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Author

Ito, Hiro

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Was Japan's Real Interest Rate Really Too High During the 1990s?

The Role of the Zero Interest Rate Bound and Other Factors[1]

Hiro Ito

Claremont McKenna College
Department of Economics

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[1] Hiro Ito, Department of Economics, Claremont McKenna College, 500 E. Ninth Street, Claremont, CA 91711. Tel/Fax: (909) 607-7261/621-8249, Email: hiro.ito@claremontmckenna.edu.

Abstract

Japan's more than a decade long "Great Recession" has presented a disconcerting case of what could happen if interest rates are bounded by zero and deflation sets in. Since Krugman (1998), the commonplace observation is that the deflationary situation combined with the zero nominal interest rate has caused elevated real interest rates, thereby nullifying monetary policy. This paper investigates this oft-cited claim and examines whether it is associated with anomalies in the way real interest rates are determined by employing an error correction model (ECM) based on the time-varying parameter model with Markov-switching variances. Using this model it is revealed that during the 1980s both ex ante and ex post rates were often lower than the equilibrium rates, indicating strong and persistent optimism among agents. However in the 1990s the ex ante real interest rate was persistently higher than the equilibrium, indicating the pessimistic expectations among agents. The time-varying speed of convergence to the equilibrium appears to slow down considerably in 1996-99, making the misalignment in the real interest rate process last twice as long as in the 1980s. In addition the analysis using the Smooth Transition Regression (STR) model shows a regime shift in the real rate process in mid-1995, three years before the implementation of the zero interest rate policy. This result suggests that a situation with an extremely low nominal interest rate, even before it reaches the zero bound, may create anomalies or nonlinearity in the effectiveness of monetary policy.

Key Words: Regime switching; real interest rates; nonlinear time series; monetary policy; zero interest rate policy; deflation; Japan; liquidity trap; state-space models

JEL Classification: C51, C52, E40

1. Introduction

1.1 Motivation

Japan's more than a decade long "Great Recession" has drawn attention from both academia and the policy community. [2] The attention has become particularly focused since early 1999 when the Bank of Japan (BOJ) officially implemented the zero interest rate policy (ZIRP), lowering the target short-term interest rate to essentially zero percent (see Figure 1) and raising the possibility of a liquidity trap, something not seriously considered since the 1930's.[3] To other industrialized countries, also experiencing relatively stable and low inflation coupled with low interest rates, Japan is a disconcerting case of what could happen if interest rates continue to fall and deflation sets in.

Krugman (1998), in analyzing Japan's dilemma, presented a Keynesian view that characterizes a liquidity trap as a situation where conventional monetary policies become impotent as the nominal interest rate approaches zero (because of the perfect substitutability between money and bonds). In his view, as was the case during the Great Depression period in the U.S. during the 1930s, a declining money multiplier can be seen as the symptom of an economy in a liquidity trap, not as a sign of tight monetary policy (as Friedman and Schwartz (1963) argued). In fact, Figure 2 shows that the Japanese money multiplier has been declining drastically since the late 1990s when monetary policy was highly expansionary, suggesting the possibility of the liquidity trap. Krugman also suggests that Japan is trapped in a deflationary spiral in which deflationary expectations, combined with the zero nominal interest rate and the lack of policy credibility that is not able to reverse these expectations, has caused elevated real interest rates.

Since Krugman's paper and the implementation of the ZIRP in Japan, much research has been undertaken on monetary policy in a low inflation/nominal interest rate environment. Benhahib, Schmitt-Grobe and Uribe (1999, 2000a, and 2000b), Krugman (2000), McCallum (1999, 2000), and Svensson (2000) among others theorized about the constraints placed on monetary policy by the zero bound on nominal interest rates. Fuhrer and Madigan (1997) and Orphanides and Wieland (1998, 2000) were among the first to present empirical evidence of the ineffectiveness of monetary policy in such an

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[2] Kenneth Rogoff used this phrase in his comment on Krugman (1998).

[3] The targeting rate was raised to around 0.25% between September 2000 and March 2001 based on the decision by the BOJ policy board that the economy was out of the deflationary situation. However, the zero interest rate policy resumed in March 2001 as the economy appeared to have begun to slow down again.

environment (using U.S. data), followed by Iwata and Wu (2001) with Japanese data. Goodfriend (2000), Buiter and Panigirtzoglou (1999), Reifschneider and Williams (2000), and Kato and Nishiyama (2001) presented their own "prescriptions" to the current Japanese conundrum and suggested drastic precautionary measures for any economy with extremely low inflation (or deflation). The lack of contemporaneous or recent liquidity trap experience has led many researchers to analyze deflationary situations from historical perspectives (Borio and Filardo (2003), Burdekin and Siklos (2002), Cargill (2001), Cargill and Parker (2003), Hutchison (2003), Orphanides (2003), and von Hagen and Hofmann (2003)).[4]

All of these studies suggest that the key to the potential liquidity trap situation in Japan is the real interest rate; an environment may arise where the nominal interest rate is essentially zero or extremely low that it prevents the real interest rate from falling enough to allow the economy to reach equilibrium output and employment level. In such an economy, monetary policy implemented by an interest rate instrument loses its effectiveness.[5] Other possible measures to guide the real interest rate to a level low enough to restore the equilibrium in employment and production may include inflation targeting (Krugman 1998, 2000) and exchange rate targeting (Svensson 2000).

If the Japanese real interest rate was "relatively high" or "not low enough" during the 1990s, then it can be surmised that the lingering recession is attributed to the "high" real interest rate. However, one must be cautious of such an assertion because questions concerning level of the real interest rate are relative. That is, if the Japanese real interest rate is *relatively* high, is it high relative to what or when? Theoretically, when the real interest rate is high, it must be high relative to the market-clearing equilibrium rate. Therefore, when one analyzes the behavior of the real interest rate, its position relative to the equilibrium level and convergence speed matters. This is one of the main subjects this paper will investigate. It will examine Japan's real interest rate during the 1990s in a historical context. The error correction model (ECM) specification based on the time-varying coefficient model with Markov-switching variances introduced in Section 2 of the paper will allow us to investigate the short-term dynamics of both ex post and ex ante real interest rates relative to the long-run equilibrium rate

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[4] See Amirault and O'Reilly (2001) for the literature review on the implication of the zero bound of nominal interest rates on monetary policy.

[5] Metlzer (2001) alternatively contends that zero short-term interest rate merely means that only one of the assets among others may be perfectly substitutable for money. Hence, as long as the returns of other assets such as long term government or corporate bonds are not zero, monetary policy can remain effective by using these assets in the open market procedures. Therefore, merely the state of zero short-term interest rate in Japan does not provide any theoretical basis for describing the economy as in a liquidity trap.

as well as how fast these rates converge to the long-run equilibrium path.

When the nominal interest rate is around zero, the real interest rate becomes susceptible more directly to expected inflation or deflation because the real interest rate now becomes a function of only expected price movements, suggesting that the real interest rate process may entail a regime shift. It also means that the real interest rate process becomes directly affected by the extent of rigidities in the price movement. If there are downward price rigidities in such an environment, the price level may not decline as much, which means deflationary expectations do not arise so particularly. Therefore, even in a zero nominal interest rate environment, when prices are sticky, real interest rates may not rise. Conversely, flexible price movement may lead to severe deflationary expectations as well as elevated real interest rates.

Using a regime shift model, this paper examines whether the zero bound of nominal interest rates can affect the nature of the real interest rate process. The analysis of the stability of the coefficients and the convergence speed show anomalies in the real interest rate process in the late 1990s, indicating a possible regime shift. Several studies have pointed out that the zero bound of nominal interest rates causes anomalies or non-linearity in terms of the term structure of the interest rates (Ruge-Murcia (2001)) or of the effectiveness of the Taylor reaction functions (Fuhrer and Madigan (1997) and Orphanides and Wieland (1998, 2000)). Because the regime shift in the real interest rate process seems to coincide with the implementation of the ZIRP, the link between the level of the nominal interest rate and the regime shift will be further tested.

1.2 Structure of the Paper

The paper proceeds as follows. Section 2 specifies the main model employed, a time-varying parameter model with Markov-switching conditional heteroskedasticity based on Bekdache (1999). This section also shows that regime shifts can be identified with a linear model in the Japanese real interest rate process. The Bekdache model is transformed into an error correction model (ECM) specification, and that allows us to calculate the equilibrium rate of real interest. Section 3 analyzes the regression results. Section 4 introduces another nonlinear time series model, the Smooth Transition Regression (STR) model, which will be specifically employed to identify the link between the level of the nominal interest rate and regime shifts in the Japanese real interest rate series in the late 1990s. The conclusion is given in Section 5.

1.3 Data

In this study, the ex post real interest rate is defined as the difference between the nominal interest rate (i_t) and the inflation rate (p_t), i.e., $r_t = i_t - p_t$. The nominal interest rate is the collateralized overnight call rate. The time period covered in this study is from January 1960 through October 2002. The data were extracted from the BOJ's website. Inflation rates are annualized month-to-month growth rates, using the CPI series reported on the website of the Japanese Government's Statistics Bureau. The CPI series are also adjusted for a rate increase in the consumption tax in April 1997. The measure of supply shock is calculated as the logarithm of the relative price of oil and related products to the wholesale price index (WPI). For more details of the variables, refer to the data index.

2. Model specifications

2.1 Overview

Theoretically, the real interest rate is assumed to be either constant or at least stationary. Although the determinants of the (long-run) real interest rate are still arguable (most typically, Solow (1956), Ramsey (1928), and Diamond (1965)), the foundations of monetary economics stand on the assumption that the long-run rate of real interest follows a constant path that brings out steady-state growth with full employment and steady inflation. [6] In the short-run, however, monetary policy makers try to influence the real interest rate, hoping that price or wage rigidities would help expand demand at least temporarily. Therefore, as long as the effect of the rigidities in wages and prices are assumed to last in the short-run, the actual path of the real interest rate can vary, but must still be stationary.

Given the assumption that the real interest rate process is stationary in the long run, we can think of the dynamics of this rate as an environment where convergence forces work to bring its short-run deviation back to the long run equilibrium path. Also, when we assume that rational expectations theory holds, the predicted (ex ante) values of the real interest rate are equivalent to the short-run equilibriums,

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[6] Refer to Allsopp and Glyn (1999) and Bliss (1999) for a review of the theoretical differences in the determination of the real interest rate.

but do not necessarily occur on the long-run equilibrium path. The actual (or ex post) real interest rates occur around the ex ante real interest rates with prediction errors whose mean is zero, but the deviation of both ex ante and ex post real interest rates from the long-run path is necessarily a temporary phenomenon. More intuitively, the long-run equilibrium occurs when both financial and goods market equilibrate while the short-run equilibrium can occur when only the financial markets clear. Because of nominal rigidities, the goods markets may take some time to clear, which allows the short-run equilibrium to be different from the long-run equilibrium. Hence, if there were no nominal rigidities in the goods market and if agents' expectations are correct, the observed real interest rate must not deviate from the long-run equilibrium path.

2.2 ECM Approach

Given the above argument, the equilibrium real interest rate (r^{eq}) can be a function of the economic variables that brings out steady-state growth with full employment and steady inflation such as:

$$(1) \quad r_t^{eq} = \boldsymbol{b}' X_t^{SS},$$

where r^{eq} is the equilibrium real interest rate and X^{SS} is the vector of variables consistent with the steady state. Because we cannot observe r^{eq}, we use the observed (ex post) real interest rates (r_t) to express it as:

$$(2) \quad r_t = \boldsymbol{b}' X + \boldsymbol{w}_t.$$

Assuming that the rational expectations hypothesis holds, $\boldsymbol{w}_t$ is a mean-zero, serially uncorrelated variable. Hence, a normal OLS estimation of equation (2) would yield consistent estimated coefficients. Also, the fitted values from this regression would be equivalent to the ex ante real interest rate ($r^{ante} = \hat{r}_t = X_t \hat{\boldsymbol{b}}$).[7] Incorporating the idea that shocks can cause the real interest rate to diverge from its equilibrium in the short-run, but that the rate should eventually converge to the long-run relationship of equation (1), equation (2) can be expressed in an error correction model (ECM) form such as:

$$(3) \quad \Delta r_t = \sum_{j=1}^{p-1} \boldsymbol{q}_j \Delta r_{t-j} + \sum_{j=0}^{q-1} \boldsymbol{b}_j \Delta X_{t-j} - B\left(r_{t-1} - b' X_{t-1}\right) + \boldsymbol{u}_t,$$

where X_t is the vector of economic variables and $\boldsymbol{u}_t$ is an independently and identically distributed,

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[7] Given the Fisher relation and the rational expectations, the difference between r_t^e and r_t is the expectation error in the market's forecast of inflation ($r_t^e - r_t = \boldsymbol{p}_t - \boldsymbol{p}_t^e = \boldsymbol{z}_t$) which has a zero-mean and is stationary.

mean-zero, stationary random variable. Assuming that all variables are either stationary or I(1), equations (2) and (3) are equivalent to each other and $0 < B < 2$ ensures that the long-run equilibrium is stable (Hinkle and Montiel (1999)). The convergence speed is the highest when $B = 1$, and becomes lower as B moves away from the value of one. When B is below one, convergence takes place monotonically, whereas it oscillates when it is above one.

This ECM specification, in fact, is merely a re-parameterization of the unrestricted autoregressive distributed lag (ADL) representation of r_t as shown as:

$$(4) \quad r_t = \sum_{j=1}^{p} \boldsymbol{q}_j r_{t-j} + \sum_{j=0}^{q} \boldsymbol{b}_j X_{t-j} + \boldsymbol{u}_t .$$

Therefore, comparing equations (3) and (4), it must hold that $B = 1 - \left(\sum_{j=1}^{p} \boldsymbol{q}_j \right)$ and $b = \dfrac{\sum_{k=0}^{q} \boldsymbol{b}_k}{B}$. Also,

the convergence takes place with the stability condition that all of the polynomial roots of the equation $B = B(L) = 1 - \sum_{j=1}^{p} \boldsymbol{q}_j L^j$, where L is the lag operator, are assumed to lie outside the unit circle.

2.3 Coefficient Instability and Nonstationarity – Preliminary Linear Tests

With the above ECM-ADL transformation, the short- and long-run dynamics of the real interest rate can be examined. However, this sort of analysis is valid only if r_t, or essentially $\boldsymbol{w}_t$, is found to be stationary. Many empirical studies have found that the real interest rate series, usually those of the U.S., are neither constant nor stationary. While Fama (1975) finds constancy in the U.S. real interest rate series for the 1953-1971 period, Mishkin (1981) strongly rejects it for both 1953-1979 and 1931-1952 periods. Rose (1988) finds that the real rate process for the U.S. and 17 other OECD countries, including Japan, is nonstationary.

Huizinga and Mishkin (1986a) present evidence that the U.S. real interest rate series contained structural breaks in the form of coefficient instability by using a simple multivariate model (the H-M model, hereafter). With their X_t containing the nominal interest rate, lagged observed inflation rates, and supply shocks, Huizinga and Mishkin check the stability of $\hat{\boldsymbol{b}}$ and conclude that in the postwar period, significant shifts in the stochastic process of U.S. real rates took place when the Fed shifted its operating

procedures in October 1979 and October 1982, which coincides with the months when the Fed emphasized monetary aggregates instead of nominal interest rates (the "monetary experiment").

Walsh (1988) points out that H-M's results from the model that includes the nominal interest rate series as one of the explanatory variables are misleading because a regime shift can be identified by the coefficient instability of the nominal interest rate variable even if the coefficient instability may have come solely from a regime shift in the inflation process unrelated to the real rate series.[8] In fact, Walsh re-estimates the H-M model altered to include lagged real rates (but not the nominal interest rate or lagged inflation variables) and finds the structural change of October 1979 as in Huizinga and Mishkin, but not the second structural break of October 1982.

Attesting to the impact of H-M, nonetheless, it thereafter became a common practice to allow for the U.S. real interest rate series containing structural breaks. Thereafter, models that incorporate structural breaks such as Garcia and Perron (1996) appeared in the analysis of the U.S. real interest rate series. It was a natural development to apply regime shifting models to the process of the real interest rates considering that the presence of structural breaks or regime shifts in the series tends to make it difficult to reject the hypothesis of a unit root in a conventional linear AR framework (Perron (1990)). Garcia and Perron's univariate, three-regime model (with regime-specific means and variances) show that once the shifts in the mean and the variance are taken into account, the autocorrelation in the series disappears, indicating that the ex-ante real interest rate is constant within each regime.[9]

When we apply the H-M model to the Japanese real interest rates, we can see that the Japanese real interest rate series also entails regime shifts in terms of coefficient instability. Using the information set that includes constant, the real interest rate lagged one, two, three, six, nine, and twelve months (r_{t-1}, r_{t-2}, r_{t-3}, r_{t-6}, r_{t-9}, r_{t-12}), two-month lagged unemployment rate (u_{t-2}), two- and three-month lagged industrial production growth rates (IPG_{t-2}, IPG_{t-3}), two-month lagged supply shock ($Supply_{t-2}$), and three-month lagged money supply growth ($Money_{t-3}$), a simple OLS is regressed in the same way as in H-M.[10, 11] Table

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[8] Walsh (1987) attempts to decompose the innovations to the U.S. nominal interest rate into those due to the real interest rate and to expected inflation during the time period 1961 through 1984 and finds that the innovations of the nominal interest rate in the 1960s and 1970s are mainly driven by drastic rises in (expected) inflation.

[9] Garcia et al. find that their 1961 – 1986 sample period contains structural breaks in 1972-3 and mid-1981 and conjecture that these two structural shifts were caused by the oil shock and a rise in the federal budget deficit, respectively.

[10] When choosing the explanatory variables, the aforementioned Walsh's critique is taken into account so that, instead of the nominal interest and inflation rates, lagged values of real interest rates are included as candidate variables. Candidate variables also include industrial production growth, unemployment rate, U.S. real interest rate series, and money supply growth. As in H-M, the variables that enter significantly in any of the seven subsample

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1 reports that this OLS model is not a correct model specification for the entire sample period of 1963:5 to 2002:10; the OLS result entails severe autocorrelation and ARCH effects.

As Appendix 1 specifies, the entire sample period is divided into seven subsample periods based on the conjectural structural break points when the events that contributed to changes in domestic and/or cross-border capital flows occurred. Most of the coefficients for each explanatory variable vary by the subsample period in terms of significance, level, and sign. In Table 2, the stability of the coefficients before and after the candidate structural break points are examined by the Chow tests, and the stability of the coefficients are strongly rejected for all candidate structural break points except for the break point of 1979:4 (column 2).

It is not surprising to see that the Japanese real interest rates contain several regime shifts in terms of coefficient stability because there are institutional or regulatory changes in both financial and goods markets that must affect the long-run path of the real interest rate. Hence, an ECM analysis based on a simple ADL framework cannot be employed because a linear model cannot yield a stable long-run relationship between the real interest rate and explanatory variables and also because it is contaminated with severe serial correlation which nullifies the rational expectations assumption. Given this consideration, we will re-estimate the Japanese real interest rate process by using the model Bekdache (1999) employed to examine the U.S. real interest rates.

2.4 The ECM Analysis Based on the Time-Varying Model

Bekdache (1999) applies to the U.S. real interest rate process a time-varying parameters model with Markov-switching heteroskedasticity developed by Kim (1993, 1994). Bekdache's multivariate model contains the same variables as does H-M, but it also allows the coefficients to be time-varying and the variances to lie in two regimes. Thus, the Bekdache model can be shown as:

(5) $r_t = W_t \boldsymbol{b}_t + \boldsymbol{e}_t$

$\quad \boldsymbol{b}_t = \boldsymbol{b}_{t-1} + \boldsymbol{n}_t$ (transition equation)

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periods specified in Appendix 1 were chosen as the variables in X_t. See Appendix 1 for more details on the process of selecting explanatory variables.

[11] Despite the preference for parsimony, longer lags in the real rate series turned out to be necessary to minimize autocorrelation.

$$\boldsymbol{e}_t \sim N(0, h_t)$$
$$\boldsymbol{n}_t \sim N(0, Q)$$
$$h_t = \boldsymbol{s}_0^2 + (\boldsymbol{s}_1^2 - \boldsymbol{s}_0^2)S_t, \quad \boldsymbol{s}_0^2 < \boldsymbol{s}_1^2.$$

Here, W_t is the vector of explanatory variables and $\boldsymbol{b}_t$ is a vector of time-varying parameters for each of the explanatory variables. Q is the variance-covariance matrix of the errors in the coefficients' transition equation. The real interest rate process can be of either low or high variance states ($\boldsymbol{s}_0^2$ and $\boldsymbol{s}_1^2$, respectively) based on the two-state Markov switching process.[12]

This model specification is used to accommodate the results from the preliminary H-M analysis that the coefficients for the Japanese real interest rate process appear to be unstable. The vector of explanatory variables, W_t, contains the same variables, namely, constant, the real interest rate lagged one, two, three, six, nine, and twelve months (r_{t-1}, r_{t-2}, r_{t-3}, r_{t-6}, r_{t-9}, r_{t-12}), two-month lagged unemployment rate (u_{t-2}), two- and three-month lagged industrial production growth rates (IPG_{t-2}, IPG_{t-3}), two-month lagged supply shock ($Supply_{t-2}$), and three-month lagged money supply growth ($Money_{t-3}$).

The merit of this approach is that, whereas the discrete regime shifting approach such as Garcia et al. and Hamilton (1988) is constrained in the sense that a regime shift must involve discrete shifts in *both* the mean and the variance of the real rate process, this model allows for two types of shifts in the process simultaneously: continuous changes in the coefficients and discrete changes in the variances. Thus, this model can represent the dynamics of the effects of the explanatory variables on the real rate process and thereby incorporate institutional or environmental change that may as well have contributed to the long-run equilibrium process of the Japanese real interest rate. Furthermore, as we will see later, this model will not suffer from serious autocorrelation or ARCH effects, both of which were observed in the linear H-M analysis.

With the above discussion of the ECM, we can also transform equation (4) into:

$$(6) \quad \Delta r_t = \sum_{j=1}^{p-1} \boldsymbol{q}_{tj} \Delta r_{t-j} + \sum_{j=0}^{q-1} \boldsymbol{b}_{tj} \Delta X_{t-j} - B_t \left[r_{t-1} - b_{t-1}{}' X_{t-1} \right] + \boldsymbol{e}_t,$$

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[12] The unobserved discrete value state variable S_t essentially functions as a dummy variable for the variances (i.e., $S_t = 1$ for the high variance regime and $S_t = 0$, otherwise).

in which the first two terms depict the short-term dynamics and the term inside the brackets indicates the long-term relationship between the real interest rate and the explanatory variables while the aforementioned stability condition holds for B_t. In this specification, the equilibrium rate of real interest is the predicted value based on the long-term equilibrium equation ($\hat{r}_t^{eq} = \hat{b}_t ' X_t$), and the observed (ex post) real interest is supposed to converge to the long-run equilibrium rate at the speed of B_t which is now time-varying. As we have seen, the characteristics of the dynamics depends upon whether B_t is above or below one (while it must lie between 0 and 2). When it is above one ($1 < B_t < 2$), the process converges to the long-run equilibrium in an oscillating manner, i.e., the actual series keeps overshooting above or below the equilibrium rate, and when it is below one ($0 < B_t < 1$), the series converges to the equilibrium monotonically.[13]

For the sake of clarity, we can show the model in scalar form,

$$
\begin{aligned}
(6)' \quad \Delta r_t &= \Delta b_{t1} r_{t-1} + \Delta b_{t2} r_{t-2} + \Delta b_{t3} r_{t-3} + \Delta b_{t6} r_{t-6} + \Delta b_{t9} r_{t-9} + \Delta b_{t12} r_{t-12} \\
&\quad + \Delta g_{t1} u_{t-2} + \Delta g_{t2} IPG_{t-2} + \Delta g_{t3} IPG_{t-3} + \Delta g_{t4} Supply_{t-2} + \Delta g_{t5} Money_{t-3} \\
&\quad - B_t \left[r_{t-1} - \hat{r}^{eq}_{t-1} \right] + e_t,
\end{aligned}
$$

$$
\hat{r}^{eq}_t = \frac{a_t}{B_t(L)} + \frac{g_{t1} u_{t-2}}{B_t(L)} + \frac{(g_{t2} + g_{t3} L) IPG_{t-2}}{B_t(L)} + \frac{g_{t4} Supply_{t-2}}{B_t(L)} + \frac{g_{t5} Money_{t-3}}{B_t(L)}
$$

where $B_t = B_t(L) = 1 - b_{t1} L - b_{t2} L^2 - b_{t3} L^3 - b_{t6} L^6 - b_{t9} L^9 - b_{t12} L^{12}$. All of the above equation's polynomial roots are assumed to lie outside the unit circle.

Lastly, we need to make one important modification. The term with the growth rate of industrial production needs to be removed from the long-run equilibrium solution because a non-zero growth rate of industrial production at the steady-state equilibrium would be contradictory to theory. When the term with IPG_{t-2} inside the square brackets in equation (6)' is assumed as zero, the equation for the equilibrium rate of real interest should reduce to:

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[13] As $B_t \to 2$, the extent of overshooting (the depth of each oscillation) becomes large, thereby making the speed of adjustment slow.

$$(7) \; \hat{r}^{eq}{}_t = \frac{a_t}{B_t(L)} + \frac{g_{t1}u_{t-2}}{B_t(L)} + \frac{g_{t4}Supply_{t-2}}{B_t(L)} + \frac{g_{t5}Money_{t-3}}{B_t(L)} .$$

3. Empirical Results and Analysis

3.1 Results with the Time-Varying Parameters/Markov-Switching Model

Figure 3 displays the results from the Bekdache (1999) model applied to the Japanese real interest rate series. Kim (1993, 1994) shows that the specification of this type of model can be checked by examining the forecast errors for serial correlation. Q-statistics for the standardized forecast errors are found to be Q(12) = 15.85, Q(24) = 30.18, and Q(36) = 45.58, all of which significantly reject autocorrelation. Hence, the model can be said to capture well the dynamics of the ex post real interest rate process.

Figure 3 shows that all the coefficients vary highly over the sample period, which is consistent with the results from the previous OLS analysis.[14] During the 1960s and 1970s, the coefficients seem to be quite unstable. The vertical lines shown in the figures indicate the candidate structural break points discussed in Appendix 1. Some of the drastic changes in the coefficients appear to take place in the times close to the candidate events, suggesting that the regulatory or environmental changes may be important contributors to regime shifts in the real rate series. Figure 4 shows that the real interest rate series belongs to the high variance regime before January 1978, and to the low variance period thereafter.[15] The variance of the high volatility state is more than four times larger than that of the low state (i.e. $s_1^2 = 55.06$ whereas $s_0^2 = 12.64$).[16] The high variance of the prediction errors before 1978 is consistent with the high

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[14] The dotted lines indicate the 95% confidence intervals. The time-varying coefficients for the lagged real interest rates other than the first lag are not displayed in the figure due to space constraint. Most of the coefficients for r_{t-i} present high variation in the 1960s and 1970s and become relatively stable since 1980 except for r_{t-9} which is consistently unstable throughout the sample period.

[15] After January 1978, there are two spikes in the probability of the high variance state. The probability exceeds fifty percent (above which we assume the series is in the high variance regime) from September 1979 through January 1980 and from October through December of 1982.

[16] Both states appear to have the same degree of strong persistence; the probabilities of the real rate series remaining in the high or low variance state are 99.75% and 99.78%, respectively. These figures indicate the transition probabilities that the data series remain in the same variance state in *two consecutive periods*. Hence, a high transition probability means a high degree of persistence. The maximum likelihood estimates for this model are available from the author upon request.

variation in the estimated time-varying coefficients in the 1960s and 1970s shown in Figure 3. The high volatility of the series before 1978 must have resulted from the high volatility of the inflation rate in the time period.

The estimated constant presents the most variation (upper-left in Figure 3).[17] The estimated coefficients for u_{t-2} and $Supply_{t-2}$ (upper-right and lower-center, respectively) vary radically in the 1960s and 1970s as well as the late 1990s. $b_t^{u_{t-2}}$, though at times significantly positive (between 1980 and mid-1982 and between 1984 and 1993), is significantly negative before the first oil shock and after 1996 when the Japanese unemployment rate is rising rapidly.[18]

The coefficients for IPG_{t-2} and $Money_{t-3}$ (bottom-left and bottom right in the figure) illustrate interesting historical characteristics of Japanese macroeconomic policy before and after the collapse of the Bretton Woods system in 1973. $b_t^{IPG_{t-2}}$ is significantly positive until 1973, indicating that Japanese monetary policy was counter-cyclical until then. This may reflect the efforts to retain the external balance under the fixed exchange rate system, which was abandoned in early 1973. Since then, the real rate has been unresponsive to industrial production growth. The coefficient for money supply growth, $b_t^{Money_{t-3}}$, on the other hand, seems to be significantly negative as theory predicts until, again, 1973, but is essentially zero since then. This result may signify that the effectiveness of money supply growth on the real interest rate has declined as financial liberalization became more prevalent after the early 1970s. However, testing these hypotheses is beyond the scope of this study.

In sum, we observed that the Japanese real interest rate series seems to contain instabilities in terms of both its reaction to the information set (i.e., the movement of the estimated coefficients of explanatory variables) and its variances, especially in the 1960s and 1970s. The real rate process experiences a turbulence again in the mid- to late 1990s in terms of changes in the coefficients, namely, the constant

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[17] The estimated constant swings radically around the time of the first oil shock (September 1973), and peaks in December 1998, when there was a banking crisis in Japan. These results suggest that the movement of the constant coefficient may reflect the effect of external shocks to the real rate series, presumably pushing up the unconditional mean of the real interest rate, though such an interpretation must be based on the assumption that other estimates remain constant.

[18] As for $b_t^{Supply_{t-2}}$, there are two different views on how the relative price of energy can affect the real interest rate. One view is that the relative price is negatively related to the real interest rate; a positive supply shock (a rise in the relative price of energy) would depress private demand and the demand for loans, thereby leading to a fall in the real interest rate (or this result may also arise as a result of a monetary policy action). Another view is that a rise in the relative price of energy may increase the returns to new capital (while the value of old capitals being reduced) and therefore raise ex ante real rates, in which case $b_t^{Supply_{t-2}}$ is predicted to be positive.

term, the two-month lagged unemployment rate, and the two-month lagged supply shock. It is interesting that the coefficients for macroeconomic variables experience a turbulence in the late 1990s. This issue will be further analyzed below.

3.2 Ex Ante and Ex Post Real Interest Rates in the 1990s

Figure 5 displays the ex ante and ex post real interest rate series.[19] The ex ante real rate series, defined as the predicted value calculated using Equations (5), seems to be more volatile in the 1960s and 1970s. It is interesting to note that the ex ante real rate appears relatively high around 1982 through 1987, when the economy was buoyant, especially compared to the mid- to late 1990s. This is consistent with Allsopp and Glyn's (1999) statement that most industrialized countries experienced high real interest rates during the 1980s relative to the decades before and after then.

The ex ante real interest rate series appears heading downward in the late 1980s, when the nominal interest rate was guided toward a lower level in reaction to a strong yen and the Black Monday stock market crash, though it rises again between 1990 and 1992 after the economy was presumably overheating by the end of the 1980s. After 1992, the ex ante real rate heads south again, as the economy started drastically slowing down. Interestingly, however, the level does not differ drastically from that in the 1980s bubble economy era until 1998, though it gradually declines toward the end of the decade. The zero interest rate policy (ZIRP) implemented in early 1999 seems to have contributed significantly to the low ex ante real rate, though it rises again fairly in 2001. Overall, as far as the ex ante real interest rate is concerned in a historical context, the real rate in the late 1990s does appear to be relatively low, or at least does not appear "too high," though not as low as during the oil shock years (1973-74).

Figure 6 displays the prediction errors ($\hat{e} = r^{ante} - r^{post}$) from this model after January 1990 (the post-bubble economy era). By construction, the prediction errors are equivalent to agents' expectation errors on the rate of inflation. The figure shows that between 1992 and 1998, the prediction errors are more often found to be positive than in the following years, indicating that the observed inflation rate was higher than what was expected ($r^{ante} > r^{post}$, i.e., $p^{post} > p^e$). This suggests that people had relatively strong and persistent expectations for lower inflation (or deflation) during this period.[20] Although the

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[19] For the sake of clarity, both series are shown with the seven-month, centered moving average, and so are the following prediction errors and the equilibrium real interest rates.

[20] Ahearne, et al. (2002) analyzed the development of Japan's economic situation in the first half of the 1990s, and concludes that the current deflationary situation was "very much unanticipated by Japanese policymakers and observers alike" such that only unconventional levels of stimulus policies could have prevented the economy from falling into the deflationary situation.

discussions on the Japanese deflationary situation became more prevalent in the last half of the 1990s, this finding suggests that the downward trend in the expected inflation already started in the first half of the 1990s. Only after the end of 1998, does r^{ante} appear to be lower than r^{post}, i.e. $\boldsymbol{p}^{post}$ was lower than $\boldsymbol{p}^{e}$, suggesting that the actual disinflation or deflation finally started at the end of the 1990s.

3.3 Dynamics of the Real Interest Rate Process – Time-Varying ECM Analysis

We now calculate the equilibrium real interest rate and its speed of adjustment by using the results from the time-varying/Markov-switching model. Figure 7 displays both the equilibrium and ex post (observed) real interest rates in Japan for 1965 – 2002. The equilibrium rate of real interest appears to be more stable compared to the ex ante (predicted) rate shown in Figure 5.[21] While $\hat{r}^{eq}$ is low in the 1960s and 1970s, it is relatively high during most of the bubble years (1982 – 87) and in the immediate post-bubble period (1991 – 92), similar to the ex ante series. Also, from the end of the 1990s on, $\hat{r}^{eq}$ is at its lowest level since the end of the second oil shock. Figure 8 focuses on the equilibrium and ex post real interest rates in the 1980s and 1990s, and Figure 9 compares the equilibrium and ex ante rates in the same period. Naturally, the deviation of the ex ante rate from the equilibrium is not so considerable compared to that of the ex post rate.

Figure 10, which is essentially equivalent to Figure 7, shows the difference between the ex post and equilibrium real interest rates ($r_t - \hat{r}_t^{eq}$). This figure helps us to observe the actual stance of Japan's monetary policy along with the time varying convergence speed. That is, when the difference between the two rates is positive, i.e., the ex post real interest rate happens to be higher than the long-run equilibrium rate ($r_t > \hat{r}_t^{eq}$), the actual effect of monetary policy is more restrictive than it could be with the equilibrium rate. Conversely, a negative difference indicates a more expansionary monetary policy. More restrictive monetary policy is found in 1977-78, 1986-87, and 1999-2001 whereas lax monetary policy seems to be in place after recessionary periods, such as 1976-77, 1988-89, 1993, and 1996-1997 except for the oil shock years of 1972-73. In Figure 10, the time varying speed of convergence is superimposed (see the right scale). As previously discussed, the convergence speed slows down as it gets farther from the value of one while a speed above one signifies an oscillating convergence (overshooting) and a speed less than one a monotonic convergence. The convergence speed appears to be very slow immediately after the first oil shock. Given that the ex post real rate is lower than the equilibrium during the first oil shock years, the

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[21] As previously discussed, the ex ante rate also reflects the short-term dynamics represented by the first difference terms outside the ECM solution term in equation (5).

slow convergence speed in the following years ensures lax monetary policy persistently. The convergence speed also starts slowing down in an oscillating manner after 1992 and becomes the slowest in 1997. However, unlike during the oil shock years, the convergence this time takes place in an oscillating motion, indicating that the real interest rate keeps overshooting the equilibrium rate.

Figures 11 and 12 display blown-up views for the 1980s and 1990s of the difference between the equilibrium and ex post real interest rates and between the equilibrium and ex ante real interest rates, respectively. One distinct difference between these two figures is that whereas Figure 11 displays the observed or ex post stance of monetary policy in Japan, Figure 12 specifically shows how agents' expectations behave with respect to the equilibrium real interest rates. Comparing Figures 11 and 12 lets us observe differences in the effect of monetary policy between the bubble economy era in the 1980s and the "Great Recession" period in the 1990s in terms of how observed and expected real interest rates behave with respect to the equilibrium. In the expansionary years during the 1980s, namely 1982-1985 and 1988-90, monetary policy appears to be expansionary in terms of both ex ante and ex post real interest rates; both rates are often lower than the equilibrium rates, indicating strong and persistent optimism among agents. Since 1990, these two rates no longer behave consistently. Except for 1994 and 2000, the ex ante rates appear to be higher than the long-run equilibrium while the ex post rates appear to be lower than the long-run equilibrium until 1999. Given that the ex ante real interest rate is the short-run equilibrium, the results in Figure 12 suggest that prices are downwardly stickier than agents expect, which is causing actual inflation to be higher than they expect. This is consistent with what Figure 6 shows.[22] Therefore, despite deflationary concerns that could lead the real interest rate higher than the equilibrium, the observed real interest rate happens not to be "too high" in the period 1992 to 1999. The real deflation appears to start in 2000 as was shown in Figure 6. Since 2001, in terms of both ex post and ex ante real interest rates, monetary policy appears to be tight, possibly indicating the seriousness of the current recessionary situation.

The diversion between the behavior of both expected and observed real interest rates in relation to the equilibrium path in the 1990s implies that the real interest rates in the late 1990s are quite unpredictable. This is consistent with the real rates overshooting the equilibrium which was shown as the convergence speed slowing down in a larger oscillating motion between 1993 and 2000. This unpredictability in the behavior of the real interest rate may indicate the lack of credibility in Japan's monetary policy makers.

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[22] Kasuya (1999) finds downward rigidities in the Japanese CPI series during the 1990s.

When there is a shock that causes deviation of the real rate from the equilibrium, the duration of the rate converging back to x% of such a "misalignment" or deviation can be calculated by solving $|B_t - 1|^t = 1 - x$ for t where B_t is the time-varying convergence speed coefficient. Figure 13 shows how many months the real interest rate takes to converge to 95% of the deviation. Based on this figure, it is clear that the "duration of misalignment" is considerably longer after 1993; the average duration of misalignment for the 1980s is 1.2 months, whereas for the 1990s is 2.1, about twice as long, and the average duration for the period of 1996-99 is found to be 3.2 months. Hence, in the late 1990s, it appears that once the real rate misses the equilibrium, it takes a longer time than during the 1980s or the first half of the decade to converge back to the equilibrium.

The above analysis suggests that the Japanese real interest rate in the late 1990s may have been experiencing anomalies on a scale not witnessed since the oil shock years. Also, the unpredictability of the real interest rate seems to be a major issue during these years, possibly reflecting the lack of policy credibility. [23] Also observed was that this period experienced relatively radical shifts in the coefficients for the explanatory variables in the time-varying coefficients model specification. With the model specification employed thus far, one can only *associate* changes in the dynamic structure, or regime shifts, to actual (economic) events, but cannot identify specific events as the direct cause of the structural changes. This is especially the case, given that the mid- or late 1990s is heavy with events that may have created shocks to the real rate process such as the ZIRP and banking crises. Thus, we need to identify the variable(s) possibly linked with regime shifts in the real rate behavior. The level of the nominal interest rate is the issue of interest. Several studies have shown that the existence of the zero bound on nominal interest rates can diminish the effectiveness of monetary policy when monetary authorities try to pursue a low inflation target (Fuhrer and Madigan (1997), Orphanides and Wieland (2000)). Now, a new model, called the Smooth Transition Regression (STR) model is introduced.

4. Further Analysis on the Real Interest Rate Series in the 1990s Using the Smooth Transition Regression (STR) Model

4.1 STR Model Specification[24]

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[23] In fact, inflation targeting policy sparked a big policy debate in the late 1990s. In these years the BOJ repeatedly rejected an implementation of such a policy as well as denied deflationary concerns and their negative impact on the economy.

[24] This section is based on Franses and van Dijk (2000), Granger and Teräsvirta (1993, 1994, 1998, 2002), and van

The Smooth Transition Autoregressive (STAR) model is a regime shift time series model in which the explained variable is a weighted average of two regime-specific linear models. The probabilities of regime shift occurring is determined by the relative position of a transition variable to a threshold.[25] Using a two-regime STAR model, the real interest rate process, r_t, can be specified as:

$$(8) \quad r_t = \mathbf{f'}_1 X_t + \mathbf{f'}_2 \tilde{Z}_t [1 - G(\mathbf{t}_{t-d};\mathbf{g},c)] + \mathbf{f'}_3 \tilde{Z}_t G(\mathbf{t}_{t-d};\mathbf{g},c) + \mathbf{e}_t,$$

where X_t is a matrix for the variables whose coefficients are not subject to nonlinear regime shifting, and $\tilde{Z}_t$ is a matrix of the variables that are.[26] In this model, the weighting is determined by the probabilities of each regime occurring that is governed by a continuous transition function $G(\mathbf{t}_{t-d};\mathbf{g},c)$ such as:

$$(9) \quad G(\mathbf{t}_{t-d};\mathbf{g},c) = \frac{1}{1 + \exp[-\mathbf{g}(\mathbf{t}_{t-d} - c)]},$$

in which the relative position of transition variable, $\mathbf{t}_{t-d}$, to a certain threshold parameter c determines the likelihood of the examined series residing in one regime or the other, and a positive slope parameter $\mathbf{g}$ indicates how rapidly the transition takes place.[27] The logistic function shown in equation (9) increases monotonically (from zero to one) in $\mathbf{t}_{t-d}$.[28]

In this model the autoregressive coefficients are included in X_t, while the macroeconomic variables are in $\tilde{Z}_t$. Also, because the previous analysis showed that the coefficients for r_{t-12}, $Money_{t-3}$, IPG_{t-2}, and IPG_{t-3} become negligible in the 1990s, these variables are not included in this STR analysis. Hence, the vectors for the explanatory variables become $X_t = (Const., r_{t-1}, r_{t-2}, r_{t-3}, r_{t-6}, r_{t-9})$ and $\tilde{Z}_t = (u_{t-2}, Supply_{t-2})$.

Unlike other regime-switching models, the STR model is rich in methods to test against linearity as well as to conduct other diagnostic checking. Linearity test can be conducted with relative ease by using LM statistics, and it is this linearity test that identifies the transition(s) variable linked to regime shifting.

Dijk et al. (2002).

[25] The number of regimes need not be constrained to two. More complex models with a higher number of regimes are discussed in Franses and van Dik (2000), Teräsvirta (1998), and van Dijk et al. (2002).

[26] $\tilde{Z}_t$ can include both AR and non-AR components. Although the same matrix, $\tilde{Z}_t$, is used for both regimes in equation (8), it is also possible to assign different Z_t's for the regimes.

[27] A higher value of $\mathbf{g}$ makes the transition function $G(.)$ more kinked or discrete, denoting a rapid transition between the two regimes. A lower $\mathbf{g}$ means a smoother transition function $G(.)$, i.e., a slow transition.

[28] Whereas a large $\mathbf{g}$ leads to a more discrete change in $G(.)$ from zero to one, $\mathbf{g} \to 0$ reduces G(.) to 0.5, leading the LSTAR model to collapse to a linear AR model with parameters $\mathbf{f}_j = (\mathbf{f}_{1,j} + \mathbf{f}_{2,j})/2, \quad j = 0,1,\ldots,p$.

For example, one can identify a most suitable transition variable or the most appropriate lag length among several candidate transition variables by examining how strongly the null linearity hypothesis is rejected. Details on the linearity test are discussed in Appendix 3.

We test the nominal interest rate (with different lag lengths) because we are interested in whether the unique situation of Japan's monetary policy has influenced the real interest rate process. If the nominal interest rate turns out to be a good transition variable, it would suggest that the level of the nominal interest rate does affect the data generating process of Japan's real interest rate.

4.2 Regression Results of the STR models

The results from the linearity test using the nominal interest rate as the transition variable with different lag length (as well as other transition variables for the robustness checks discussed later) are given in Table 3. The testing period examined is 1994:10 to 2002:10. The starting date of 1994:10 is chosen based on the previous analysis, showing 1994:10 and 1998:8 as structural break points for the 1990s.[29] The p-values based on LM_3 statistics for the null hypothesis $H''_0: b_1 = b_2 = b_3 = 0$ in equation (A-2) in Appendix 3 are shown in Table 3. For each transition variable as well as each lag length, p-values are reported; figures in boldface are the smallest and statistically significant. In the table, the five-month lagged nominal interest rate is the most significant, indicating that the level of the nominal interest rate does seem to cause the nonlinearity in the process of the Japanese real interest rate.

The estimation results of both linear and STR models are in Table 4.[30] In this table we can see that the nonlinear STR model with the nominal interest rate as the transition variable (column (2)) has lower SSRs, residual standard deviation ($\hat{s}_e$), and AIC than the linear model (column (1)) despite an increase in the number of parameters estimated in the model. That is, the Japanese real interest rate process does appear to have nonlinear characteristics during the 1990s. The threshold variable, c, is statistically significant for the STR model, as is the variable for the speed of regime switching g. The results in column (2) indicates that a rapid transition between the regimes (shown as a high g) takes place around the threshold of 0.62% for the nominal interest rate. The implication of g and c can be more clearly observed in Figure 12; the

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[29] Whereas the simple OLS analysis showed the previous structural break point can go back to as far as 1985, the main focus here is the real interest rate series in the 1990s, especially given the finding that many of the coefficients displayed relatively high variation in the mid - to late 1990s. Considering also that the model we are about to employ is a two-state model, and that two regimes can be suspected to exist after 1994, I consider 1994:10 a good beginning of the testing period.

[30] To maintain parallelism with the STR models, the linear model contains the same explanatory variables.

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upper figure (a) shows the estimated values of $G(.)$ – actual development of the probability of regime 2 occurring – over time for the STR model with the actual development of the nominal interest rate superimposed (dotted line with the right scale), and the lower figure displays the estimated values of $G(.)$ with respect to the level of the nominal interest rate. The figure shows that a regime shift occurs in the real interest rate series in May 1995, when the nominal interest rate (five-month lagged) falls below 0.62, about three years before the implementation of the ZIRP.[31] This result may indicate that an extremely low level of, but not zero, nominal interest rate may create a regime shift in the real interest rate process.

To check the robustness of the results, other variables are also tested as transition variables. The candidate variables are the yearly and monthly inflation rates, the money multiplier, and the financial shock variable. Month-to-month and year-to-year inflation rates are tested because they may answer the question of whether the Japanese real interest rate might be influenced by the inflation level per se. The money multiplier, defined as the log of money supply (M2) relative to monetary base shown in Figure 2, represents the extent to which the Japanese economy is experiencing a liquidity trap (Krugman (1998)). The financial shock transition variable used here is the series of prediction errors when the monthly growth rates in total lending provided by Japanese depository financial institutions are regressed on the growth rates of industrial production.[32,33]

The results from the nonlinearity tests using these candidate transition variables are shown in Table 3 (columns (3) to (5)). In the table, the one-month lagged month-to-month inflation rate and one-month lagged money multiplier variable, interestingly the ones that can be related to the liquidity trap situation in Japan, are the statistically most significant. By contrast, no financial shock variable appears to be significant. However, when the STR analysis is actually conducted, the models with these transition variables do not yield significant results. The model with the one-month lagged month-to-month inflation rate (column (3)) does not reduce the SSR or the residual standard deviation ($\hat{s}_e$) compared to the OLS model while AIC does actually increase for this model. The values g, c, and most of the estimated coefficients do not appear to be statistically significant. The model with the money multiplier performs

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[31] The same structural breakpoint is observed even when the same exercise is repeated using 1991:4 as the beginning of the sample period. 1991:4 could be of interest because since then, the short-term nominal interest rate has been consistently in a downward trend until present.

[32] As for the Japanese financial markets, many researchers use the so-called *Japan premium*, the difference between the London Interbank Offered Rate (LIBOR) and the prime rate offered by a Japanese major bank, i.e., additional basis points charged on international loans to Japanese banks. This variable is not used because of the unavailability of data.

[33] Here, the monthly growth rate of total lending (ldg_t) is regressed on one-, four-, and five-month lagged monthly growth rate of industrial production and time trend after dropping insignificant lags.

better than the one with the inflation rate, but clearly it does not present as good a fit as the model with the nominal interest rate.

These robustness checks confirm that the nominal interest rate is a good transition variable for this STR analysis. Hence, the level of the nominal interest rate does seem to matter to the real rate process, but neither the level of inflation rate nor the disarray in the financial system. Fuhrer and Madigan (1997) and Orphanides and Wieland (1998, 2000) show empirically that monetary policy may lose its effectiveness when low inflation target policy is conducted with the existence of the zero bound on (short-term) nominal interest rates. Black (1995) applies option price theory to the movement of the short-term interest rate and predicts that the zero floor does have an effect on the interest rate as it approaches zero. This is because the existence of the floor will influence agents' expectations, making the rate behave nonlinearly as it nears the floor, just as the option price does as it moves toward the strike price. Ruge-Murcia (2001) formalizes this idea, showing that the term structure relationship between the long and short nominal interest rates becomes nonlinear as the short-term rate approaches the zero floor. The finding from the above STR analysis may indicate that the real interest rate process represents the nonlinearity of the Taylor rule or the term structure of the nominal interest rate when it reaches an extremely low level. The occurrence of nonlinearity or anomalies in the Japanese real interest process in this model analysis is consistent with the drastic change in the coefficients and the convergence speed in the mid- to late 1990s that we observed in the time-varying/MRS ECM analysis. Another possible explanation for this finding is that, until the nominal interest rate hits the threshold point shown above, the level of the nominal interest rate dominates the expected inflation in the movement of the ex ante real interest rate, but that once it hits the threshold level, the effect of the expected inflation rate becomes dominant. Because of the price stickiness, however, the observed real interest rate does not end up as high as expected in some years. However, as far as the period after 2000 is concerned, the real interest rate does appear to be higher than the equilibrium in both ex ante and ex post.

5. Concluding Remarks

This paper investigates the continuing economic slump in Japan from the point of view of real interest rate behavior. The main objective of this paper has been to examine the premise often seen in studies assessing the recent situation of Japan's macroeconomy: whether the real interest rate is too high and whether it is associated with anomalies in the way real rates are determined. The paper examines the Japanese real interest rate process in an ECM framework, both ex ante and ex post, in comparison to the

long-run equilibrium rate of real interest. The analysis with the time-varying/MRS finds that the Japanese real interest rate process has gone through several regime shifts over the years, especially in 1973-74, 1979-80, and 1996-97, which we conjecture may be explained by the two oil shocks in the 1970s, and financial instabilities or deflationary situation in the late 1990s.

The ECM analysis based on the Bekdache model allowed us to calculate the equilibrium rate of real interest which accounts for regime shifts as well as the time-varying speed of adjustment. When the ex post real interest rate is compared to the equilibrium rate, we saw that, generally, the ex post real interest rate appears to be lower than the equilibrium after the recessionary periods in the last three decades. When we observed the time-varying convergence speeds, we found that there are two time periods when the convergence speed slows down considerably in the sample period: the oil shock years and the mid- to late 1990s.

We particularly compared the 1980s and the 1990s, the glory boom years and the "Great Recession" years. The ECM analysis showed a difference between the movement of the ex post (observed) and ex ante (predicted or short-term equilibrium) real interest rates relative to the long-run equilibrium. During the 1980s, both ex ante and ex post rates were often lower than the equilibrium rates, indicating strong and persistent optimism among agents. After 1990, on the other hand, the ex ante real interest rate is persistently higher than the equilibrium, indicating the pessimistic expectations among agents, even though the observed real interest rate is often lower than the equilibrium. The low ex post real interest rate compared to both ex ante and equilibrium rates may be explained by price stickiness. In fact, after 2000, both ex post and ex ante rates appear to be higher than the equilibrium rate, suggesting that prices are slowly adjusting and making deflationary concerns real. The time-varying speed of convergence illustrates that the convergence speed considerably slows down in 1996-99, which makes the duration of misalignment in the real interest rate process in these years twice as long as the misalignment in the 1980s. Thus, in these years, once the real interest rate misses the equilibrium rate, it would take a longer time to converge to the equilibrium, which may be reflecting the lack of credibility among monetary policy makers.

We also investigated whether the anomalies in the real interest process, or its regime shifting, can be linked with the level of the nominal interest rate by employing the Smooth Transition Regression (STR) model. The results with this model showed that, like the Markov model analysis, the real interest rate process entails regime shifts in the mid-1990s when the nominal interest rate (five-month lagged) was used as the transition variable. The real interest rate appears to have experienced a regime shift in May

1995, three years before the implementation of the zero interest rate policy in September 1998. This result interestingly bolsters the findings of Fuhrer et al. (1997) and Orphanides et al. (1998, 2000) in the sense that the zero bound on nominal interest rates with a low inflation targeting policy may create anomalies or nonlinearity in the effectiveness of monetary policy. However, this study suggests the anomalies arise even before the nominal interest rate reaches the zero bound.

This research finds that the recession in Japan during the 1990s is "expectation-driven" in that pessimism among agents seems to have contributed to the rise in the (ex ante) real interest rate. In the case of the Japanese recession since the 1990s, policy consistency to guide the real interest rate downward appears to be an important issue, especially because we have seen that the unpredictability of the monetary policy stance may have made it difficult for the real rate to converge to the equilibrium, thereby worsening the situation. Also, as other studies have shown, the price stability efforts with the conventional Taylor rule may need reconsideration when the nominal interest rate approaches an extremely low level because a different dynamics in the process of the real interest rate seems to be at work in such an environment.

Appendices

Data Appendix:

The WPI series are obtained from the Bank of Japan's web site **www.boj.or.jp**, as is the series for the WPI for oil and its related products. The industrial production index can be retrieved from the Ministry of Economy, Trade, and Industry's (METI) web site **www.meti.go.jp** and the unemployment series from the Ministry of Health, Labour, and Welfare's **www.mhlw.go.jp/**. The Bank of Japan also has the series of money supply and total lending available on its site. The M2 series are only available after January 1963.

Total lending is the "total loans and discounts provided by banks and *shinkin* banks," i.e., Japanese depository financial institutions, as published in the Bank of Japan's data (www.boj.org). The "banks" include city banks, long-term credit banks, trust banks, and regional banks whereas "*shinkin* banks" are equivalent to U.S. credit unions.

The U.S. real interest rate series are calculated in the same manner as the Japanese counterpart. In order to maintain parallelism with the Japanese data, this study uses the federal fund rates as the nominal interest rate and the inflation rates are annualized month-to-month growth rates using the CPI series available in the FRED database **http://research.stlouisfed.org/fred/**.

Price indexes, industrial production index, monetary base, M2, unemployment rates, and total lending are seasonally adjusted using the X-12 routine based on the U.S. Census Bureau method. Growth rates are calculated as the log differences.

Appendix 1: Selection of the Explanatory Variables

As H-M do with the U.S. data, I divide the entire period into subsample periods based on an initial conjecture of structural breaks. For each subsample period, the significance of the candidate variables is tested, and the variables which entered significantly in any of the subsample periods will be included in the final model specification. With the chosen variables, the coefficients' stability will be tested to detect linearly regime shifts in the Japanese real interest rate series.

While H-M suspect that the Fed regime change (i.e., the "monetarist experiment" from 1979 to 1982) may be associated with the regime shifts in the U.S. real rates, regulatory and institutional changes surrounding the Japanese financial markets/system are considered as possible factors causing structural breaks. Unlike U.S. financial markets that have been relatively free of government regulations for the last decades, it is well-known that the Japanese capital and money markets had been regulated for a long time, and only recently have they been mostly liberalized.[34] Hence, it becomes essential to incorporate regulatory/institutional factors that may affect the data generating process.

Given this consideration, I have selected the following six events as possible structural break points in the Japanese real rate series since 1960, believing that these events contributed to changes in either or both of domestic and cross-border capital flows.

List of possible structural break points:[35]

1. 1964:4 – Japan becomes an IMF's Article-Eight country and an OECD member.[36]
2. 1973:2 – Abandonment of the fixed exchange regime.
3. 1979:4 – Deregulation of the call rate market.
4. 1985:9 – The Plaza Accord.

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[34] Of course, U.S. financial markets have not been totally open or completely free of government regulations. Besides the regulations on the types of businesses ("firewall" regulations) and the locations (national and state charter systems) binding U.S. financial institutions, the most known regulation was Regulation Q. This regulation essentially controlled the levels of interest rates and was in effect until it was completely abolished in 1980. However, comparably, it is fair to say that the Japanese financial system has experienced more financially repressive environment in the post-war period because of tight and protective regulations, and that more recently, regulatory changes noticeably affected Japanese finance.

[35] For the details of these events, refer to Flath (2001).

[36] Consequently, Japan became obliged to abandon its foreign currency rationing policy. This break point, as shown later, will not be tested, however, because three-month lagged money supply (M2) growth is chosen as one of the explanatory variables for the model and its data is available only after May 1963.

5. 1994:10 – Completion of the gradual abolishment of the ceiling on bank deposit rates.
6. 1998:8 – Implementation of the zero interest rate policy.

The entire sample period is divided into seven subsample periods according to the above dates, and the statistical significance of the candidate variables are tested in selecting the information set matrix, X_t. The aforementioned Walsh's critique on the H-M method is taken into account so that instead of the nominal interest and inflation rates, lagged values of real interest rates are included as candidate variables. In addition to these lagged real interest rates, candidate variables include industrial production growth, unemployment rate, U.S. real interest rate series, and money supply growth.[37, 38]

Since the three-month lagged money supply growth variable is chosen for the information set, the full sample starts in May, 1963. Consequently, the first candidate structural break of April 1964 is now untested.

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[37] H-M list as candidate variables the detrended series of industrial production (IP) or its deviation from the trend, as well as investment and budget deficits. Investment and budget deficit series are not used as candidate variables due to data unavailability. The detrended IP series is found to be problematic because of its calculation methods. As Krugman (1998) points out, when the Hodrick-Prescott filter is applied to the Japanese IP series, the filtered series gets biased toward the late 1990s sustained economic slowdown. In fact, when the observed and H-P filtered IP series are plotted together, the former series appears to outperform the latter by far in the early 1990s, making it look as if the economy experienced a boom in the early 1990s (contrary to the reality). Hence, both detrended IP series and deviation from the trend are dropped from the candidate basket. Kuttner and Posen (2003) present interesting analysis on how different the Japanese potential output level can appear depending on the detrending technique.

[38] For the sake of parsimony of the model, if the coefficient of a variable turns out to be statistically significant only in one of the subsample periods, but if dropping the variable would not change, or lead to a reduction of, the Schwarz Bayesian Information Criterion (SBIC), the variable is omitted from X_t.

Appendix 2: Estimation of the parameters in the STR model

Unlike the intricate process in parameter estimation for the Markov-switching type of models, estimation of the parameters in the STAR model ((17), (19), or (20)) is relatively straightforward. If the errors $\boldsymbol{e}_t$ are normally distributed, the model becomes a simple maximum likelihood model. Even without this assumption, the STAR model is an application of nonlinear least squares (NLS). Hence, the parameters $\boldsymbol{q} = (\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{g}, c)'$ can be estimated as:

$$(\text{A-1}) \quad \hat{\boldsymbol{q}} = \arg\min_{\boldsymbol{q}} \sum_{t=1}^{n} [y_t - F(x_t; \boldsymbol{q})]^2 \,,$$

where $F(x_t; \boldsymbol{q})$ is the *skeleton* of the model, i.e., the deterministic and predictable part of the model, that is, $F(x_t; \boldsymbol{q}) = \boldsymbol{f}_1 x_t [1 - G(\boldsymbol{t}_{t-d}; \boldsymbol{g}, c)] + \boldsymbol{f}_2 x_t G(\boldsymbol{t}_{t-d}; \boldsymbol{g}, c)$ (in case of equation (17)).

The NLS estimates are consistent and asymptotically normal. The asymptotic covariance-matrix C of $\hat{\boldsymbol{q}}$ can be estimated consistently using the Hessian and gradient of the squared residuals of the NLS (i.e., $[y_t - F(x_t; \hat{\boldsymbol{q}})]^2$). Further details on the properties of the estimated maximum likelihood parameters in this model can be found in Franses et al. (2000).

Appendix 3: Nonlinearity testing methods

The linearity testing method against STR could be conducted with the null hypothesis $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2$ against the alternative hypothesis $H_1 : \boldsymbol{f}_1 \neq \boldsymbol{f}_2$. However, this testing method is complicated by the presence of *unidentified nuisance parameters,* $\boldsymbol{g}$ and c. That is, the STR model contains parameters which are not restricted by the null hypothesis $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2$. Moreover, the likelihood is independent of the values of $\boldsymbol{g}$ and c when the null hypothesis holds true (i.e., $\boldsymbol{f}_1 = \boldsymbol{f}_2$). In short, because $\boldsymbol{g}$ and c are meaningful variables only if the null hypothesis does not hold true, the null hypothesis $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2$ cannot be tested simply against the alternative hypothesis $H_1 : \boldsymbol{f}_1 \neq \boldsymbol{f}_2$ using conventional statistical theory.

Luukkonen et al. (1988) suggest approximating the logistic function $G(\boldsymbol{t}_t ; \boldsymbol{g}, c)$ with a third-order Taylor approximation. This auxiliary regression, consisting of the regressors multiplied by the transition variable, can be tested for nonlinearity.

$$(\text{A-2}) \quad y_t = \boldsymbol{a}' X_t + \boldsymbol{b}'_0 \tilde{Z} + \boldsymbol{b}'_1 \tilde{Z}_t \boldsymbol{t}_t + \boldsymbol{b}'_2 \tilde{Z}_t \boldsymbol{t}_t^2 + \boldsymbol{b}'_3 \tilde{Z}_t \boldsymbol{t}_t^3 + e_t,$$

where $e_t = \boldsymbol{e}_t + (\boldsymbol{f}_2 - \boldsymbol{f}_1)' \tilde{Z}_t R_3(\boldsymbol{t}_t ; \boldsymbol{g}, c)$ with $R_3(\boldsymbol{t}_t ; \boldsymbol{g}, c)$ the remainder term from the Taylor expansion.[39] With this auxiliary regression, testing the null linear hypothesis $H_0 : \boldsymbol{f}_1 = \boldsymbol{f}_2$ (or $H'_0 : \boldsymbol{g} = 0$) in (A-2) is equivalent to testing the null hypothesis $H''_0 : \boldsymbol{b}_1 = \boldsymbol{b}_2 = \boldsymbol{b}_3 = 0$. Here, the corresponding LM test statistic (LM$_3$) has an asymptotic $\boldsymbol{c}^2$ distribution with $3(k+1)$ degrees of freedom (k is the number of columns in $\tilde{Z}_t$). Table 4 lists the *p*-values based on LM$_3$.

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[39] Under the null hypothesis, $R_3(\boldsymbol{t}_t ; \boldsymbol{g}, c) \equiv 0$ and $e_t = \boldsymbol{e}_t$.

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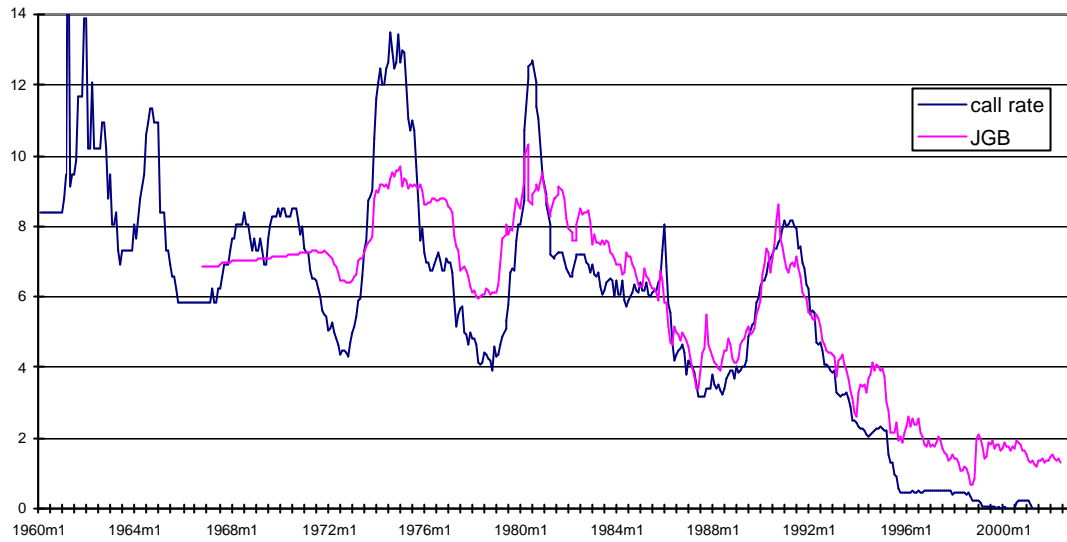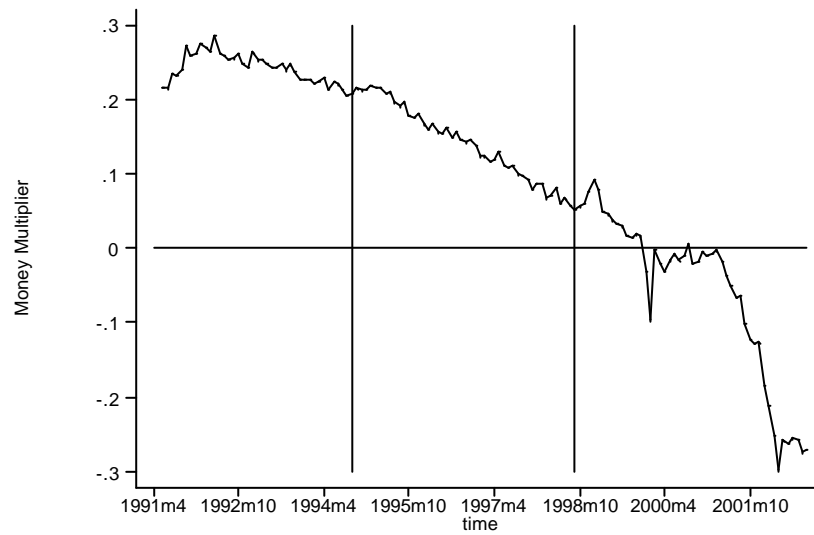
Figure 1: Japan's Short and Long Nominal Interest Rates

Figure 2: Money Multiplier, 1991:4 – 2002:10

Note: Money multiplier = ln(M2) – ln(MB). Both M2 and MB are seasonally adjusted by the X-12 routine.

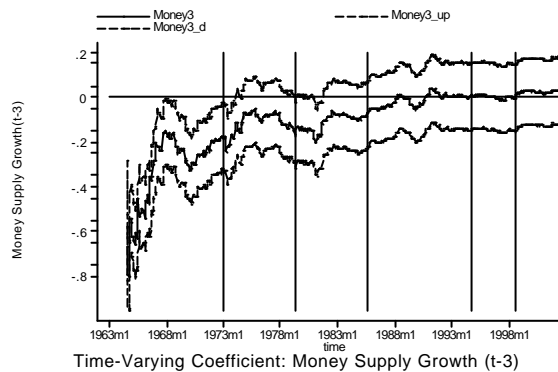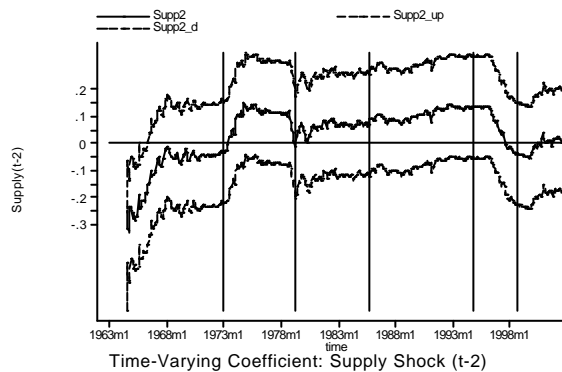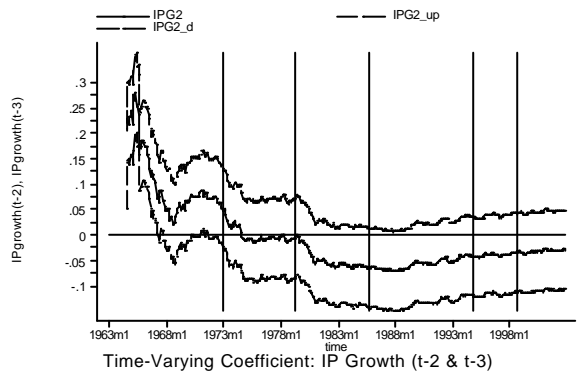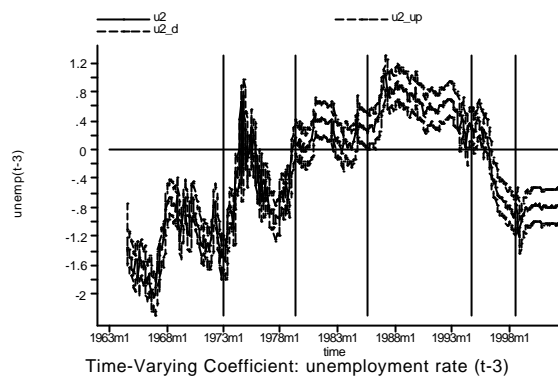# Figure 3: Time -varying Regression Coefficients

Time-Varying Coefficient: Constant

Time-Varying Coefficient: r(t-1), r(t-2), r(t-3)

Time-Varying Coefficient: unemployment rate (t-3)

Time-Varying Coefficient: IP Growth (t-2 & t-3)

Time-Varying Coefficient: Supply Shock (t-2)

Time-Varying Coefficient: Money Supply Growth (t-3)

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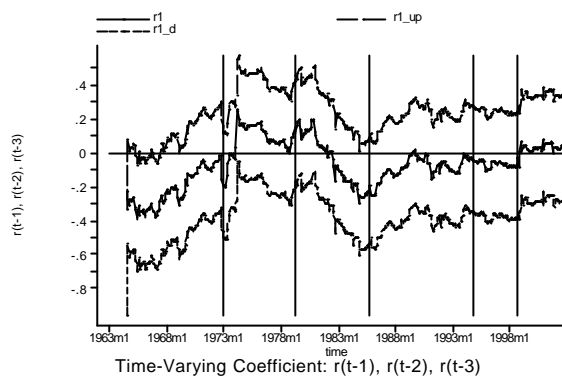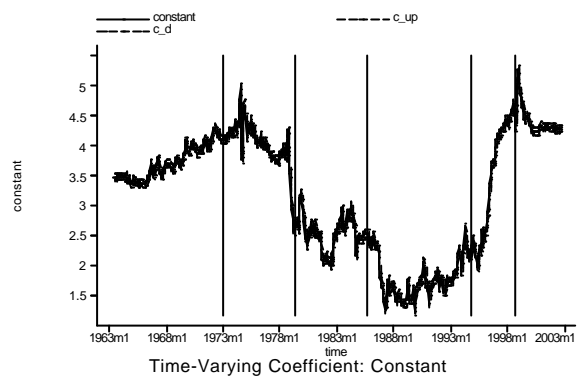
Figure 4: Probability of a High Variance State
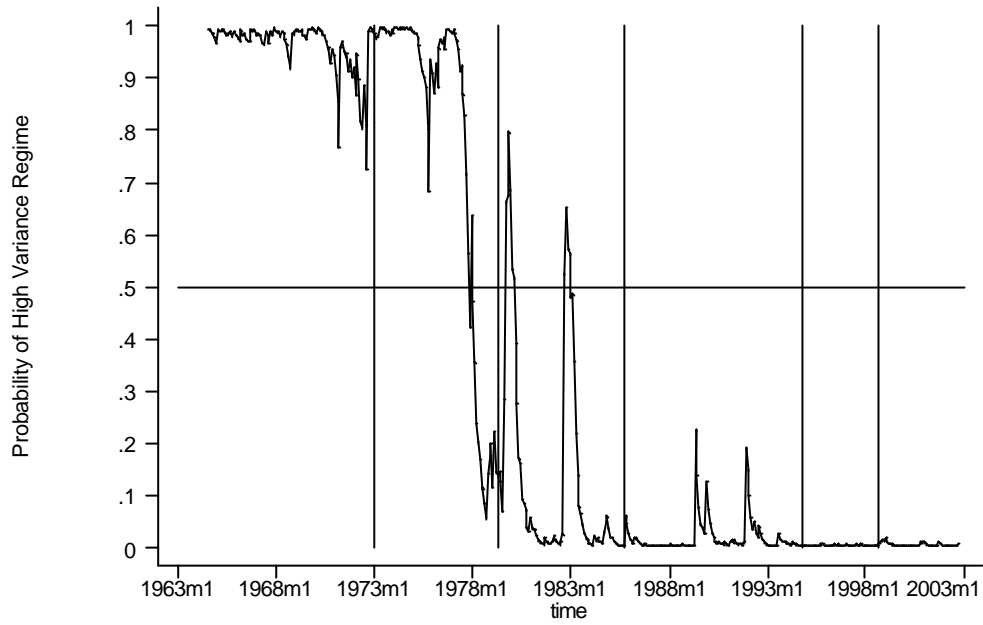
Figure 5: *Ex ante* and *Ex-post* Real Rates: 1965 – 2002
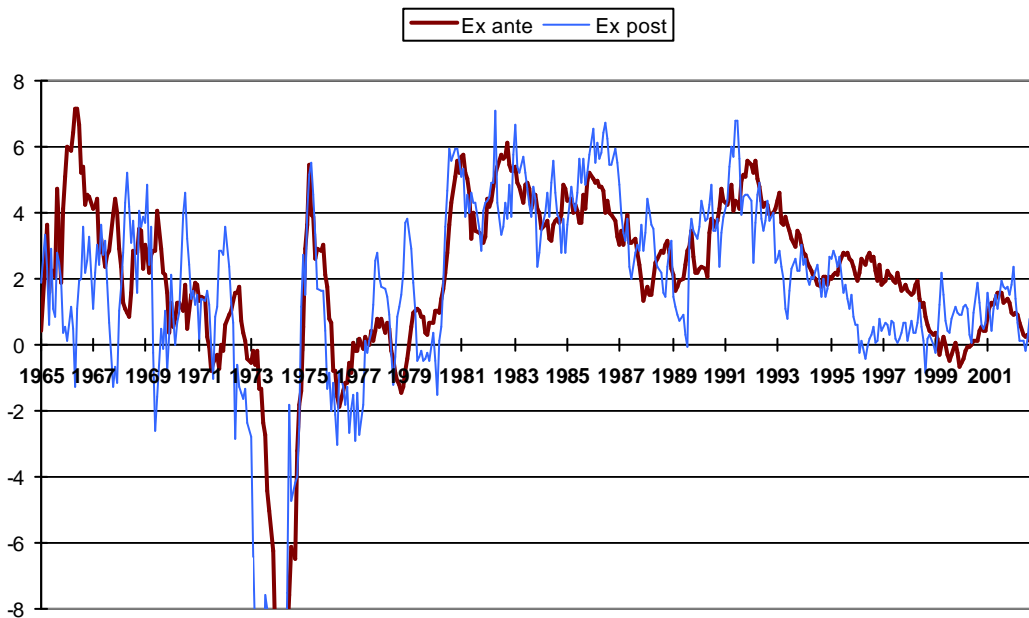
Figure 6: Prediction Errors in the 1990s

$$\hat{e} = r^{ante} - r^{post} = p\,(\text{actual}) - p^{e}\,(\text{expected})$$
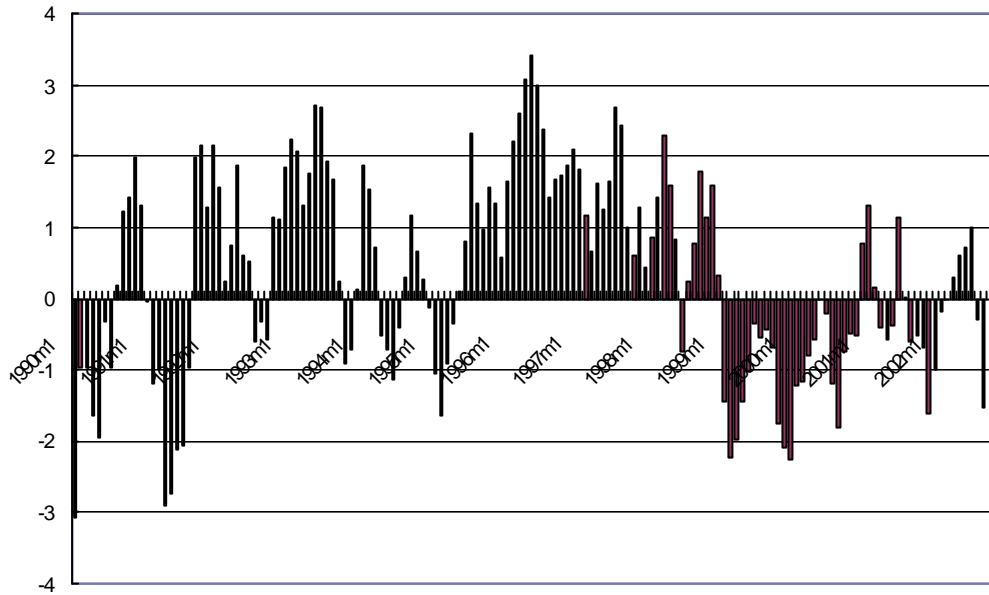
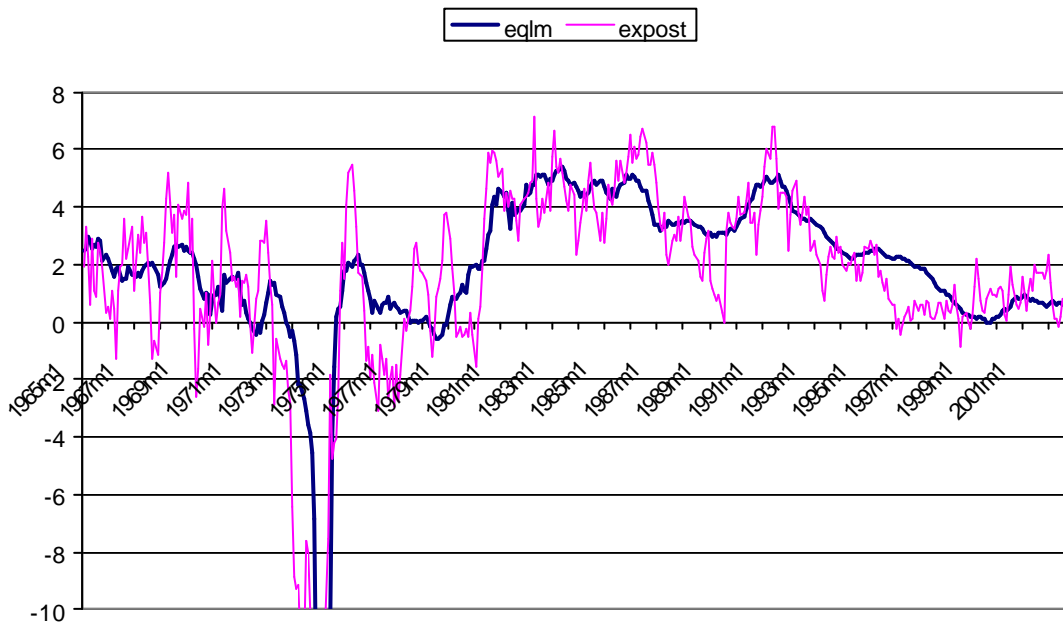**Figure 7: The Equilibrium vs. Ex Post Real Rates: 1965 - 2001**

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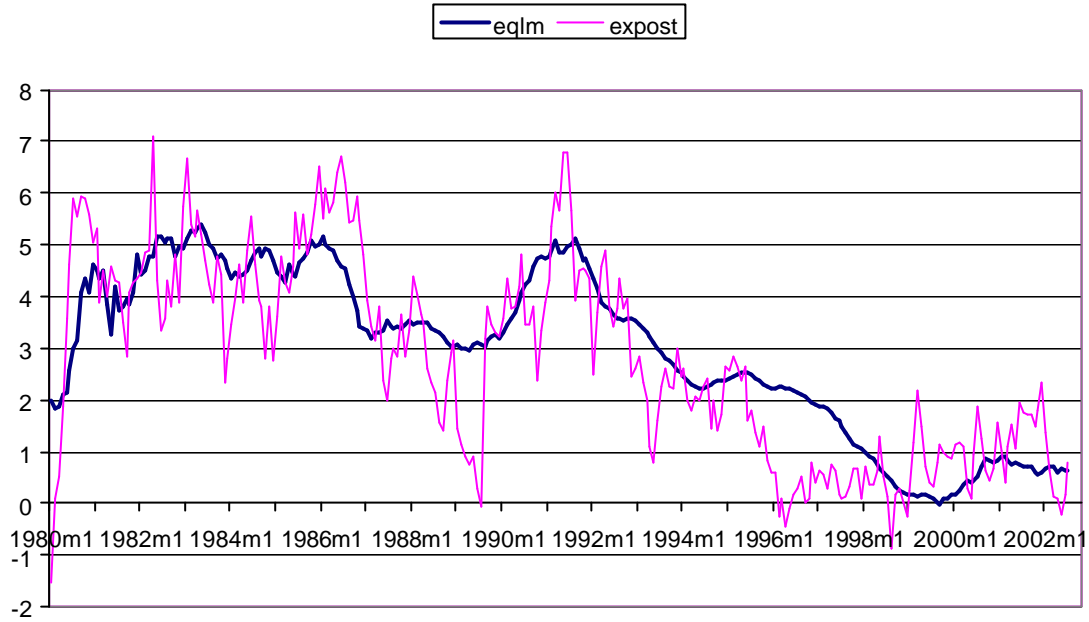**Figure 8: The Equilibrium and Ex Post Real Rates: 1980 - 2002**

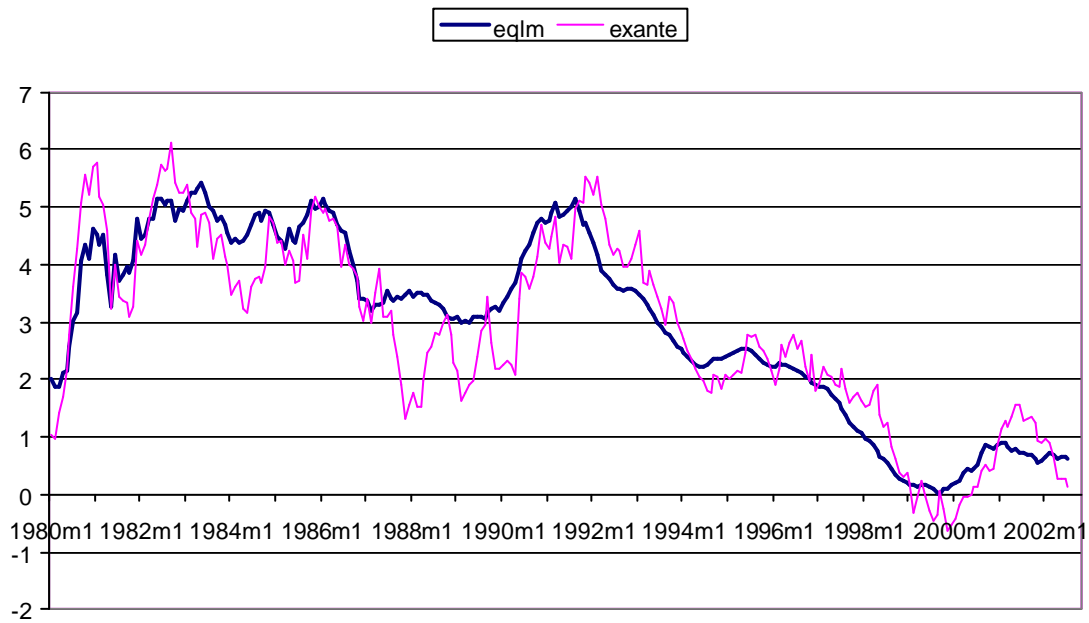**Figure 9: The Equilibrium and Ex Ante Real Rates: 1980 - 2002**

Figure 10: The Dynamics of the Real Interest Rates: 1970 – 2002

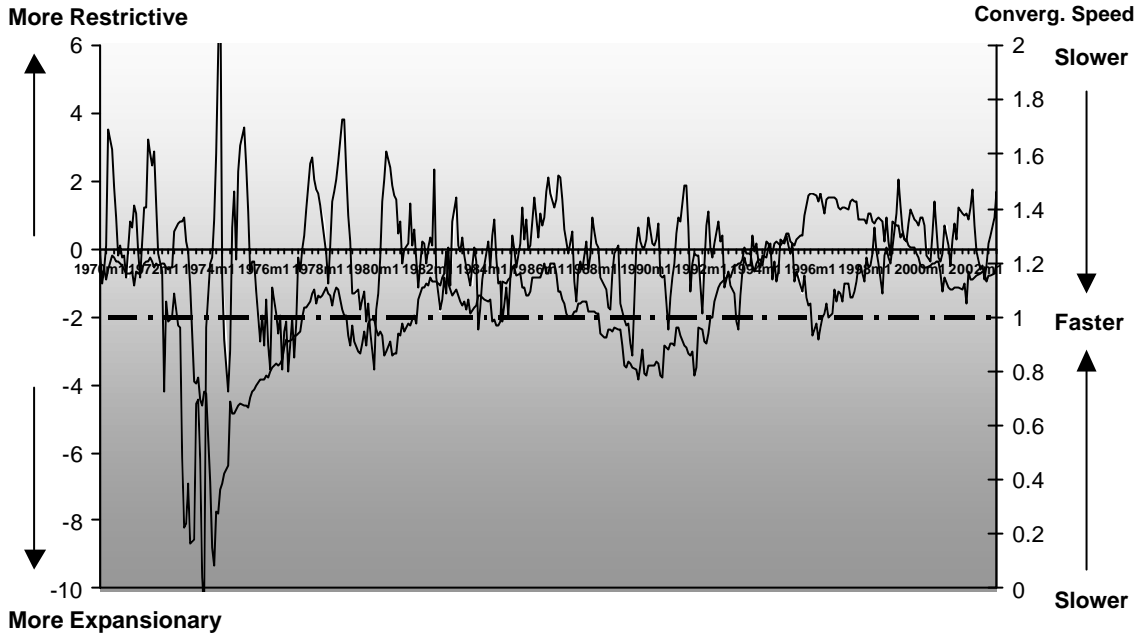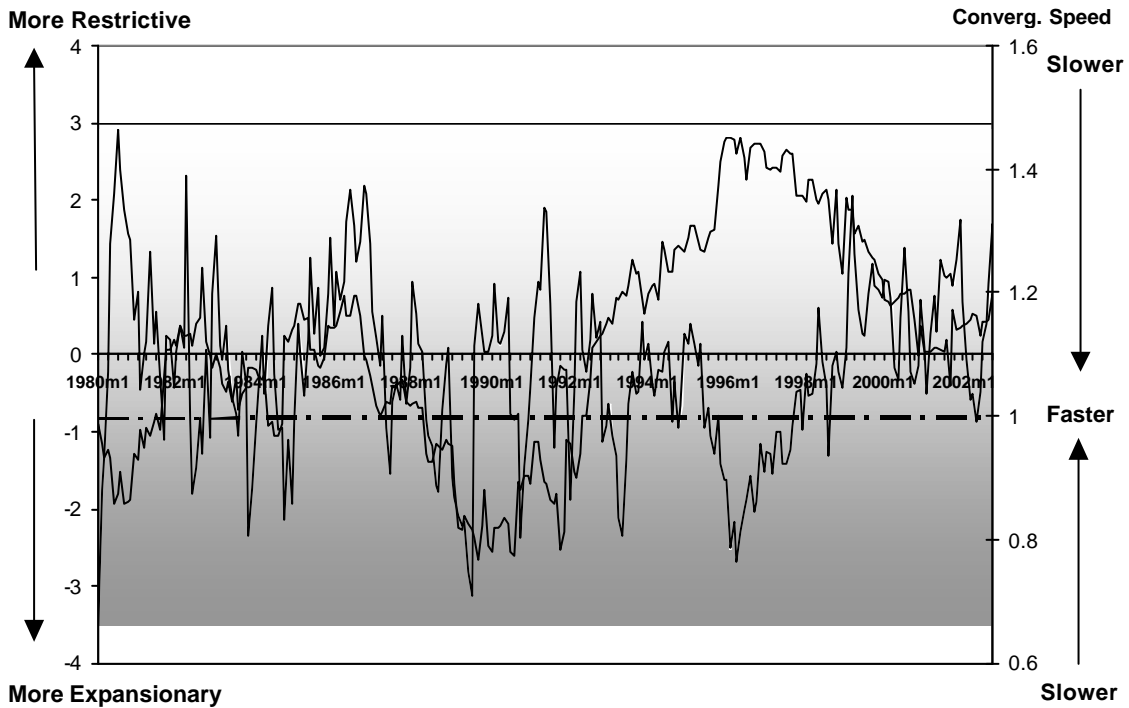Difference between r_t and $\hat{r}_t^{eq}$ and the Speed of Adjustment (B_t)

**Figure 11: The Bubble Years vs. The "Great Recession" in *Ex Post*: 1980 - 2002**

Difference between the equilibiru m ($\hat{r}_t^{eq}$) and ex post (r_t) rates and the Speed of Adjustment (B_t)

NOTE: The left scale is for the difference between r_t and $\hat{r}_t^{eq}$ (i.e., $r_t - \hat{r}_t^{eq}$), and the right scale for B_t.

Figure 12: The Bubble Years vs. The "Great Recession" in *Ex Ante*: 1980 - 2002

Difference between the equilibiru m ($\hat{r}_t^{eq}$) and ex ante (r_t^{ante}) rates and the Speed of Adjustment (B_t)
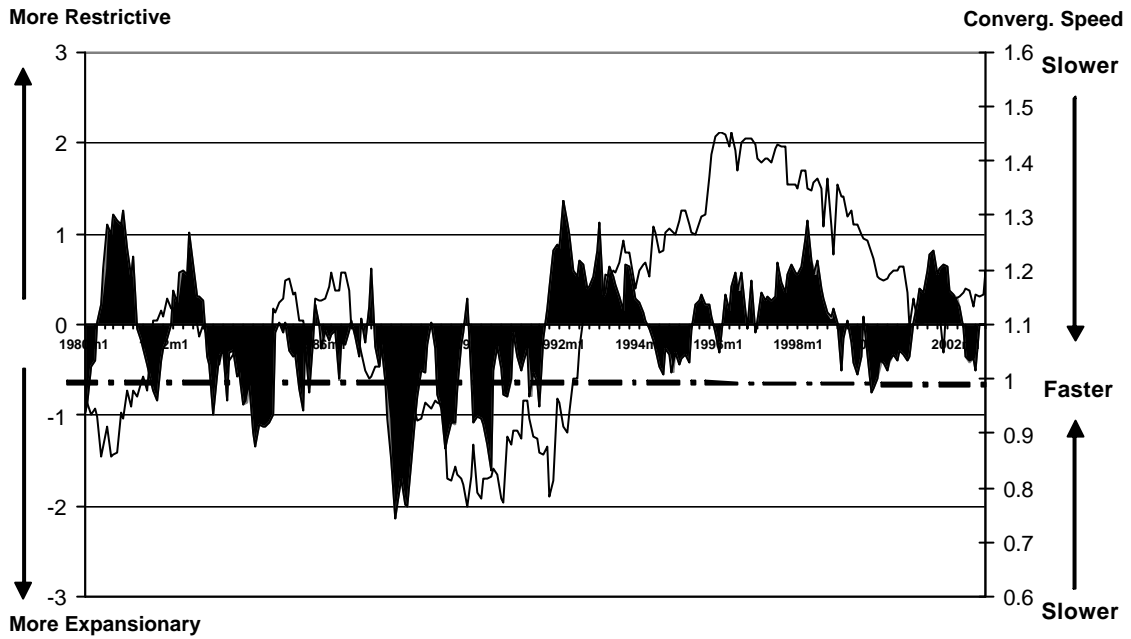
NOTE: The left scale is for the difference between r_t^{ante} and $\hat{r}_t^{eq}$ (i.e., $r_t^{ante} - \hat{r}_t^{eq}$), and the right scale for B_t.

Figure 13: The Duration of "Misalignment": 1980 – 2002
Number of months for the real interest rate to take to converge to the equilibrium
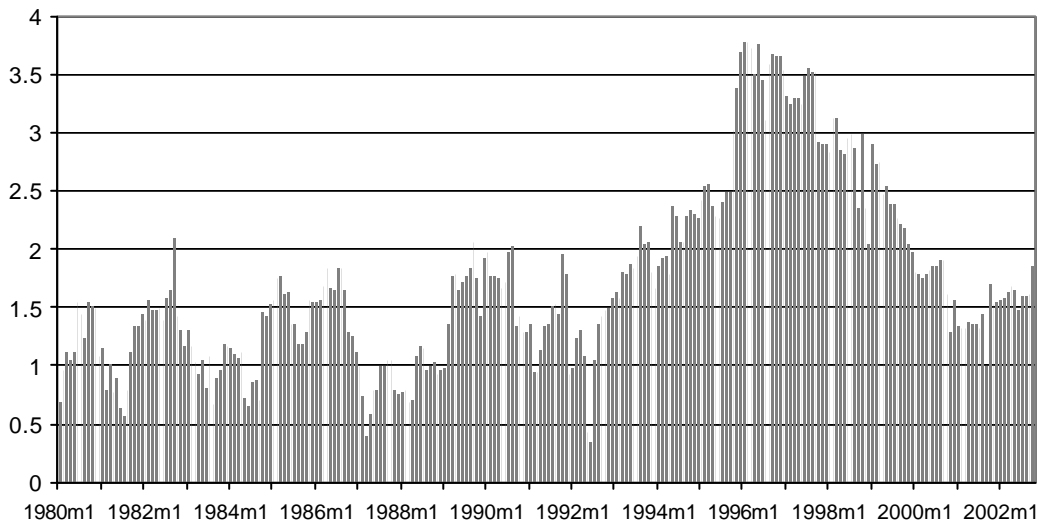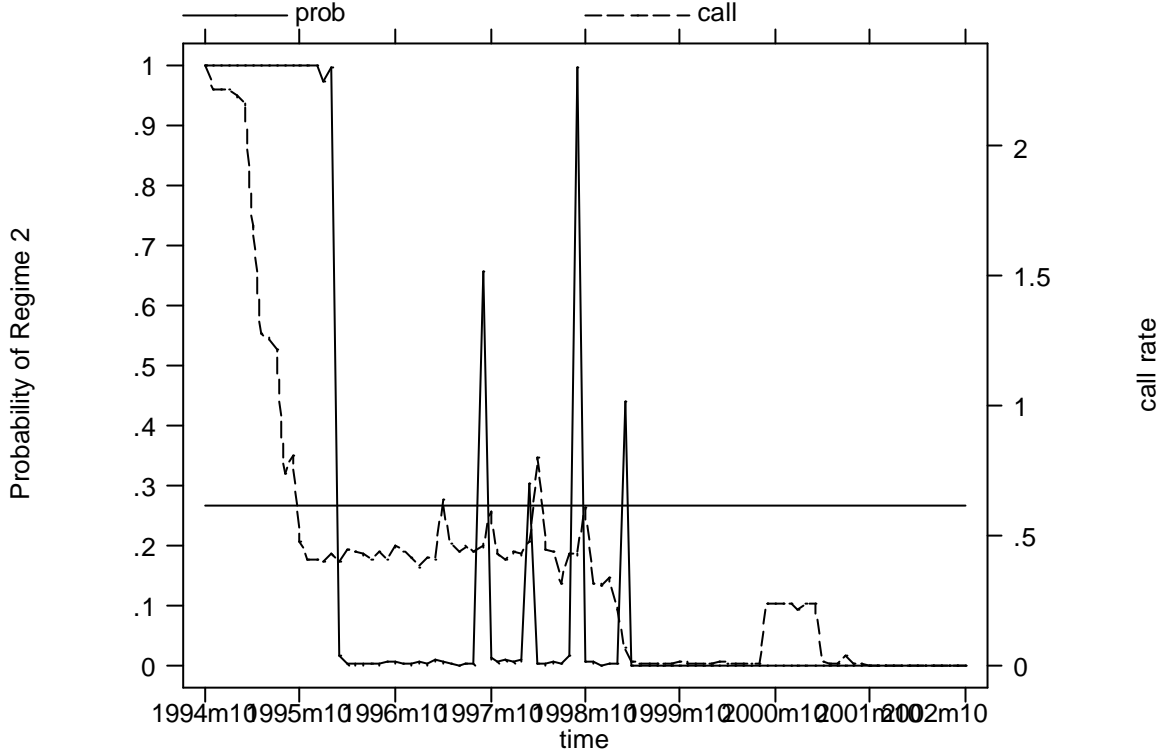

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Figure 12: Nominal Interest Rate (*t-5*) as the Transition Variable

(a) Transition Function G(.) vs. Time
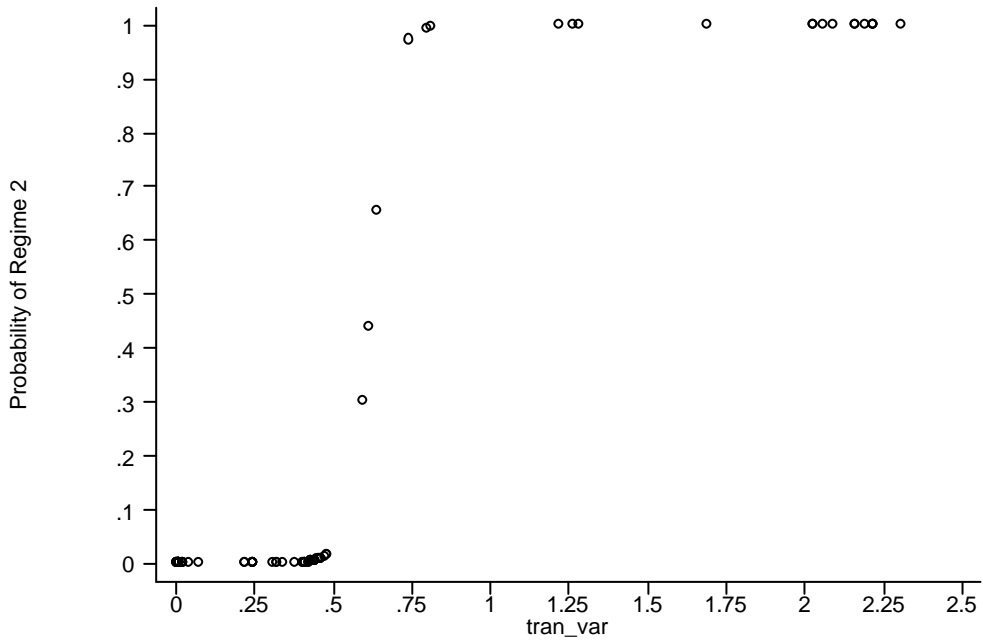

(b) Transition Function G(.) vs. Level of i_{t-5}

Table 1: OLS Regression of the Ex-post Real Rate

				Dependent Variable: Ex post real interest rate			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Period	1963:5 – 2002:10	1963:5 – 1973:1	1973:2 – 1979:3	1979:4 – 1985:8	1985:9 – 1994:9	1994:10 – 1998:8	1998:9 – 2002:10
$N=$	474	117	74	77	109	47	50
Constant	3.4774 [1.1430]***	36.4031 [15.6415]**	-19.3273 [9.9104]*	-0.3894 [4.2353]	2.588 [2.9290]	1.5313 [5.9002]	-8.6564 [6.6724]
r_{t-1}	0.0804 [0.0881]	-0.1579 [0.1045]	0.1381 [0.1439]	-0.3813 [0.1404]***	-0.0665 [0.0978]	-0.0606 [0.1529]	0.0931 [0.1593]
r_{t-2}	0.1012 [0.0699]	-0.0563 [0.1071]	-0.1063 [0.1707]	-0.2113 [0.1026]**	-0.1258 [0.0910]	0.1381 [0.1737]	-0.3032 [0.1536]*
r_{t-3}	0.1358 [0.0643]**	0.0335 [0.0954]	-0.0129 [0.1383]	-0.2283 [0.0947]**	-0.0333 [0.0959]	0.0214 [0.1583]	-0.4839 [0.1278]***
r_{t-6}	0.1312 [0.0576]**	0.0147 [0.1020]	-0.0035 [0.0995]	-0.1658 [0.1257]	0.1702 [0.0848]**	-0.2284 [0.1324]*	-0.1892 [0.1607]
r_{t-9}	0.1524 [0.0654]**	0.0126 [0.1071]	0.0909 [0.1420]	0.1579 [0.0850]*	0.1105 [0.0879]	-0.2607 [0.1315]*	0.1465 [0.1814]
r_{t-12}	-0.1506 [0.0472]***	-0.1849 [0.0861]**	-0.2718 [0.0890]***	-0.0682 [0.0947]	-0.0806 [0.0881]	-0.4429 [0.1367]***	0.1884 [0.1612]
u_{t-2}	-0.7972 [0.2884]***	-11.2709 [7.3076]	10.4567 [4.4963]**	2.1771 [2.1280]	0.0633 [1.1589]	0.5654 [1.7644]	2.1884 [1.4898]
IPG_{t-2}	-0.0169 [0.0131]	0.0789 [0.0443]*	-0.0715 [0.0705]	-0.1338 [0.0384]***	-0.0174 [0.0199]	-0.0231 [0.0175]	-0.0371 [0.0178]**
IPG_{t-3}	-0.0152 [0.0154]	-0.0283 [0.0474]	-0.0558 [0.0659]	-0.1418 [0.0427]***	-0.0004 [0.0237]	0.0019 [0.0189]	-0.024 [0.0183]
$Supply_{t-2}$	0.0325 [0.0112]***	0.2743 [0.1588]*	0.0774 [0.0974]	0.1388 [0.0336]***	0.1277 [0.0416]***	-0.2414 [0.0783]***	-0.0269 [0.0471]
$Money_{t-3}$	-0.0212 [0.0285]	-0.2266 [0.1190]*	-0.0008 [0.1456]	-0.118 [0.0721]	0.0377 [0.0333]	0.0562 [0.0499]	0.0254 [0.0530]
Adj. R^2	0.15	0.07	0.33	0.31	0.11	0.21	0.21
$Q(12)$, [p], $Q(24)$, [p]	.001 .000	.362 .079	.468 .732	.980 .942	.186 .016	.735 .645	.798 .460
Durbin's h alt. [p]	.009	.026	.195	.155	.184	.556	.088
ARCH [p]	.000	.325	.093	.070	.922	.691	.269

NOTE: Standard errors are in square brackets and heteroskedastic-consistent. * significant at 10%; ** at 5%; *** at 1%. Q(12) and Q(24) are the Ljung-Box Q-statistics for the estimated errors with the lag length of 12 and 24, respectively. "Durbin's h alt." is the Durbin-Watson statistics adjusted to allow lagged dependent variables in the model.

Table 2: Stability Tests with the Linear Models

F-Test (Chow Test) for Equality of Coefficients

1963:5 – 1973:1 and 1973:2 – 1979:3	1973:2 – 1979:3 and 1979:4 – 1985:8	1979:4 – 1985:8 and 1985:9 – 1994:9	1985:9 – 1994:9 and 1994:10 – 1998:8	1994:10 – 1998:8 and 1998:9 – 2002:10
$F(12, 191) =$ 3.785	$F(12, 151) =$ 1.542	$F(12, 186) =$ 2.669	$F(12, 156) =$ 1.801	$F(12, 97) =$ 2.968
(0.000)	(0.117)	(0.003)	(0.054)	(0.002)

NOTES: The ex-post real rate is regressed on the real interest rates one-, two-, three-, six-, nine-, and twelve-month lagged (r_{t-1}, r_{t-2}, r_{t-3}, r_{t-6}, r_{t-9}, and r_{t-12},), three-month lagged unemployment rate (u_{t-2}), two- and three-month lagged industrial production growth (IPG_{t-2}, IPG_{t-3}), two-month lagged supply shock ($Supply_{t-2}$), and three-month lagged money supply growth ($Money_{t-3}$). The probability of obtaining a value of the F-statistics or higher under the null hypothesis that the coefficients in the ex post real rate regression are equal in the two subsample periods is in parentheses. For the discussions on the structural break points, refer to Appendix 1: *Selection of the Explanatory Variables*.

Table 3: Tests for Nonlinearity (*p*-values)

Transition Variable	(1) Nominal Interest Rate	(2) M-to-M Inflation Rate	(3) Y-to-Y Inflation Rate	(4) Money Multiplier	(5) Financial Shock
lags					
t-1	0.121	**0.057**	0.264	**0.003**	0.138
t-2	0.118	0.184	0.755	0.015	0.527
t-3	0.053	0.733	0.216	0.029	0.849
t-4	0.014	0.889	0.612	0.029	0.819
t-5	**0.011**	0.694	0.156	0.014	0.822
t-6	0.017	0.685	0.18	0.019	0.552

NOTES: See Appendix 3 for the details on the testing method. The *p*-value is based on the LM_3 statistics for the null hypothesis $H'_0 : b_1 = b_2 = b_3 = 0$ in equation (A-2). Figures in bold face are the smallest *p*-value (among the statistically significant ones) in each column.

Table 4: Estimation Results (Linear and STR Models)

	(1) Linear Model		Lin. comp., f_1	(2) Nominal Interest Rate (*t*-5)		(3) M-to-M Inflation Rate (*t*-1)		(4) Money Multiplier (*t*-1)	
Dependent variable: Ex post real interest rate (1994:10 – 2002:10)								
	(1) **Linear Model**				**STR Models**				
Constant	0.945 [1.687]		**Constant**	-0.196 [1.629]		-0.100 [2.122]		0.661 [2.158]	
$r_{(t-1)}$	0.039 [0.122]		$r_{(t-1)}$	0.032 [0.117]		0.193 [0.166]*		0.120 [0.1060]	
$r_{(t-2)}$	-0.147 [0.110]		$r_{(t-2)}$	-0.186 [0.112]*		-0.160 [0.118]		-0.174 [0.118]	
$r_{(t-3)}$	-0.146 [0.099]*		$r_{(t-3)}$	-0.221 [0.097]**		-0.126 [0.093]		-0.243 [0.095]**	
$r_{(t-6)}$	-0.026 [0.101]		$r_{(t-6)}$	-0.170 [0.094]*		-0.005 [0.102]		-0.18 [0.098]	
$r_{(t-9)}$	-0.082 [0.098]		$r_{(t-9)}$	-0.201 [0.100]**		-0.060 [0.099]		-0.079 [0.090]	
$u_{(t-2)}$	0.0805 [0.507]		*Non-lin. comp.,* f_2, f_3	**Regime 1,** f_2	**Regime 2,** f_3	**Regime 1,** f_2	**Regime 2,** f_3	**Regime 1,** f_2	**Regime 2,** f_3
$Supply_{(t-2)}$	-0.004 [0.035]		$u_{(t-2)}$	0.274 [0.489]	1.059 [0.636]*	0.0547 [0.596]	0.889 [0.982]	0.093 [0.571]	2.208 [3.694]
			$Supply_{(t-2)}$	0.023 [0.037]	-0.831 [0.218]***	0.033 [0.041]	-0.112 [0.107]	0.016 [0.035]	-5.998 [13.074]
			g	19.45 [10.62]*		8.083 [14.674]		6.688 [4.577]	
			c	0.62 [0.02]***		100.64 [71.16]		0.232 [0.0762]***	
SSR	664.96		**SSR**	557.24		630.44		575.15	
$\hat{s}_e$	2.618		$\hat{s}_e$	2.397		2.549		2.435	
AIC	2.090		**AIC**	1.996		2.119		2.027	
Autocorr. (12), [*p*]	0.13		**Autocorr. (12)**, [*p*]	0.13		0.03		0.07	
ARCH. (12), [*p*]	0.06		**ARCH. (12)**, [*p*]	0.02		0.14		0.02	