}Fe as a guiding example. In fact our results show that the higher the excitation energy of the parent state the more dispersed the strength distribution becomes up to the point where the width of the distribution becomes about 4 MeV. Beyond this the character of the distribution should not change much with excitation energy, though ref. ²²) and the comments in sect. 3 following the $^{54}\text{Fe} \rightarrow ^{54}\text{Mn}$ discussion should also be taken into account. Good examples of such "standard" distributions are those based on the first excited states of ^{56}Fe and ^{60}Fe . In such cases one would expect the centroid of the distribution to shift with the excitation energy of the parent, in first order. Of course these are at best rough characterizations but they are still useful in situations where more detailed information would be essentially impossible to acquire.

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