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Author
Korneev, V.A.

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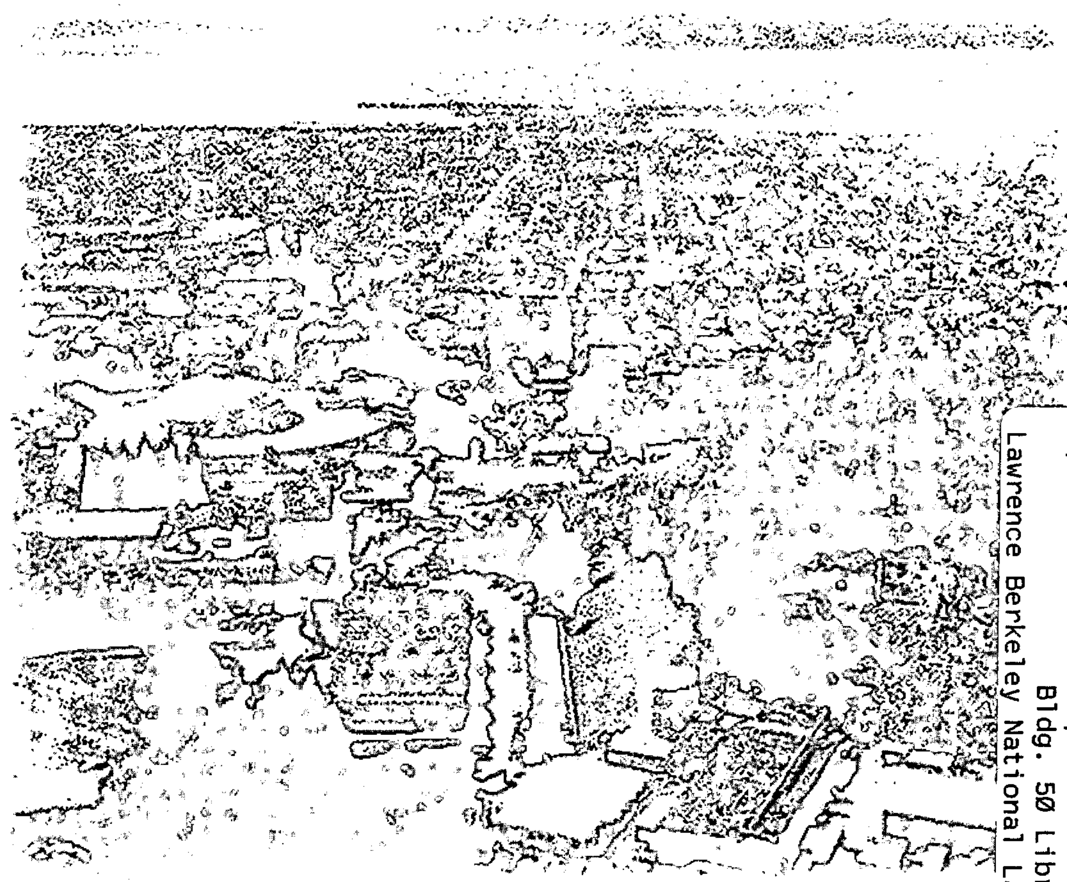

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Time-domain solutions for nonlinear elastic 1-D plane wave propagation

Valeri A Korneev

Earth Sciences Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley , California 94720

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SUMMARY

Time-domain solutions are obtained for 1-D nonlinear elastic wave propagation problems using a five-constant nonlinear theory. The assumption of weak attenuation was used throughout the development. The strongest nonlinear effects are obtained for the case of single compressional wave propagation, for single compressional or shear wave propagation through a longitudinally pre-stressed elastic material, and for shear wave propagation in a shear pre-stressed elastic material. Estimates of the size of these effects indicate that nonlinear phenomena are likely to be observable in real seismic data. The results may be useful for the measurement of nonlinear constants in elastic materials, for explaining the frequency content of seismograms, and for monitoring strain fields in the earth's crust.

Key words: nonlinear, elastic, wave propagation.

1. Introduction

Fundamentals of the nonlinear theory of elasticity were developed by Novozhilov (1948) and Murnaghan (1951). This theory in its first approximation for the isotropic case introduces three (l, m, n) nonlinear elastic moduli in addition to the Lame constants λ and μ to describe the elastic properties of a material. In the literature this theory is usually referred to as the five-constant theory of Murnaghan. Later Landau and Lifshitz (1954) proposed an elegant method of deriving the equations of motion for nonlinear elastic media and qualitatively described the basic wave phenomena, such as the generation of multiple frequencies by a single harmonic wave and the generation of combination frequencies due to the interaction of two or more harmonic waves. They developed their approach using the fact that the internal energy of the deformed elastic body is independent of the choice of the coordinate system. This is possible if the internal energy is a function only of the invariants of the strain tensor. Since the strains are small, the internal energy can be expanded in a series about the undeformed state, where the expansion begins with the quadratic terms (undeformed state considered as equilibrium). The coefficients of the quadratic terms specify the second-order moduli (Lame constants), the coefficients of the cubic terms specify three third-order constants and so on. Retaining terms other than the quadratic ones in the internal energy results in nonlinear equations for the elastic motion.

In most applications of the nonlinear theory only the cubic terms in the internal energy have been retained (first order nonlinearity). A first order nonlinear approximation was used by Gol'dberg (1960) to obtain solutions for 1-D nonlinear propagation problems for harmonic primary signals. Since resonant excitation of the nonlinear field sometimes takes place, it was important to involve intrinsic attenuation in this problem (Polyakova, 1964). The relaxation mechanism of attenuation was introduced into the problem by McCall (1993). Multiple frequency generation has been detected in a number of experimental investigations (Breazeale and Thompson, 1963; Moriamez et al, 1968; Thery et al, 1969; Ermilin et al, 1970; Zarembo and Krasil'nikov, 1971; Carr, 1966, 1968; Gedroits and Krasil'nikov, 1962; Johnson et al, 1993). Nonlinear interaction of elastic waves may take place under certain experimental conditions (Jones and Kobett, 1963; Taylor and Rollings, 1964; Zarembo and Krasil'nikov, 1971). This phenomena was observed in metals and crystals (Rollings et al, 1964; Zarembo and Krasil'nikov, 1971) and also in rocks (Johnson et al, 1987; Johnson and Shankland, 1989) .

In previous investigations (Gol'dberg,1960;Polyakova,1964;McCall,1993) the nonlinear elastic 1-D propagation problem was treated using the five-constant nonlinear theory of Murnaghan for the case of harmonic primary waves. Providing the relevant laboratory conditions for verifying these results encounters difficulties in eliminating the reflection and scattering phenomena caused by the edges and sides of the sample. The lack of purely longitudinal or transverse sources also causes difficulties when separating the nonlinear effects. The use of finite duration signals (pulses) could alleviate some of these difficulties by providing better conditions for avoiding the interference that causes problems in detecting the nonlinear waves. In this paper we present the solution of the 1-D propagation problem in the time domain. The results are illustrated with a specific application to the case where the primary wave is the first derivative of the gaussian curve. It is shown that for such a signal, the nonlinear component of the field can be detected in the frequency domain by isolating the real part of the spectrum.

2. Basic equations

Following Gol'dberg (1960) and Polyakova (1964), we consider the nonlinear equation of motion for the displacement field **u** for an isotropic elastic material with viscous dissipation (Landau and Lifshitz, 1954) in the form

$$\rho\frac{\partial^2 u_i}{\partial t^2} - \mu\frac{\partial^2 u_i}{\partial x_k^2} - (\lambda+\mu)\frac{\partial^2 u_k}{\partial x_k \partial x_i} - \frac{\partial}{\partial t}\left[\eta\frac{\partial^2 u_i}{\partial x_k \partial x_k} + (\frac{\eta}{3}+\zeta)\frac{\partial^2 u_k}{\partial x_k \partial x_i}\right] = F_i \quad (1)$$

where F_i contains all the nonlinear terms which are second order in smallness with respect to the components of the strain tensor

$$F_i = \left[\mu + \frac{n}{4}\right]\left[\frac{\partial^2 u_s}{\partial x_k^2}\frac{\partial u_s}{\partial x_i} + \frac{\partial^2 u_s}{\partial x_k^2}\frac{\partial u_i}{\partial x_s} + 2\frac{\partial^2 u_i}{\partial x_s \partial x_k}\frac{\partial u_s}{\partial x_k}\right]$$

$$+ \left[m - \frac{n}{4}\right]\left[\frac{\partial^2 u_s}{\partial x_i \partial x_k}\frac{\partial u_s}{\partial x_k} + \frac{\partial^2 u_k}{\partial x_s \partial x_k}\frac{\partial u_i}{\partial x_s}\right] + \left[\lambda + m - \frac{n}{2}\right]\frac{\partial^2 u_i}{\partial x_k^2}\frac{\partial u_s}{\partial x_s}$$

$$+ \left[m - \frac{n}{4}\right]\left[\frac{\partial^2 u_k}{\partial x_s \partial x_k}\frac{\partial u_s}{\partial x_i} + \frac{\partial^2 u_s}{\partial x_i \partial x_k}\frac{\partial u_k}{\partial x_s}\right] + \left[2l - m + \frac{n}{2}\right]\frac{\partial^2 u_k}{\partial x_i \partial x_k}\frac{\partial u_s}{\partial x_s} \quad (2)$$

The terms x_1, x_2, and x_3 are rectangular coordinates, and subscripts appearing twice in a single term indicate summation over the values 1,2,3. The parameter ρ in (1) is the density of the material, and η and ζ are the coefficients of the shear and volume viscosities, respectively. Viscous terms are omitted in (2) because they are assumed to be very small. It is obvious that the nonlinearity does not vanish even if coefficients the l,m,n are all equal to zero. This is the result of nonlinearity in the strain tensor itself. Nevertheless, as measurements show (Jones and Kobett, 1963), the nonlinear constants are several orders of magnitude larger than the Lame constants, and so the real elastic nonlinearity vanishes together with the nonlinear constants.

In this paper we consider only plane elastic wave fields propagating along the x axis with a displacement of the form

$$\mathbf{u}(x,t) = u_x(x,t)\hat{\mathbf{x}} + u_y(x,t)\hat{\mathbf{y}} + u_z(x,t)\hat{\mathbf{z}} \quad (3)$$

From (1) we obtain the system of equations (Polyakova, 1964)

$$\frac{\partial^2 u_x}{\partial t^2} - v_p^2 \frac{\partial^2 u_x}{\partial x^2} - \chi_p \frac{\partial^3 u_x}{\partial t \, \partial x^2} = v_p^2 \beta_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x}{\partial x}\right]^2 + v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\left[\frac{\partial u_y}{\partial x}\right]^2 + \left[\frac{\partial u_z}{\partial x}\right]^2\right] \tag{4}$$

$$\frac{\partial^2 u_y}{\partial t^2} - v_s^2 \frac{\partial^2 u_y}{\partial x^2} - \chi_s \frac{\partial^3 u_y}{\partial t \, \partial x^2} = 2 v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x}{\partial x}\frac{\partial u_y}{\partial x}\right] \tag{5}$$

$$\frac{\partial^2 u_z}{\partial t^2} - v_s^2 \frac{\partial^2 u_z}{\partial x^2} - \chi_s \frac{\partial^3 u_z}{\partial t \, \partial x^2} = 2 v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x}{\partial x}\frac{\partial u_z}{\partial x}\right] \tag{6}$$

where the following notation has been used

$$v_p^2 = \frac{\lambda + 2\mu}{\rho}, \quad v_s^2 = \frac{\mu}{\rho}, \quad \chi_p = \frac{1}{\rho}\left[\frac{4}{3}\eta + \zeta\right], \quad \chi_s = \frac{\eta}{\rho}$$

$$\beta_0 = \frac{3(\lambda + 2\mu) + 2(2m + l)}{2(\lambda + 2\mu)}, \qquad \gamma_0 = \frac{\lambda + 2\mu + m}{2(\lambda + 2\mu)}$$

Because of the assumption about the smallness of the strain components, the nonlinear right hand sides of eqs (4)-(6) are small and we can seek solutions of these equations in the form

$$\mathbf{u}(x,t) = \mathbf{u}^0(x,t) + \mathbf{u}^1(x,t) \tag{7}$$

where the field $\mathbf{u}^0$ represents the primary wave, satisfying reduced versions of the system (4)-(6). The field $\mathbf{u}^1$ therefore represents a small correction to the primary wave which is due to the presence of the nonlinear components in the equations. Thus

$$\left|\mathbf{u}^1\right| \ll \left|\mathbf{u}^0\right| \tag{8}$$

Throughout this paper we will refer to the field $\mathbf{u}^1$ as the nonlinear field.

Substituting (7) into the system (4)-(6) and neglecting the smallest components we obtain the system

$$\frac{\partial^2 u_x^1}{\partial t^2} - v_p^2 \frac{\partial^2 u_x^1}{\partial x^2} - \chi_p \frac{\partial^3 u_x^1}{\partial t \, \partial x^2} = v_p^2 \beta_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x^0}{\partial x}\right]^2 + v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\left[\frac{\partial u_y^0}{\partial x}\right]^2 + \left[\frac{\partial u_z^0}{\partial x}\right]^2\right] \tag{9}$$

$$\frac{\partial^2 u_y^1}{\partial t^2} - v_s^2 \frac{\partial^2 u_y^1}{\partial x^2} - \chi_s \frac{\partial^3 u_y^1}{\partial t \, \partial x^2} = 2 v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x^0}{\partial x}\frac{\partial u_y^0}{\partial x}\right] \tag{10}$$

$$\frac{\partial^2 u_z^1}{\partial t^2} - v_s^2 \frac{\partial^2 u_z^1}{\partial x^2} - \chi_s \frac{\partial^3 u_z^1}{\partial t \, \partial x^2} = 2 v_p^2 \gamma_0 \frac{\partial}{\partial x}\left[\frac{\partial u_x^0}{\partial x}\frac{\partial u_z^0}{\partial x}\right] \tag{11}$$

of linear differential equations for the determination of $\mathbf{u}^1$. These equations turn out to be independent of one another in the approximation considered. The general solution of any of these equations is a sum of a particular solution defined by the right hand side and an arbitrary combination of solutions of the reduced equation. The boundary conditions chosen for the problem must be sufficient to specify two coefficients, and this gives uniqueness to the solution. Because solutions of the reduced equations are trivial and there are different boundary conditions that could be imposed in various possible applications, we restrict ourselves here to derivation of the particular solutions. This form of the solutions is also favored by the absence of purely compressive or shear sources in practice.

From eqs (9)-(11) it follows that the compressional nonlinear wave appears in the medium when any of compressional or shear waves is taken as the primary wave. The shear nonlinear wave is generated only in the case where both types of primary waves are present. The symmetry of equations relative to polarization of the primary shear wave allows one to study just one type of polarization, with the results for the other case being obvious.

The eqs (9)-(11) contain products of derivatives of the primary field $\mathbf{u}^0$, and therefore this field must eventually be specified in the form of a real function. On the other hand, these equations are linear with respective to the unknown "nonlinear" field $\mathbf{u}^1$, and thus we can use the complex Fourier transform method to solve them. In the next sections this method will be applied to obtain time-domain solutions of eqs (9)-(11). In all cases it will be assumed that the attenuation is small.

3. Solutions for primary waves

Primary waves are the solutions $u^0(x,t)$ of correspondent reduced eq. (4)-(6), all of which may be put in general form

$$\frac{\partial^2 u^0}{\partial t^2} - v^2 \frac{\partial^2 u^0}{\partial x^2} - \chi \frac{\partial^3 u^0}{\partial t\, \partial x^2} = 0 \tag{12}$$

Assuming relative smallness of the third component of (12) compared to other two components (weak attenuation) we will seek a solution in the form

$$u^0(t,x) = u_0\left(t - \frac{x}{v}\right) e^{-\alpha x} \equiv u_0(\tau) e^{-\alpha x}, \qquad \tau = t - \frac{x}{v} \tag{13}$$

where α is effective attenuation coefficient. Substituting (13) in (12) we get the equation

$$R(\tau,x) \equiv -e^{-\alpha x}\left[v^2\alpha^2 u_0(\tau) + \chi\alpha^2\frac{\partial u_0(\tau)}{\partial\tau} + \chi\alpha\frac{2}{v}\frac{\partial^2 u_0(\tau)}{\partial\tau^2} + \right.$$

$$\left.\alpha 2v\frac{\partial u_0(\tau)}{\partial\tau} + \frac{\chi}{v^2}\frac{\partial^3 u_0(\tau)}{\partial\tau^3}\right] = 0 \qquad (14)$$

Since α is a constant the residual function $R(\tau,x)$ can't stay equal to zero for arbitrary waveform $u_0(\tau)$. However, approximate character of the whole approach and condition (8) require just keeping function (14) being much smaller than correspondent nonlinear left hand sides $N(\tau,x)$ of (5)-(11) in absolute values. We assume that spectrum $S(\omega)$ of the function $u_0(\tau)$ has a limited frequency band, and for each single frequency from that band the usual condition of the small attenuation

$$\alpha \ll \frac{\omega}{v} \qquad (15)$$

is satisfied. Condition (15) means that amplitude difference between two consequent cycles is small compare to the amplitude itself, and will be used throughout the whole paper. When condition (15) applied to (14) the three first terms vanish

$$R(\tau,x) = -e^{-\alpha x}\left[\alpha_p 2v\frac{\partial u_0(\tau)}{\partial\tau} + \frac{\chi}{v^2}\frac{\partial^3 u_0(\tau)}{\partial\tau^3}\right] \qquad (16)$$

The best estimation of α may be found as a value which delivers a minimum to the function

$$\delta(\alpha) = \frac{\int_0^\infty R^2(\tau,x)\,d\tau}{e^{-2\alpha x}\int_0^\infty\left[\frac{\partial^2 u_0(\tau)}{\partial\tau^2}\right]^2 d\tau} \qquad (17)$$

This problem has the least square solution

$$\alpha = \frac{\chi}{2v^3}\frac{\int_0^\infty\left|S(\omega)\right|^2\omega^4\,d\omega}{\int_0^\infty\left|S(\omega)\right|^2\omega^2\,d\omega} \qquad (18)$$

which is proportional to viscosity factor χ . The narrower the frequency band around the central frequency ω_0 the closer becomes the estimation (18) to the formula

$$\alpha = \frac{\chi \omega_0^2}{2v^3} \tag{19}$$

for the harmonical primary wave (Polyakova,1964).

To estimate the quality of the solution (13) we must compare the residue function $R(\tau,x)$ with nonlinear part $N(\tau,x)$ of the equations (9)-(11) , which for the case of single primary wave has a form

$$N(\tau,x) = v^2\beta \frac{\partial}{\partial x}\left[\frac{\partial u^0}{\partial x}\right]^2 e^{-2\alpha x} \approx -\frac{\beta}{v}\frac{\partial}{\partial \tau}\left[\frac{\partial u^0}{\partial \tau}\right]^2 e^{-2\alpha x} \tag{20}$$

Value of quality ratio

$$C = \left[\frac{\int_0^\infty N^2(\tau,x)\,d\tau}{\int_0^\infty R^2(\tau,x)\,d\tau}\right]^{\frac{1}{2}} \tag{21}$$

should remain large for chosen set of parameters, and condition $C \gg 1$ together with (8) may be considered as necessary for taking primary wave in the form (13). The results of numerical examples in section 8 show good quality of estimation (18) for reasonable sets of parameters. This justifies the usage of the form (13) for primary waves representation in following sections , where we discuss different cases of solutions of the system (9)-(11) for nonlinear field $\mathbf{u}^1$.

4. Single P-wave propagation

Let the primary field $\mathbf{u}^0$ be a single plane P-wave

$$\mathbf{u}^0(t,x) = u_0\left[t - \frac{x}{v_p}\right] e^{-\alpha_p x}\,\hat{\mathbf{x}} \equiv u_0(\tau) e^{-\alpha_p x}\,\hat{\mathbf{x}} , \qquad \tau = t - \frac{x}{v_p} \tag{22}$$

The system (9)-(11) in this case reduces to the single equation

$$\frac{\partial^2 u_x^1}{\partial t^2} - v_p^2 \frac{\partial^2 u_x^1}{\partial x^2} - \chi_p \frac{\partial^3 u_x^1}{\partial t\,\partial x^2} = v_p^2 \beta_0 \frac{\partial}{\partial x}\left[\frac{\partial u^0}{\partial x}\right]^2 e^{-2\alpha_p x} \equiv f(x,t) \tag{23}$$

This can be solved by taking the spatial Fourier transform $w(k,t)$ of the field u_x^1. Noting that the spatial Fourier transformation of $f(x,t)$ is

$$w_o(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x,t)e^{-ikx}dx = \frac{i\beta_0 k v_p^2}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\partial u_0(t-\frac{x}{v_p})}{\partial x}\right]^2 e^{-2\alpha_p x}e^{-ikx}dx$$

$$= \frac{i\beta_0 k v_p}{\sqrt{2\pi}}e^{-v_p(ik+2\alpha_p)t}\int_{-\infty}^{+\infty}\left[\frac{\partial u_0(\tau)}{\partial\tau}\right]^2 e^{v_p(ik+2\alpha_p)\tau}d\tau \tag{24}$$

eq. (23) becomes the following ordinary differential equation of the second order

$$\frac{d^2w}{dt^2} + \chi_p k^2\frac{dw}{dt} + v_p^2k^2 w = c(k)e^{-v_p(ik+2\alpha_p)t} \tag{25}$$

where

$$c(k) = \frac{i\beta_0 k v_p}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\left[\frac{\partial u_0(\tau)}{\partial\tau}\right]^2 e^{v_p(ik+2\alpha_p)\tau}d\tau \tag{26}$$

Equation (25) has a solution

$$w(k,t) = \frac{c(k)}{iv_p k\left[4\alpha v_p-\chi_p k^2\right]}\left[e^{-(2\alpha_p v_p-\frac{\chi_p k^2}{2})t} - 1\right]e^{-(v_p ik+\frac{\chi_p k^2}{2})t} \tag{27}$$

An inverse Fourier transform then yields an expression for the nonlinear correction

$$\mathbf{u}^1(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} w(k,t)\ e^{ikx}dk\ \hat{\mathbf{x}} =$$

$$\frac{\beta_0}{2\pi}\int_{-\infty}^{+\infty}\left[\frac{\partial u_0(\tau)}{\partial\tau}\right]^2 e^{2v_p\alpha_p\tau}\int_{-\infty}^{+\infty}\frac{\left[e^{-(\chi_p k^2+4\alpha_p v_p-\chi_p k^2)\frac{t}{2}} - 1\right]}{4\alpha_p v_p - \chi_p k^2}e^{-\frac{\chi_p k^2}{2}t}e^{ik(x-v_p t+v_p\tau)}dk\ d\tau\ \hat{\mathbf{x}} \tag{28}$$

The function in the integral over the k obviously contains no singularities. In the low attenuation limit, when $\chi_p \to 0$ eq.(28) gives

$$\mathbf{u}^1(x,t) = -x\frac{\beta_0}{2}\left[\frac{\partial u_0(t-\frac{x}{v_p})}{\partial x}\right]^2\hat{\mathbf{x}} \tag{29}$$

The solution (29) shows that the nonlinear component of the primary field (22) is also a plane P-wave with the same wavefront. The amplitude of this wave is proportional to the square of the strain level and to the distance of propagation x . The sign of the displacement of the nonlinear wave is opposite to that of the coefficient β_0 . For harmonical primary wave solution (29) gives the result of Polyakova (1964).

Consider the case where a constant longitudinal strain S_x is present in the material, in which the primary field is

$$\mathbf{u}^0(t,x) = u_0(t - \frac{x}{v_p})e^{-\alpha_p x}\,\hat{\mathbf{x}} + S_x x \hat{\mathbf{x}} , \tag{30}$$

The nonlinear part of the field then becomes

$$\mathbf{u}^1(x,t) = -x\frac{\beta_0}{2v_p^2}\left[\left(\frac{\partial u_0(\tau)}{\partial \tau}\right)^2 - 2v_p S_x \frac{\partial u_0(\tau)}{\partial \tau} + v_p^2 S_x^2\right]\hat{\mathbf{x}} , \quad \tau = t - \frac{x}{v_p} \tag{31}$$

The presence of the constant strain S_x in the elastic material causes, in addition to the nonlinear field (29), the appearance of dynamic and static components connected with this constant strain. The dynamic component is proportional to S_x and the waveform is the first derivative of the primary signal, whereas the static component is proportional to the square of S_x .

5. Single S-wave propagation

Now let the primary field $\mathbf{u}^0$ be a single plane S-wave

$$\mathbf{u}^0(t,x) = u_0\left(t - \frac{x}{v_s}\right)e^{-\alpha_s x}\,\hat{\mathbf{y}} \equiv u_0(\tau)e^{-\alpha_s x}\,\hat{\mathbf{y}} , \qquad \tau = t - \frac{x}{v_s} \tag{32}$$

Substituting this into the system (4)-(6) yields the equation

$$\frac{\partial^2 u_x^1}{\partial t^2} - v_p^2\frac{\partial^2 u_x^1}{\partial x^2} - \chi_p \frac{\partial^3 u_x^1}{\partial t\, \partial x^2} = v_p^2 \gamma_0 \frac{\partial}{\partial x}\left(\frac{\partial u^0}{\partial x}\right)^2 e^{-2\alpha_s x} \equiv f(x,t) \tag{33}$$

It is obvious that in the approximation being considered, a single S-wave does not excite nonlinear shear waves, but does generate a nonlinear compressional component. The structure of eq. (33) is very similar to that of eq. (23) for the single P-wave case. Using the same approach and notation of section

4, the following spectrum of the nonlinear field is obtained

$$w(k,t) = \frac{i\gamma_0}{\sqrt{2\pi}v_s(v_p^2-v_s^2)k} e^{-(ikv_s+\beta+2\alpha_s v_s)t} \int_{-\infty}^{+\infty} \left[\frac{\partial u_0(\tau)}{\partial \tau}\right]^2 e^{v_s(ik+2\alpha_p)\tau} d\tau \tag{34}$$

Taking the inverse Fourier transform of (28) results in

$$\mathbf{u}^1(x,t) = \frac{i\gamma_0}{2\pi v_s(v_p^2-v_s^2)} \int_{-\infty}^{+\infty} \frac{1}{k} e^{-(ikv_s+\beta+2\alpha_s v_s)t+ikx} \int_{-\infty}^{+\infty} \left[\frac{\partial u_0(\tau)}{\partial \tau}\right]^2 e^{v_s(ik+2\alpha_p)\tau} d\tau\, dk\, \hat{\mathbf{x}}$$

$$\approx \frac{\gamma_0 v_p^2}{4v_s(v_p^2-v_s^2)} \int_{-\infty}^{+\infty} \left[\frac{\partial u_0(\tau)}{\partial \tau}\right]^2 e^{2\alpha_p(\tau-t)v_p} sign\left[t-\frac{x}{v_s}-\tau\right] d\tau\, \hat{\mathbf{x}} \tag{35}$$

which is a compressional wave propagating with the shear velocity. The amplitude of this wave is relatively small compared to the case of single P-wave propagation, but it is important to note that in this case the polarization of the nonlinear field is orthogonal to the polarization of the primary wave, which may simplify their separation. For the case of harmonic primary S-waves some authors (Polyakova,1964; Zarembo and Krasil'nikov,1967) emphasize an additional nonlinear P-wave propagating with the P velocity, which causes the phenomena of spatial modulation for the combined nonlinear field. This second wave appears in the solution because of specific boundary conditions (for instance for the point $x = 0$) imposed on the reduced equation. The generation of this wave occurs at the point $x = 0$ and is not connected with the nonlinear propagation of the primary wave itself.

6. Cross-through propagation of two P-pulses

In this section we consider the nonlinear interaction of two plane P-waves propagating along the same axis in opposite directions. Taking the primary field in the form

$$\mathbf{u}^0(t,x) = u_1\left[t - \frac{x}{v_p}\right] e^{-\alpha_1 x}\, \hat{\mathbf{x}} + u_2\left[t + \frac{x}{v_p}\right] e^{\alpha_2 x}\, \hat{\mathbf{x}} \tag{36}$$

and substituting it into the system (9)-(11), we obtain an equation of the form (23) where the right hand side now includes three components. Two of these components correspond to nonlinear waves generated directly by the two parts of (36), and the solutions for these waves were discussed earlier in section 3. Here we will be interested in the nonlinear wave arising due to mutual interaction of the components of (36). The equation of motion for this wave is

$$\frac{\partial^2 u_x^1}{\partial t^2} - v_p^2 \frac{\partial^2 u_x^1}{\partial x^2} - \chi_p \frac{\partial^3 u_x^1}{\partial t\, \partial x^2} = 2 v_p^2 \beta_0 \frac{\partial}{\partial x}\left[\frac{\partial u_1}{\partial x}\frac{\partial u_2}{\partial x}\right] e^{(\alpha_2-\alpha_1)x} \quad (37)$$

Taking the double Fourier transform of (37) over space and time variables, we have

$$\left[-\omega^2 + v_p^2 k^2 + i\chi_p \omega k^2\right] R(k,\omega) = R_0(k,\omega) \quad (38)$$

where

$$R(k,\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_x^1(x,t)\; e^{-ikx-i\omega t} dx\; dt \quad (39)$$

$$R_0(k,\omega) = -\frac{ik\beta_0}{\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\partial u_1}{\partial x}\frac{\partial u_2}{\partial x} e^{(\alpha_2-\alpha_1)x}\, e^{-ikx-i\omega t} dx\; dt \quad (40)$$

Introducing functions $F_1(\omega)$ and $F_2(\omega)$ as

$$F_1(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \frac{\partial u_1(\tau)}{\partial \tau} e^{-i\omega\tau} d\tau \quad (41)$$

$$F_2(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \frac{\partial u_2(\tau)}{\partial \tau} e^{-i\omega\tau} d\tau \quad (42)$$

and using the convolution theorem, it is possible to obtain

$$R_0(k,\omega) = -ik\beta_0 v_p F_1\left[\frac{\omega - v_p k}{2}\right] F_2\left[\frac{\omega + v_p k}{2}\right] \quad (43)$$

The solution for nonlinear component therefore has the form

$$\mathbf{u}^1(t,x) = \hat{\mathbf{x}}\frac{i\beta_0 v_p}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{F_1\left[\frac{\omega - v_p k}{2}\right] F_2\left[\frac{\omega + v_p k}{2}\right]}{\omega^2 - v_p^2 k^2 - i\chi_p \omega k^2} e^{ikx+i\omega t} k\; dk\; d\omega \quad (44)$$

The denominator in (44) has two roots in the upper half of the complex ω plane located at

$$\omega_1 \approx kv + i\frac{\chi_p}{2}v_p^2 \ , \qquad \omega_2 \approx -kv + i\frac{\chi_p}{2}v_p^2 \tag{45}$$

Let us assume that the functions $F_1(\omega)$ and $F_2(\omega)$ contain no poles and that both signals in (36) are different from zero in the time interval between zero and some finite duration of the signal

$$u_i(\tau) = 0 \ , \quad \text{for } \tau < 0 \quad \text{and} \quad \text{for } \tau > T_i \ , \quad (i=1,2) \tag{46}$$

Then for $t < 0$ we take a contour consisting of the real axis and a semicircle in the lower half plane in evaluating the ω integral in (44). In this case the integral and therefore the nonlinear component of the field is equal to zero. Physically, this means that the two waves from (36) have not yet started to interact.

If $t > T_1 + T_2$ the waves have already interacted and separated, and the contour of integration includes the real axis and a semicircle in the upper half plane. Evaluating the residues at the poles given by (45), we have

$$\mathbf{u}^1(t,x) = \hat{\mathbf{x}}\frac{k\beta_0\sqrt{2\pi}}{2}e^{-\frac{\chi_p}{2}v_p^2 t}\left[F_1(i\frac{\chi_p}{2}v_p^2)\frac{\partial u_2(t-\frac{x}{v_p})}{\partial t} - F_2(i\frac{\chi_p}{2}v_p^2)\frac{\partial u_1(t+\frac{x}{v_p})}{\partial t}\right] \tag{47}$$

The result (47) shows that after interaction each signal travels together with a nonlinear wave having the shape of the first derivative of the interacting wave. The amplitudes of these nonlinear fields are rather small because $F_i(\omega) \approx 0$ when $\omega \approx 0$, except in the case of a prestressed material similar to that discussed earlier in section 3. It is important to note that for this case of a prestressed material, one of the functions in (44) possesses an additional pole in the upper half of the ω plane which must be taken into account.

7. Generation of nonlinear shear waves

The generation of nonlinear shear waves becomes possible when we have the simultaneous presence of shear and compressional primary waves in the nonlinear elastic material, which can be seen in (10) and (11). The propagation of each of these waves also causes the excitation of compressional waves, which was considered in the previous sections. Therefore, in the present section we consider only the shear part of the total nonlinear field, which satisfies (10). Three cases are examined: 1) compressionally pre-stressed material; 2) shear pre-stressed material; 3) compressional pulse passing through a shear pulse.

1) Compressionally pre-stressed material We take the primary field in the form

$$\mathbf{u}^0(t,x) = u_0(t - \frac{x}{v_s})e^{-\alpha_s x}\,\hat{\mathbf{y}} + S_x x \hat{\mathbf{x}}\ , \qquad S_x = const. \tag{48}$$

Then (10) becomes

$$\frac{\partial^2 u_y^1}{\partial t^2} - v_s^2 \frac{\partial^2 u_y^1}{\partial x^2} - \chi_s \frac{\partial^3 u_y^1}{\partial t\, \partial x^2} = 2 v_p^2 \gamma_0 S_x \frac{\partial^2 u_0\left[t - \frac{x}{v_s}\right]}{\partial x^2} \tag{49}$$

This equation has the same structure as (23). Following the same approach used in section 4, we obtain the solution

$$\mathbf{u}^1(x,t) = x \frac{\gamma_0}{v_s} S_x \frac{\partial u^0(\tau)}{\partial \tau}\,\hat{\mathbf{y}}\ , \quad \tau = t - \frac{x}{v_s} \tag{50}$$

which is similar to the dynamic component from (31). The wave (50) propagates along with the shear primary wave and has the same polarization. The amplitude of the nonlinear field grows proportionally to the distance x of propagation.

2) Shear pre-stressed material

Now the primary field has the form

$$\mathbf{u}^0(t,x) = u_0(t - \frac{x}{v_p})e^{-\alpha_p x}\,\hat{\mathbf{x}} + S_y x \hat{\mathbf{y}} \tag{51}$$

where S_y is the constant shear strain in the material arising due to the presense of the transverse stress and the equation of motion is

$$\frac{\partial^2 u_y^1}{\partial t^2} - v_s^2 \frac{\partial^2 u_y^1}{\partial x^2} - \chi_s \frac{\partial^3 u_y^1}{\partial t\, \partial x^2} = 2 v_p^2 \gamma_0 S_y \frac{\partial^2 u_0\left[t - \frac{x}{v_p}\right]}{\partial x^2} \tag{52}$$

This problem is similar to that considered in section 5 and the solution is

$$\mathbf{u}^1(x,t) \;=\; \hat{\mathbf{y}}\frac{\gamma_0 v_p^3}{(v_p^2-v_s^2)}S_y\int_{-\infty}^{+\infty}\frac{\partial u_0(\tau)}{\partial\tau}sign\left[t-\frac{x}{v_p}-\tau\right]d\tau \;=\; \hat{\mathbf{y}}\frac{\gamma_0 v_p^3}{(v_p^2-v_s^2)}S_y u_0\left[t-\frac{x}{v_p}\right] \tag{53}$$

This wave has shear polarization, propagates along with the primary dynamic field at the P-wave velocity, and has the same waveform as the dynamic component $u_0(\tau)$ from (51).

3) <u>Compressional pulse passing through a shear pulse</u>

Consider the general case of both a P-wave and an S-wave propagating in the same direction

$$\mathbf{u}^0(t,x) \;=\; u_0(x,t)\,\hat{\mathbf{x}} \;=\; u_1\left[t-\frac{x}{v_p}\right]e^{-\alpha_p x}\,\hat{\mathbf{x}} \;+\; u_2\left[t-\frac{x}{v_s}\right]e^{-\alpha_s x}\,\hat{\mathbf{y}} \tag{54}$$

Following the approach of section 6, we have for the double Fourier transform of (10)

$$\left[-\omega^2 + v_p^2k^2 + i\chi_p\omega k^2\right]R(k,\omega) \;=$$

$$-\frac{ik\gamma_0}{\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\partial u_1(t-\frac{x}{v_p})}{\partial x}\frac{\partial u_2(t-\frac{x}{v_s})}{\partial x}e^{-(\alpha_p+\alpha_s)x}e^{-ikx-i\omega t}dx\,dt \tag{55}$$

Introducing the spectra $F_1(\omega)$ and $F_2(\omega)$ as in (41) and (42), we obtain the solution

$$\mathbf{u}^1(t,x) \;=\; \hat{\mathbf{y}}\frac{i\gamma_0 v_p v_s}{\pi(v_p-v_s)}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{F_1\left[\omega+\omega'\right]F_2\left[-\omega'\right]}{\omega^2 - v_p^2k^2 - i\chi_p\omega k^2}e^{ikx+i\omega t}k\,dk\,d\omega \tag{56}$$

where

$$\omega' \;=\; \frac{v_p v_s}{v_p-v_s}\left[k+\frac{\omega}{v_p}-(\alpha_p+\alpha_s)\right] \tag{57}$$

The denominator in (56) has the same roots (45) as in section 6, and, assuming that $F_1(\omega)$ and $F_2(\omega)$ have no poles, we can take the contour of integration with a semicircle in the upper half plane when the interaction is completed.

After calculating the residues we have the result

$$\mathbf{u}^1(t,x) \;=\; \hat{\mathbf{y}}\frac{\gamma_0 v_p}{(v_p-v_s)}\int_{-\infty}^{+\infty}F_1\left[2k\frac{v_p v_s}{v_p-v_s}\right]F_2\left[-kv_s\frac{v_p+v_s}{v_p-v_s}\right]e^{ik(x+tv_s)}dk$$

$$- \hat{y}\sqrt{2\pi}\frac{\gamma_0 v_p}{(v_p - v_s)} F_1\left[i\frac{v_s \chi_s}{2(v_p - v_s)} - \alpha_p - \alpha_s \right] \frac{\partial u_2(t - \frac{x}{v_s})}{\partial x} \tag{58}$$

which includes a wave propagating along with the primary S-wave and a backscattering disturbance of the same polarization. The field (58) is obviously very small for most real elastic pulses.

7. Discussion and numerical results

The results of the previous sections show that we should expect the strongest nonlinear effects to occur in four cases: 1) Propagation of a single compressional pulse; 2) Propagation of a single compressional pulse in a longitudinally pre-stressed material; 3) Propagation of a single shear pulse in a longitudinally pre-stressed material; 4) Propagation of a single compressional pulse in a shear pre-stressed material. In the last case the polarization of the nonlinear component is orthogonal to the polarization of the primary wave. For first three cases the amplitude of the nonlinear field is proportional to the distance of propagation for distances which are not too large. The case of single shear wave propagation gives weaker interaction, but here the nonlinear field has polarization orthogonal to the polarization of the primary wave, which may have advantages for the detection of the nonlinearity. When two pulses propagate in opposite directions or pulses propagate in the same direction with different velocities, the results show a very weak nonlinear interaction which is unlikely to be observed in real experiments.

As it follows from obtained results the nonlinear effect depends on values of nonlinear constants,strain levels and attenuation. Using data from Huges and Kelly (1953) for solid materials we have: $\beta_0 = -6.5$, $\gamma_0 = -0.7$ for polystyrene, $\beta_0 = -7.3$, $\gamma_0 = -1.4$ for armco iron, $\beta_0 = 4.4$, $\gamma_0 = 1.2$ for pyrex. Observations in rocks showed much higher nonlinear coefficients. For sandstone β_0 can be as high as 7000 (Johnson et al.,1993). The strain level of the propagating signal is usually a value of the order of 10^{-5} for body waves and it can reach the level of $10^{-4} - 10^{-3}$ in the regions of strong earthquakes and explosion sources. The constant strain always exists in the earth's crust, generally increasing with depth. Areas with a high strain level also surround fault zones that are potential sites for earthquakes. A level of the strain $\approx 10^{-4} - 10^{-3}$ is quite common in such areas. The same level of strain can also be produced in laboratory measurements. The quality factor Q due to intrinsic attenuation in seismic experiment varies basically in the range 10 – 1000 , and therefore propagation distance may reach and exceed a value of many of wavelengths before dissipation processes begin to dominate.

To demonstrate how the nonlinear phenomena affects a propagating signal, we we created a set of numerical calculations of the nonlinear response of a material representing a rock sample with P-wave velocity of $4\ km/sec$, S-wave velocity of $2\ km/sec$ and with nonlinear coefficients $\beta_0 = 500$, $\gamma_0 = 500$. The dominant frequency in the primary wave was taken 100 Hz. Attenuation coefficient was taken equal to $\alpha = 1.6{\cdot}10^{-3}m^{-1}$,*whichcorresponds* ($Q = 50$) .

First we illustrate the validity of the form (13) of the primary wave applying the Berlage impulse,as a commonly used synthetic waveform in seismic numerical simulations

$$u_0(\tau) = w \cdot e^{-w/2.5} sin(w) \quad , \quad w = \omega\tau > 0 \tag{59}$$

On the Fig.1 the comparison between nonlinear component $N(\tau,x)$ from (20) and residual $R(\tau,x)$ from (14) for the primary signal (59) is shown, where propagation distance $x = 200\ m.$ and maximum strain level $w_0 = 10^{-4}$ were used. The quality ratio C from (21) for this example is equal to 43 , which is more than enough for high accuracy. Nevertheless, for nonlinear distortion computations for propagating single P-pulse we use broadband signal

$$\mathbf{u}^0 = \hat{\mathbf{x}}\frac{A}{T}(t - \frac{x}{v_p})e^{-\frac{\left[t-\frac{x}{v_p}\right]^2}{2T^2}} e^{-\alpha_p x} \quad , \qquad A = const. \tag{60}$$

since its simple analitical structure allows us to obtain proper and observable formulas. On Fig.2 the comparison between $N(\tau,x)$ and $R(\tau,x)$ is shown for the signal (60), for the same set of parameters. The quality ratio C here is equal to 12, which is still good enough agreement with condition $C \gg 1$. The attenuation coefficient for this wave has analitical expression

$$\alpha_p = \frac{5}{4}\frac{\chi_p}{T^2 v_p^3} . \tag{61}$$

The parameter T in (60) and (61) is a characteristic duration of the signal, which has a spectrum

$$F_0(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} u_0 e^{-i\omega t} dt = -iAT^2\omega \, e^{-i\frac{x}{v}\omega} e^{-\frac{\omega^2 T^2}{2}} e^{-\alpha x} \tag{62}$$

with a maximum at $\omega_{\max} = T^{-1}$

Considering the case of a single propagating P-wave and using (29), the total solution has the form

$$\mathbf{u}(x,t) = \hat{\mathbf{x}}\frac{A}{T}\tau e^{-\frac{\tau^2}{2T^2}} e^{-\alpha_p x} - \hat{\mathbf{x}}x\frac{\beta_0 A^2}{2T^2 v^2} e^{-\frac{\tau^2}{T^2}} \left[1 - \frac{\tau^2}{T^2}\right]^2 \tag{63}$$

The spectrum of the nonlinear component u^1 is

$$S_1(\omega) = -x\frac{\beta_0 A^2}{32\sqrt{2}Tv_p} e^{-i\frac{x}{v_p}\omega} e^{-\frac{\omega^2 T^2}{4}} \left[\omega^4 T^4 - 4\omega^2 T^2 + 12\right] \tag{64}$$

Extracting the phase factor $\exp(-i\omega x/v_p)$ from (64) and (62), we see that the spectrum of the nonlinear signal is a real function, whereas the spectrum of the primary signal (62) is a purely imaginary function, the latter result coming from the fact that (60) is an odd function with respect to the argument $\tau = t - x/v_p$.

The constant A may be related to the maximum strain level of the signal (60)

$$W_0 = \left|\frac{\partial u_0}{\partial x}\right|_{\max} \approx \frac{A}{Tv_p} \tag{65}$$

Defining the nonlinear ratio q as the ratio of the maximum of the nonlinear field $\mathbf{u}^1$ to the maximum of the primary field, we obtain the expression

$$q = \frac{\left|\mathbf{u}^1\right|_{\max}}{\left|\mathbf{u}^0\right|_{\max}} = \frac{\sqrt{e}\,\beta_0}{2}\frac{x}{\lambda_0}W_0 \tag{66}$$

where $\lambda_0 = v_p T$ is the dominant wavelength.

The treatment of the prestressed case is obvious and follows directly from (31). Here we assume for simplicity that the constant strain S_x in the material is much larger than the strain W_0 in the pulse. The total field for this case is

$$\mathbf{u}(x,t) = \hat{\mathbf{x}}\frac{A}{T}\tau e^{-\frac{\tau^2}{2T^2}}e^{-\alpha_p x} - \hat{\mathbf{x}}x\frac{\beta_0 A}{2Tv_p}e^{-\frac{\tau^2}{2T^2}}\left[1-\frac{\tau^2}{T^2}\right]S_x \tag{67}$$

and the nonlinear ratio is

$$q = \sqrt{e}\,\beta_0\frac{x}{\lambda_0}S_x \tag{68}$$

The expressions (66) and (68) allow one to estimate the nonlinear ratio, if the nonlinear coefficient of the material and the strain level are known. Taking the maximum strain level W_0 equal to $5\cdot 10^{-5}$ and the distance of observation $x = 500\ m.$, we see that nonlinear ratios from (66) and (68) may reach the levels of tens of percent, providing a strong implication for nonlinear effects in the phenomena of elastic wave propagation in the earth.

Shown in Figure 3 is the total field together with both the primary and nonlinear components of the field for the case of single P-wave propagation in an unstressed material. The results for P-wave propagation in a pre-stressed medium with a strain level of $S_x = 2\cdot 10^{-5}$ are presented in Figure 4. The spectra of all components of the field in the prestressed material are presented in Figure 5, which shows that the spectrum of the total field in the presence of the nonlinearity becomes broader, increasing the level of both low-frequency and high-frequency parts. The signal for S-wave propagation in a pre-stressed medium with a strain level $S_x = 2\cdot 10^{-5}$ is shown in Figure 6. As can be observed in these figures, the nonlinear component of the field for a chosen set of parameters is large enough to violate the assumption (8). This means that nonlinear phenomena should be taken into account for even smaller values of strains and nonlinear coefficients and that the theoretical development involving the next orders of nonlinearity is necessary.

8. Conclusions

Time-domain solutions for 1-D nonlinear elastic propagation problems have been obtained. Numerical estimates suggest significant contributions from the nonlinear component to the total field in rocks when strain level of the primary signal is of the order of 10^{-5} and greater. The strongest nonlinear effects are obtained for single compressional wave propagation with or without compressional stress in the elastic material, and for single shear wave propagation through a pre-stressed elastic material. The values of parameters of strain levels, nonlinear elastic constants and quality factor Q used for computations are far from exceeding their really possible limits , which suggests the existence of more profound nonlinear elastic phenomena. These results may be useful for the measurement of nonlinear constants of elastic materials, for explaining the frequency content of seismograms, and for monitoring the strain field in the earth's crust.

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Figure Captions

Figure 1 Comparison between nonliner term $N(\tau,x)$ (solid line) and residue function $R(\tau,x)$ (dashed line) for Berlage impulse. Attenuation coefficient $\alpha = 1.6 \cdot 10^{-3} m^{-1}$, ($Q \approx 50$) ,propagation distance $x = 200\ m.$, maximum strain level $W_0 = 10^{-4}$. The quality ratio $C = 43$.

Figure 2 Comparison between nonliner term $N(\tau,x)$ (solid line) and residue function $R(\tau,x)$ (dashed line) for the signal from (60). Attenuation coefficient $\alpha = 1.6 \cdot 10^{-3} m^{-1}$, ($Q \approx 50$) ,propagation distance $x = 200\ m.$, maximum strain level $W_0 = 10^{-4}$. The quality ratio $C = 12$.

Figure 3 Nonlinear single P-wave propagation. Solid line - the total field, dashed line - primary wave, dotted line - nonlinear component.

Figure 4 Nonlinear single P-wave propagation in a pre-stressed material. Solid line - the total field, dashed line - primary wave, dotted line - nonlinear component.

Figure 5 Spectra of the components of the total field for single P-wave propagation in a pre-stressed material. Solid line - primary wave, dashed line - nonlinear component of the primary wave itself, dotted line - dynamic nonlinear component due to the constant strain level.

Figure 6 Nonlinear single S-wave propagation in pre-stressed material. Solid line - the total field, dashed line - primary wave, dotted line - nonlinear component.

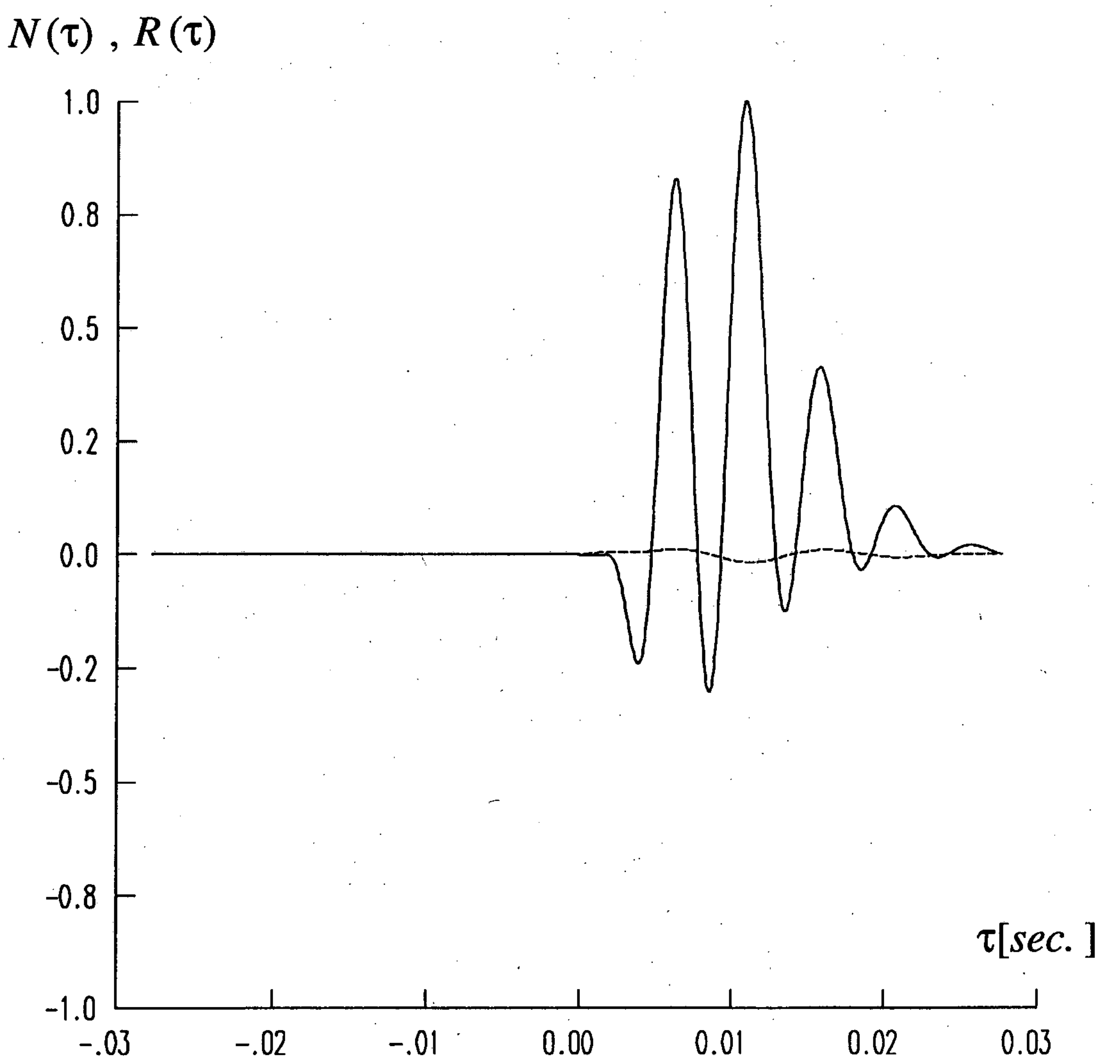


Fig. 1

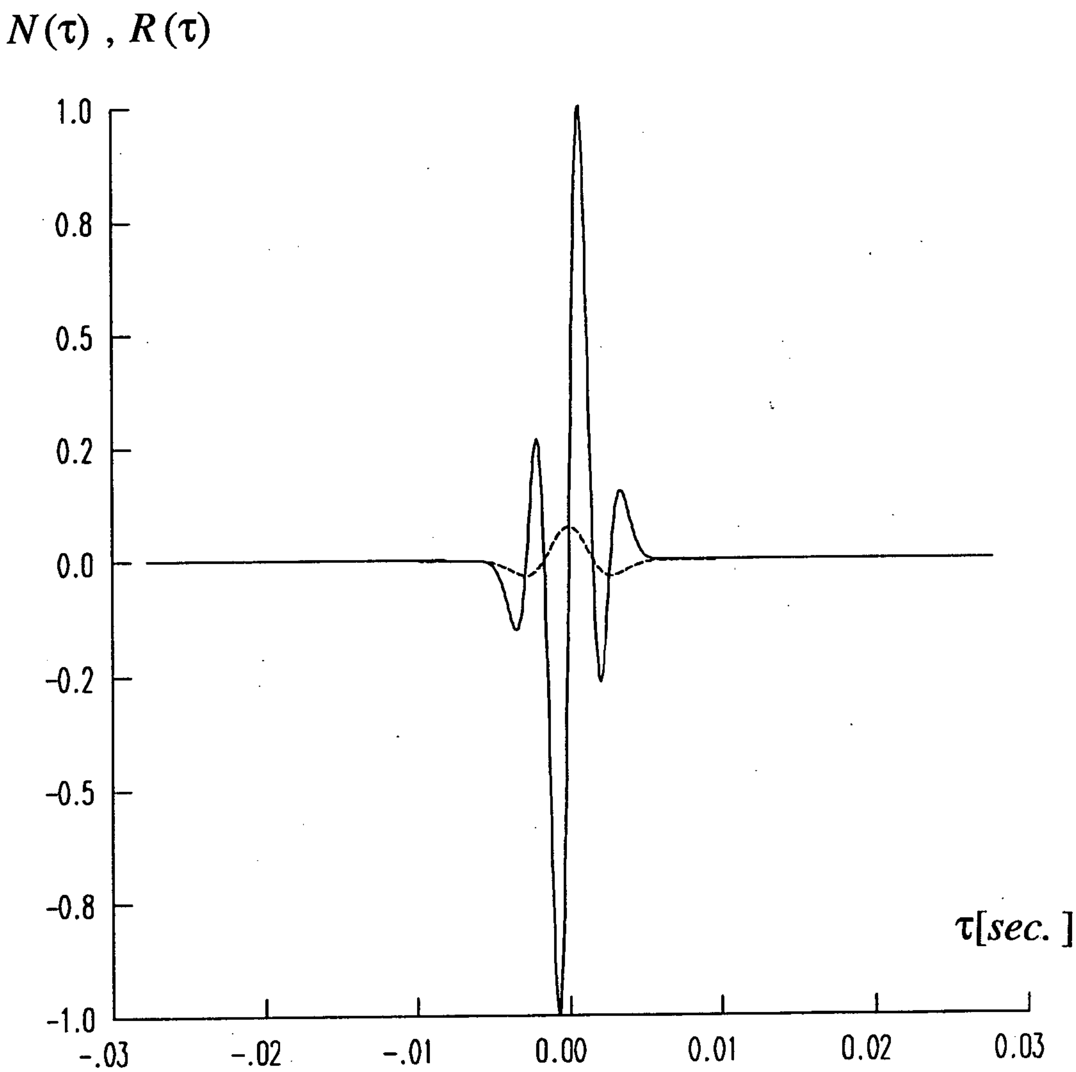
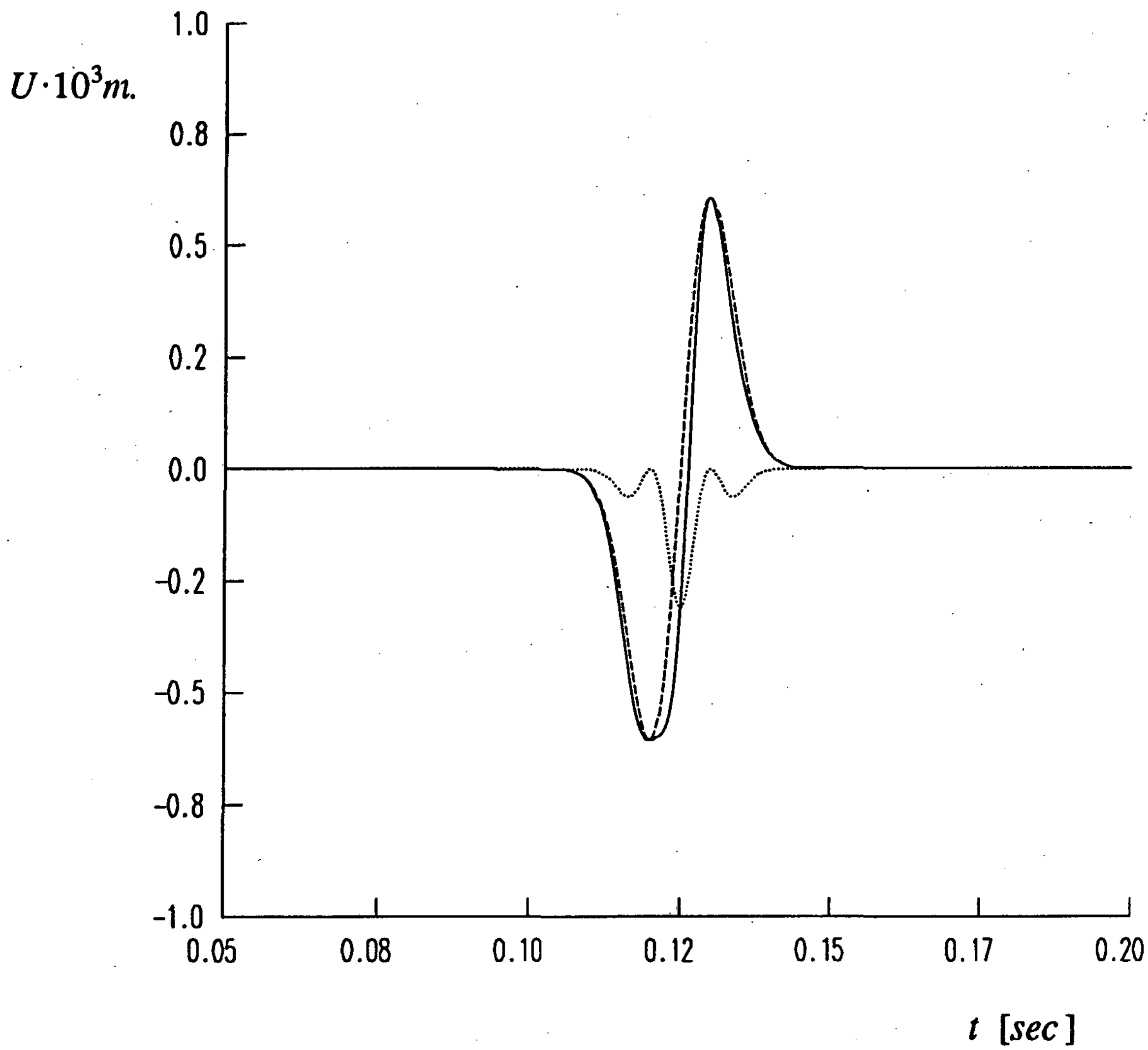


Fig.2



1.0
$U \cdot 10^3 m.$
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0.0
-0.2
-0.5
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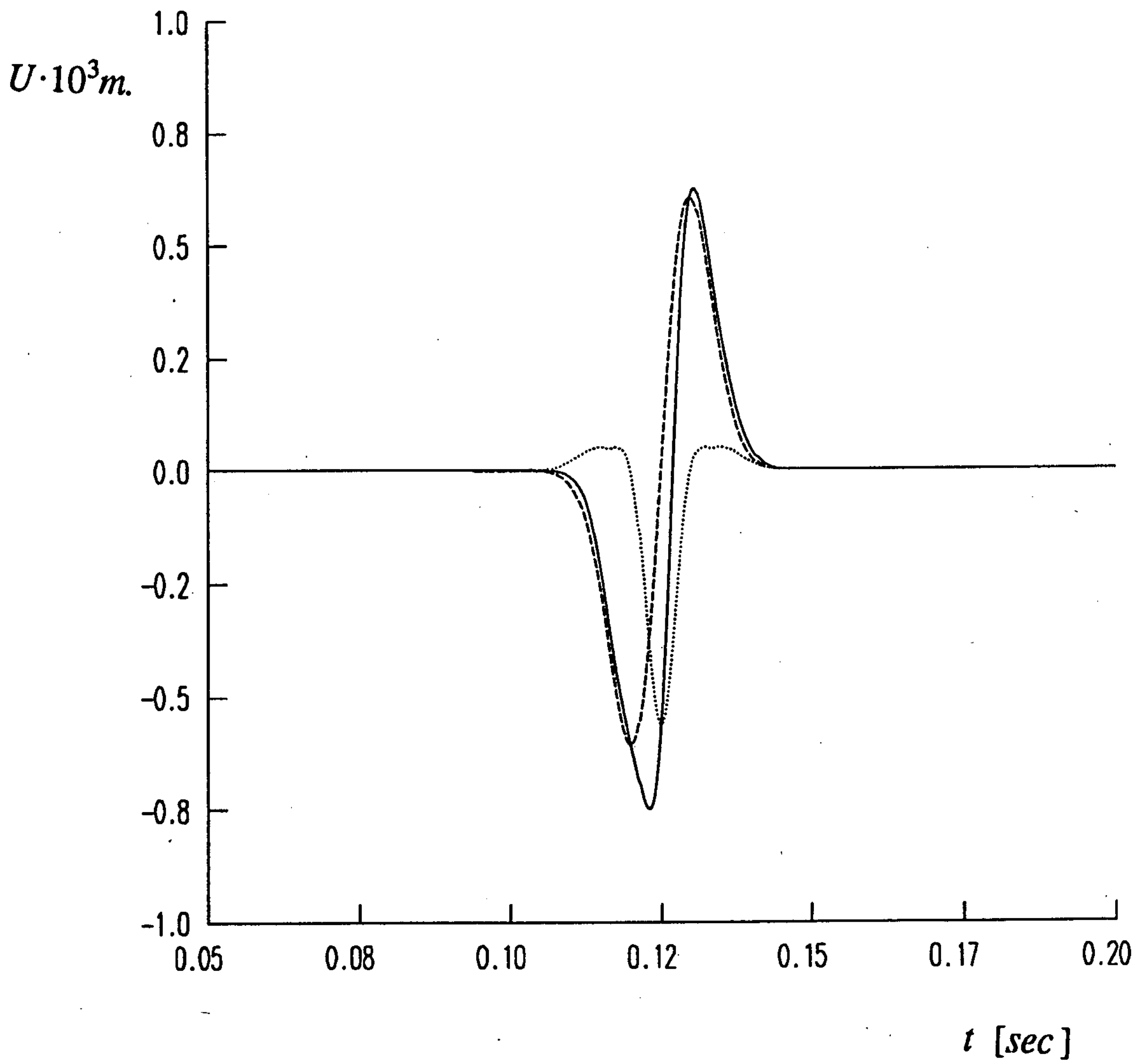
t [sec]

Fig. 3



$U \cdot 10^3 m.$
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t [sec]

Fig.4

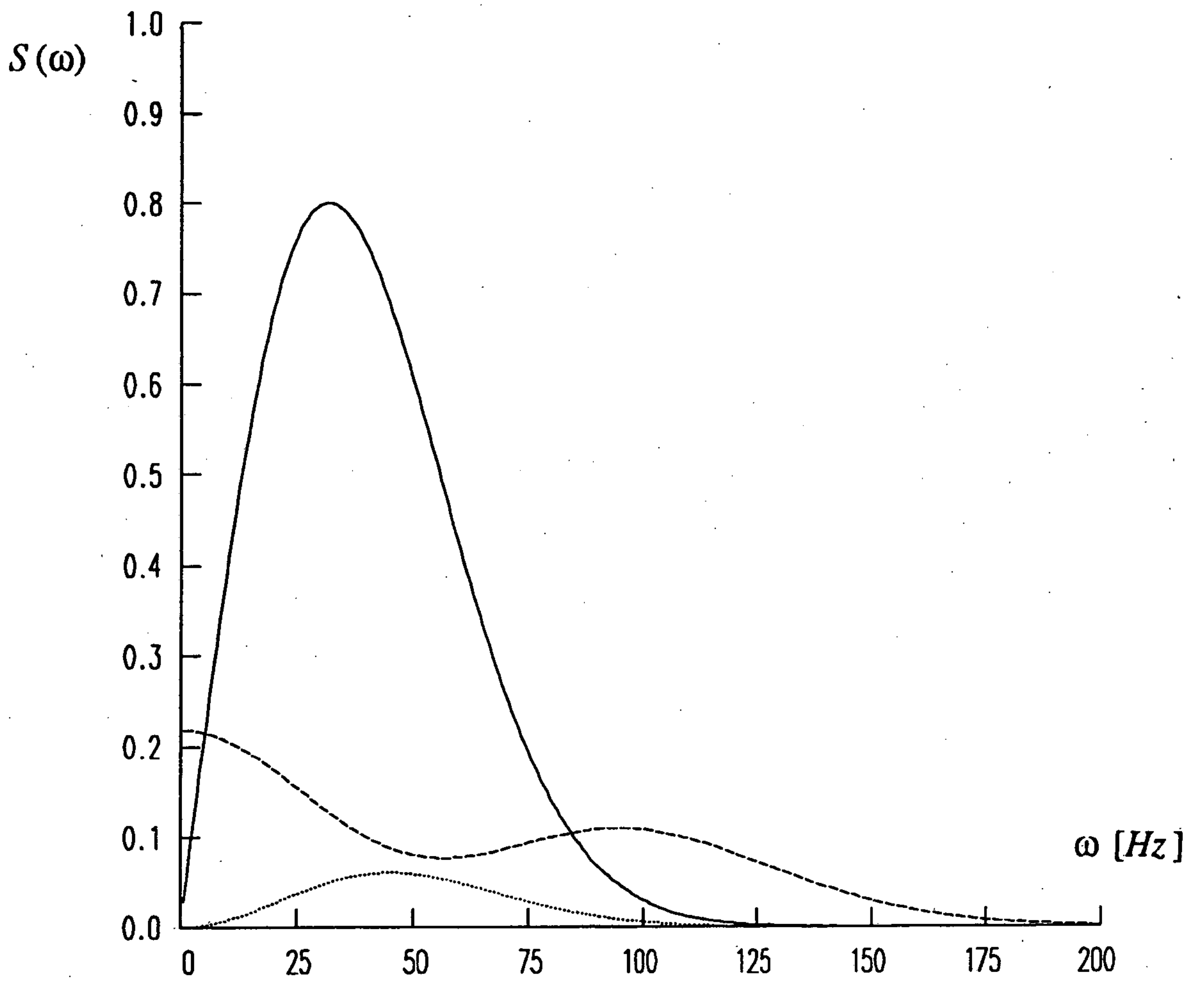
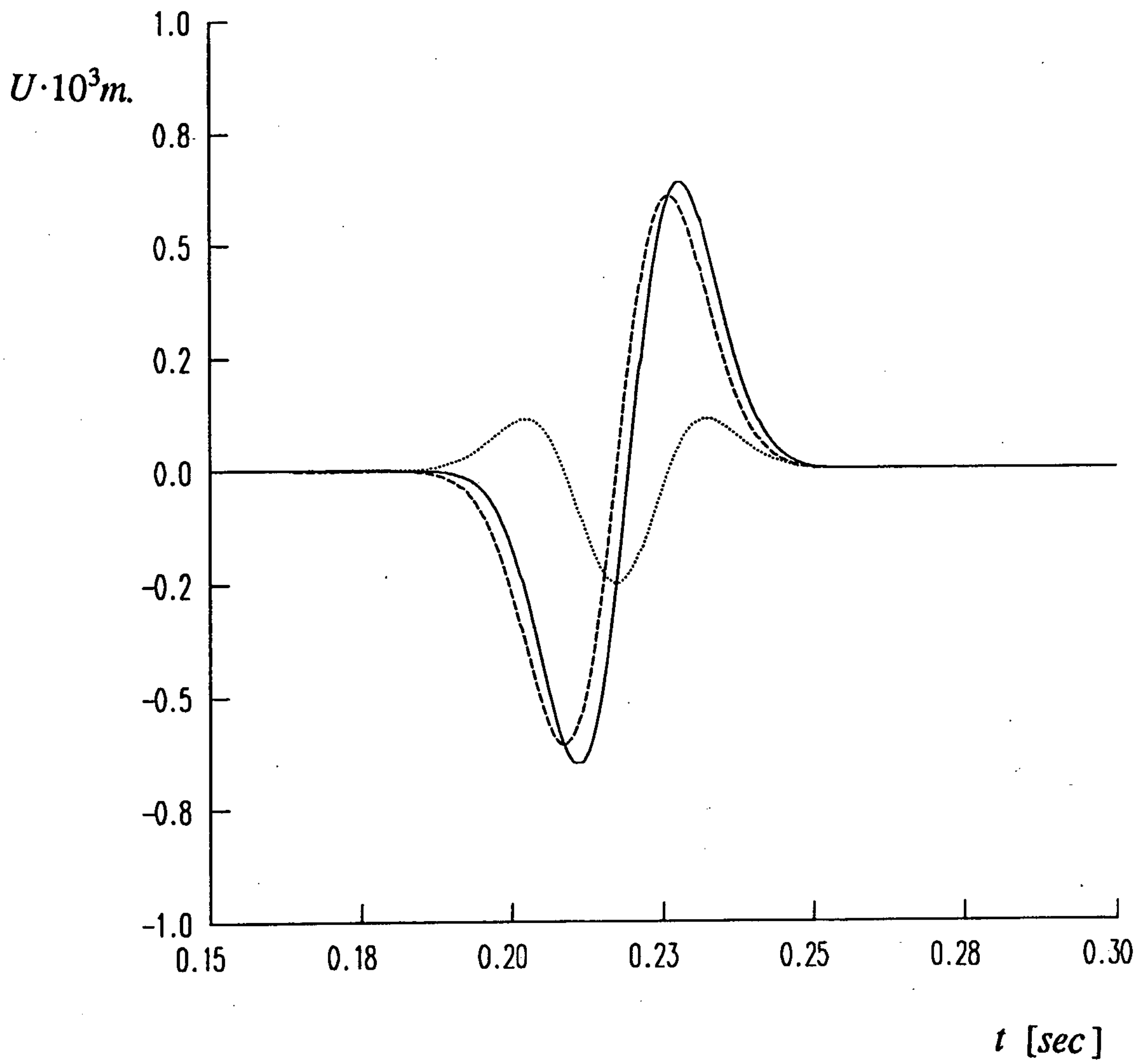


Fig. 5



1.0
$U \cdot 10^3 m.$
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0.2
0.0
-0.2
-0.5
-0.8
-1.0
0.15
0.18
0.20
0.23
0.25
0.28
0.30
t [sec]

Fig.6

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ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

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